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42380 Supply Chain Analytics

Group 24

Assignment 2

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1 Introduction

The objective of this report is to use network design to optimize the future expansion of *Indiana Jeans* and optimize their truck routes to minimize transportation costs.

2 Part 1: Network design

In Part 1, we seek to find what plants and DC's to open for Indiana Jeans in order to satisfy customers demand. We then compare it to a more centralized plant version to see which is more economical.

2.1 Question 1

To formulate this model, we used the Network design model that is thoroughly described in lecture 7. The only difference is that we have one product so we can ignore L for products.
Data:

- C: Number of customers.
- DC: Number of possible DC.
- P: Number of possible plants.
- $distance\_DC\_Customer_{d,c}$: Distance between DC and customer.
- $distance\_Plant\_DC_{p,d}$: Distance between Plant and DC.
- $f\_dc\_open_d$: Fixed cost of opening DC.
- $f\_plant\_open_p$: Fixed cost of opening Plant.
- $f\_dc\_close_d$: Fixed cost of closing DC
- $f\_plant\_close_p$: Fixed cost of closing Plant
- dc_cap_d: Capacity of each DC
- $plant\_cap_p$: Capacity of each Plant
- $demand_c$: Customers demand

Decision variables:

- Binary, if we open a DC dc: $x_d \in \{0, 1\}$
- Binary, if we open Plant p: $z_p \in \{0, 1\}$
- Number of units transported from plant to DC: $w_{p,d} \in R^+$
- Number of units transported from DC to Customer: $y_{d,c} \in R^+$

Objective:

$$\begin{aligned}\min \ & \sum_{d=1}^{DC}(f\_DC\_open_d \cdot x_d) + \sum_{d=1}^{DC}(f\_Plant\_open_p \cdot z_p) + \sum_{d=1}^{DC}(f\_DC\_close_d \cdot (1 - x_d)) \\ & + \sum_{d=1}^{DC}(f\_Plant\_close_p \cdot (1 - z_p)) + \sum_{d=1}^{DC}(\sum_{c=1}^{C}(0.2 \cdot dist\_DC\_customer_{d,c} \cdot y_{d,c})) \\ & + \sum_{p=1}^{P}(\sum_{d=1}^{DC}(0.1 \cdot dist\_Plant\_DC_{p,d} \cdot w_{p,d}))\end{aligned} \tag{1}$$

Subject to:

- Make sure we fulfill the demand for each customer:

$$\sum_{d=1}^{DC} y_{d,c} = Demand_c \quad \forall \ c = 1 : C \tag{2}$$

- Make sure we never go over allowed capacity for the DC's

$$\sum_{c=1}^{C} y_{d,c} \leq dc\_cap_d \cdot x_d \quad \forall \ d = 1 : DC \tag{3}$$

- Make sure we never go over allowed capacity for the Plants

$$\sum_{d=1}^{DC} w_{p,d} \leq plant\_cap_p \cdot z_p \quad \forall \ p = 1 : P \tag{4}$$

- Balance constraint, to make sure that everything that goes into the DC's, comes out as well

$$\sum_{p=1}^{P} w_{p,d} = \sum_{c=1}^{C} y_{d,c} \quad \forall \ d = 1 : DC \tag{5}$$

2.2 Question 2

The model described in question 1 was implemented and solved in Julia, the code can be seen in the julia file: *P1Q2.jl*

2.3 Question 3

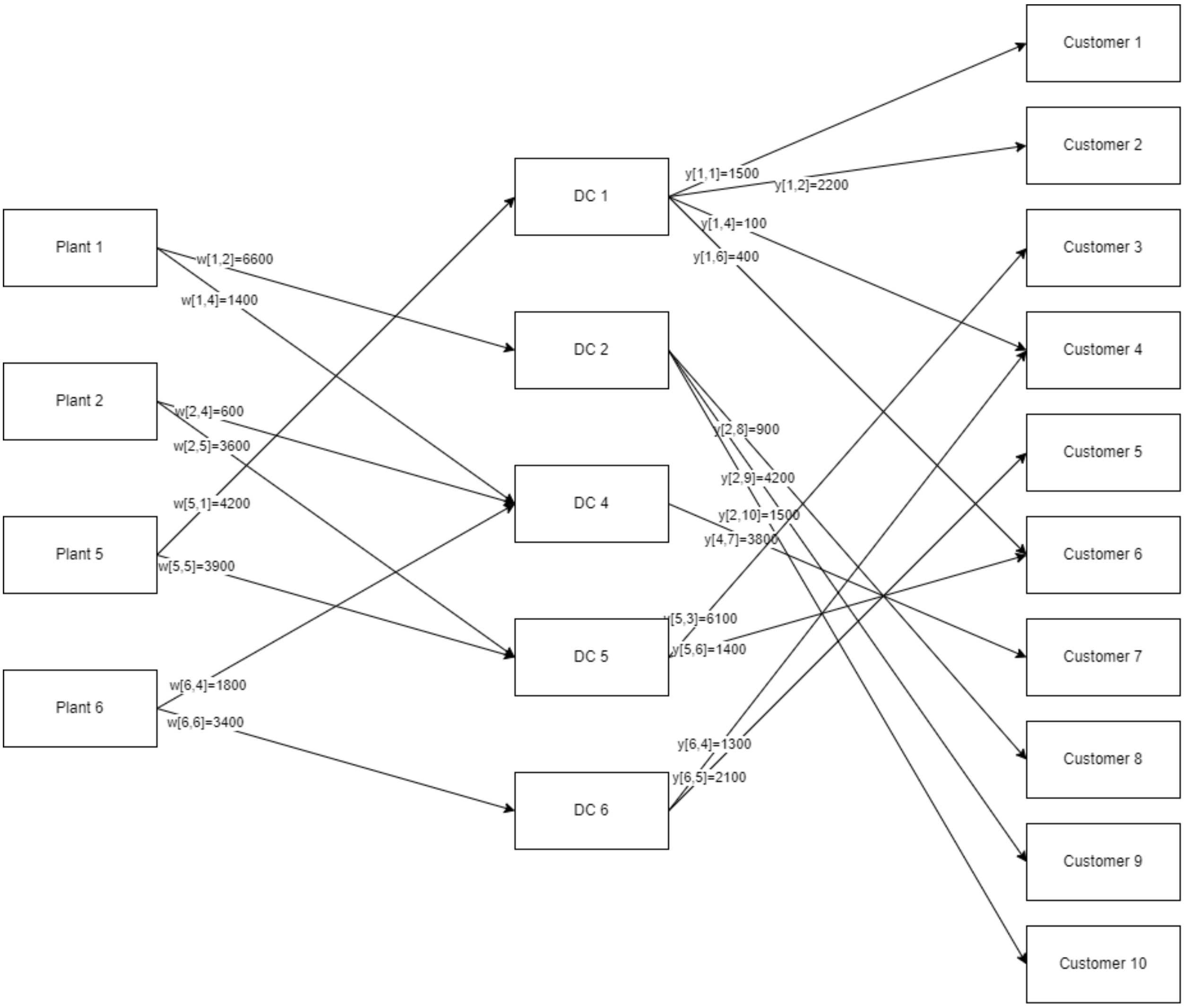

Figure 1: Network diagram

Figure 1, shows us the connections between all active plants, distribution centers and customers after the expansion. We can see what plants service what DC's and which DC's service each customer along with the quantity of goods in every connection. For instance: Plant 1 sends 6.600 goods to DC 2, which in return, sends 900 goods to customer 8, 4.200

goods to customer 9 and finally, 1.500 goods to customer 10.
The solution of having plants 1,2,5 and 6 and DC 1,2,4,5,6 gives us the the total cost of *10,154,760$*

2.4 Question 4

Are all plants and distribution centres being used at full capacity? no
DC number 1 is using 4200.0 of max capacity 6000
DC number 2 is using 6600.0 of max capacity 7000
DC number 4 is using 3800.0 of max capacity 4000
DC number 5 is using 7500.0 of max capacity 7500, using full capacity
DC number 6 is using 3400.0 of max capacity 3400, using full capacity

Plant number 1 is using 8000.0 of max capacity 8000, using full capacity
Plant number 2 is using 4200.0 of max capacity 6500
Plant number 5 is using 8100.0 of max capacity 8100, using full capacity
Plant number 6 is using 5200.0 of max capacity 5200, using full capacity

Assuming that any plant can get disrupted with equal probability, which DC's are in a riskier position and why?
As we can see on *Figure 1*, DC's 1, 2 and 6 could be considered at more of a risk since they only have one supplier, while all the other DC's have more than one Plant supplying them with goods. And out of the riskier DC's, DC 2 has 3 different customers who are not being serviced by any other DC. Similarly, DC 1 has 4 different customers, two of which are solely relying on DC 1. Lastly, DC 6 has two different customers, one is relying solely on them.

2.5 Question 5

In this question, we are, in a way comparing having a centralized or a decentralized model. In previous questions, we found the total cost of having decentralized plants but now only using two plants to centralize.
We used the same model as before in Julia but with only two possible plants $P = 2$. From that, the adjusted the data, doubled the capacity at the plants, and added the cost of $2 \cdot 1,500,000$ to the objective, the model can be seen in *P1Q5.jl*.
The solution to this model is then *11,057,020 $*. From this, we can see that the decentralized model is better since it has a lower total cost.

3 Part 2: Distribution planning

In part 2, the truck routes of 15 shops served by the DC in Aarhus were optimized to minimize transportation costs. The following assumptions were used in the calculation:

- All vehicles depart and end at their route at the depot (DC)
- Each route is driven by a different vehicle and a different driver
- Truck Capacity is $C = 80$
- Transportation cost is *1$ per unit and km*
- Fixed cost for each driver for each route is 200$

3.1 Question 1

The *Clarke-Wright heuristic* was used to optimize the truck routes to minimize transportation costs.
Equation 6 was used to calculating the savings from merging two routes with each other.

$$s_{ij} = c_{i0} + c_{0j} - c_{ij} \tag{6}$$

The result from the saving list was sorted in descending order and each merge was checked by condition until no more routes could be merged without exceeding the truck capacity or had 0 savings. Each merge was checked if the resulting route was feasible with respect to the truck capacity and if the route was already in the previous merging route and therefore a sub-tour would occur.
The resulting sets of routes were:

$$(0, 1, 10, 12, 0), (0, 3, 9, 7, 14, 0), (0, 11, 13, 0), (0, 4, 15, 0), (0, 2, 8, 0), (0, 5, 6, 0),$$

The total transportation cost was calculated by adding together the distance of all of the resulting routes together. *The total transportation cost = 3,193 $*

3.2 Question 2

The new sets of routes result in a saving for the company. Previously the company had to pay the fixed cost for 15 drivers and a transportation cost of *5,724 $* but with the new route the company has to pay 6 drivers and a transportation cost of *3,193 $* which results in a total saving of *3,131 $*

3.3 Question 3

To formulate the model in Julia, we used the vehicle routing problem (VRP) and the load-variable formulation to build the full network where we use a new variable, $z_{i,k}$, to track the load of the vehicles. The VRP optimizes the routes where all routes serve a set of customers and begin and end at the same depot. The model is formulated as follows and can also be seen in file *P2Q3.jl*

Data:

- K: Number of vehicles
- N: Number of nodes, node 1 is the depot.
- C: Capacity of each vehicle
- d_i: demand of each customer
- c_{ij}: transportation cost of going directly from node i to node j, since *1 km = 1 $*

Decision variables:

- Binary, if vehicle k goes directly from node i to node j : $x_{ijk} \in \{0,1\}$
- Binary, if vehicle k visits customer i: $y_{ik} \in \{0,1\}$
- The load carried by vehicle k when arriving at node i: $z_{ik} \in R^+$

Objective:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{K} c_{i,j} \cdot x_{i,j,k} \tag{7}$$

Subject to:

- Only visit every customer once:

$$\sum_{k=1}^{K} y_{i,k} = 1, \; \forall \; i = 2 : N \tag{8}$$

- Vehicle flow balance constraints

$$\sum_{j \in N} x_{j,i,k} = \sum_{j \in N} x_{i,j,k} \tag{9}$$

- Subtour elimination:

$$z_{i,k} - z_{j,k} \geq d_i - (1 - x_{i,j,k}) \cdot D \tag{10}$$

Since the model is too large to solve in a short amount of time we used the following time limit of 90 seconds.

- $set\_optimizer\_attribute(model, "TimeLimit", 90)$

Since the time limit was used the best-found solution, after 90 seconds, was:

$$(0, 10, 1, 4, 0), (0, 11, 13, 0), (0, 5, 2, 0), (0, 15, 12, 0), (0, 14, 7, 9, 3, 0), (0, 6, 8, 0) \tag{11}$$

With total cost of *3,122 $* and with drivers cost *4,322 $*
In comparison, with the heuristic method. The Julia solution gave a slightly better value for total cost. We still have six routes but only two of them, (0,14,7,9,3,0) & (0,11,13,0), were the same.
We assume that we recalculate the cost of the same solution by using *0.1$ * number of units between nodes * km between nodes* for each route and sum them all up. The calculations are the following:

- Route (0,10,1,4,0) starts with the total vehicle load of $D = 19 + 20 + 41 = 80$ units
  From 0->10 the vehicle carries load of 80 for 295 km equals to $0.1\$ * 80 units * 295 km = 2,360$ in cost. The demand in node 10 is 19 the the vehicle leaves node 10 with load of 61
  From 10->1 the vehicle carries load of 61 unit for 55 km equals to $0.1\$ * 61 units * 55 km = 335.5$ in cost. The demand in node 1 is 20 the the vehicle leaves node 1 with load of 41
  From 1->4 the vehicle carries load of 41 for 197 km equals to $0.1\$ * 41 units * 197 km = 807.7$ in cost. The demand in node 4 is 41 the the vehicle leaves node 4 with load of 0
  From 1->0 the vehicle is empty so there is no transportation cost.
  The transportation cost for this route is 3,503.2 $

The same calculation were applied for the other routes:

- Route (0,11,13,0) starts with the total vehicle load of $D = 32 + 33 = 65$
  From 0->11 $0.1\$ * 65 units * 221 km = 1436.5$ in cost. Node 11 has 32 units in demand
  From 11->13 $0.1\$ * 33 units * 133 km = 438.9$ in cost. Node 13 has 33 units in demand
  From 13->0 has no cost since the vehicle is empty.
  The transportation cost for this route is 1,875.4 $

- Route (0,5,2,0) starts with the total vehicle load of $D = 19 + 38 = 57$
  From 0->5 $0.1\$ * 57 units * 36 km = 203$ in cost. Node 5 has 19 units in demand.
  From 5->2 $0.1\$ * 38 units * 61 km = 231.3$ in cost. Node 2 has 38 units in demand.
  From 2->0 has no cost since the vehicle is empty.
  The transportation cost for this route is 434.3$

- Route (0,15,12,0) starts with the total vehicle load of $D = 31 + 25 = 56$
  From 0->15 $0.1 * 56 units * 211 km = 1181.6$ in cost. Node 15 has 31 units in demand.
  From 15->12 $0.1 * 25 units * 118 km = 295$ in cost. Node 12 has 25 units in demand.
  From 12->0 has no cost since the vehicle is empty.
  The transportation cost for this route is 1,476.6 $

- Route (0,14,7,9,3,0) starts with the total vehicle load of $D = 14 + 17 + 26 + 16 = 73$
  From 0->14 $0.1\$ * 73 units * 190 km = 1387$ in cost. Node 14 has 14 units in demand.
  From 14->7 $0.1\$ * 59 units * 161 km = 949.9$ in cost. Node 7 has 17 units in demand.
  From 7->9 $0.1\$ * 42 units * 314 km = 1318.8$ in cost. Node 9 has 26 units in demand.
  From 9->3 $0.1\$ * 16 units * 106 = 169.6$ in cost. Node 3 has 16 units in demand.
  From 3->0 has no cost since the vehicle is empty.
  The transportation cost for this route is 3,825.3 $

- Route (0,6,8,0) starts with the total vehicle load of $D = 30 + 38 = 68$
  From 0->6 $0.1\$ * 68 units * 59 km = 401.2$ in cost. Node 6 has 30 units in demand.
  From 6->8 $0.1\$ * 38 units * 87 km = 330.6$ in cost. Node 8 has 38 units in demand.
  From 8->0 has no cost since the vehicle is empty.
  The transportation cost for this route is 731.8 $

The total cost of all the routes is therefore $3,503.2 + 1,875.4 + 434.3 + 1,476.6 + 3,825.3 + 731.8 =$ *11,846.6$*
From this solution, we can see that the transportation cost increases a lot by having cost load-dependent though the unit cost is lowered.

3.4 Question 4

In this question, we reuse the model previously described in question 3. The new code can be seen in *P2Q4.jl*. The only change is that capacity and the objective.

- New capacity: $C = 50$ instead of $C = 80$
- New objective has transportation cost of 0.5$ per km instead of 1$ per km:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{K} 0.5 \cdot c_{i,j} \cdot x_{i,j,k} \tag{12}$$

Since, most of the customers have a high demand it doesn't come as a surprise that the solution becomes:

$$(0,4,0),(0,10,15,0),(0,6,0),(0,9,3,0),(0,1,12,0),(0,13,0),(0,11,0),(0,8,0),(0,7,14,5,0),(0,2,0)$$

As an example, customers 2 and 4 cannot share a route with another customer with the new vehicle capacity. Compared to our previous result we have gone from 6 routes to 10 routes with lower vehicle capacity. However, we have managed to lower our transportation costs to *2,066 $*. With 10 drivers, each paid 200$ the total transportation cost is *4,066 $*.
We assume by the meaning of "distance" in question 4 that we are supposed to calculate the total distance of our new network and multiply it by 0.5.
Total distance is 4130.6 km which leads to distance cost of *2,065.3 $* and with the 10 drivers the distance cost is *4,065.3 $*

3.5 Question 5

In this question, we continue to use the same model as in questions 2 and 3, see file *P2Q5.jl*.
We needed to split the index $k \in K$ into two indexes one for the trucks and one for electric vehicles. The same was done with the parameter for the vehicle's capacity.
All of the variables and constraints were adjusted to these changes, for example, $x_{i,j,k}$ was changed to variables $xt_{i,j,kt}$ and $xe_{i,j,ke}$.
We also added a new constraint to assure that the driving distance for the electric vehicles was max 500 km.

- $$\sum_{i=1}^{N} \sum_{j=1}^{N} c_{i,j} \cdot xe_{i,j,ke} \leq 500 \quad \forall \; ke = 1 : K_{El} \tag{13}$$

This model gave us the following routes, for electric vehicles:

$$(0, 6, 5, 0), (0, 11, 0), (0, 13, 0), (0, 15, 0), (0, 4, 0) \tag{14}$$

And the routes for the trucks were:

$$(0, 2, 8, 0), (0, 1, 10, 12, 0), (0, 14, 7, 9, 3, 0) \tag{15}$$

With the total cost of *2,702.5 $* and with eight drivers the total cost is *4,302.5 $* We can see that this solution has five electric cars and three trucks, and is therefore between the only trucks solution of *4,322 $* and only electric solution of *4,066 $*.