

Exercise 1: Asymptotic complexity of the functions

- $6n^3 + 10n^2 + 5n + 2 \rightarrow O(n^3)$.
- $3\log_2(n) + 4 \rightarrow O(\log_2(n))$.
- $2^n + 6n^2 + 7n \rightarrow O(2^n)$.
- $4k + 2 \rightarrow O(1)$.
- $4\log(n) + n \rightarrow O(n)$.
- $2\log_{10}(k) + kn^2 \rightarrow n^2$

Exercise 2: Compare the functions near infinity

$$\left(\frac{1}{3}\right)^n < 17 < \log(n) < \frac{n}{\log(n)} <$$

$$\sqrt{n} < n < \left(\frac{3}{2}\right)^n < 2^n$$

Exercise 3: Find number of operations and complexity

- A: $1 + 2n \rightarrow O(n)$.
- B: $1 + 2(2n + n) \rightarrow O(n)$.
- C: $2\frac{n(n+1)}{2}n \rightarrow O(n^3)$.

Exercise 4: Find number of operations and complexity

- $f_1: 1 + 2\frac{n(n+1)}{2} \rightarrow O(n^2)$.
- $f_2: 2 + 4\log_2(n) \rightarrow O(\log_2(n))$.
- $f_3: 2 + 4\log_2(n) \rightarrow O(\log_2(n))$.
- $f_4: 2 + 2n\log_2(n) + 2\log_2(n) \rightarrow O(n\log_2(n))$.

Exercise 5:

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1 - input: n > 0
2 - output: n!
3 -
4 - S := 1
5 - for i = 2 to n do
6 -   S = S*i
7 - end
8 - return S

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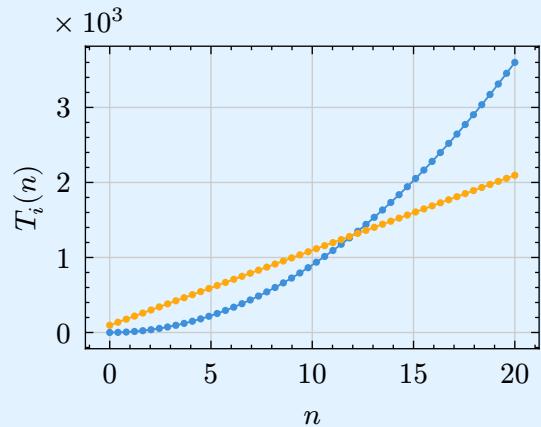
2. The complexity is $1 + 2(n - 1) = O(n)$.

Exercise 6:

1. $T_1(n) = 9n^2 = O(n^2)$.
- $T_2(n) = 100n + 96 = O(n)$.
1. For $T_1: (n_0, c) = (1, 9)$ clearly.
2. For $T_2: (n_0, c) = (1, 200)$ since $200n = 100n + 100n > 100n + 100 > 100n + 96$.

$T_i \setminus n$	1	3	5	10	14
T_1	9	81	225	900	1764
T_2	196	396	596	1096	1496

4.



- for $n < 12$: T_1 is better, otherwise T_2 is better.
- calling the algorithms with complexity T_1 and T_2 we get an algorithm of complexity $T_1 + T_2 + c$ where c is a constant, and its complexity would be $O(n^2)$.

Exercise 7:

1.

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1 - input:
2 -   - n: size
3 -   - A: matrix of size n
4 -   - B: matrix of size n
5 - output:
6 -   - C: matrix product of A*B
7 -
8 - C = []
9 - for i = 0 to n-1
10 -   for j = 0 to n-1
11 -     for k = 0 to n-1
12 -       C[i][j] += A[i][k] * B[k][j]
13 -     end
14 -   end
15 - end
16 - return C

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2. $T(n) = 3 * (n - 1)^3 \rightarrow O(n)$.