2.3.5 Carry Save Adder

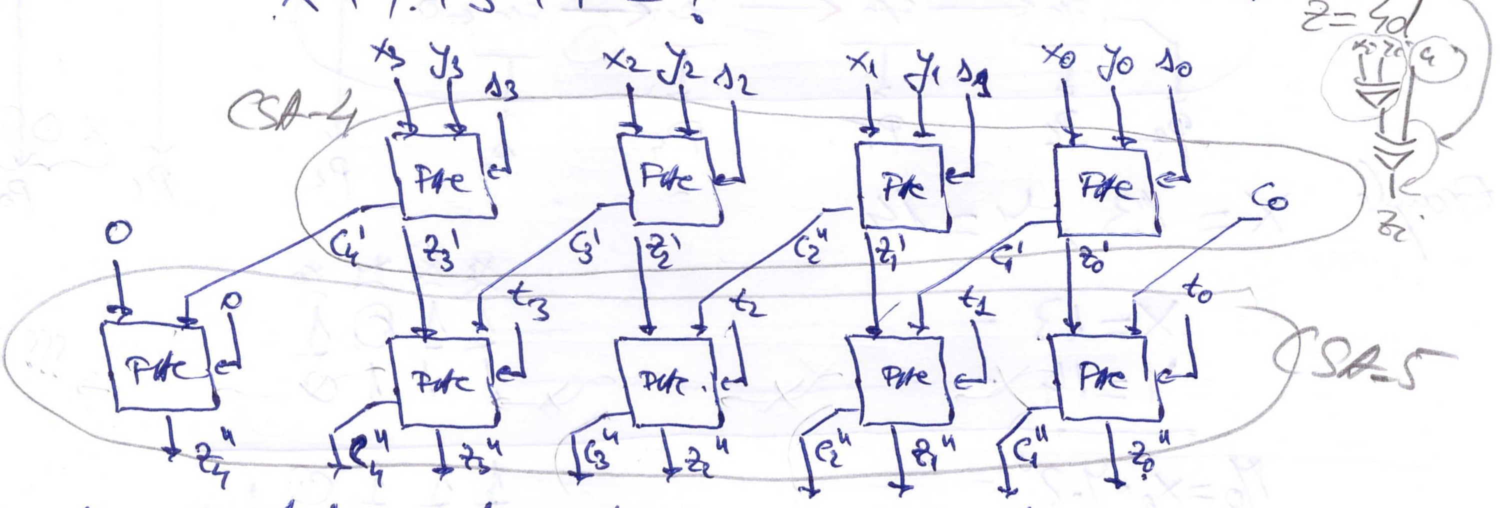
- results in redundant format ⟨ sum vector / carry vector
  - carry vector is 1-bit more significant than the sum vector
- used for multi-operand additions.
  - multiplication.

Let X, Y, S, T 4-bit operands

$X + Y + S + T = ?$

$D_{CSA}^{Z/C_{out}} = 2d$

$z = 4d$

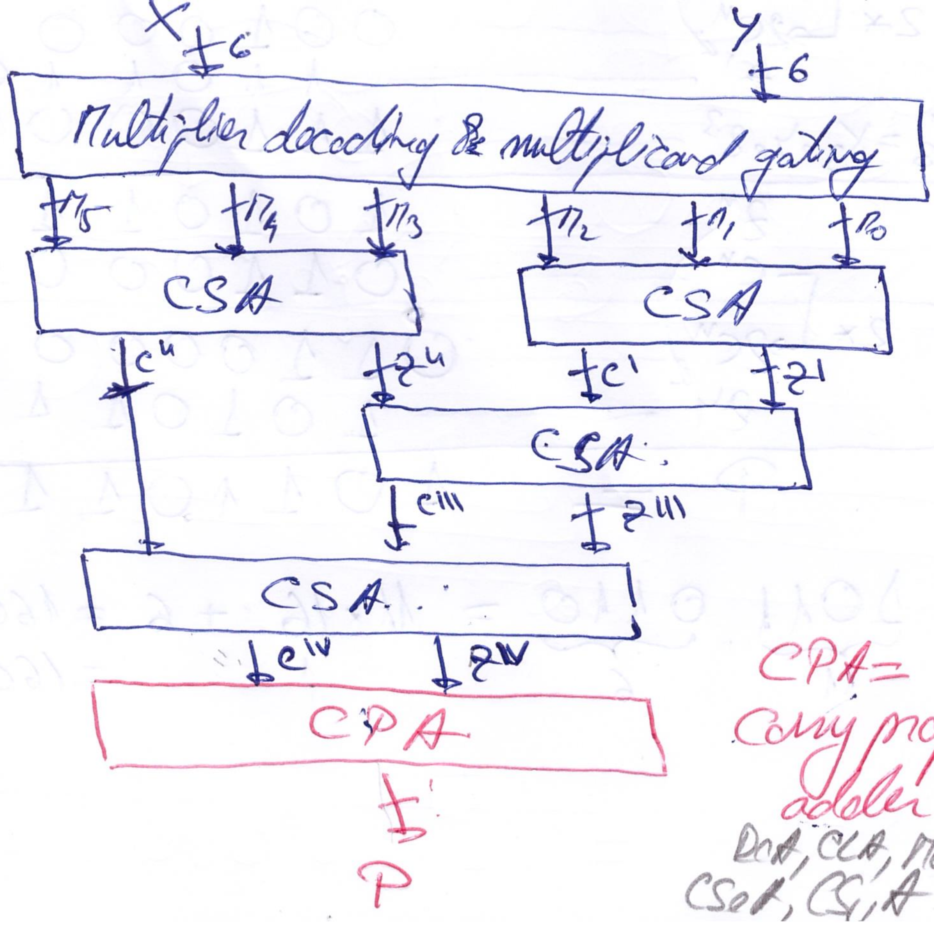


CSA – used for combinational multiplication.

Let X, Y – 6 bits, unsigned.

$P = X \cdot Y = \sum_{i=0}^{5} \Pi_i$ Π_i – 1 bit product

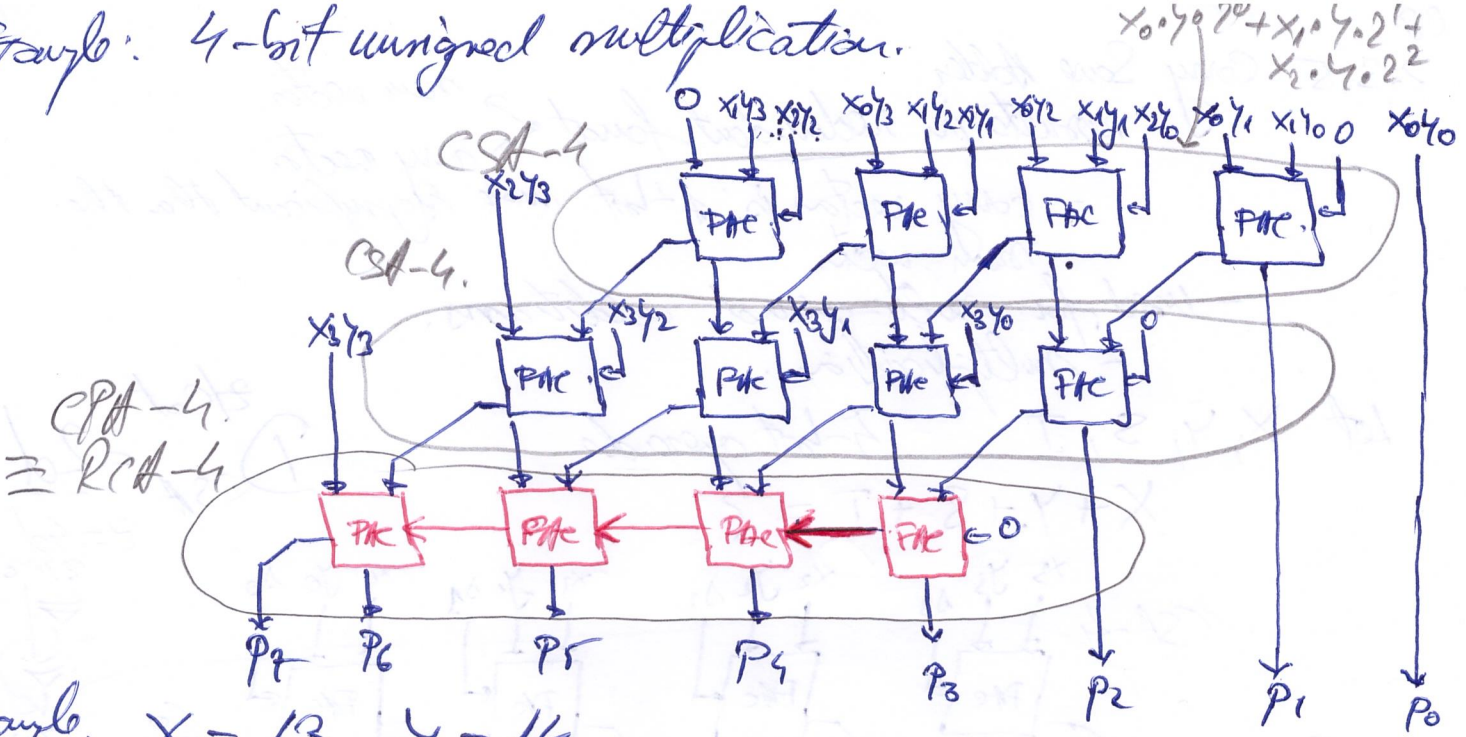
$\Pi_i = x_i \cdot Y \cdot 2^i$.



CPA = carry propagate adder

RCA, CLA, ML-CLA, CSkA, CSelA, CGA

Exemplu: 4-bit unsigned multiplication.



Exemplu. $X = 13, Y = 14$

$X = 13$ — 1 1 0 1 (x_3 x_2 x_1 x_0)

$Y = 14$ — 1 1 1 0

$M_0 = x_0 \cdot Y \cdot 2^0$ — 1 1 1 0

$M_1 = x_1 \cdot Y \cdot 2^1$ — 0 0 0 0 0

$M_2 = x_2 \cdot Y \cdot 2^2$ — 1 1 1 0 0 0 | + CSA.

Z^1 — 1 1 0 1 1 0

C^1 — 0 0 1 0 0 0

$2 * 2C^1$ — 0 0 1 0 0 0

Z^1 — 1 1 0 1 1 0

$M_3 = x_3 \cdot Y \cdot 2^3$ — 1 1 1 0 0 0 | + CSA.

Z^4 — 1 0 1 0 1 1 0

C^4 — 0 1 1 0 0 0 0

$2 * 2C^4$ — 0 1 1 0 0 0 0

Z^4 — 1 0 1 0 1 1 0 | + CPA

P = 1 0 1 1 0 1 1 0

$P = \underbrace{1011}_{11}\ \underbrace{0110}_{6} = 11 \cdot 16 + 6 = 160 + 16 + 6 = 160 + 22 = 182$

14
13
42
14
182

2.4. Reliable computing

- performance
- power consumption ↑ "mobile" computing
- reliability !!!

Attributes of a reliable computing:
- availability: readiness of system service
- reliability: continuity of system service.
- maintainability: ability to undergo system repairs and modifications.

2.4.1 Binary adders with parity control.

— fault —activates→ error —propagation→ failure —causality→ fault — (ATM)

→ increase reliability by using parity control.
- attach an even parity bit to all operands

$\underline{X}$, $\underline{Y}$, c_{in}

$$X + Y + c_0 \to \underline{Z},\ c_{out}$$

$$x_p = x_{n-1} \oplus x_{n-2} \oplus \dots \oplus x_1 \oplus x_0$$

$$y_p = y_{n-1} \oplus y_{n-2} \oplus \dots \oplus y_1 \oplus y_0$$

(1) $$\underline{z_p} = z_{n-1} \oplus z_{n-2} \oplus \dots \oplus z_1 \oplus z_0$$

↳ verify parity of z

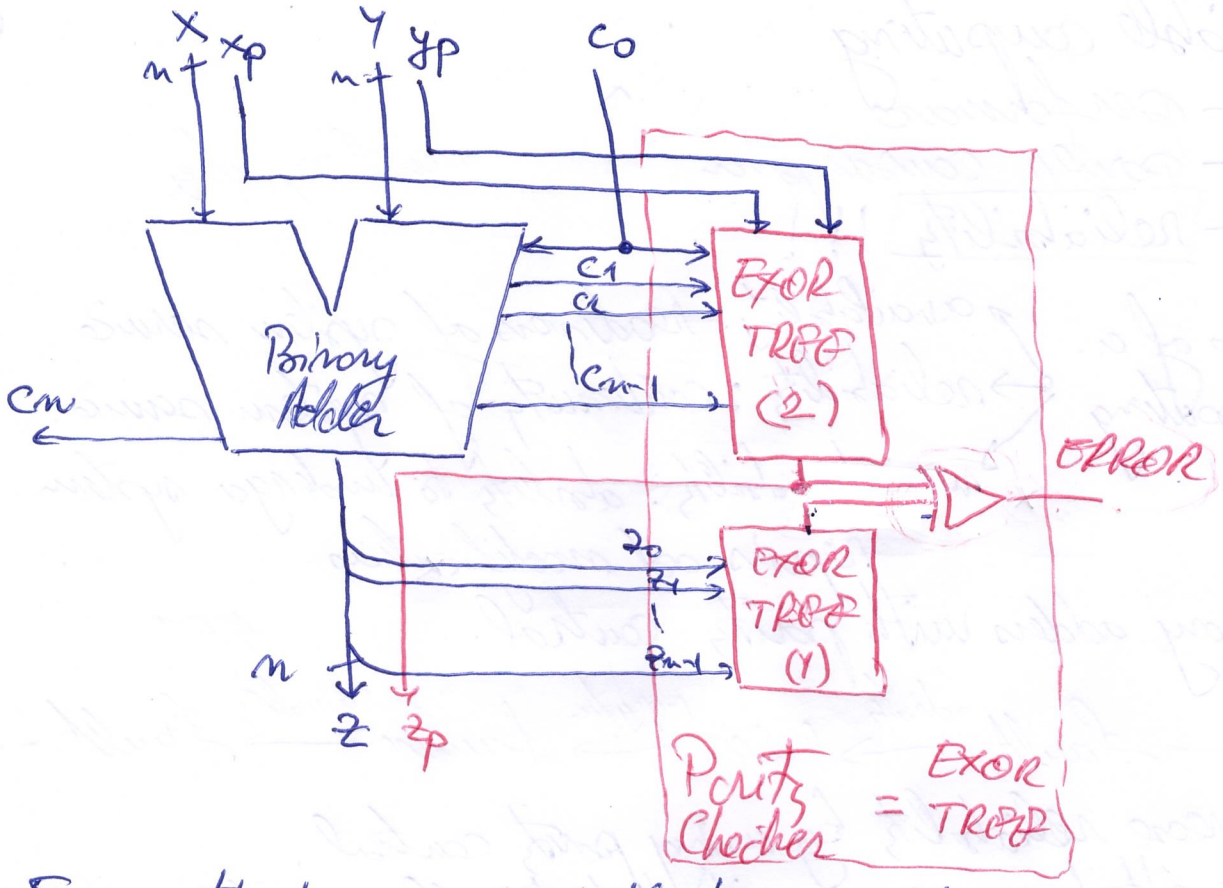
$z_i = x_i \oplus y_i \oplus c_i \to (1)$ Let $c_p = c_{n-1} \oplus c_{n-2} \oplus \dots \oplus c_0$ (all carries without carry out)

$$\Rightarrow z_p = x_{n-1} \oplus y_{n-1} \oplus c_{n-1} \oplus x_{n-2} \oplus y_{n-2} \oplus c_{n-2} \oplus \dots \oplus x_0 \oplus y_0 \oplus c_0$$

$$z_p = x_p \oplus y_p \oplus \underbrace{c_{n-1} \oplus c_{n-2} \oplus \dots \oplus c_0}_{c_p}$$

(2) $$z_p = x_p \oplus y_p \oplus c_p$$

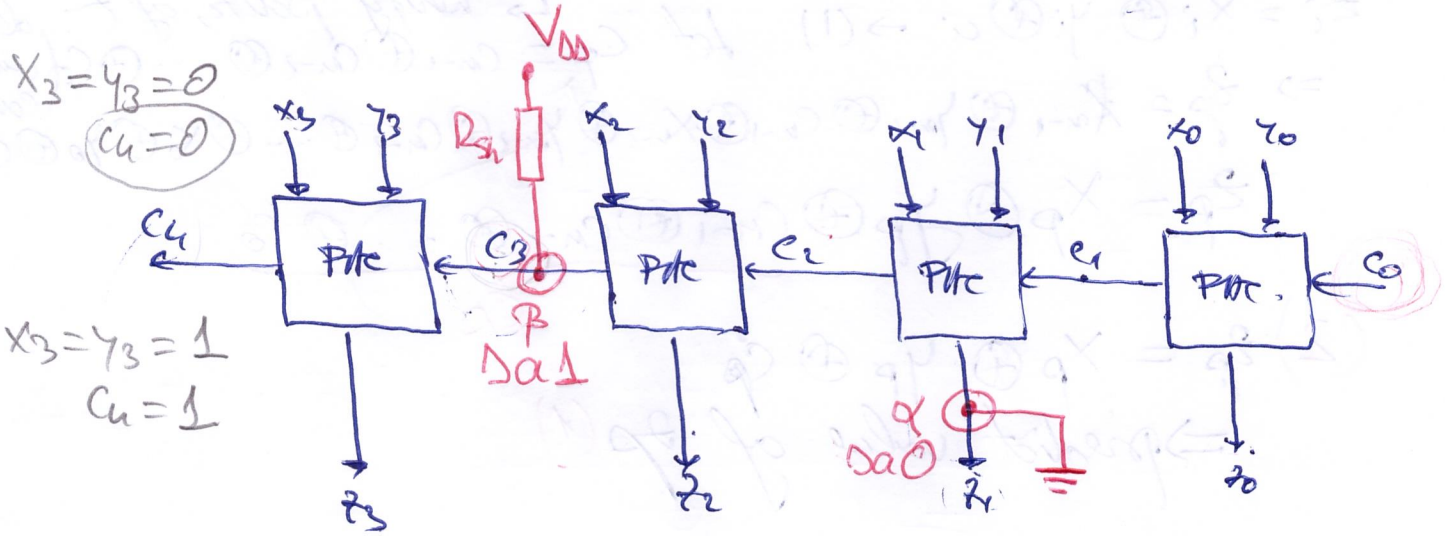
⇒ predict value of z_p (1)



Errors that can affect the binary adder.

- multiple faults
- single faults ↔ harder to detect / higher probability } Single Stuck-at Fault (SSaF)
- logic manifestation

RCA on 4 bits.



α: Z_1 Sa0 → only Z_1 is affected.
↓ C_2 is not affected
⇒ odd no of bits affected.

β: C_3 Sa1 → C_3 is affected ⇒ Z_3 is affected.
⇒ C_4 can be affected ⇒ Z_4 can be affected
⇒ even no of bits are affected.

Can parity control detect both α & β SSaF

fault-free case.

$X = 0\ 0\ 1\ 1$ (input bits)

$Y = 0\ 0\ 1\ 1$

$c = c_4\ 0\ 1\ 1\ |\ 0_{c_0}$

$z = 0\ 1\ 1\ 0$

$x_p = 0 \oplus 0 \oplus 1 \oplus 1 = 0$

$y_p = 0$

$c_p = 0$

$\Rightarrow z_p(2) = 0 \oplus 0 \oplus 0 = 0$

$\Rightarrow z_p(1) = 0 \oplus 1 \oplus 1 \oplus 0 = 0$

no error!!

Fault α

$X = 0\ 0\ 1\ 1$

$Y = 0\ 0\ 1\ 1$

$c = c_4\ 0\ 1\ 1\ |\ 0_{c_0}$

$z = 0\ 1\ 0\ 0$

$x_p = 0$

$y_p = 0$

$c_p = 0$

$z_p(1) = 1$

$\Rightarrow z_p(2) = 0$

difference!!!

$\Rightarrow$ Error (✓)

Fault β

$X = 0\ 0\ 1\ 1$

$Y = 0\ 0\ 1\ 1$

$c = c_4\ 0\ 1\ 1\ |\ 0_{c_0}$

$z = 1\ 1\ 1\ 0$

$x_p = 0$

$y_p = 0$

$c_p = 1$

$z_p(1) = 1$

$\Rightarrow z_p(2) = 1$

NO difference!!!

No Error?!?

Single parity bits cannot detect all possible SSaF.

- cannot detect errors in the carry chain