CSP A

z_i's calculation depends on: $C^* + z_3^* z_2^* + z_3^* z_1^*$

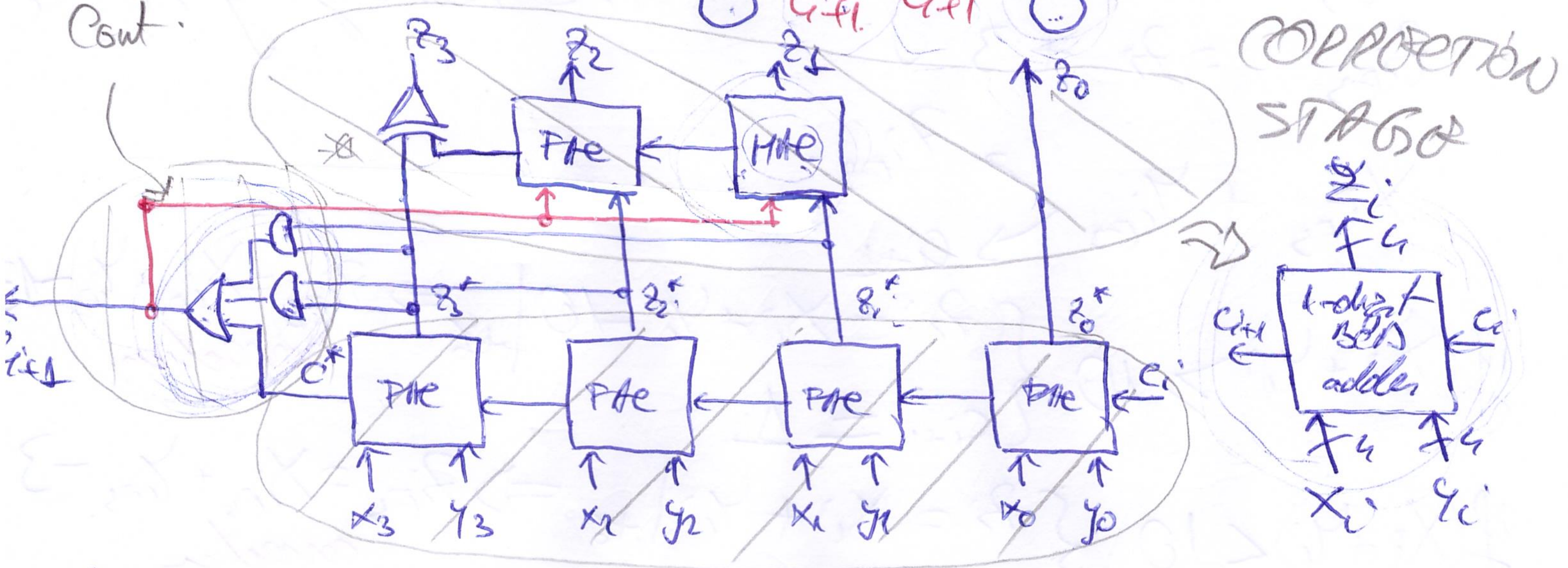
→ 1 $(x_i + y_i \geq 10)$:
- $z_i = z_3^* z_2^* z_1^* z_0^* + 0110$ ← correction
- $c_{i+1} = 1$

→ 0 $(x_i + y_i < 10)$:
- $z_i = z_3^* z_2^* z_1^* z_0^* + 0000$
- $c_{i+1} = 0$

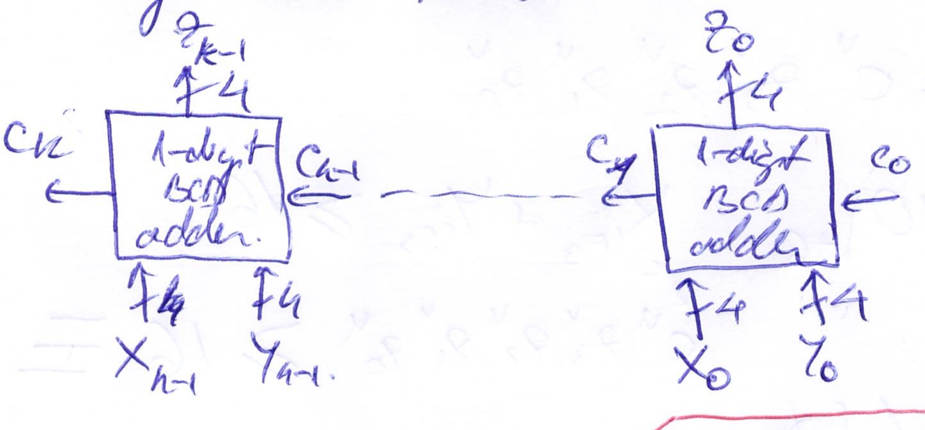
z_i's correction stage:

$$c_{i+1} = C^* + z_3^* z_2^* + z_3^* z_1^*$$

$$z_i = z_3^* \; z_2^* \; z_1^* \; z_0^* \; + \; 0 \; c_{i+1} \; c_{i+1} \; 0$$



Add 2 k-digit BCD numbers.



Example:

196 +
587
———
783

1	1	
0001 +	1001 +	0110 +
0101	1000	0111
0111 (<10)	10010 (≥10)	1101 (≥10)
0000	0110	0110
0111	1000	0011
7	8	3

(B) Excess-of-three addition

$X_{i_{E3}}, Y_{i_{E3}}, Z_{i_{E3}}$ $Z_{i_{E3}}$ = sum digit in E3 of $X_{i_{E3}} + Y_{i_{E3}}$

$X_{i_{E3}} = x_3 x_2 x_1 x_0$ $\underline{Z_{i_{E3}}} = z_3 z_2 z_1 z_0$

$Y_{i_{E3}} = y_3 y_2 y_1 y_0$

$$\left.\begin{array}{l} X_{i_{E3}} = X_i + 3 \\ Y_{i_{E3}} = Y_i + 3 \\ Z_{i_{E3}} = Z_i + 3 \end{array}\right\}$$ with X_i, Y_i, Z_i – BCD digits, Z_i = sum digit of $X_i + Y_i$

$X_{i_{E3}} + Y_{i_{E3}} \to Z_{i_{E3}}, c_{i+1}$

if $X_i + Y_i \geq 10$ $\begin{cases} Z_i = X_i + Y_i - 10 \\ c_{i+1} = 1 \end{cases}$ $\Big|_{+6} \Rightarrow Z_{i_{E3}} = X_{i_{E3}} + Y_{i_{E3}} - 13$ correction

if $X_i + Y_i < 10$ $\begin{cases} Z_i = X_i + Y_i \\ c_{i+1} = 0 \end{cases}$ $\Big|_{+6} \Rightarrow Z_{i_{E3}} = X_{i_{E3}} + Y_{i_{E3}} - 3$ correction

$X_{i_{E3}} + Y_{i_{E3}} = \underline{c^u\, z_3^u\, z_2^u\, z_1^u\, z_0^u}$

$X_i + Y_i \geq 10 \Big|_{+6}$ $X_{i_{E3}} + Y_{i_{E3}} \geq 16$

$\overset{2^4}{c^u}\, z_3^u z_2^u z_1^u z_0^u \geq 16 \equiv c^u$

Subtract 13 on 4 bits.

$(c^u z_3^u z_2^u z_1^u z_0^u - 13) \bmod 2^4 =$

$(\cancel{c^u} z_3^u z_2^u z_1^u z_0^u + 3) \bmod 2^4 =$

$z_3^u z_2^u z_1^u z_0^u +$
$0\ 0\ 1\ 1$

Similarly, subtracting 3 on 4 bits $\equiv$ adding 13

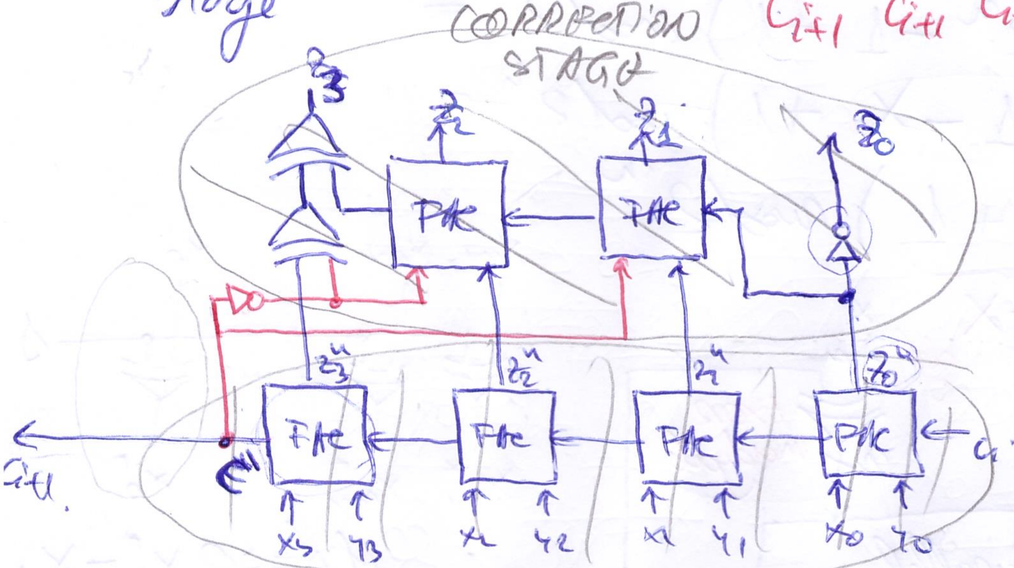
z_{iE3}'s correction depends on: C^u

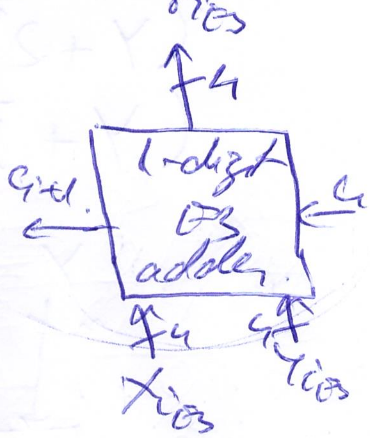
- $C^u = 1$ ($x_i + y_i \geq 10$):
  - $z_{iE3} = z_3^u\ z_2^u\ z_1^u\ z_0^u +$ $0\ 0\ 1\ 1$ correction
  - $c_{i+1} = 1$
- $C^u = 0$ ($x_i + y_i < 10$):
  - $z_{iE3} = z_3^u\ z_2^u\ z_1^u\ z_0^u +$ $1\ 1\ 0\ 1$ correction
  - $c_{i+1} = 0$

$$c_{i+1} = C^u$$

z_{iE3}'s correction stage:

$$z_{iE3} = z_3^u\ z_2^u\ z_1^u\ z_0^u + \overline{c_{i+1}}\ \overline{c_{i+1}}\ c_{i+1}\ 1$$





Example:

$$\begin{array}{r} 196 + \\ 587 \\ \hline 783 \end{array}$$

0 ←	0100	1 ← 1100	1 ← 1001 +
	1000	1011	1010
0	1101	1 1000	1 0011
	↳ 1101	↳ 0011	↳ 0011
*	1010	1011	0110
	7	8	3

1.3. Subtractors based on serial carry/borrow propagation

Y = subtrahend.

X = subtractor.

$Z = Y - X$

Ⓐ Use binary adders. – n-bit operands, unsigned.

$$X = x_{n-1}x_{n-2} \cdots x_1x_0$$

$$\overline{X} = \overline{x_{n-1}}\,\overline{x_{n-2}} \cdots \overline{x_1}\,\overline{x_0} \qquad \overline{x_i} = 1 - x_i$$

$$= \underbrace{1\ 1 \cdots 1\ 1}_{\langle n \rangle} - x_{n-1}x_{n-2} \cdots x_1x_0$$

– no borrow is generated.

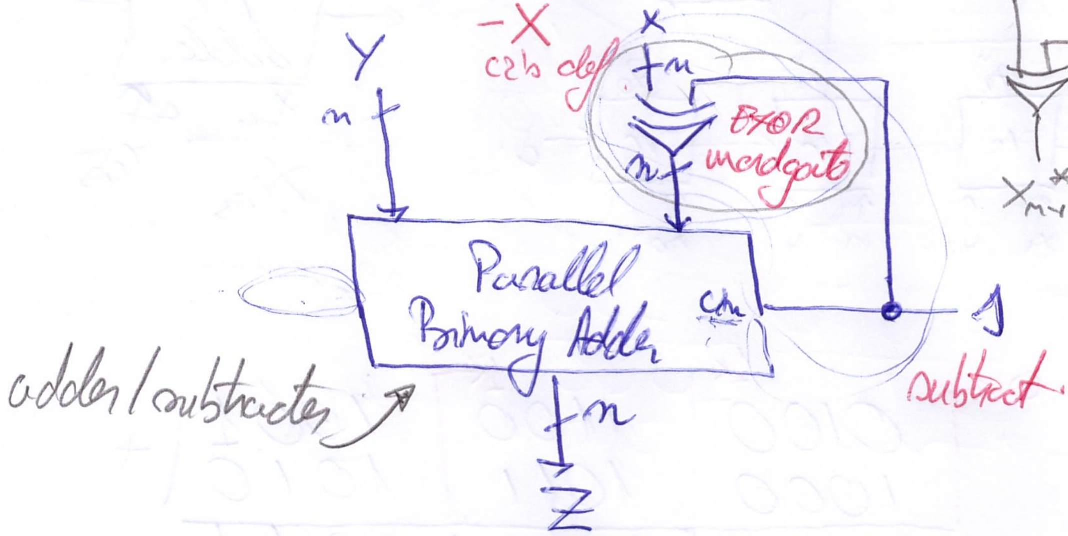
$$= 2^n - 1 - X$$

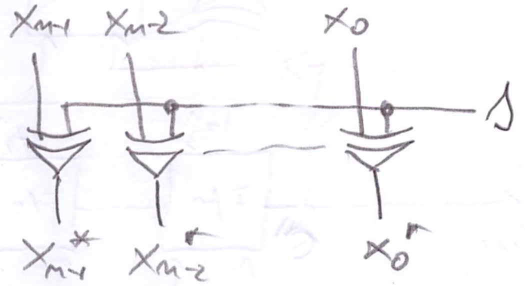
$$Z = (Y - X) \bmod 2^n =$$

$$(Y - X + 2^n - 1 + 1) \bmod 2^n =$$

$$(Y + 2^n - 1 - X + 1) \bmod 2^n =$$

$$(Y + \underbrace{\overline{X} + 1}_{-X}) \bmod 2^n$$





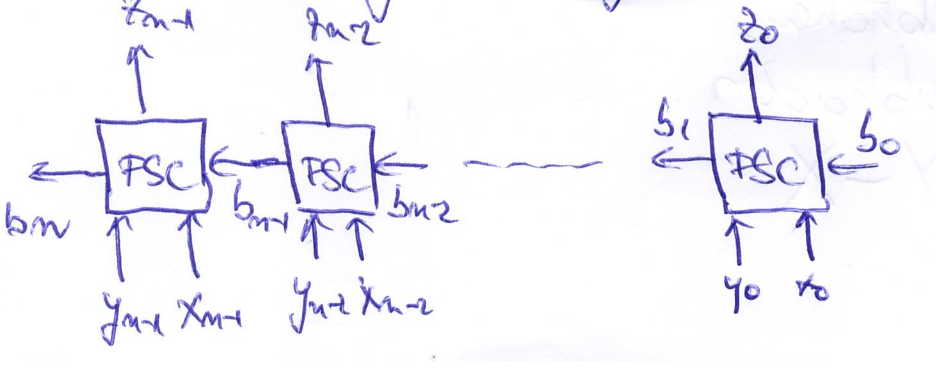
If $s == 0$ $x^*_i = x_i$

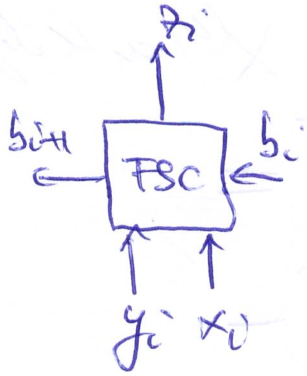
If $s == 1$ $x^*_i = \overline{x_i}$

$$s \begin{cases} 1: Z = Y + \overline{X} + 1 = Y - X \\ 0: Z = Y + X + 0 = Y + X \end{cases}$$

Ⓑ Using dedicated subtracters: – n bit operands

– like RCA: serially connecting Full Subtracter Cells (FSCs)





$y_i - x_i - b_i \to z_i,\ b_{i+1}$

inputs			Output	
y_i	x_i	b_i	z_i	b_{i+1}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$y_i \quad x_i \quad b_i$

$0 - 0 - 1 =$

b_{i+1}

$1 \; 0 = 2 -$

$\underline{\quad 1 \quad}$

1_{z_i}

$0-1-0 \equiv 0-0-1$

$1-1-1 = 0-0-1$

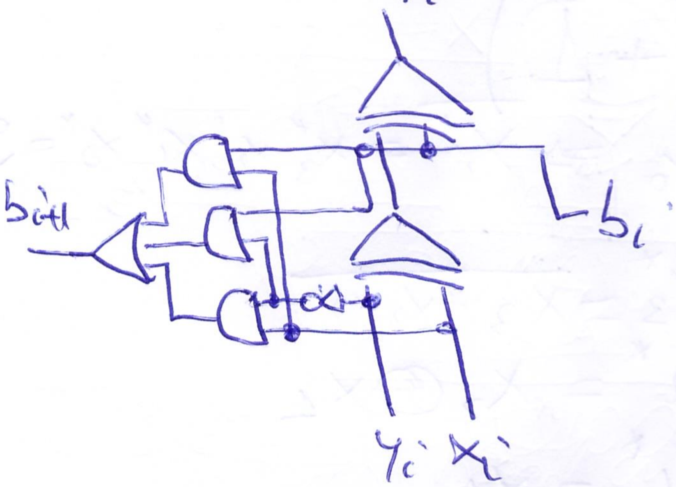
z_i: $y_i \backslash x_i b_i$

$y_i \backslash x_i b_i$	00	01	11	10
0	0	1	0	1
1	1	0	1	0

b_{i+1}: $y_i \backslash x_i b_i$

$y_i \backslash x_i b_i$	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$z_i = y_i \oplus x_i \oplus b_i$ $\qquad$ $b_{i+1} = \overline{y_i}\, x_i + \overline{y_i}\, b_i + x_i b_i$



© BCD subtraction:

Let $Y^{(k)} = Y_{n-1} Y_{n-2} \_\_\_ Y_0$ $\qquad$ X_i, Y_i - BCD digit on 4 bits.

$X^{(k)} = X_{n-1} X_{n-2} \_\_\_ X_0$

$Z^{(k)} = Y^{(k)} - X^{(k)}$

Let X_i is a BCD digit $X_i = x_3 x_2 x_1 x_0$

$\to \overline{X_i^*} = 9$'s complement of X_i

$\overline{X_i^*} = 9 - X_i \Rightarrow$ NO borrow!

Notation $\overline{X^{*(k)}}$ = 9's complement of number $X^{(k)}$ on k digits

$$\overline{X^{*(k)}} = \overline{X^*_{k-1}}\;\overline{X^*_{k-2}} \;---\; \overline{X^*_0} =$$

$$\overset{\text{<k-digits>}}{\begin{array}{cccc} 9 & 9 & \text{———} & 9 \\ X_{k-1} & X_{k-2} & \text{———} & X_0 \end{array}} -$$

$$= 10^k - 1 - X^{(k)}$$

$$Z^{(k)} = \left(Y^{(k)} - X^{(k)} \right) \bmod 10^k$$

$$= \left(Y^{(k)} + 10^k - 1 - X^{(k)} + 1 \right) \bmod 10^k$$

$$= \left(Y^{(k)} + \overline{X^{*(k)}} + 1 \right)$$

9's complement of $X_i = x_3 x_2 x_1 x_0 \Rightarrow \overline{X_i^*} = x_3^* x_2^* x_1^* x_0^* = 9 - X_i$

Inputs X_i				Outputs $\overline{X_i^*}$			
x_3	x_2	x_1	x_0	x_3^*	x_2^*	x_1^*	x_0^*
0	0	0	0	1	0	0	1
0	0	0	1	1	0	0	0
0	0	1	0	0	1	1	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	0	0
0	1	1	0	0	0	1	1
0	1	1	1	0	0	1	0
1	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0

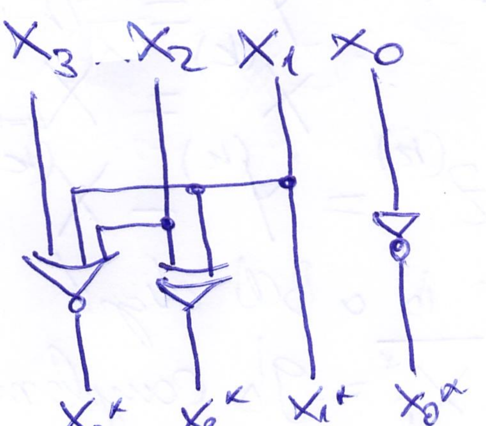
$$x_3^* = \overline{x_3 + x_2 + x_1}$$

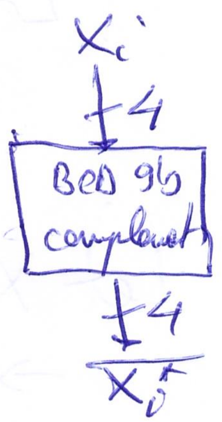
$$x_2^* = x_2 \oplus x_1$$

$$x_1^* = x_1$$

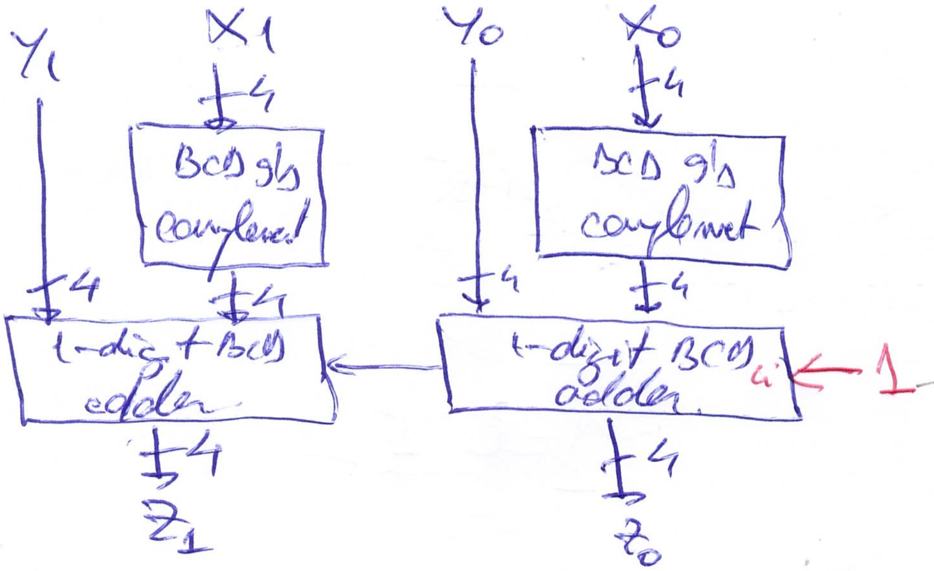
$$x_0^* = \overline{x_0}$$

* for all input $x_3x_2x_1x_0 \in [10:15]$, outputs ≡ don't cares





Subtracting 2-digit numbers $Y^{(2)} - X^{(2)}$



$$\begin{array}{r} 703 \; - \\ 389 \\ \hline 314 \end{array}$$

9's complement of 389

$$\begin{array}{rrr} & & 1 \\ 0111 & 0000 & 0011 \; + \\ 0110 & 0001 & 0000 \\ \hline 1101 \; + & 0001 & 0100 \\ 0110 & \underbrace{}_{1} & \underbrace{}_{4} \\ \hline \underbrace{0011}_{3} & & \end{array}$$