Discrete Mathematics

Program Learning Outcome (PLO)

- **PLO-1**
  Able to analyze complex problems in the field of informatics and apply principles of informatics and other relevant disciplines to identify their solutions by taking into account insights from the advancements of trans-disciplinary fields.
- **PLO-6**
  Able to apply computer science theories and the basis of software development to develop computing-based solutions.

Course Learning Outcomes (CLO)

		Supported PLO
CLO1	Able to analyze complex problems in the field of informatics and apply Discrete Mathematics principles to identify their solutions by taking into account insights from the advancements of trans-disciplinary fields.	PLO1
CLO2	Able to apply Discrete Mathematics theories and the basis of software development to develop computing-based solutions.	PLO2

Assessments

ID CLO	Weight per Evaluation				
	Assignment 1	Assignment 2	Midterm Exam	Final Exam	
CLO-1	10	10	15	20	55
CLO-2		10	15	20	45
Total per penilaian	**10**	**20**	**30**	**40**	**100**

Course syllabus

Week 1	Discrete Mathematics – The Foundations (Proportional Logic).
Week 2	Applications of Propositional Logic
Week 3	Propositional Equivalences
Week 4	Predicates and Quantifiers
Week 5	Nested Quantifiers
Week 6	Rules of Inference
Week 7	Proof Methods and Strategy
Week 8	**Mid Exam.**
Week 9	Sets and set operations
Week 10	Sets and set operations: cont. Function I
Week 11	Sets and set operations: cont. Function II
Week 12	Sequences and summations
Week 13	Integers and division
Week 14	Mathematical induction & Recursion
Week 15	Counting: Binomial Coefficients and Identities
Week 16	**Final Exam.**

Week 1 The Foundations (Proportional Logic).

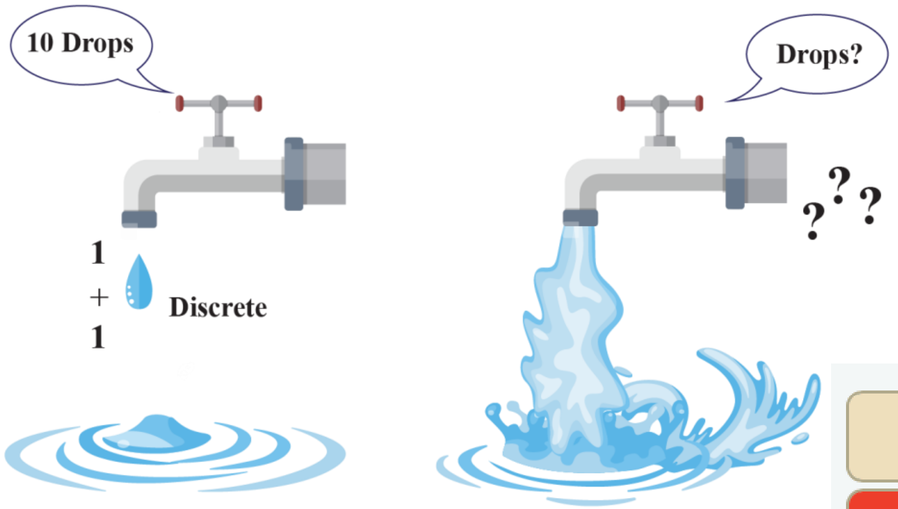
Discrete Mathematics - Introduction

Mathematics can be broadly classified into two categories –

- **Continuous Mathematics** – It is based upon continuous number line or the real numbers. It is characterized by the fact that between any two numbers, there are almost always an infinite set of numbers. For example, a function in continuous mathematics can be plotted in a smooth curve without breaks.
- **Discrete Mathematics** – It involves distinct values; i.e. between any two points, there are a countable number of points. For example, if we have a finite set of objects, the function can be defined as a list of ordered pairs having these objects, and can be presented as a complete list of those pairs.

Continuous vs Discrete Data

Continuous		Discrete
Any Value		Specific Values
"Measured"		"Counted"
5.6 , 2.489	vs	1, 2, 3, 4, 5, 6
Temperature		# of cats



Examples	
Discrete	**Continuous**
• # of eggs in a basket • # of kids in a class • # of Facebook likes • # of diaper changes in a day • # of wins in a season • # of votes in an election	• Weight difference to 8 decimals before and after cookie binge. • Wind speed • Water temperature • Volts of electricity

Logic

Logic:

- defines a formal language for representing knowledge and for making logical inferences
- It helps us to understand how to construct a valid argument

Logic defines:

- Syntax of statements
- The meaning of statements
- The rules of logical inference (manipulation)

Propositional logic

- The simplest logic
- Definition:
  - – A proposition **is a statement that is either true or false.**
- Examples:
  - – President university is located in Cikarang
    - (T)
  - – 5 + 2 = 8.
    - (F)
  - – It is raining today.
    - (either T or F)

Examples (cont.):

- – How are you?
    - a question is not a proposition
- – $x + 5 = 3$
    - since x is not specified, neither true nor false
- – 2 is a prime number.
    - (T)
- – She is very talented.
    - since she is not specified, neither true nor false
- – There are other life forms on other planets in the universe.

either T or F

Composite statements

More complex **propositional statements can be build from elementary statements using logical connectives.**

Example:

- • Proposition A: It rains outside
- • Proposition B: We will see a movie
- • A new (combined) proposition:
- If it rains outside then we will see a movie

Composite statements

More complex propositional statements can be build from elementary statements using logical connectives.

Logical connectives:

- Negation
- Conjunction
- Disjunction
- Exclusive or
- Implication
- Biconditional

Negation

Definition: Let **p** be a proposition. The statement "It is not the case that p." is another proposition, called the **negation of p**. **The negation of p is denoted by ¬ p** and read as "not p."

Example:

- Pitt is located in the Oakland section of Pittsburgh.
- It is not the case that Pitt is located in the Oakland section of Pittsburgh.

Other examples:

- $5 + 2 \neq 8$.
- 10 is not a prime number.
- It is not the case that buses stop running at 9:00pm.

Negate the following propositions:

- It is raining today.
  - It is not raining today.
- 2 is a prime number.
  - 2 is not a prime number
- There are other life forms on other planets in the universe.
  - It is not the case that there are other life forms on other planets in the universe.

Negation

A truth table displays **the relationships between truth values** (T or F) of different propositions.

p	¬ p
T	F
F	T

Conjunction

Conjunction means Anding of two statements. If p, q are two statements, then "p and q" is a compound statement, denoted by **p ∧ q** and referred as **the conjunction of p and q**. **The conjunction of p and q is true only when both p and q are true**. Otherwise, it is false.

Examples:

- Pitt is located in the Oakland section of Pittsburgh and 5 + 2 = 8
- It is raining today and 2 is a prime number.
- 2 is a prime number and 5 + 2 ≠ 8.
- 13 is a perfect square and 9 is a prime.

Conjunction

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

Disjunction means Oring of two statements. If p, q are two statements, then "p or q" is a compound statement, denoted by p ∨ q and referred to as the disjunction of p and q. The disjunction of p and q is true whenever at least one of the two statements is true, and it is false only when both p and q are false.

Examples:

- Pitt is located in the Oakland section of Pittsburgh or 5 + 2 = 8.
- It is raining today or 2 is a prime number.
- 2 is a prime number or 5 + 2 8.
- 13 is a perfect square or 9 is a prime.

Disjunction

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Truth tables

- Conjunction and disjunction
- Four different combinations of values for p and q

p	q	$p \land q$	$p \lor q$
T	T		
T	F		
F	T		
F	F		

Exclusive or

Definition: Let p and q be propositions. The proposition "**p exclusive or q**" denoted by $p \oplus q$, **is true when exactly one of p and q is true** and it is false otherwise.

p	q	**$p \oplus q$**
T	T	F
T	F	T
F	T	T
F	F	F

Implication

- An implication $p \longrightarrow q$ is the proposition "if p, then q." It is false if p is true and q is false. The rest cases are true.
- In $\mathbf{p \rightarrow q}$, p is called the hypothesis and q is called the conclusion.

p	q	$\mathbf{p \rightarrow q}$
T	T	T
T	F	F
F	T	T
F	F	T

Implication

$p \rightarrow q$ is read in a variety of equivalent ways:

- if p then q
- p only if q
- p is sufficient for q
- q whenever p

Examples:

- if Steelers win the Super Bowl in 2013 then 2 is a prime.
- If F then T ?

Implication

- The **converse of $p \to q$** is $q \to p$.
- The **contrapositive of $p \to q$** is $\neg q \to \neg p$
- The **inverse** of **$p \to q$** is $\neg p \to \neg q$

- **<u>Examples:</u>**
    - If it snows, the traffic moves slowly.
    - p: it snows   q: traffic moves slowly.
    - $p \to q$
  - **The converse:**

    If the traffic moves slowly then it snows.
    - $q \to p$

Implication

- The **contrapositive of** $\mathbf{p \rightarrow q}$ is $\neg q \rightarrow \neg p$
- The **inverse** of $\mathbf{p \rightarrow q}$ is $\neg p \rightarrow \neg q$

- **<u>Examples:</u>**
    - If it snows, the traffic moves slowly.
  - **The contrapositive:**
    - If the traffic does not move slowly then it does not snow.
    - $\neg q \rightarrow \neg p$
  - **The inverse:**
    - If it does not snow the traffic moves quickly.
    - $\neg p \rightarrow \neg q$

Biconditional

- **Definition**: Let p and q be propositions. The biconditional $p \leftrightarrow q$ (**read p if and only if q**), is true when p and q have the same truth values and is false otherwise.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- **Note:** two truth values always agree.

Constructing the truth table

- **Example: Construct a truth table for**
  $(p \to q) \wedge (\neg p \leftrightarrow q)$
- Simpler if we decompose the sentence to elementary and intermediate propositions

p	q	$\neg p$	$p \to q$	$\neg p \leftrightarrow q$	$(p \to q) \wedge (\neg p \leftrightarrow q)$
T	T				
T	F				
F	T				
F	F				

Constructing the truth table

- Example: Construct the truth table for
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

Rows: all possible combinations of values for elementary propositions: 2^n values

p	q	$\neg p$			$(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$
T	T				
T	F				
F	T				
F	F				

Constructing the truth table

- Example: Construct the truth table for $(p \to q) \wedge (\neg p \leftrightarrow q)$

Typically the target (unknown) compound proposition and its values

p	q	$\neg p$	$p \to q$	$\neg p \leftrightarrow q$	$(p \to q) \wedge (\neg p \leftrightarrow q)$
T	T				
T	F				
F	T				
F	F				

Auxiliary compound propositions and their values

Constructing the truth table

- Examples: Construct a truth table for
  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$
T	T	F			
T	F	F			
F	T	T			
F	F	T			

Constructing the truth table

- **Examples: Construct a truth table for**

  $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$

  Simpler if we decompose the sentence to elementary and intermediate propositions

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F