

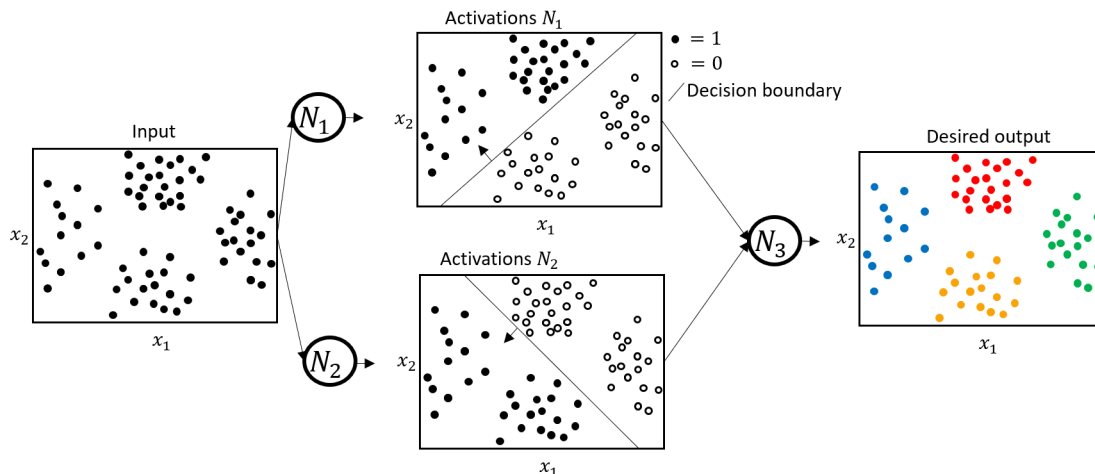
Problem 1 (Perceptrons as logic gates) (15 pts).

Perceptrons with step activation functions can implement logic gates. The step-function is given by $a(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z \leq 0 \end{cases}$. E.g., a NOT gate for a single input $x \in \{0, 1\}$ is implemented by a perceptron with weight -1, bias 0.5, and step activation function, as $a(-1 \cdot x + 0.5) = \neg x$. Construct a perceptron that implements the following logic gates with n inputs $x_1, \dots, x_n \in \{0, 1\}$ by precising suitable weights and bias:

- (a) (2pt) $\text{OR}(x_1, \dots, x_n)$
- (b) (2pt) $\text{AND}(x_1, \dots, x_n)$

Below is sketch of a two-layer Perceptron with two Neurons N_1, N_2 in the first layer and their (step-) activations. Find weights and bias for neuron N_3 in the second layer w.r.t. N_1 and N_2 such that N_3 (with step activation function) outputs

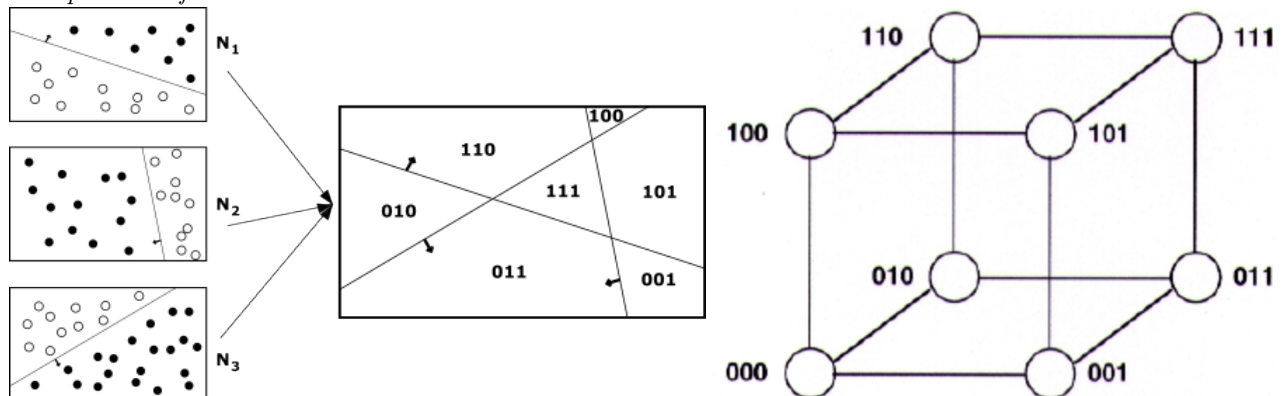
- (c) (4pt) 1 for the green dots and 0 for all others.
- (d) (4pt) 1 for the red dots and 0 for all others.



Now imagine a generalization of the above sketch into a two-layer perceptron with M neurons in the 1st layer. As above, each such neuron defines a decision boundary in input feature space.

- (e) (3pt) Specify a general rule for constructing weights and bias for a neuron in the 2nd layer that outputs 1 for samples that lie inside one specific polygonal region of the input feature space, and 0 for all others.

Hint: You can identify each such polygonal region with a bit string of length M as shown in the example below for $M = 3$.



Exercise Sheet #2: Neural Networks

Due date: April 30, 2024, before 11 am

Problem 2 (XOR function) (10 pt).

Consider the logical XOR operation on the following data set

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

- (a) (1pt) Is there a single-layer perceptron model using ReLU as activation function which has a 100% accuracy on the dataset? Justify your answer.
- (b) (1pt) Show that the binary XOR function can be realized as a combination of the binary AND and OR functions plus negations.
- (c) (8pt) Design a Neural Network consisting of 3 neurons which realizes the binary XOR function. Provide a full explicit description of the network! (activation function, weights, bias etc.)