HANDOUT ON SETS, SEQUENCES, and STRINGS

finite Set:	A = {1, 2, 3, 4}
infinite Set:	B = {x | x is a positive, even integer} "B equals the set of all x such that x is a positive, even integer"
empty (or **null** or **void**) **set**:	$\varnothing = \{\ \}$ but $\{\varnothing\} \neq \varnothing$
cardinality of X	| X | = number of elements in X
subset:	$X \subseteq Y$ Every element of X is an element of Y.
proper subset:	$X \subset Y$ X is a subset of X and X does not equal Y.
power set of X:	$\mathcal{P}(X)$ The set of all subsets (proper or not) of a set X.
union:	$X \cup Y = \{ x \mid x \in X \text{ or } x \in Y \}$ All the elements belonging to either X or Y (or both).
intersection:	$X \cap Y = \{ x \mid x \in X \text{ and } x \in Y \}$ All elements belonging to both X and Y.
disjoint:	X and Y are disjoint if $X \cap Y = \varnothing$
pairwise disjoint:	A collection of sets S is said to be pairwise disjoint if whenever X and Y are distinct sets in S, X and Y are disjoint.
difference: (or **relative complement**)	$X - Y = \{ x \mid x \in X \text{ and } x \notin Y \}$ All elements in X that are not in Y.
universal set (or **universe**)	U contains all the objects under consideration. It must be explicitly given or inferred from the context.
complement:	X or X^c or X' Given a universal set U and a subset X of U, the set U - X is called the complement of X.
union of a family of sets S	$\cup S = \{ x \mid x \in X \text{ for some } x \in S \}$ The elements x belonging to at least one set X in S.
intersection of a family of sets S	$\cap S = \{ x \mid x \in X \text{ for all } x \in S \}$ The elements x belonging to every set X in S.
partition:	A collection S of nonempty subsets of X for which every element in X belongs to exactly one member of S. (i.e. A partition of a set X divides X into non-overlapping subsets.) Notice: If X is a partition of X, S is pairwise disjoint and $\cup S = X$.

Cartesian product $X \times Y$ Set of all ordered pairs (x, y) where $x \in X$ and $y \in Y$.

SET PROPERTIES

Venn diagrams can be used to establish certain properties of sets.

(a) Associative laws:
$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$

(b) Commutative laws:
$A \cup B = B \cup A$ $A \cap B = B \cap A$

(c) Distributive laws:
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(d) Identity laws:
$A \cup \emptyset = A$ $A \cap U = A$

(e) Complement laws:
$A \cup A^c = U$ $A \cap A^c = \emptyset$

(f) Idempotent laws:
$A \cup A = A$ $A \cap A = A$

(g) Bound laws:
$A \cup U = U$ $A \cap \emptyset = \emptyset$

(h) Absorption laws:
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$

(i) Involution law:
$(A^c)^c = A$

(j) 0 / 1 laws:
$\emptyset^c = U$ $U^c = \emptyset$

(k) De Morgan's laws for sets:
$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$

SEQUENCES and STRINGS

sequence: A list in which order is taken into account.

index: For sequence s_n, n is the index of the sequence.

increasing sequence: A sequence s is increasing if $s_n \leq s_{n+1}$ for all n.

decreasing sequence: A sequence s is decreasing if $s_n \geq s_{n+1}$ for all n.

If $\{a_i\}_{i=m}^{n}$ is a sequence, we define

sum (or sigma) notation: $\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \cdots + a_n$

product notation: $\prod_{i=m}^{n} a_i = a_m \cdot a_{m+1} \cdots a_n$

i is called the **index** m is called the **lower limit** n is called the **upper limit**

string: A *string over X* is a finite sequence of elements from X.

Repetitions in a string can be specified by superscripts.

example: string *bbaaac* may be written b^2a^3c

null string: λ String with no elements. Sometimes denoted as ε.

X^* denotes the set of all strings over X including the null string..

X^+ denotes the set of all nonnull strings over X.

length of a string $|\alpha|$ the number of elements in the string α.

concatenation: αβ The concatenation of the two strings α and β is the string consisting of α followed by β.