2. exponential $\left(\frac{1}{3}\right)$

$$\Rightarrow f_T(t) = \begin{cases} \frac{1}{3} e^{-\frac{t}{3}}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow F_T(t) = \begin{cases} 1 - e^{-\frac{t}{3}}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$f_{T|(t\geq 2)}(t) = \begin{cases} \frac{f_T(t)}{P((t\geq 2))}, & t \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned} P((t\geq 2)) &= 1 - P((t<2)) \\ &= 1 - \int_0^2 f_T(t)\,dt \\ &= 1 - \int_0^2 \frac{1}{3} e^{-\frac{t}{3}}\,dt \\ &= 1 + \int_0^2 -\frac{1}{3} e^{-\frac{1}{3}\cdot t}\,dt \\ &= 1 + e^{-\frac{t}{3}}\Big|_0^2 = 1 + \left(e^{-\frac{2}{3}} - e^{-\frac{0}{3}}\right) \\ &= 1 + e^{-\frac{2}{3}} - 1 = e^{-\frac{2}{3}} = \frac{1}{e^{\frac{2}{3}}} \end{aligned}$$

$$\Rightarrow f_{T|(t\geq 2)}(t) = \begin{cases} \frac{1}{3} e^{-\frac{t}{3}} \cdot e^{\frac{2}{3}}, & t \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{3} e^{\frac{2-t}{3}}, & t \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

$$F_{T|(t\geq 2)}(t) = \int_2^t f_{T|x\geq 2}(x)\,dx$$

$$= \int_2^t \frac{1}{3} e^{\frac{2-x}{3}} dx$$

$$= -e^{\frac{2-x}{3}} \Big|_2^t = (-e^{\frac{2-t}{3}} + e^0)$$

$$= 1 - e^{\frac{2-x}{3}}$$

$$\Rightarrow F_{T|(t\geq 2)}(t) = \begin{cases} 1 - e^{\frac{2-x}{3}}, & t \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

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CTI - EN. 3.1

EXAM SM

1. $f_{X,Y}(x,y) = \begin{cases} \frac{5x^2}{2} & -1 \le x \le 1,\ 0 \le y \le x^2 \\ 0, \text{ otherwise} \end{cases}$

$x^2 \ge y$
$\Rightarrow |x| \ge \sqrt{y}$
$\Rightarrow x \in (-\infty, -\sqrt{y}) \cup (\sqrt{y}, \infty)$
$-1 \le x \le 1$
$\Rightarrow x \in (-1, -\sqrt{y}) \cup (\sqrt{y}, 1)$

a) $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$

$= \int_{-1}^{-\sqrt{y}} f_{X,Y}(x,y)\,dx + \int_{\sqrt{y}}^{1} f_{X,Y}(x,y)\,dx$

$= \int_{-1}^{-\sqrt{y}} \frac{5x^2}{2}\,dx + \int_{\sqrt{y}}^{1} \frac{5x^2}{2}\,dx$

$= \frac{5x^3}{6}\Big|_{-1}^{-\sqrt{y}} + \frac{5x^3}{6}\Big|_{\sqrt{y}}^{1}$

$= \frac{-5y^{\frac{3}{2}}}{6} + \frac{5}{6} + \frac{5}{6} - \frac{5y^{\frac{3}{2}}}{6}$

$= \frac{10}{6}\left(-y^{\frac{3}{2}} + 1\right)$

$\Rightarrow f_Y(y) = \begin{cases} \frac{10}{6}\left(-y^{\frac{3}{2}} + 1\right), & 0 \le y \le 1 \\ 0, \text{ otherwise} \end{cases}$

c) $f_{Y|A}(y) = \begin{cases} \frac{f_Y(y)}{P(Y \le \frac{1}{4})}, & 0 \le y \le \frac{1}{4} \\ 0, \text{ otherwise} \end{cases}$

$$P(Y \le \tfrac{1}{4}) = \int_0^{\frac{1}{4}} f_Y(y)\,dy = \int_0^{\frac{1}{4}} \frac{10}{6}(-y^{\frac{3}{2}}+1)\,dy$$

$$= -\frac{2y^{\frac{5}{2}}}{3} + \frac{5y}{3}\Big|_0^{\frac{1}{4}}$$

$$= \frac{19}{48}$$

$$\Rightarrow f_{Y|(Y\le\frac{1}{4})}(y) = \begin{cases} \frac{5}{3}(-y^{\frac{3}{2}}+1)\cdot\frac{48}{19}, & 0 \le y \le \frac{1}{4} \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{80}{19}(-y^{\frac{3}{2}}+1), & 0 \le y \le \frac{1}{4} \\ 0, & \text{otherwise} \end{cases}$$

b) $\mathrm{cov}(X,Y) = E(X\cdot Y) - E(X)E(Y)$

$$E(X\cdot Y) = \iint_{-\infty}^{\infty} x\cdot y\, f_{X,Y}(x,y)\,dx\,dy$$

$$= \int dx \int_0^{x^2} y\cdot\frac{5x^3}{2}\,dy$$

$$= \int dx \cdot \frac{5x^3y^2}{4}\Big|_0^{x^2}$$

$$= \int \frac{5x^7}{4}\,dx$$

$$= \frac{5x^8}{32}\Big|_{-1}^{1} = \frac{10}{32} = \frac{5}{16}$$