INTRODUCTION

❖ *Joint Cumulative Distribution Function*

$$F_{X,Y}(x,y) = P(X \le x, Y \le y)$$

$\boldsymbol{x-variable}$	$\boldsymbol{y-variable}$
$0 \le F_{X,Y}(x,y) \le 1$	
$F_X(x) = \lim_{y\to\infty} F_{X,Y}(x,y)$	$F_Y(y) = \lim_{x\to\infty} F_{X,Y}(x,y)$
$\lim_{x\to-\infty} F_{X,Y}(x,y) = 0$	$\lim_{y\to-\infty} F_{X,Y}(x,y) = 0$
$\lim_{\substack{x\to\infty \\ y\to\infty}} F_{X,Y}(x,y) = 1$	

❖ *Joint Probability Mass Function*

$$P_{X,Y}(x,y) = P[X = x, Y = y]$$

- ***Possible Value Range***

$$S_{X,Y}(x,y) = \{(x,y) \mid P(x,y) > 0\}$$

- ***Marginal PMF***

$\boldsymbol{x-variable}$	$\boldsymbol{y-variable}$
$P_X(x) = \sum_{y\in S_Y} P_{X,Y}(x,y)$	$P_Y(y) = \sum_{x\in S_X} P_{X,Y}(x,y)$

❖ *Joint Probability Density Function*

$$f_{X,Y}(x,y) - given(family\ of\ joint\ CRV)$$

Probability Density Function	*Cumulative Distribution Function*
$f_X(x) = \dfrac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$	$F_{X,Y}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y}(u,v)\,dudv$
$P[a \le x \le b, c \le y \le d]$	
$\int_a^b \int_c^d f_{X,Y}(x,y)\,dxdy$	$F_{X,Y}(b,d) - F_{X,Y}(b,c) - [F_{X,Y}(a,d) - F_{X,Y}(a,c)]$
$f_{X,Y}(x,y) \ge 0$	$\lim_{x\to-\infty} F_{X,Y}(x,y) = \lim_{y\to-\infty} F_{X,Y}(x,y) = 0$
$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dxdy = 1$	$\lim_{\substack{x\to\infty \\ y\to\infty}} F_{X,Y}(x,y) = 1$

- ***Marginal PDF***

$\boldsymbol{x-variable}$	$\boldsymbol{y-variable}$
$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$	$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$

FUNCTION OF 2RV

- $\boldsymbol{W = g(x, y)}$
  - $Discrete\ R.V.$

$$P_W(w) = \sum_{g(x,y)=w} P_{X,Y}(x, y)$$

  - $Continuous\ R.V.$

$$F_W(w) = P_W(W \le w) = \iint_{g(x,y)\le w} P_{X,Y}(x, y)\, dxdy$$

- ***Expected Value***
  - $Discrete\ R.V.$

$$E(W) = \sum_{x\in S_X} \sum_{y\in S_Y} g(x, y) \cdot P_{X,Y}(x, y)$$

  - $Continuous\ R.V.$

$$E(W) = \iint_{\mathbb{R}^2} g(x, y) \cdot P_{X,Y}(x, y)\, dxdy$$

- ***Variance***

$$Var(X + Y) = Var(X) + Var(Y) + 2 \cdot Cov(X, Y)$$

  - *Covariance*

$$Cov(X, Y) = E((X - E(X)) \cdot (Y - E(Y)))$$

$$Cov(X, Y) = r_{X,Y} - E(X)E(Y)$$

  - *Correlation*

$$r_{X,Y} = E(X \cdot Y)$$

- *Orthogonal X & Y:* $r_{X,Y} = 0$
- *Uncorrelated X & Y:* $Cov(X, Y) = 0$

  - *Correlation Coefficient*

$$\rho_{X,Y} = \frac{cov(X, Y)}{\sqrt{Var(X) \cdot Var(Y)}} = \frac{cov(X, Y)}{\sigma(X) \cdot \sigma(Y)}$$

CONDITIONING BY EVENT

- ***Probability Mass/Density Function***
  - ***Discrete***

$$P_{X,Y|B}(x,y) = \begin{cases} \dfrac{P_{X,Y|B}(x,y)}{P(B)}, (x,y) \in B \\ 0, otherwise \end{cases}$$

  - ***Continuous***

$$f_{X,Y|B}(x,y) = \begin{cases} \dfrac{f_{X,Y|B}(x,y)}{P(B)}, (x,y) \in B \\ 0, otherwise \end{cases}$$

- ***Expected Value $\boldsymbol{W = g(x, y)}$***
  - ***Discrete***

$$E(W|B) = \sum_{x \in S_X} \sum_{y \in S_Y} g(x,y) \cdot P_{X,Y|B}(x,y)$$

  - ***Continuous***

$$E(W|B) = \iint_{\mathbb{R}^2} g(x,y) \cdot P_{X,Y|B}(x,y)\, dxdy$$

- ***Variance***

$$Var(W|B) = E(W^2|B) - [E(W|B)]^2$$