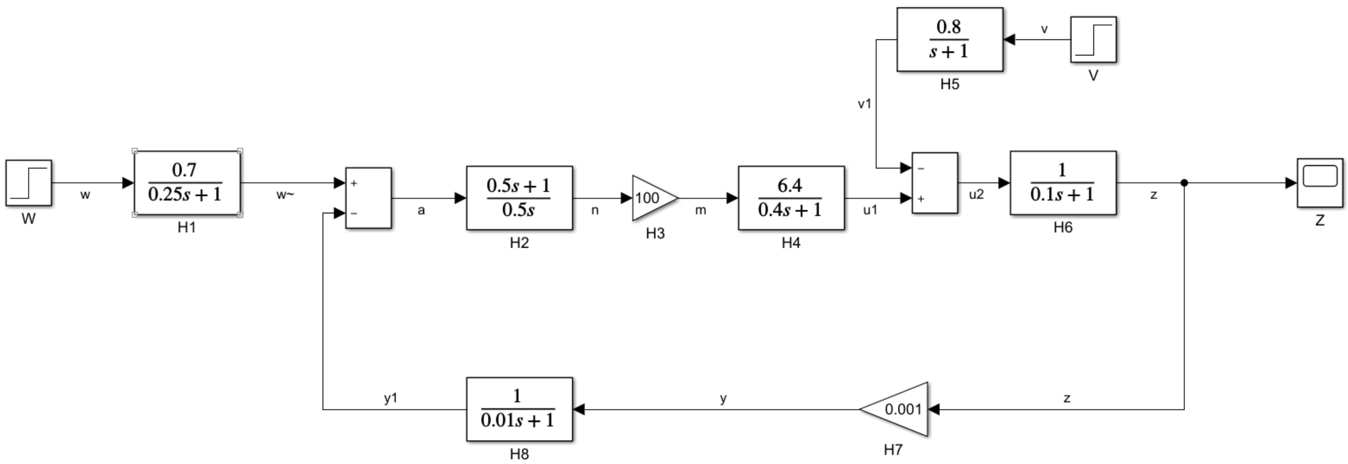


W
w
0.7
0.25s + 1
H1
w~
+
−
a
0.5s + 1
0.5s
H2
n
100
H3
m
6.4
0.4s + 1
H4
u1
0.8
s + 1
H5
v
V
v1
−
+
u2
1
0.1s + 1
H6
z
Z
y1
1
0.01s + 1
H8
y
0.001
H7
z

EX CS-2 • PI-Type controller

a) $w_\infty = 10$
$v_\infty = 1250$

$H_1 \to PT1 \overset{FVT}{\Rightarrow} \tilde{w}_\infty = H_1(0) \cdot w_\infty = 0.7 \cdot w_\infty = 7$

• PI controller $\Rightarrow$ $a_\infty = 0$
$a_\infty = \tilde{w}_\infty - y_\infty$
$\tilde{w}_\infty = 7$ $\Big| \Rightarrow y_\infty = 7$

$H_8 \to PT1 \overset{FVT}{\Rightarrow} y_\infty = H_8(0) \cdot y_{1\infty} \Rightarrow y_{1\infty} = \frac{y_\infty}{H_8(0)} = \frac{7}{1} = 7$

$H_7 \to P \overset{FVT}{\Rightarrow} y_{1\infty} = H_7(0) \cdot z_\infty \Rightarrow z_\infty = \frac{y_{1\infty}}{H_7(0)} = \frac{7}{0.001} = 7000$

$H_6 \to PT1 \overset{FVT}{\Rightarrow} z_\infty = H_6(0) \cdot u_{2\infty} \Rightarrow u_{2\infty} = \frac{7000}{1} = 7000$

$H_5 \to PT1 \overset{FVT}{\Rightarrow} v_{1\infty} = H_5(0) \cdot v_\infty = 0.8 \cdot 1250 = 1000$

• $u_{2\infty} = u_{1\infty} - v_{1\infty} \Rightarrow u_{1\infty} = u_{2\infty} + v_{1\infty} = 8000$

$H_4 \to PT1 \overset{FVT}{\Rightarrow} u_{1\infty} = H_4(0) \cdot m_\infty \Rightarrow m_\infty = \frac{8000}{6.4} = 1250$

$H_3 \to P \overset{FVT}{\Rightarrow} m_\infty = H_3(0) \cdot n_\infty \Rightarrow n_\infty = \frac{1250}{100} = 12.5$

• SS values:

$\tilde{w}_\infty = 7$
$a_\infty = 0$
$n_\infty = 12.5$
$m_\infty = 1250$
$u_{1\infty} = 8000$
$u_{2\infty} = 7000$
$v_{1\infty} = 1000$

$y_{1\infty} = 7$
$y_\infty = 7$
$w_\infty = 10$
$v_\infty = 1250$
$z_\infty = 7000$

b) $z_\infty = 6000$
$v_\infty = 1000$

$H_7 \to P \overset{FVT}{\Rightarrow} y_{1\infty} = H_7(0) \cdot z_\infty = 0.001 \cdot 6000 = 6$

$H_8 \to PT1 \overset{FVT}{\Rightarrow} y_\infty = H_8(0) \cdot y_{1\infty} = 1 \cdot 6 = 6$

$H_6 \to PT1 \overset{FVT}{\Rightarrow} z_\infty = H_6(0) \cdot u_{2\infty} \Rightarrow u_{2\infty} = \frac{6000}{1} = 6000$

$H_5 \to PT1 \overset{FVT}{\Rightarrow} v_{1\infty} = H_5(0) \cdot v_\infty = 0.8 \cdot 1000 = 800$

- $u_{2\infty} = u_{1\infty} - v_{1\infty} \Rightarrow u_{1\infty} = u_{2\infty} + v_{1\infty} = 6800$

$H_4 \to PT1 \overset{FVT}{\Rightarrow} u_{1\infty} = H_4(0) \cdot m_\infty \Rightarrow m_\infty = \frac{6800}{6.4} = 1062.5$

$H_3 \to P \overset{FVT}{\Rightarrow} m_\infty = H_3(0) \cdot n_\infty \Rightarrow n_\infty = \frac{1062.5}{100} = 10.625$

- PI controller $\Rightarrow a_\infty = 0$
$a_\infty = \tilde{w}_\infty - y_\infty$
$y_\infty = 6$
$\Rightarrow \tilde{w}_\infty = 6$

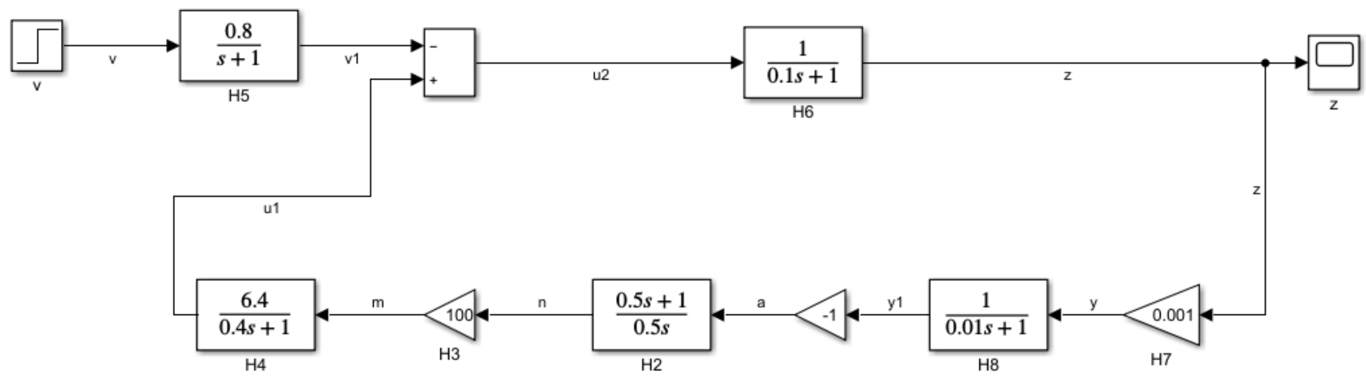
$H_1 \to PT1 \overset{FVT}{\Rightarrow} \tilde{w}_\infty = H_1(0) \cdot w_\infty \Rightarrow w_\infty = \frac{6}{0.7} = 8.571$

* SS values:

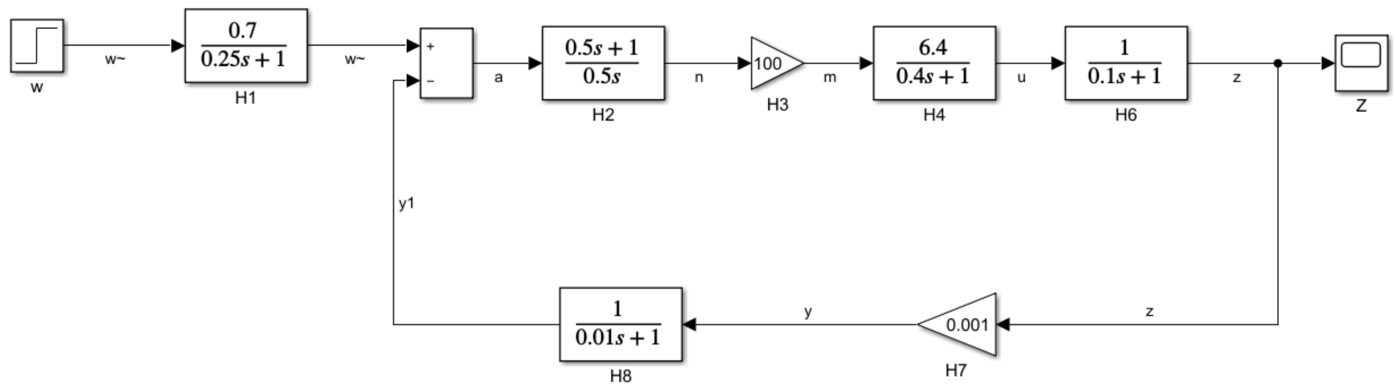
$\tilde{w}_\infty = 6$
$a_\infty = 0$
$n_\infty = 10.625$
$m_\infty = 1062.5$
$u_{1\infty} = 6800$
$u_{2\infty} = 6000$
$v_{1\infty} = 800$

$y_{1\infty} = 6$
$y_\infty = 6$

$w_\infty = 8.571$
$v_\infty = 1000$
$z_\infty = 6000$



v
v
0.8 / (s + 1)
H5
v1
u2
1 / (0.1s + 1)
H6
z
z
z
u1
6.4 / (0.4s + 1)
H4
m
100
H3
n
0.5s + 1 / 0.5s
H2
a
-1
y1
1 / (0.01s + 1)
H8
y
0.001
H7



w
w~
0.7 / (0.25s + 1)
H1
w~
a
0.5s + 1 / 0.5s
H2
n
100
H3
m
6.4 / (0.4s + 1)
H4
u
1 / (0.1s + 1)
H6
z
Z
y1
1 / (0.01s + 1)
H8
y
0.001
H7
z

c) $$H_{z-w}(s)\Big|_{v=0} = H_1(s)\cdot\frac{(H_2(s)\cdot H_3(s)\cdot H_4(s)\cdot H_6(s))}{1+(H_7(s)\cdot H_8(7))(H_2(s)H_3(s)H_4(s)H_6(s))}$$

$$= \frac{0.7}{0.25s+1}\cdot\frac{\left(\frac{0.5s+1}{0.5s}\cdot 100\cdot\frac{6.4}{0.4s+1}\cdot\frac{1}{0.1s+1}\right)}{1+\left(0.001\cdot\frac{1}{0.01s+1}\right)\left(\frac{0.5s+1}{0.5s}\cdot 100\cdot\frac{6.4}{0.4s+1}\cdot\frac{1}{0.1s+1}\right)}$$

$$= \frac{4.48s^2+456.96s+896}{0.0001s^5+0.001165s^4+0.1725s^3+0.92s^2+1.96s+1.28}$$

$$H_{z-v}(s)\Big|_{w=0} = -H_5\cdot\frac{H_6}{1-(H_7\cdot H_8\cdot(-1)\cdot H_2\cdot H_3\cdot H_4)(H_6)}$$

$$= -\frac{0.8}{s+1}\cdot\frac{\left(\frac{1}{0.1s+1}\right)}{1+\left(0.001\cdot\frac{1}{0.01s+1}\cdot\frac{0.5s+1}{0.5s}\cdot 100\cdot\frac{6.4}{0.4s+1}\right)\left(\frac{1}{0.1s+1}\right)}$$

$$= -\frac{0.0032s^3+0.328s^2+0.8s}{0.0004s^5+0.0454s^4+0.555s^3+2.15s^2+2.92s+1.28}$$

- For $H_{z-w}(s)\,|\,v=0$:

$$\Delta(s) = 0.0001s^5+0.001165s^4+0.1725s^3+0.92s^2+1.96s+1.28$$

$a_5 = 0.0001 > 0$
$a_4 = 0.001165 >$
$a_3 = 0.1725 > 0$
$a_2 = 0.92 > 0$
$a_1 = 1.96 > 0$
$a_0 = 1.28 > 0$

$$H = \begin{pmatrix} a_4 & a_2 & a_0 & 0 & 0 \\ a_5 & a_3 & a_1 & 0 & 0 \\ 0 & a_4 & a_2 & a_0 & 0 \\ 0 & a_5 & a_3 & a_1 & 0 \\ 0 & 0 & a_4 & a_2 & a_0 \end{pmatrix}$$

- $\det H_1 = a_4 = 0.001165 > 0$
- $\det H_2 = (a_4 a_3) - (a_5 a_2) = \cancel{0.1 > 0}\ 0.0019 > 0$
- $\det H_3 = (a_4 a_3 a_2) + 0 + (a_5 a_4 a_0) - 0 - (a_4 a_4 a_1) - (a_2 a_2 a_5)$

~~= 0.0011 < 0 => SYSTEM IS UNSTABLE~~

$= 0.0014 > 0$

- $\det H_4 = 0.0025 > 0$
- $\det H_5 = 0.0032 > 0$

=> SYSTEM IS STABLE

• For $H_{z-v}(s)\,|_{w=0}$:

$$\Delta(s) = 0.0004s^5 + 0.0454s^4 + 0.555s^3 + 2.15s^2 + 2.92s + 1.28$$

$a_5 = 0.0004 > 0$
$a_4 = 0.0454 > 0$
$a_3 = 0.555 > 0$
$a_2 = 2.15 > 0$
$a_1 = 2.92 > 0$
$a_0 = 1.28 > 0$

$$H = \begin{pmatrix} a_4 & a_2 & a_0 & 0 & 0 \\ a_5 & a_3 & a_1 & 0 & 0 \\ 0 & a_4 & a_2 & a_0 & 0 \\ 0 & a_5 & a_3 & a_1 & 0 \\ 0 & 0 & a_4 & a_2 & a_0 \end{pmatrix}$$

• $\det H_1 = a_4 = 0.0004 > 0$
• $\det H_2 = 0.0243 > 0$
• $\det H_3 = 0.0463 > 0$
• $\det H_4 = 0.1180 > 0$
• $\det H_5 = 0.1511 > 0$

$\Rightarrow$ THE SYSTEM IS STABLE

$$H_{z-w}(0) = \frac{896\cancel{K_c}}{1.28\cancel{K_c}} = 700$$

$$H_{z-v}(0) = \frac{-0}{1.28K_c} = 0$$

$\Rightarrow$ the system's static coefficients do not depend on K_c

$\Rightarrow$ NOT INFLUENCED BY INCREASE

$$H_{z-w}(s) = \frac{0.7}{0.25s+1} \cdot \frac{\left(\frac{0.5s+K_c}{0.5s} \cdot 100 \cdot \frac{6.4}{0.4s+1} \cdot \frac{1}{0.1s+1}\right)}{1+\left(\frac{0.5s+K_c}{0.5s} \cdot 100 \cdot \frac{6.4}{0.4s+1} \cdot \frac{1}{0.1s+1}\right)\left(\frac{1}{0.01s+1} \cdot 0.001\right)}$$

$$H_{z-v}(s) = -\frac{0.8}{s+1} \cdot \frac{\frac{1}{0.1s+1}}{1+\left(\frac{1}{0.1s+1}\right)\left(0.001 \cdot \frac{1}{0.01s+1} \cdot 100 \cdot \frac{0.5s+K_c}{0.5s} \cdot 100 \cdot \frac{6.4}{0.4s+1}\right)}$$