Packages

Homework 8.1: Vehicle Dynamics and Stability

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8.1.1) Vehicle parameter list

For a 1998 Honda Civic.
Most vehicle parameters are found from
https://www.eng.auburn.edu/~dmbevly/mech4420/vehicle_params.pdf (see Vehicle No. 452)

Parameter	**Value**
Mass, m	$m = 1143\,\text{kg}$
Length of car, L	$L = 2.621\,\text{m}$
Distance to front wheels from CG, a	$a = 1.038\,\text{m}$
Yaw moment of inertia, I_{zz}	$I_{zz} = 1785\,\text{kg}\cdot\text{m}^2$
Cornering stiffnesses, $C_{\alpha f}$ & $C_{\alpha r}$	$C_{\alpha f} = C_{\alpha r} = 50990.278\,\dfrac{\text{N}}{\text{rad}}$
Proportioning of front to rear lateral load transfer, $\overline{p}$	$\overline{p} = 0.5$
Height to center of mass, H	$H = 0.513\,\text{m}$
Width of car, T	$T = 1.695\,\text{m}$
Roll gain, $\overline{\phi}$	$\overline{\phi} = -4\,\dfrac{\text{deg}}{\text{g}}$
Roll Frequency	$freq\_R = 1.5\,\text{Hz}$
Pitch Frequency	$freq\_P = 1\,\text{Hz}$
Brake limit (from HW6)	$BLIMIT = 1787.88\,\text{N}$
Braking ratio (from HW6), Q	$Q = 2.48$
Pitch gain, $\overline{\theta}$	$\overline{\theta} = -0.1\,\dfrac{\text{rad}}{\text{g}}$

8.1.2)

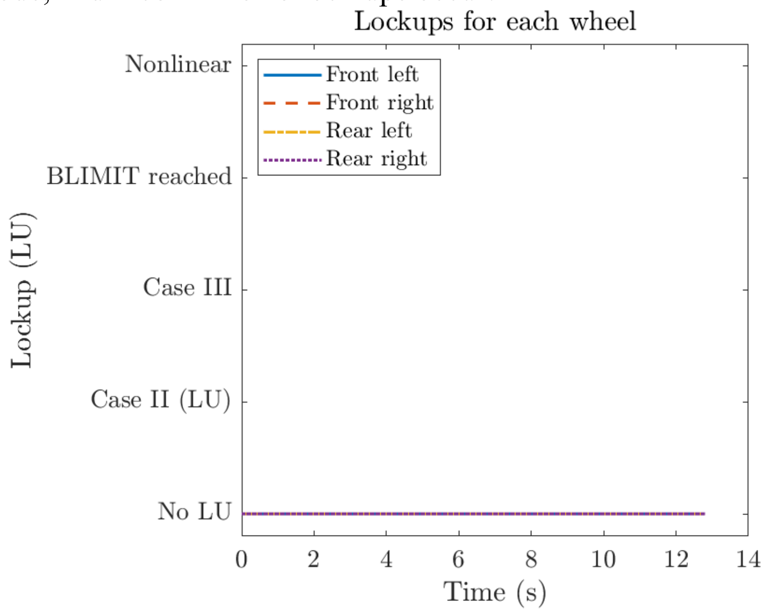
a)

Defining needed parameters:

$H := 0.513\,\text{m} : L := 2.621\,\text{m} : m := 1143\,\text{kg} : g := 9.81\,\frac{\text{m}}{\text{s}^2} : decel := 0.3 : a := 1.038\,\text{m} : b := L - a :$

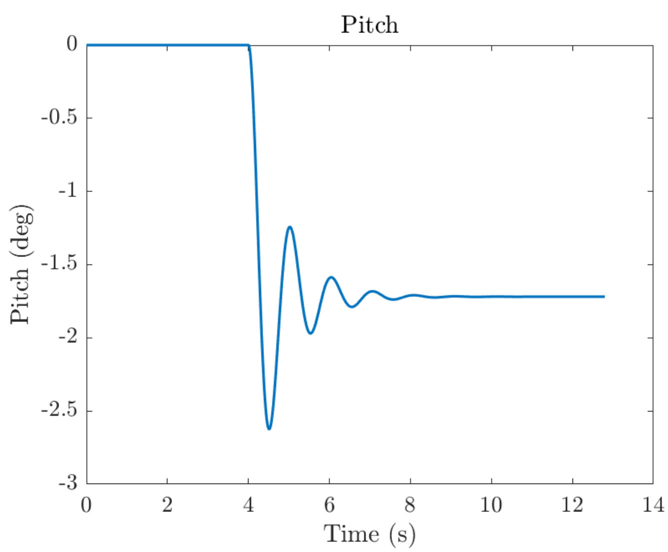
Note that it is concluded that the integration reaches steady state 3 seconds in, and so the braking begins after 3 seconds.

It is found that $\mu = 0.36$ just barely ensures no lock ups occur. The following figure is used to monitor what happens in the code, and it confirms no lock ups occur:



Thus the following plots are created for 60 mph 0.3g braking:

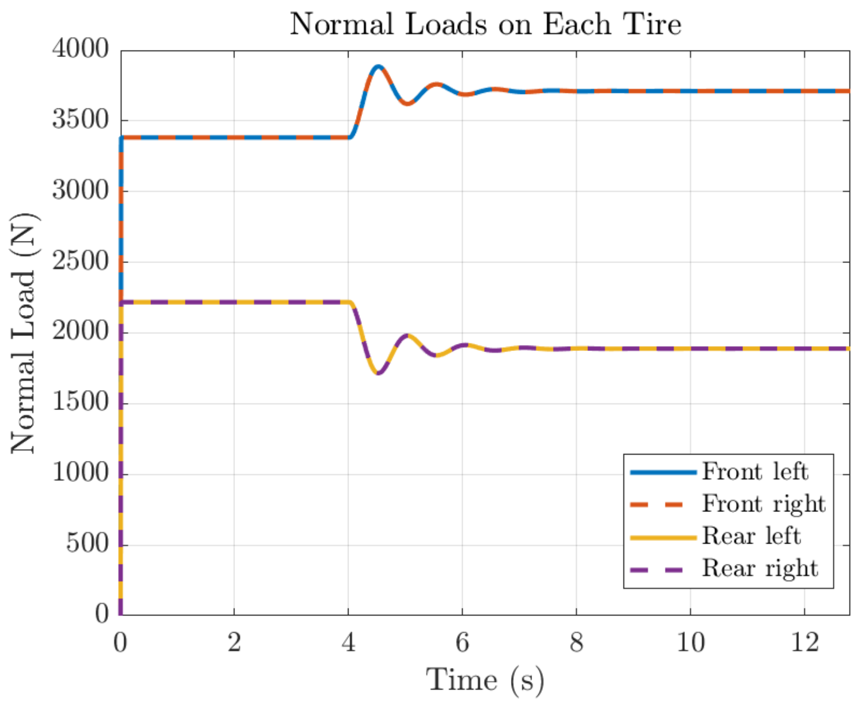
The pitch frequency:



At steady state the pitch is approximately 1.72 degrees which is ~ 0.03 radians. Since the steady state pitch is $0.1\,\frac{\text{rad}}{\text{g}}$, then 0.03 radians at 0.3g verifies the pitch gain. The pitch frequency can be calculated by looking at the duration between peaks or troughs (the period):

$$f := \frac{1}{5.53\,\mathrm{s} - 4.49\,\mathrm{s}} = 0.962\ \mathrm{Hz}$$, thus it approximately matches the pitch frequency of 1 Hz.

Normal loads:



The front and rear static load (for a single tire):

$$N_{f,\,stat} := \frac{m \cdot g \cdot b}{2 \cdot L} = 3386.095\ \mathrm{N}$$

$$N_{r,\,stat} := \frac{m \cdot g \cdot a}{2 \cdot L} = 2220.320\ \mathrm{N}$$

For each of the front tires the steady state load is:

$$\Delta N_f := N_{f,\,stat} + \frac{H \cdot m \cdot g \cdot decel}{2 \cdot L} = 3715.293\ \mathrm{N}$$

Which matches the graph:

$N_{ss,f} = 3711.51\ \mathrm{N}$

For each of the rear wheels the steady state load is:

$$\Delta N_r := N_{r,\,stat} - \frac{H \cdot m \cdot g \cdot decel}{2 \cdot L} = 1891.122\ \mathrm{N}$$

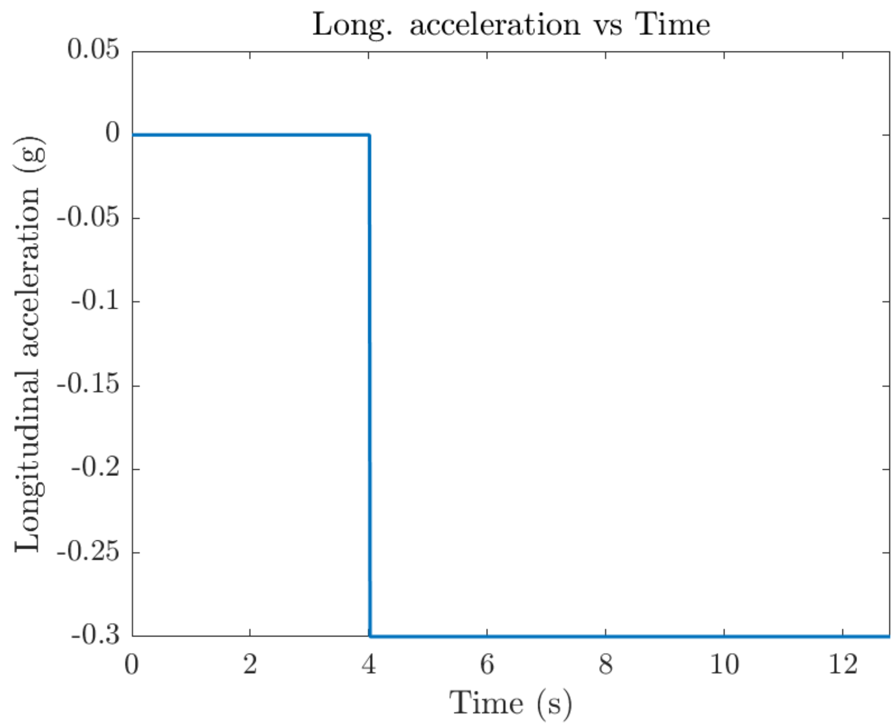
Which matches the graph aswell:

$N_{ss,r} = 1889.18\ \mathrm{N}$

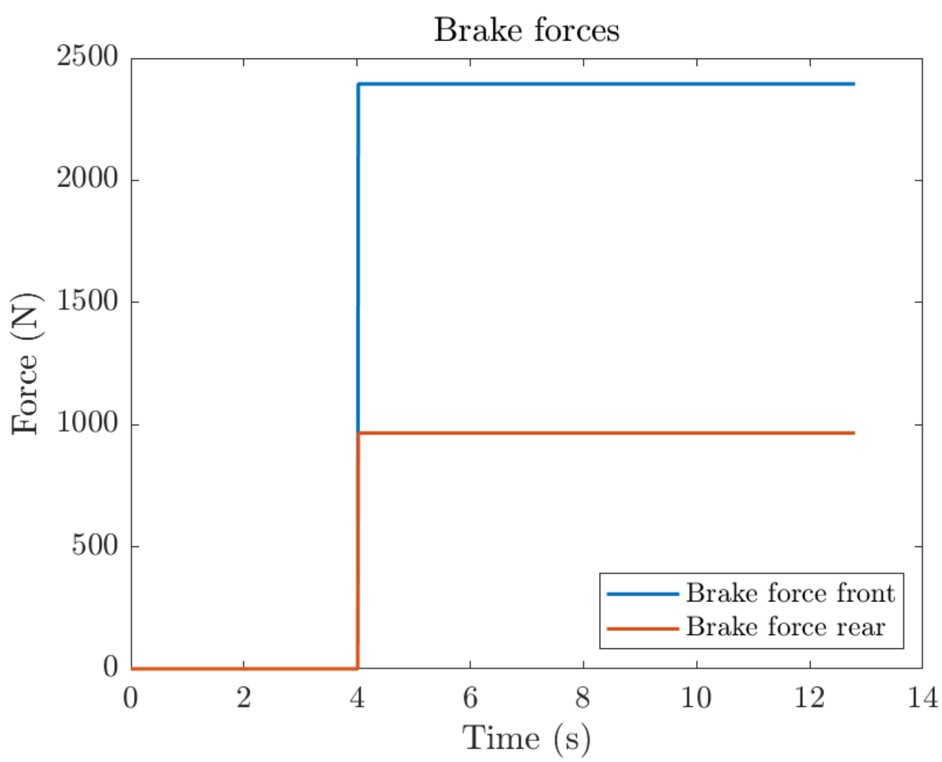
Thus the normal loads are confirmed to match the quasi static calculations.

Providing the remaining requested plots for longitudinal deceleration and brake forces:

Longitudinal deceleration:



Brake forces:



The plot of the normal forces have already been provided.

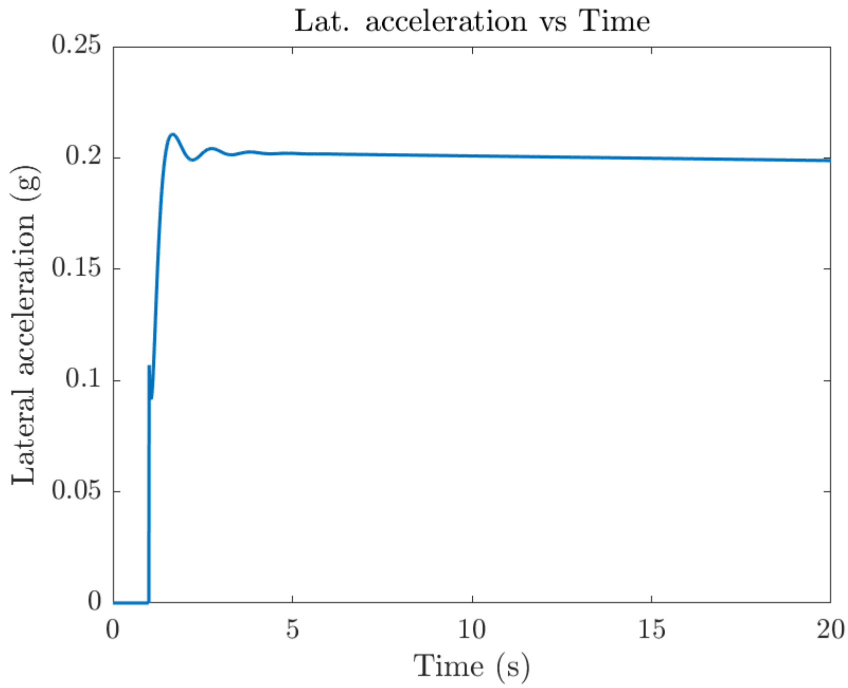
b)

Defining the additionally needed parameters for this task:

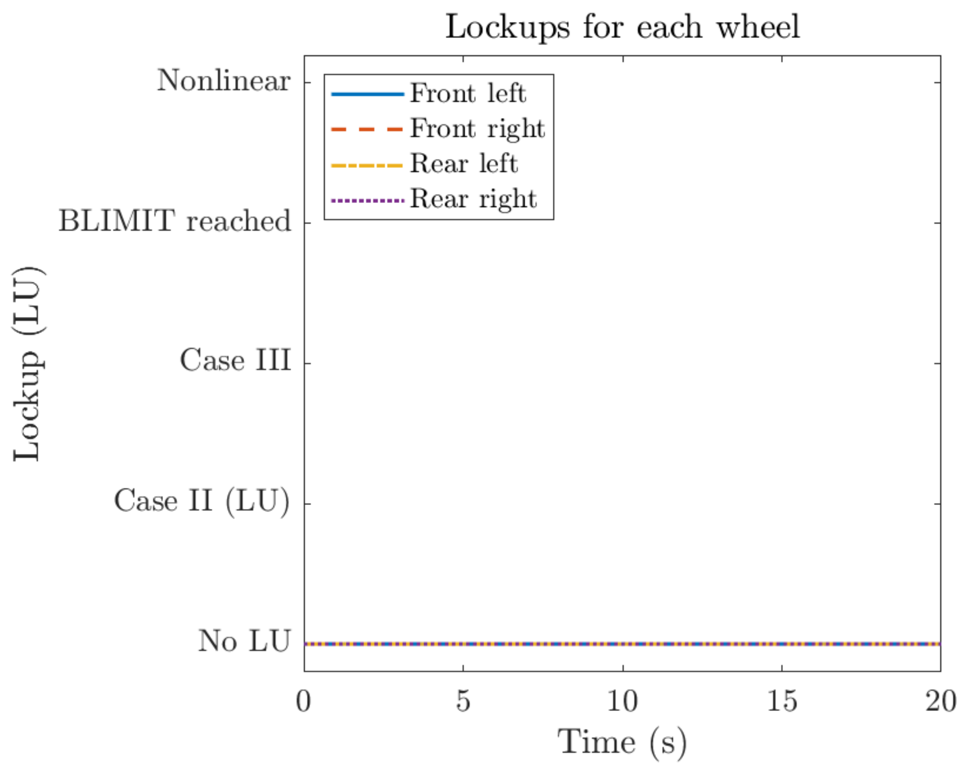
$$u := 26.8224\,\frac{\mathrm{m}}{\mathrm{s}} : C_{\alpha f} := 50990.278\,\mathrm{N} : C_{\alpha r} := 50990.278\,\mathrm{N} :$$

It is found that a 0.68 step steer results in 0.2g turn. In radians:

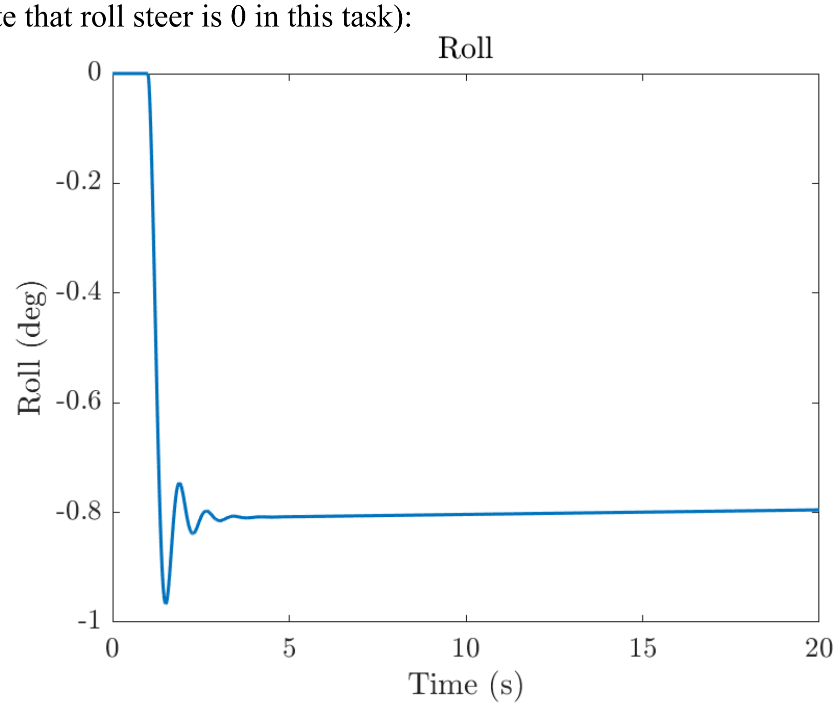
$$\delta_f := 0.68 \cdot \frac{\pi}{180} = 0.012\ \mathrm{rad}$$



It is found that for this task $\mu = 0.56$ is the smallest friction coefficient that ensures the run is linear. This figure shows what happens in the code, and finds that the nonlinear region is not reached



The roll angle (note that roll steer is 0 in this task):



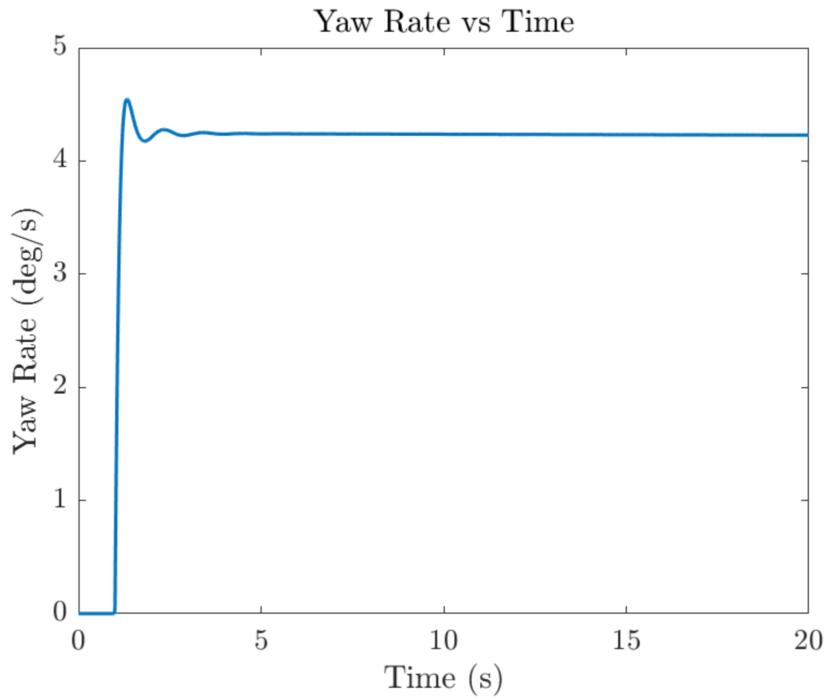
The roll gain is $(-4)\dfrac{\text{deg}}{\text{g}}$

The steady state roll is $-0.8\,\text{deg}$ and since it is a 0.2g steady turn, then it matches the roll gain.

The frequency:

$f := \dfrac{1}{2.6\,\text{s} - 1.9\,\text{s}} = 1.429\ \text{Hz}$, thus it approximately matches the roll frequency of 1.5 hz

Plotting yaw rate:



The steady state yaw rate is found to be $4.23\ \dfrac{\text{deg}}{\text{s}}$

This can be confirmed by calculating the steady state yaw rate analytically.
Firstly the front and rear weights and the K factor is calculated:

$$W_f := \frac{m \cdot b \cdot g}{L} = 6772.190\ \text{N}, \qquad W_r := \frac{m \cdot a \cdot g}{L} = 4440.640\ \text{N}$$

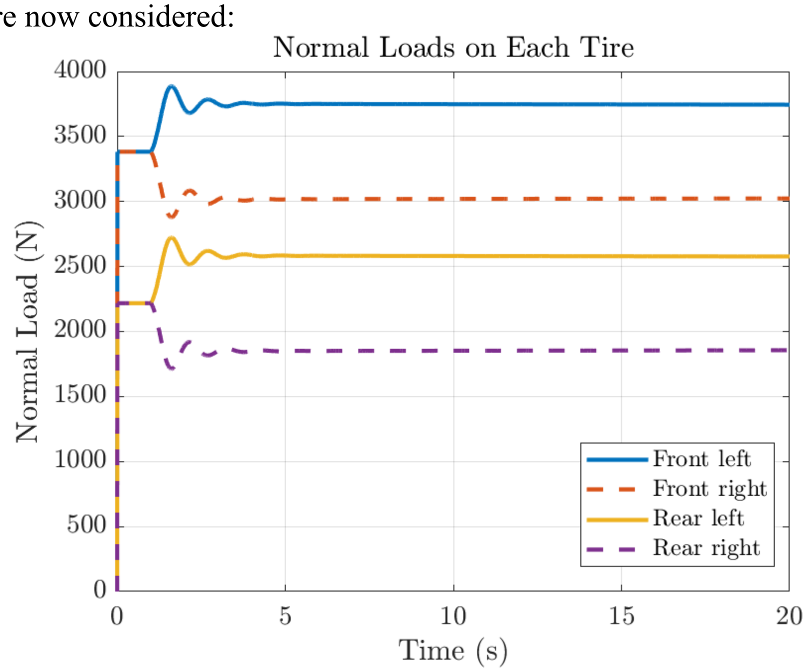
$$K := \frac{W_f}{2 \cdot C_{\alpha f}} - \frac{W_r}{2 \cdot C_{\alpha r}} = 0.023$$

Now the steady state yaw rate is calculated analytically:

$$r := \frac{\frac{u}{L}}{1 + \frac{K \cdot u^2}{L \cdot g}} \cdot \delta_f \cdot \frac{180}{\pi} = 4.2440\ \frac{\text{rad}}{\text{s}}$$

Thus the analytically steady state yaw rate is found to match the numerically found value.

The normal loads are now considered:



From the graph the steady state loads and initial loads for all the tires:

$N_{ss,fl} := 3745.66\text{N} : N_{i,fl} := 3382.64\text{N} :$

$N_{ss,fr} := 3021.75\text{N} : N_{i,fr} := 3382.64\text{N} :$

$N_{ss,rl} := 2582.93\text{N} : N_{i,rl} := 2218.06\text{N} :$

$N_{ss,rr} := 1851.59\text{N} : N_{i,rr} := 2218.06\text{N} :$

Checking the initial values for a single tire (as done previously):

$$\Delta N_{f,static} := \frac{m \cdot g \cdot b}{2 \cdot L} = 3386.095\ \text{N}$$

$\Delta N_{r,\,static} := \dfrac{m \cdot g \cdot a}{2 \cdot L} = 2220.320 \text{ N}$

The initial values match nicely with the hand calculations.

To check the normal loads at the steady state, additional parameters must be defined:

$a_y := 0.2 : T := 1.695\text{m} : \overline{p} := 0.5 : W := m \cdot g :$

Calculating:

$\Delta N_{LLT} := W \cdot a_y \cdot \dfrac{2 \cdot H}{T} = 1357.447 \text{ N}$ (1)

This value should be slightly smaller than the normal forces of the two left tires minus the two right tires:

$\Delta N := \left(N_{ss,\,fl} + N_{ss,\,rl}\right) - \left(N_{ss,\,fr} + N_{ss,\,rr}\right) = 1455.25 \text{ N}$ (2)

As it can be seen (1) is only slightly smaller than (2).

Considering just the front axle now. The normal force difference from the graph is.

$\Delta N_{f1} := N_{ss,\,fl} - N_{ss,\,fr} = 723.910 \text{ N}$

This should be equal to (2) multiplied with $\overline{p}$:

$\Delta N_{f2} := \Delta N \cdot \overline{p} = 727.625 \text{ N}$

And similarly for the rear axle:

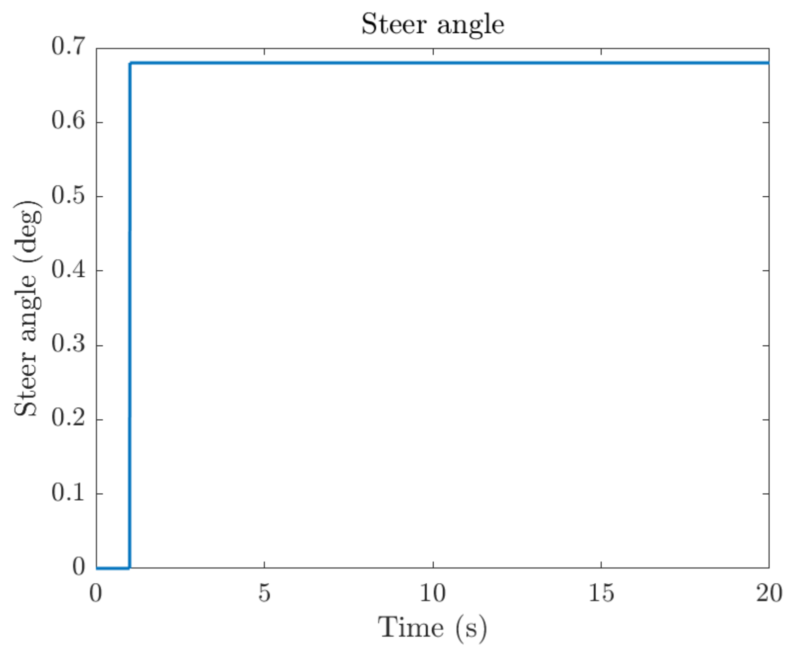
$\Delta N_{r1} := N_{ss,\,rl} - N_{ss,\,fr} = 723.910 \text{ N}$

Taking $\overline{p}$ into account for the rear axle aswell (note in this case $\overline{p}$ is 0.5, so the rear and front axle normal load calculations are equal)

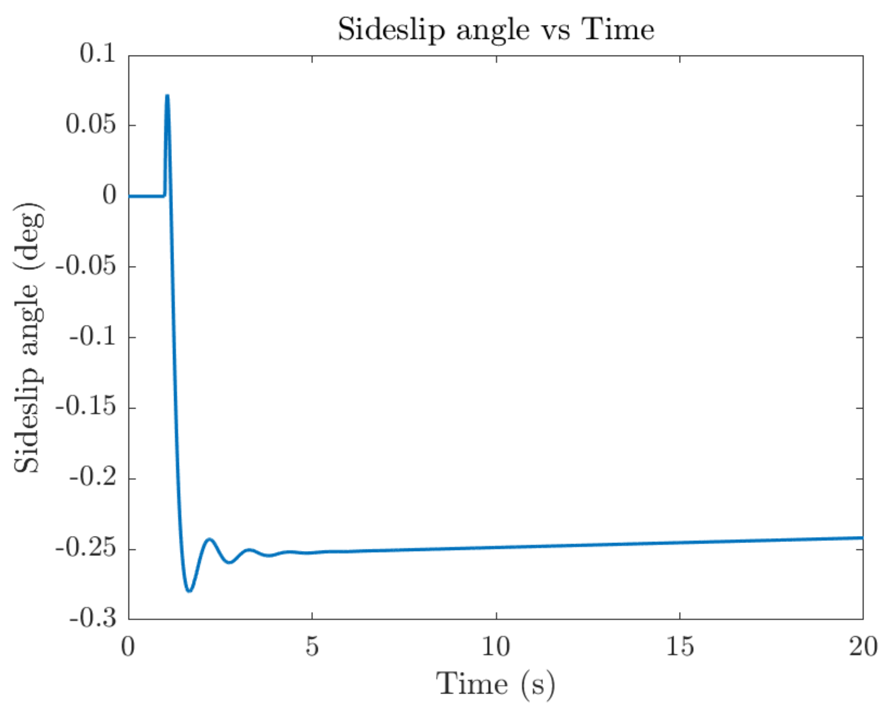
$\Delta N_{r2} := \Delta N \cdot \overline{p} = 727.625 \text{ N}$

Providing all the remaining requested plots.

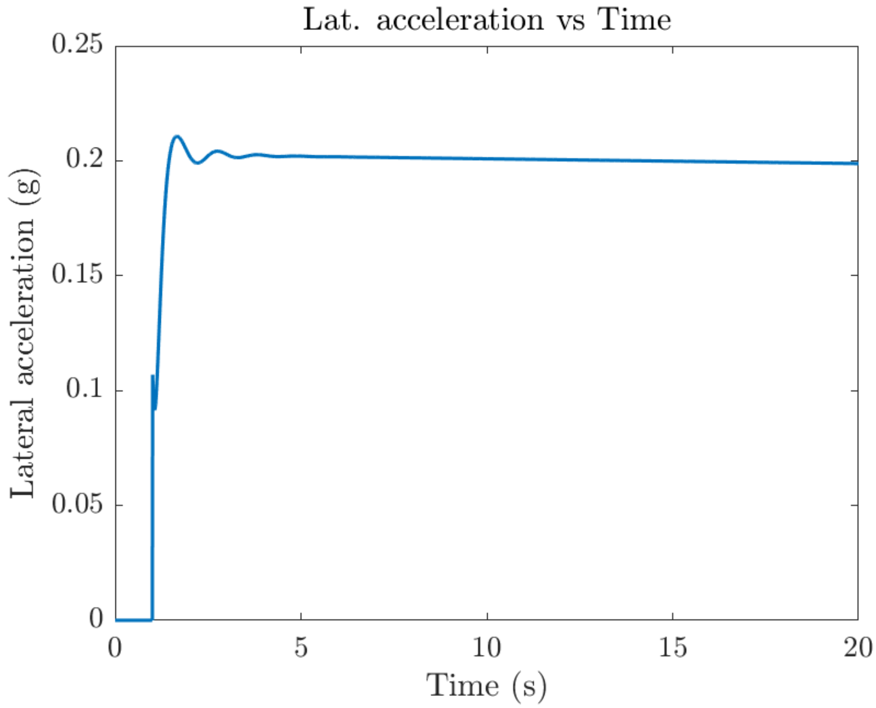
Steer angle:



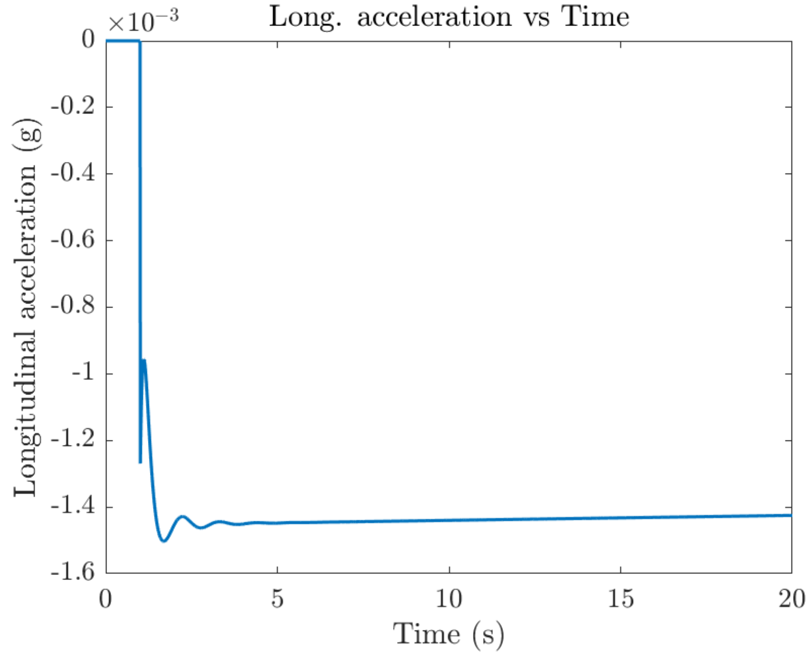
Sideslip angle:



Lateral acceleration:


Lat. acceleration vs Time
Lateral acceleration (g)
Time (s)
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20

Longitudinal acceleration



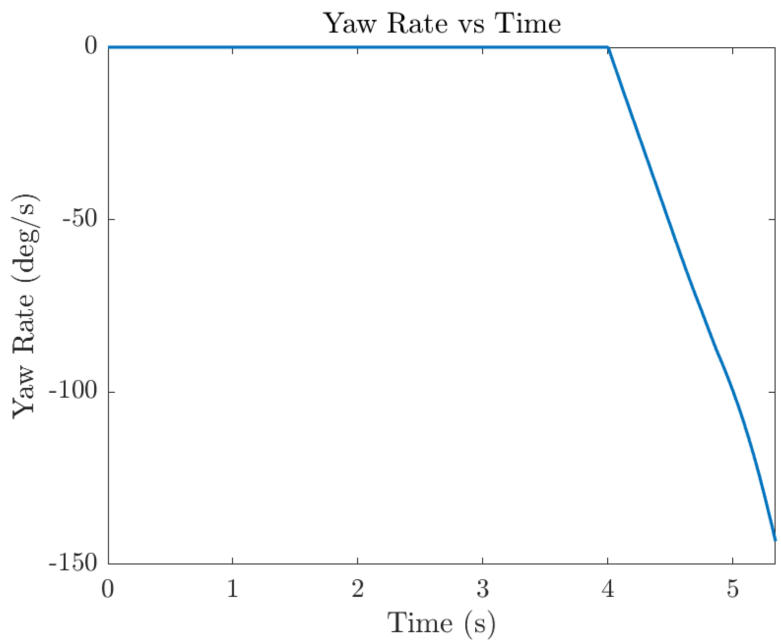
The normal loads for all four tires have already been provided aswell as the roll angle.

8.1.3)

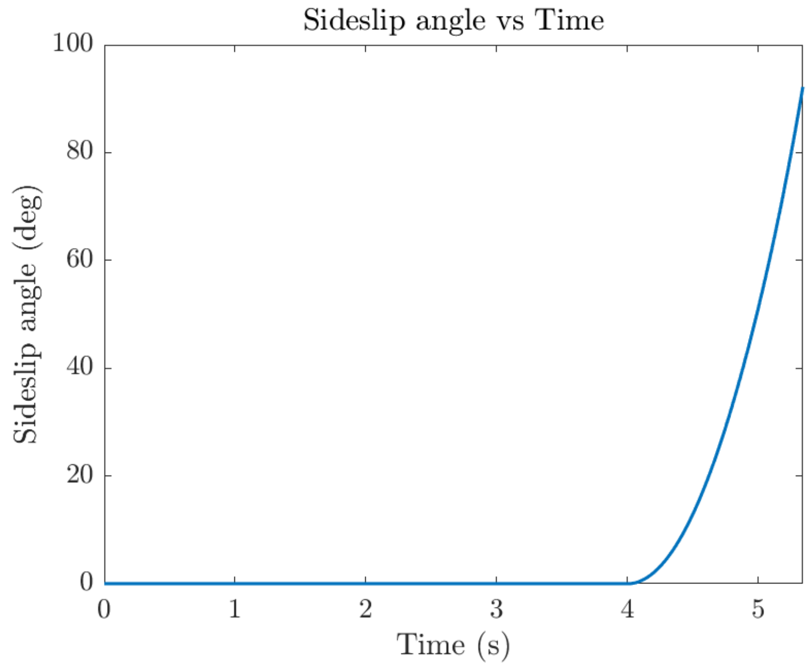
In this task a split μ straight line braking event is done. To do this it is necessary to know the position of each tire at all times. The function in the code that does that can be seen in appendix A

Braking begins after 4 seconds.

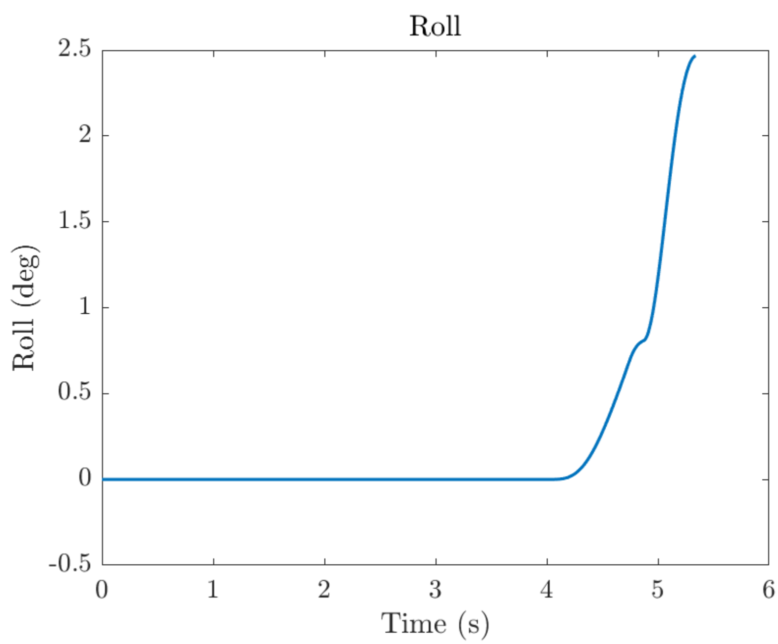
Providing the requested plots:
Yaw rate:



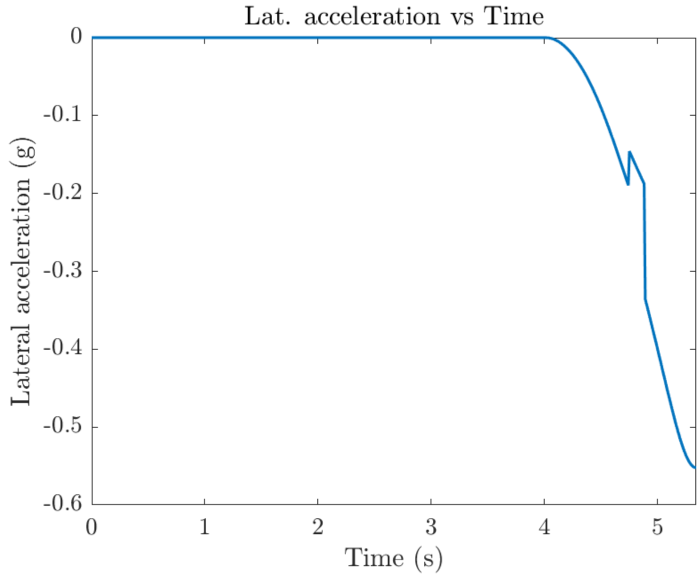
Sideslip angle:



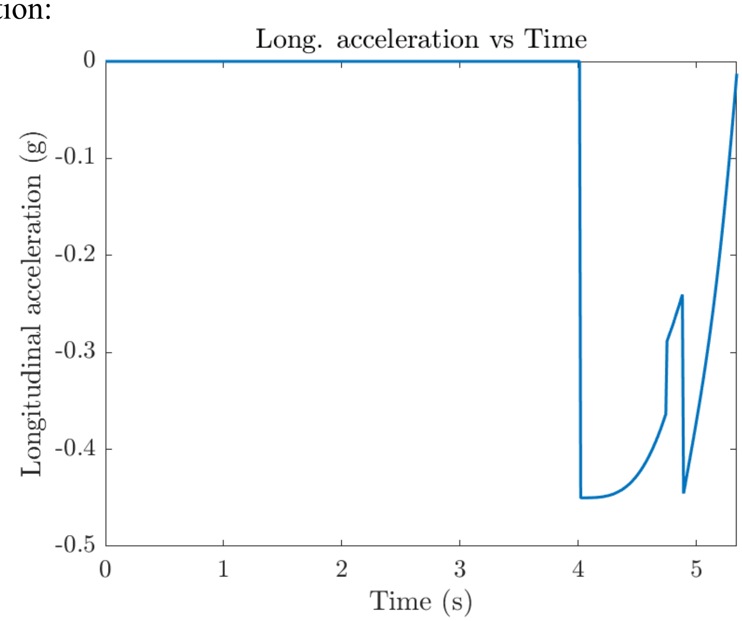
Roll:



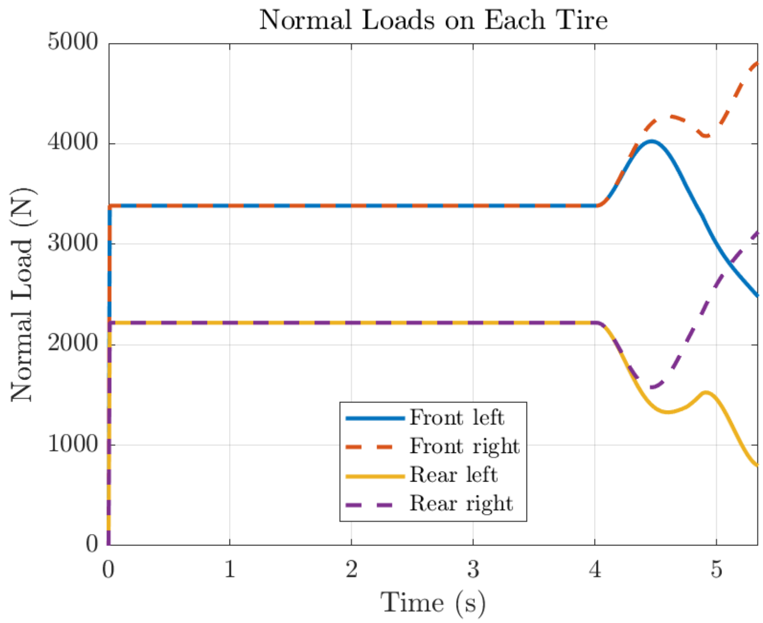
Lateral accleration:



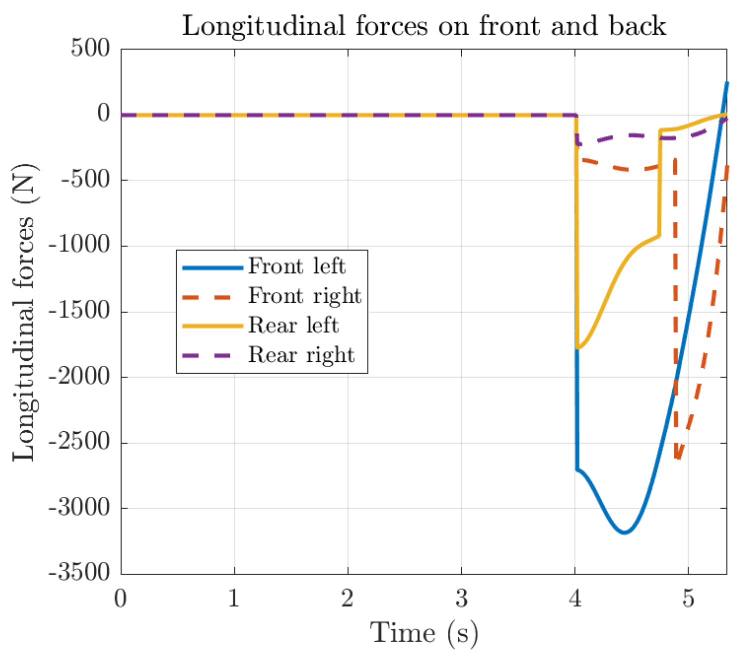
Longitudinal acceleration:



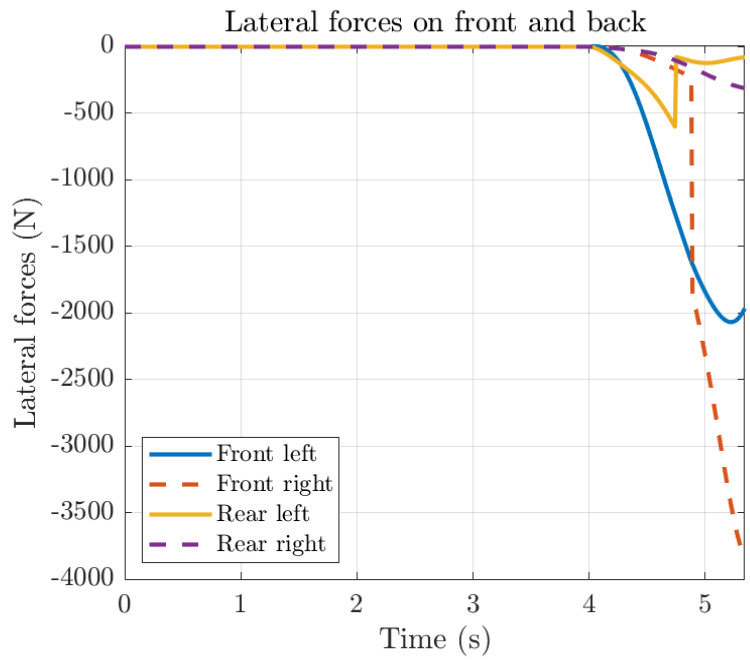
Normal loads on each tire:



Longitudinal forces:

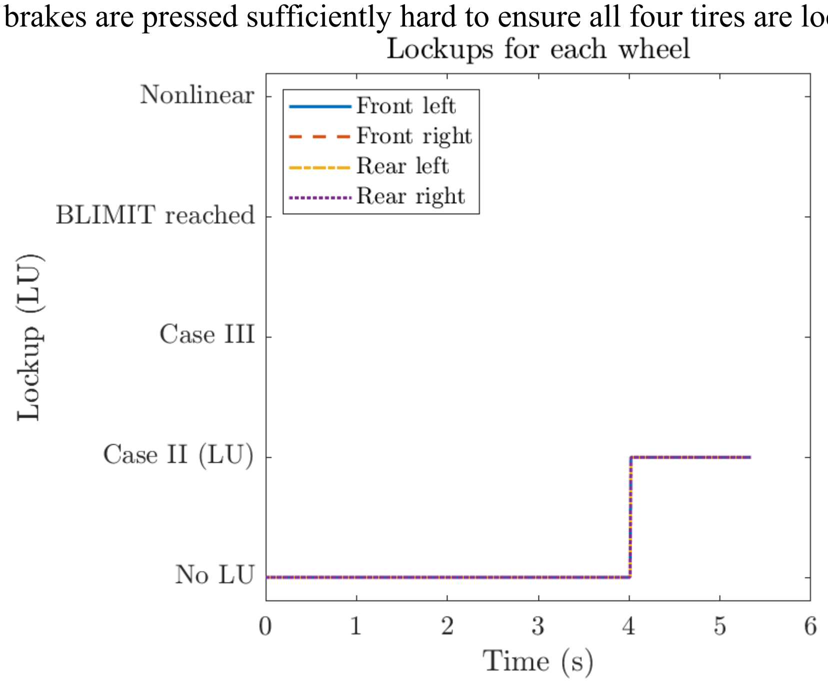


Lateral forces:

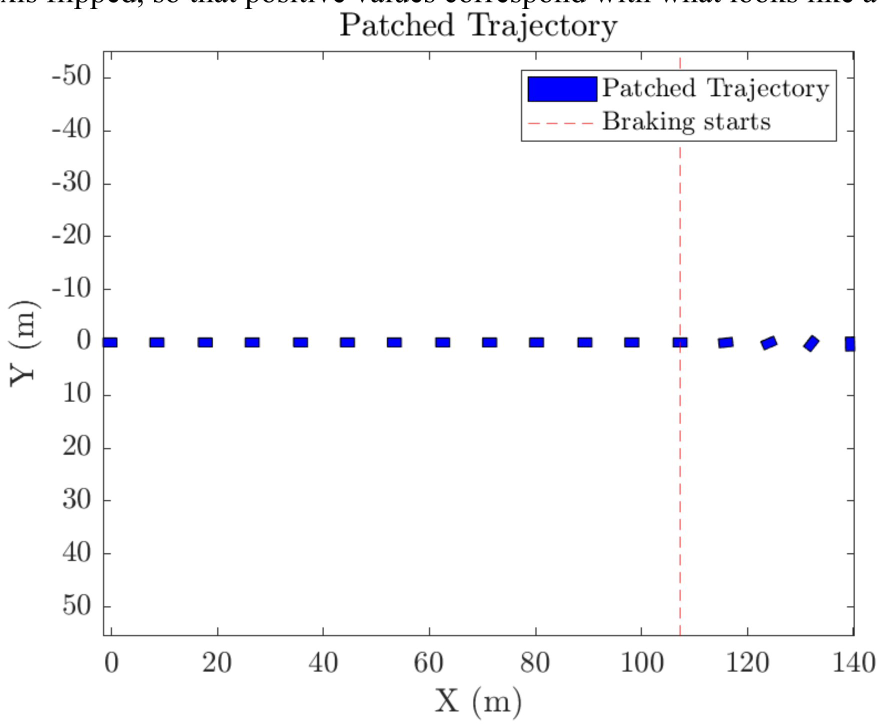


Extra plots.

Plot to confirm the brakes are pressed sufficiently hard to ensure all four tires are locked:



The patched trajectory to see the car start spinning once braking, and it stopping at 90 degrees:
Note that the Y-axis flipped, so that positive values correspond with what looks like a right hand turn.



Appendix A: Code to determine tire positions

```
function [mu_fL, mu_fR, mu_rL, mu_rR] = fric_coeff(Y, psi, car, X)
T = car.T; a = car.a; b=car.b;

% Distance between CP and a front wheel
r_a=(a^2+(T/2)^2)^0.5;
% Distance between CP and a rear wheel
r_b=(b^2+(T/2)^2)^0.5;

% Angle between the x-axis of the car and the distance vector to the front
% wheel
v_a=atan(T/(2*a));
% Angle between the x-axis of the car and the distance vector to the rear
% wheel
v_b=atan(T/(2*b));

% Orientation equation
FR_wy=(Y+r_a*(cos(v_a)*sin(psi)+sin(v_a)*cos(psi))); %Flipped left and rights to take the flipped y-axis of the patched trajectory into account
RR_wy=(Y+r_b*(-cos(v_b)*sin(psi)+sin(v_b)*cos(psi)));
RL_wy=(Y+r_b*(-cos(v_b)*sin(psi)-sin(v_b)*cos(psi)));
FL_wy=(Y+r_a*(cos(v_a)*sin(psi)-sin(v_a)*cos(psi)));

% Set friction coefficients based on wheel locations
if  (FL_wy < 0) %If the front left positive then i
    mu_fL = 0.8;
else
    mu_fL = 0.1;
    fprintf("Front left reached right side at X=%.5f \n", glob_X)
end
if  (FR_wy < 0)
    mu_fR = 0.8;
    fprintf("Front right reached left side at X=%.5f \n", glob_X)
else
    mu_fR = 0.1;
end
if  (RL_wy < 0)
    mu_rL = 0.8;
else
    mu_rL = 0.1;
    fprintf("Rear left reached right side at X=%.5f \n", glob_X)
end
if  (RR_wy < 0)
    mu_rR = 0.8;
    fprintf("Rear right reached left side at X=%.5f \n", glob_X)
else
    mu_rR = 0.1;
end
end
```