5. Berechnen Sie (mit Herleitung/Begründung, keine fertigen Ergebnisse aus Nachschlagewerken/Integralrechnern) die folgenden bestimmten oder unbestimmten Integrale:

   (a) $\int \sin^2(x)\cos(x)\,\mathrm{d}x$

$$\begin{aligned}
u &= \sin(x) \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= \cos(x) \\
\mathrm{d}u &= \cos(x)\,\mathrm{d}x \\
\int \sin^2(x)\cos(x)\,\mathrm{d}x &= \int u^2 \cos(x)\,\mathrm{d}x \\
&= \int u^2\,\mathrm{d}u \\
&= \frac{u^3}{3} \\
&= \frac{\sin^3(x)}{3} + C
\end{aligned}$$

   (b) $\int\limits_0^\pi \sin^3(x)\,\mathrm{d}x$

$$\begin{aligned}
\int_0^\pi \sin^3(x)\,\mathrm{d}x &= \int_0^\pi \sin^2(x)\sin(x)\,\mathrm{d}x \\
&= \int_0^\pi (1-\cos^2(x))\sin(x)\,\mathrm{d}x \\
u &= \cos(x) \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= -\sin(x) \\
\mathrm{d}u &= -\sin(x)\mathrm{d}x \\
\int_0^\pi (1-\cos^2(x))\sin(x)\,\mathrm{d}x &= -\int_0^\pi (1-\cos^2(x))(-\sin(x))\,\mathrm{d}x \\
&= -\int_1^{-1} (1-u^2)\,\mathrm{d}u \\
&= -\left(\int_1^{-1} 1\,\mathrm{d}u - \int_1^{-1} u^2\,\mathrm{d}u\right) \\
&= -\left([u]_1^{-1} - \left[\frac{u^3}{3}\right]_1^{-1}\right) \\
&= -\left((-1-1) - \left(\frac{-1}{3} - \frac{1}{3}\right)\right) \\
&= \frac{4}{3}
\end{aligned}$$

(c) $\int\limits_0^{2\pi} \sin^4(x)\,\mathrm{d}x$

$$
\begin{aligned}
\int \sin^4(x)\,\mathrm{d}x &= \int \sin(x)\sin^3(x)\,\mathrm{d}x \\
f' &= \sin(x) \\
g &= \sin^3(x) \\
\int \sin^4(x)\,\mathrm{d}x &= -\cos(x)\sin^3(x) - \int -\cos(x)\frac{\mathrm{d}(\sin^3(x))}{\mathrm{d}x}\,\mathrm{d}x \\
&= -\cos(x)\sin^3(x) + 3\int \cos(x)\sin^2(x)\cos(x)\,\mathrm{d}x \\
&= -\cos(x)\sin^3(x) + 3\int \cos^2(x)\sin^2(x)\,\mathrm{d}x \\
&= -\cos(x)\sin^3(x) + 3\int (1-\sin^2(x))\sin^2(x)\,\mathrm{d}x \\
&= -\cos(x)\sin^3(x) + 3\int \sin^2(x) - \sin^4(x)\,\mathrm{d}x \\
&= -\cos(x)\sin^3(x) + 3\left(\int \sin^2(x)\,\mathrm{d}x - \int \sin^4(x)\,\mathrm{d}x\right) \\
&= -\cos(x)\sin^3(x) + 3\int \sin^2(x)\,\mathrm{d}x - 3\int \sin^4(x)\,\mathrm{d}x \\
4\int \sin^4(x)\,\mathrm{d}x &= -\cos(x)\sin^3(x) + 3\int \sin^2(x)\,\mathrm{d}x \\
f'(x) &= \sin(x) \\
g(x) &= \sin(x) \\
\int \sin^2(x)\,\mathrm{d}x &= -\cos(x)\sin(x) - \int -\cos(x)\cos(x)\mathrm{d}x \\
&= -\cos(x)\sin(x) + \int (1-\sin^2(x))\,\mathrm{d}x \\
&= -\cos(x)\sin(x) + \int 1\,\mathrm{d}x - \int \sin^2(x)\,\mathrm{d}x \\
2\int \sin^2(x)\,\mathrm{d}x &= -\cos(x)\sin(x) + \int 1\,\mathrm{d}x \\
\int \sin^2(x)\,\mathrm{d}x &= \frac{-\cos(x)\sin(x)}{2} + \frac{x}{2} \\
4\int \sin^4(x)\,\mathrm{d}x &= -\cos(x)\sin^3(x) - \frac{3\cos(x)\sin(x)}{2} + \frac{3x}{2} \\
\int \sin^4(x)\,\mathrm{d}x &= \frac{-\cos(x)\sin^3(x)}{4} - \frac{3\cos(x)\sin(x)}{8} + \frac{3x}{8} \\
\int\limits_0^{2\pi} \sin^4(x)\,\mathrm{d}x &= \left[\frac{-\cos(x)\sin^3(x)}{4} - \frac{3\cos(x)\sin(x)}{8} + \frac{3x}{8}\right]_0^{2\pi} \\
&= \left(\frac{-1\cdot 0}{4} - \frac{3\cdot 1\cdot 0}{8} + \frac{3\cdot 2\pi}{8}\right) - \left(\frac{-1\cdot 0}{4} - \frac{3\cdot 1\cdot 0}{8} + \frac{3\cdot 0}{8}\right) \\
&= \frac{3\pi}{4}
\end{aligned}
$$

(d) $\int e^{-2x}\sin(x)\,\mathrm{d}x$

$$\begin{aligned}
f' &= e^{-2x}\\
g &= \sin(x)\\
\int e^{-2x}\sin(x)\,\mathrm{d}x &= -\frac{e^{-2x}\sin(x)}{2} - \int -\frac{e^{-2x}\cos(x)}{2}\,\mathrm{d}x\\
&= -\frac{e^{-2x}\sin(x)}{2} + \frac{1}{2}\int e^{-2x}\cos(x)\,\mathrm{d}x\\
f' &= e^{-2x}\\
g &= \cos(x)\\
\int e^{-2x}\cos(x)\,\mathrm{d}x &= -\frac{e^{-2x}\cos(x)}{2} - \int -\frac{e^{-2x}(-\sin(x))}{2}\,\mathrm{d}x\\
&= -\frac{e^{-2x}\cos(x)}{2} - \frac{1}{2}\int e^{-2x}\sin(x)\,\mathrm{d}x\\
\int e^{-2x}\sin(x)\,\mathrm{d}x &= -\frac{e^{-2x}\cos(x)}{2} + \frac{1}{2}\left(-\frac{e^{-2x}\cos(x)}{2} - \frac{1}{2}\int e^{-2x}\sin(x)\,\mathrm{d}x\right)\\
&= -\frac{e^{-2x}\sin(x)}{2} - \frac{e^{-2x}\cos(x)}{4} - \frac{1}{4}\int e^{-2x}\sin(x)\,\mathrm{d}x\\
\frac{5\int e^{-2x}\sin(x)\,\mathrm{d}x}{4} &= -\frac{e^{-2x}\sin(x)}{2} - \frac{e^{-2x}\cos(x)}{4}\\
\int e^{-2x}\sin(x)\,\mathrm{d}x &= -\frac{2e^{-2x}\sin(x) - e^{-2x}\cos(x)}{5} + C\\
&= -\frac{e^{-2x}(2\sin(x)+\cos(x))}{5} + C
\end{aligned}$$

(e) $\int\limits_0^1 \frac{4x^3-3x^2+6x-1}{x^4-x^3+3x^2-x+2}\,\mathrm{d}x$

$$\begin{aligned}
u &= x^4 - x^3 + 3x^2 - x + 2\\
\frac{\mathrm{d}u}{\mathrm{d}x} &= 4x^3 - 3x^2 + 6x - 1\\
\mathrm{d}u &= (4x^3 - 3x^2 + 6x - 1)\,\mathrm{d}x\\
\int\limits_0^1 \frac{4x^3-3x^2+6x-1}{x^4-x^3+3x^2-x+2}\,\mathrm{d}x &= \int\limits_0^1 \frac{4x^3-3x^2+6x-1}{u}\,\mathrm{d}x\\
&= \int\limits_2^4 \frac{\mathrm{d}u}{u}\\
&= [\ln|u|]_2^4\\
&= \ln(4) - \ln(2)\\
&= \ln(2)
\end{aligned}$$

(f) $\int\limits_{0}^{\frac{e-\frac{1}{e}}{2}} \frac{1}{\sqrt{x^2+1}}\,\mathrm{d}x$

$$\begin{aligned}
x &= \sinh(u) \\
u &= \operatorname{arsinh}(x) \\
\frac{\mathrm{d}x}{\mathrm{d}u} &= \cosh(u) \\
\mathrm{d}x &= \cosh(u)\,\mathrm{d}u \\
\int \frac{1}{\sqrt{x^2+1}}\,\mathrm{d}x &= \int \frac{\cosh(u)}{\sqrt{\sinh^2(u)+1}}\,\mathrm{d}u \\
&= \int \frac{\cosh(u)}{\sqrt{\cosh^2(u)}}\,\mathrm{d}u \\
&= \int \mathrm{d}u \\
&= u \\
&= \operatorname{arsinh}(x) + C \\
\int\limits_{0}^{\frac{e-\frac{1}{e}}{2}} \frac{1}{\sqrt{x^2+1}}\,\mathrm{d}x &= \left[\operatorname{arsinh}(x)\right]_0^{\frac{e-\frac{1}{e}}{2}} \\
\frac{e-\frac{1}{e}}{2} &= \sinh(y) \\
\frac{e-\frac{1}{e}}{2} &= \frac{e^y-e^{-y}}{2} \\
\frac{e-\frac{1}{e}}{2} &= \frac{e^1-e^{-1}}{2} \\
\operatorname{arsinh}\left(\frac{e-\frac{1}{e}}{2}\right) &= 1 \\
\operatorname{arsinh}(0) &= 0 \text{ , da } \sinh(0) \text{ ist} \\
\left[\operatorname{arsinh}(x)\right]_0^{\frac{e-\frac{1}{e}}{2}} &= 1 \\
\int\limits_{0}^{\frac{e-\frac{1}{e}}{2}} \frac{1}{\sqrt{x^2+1}}\,\mathrm{d}x &= 1
\end{aligned}$$

(g) $\int \frac{4x\arctan(x^2)}{1+x^4}\,\mathrm{d}x$

$$\begin{aligned}
u &= \arctan(x^2) \\
\frac{\mathrm{d}[\arctan(f(x))]}{\mathrm{d}x} &= \frac{1}{1+(f(x))^2}\cdot f'(x) \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= \frac{1}{1+x^4}2x \\
\mathrm{d}u &= \frac{2x}{1+x^4}\,\mathrm{d}x \\
\int \frac{4x\arctan(x^2)}{1+x^4}\,\mathrm{d}x &= 2\int \frac{2x\cdot u}{1+x^4}\,\mathrm{d}x \\
&= 2\int u\,\mathrm{d}u \\
&= 2\frac{u^2}{2} \\
&= u^2 \\
&= \arctan^2(x^2)+C
\end{aligned}$$

(h) $\int\limits_1^2 x\sqrt{1+x^2}\,\mathrm{d}x$

$$\begin{aligned}
u &= 1+x^2 \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= 2x \\
\mathrm{d}u &= 2x\,\mathrm{d}x \\
\int\limits_1^2 x\sqrt{1+x^2}\,\mathrm{d}x &= \frac{1}{2}\int\limits_1^2 2x\sqrt{u}\,\mathrm{d}x \\
&= \frac{1}{2}\int\limits_2^5 \sqrt{u}\,\mathrm{d}u \\
&= \left[\frac{u^{\frac{3}{2}}}{3}\right]_2^5 \\
&= \frac{5^{\frac{3}{2}}}{3}-\frac{2^{\frac{3}{2}}}{3} \\
&= \frac{5^{\frac{3}{2}}-2^{\frac{3}{2}}}{3}
\end{aligned}$$

(i) $\int\limits_2^3 4x\ln(2x)\,\mathrm{d}x$

$$
\begin{aligned}
\int\limits_2^3 4x\ln(2x)\,\mathrm{d}x &= 4\int\limits_2^3 x\ln(2x)\,\mathrm{d}x \\
f' &= x \\
g &= \ln(2x) \\
\int x\ln(2x)\,\mathrm{d}x &= \frac{1}{2}x^2\ln(2x) - \int \frac{1}{2}x^2\frac{1}{x}\,\mathrm{d}x \\
&= \frac{1}{2}x^2\ln(2x) - \frac{1}{2}\int x\,\mathrm{d}x \\
&= \frac{1}{2}x^2\ln(2x) - \frac{1}{2}\frac{1}{2}x^2 \\
&= \frac{x^2\ln(2x)}{2} - \frac{x^2}{4} \\
\int\limits_2^3 4x\ln(2x)\,\mathrm{d}x &= 4\left[\frac{x^2\ln(2x)}{2} - \frac{x^2}{4}\right]_2^3 \\
&= \left[2x^2\ln(2x) - x^2\right]_2^3 \\
&= (2\cdot 3^2\cdot\ln(2\cdot 3) - 3^2) - (2\cdot 2^2\cdot\ln(2\cdot 2) - 2^2) \\
&= (18\ln(6) - 9) - (8\ln(4) - 4) \\
&= 18\ln(6) - 8\ln(4) - 5
\end{aligned}
$$

(j) $\int 3e^x\sqrt{e^x+1}\,\mathrm{d}x$

$$
\begin{aligned}
u &= e^x + 1 \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= e^x \\
\mathrm{d}u &= e^x\,\mathrm{d}x \\
\int 3e^x\sqrt{e^x+1}\,\mathrm{d}x &= 3\int e^x\sqrt{u}\,\mathrm{d}x \\
&= 3\int\sqrt{u}\,\mathrm{d}u \\
&= 3\frac{2u^{\frac{3}{2}}}{3} \\
&= 2u^{\frac{3}{2}} \\
&= 2(e^x+1)^{\frac{3}{2}} + C
\end{aligned}
$$

(k) $\int \tan(x)\,\mathrm{d}x$

$$\begin{aligned}
\int \tan(x)\,\mathrm{d}x &= \int \frac{\sin(x)}{\cos(x)}\,\mathrm{d}x \\
u &= \cos(x) \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= -\sin(x) \\
\mathrm{d}u &= -\sin(x)\,\mathrm{d}x \\
\int \frac{\sin(x)}{\cos(x)}\,\mathrm{d}x &= -\int \frac{-\sin(x)}{\cos(x)}\,\mathrm{d}x \\
&= -\int \frac{\mathrm{d}u}{u} \\
&= -\ln(|u|) \\
&= -\ln(|\cos(x)|) + C
\end{aligned}$$

(l) $\int \frac{\sin(x)}{1+\cos^2(x)}\,\mathrm{d}x$

$$\begin{aligned}
u &= \cos(x) \\
\frac{\mathrm{d}u}{\mathrm{d}x} &= -\sin(x) \\
\mathrm{d}u &= -\sin(x)\,\mathrm{d}x \\
\int \frac{\sin(x)}{1+\cos^2(x)}\,\mathrm{d}x &= -\int \frac{-\sin(x)}{1+\cos^2(x)}\,\mathrm{d}x \\
&= -\int \frac{1}{1+u^2}\,\mathrm{d}u \\
&= -\arctan(u) \\
&= -\arctan(\cos(x))
\end{aligned}$$

(m) $\int \left(\frac{(2\sqrt{x}+1)^2}{x^2} - \frac{1}{x\sqrt{x}}\right)\,\mathrm{d}x$

$$\begin{aligned}
\int \left(\frac{(2\sqrt{x}+1)^2}{x^2} - \frac{1}{x\sqrt{x}}\right)\mathrm{d}x &= \int \frac{(2\sqrt{x}+1)^2}{x^2}\,\mathrm{d}x - \int \frac{1}{x\sqrt{x}}\,\mathrm{d}x \\
\int \frac{(2\sqrt{x}+1)^2}{x^2}\,\mathrm{d}x &= \int \frac{4x+4\sqrt{x}+1}{x^2}\,\mathrm{d}x \\
&= \int \frac{4x}{x^2}\,\mathrm{d}x + \int \frac{4\sqrt{x}}{x^2}\,\mathrm{d}x + \int \frac{1}{x^2}\,\mathrm{d}x \\
&= 4\int \frac{1}{x}\,\mathrm{d}x + 4\int \frac{1}{x^{\frac{3}{2}}}\,\mathrm{d}x + \int \frac{1}{x^2}\,\mathrm{d}x \\
&= 4\ln(|x|) - \frac{8}{\sqrt{x}} - \frac{1}{x} \\
\int \frac{1}{x\sqrt{x}}\,\mathrm{d}x &= \int \frac{1}{x^{\frac{3}{2}}}\,\mathrm{d}x \\
&= -\frac{2}{\sqrt{x}} \\
\int \frac{(2\sqrt{x}+1)^2}{x^2}\,\mathrm{d}x - \int \frac{1}{x\sqrt{x}}\,\mathrm{d}x &= 4\ln(|x|) - \frac{8}{\sqrt{x}} - \frac{1}{x} - \left(-\frac{2}{\sqrt{x}}\right) \\
\int \left(\frac{(2\sqrt{x}+1)^2}{x^2} - \frac{1}{x\sqrt{x}}\right)\mathrm{d}x &= 4\ln(|x|) - \frac{6}{\sqrt{x}} - \frac{1}{x} + C
\end{aligned}$$

(n) $\int x^2 \sin(x)\,\mathrm{d}x$

$$
\begin{aligned}
f' &= \sin(x) \\
g &= x^2 \\
\int x^2 \sin(x)\,\mathrm{d}x &= -x^2\cos(x) - \int -2x\cos(x)\,\mathrm{d}x \\
&= -x^2\cos(x) + 2\int x\cos(x)\,\mathrm{d}x \\
f' &= \cos(x) \\
g &= x \\
\int x^2 \sin(x)\,\mathrm{d}x &= -x^2\cos(x) + 2(x\sin(x) - \int \sin(x)\cdot 1\,\mathrm{d}x) \\
&= -x^2\cos(x) + 2(x\sin(x) - (-\cos(x))) \\
&= -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C \\
&= 2x\sin(x) + \cos(x)(2 - x^2) + C
\end{aligned}
$$

(o) $\int e^{ax}\sin(bx)\,\mathrm{d}x$, für $a, b \in \mathbb{R} \setminus \{0\}$

$$
\begin{aligned}
f' &= e^{ax} \\
g &= \sin(bx) \\
\int e^{ax}\sin(bx)\,\mathrm{d}x &= \frac{e^{ax}\sin(bx)}{a} - \int \frac{e^{ax}b\cos(bx)}{a}\,\mathrm{d}x \\
&= \frac{e^{ax}\sin(bx)}{a} - \frac{b}{a}\int e^{ax}\cos(bx)\,\mathrm{d}x \\
f' &= e^{ax} \\
g &= \cos(bx) \\
\int e^{ax}\sin(bx)\,\mathrm{d}x &= \frac{e^{ax}\sin(bx)}{a} - \frac{b}{a}\left(\frac{e^{ax}\cos(bx)}{a} - \int \frac{-e^{ax}b\sin(bx)}{a}\,\mathrm{d}x\right) \\
&= \frac{e^{ax}\sin(bx)}{a} - \frac{b}{a}\left(\frac{e^{ax}\cos(bx)}{a} + \frac{b}{a}\int e^{ax}\sin(bx)\,\mathrm{d}x\right) \\
&= \frac{e^{ax}\sin(bx)}{a} - \frac{be^{ax}\cos(bx)}{a^2} - \frac{b^2}{a^2}\int e^{ax}\sin(bx)\,\mathrm{d}x \\
\int e^{ax}\sin(bx)\,\mathrm{d}x + \frac{b^2}{a^2}\int e^{ax}\sin(bx)\,\mathrm{d}x &= \frac{e^{ax}\sin(bx)}{a} - \frac{be^{ax}\cos(bx)}{a^2} \\
\frac{a^2\int e^{ax}\sin(bx)\,\mathrm{d}x + b^2\int e^{ax}\sin(bx)\,\mathrm{d}x}{a^2} &= \frac{e^{ax}\sin(bx)}{a} - \frac{be^{ax}\cos(bx)}{a^2} \\
a^2\int e^{ax}\sin(bx)\,\mathrm{d}x + b^2\int e^{ax}\sin(bx)\,\mathrm{d}x &= ae^{ax}\sin(bx) - be^{ax}\cos(bx) \\
(a^2 + b^2)\int e^{ax}\sin(bx)\,\mathrm{d}x &= ae^{ax}\sin(bx) - be^{ax}\cos(bx) \\
\int e^{ax}\sin(bx)\,\mathrm{d}x &= \frac{ae^{ax}\sin(bx)}{a^2 + b^2} - \frac{be^{ax}\cos(bx)}{a^2 + b^2} \\
&= \frac{ae^{ax}\sin(bx) - be^{ax}\cos(bx)}{a^2 + b^2} + C
\end{aligned}
$$

(p) $\int (x^2+2x)\sinh\left(\frac{x}{2}\right)\ \mathrm{d}x$

$$\begin{aligned}
f' &= \sinh\left(\frac{x}{2}\right) \\
g &= x^2+2x \\
\int (x^2+2x)\sinh\left(\frac{x}{2}\right)\ \mathrm{d}x &= 2\cosh\left(\frac{x}{2}\right)(x^2+2x) - \int 2\cosh\left(\frac{x}{2}\right)(2x+2)\,\mathrm{d}x \\
&= 2\cosh\left(\frac{x}{2}\right)(x^2+2x) - 2\int \cosh\left(\frac{x}{2}\right)(2x+2)\,\mathrm{d}x \\
f' &= \cosh\left(\frac{x}{2}\right) \\
g &= 2x+2 \\
\int \cosh\left(\frac{x}{2}\right)(2x+2)\,\mathrm{d}x &= 2\sinh\left(\frac{x}{2}\right)(2x+2) - \int 2\sinh\left(\frac{x}{2}\right) 2\,\mathrm{d}x \\
&= 2\sinh\left(\frac{x}{2}\right)(2x+2) - 4\int \sinh\left(\frac{x}{2}\right)\ \mathrm{d}x \\
&= 2\sinh\left(\frac{x}{2}\right)(2x+2) - 8\cosh\left(\frac{x}{2}\right) \\
\int (x^2+2x)\sinh\left(\frac{x}{2}\right)\ \mathrm{d}x &= 2\cosh\left(\frac{x}{2}\right)(x^2+2x) - 2\int \cosh\left(\frac{x}{2}\right)(2x+2)\,\mathrm{d}x \\
&= 2\cosh\left(\frac{x}{2}\right)(x^2+2x) - 2(2\sinh\left(\frac{x}{2}\right)(2x+2) - 8\cosh\left(\frac{x}{2}\right)) \\
&= 2\cosh\left(\frac{x}{2}\right)(x^2+2x) - 8\sinh\left(\frac{x}{2}\right)(x+1) + 16\cosh\left(\frac{x}{2}\right) + C
\end{aligned}$$

(q) $\int \sin(ax)\cos(bx)\,\mathrm{d}x$, für $a, b \in \mathbb{R} \setminus \{0\}$ mit $|a| \neq |b|$

$$
\begin{aligned}
\sin(\alpha)\cos(\beta) &= \frac{1}{2}(\sin(\beta+\alpha) - \sin(\beta-\alpha)) \\
\alpha &= ax \\
\beta &= bx \\
\int \sin(ax)\cos(bx)\,\mathrm{d}x &= \int \left(\frac{1}{2}(\sin(bx+ax) - \sin(bx-ax))\right)\,\mathrm{d}x \\
&= \frac{1}{2}\left(\int \sin(x(b+a))\,\mathrm{d}x - \int \sin(x(b-a))\,\mathrm{d}x\right) \\
u_1 &= x(b+a) \\
\frac{\mathrm{d}u_1}{\mathrm{d}x} &= b+a \\
\mathrm{d}u_1 &= (b+a)\,\mathrm{d}x \\
\int \sin(x(b+a))\,\mathrm{d}x &= \frac{1}{b+a}\int (b+a)\sin(x(b+a))\,\mathrm{d}x \\
&= \frac{1}{b+a}\int \sin(u_1)\,\mathrm{d}u_1 \\
&= \frac{-\cos(u_1)}{b+a} \\
&= \frac{-\cos(x(b+a))}{b+a} \\
u_2 &= x(b-a) \\
\frac{\mathrm{d}u_2}{\mathrm{d}x} &= b-a \\
\mathrm{d}u_2 &= (b-a)\,\mathrm{d}x \\
\int \sin(x(b-a))\,\mathrm{d}x &= \frac{1}{b-a}\int (b-a)\sin(x(b-a))\,\mathrm{d}x \\
&= \frac{1}{b-a}\int \sin(u_2)\,\mathrm{d}u_2 \\
&= \frac{-\cos(u_2)}{b-a} \\
&= \frac{-\cos(x(b-a))}{b-a} \\
\int \sin(ax)\cos(bx)\,\mathrm{d}x &= \frac{1}{2}\left(\frac{-\cos(x(b+a))}{b+a} + \frac{\cos(x(b-a))}{b-a}\right) \\
&= \frac{\cos(x(b-a))}{2(b-a)} - \frac{\cos(x(b+a))}{2(b+a)} + C
\end{aligned}
$$

(r) $\int \sin(ax)\cos(bx)\,\mathrm{d}x$, für $a, b \in \mathbb{R} \setminus \{0\}$ mit $|a| = |b|$

$$(1)\ a = b$$

$$\int \sin(ax)\cos(ax)\,\mathrm{d}x$$

$$\begin{aligned}
u_1 &= \sin(ax) \\
\frac{\mathrm{d}u_1}{\mathrm{d}x} &= a\cos(ax) \\
\mathrm{d}u_1 &= a\cos(ax)\mathrm{d}x \\
\int \sin(ax)\cos(ax)\,\mathrm{d}x &= \frac{1}{a}\int \sin(ax)a\cos(ax)\,\mathrm{d}x \\
&= \frac{1}{a}\int u_1\,\mathrm{d}u_1 \\
&= \frac{u^2}{2a} \\
&= \frac{\sin^2(ax)}{2a} + C
\end{aligned}$$

$$(2)\ a = -b$$

$$\int \sin(-ax)\cos(ax)\,\mathrm{d}x$$

$$\begin{aligned}
u_2 &= \cos(ax) \\
\frac{\mathrm{d}u_2}{\mathrm{d}x} &= -a\sin(ax) \\
\mathrm{d}u_2 &= -a\sin(ax)\mathrm{d}x \\
\int \sin(ax)\cos(ax)\,\mathrm{d}x &= \frac{-1}{a}\int -a\sin(ax)\cos(ax)\,\mathrm{d}x \\
&= \frac{-1}{a}\int u_2\,\mathrm{d}u_2 \\
&= \frac{-u^2}{2a} \\
&= \frac{-\sin^2(ax)}{2a} + C
\end{aligned}$$