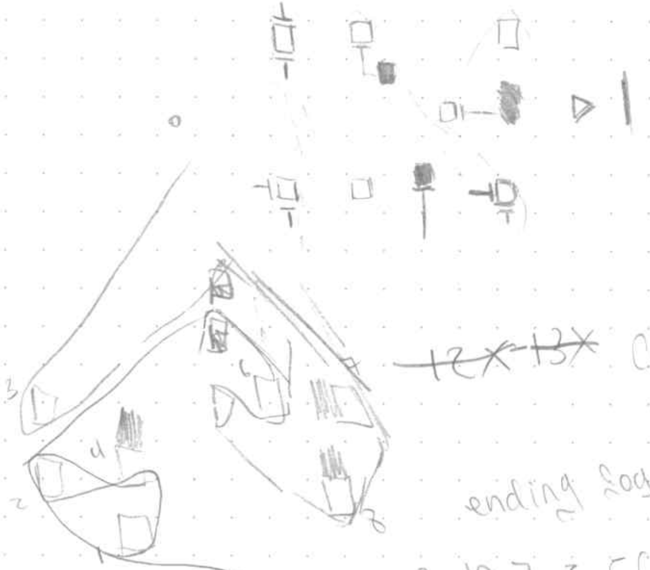


~~12X 13X~~ Cannot start with 1

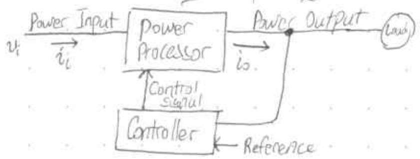
ending sequence

3, 10, 7, 8, 5, 6, 9

Chapter 1 power problems.

1-1 In the power processor of Fig 1-1, the energy efficiency is 95%. The output to the three-phase load is as follows: 200 V Line-to-line (Rms) sinusoidal voltages at 52hz and line current of 10A at a power factor of 0.8(lagging). The input to the power processor is a single phase utility voltage at 60Hz. The input power is drawn at unity power factor. Calculate the input current and the input power.

First we start by depicting the mentioned diagram



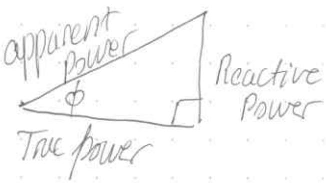
The power factor (p.f.) is the Ratio of the True power to the apparent power. We can Represent this ~~power~~ factor as $\cos\phi$

$$p.f = \frac{\text{True power}}{\text{apparent power}} = \cos\phi$$

A power factor of 1 implies there are no energy losses. p.f. = 1 means all the power that is consumed, is consumed by the Resistive element R.

It can also be Represented as the Resistance R over the impedance Z

The apparent power is the squareroot of the sum of the True power squared, and the Reactive power squared. Refer to the triangle



$$\cos\phi = \frac{\text{True power}}{\text{apparent power}}$$

52hz

Pf = 1
f = 60hz

Pf = 0.8
f = 52hz

Output: $V_{LL} = 200V$ (3 phase, Rms) at 52Hz
$I_o = 10A$ at 0.8 lagging pf.

efficiency is at $\eta = 95\%$

The output power can be described as $\sqrt{3}\, V_{LL} I_o \cdot pf$.

$$\Rightarrow P_{out} = \sqrt{3}\, V_{LL} I_o (\text{Power factor}) = \sqrt{3}(200V)(10A)(0.8)$$
$$\Rightarrow P_{out} = 2.77\ kW = 2771.28\ \text{watts}.$$

from the output power, we can find the input power since we were given the efficiency.

$$P_{out} = \eta\, P_{in} \Rightarrow P_{in} = P_{out}\left(\frac{1}{\eta}\right) = \frac{2.77e3(\text{Watts})}{(0.95)}$$

This gives us a Result of the input power being
2917 Watts

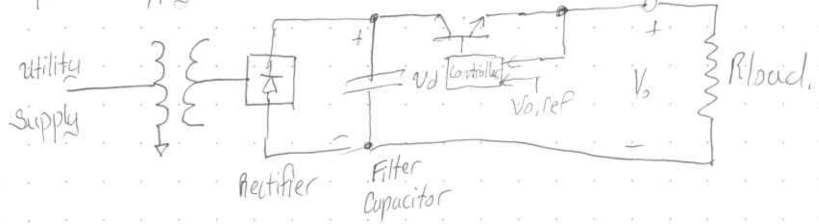
Input	$\eta = 0.95$	Output
Power: 2917 W	$\longrightarrow$	Power: 2771
P.f = 1		Pf = 0.8 lagging
$V_{in} = 230V$		$V_{LL} = 200V$ (3 phases, Rms)
f = 60hz		f = 52hz

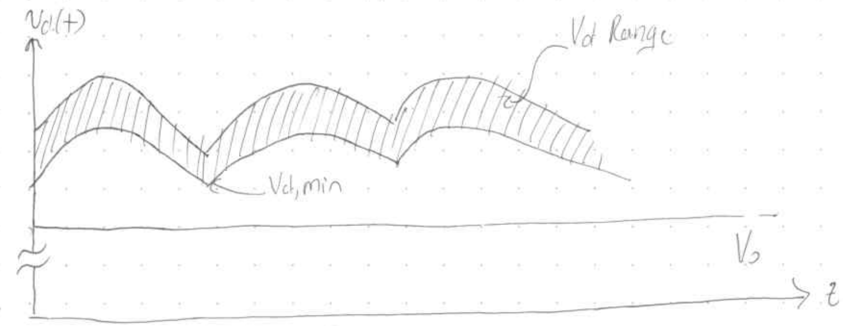
The Square Root three is attained by the out of phase connections of the ~~pf~~. Δ or Y connection.

We know the input power to be $P_{in} = I_{in} V_{in} (P.f)$
We have all but the current, so we can solve the current simply by dividing the power by the voltage.

$$I_{in} = \frac{P_{in}}{V_{in}(P.f)} = \frac{P_{in}}{V_{in} \cdot 1} = \frac{2917\ \text{Watt}}{230V} = 12.68\ \text{Amps (Rms)}$$

1-2 Consider a linear Regulated dc power Supply (Fig. 1-2a). The instantaneous input voltage corresponds to the waveform in Fig 1-2b, where $V_{d,min} = 20V$ and $V_{d,max} = 30V$. Approximate this waveform by a triangular wave consisting of two linear segments between the above two values. Let $V_o = 15$ and assume and assume the output load is constant. Calculate the energy efficiency in this part of the power supply due to losses in the transistor.





Linear dc power Supply

Lets look at the lower bound of the input voltage, and for simplicity, we will assume it to be triangular.

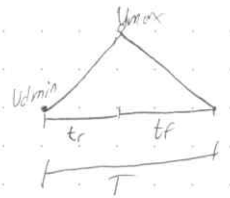
where the Regulated output voltage $V_o = 15V$.

where one waveform may be Represented as a triangle of two linear segments.

The average waveform for the voltage over one period is

$$V_{avg}(t)_T = \frac{1}{T}\int_0^T V_{in}(t) \cdot dt$$

where we will now try and create the waveform

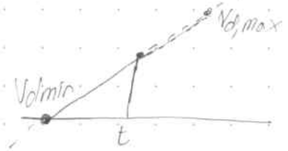


where t_r and t_f are the rise and fall times

$$T = t_r + t_f \Rightarrow t_f = T - t_r$$

where $$v(t) = \begin{cases} \frac{(V_{d,max} - V_{d,min})}{t_r} + V_{d,min} & 0<t<t_r \\ \frac{(V_{d,min} - V_{d,max})}{t_f} + V_{d,max} & t_r<t<T \end{cases}$$

I have all ready made a mistake, and need to express each line segment parametrically with time.



The voltage of the current position at time t can be represented parametrically as the slope and Δt product plus the initial condition.

$v_{rise}(t) = V_{d,min} + \frac{\Delta V_d}{\Delta t}\cdot t$ which doing so continuously

gives $v_{rise}(t) = \frac{dv_d}{dt}\cdot t + v_{d,min}$ $\quad 0<t<t_r$

Likewise $v_{fall}(t) = \frac{dv_{df}}{dt}\cdot t + v_{d,max}$ $\quad t_r<t<T$

$$\Rightarrow v(t) = \begin{cases} \frac{(V_{d,max} - v_{d,min})}{t_r}\cdot t + V_{d,min} & 0<t<t_r \\ \frac{(V_{d,min} - v_{d,max})}{T-t_r}\cdot t + V_{d,max} & t_r<t<T \end{cases}$$

where t_r is the rise time, and T is the period

To take the average voltage $v_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$

however, we can simplify the integral by splitting up the portions with each corresponding portion.

$$\Rightarrow v_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T}\left[\int_0^{t_r}\left(\left[\frac{V_{d,max}-V_{d,min}}{t_r}\right]\cdot t + V_{d,min}\right)dt + \int_{t_r}^{T}\left(\left[\frac{V_{d,min}-V_{d,max}}{T-t_r}\right]\cdot t + V_{d,max}\right)dt\right]$$

Evaluating the first integral:

$$\int_0^{t_r}\left(\frac{V_{max}-V_{min}}{t_r}\right)\cdot t + V_{min}\,dt \Rightarrow \frac{1}{2}\left(\frac{V_{max}-V_{min}}{t_r}\right)t^2 + V_{min}t\,\Big|_0^{t_r}$$

Simplifying, we get the first integral to be $\frac{1}{2}\left(\frac{V_{max}-V_{min}}{t_r}\right)t_r^2 + V_{min}t_r$

$$\Rightarrow \boxed{\frac{1}{2}(V_{max}-V_{min})t_r + V_{min}t_r} = \frac{1}{2}V_{max}t_r + \frac{1}{2}V_{min}t_r \Rightarrow \frac{1}{2}t_r(V_{max}+V_{min})$$

Evaluating the falling integral, we get:

$$\int_{t_r}^{T}\left(\frac{V_{min}-V_{max}}{T-t_r}\right)\cdot t + V_{max}\,dt = \left(\frac{V_{min}-V_{max}}{T-t_r}\right)\cdot\frac{1}{2}t^2 + V_{max}t\,\Big|_{t_r}^{T}$$

$$\Rightarrow \left(\frac{V_{min}-V_{max}}{T-t_r}\right)\cdot\frac{1}{2}t^2 + V_{max}\cdot t\,\Big|_{t_r}^{T} = \left(\frac{V_{min}-V_{max}}{T-t_r}\right)\cdot\frac{1}{2}T^2 + V_{max}\cdot T \;\ldots$$

$$\ldots - \left[\left(\frac{V_{min}-V_{max}}{T-t_r}\right)\cdot\frac{1}{2}t_r^2 + V_{max}t_r\right]$$

That's where

$$\Rightarrow \left(\frac{V_{min}-V_{max}}{T-t_r}\right)\frac{1}{2}\left(T^2 - t_r^2\right) + V_{max}(T-t_r)$$

the ~~product of difference~~ ? The difference of squares $(T^2 - t_r^2) = (T+t_r)(T-t_r)$

$$\Rightarrow \frac{1}{2}(V_{min}-V_{max})(T+t_r) + V_{max}(T-t_r)$$

expanding ~~Simplifying~~, we get: $\frac{1}{2}(V_{min}-V_{max})t_r + \frac{1}{2}(V_{min}-V_{max})T + V_{max}T - V_{max}t_r$

$$\Rightarrow \frac{1}{2}V_{min}t_r - \frac{1}{2}V_{max}t_r - V_{max}t_r + \frac{1}{2}V_{min}T - \frac{1}{2}V_{max}T + V_{max}T$$

$$\Rightarrow \frac{1}{2}t_r(V_{min} - 3V_{max}) + \frac{T}{2}(V_{min}+V_{max})$$

Combining all the information we derived, we get

$$v_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T}\left[\frac{1}{2}(V_{max}+V_{min})t_r + \frac{1}{2}(V_{max}+V_{min})T + \dots\right.$$

$$\left.\dots + \frac{1}{2}(V_{min}-3V_{max})t_r\right]$$

Combining the tr terms, we get:

$$\frac{1}{2}t_r\left(V_{max}-3V_{max}+2V_{min}\right) = \frac{1}{2}\left(-2V_{max}+2V_{min}\right)t_r$$

$$\Rightarrow (V_{min}-V_{max})t_r$$

why. So $V_{avg} = \frac{1}{T}\left[(V_{min}-V_{max})t_r + \frac{1}{2}(V_{max}+V_{min})T\right]$

$$\Rightarrow V_{avg} = (V_{min}-V_{max})\cdot\frac{t_r}{T} + \frac{1}{2}(V_{max}+V_{min})$$

$$\Rightarrow V_{avg} = (V_{min}-V_{max})\frac{t_r}{T} + \frac{1}{2}V_{max} + \frac{1}{2}V_{min}$$

$$\Rightarrow V_{avg} = V_{min}\cdot\frac{t_r}{T} - V_{max}\cdot\frac{t_r}{T} + \frac{1}{2}V_{max} + \frac{1}{2}V_{min}$$

$$\Rightarrow V_{avg} = V_{min}\left(\frac{1}{2}+\frac{t_r}{T}\right) + V_{max}\left(\frac{1}{2}-\frac{t_r}{T}\right)$$

If we assume the case where the triangle is symmetric, so that $t_r = t_f = \frac{T}{2}$ we end up getting $V_{avg} = V_{min}$.

does this make sense? No, we should get $\frac{V_{max}-V_{min}}{2}$

Lets look at the second integral

$$\frac{1}{2}\left(\frac{V_{min}-V_{max}}{T-T/2}\right)\cdot t^2 + V_{max}\cdot t\,\Big|_{T/2}^{T} \Rightarrow \frac{1}{2}\frac{(V_{min}-V_{max})}{\frac{T}{2}}\cdot t^2 + V_{max}\cdot t\,\Big|_{T/2}^{T}$$

$$\Rightarrow \frac{(V_{min}-V_{max})}{T}(T^2)(1) + V_{max}\cdot T + \frac{(V_{min}-V_{max})}{T}\cdot\frac{T^2}{2} + V_{max}\frac{T}{2}$$

$$\Rightarrow (V_{min}-V_{max})T + V_{max}T = \frac{T^2}{2}\left(\frac{(V_{min}-V_{max})}{T} + V_{max}\right)$$

$$\Rightarrow V_{min}T - \frac{1}{2}V_{min}T + \frac{1}{2}V_{max}T \Rightarrow \frac{1}{2}V_{min}T + \frac{1}{2}V_{max}T$$

adding the first integrand, we get

$$\frac{1}{2}\left(\frac{T}{2}\right)(V_{min}+V_{max}) + \frac{T}{2}(V_{min}+V_{max}) = (V_{min}+V_{max})T\left(1+\frac{1}{2}\right)$$

and we remember we divide by T so $V_{avg}\big|_{t_r=\frac{T}{2}} = \frac{3}{2}(V_{min}+V_{max})$

which is still <u>wrong</u>. The only conclusion is a mistake.

Reevaluating ~~the first~~, we look at when we split the integral,

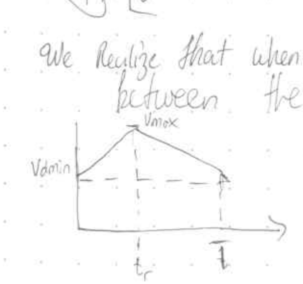
$$v_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T\cdot 2}\int_0^{T/2}\frac{(V_{d,max}-V_{d,min})}{\frac{T}{2}}t + V_{d,min}\,dt + \dots$$

$$\dots + \frac{1}{2T}\int_{T/2}^{T}\frac{(V_{d,min}-V_{d,max})}{\frac{T}{2}}t + V_{d,max}\,dt \quad \frac{1}{2}$$

$$V_{avg} = \frac{1}{4T}\left[\frac{V_{d,max}-V_{d,min}}{T/2}\left(\frac{T}{2}\right)^2 + V_{d,min}\left(\frac{T}{2}\right)\right]$$

$$V_{avg} = \frac{1}{4T}\left[(V_{max}-V_{min})\frac{T}{2} + V_{d,min}\cdot\frac{T}{2}\right]$$

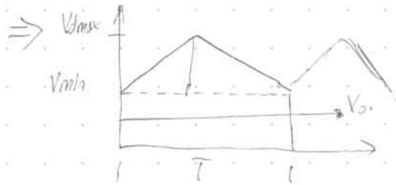
We realize that when we split the integral the average splits between the two, so we need to



Looking at the waveform the area under the curve. I realize I forgot to account for the offset!!

The area of the rectangle $V_{d,min}\cdot T$ and the area of the triangle is $\frac{1}{2}B\cdot h = \frac{1}{2}T(V_d - V_{max})$ and we normalize ~~the~~ the duration T.
So this tells us that the average is independent of the rise time.

$\Rightarrow$



$Area = \int_0^T V(t)dt = \left[V_{d,min} \cdot T + \frac{1}{2}T(V_{d,max} - V_{d,min})\right]$

The average is $V_{avg}(t) = \frac{1}{T}\left[V_{d,min} \cdot T + \frac{1}{2}T(V_{d,max} - V_{d,min})\right]$

$\Rightarrow V_{avg}(t) = V_{d,min} + \frac{1}{2}(V_{d,max} - V_{d,min})$

$\Rightarrow V_{avg}(t) = \frac{(V_{d,min} + V_{d,max})}{2}$

On average the input power would be $P_{in} = I_o V_{avg}$.

$\Rightarrow P_{in} = I_o \frac{1}{2}[V_{d,min} + V_{d,max}]$

We were given $V_{d,min} = 20v$, $V_{d,max} = 30v$

$P_{in} = \frac{(30+20)}{2} I_o = \frac{50}{2} I_o = 25 I_o$

The energy efficiency ideally for this triangular case

is $\eta = \frac{P_{out}}{P_{in}} = \frac{V_o I_o}{V_{in}} = \frac{15v I_o}{25v I_o} = 0.6$

$\Rightarrow \eta = 0.6$

The average voltage as I have been attempting earlier

$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{(V_{d,min} + V_{d,max})}{2}$

Lets try $V_{avg} = \frac{1}{T}\left[a\int_0^{t_r} v(t)dt + b\int_{t_r}^T v(t)\,dt\right]$

where we know $a\int_0^{t_r} v(t)\,dt = \underbrace{\frac{1}{2}t_r \cdot [V_{max} - V_{min}]}_{\triangle} + \underbrace{V_{min} \cdot t_r}_{\square}$

$b\int_{t_r}^T v(t)\,dt = \underbrace{\frac{1}{2}(T-t_r)[V_{max} - V_{min}]}_{\triangle} + \underbrace{V_{min}(T-t_r)}_{\square}$

Lets evaluate $a\int_0^{t_r} V_a(t)\,dt = \frac{1}{2}t_r \cdot [V_{max} - V_{min}]$

$a\int_0^{t_r} V_a(t)\,dt$ where $0<t<t_r$

Where before, I wanted to $\frac{(V_{d,max} - V_{d,min})t}{t_r} + V_{d,min}$ $\quad 0<t<t_r$

$V(t) = (V_{max} - V_{min})\frac{t}{t_r} + V_{min} \quad 0<t<t_r$.

Validating, $V(0) = V_{min}$, $V(t_r) = V_{max} - \cancel{V_{min}} + V_{min} = V_{max}$

So ~~then~~ Lets evaluate the integral

$a\int_0^{t_r} V(t)\,dt = a\int_0^{t_r} [V_{max} - V_{min}]\frac{t}{t_r} + V_{min}\,dt$

$= a\left[\frac{1}{2}(V_{max} - V_{min})\frac{t^2}{t_r} + V_{min} \cdot t\right]_0^{t_r} \Rightarrow a\left[\underbrace{\frac{t_r}{2}(V_{max} - V_{min})}_{\triangle} + \underbrace{V_{min} \cdot t_r}_{\square}\right]$

$\Rightarrow a\left[t_r\left(\frac{1}{2}V_{max} + \frac{1}{2}V_{min}\right)\right] = a\frac{1}{2}(V_{max} + V_{min}) \cdot t_r$

So a and b are 1.

~~that~~ ~~Comparing the two Results we get~~

~~$a \cdot \frac{1}{2}t_r(V_{max} + V_{min}) = \frac{1}{2}t_r[V_{max} - V_{min})$~~

~~$\Rightarrow a(V_{max} + V_{min}) = V_{max} - V_{min}$~~ $\quad a = \alpha + \beta$

~~$(\alpha+\beta)(V_{max} + V_{min}) = V_{max} - V_{min}$~~

~~$(\alpha+\beta)(V_{max} + V_{min}) = V_{max} - V_{min}$~~

~~$\alpha V_{max} + \alpha V_{min}$~~

So... we find we got the correct Result.

$\int_0^{t_r} v(t)\,dt = t_r \cdot \frac{1}{2}(V_{max} - V_{min}) + V_{min} \cdot t_r$

$\int_{t_r}^T v(t)\,dt = \frac{1}{2}(T - t_r)(V_{max} - V_{min}) + V_{min}(T - t_r)$

$V_{avg} = \frac{1}{T}\left(\int_{t_r}^T v(t)dt + \int_0^{t_r} v(t)\,dt\right) = \frac{1}{T}\left[t_r \cdot \frac{1}{2}(V_{max} - V_{min}) + V_{min} \cdot t_r + \frac{1}{2}(T - t_r)(V_{max} - V_{min}) + V_{min}(T - t_r)\right]$

$\Rightarrow V_{avg} = \frac{1}{2}\frac{T}{T}(V_{max} - V_{min}) + V_{min} \cdot T = \frac{T}{T}\frac{}{2}(V_{max} - V_{min})$

... So I am finding my integrals correct now.

1-3 Consider a switch mode dc - power supply represented by the circuit in Fig. 1-4a. The input dc voltage $V_d = 20v$ and the switch duty ratio $D = 0.75$. Calculate the fourier components of v_{oi} using the description of fourier analysis.

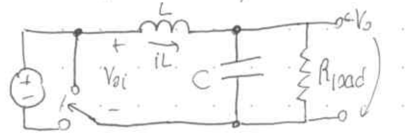
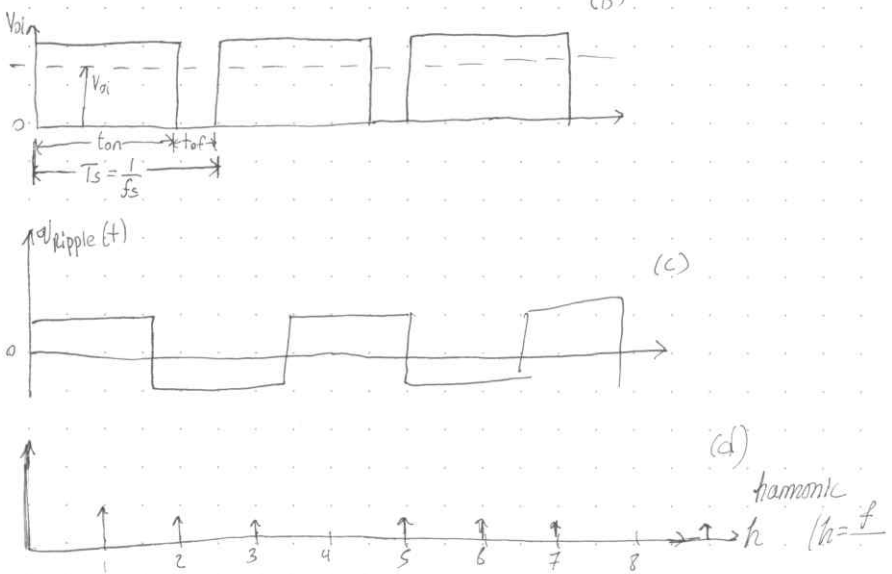
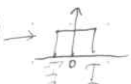


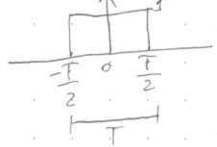
Figure 1-4 Equivalent circuit, waveforms and frequency spectrum of the supply in Fig 1-3.



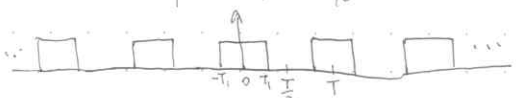
Let's review our signals. We have a rectangular function $\Pi(t) \rightarrow$



$$\Pi(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & |t| > T/2 \end{cases} \longrightarrow$$



For a periodic, rectangular function



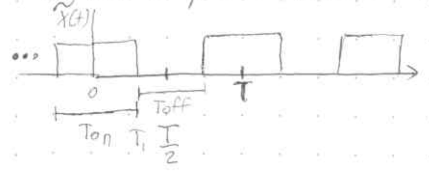
where the fourier series coefficients a_k are found using

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_T x(t) e^{-jk(2\pi/T)t}\, dt \qquad (3.39)$$

Likewise, we have previously

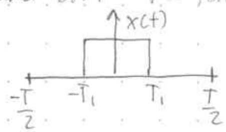
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \qquad (3.38)$$

Lets carefully define our variable function $x(t)$ to have a ~~periodic function~~ variable duty cycle.



where the period T is the sum of durations when the duty is on and off. $T = T_{on} + T_{off}$

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$



Evaluating the DC fourier component ($k=0$) (average) $a_k = \frac{1}{T}\int_T x(t) e^{-jk(2\pi/T)\cdot t}\, dt$

So, we have $a_0 = \frac{1}{T}\int_T x(t) e^0\, dt$

We defined the limits of the square interval as $\pm|T_1|$

$$\text{So} \quad a_0 = \frac{1}{T}\int_{-T_1}^{T_1} 1\, dt = \frac{1}{T}\left(T_1 - (-T_1)\right) = \frac{2T_1}{T}$$

a_k for $k \neq 0$ $\quad a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\, dt = -\frac{1}{jk\omega_0 T} e^{-jk\omega_0 t}\Big|_{-T_1}^{T_1}$

evaluating the bounds of the integral, we get the fourier series components for when $k \neq 0$

$$a_k = \begin{cases} \frac{2T_1}{T} & k = 0 \\ \frac{2}{k\omega_0 T}\left[\frac{e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}}{2j}\right] & k \neq 0 \end{cases}$$

and we realize that the duty cycle is only modified by the bounds

Using eulers identity, we can simplify the case for this non-variable $x(t)$

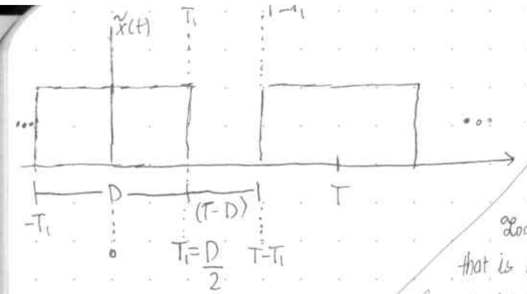
$$\Rightarrow a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T} = \frac{\sin(k\omega_0 T_1)}{k\pi}, \quad k \neq 0$$

where we have used the fact that $\omega_0 T = 2\pi$.

~~and~~ We can interpret these as samples of an envelope function

$$\Rightarrow Ta_k = \frac{2\sin \omega T_1}{\omega}\Big|_{\omega = k\omega_0} \qquad \Big| \; \tilde{x}(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

and this gives us a good review of oppenheim.



The periodic function $\tilde{x}(t)$ variable with duty D.

$$a_k = \frac{1}{T}\int_{-\infty}^{\infty} \tilde{x}(t)\, e^{-jk\omega_0 t}\, dt$$

Looking at it simpler, $x(t)$ is a finite duration signal that is equal to $\tilde{x}(t)$ over exactly one period — says, for $s \le t \le s+T$ for some value s — and zero otherwise. Then, since eq. (3.39) allows us to compute the Fourier components of $\tilde{x}(t)$ by integrating over any period, we can write:

$$\cancel{a_k = \frac{1}{T}\int_{-\infty}^{\infty} x(t)\, e^{-jk\omega_0 t}\, dt}$$

$$a_k = \frac{1}{T}\int_{s}^{s+T} \tilde{x}(t)\, e^{-j\omega k_0 t}\, dt = \frac{1}{T}\int_{s}^{s+T} x(t)\, e^{-jk\omega_0 t}\, dt$$

Since $x(t)$ is zero outside the range $s \le t \le s+T$

$\Rightarrow a_k = \frac{1}{T}\int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt$ Comparing to the Fourier transform,

$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ we conclude $a_k = \frac{1}{T} X(j\omega)\Big|_{\omega = k\omega_0}$

where $X(j\omega)$ is the fourier Transform of $x(t)$. This states that the Fourier coefficients of $\tilde{x}(t)$ are proportional to samples of the Fourier transform of one period of $\tilde{x}(t)$. This alludes to the concept of duality.

Solving for the first component, $a_0 = \frac{1}{T}\int_{-D/2}^{D/2} x(t)\, e^{0}\, dt$. This is an average. We essentially are adding up all the infintesimal components where the square pulse exists as 1.

$$\Rightarrow a_0 = \frac{t}{T}\Big|_{-D/2}^{D/2} = \frac{1}{T}\left(\frac{D}{2} - \left(\frac{-D}{2}\right)\right) = \frac{D}{T} \Rightarrow a_0 = \frac{D}{T}, \quad K = 0$$

$$a_k = \frac{1}{T}\int_{-D/2}^{D/2} e^{-jk\omega_0 t}\, dt = \frac{-1}{jk\omega_0 T} e^{-jk\omega_0 t}\Big|_{-D/2}^{D/2} = \frac{-1}{jk\omega_0 T}\left[e^{+jk\omega_0\left(\frac{D}{2}\right)} - e^{-jk\omega_0\left(\frac{D}{2}\right)}\right]$$

multiplying by one in the form $\frac{2}{2}$ leads us to having $\frac{-2}{j2k\omega_0 T}[\ldots]$

$\Rightarrow a_k = \frac{-2}{k\omega_0 T}\,\frac{e^{+jk\omega_0\left(\frac{D}{2}\right)} - e^{-jk\omega_0 D/2}}{2j}$ converting with eulers identity, back to the sinusoidal form

$$\Rightarrow a_k = \frac{-2}{k\omega_0 T}\sin(k\omega_0 D/2)$$

$$a_k = \begin{cases} \frac{D}{T}, & K = 0 \\ \frac{-2}{k\omega_0 T}\sin(k\omega_0 D/2), & K \neq 0 \end{cases}$$

If we were to offset the periodic function $\tilde{x}(t)$ we would end up with the same fourier components. This is because we only care about the integration of one period, for our purpose, whether it is offset or not, the same amount gets covered in a period. To account for the change in voltage, we can simply scale these constants by an amplitude.

So returning to our example, the average voltage will be

$V_{oi} = \frac{1}{T}\cdot D \cdot V_d$ where we were given $V_d = 20V$ and $D = 0.75$

Now if we look at our argument D is the portion of the period we have on. If we had a full duty-cycle $D = 1$, we would have a dc of $V_d = 20V$. and we know the average is linear with D So we can use D to represent the fraction of time the period is filled over one period.

$\Rightarrow T = 1$, $V_{oi,avg} = D \cdot V_d = 20 \cdot 0.75 = 15V$

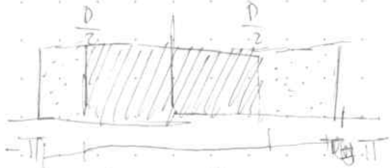
using the same methodology we can calculate the fourier components.

$a_k = \frac{-2}{k\omega_0 T}\sin(k\omega_0 D/2)$ where we recognize $\omega_0 T$ as 2π

$$\Rightarrow a_k = \frac{-2}{2\pi K}\sin(2\pi K D/2) \Rightarrow a_k = \frac{-1}{\pi K}\sin(\pi K D)$$

So, for the voltage $a_k = \frac{V_d}{\pi K}\sin(\pi K D)$

This essentially means that the period T is over a value of π, the half a natural cycle, however π time is unable to satisfy the duration.



We want the integration over the full Range $0-T$ including the empty Space

In our integration we missed this fact by forgetting $\omega_0 T = 2\pi$

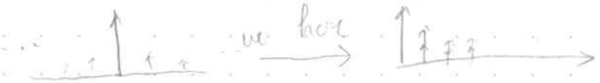
$$\Rightarrow a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int x(t)\, e^{-jk(2\pi/T)\cdot t}\, dt$$

So, Reworking the integral $\Rightarrow a_k = \frac{V_d}{T}\cdot\int_{-D/2}^{D/2} e^{-jk(2\pi/T)\cdot t}\, dt$

$$\Rightarrow a_k = \frac{V_d}{T}\cdot\frac{-T}{jk2\pi}\, e^{-jk(2\pi/T)\cdot t}\Big|_{-D/2}^{D/2}$$

$$\Rightarrow a_k = \frac{-V_d}{\pi}\,\frac{e^{-jk\frac{\pi D}{T}} - e^{+jk\frac{\pi D}{T}}}{2j} = \frac{V_d}{\pi}\,\frac{e^{jk\frac{\pi D}{T}} - e^{-jk\frac{\pi D}{T}}}{2j} = \frac{V_d}{\pi}\sin\left(\frac{k\pi D}{T}\right)$$

[illegible] answer that we [illegible] for [illegible] coefficient for inverse frequency, we can Rectify these frequencies by choosing only the positive values of k components and doubling the Results, so instead of having

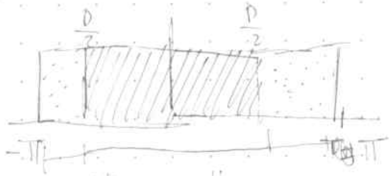


So by choosing our symmetry this way

$$a_k = \frac{2V_d}{\pi k}\sin(\pi k D), \quad k \neq 0 \quad \Big| \quad a_0 = DV_d \quad k = 1$$

$\therefore$ $a_k = 15V, 9V, 6.37V, 3V, 0V, 1.8V, \cdots$ for $k = 0 \ldots 5 \cdots$

This is because the area under ⋀ is the same as ⊥ so the height gets doubled.



We want the integration over the full Range 0–π including the empty Space

In our integration we missed this fact by forgetting $\omega_0 T = 2\pi$

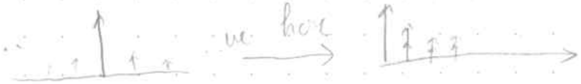
$$\Rightarrow a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int x(t)\, e^{-jk(2\pi/T)\cdot t}\, dt$$

So, Reworking the integral $\Rightarrow a_k = \frac{V_d}{T}\cdot\int_{-D/2}^{D/2} e^{-jk(2\pi/T)\cdot t}\, dt$

$$\Rightarrow a_k = \frac{V_d}{T}\cdot\frac{-T}{jk2\pi}\, e^{-jk(2\pi/T)\cdot t}\Big|_{-D/2}^{D/2}$$

$$\Rightarrow a_k = -\frac{V_d}{\pi}\,\frac{e^{-jk\frac{\pi D}{T}} - e^{+jk\frac{\pi D}{T}}}{2j} = \frac{V_d}{\pi}\,\frac{e^{jk\frac{\pi D}{T}} - e^{-jk\frac{\pi D}{T}}}{2j} = \frac{V_d}{\pi}\sin\left(\frac{k\pi D}{T}\right)$$

[illegible] that we [illegible] coefficient for inverse frequency, we can Rectify these frequencies by choosing only the positive values of k components and doubling the Results, so instead of having

K ... 

So by choosing an symmetry this way

$a_k = \frac{2V_d}{\pi k}\sin(\pi k D)$, $k \neq 0$ | $a_0 = DV_d$

$\therefore a_k = 15v, 8v, 6.37V, 3V, 0V, 1.8v, \ldots$ for $k = 0 \ldots 5 \ldots$

This is because the area under is the same as so the height gets doubled.

Mikhail Tkachenko

again continuing with the last problems book, 1-3

we find that the fourier components are

~~$a_k = \overset{-\infty}{\leftarrow} \dots 1.8V, 0V, 3V, 6.37V, +9V,$~~ 15V

for $k = \cdots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots$

$a_k = 0.9V, 0V, 1.5V, 3.185V, 4.5V, 15V, 4.5V, 3.185V, 1.5V, 0V, 0.9V.$

for $-\infty < k < \infty$

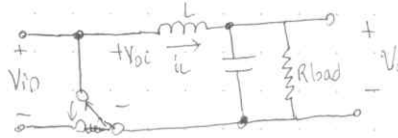
which was ~~found~~ derived from $a_k = \begin{cases} \frac{V_d}{\pi k} \sin(\pi k D) & k \neq 0 \\ V_d \cdot D & k = 0 \end{cases}$

This is for the bilateral fourier components.
For the unilateral fourier components we simply double the non-dc, average component as we fold the negative k components with the positive.

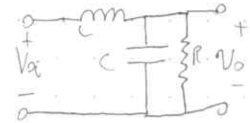
$$\Rightarrow a_{k,\text{unilateral}} = \begin{cases} \frac{2 \cdot V_d}{\pi k} \sin(\pi k D) & k > 0 \\ V_d \cdot D & k = 0 \end{cases}$$

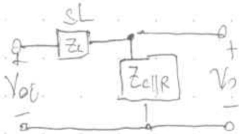
$$\boxed{\begin{array}{l} \text{for duty } D = 0.75, \text{ and } V_d = 20V. \qquad k \geq 0 \\ \Rightarrow |a_k| = 15V, 9.0V, 6.37V, 3.0V, 0.0V, 1.8V, 2.12V \end{array}}$$

1-4
In problem 1-3, the switching frequency $f_s = 300$ kHz and the resistive load draws 240 W. The filter components corresponding to Fig. 1-4a are $L = 1.3\,\mu H$ and $C = 50\,\mu F$. Calculate the attenuation in decibels of the ripple voltage in V_{oi} at various harmonic frequencies. (Hint: To calculate the load resistance, assume the output voltage to be constant DC, without any ripple.



$\longrightarrow$

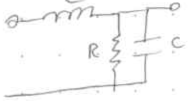




$\Rightarrow V_{oi} \longrightarrow \boxed{H(s)} \longrightarrow V_o$ where $H(s)$ is voltage circuit division

$$\Rightarrow H(s) = \frac{V_o(s)}{V_{oi}(s)} = \frac{R \parallel \left(\frac{1}{Cs}\right)}{sL + \left[R \parallel \left(\frac{1}{Cs}\right)\right]}$$

~~Using~~ we have the transfer function $H(s)$ of the circuit



$V_0 = 20V$, $D = 0.75$, $f_s = 300kHz$,
$L = 1.3\mu H$, $C = 50\mu F$,

where $V_{oi}(t)$ was our duty cycled $V_d \cdot \tilde{x}(t)$ from before.

Calculating $H(s) = \dfrac{R \| \frac{1}{sC}}{sL + [R \| (\frac{1}{sC})]}$ where $R \| \frac{1}{sC} = \dfrac{\frac{R}{sC}}{R + \frac{1}{sC}}$

$$R \| \frac{1}{sC} = \frac{R}{sC}\left(\frac{1}{R + \frac{1}{sC}}\right) = \frac{R}{sC}\left(\frac{sC}{RsC + 1}\right) = \frac{R}{RC \cdot s + 1}$$

$$\Rightarrow H(s) = \frac{\frac{R}{sC} \cdot \frac{1}{R + \frac{1}{sC}}}{sL + [R \| (\frac{1}{sC})]} = \frac{\frac{R}{RC \cdot s + 1}}{sL + \frac{R}{RC \cdot s + 1}}$$

$$H(s) = \frac{\frac{R}{RC \cdot s + 1}}{\frac{sL(RC \cdot s + 1) + R}{RC \cdot s + 1}} = \frac{R}{sL \cdot (RC \cdot s + 1) + R} = \frac{R}{(RLC)s^2 + Ls + R}$$

For our harmonic components, as before, we sample at a period $k\omega_s$ $T \cdot a_k$ and since we are assuming a steady state σ is $0 \longrightarrow s = j\omega = j(k2\pi f_s)$

$\therefore$ To calculate the attenuation ~~then~~ for each component we take $20\log_{10}|H(s)|$

Lets find the Resistance R. Pout is assumed constant.

$P_{out} = \frac{V_o^2}{R} \Rightarrow R = \frac{V_o^2}{P_o}$ The average output dc voltage we calculated to be 15V. $\rightarrow V_{o,avg} = V_o = 15V$

We were given the power draw to be a constant 240W

$$R = \frac{V_o^2}{P_{out}} = \frac{15^2}{240} = 0.9375\,\Omega.$$

$$\therefore H(s) = \frac{V_o(s)}{V_{oi}(s)} = \frac{R}{(RLC) \cdot s^2 + L \cdot s + R} = \frac{0.9375\,\Omega}{6.094e{-}11 \cdot s^2 + 1.3e{-}6 \cdot s + 0.9375}$$

The first sampling ~~component~~ frequency gives our ~~fourier co~~ harmonic attenuations at the specific frequencies

$s = j\omega_s = j(2\pi \cdot h \cdot f_s)$ where our frequency was 300khz

$$H(\omega) = R\left(\frac{1}{-RLC(2\pi \cdot h \cdot f_s)^2 + 2\pi \cdot h \cdot f_s \cdot L + R}\right)$$

Tabulating the numbers in a computer. Refer to problem_1-4.py

As a Result we have our harmonic components.

h	$20\log_{10}\lvert H(s)\rvert$
0	0
1	-47.1333db
2	-59.252db
3	-66.31813654db
4	-71.32574325 db
5	-75.20792118 db
6	-78.37891582 db
7	-81.05941002 db
8	-83.38102631 db
9	-85.42861796 db

at $f_s = 300$khz.

where $H(\omega_s, h) = \dfrac{R}{-RLC \cdot h^2\omega_s^2 + R + \omega_s \cdot L \cdot h}$

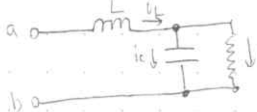
Adding the numbers, we get

$$H(f_s, h) = \frac{0.9375}{-2.4057e{-}09\, f^2h^2 + 8.168e{-}06\, f \cdot h \cdot 4\ 0.9375}$$

	db
1	-47.2
2	-59.3
3	-66.34
4	-71.4
5	-75.2

Minor correction I forgot the i component.

1-5 In problem 1-4, assume the output voltage to be a pure dc = 15V. Calculate and draw the voltage and current associated with the filter inductor L, and the current through C. Using the capacitor current obtained above, estimate the p-p voltage across C ~~when~~ which we previously assumed to be zero.
(Hint: Note that under steady state conditions, the average value of the current through C is zero.



Using Kirchoff's current law, we ~~assume~~ know that $\boxed{i_L = i_C + i_R}$
If we assume the output voltage to be constant, we find that $V_o = 15 = I \cdot R$
and we calculate the power sink in the Resistor
$I_R = \frac{240\ W}{15V} = 16\ A.$

This essentially enforces the current in the Resistor to be constant.

$I_R = 16A,\ V_R = 15,\ R = 0.9375\,\Omega$

So any Ripple current and voltage will be exchanged only through the inductor L and the capacitor C.

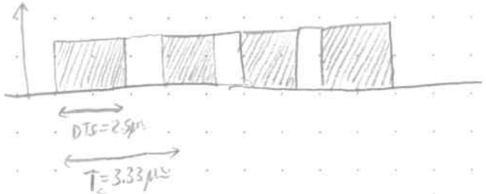
This also means the average Capacitor current will be zero, and the average inductor current will be

i_R $\therefore$ On average $i_L = I_L = I_R = i_R \Rightarrow I_C = 0$
where I_R is fixed at 16 amps due to the problems assertion.

The switching frequency was given as $f_s = 300\,kHz$
The period is thus $T = \frac{1}{f_s} = 3.3\bar{3}\,\mu s$

The duty period with a duty of $D = 0.75$ is calculated as

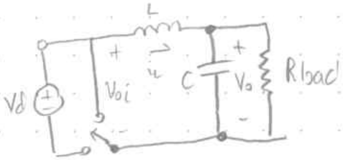
$D \cdot T = 0.75 \cdot 3.3\bar{3}\,\mu s = 2.5\,\mu s$



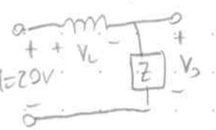
During $0 \le t < 2.5\mu s$ we have an active Duty Cycle and during $2.5\mu s < t < T$ we are off for one period

$0 < t_{on} < 2.5 < t_{off} < T_s$

Redrawing the circuit,



When the switch is on $V_d = 20$, and is the same voltage as V_{oi}. The output voltage was asserted by the problem to be 15.



Using Kirchow voltage law,

$-V_d + V_L + V_o = 0 \Rightarrow V_d - V_o = V_L$

$\therefore V_L = V_d - V_o = 20 - 15V = 5V$

a Constant 5 volts is asserted in the On State.

I am assuming 5 volts always across the inductor in the on state since by definition of an ideal voltage Source, the voltage is asserted by this Source.

When in the on State $V_L = 5V$.
When in the off state, no power is flowing and our superior ideal Resistor takes the current of the capacitor making the ~~current~~ Voltage of the Capacitor 0.

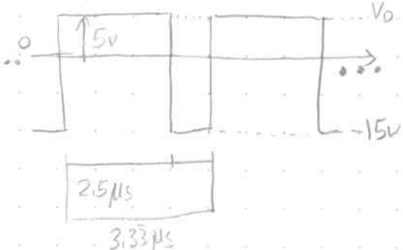
However in Reality, we will have a transient
So $V_L = -\left(C\frac{dv}{dt} \cdot t + 15\right)$

Notice how we spike up to 15V instantaneously when the switch is thrown, shorting the voltage source.

This is still somewhat of an idealization though. The circuit ~~capp~~ dampens the transient as it prevents it from changing instantly.

However, we can continue this logic and assert that power is always drawn from the power supply, even when switched off. This idealization with strong assertions is better comforted with the fact that these are very small periods of time, and another wave will soon fill its place

$\therefore$ The ideal switching inductor voltage is Represented below.



In Reality we will have [illegible] ... [illegible] due to the [illegible] Capacitor C.

We can Calculate the inductor current in the inductor

$V_L = L\frac{di}{dt} + k,\qquad i_L = i_C + i_R = \frac{1}{L}\int_0^{t_{on}} v(\tau)\,d\tau + i_{C_0}$

If we assume [illegible] Current [illegible] in the [illegible] of this Ideal

$V_L = L\frac{di}{dt}(t) + V_{L0}$ gives us the voltage with changing current. We know the inductor voltage. Let us integrate it to get the current for the [illegible] of this [illegible]

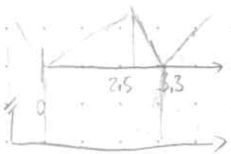
$\Rightarrow \frac{di_L}{dt}(t) = \frac{V_L}{L} - V_{L0}$ integrating both sides by t

$$i_L = \int \frac{V_L}{L}\,dt + V_{L0} = \frac{1}{L}\int V_L\,dt + \int V_{L0}\,dt$$

We have our interval as from $0 \rightarrow 2.5\mu$, the duty cycle we are on.

evaluating the starting current at $t=0$?

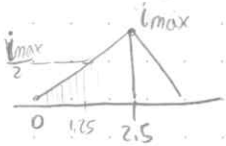
$\frac{di_L}{dt}(t) = \frac{V_L^{(t)} - V_0}{L}$ we have a constant 5 volts across the inductor when the inductor is turned on.



We already know the average inductor current will be $\frac{V_{0}}{R} = 16$ amps

If we asume the inductor current Ripple as a triangle and a square we remember we derived the average voltage as $V_{avg} = \frac{V_{dmin} + V_{dmax}}{2}$

Just calculating the Δ component, we assert the initial condition current being δi.



The p-p voltage is gotten by integrating the voltage with respect to time

$$i_L = \int_0^{2.5e\text{-}6} V_L \cdot dt = \frac{5 \cdot 2.5e\text{-}6}{1.3e\text{-}6}$$

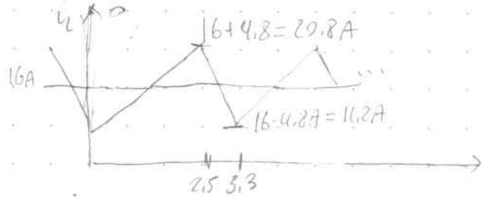
$i_L = 9.6A$

The intuition is this: If the voltage in an inductor is constant, the current is constantly changing.

$V_L = \frac{L\,di(t)}{dt}$

So the current spanned during this on time is 9.6A.

We already know the average. So the Ripple, centered at 16 amps swings 4.8 Amps.

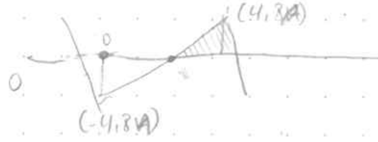


Since we recognize that when the inductor current is not zero, and we have had the Resistor current asserted constant. The Capacitor current must change in proportion to the inductor current.

$v_L = \frac{L\,di(t)}{dt} + V_{L0}$

$\Rightarrow i_L(t) = i_C(t) + I_R \Rightarrow i_C(t) = i_L(t) - I_R$

We know the average capacitor current as follows



we know the current of the Capacitor will be

$i_C = C\frac{dv}{dt}$

So we have a maximum magnitude of current running through the Capacitor to be 4.8A.

the voltage spanned by the changing voltage ends up being proportional to the area under the triangle, with a base $\frac{T}{2}$

$$\therefore \Delta V_C = \frac{1}{C}\ \text{integrate current} \rightarrow \frac{1}{C}\int_{\frac{T}{2}}^{T} i_C \cdot dt$$

$$= \frac{1}{C} \cdot \frac{1}{2}\,\frac{T}{2} \cdot 4.8A = \frac{1}{50e\text{-}6} \cdot \frac{1}{4} \cdot 4.8 \cdot 3.3e\text{-}6$$

$\Rightarrow \Delta V_C = 0.0792v \approx 80mV$

problem 1-6

Considering only the switching frequency component in v_{oi} in problems 1-3 and 1-4, calculate the peak to peak ripple in the output voltage across C_o. Compare the result with that obtained in problem 1-5.

The switching ~~frequency~~ Ripple of a square wave is the first frequency component of its Fourier space, excluding the

The Fourier component has been calculated to have a voltage of 9V.

So the first component of the input voltage $v_{oi_1} = 9.023V$

Next, we calculated the attenuation at the first harmonic to be -47,13db.

$$\Rightarrow 20\log_{10}\left|\frac{V_o(s)}{V_{oi}(s)}\right| = -47{,}133db = 20\log_{10}|H(s)|$$

However, I can do better, since I already know the transfer function at the first component as

$$H_1(f = 300khz) = -0.004348 - 4.9426e{-}05 \cdot j$$

The magnitude is calculated to be 0.0043485

So the voltage span is the first component

$$(\hat{V}_{o_1}) = 0{,}004348 \cdot (\hat{V}_{oi})_1 = 39mV$$

To get the p-p voltage of the cap we double this number since we also have the segment which goes negative.

$\Rightarrow$ Therefor $\Delta V_{c\,p\text{-}p} = 2 \cdot H_1(s) \cdot (V_{oi})_1 = 0{,}07830076V$

The span is $\boxed{78{,}3mV}$

Comparing to the previous result of 0,0792, we are very close. It is only a 0,1% difference !!!

17 Reference 4 refers to a U.S. Department of Energy Report that estimated that over 100 billion kWh/year can be saved in the united States by various energy conservation techniques applied to the pump driven systems. Calculate how many 1000 MW generating plants running constantly supply this wasted energy, which could be saved, and (b) the savings in dollars if the cost of electricity is 0.1 \$/kWh.

[4] N. Mohan and R.J. Ferraro, "Techniques for energy conservation in AC Motor driven Systems," EPRI Final Report EM-2037 project 1201-1213, September 1981

(a) There is 100 billion kWh saved in a year

$\Rightarrow 100 \cdot 10^9 \cdot 10^3$ W-hr

a year has 8760 hours in a year

$$\frac{100 \text{ billion kWh}}{8760h} = 11{,}4 \text{ million watts}$$

$$\frac{1 \text{ Kwh}}{1000 \text{ Wh}}$$

This is about the same as 4-5 power plants.

The savings off thes 100 billion kWhs is simple

$$\text{Saving} = 100e9 \text{ kWh} \frac{0{,}1\$}{\text{KWh}} = 10 \cdot 10^9 \$$$

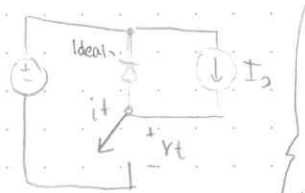
which is 10 Billion dollars.

Chapter 2 Problems

2-1 The data sheets of a switching specify the following switching times corresponding to the linearized charachteristics shown in Fig. 2.6b for clamped-inductive switchings:

$t_{ri} = 100ns$ $t_{fv} = 50ns$ $t_{rv} = 100ns$ $t_{fi} = 200ns$

Calculate and plot the switching power loss as a function of frequency in a range 25-100 khz, assuming $V_d = 300V$ and $I_o = 4A$ in the circuit.



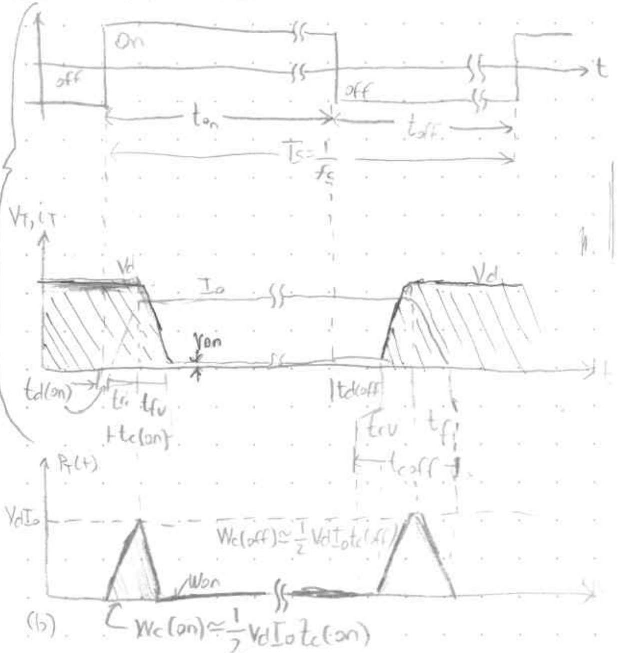


Fig 2-6 Generic switch switching charachteristics (linearized): (a) Simplified clamped-inductive-switching circuit, (b) Switch waveforms (c) instantaneous switch power loss.

In order to consider power dissipation in a semiconductor device, a controllable switch is connected in the simple circuit shown in Fig. 2.6A. This circuit models a very commonly encountered situation in power electronics; the current flowing through a switch must also flow through some series inductance(s). This circuit is similair to the circuit of Fig. 1-3b, which was used to introduce switch mode power electronic circuits. The dc current source approximates the current that would actually flow due to inductive energy storage. The diode is assumed to be ideal because our focus is on the switch charachteristics, though in practice the diode reverse recovery current can significantly affect the stresses on the switch.

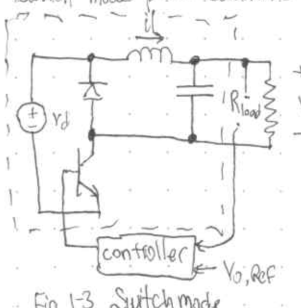


Fig 1-3 Switch mode dc power supply

when the switch is on, the entire current I_o flows through the switch and the diode is reverse biased. When the switch is turned off, I_o flows through the diode and a voltage equal to the input voltage appears across the open switch. assuming zero voltage drop across the ideal diode. Figure 2-6b shows the waveforms for the current through the switch and ~~across~~ the voltage across the switch when it is being operated at a repetition rate or switching frequency of $f_s = 1/T_s$, with T_s being the switching time period. The switching waveforms are represented by linear approximations to the actual waveforms in order to simplify the discussion.

When the switch has been off for a while, it is turned on by applying a positive control signal to the switch, as is shown in Fig 2-6-B ~~shows the waveforms for the current through the switch and the voltage across the switch when it is being operated at a repetition rate or switching frequency of $f_s = 1/T_s$, with T_s being the switching time period. The switching waveforms are represented by linear approximations to the actual waveforms in order to simplify the discussion.~~

~~When the switch has been off for a while~~

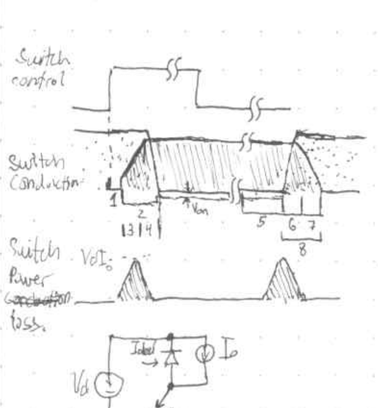
During the turn-on transition of this generic switch, the current build up consists of a short delay time $t_{d(on)}$ followed by the current rise time t_{ri}. Only after the current I_o flows entirely through the switch can the diode become reverse biased and the switch voltage fall to a small on state value of V_{on} with a voltage fall time of t_{fv}. The waveforms in Fig 2-6b indicate that large values of switch voltage and current are present simultaneosly during the crossover interval $t_{c(on)}$, where:

$$t_{c(on)} = t_{ri} + t_{fv} \quad (2-1)$$

The energy dissipated in the device during this turn-on transition can be approximated from Fig. 2-6c as:

$$W_{c(on)} = \frac{1}{2} V_d I_o t_{c(on)}$$

where it is recognized that no energy dissipation occurs during the turn-on delay interval $t_{d(on)}$.



1 → $t_{d(on)}$
2 → $t_{c(on)}$
3 → t_{ri}
4 → t_{fv}
5 → t_{doff}
6 → t_{rv}
7 → t_{fi}
8 → t_{coff}

Once the switch is fully on, the on-state voltage V_{on} will be on order ~~of magnitude a volt~~ of a volt or so depending on the ~~service~~ device, and it will be conducting a current I_o. The switch remains in conduction during the interval t_{on}, which in general is much longer than the turn-on and off transision times. The energy dissipation W_{on} in the switch during this on-state interval can be approximated as

$$W_{on} = V_{on} I_o t_{on} \quad (2-3)$$

where $t_{on} \gg t_{c(on)}, t_{c(off)}$.

In order to turn the switch off, a negative control signal is applied to the control signal on the switch. During the turn-off transition period of ~~the switch~~ generic switch, the voltage build up consists of a turn-off delay time $t_{d(off)}$ and a voltage rise time t_{rv}. Once the voltage reaches its final value of V_d. (See Fig. 2-6a), the diode can become forward biased and begin to conduct current. The current in the switch falls to zero with a current fall time t_{fi} as the current I_o commutates from the switch to the diode. Large values of switch voltage and switch current occur simutonessly during the crossover interval $t_{c(off)}$, where:

$$t_{c(off)} = t_{rv} + t_{fi} \qquad (2\text{-}4)$$

The energy dissipated in the switch during this turn-off transition can be written, using Fig. 2-6c, as:

$$W_{c(off)} = \frac{1}{2} V_d I_o t_{c(off)} \qquad (2\text{-}5)$$

where any energy dissipation during the turn-off delay interval $t_{d(off)}$ is ignored since it is compared to $W_{c(off)}$.

The instantaneous power dissipation $p_T(t) = v_T i_T$ plotted in Fig. 2-6c makes it clear that a large instantaneous power dissipation occurs in the switch during the turn-on an turn-off intervals. There are f_s such turn-on and turn-off transitions per second. Hence the average switching power loss P_s in the switch due to these transitions can be approximated from Eqs. 2-2 and 2-5 as:

$$P_s = \frac{1}{2} V_d I_o f_s \left(t_{c(on)} + t_{c(off)}\right) \qquad (2\text{-}6)$$

This is an important result because it shows that the switching power loss in a semiconductor switch varies linearly with the switching frequency and the switching times. Therefore, if devices with short switching times are available, it is possible to operate at high switching frequencies in order to reduce filtering requirements and at the same time keep the switching power loss in the device from being excessive.

The other major contribution to the power loss in the switch is the average power dissipated during the on-state P_{on}, which varies in proportion to the on state voltage. From Eq. 2-3, P_{on} is given by:

$$P_{on} = V_{on} I_o t_{on} / T_s \qquad (2\text{-}7)$$

which shows that the on-state voltage in a switch should be as small as possible.

The leakage current during the off state (switch open) of controllable switches is negligibly small, and therefore the power loss during the off state can be neglected in practice. Therefore, the total average power dissipation P_T in a switch is the sum of P_s and P_{on}.

From the considerations discussed in the preceding paragraphs, and following characteristics in a controllable switch, are desireable:

(1.) Small leakage current in the off state.

(2.) Small on state voltage V_{on} to minimize on-state power loss.

(3.) Short turn-on and turn-off times. This permits the usage at high frequencies

(4.) Large forward- and reverse-voltage-blocking capability. This will ~~need to~~ minimize the need for series connection of several devices, which complicates the control and protection on the switches. Moreover, most of the devices types have a minimum on-state voltage regardless of their blocking voltage rating. A series connection of several such devices would lead to a higher total on-state voltage and hence higher conduction losses. In most (but not all) converter circuits, a diode is placed across the controllable switch to allow the current to flow in the reverse direction. In these circuits, controllable switches are not required to have any significant reverse-voltage blocking capability.

(5) High on state current rating. In high-current applications, this would minimize the need to connect several devices in parallel, thereby avoiding the problem of current sharing.

(6.) Positive temperature coefficient of on-state resistance. This ensures that paralleled devices will share the total current equally.

(7) Small control power required to switch the device. This will simplify the control circuit design.

(8) Capability to withstand rated voltage and rated current simultaneously while switching. This will elimenate the need for external protection (snubber) circuits across the device.

(9) Large $\frac{dv}{dt}$ and $\frac{di}{dt}$ ratings. This will minimize the need for external circuits otherwise needed to limit dv/dt and di/dt in the device so that it is not damaged.

We should note that the clamped-inductive-switching circuit of Fig. 2-6a results in higher switching power loss and puts higher stresses on the switch in comparison to the resistive switching circuit shown in problem 2-2 (Fig P2-2)

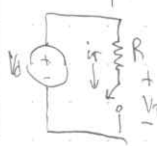


Figure P2-2

Now we will continue with the problem after that lengthy review of considerations.

Given:

- $t_{ri} = 100ns$
- $t_{fv} = 50ns$
- $t_{rv} = 100ns$
- $t_{fi} = 200ns$

The portion of time when the switch is is consuming power is given by $t_c(on)$. The time of rising current is given by t_{ri}, and the time of falling voltage, t_{fv}. When these dynamics are en play, before the switch has settled, a larger amount of power is drawn, for this duration. The time during this power spike is trivially calculated as:

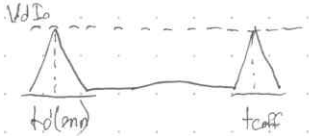
$$t_{c(on)} = t_{ri} + t_{fv} = 100ns + 50ns = 150ns$$

The time when the switch closes is given by $t_c(off)$ and it also consumes power during this switch. The duration is given by ~~t_c(off)~~ t_{rv}, the time of the rising voltage, and ~~once~~ once the voltage charge accumulates the current drains in the diode. ~~Now~~ t_{fi} is the time of the falling current ~~duration~~. Combined these give us $t_{c(off)}$:

$$t_c(off) = t_{rv} + t_{fi} = 100ns + 200ns = 300ns$$

Notice how the off switch time is double the on switch time. This shows us how we cannot simply double the on time, and assume the off time is summetric.

The power can be represented as the area underneath the VI curve, which are triangular. We can approximate these as triangles. The peak power represents the height of both triangles.



The area of the first triangle is

$$P_{s,on} = \frac{1}{2}\underbrace{(V_d I_o)}_{height}\underbrace{t_c(on)}_{base}\cdot 1hz.$$

likewise the second triangle

$$P_{s,off}\ \frac{1}{2}(V_d I_o)\underbrace{t_c(off)}_{base}\cdot 1hz.$$

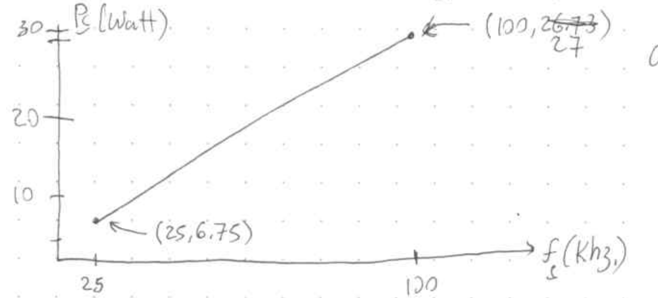
We can combine the common terms and thus get the power consumed during the switch as

$$P_s = \frac{1}{2}V_d \cdot I_o\left[t_c(on) + t_c(off)\right]\cdot f_s \quad (S\cdot\tfrac{1}{s})\text{ Watt.}$$

$$\Rightarrow P_s = \frac{1}{2}(300V)(4A)\cdot[450e\text{-}9]\cdot f_s \text{ Watt}$$

$$\Rightarrow P_s = 2.7\cdot 10^{-4} W\cdot f_s$$ which is 270 μW per cycle.

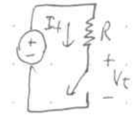
We were instructed to plot the power, and for our model, the power is consumed linearly with frequency. For every cycle we consume 270μW a cycle, so in a khz we will consume 270 mW. We were instructed to use the range 25khz to 100khz.



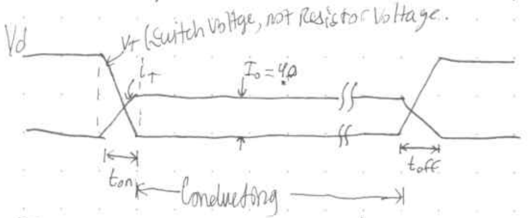
At 25khz we consume about 7 watt, at 100khz we consume is ~~about~~ ~~to~~ 27 Watt.

Whether this linear case is what happens or not is not to be determined by this model. This model is highly idealized and simple to fit the pedanticness of the textbook problem.

2-2 Consider the resistive switching circuit shown in Fig. P2-2. $V_d = 300V$, $f_s = 100khz$ and $R = 75\Omega$, so that the on state current is the same as in problem 2-1. Assume the switch turn-on time to be the sum of t_{ri} and t_{fv} in problem 2-1. Similarly, assume the turn off time to be the sum of t_{rv} and t_{fi}.



Assume linear voltage and current switching characteristics, plot the switch voltage and current and the switching power loss as a function of time. Compare the average power loss with that in problem 2-1.



The current consumed by the resistor is simply

$$I_o = \frac{V_d}{R} = \frac{300V}{75\Omega} = 4.0 \text{ Amps.}$$

this occurs when:

$t_{on} < t < t_{off}$

We again assume that

$t_{on} = t_{ri} + t_{fv} = 150ns$ &

$t_{off} = t_{rv} + t_{fi} = 300ns.$

~~The power consumption during this conducting time is 1.2 kW.~~

The power consumed by the resistor when on is 1.2KW.

The current is assumed to be linear along with the voltage for the transient periods between the conduction and off states.

The current and voltage is being represented linearly in this idealistic model such that

$$i_T = I_o\left(\frac{t}{t_{on}}\right)$$ where the fraction bounds are $(0 < t < t_{on})$

for the Range when the transient enables $[0<t<t_{on}]$

$i_T = I_o\,(t/t_{on})$, $V_T = V_d\,(1-\frac{t}{t_{on}})$

The voltage is also Represented As a linear drop in this model.

So the power is the VI product of the voltages for this time Range.

$$p_{T_{on}} = V_d I_o \cdot \left(1-\frac{t}{t_{on}}\right) t = (300\,\text{volts})(4\,\text{Amps})\left(t-\frac{t^2}{t_{on}}\right)\frac{t}{t_{on}}$$

The actual work energy is the integration of power.

$$\text{Work consumed to switch on} = \int_0^{t_{on}} (P_T)_{on}\cdot dt$$

$$\Rightarrow W_{on} = \frac{V_d I_o}{t_{on}} \int_0^{t_{on}} t - \frac{t^2}{t_{on}} \cdot dt = \frac{V_d I_o}{t_{on}}\left[\int_0^{t_{on}} t\,dt - \int_0^{t_{on}} \frac{t^2}{t_{on}}\,dt\right]$$

$$\cancel{W_{on} = \frac{V_d I_o}{t_{on}}\left[\frac{1}{2}t^2\Big|_0^{t_{on}} - \left(\frac{1}{3t_{on}}\right)t^3\Big|_0^{t_{on}}\right] = \frac{V_d I_o}{t_{on}}\left[\frac{1}{2}t_{on}^2 - \frac{1}{3}t_{on}^3\right]}$$

We remember our chain Rule for integration.

$$W_{on} = \frac{V_d I_o}{t_{on}} \left[\frac{1}{2}t^2 - \frac{1}{3}t^3 \cdot \frac{1}{t_{on}}\right]_0^{t_{on}} = \frac{V_d I_o}{t_{on}}\left[\frac{1}{2}t_{on}^2 - \frac{1}{3}\frac{t_{on}^3}{t_{on}}\right]$$

$$\Rightarrow W_{on} = V_d I_o \cdot t_{on}\left[\frac{1}{2}-\frac{1}{3}\right] = V_d I_o \cdot t_{on}\left[\frac{3}{6}-\frac{2}{6}\right] = \frac{V_d I_o\, t_{on}}{6}$$

$$\boxed{W_{on} = \frac{V_d I_o \cdot t_{on}}{6}}$$

Similarly, for the off switch, we can Represent the current linearly as $i_T = I_o(1-\frac{t}{t_{off}})$, $V_T = V_d(\frac{t}{t_{off}})$

Resulting in a power $P_{off} = V_d I_o \cdot \left(1-\frac{t}{t_{off}}\right)\cdot t \cdot \frac{1}{t_{off}}$

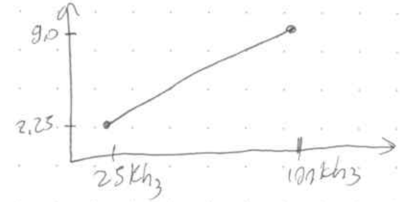
and the work ends up being an analogous Result, only t_{on} is Replaced with t_{off}. $\Rightarrow \boxed{W_{off} = \frac{V_d I_o}{6} \cdot t_{off}}$

The work consumed by the switch on and switch off factors out the common elements.

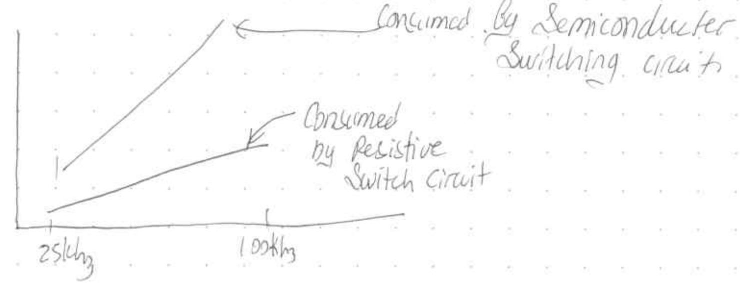
$$W_{switch} = W_{on} + W_{off} = \frac{V_d I_o}{6}\left[t_{off} + t_{on}\right]$$

the average power consumed is determined by dividing by the seconds a cycle occurs, or in other words multiplying by frequency. The higher frequency the better averages

So the average switching power loss as a function of switching frequency is simply: $P_s(f_s) = f_s \cdot W_s = \frac{V_d I_o}{6}\left[t_{on} + t_{off}\right] \overset{f_s}{=} \frac{300 \cdot 4}{6}\left[450e{-9}\right]$ Watt



The power consumption also varies linearly with frequency.



at 100 Khz, the P2-1 circuit gives 27 watt, this Resistive Switching circuit is 9 watt.

$$\frac{1}{2}\,v \cdot i \cdot t \cdot f \quad vs \quad \frac{1}{6}\,v \cdot i \cdot t \cdot f.$$

~~The Resistive Switch consumes a third of the power, however what I would consider more concerning is that the Resistive Switch stays on, burning current, while the diode probably has less Resistance when it is in steady state on. This is probably~~