CA C14 $\qquad -8;7$ $\qquad$ (1)

$X = -4 \quad Y = +7$

$X = -4 = 1100_{C2}$

$Y = +7 = 0111_{C2}$

$16 \cdot 6 = 96$

$P = 11100100_{C2}$

$= -128 + \underbrace{110}_{6\times}\underbrace{0100}_{4}$

$= -128 + 6\times16 + 4$

$= -128 + 100 = -28.$

$P = X * Y = -4 * 7 = -28$

A	Q	M	COUNT
0000		0111	00
	11000		
0000	01100		01
			10
0000	00110		
- 0111			
1001			11
1100	10011		←COUNT3 = 1
1110	01001		

4.6. Combinational array structures for binary multiplication

$X, Y =$ unsigned, 4 bits, integer $\quad X = x_3x_2x_1x_0 \quad Y = y_3y_2y_1y_0$

$$P = X * Y = \left(\sum_{i=0}^{3} x_i \cdot 2^i\right) * \left(\sum_{j=0}^{3} y_j \cdot 2^j\right) =$$

$$= \sum_{i=0}^{3} 2^i \cdot \left(\sum_{j=0}^{3} x_i \cdot y_j \cdot 2^j\right)$$

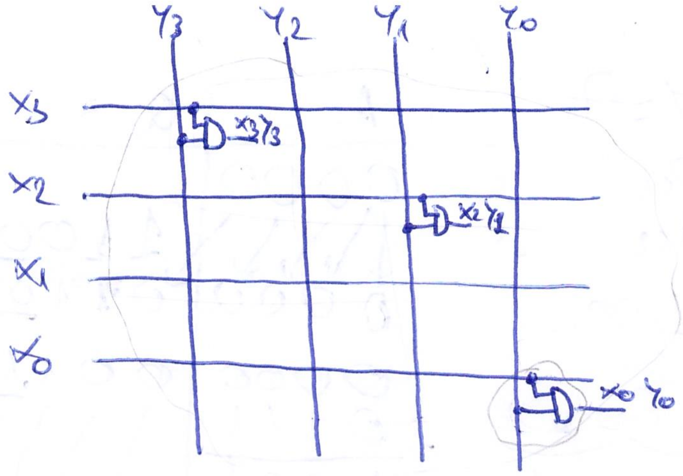
$$\begin{aligned} P = {} & 2^0(x_0 y_0 \cdot 2^0 + x_0 \cdot y_1 \cdot 2^1 + x_0 \cdot y_2 \cdot 2^2 + x_0 y_3 2^3) + \\ & 2^1(x_1 \cdot y_0 \cdot 2^0 + x_1 \cdot y_1 \cdot 2^1 + x_1 \cdot y_2 \cdot 2^2 + x_1 \cdot y_3 \cdot 2^3) + \\ & 2^2(x_2 \cdot y_0 \cdot 2^0 + x_2 \cdot y_1 \cdot 2^1 + x_2 \cdot y_2 \cdot 2^2 + x_2 \cdot y_3 \cdot 2^3) + \\ & 2^3(x_3 \cdot y_0 \cdot 2^0 + x_3 \cdot y_1 \cdot 2^1 + x_3 \cdot y_2 \cdot 2^2 + x_3 \cdot y_3 \cdot 2^3) \end{aligned}$$

$$\begin{aligned} P = {} & 2^6 \cdot x_3 y_3 + \\ & 2^5 \cdot (x_3 y_2 + x_2 y_3) + \\ & 2^4 \cdot (x_3 y_1 + x_2 y_2 + x_1 y_3) + \\ & 2^3 \cdot (x_3 y_0 + x_2 y_1 + x_1 y_2 + x_0 y_3) + \\ & 2^2 \cdot (x_2 y_0 + x_1 y_1 + x_0 y_2) + \\ & 2^1 \cdot (x_1 y_0 + x_0 y_1) + \\ & 2^0 \cdot x_0 y_0 \end{aligned}$$

P: 2 matrices

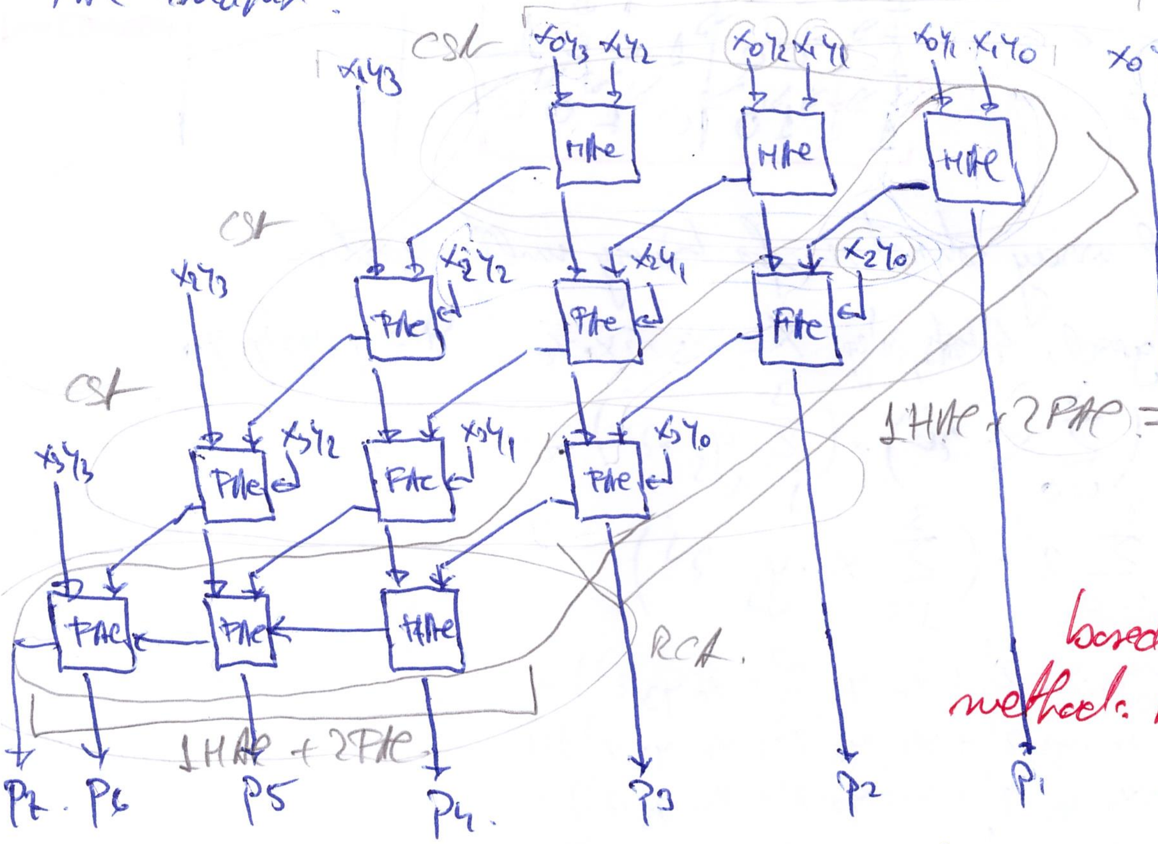
- AND matrix $\to x_i \cdot y_j$
- FAC matrix $\to x_3 y_2 + x_2 y_3$

AND matrix:



$n-1$ cells
1 HAe.
$n-2$ PAe.
$((n-2)\cdot 2+1)d.$
$(2n-3)d.$

PAe matrix.



n bit $\to 2\cdot n-3$

4 bit $\to 5d.$
$(2\cdot 4-3)$

1 HAe + 2PAe = $1d+4d.=5d.$

the structure is based on the second mult. method. Keeping the partial products fixed

Latency $D = D_{\substack{AND\\matrix}} + D_{\substack{PAE\\matrix}} = 1d + 2(2n-3)d = (4n-5)d.$

Area $A = A_{\substack{AND\\matrix}} + A_{\substack{PAE\\matrix}} = n^2 \cdot A_{\substack{AND\\gate}} + n(n-1)\cdot A_{PAE/HAE}$

Not favorable for VLSI implementation.
- wire crossing.
- lack of regularity

Signed operands multiplication.
- use Booth method.
  - each layer can either add to, subtract from or not modify the previous partial product.
  $\Rightarrow$ at the output of each layer, a partial product is delivered.

input $x_i\ x_{i-1}$ pair → decide the operation of the layer.

- if $x_i x_{i-1} == 01$ → add $y_j \cdot 2^j$ to the previous p.product
- if $x_i x_{i-1} == 10$ → subtract $y_j \cdot 2^j$ from — " —
- if $x_i x_{i-1} == 00$, or 11 → leave the previous p.product unchanged.

Design a cell operating like ↗

- only use y_j
- perform either +, −, or not modify the input
  - → + and − : carry chain adder/subtracter
    - → use carries
  - need 2 control lines for selecting one of the 3 operations (+, −, NOP)
- receive one bit of the previous partial product → z
- generate one bit of the next partial product.

Operation	Outputs w	c_{out}
Addition	$w = y_j \oplus z \oplus c_{in}$	$c_{out} = y_j \cdot z + y_j \cdot c_{in} + z \cdot c_{in}$
Subtraction	$w = y_j \oplus z \oplus c_{in}$	$c_{out} = y_j \cdot \bar{z} + y_j \cdot c_{in} + \bar{z} \cdot c_{in}$
NOP	$w = z$	irrelevant

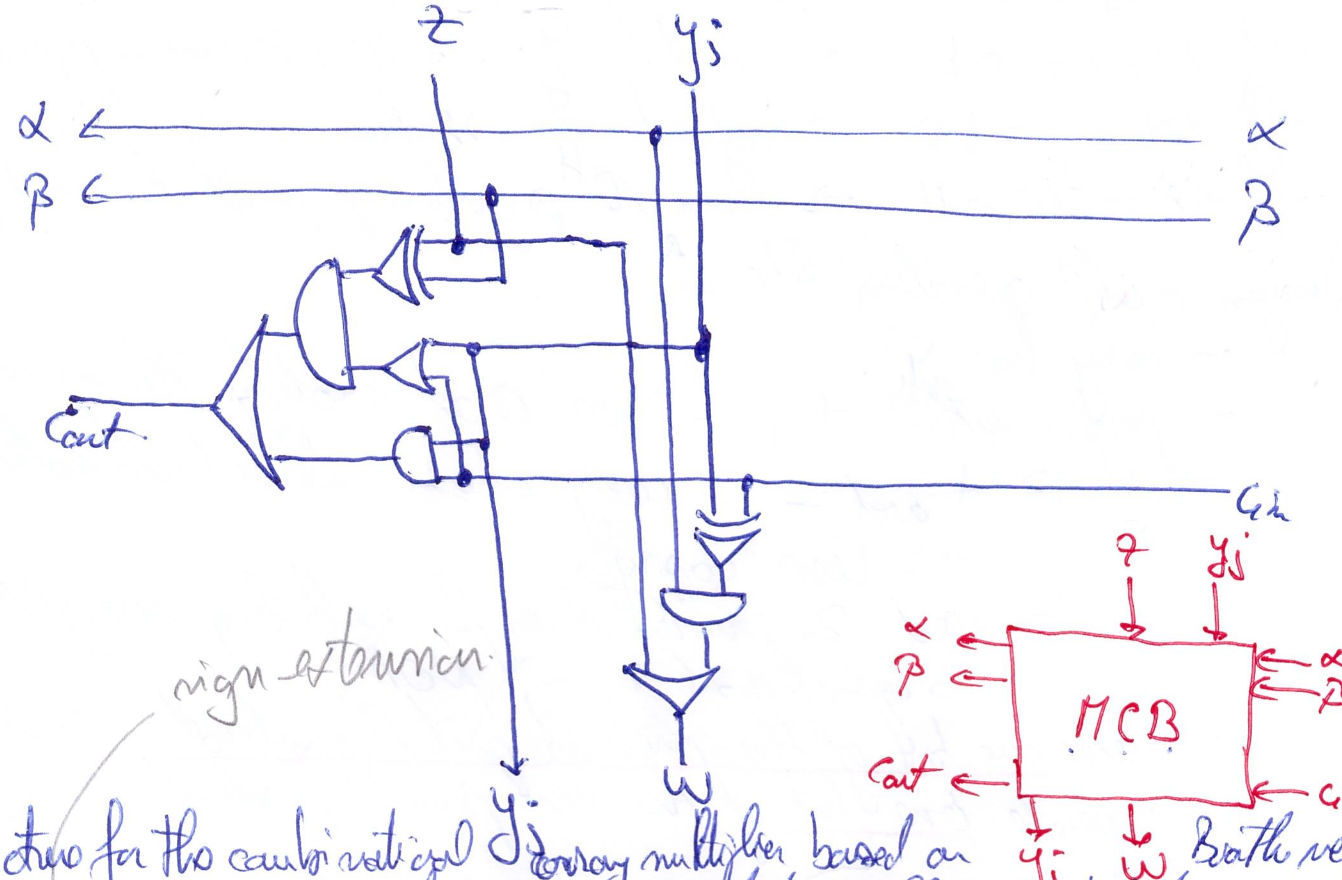
Encode the 3 operations as 2 control variables α, β

Compute w and c_{out}, using α, β.

Operation	Control variable α	β
Addition	1	0
Subtraction	1	1
NOP	0	d

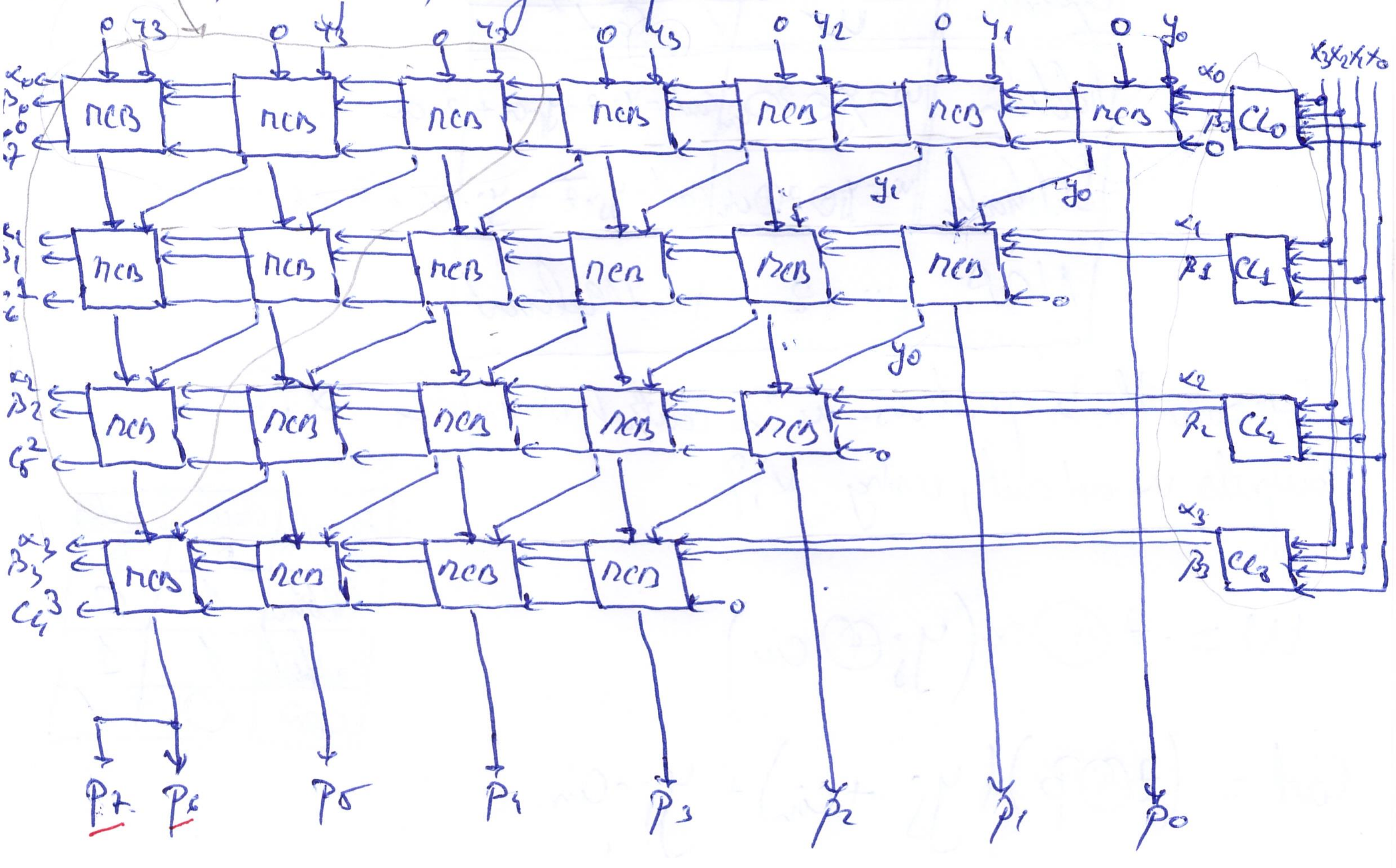
$$w = z \oplus \alpha \cdot (y_j \oplus c_{in})$$

$$c_{out} = (z \oplus \beta)(y_j + c_{in}) + y_j \cdot c_{in}$$



sign extension

Architecture for the combinational array multiplier based on Booth method.
Consider 2, , integers represented in C2, on 4 bits.



Ex: $x = +5 = x_3x_2x_1x_0 = 0\,1\,0\,1\,0$

$y = -6 = 1\,0\,1\,0$

0 0 0 0 0 0 0 0	$P_0 := 0$	−
1 1 1 1 0 1 0	$Y \cdot 2^0$	
0 0 0 0 0 1 1 0	$P_1 := P_0 - Y \cdot 2^0$	+
1 1 1 0 1 0	$Y \cdot 2^1$	
1 1 1 1 0 1 0	$P_2 := P_1 + Y \cdot 2^1$	−
1 1 0 1 0	$Y \cdot 2^2$	
0 0 1 0 0 0 1 0	$P_3 := P_2 - Y \cdot 2^2$	+
1 0 1 0	$Y \cdot 2^3$	
1 1 1 0 0 0 1 0	$P_4 := P_3 + Y \cdot 2^3 = P$	

sign extension

$P = 11100010 = -128 + \underbrace{110}_{6}\underbrace{0010}_{2} = -128 + 96 + 2 = -128 + 98 = -30$

$P = x * y = +5 * (-6) = -30$

Generating α_i, β_i

NOP $\alpha_i \beta_i = 0\,d$ for NOP ($x_i x_{i-1} = 00$, $x_i x_{i-1} = 11$)

ADD $\alpha_i \beta_i = 1\,0$ for ADD ($x_i x_{i-1} = 01$)

SUB $\alpha_i \beta_i = 1\,1$ for SUB ($x_i x_{i-1} = 10$)

Input				Booth recoded				Output							
x_3	x_2	x_1	x_0	x_{3B}	x_{2B}	x_{1B}	x_{0B}	α_3	β_3	α_2	β_2	α_1	β_1	α_0	β_0
0	0	0	00	0	0	0	0	0	d	0	d	0	d	0	d
0	0	0	10	0	0	1	$\bar{1}$	0	d	0	d	1	0	1	1
0	0	1	0	0	1	$\bar{1}$	0	0	d	1	0	1	1	0	d
0	0	1	1	0	1	0	$\bar{1}$	0	d	1	0	0	d	1	1
0	1	0	0	1	$\bar{1}$	0	0	1	0	1	1	0	d	0	d
0	1	0	1	1	$\bar{1}$	1	$\bar{1}$	1	0	1	1	1	0	1	1
1	1	0	0	0	$\bar{1}$	0	0	0	d	1	1	0	d	0	d
1	1	0	1	0	$\bar{1}$	1	$\bar{1}$	0	d	1	1	1	0	1	1
1	1	1	0	0	0	$\bar{1}$	0	0	d	0	d	1	1	0	d
1	1	1	1	0	0	0	$\bar{1}$	0	d	0	d	0	d	1	1

After minimization:

$\alpha_0 = x_0$

$\beta_0 = 1$

$\alpha_1 = x_1 \oplus x_0$

$\beta_1 = x_1$

$\alpha_2 = x_2 \oplus x_1$

$\beta_2 = x_2$

$\alpha_3 = x_3 \oplus x_2$

$\beta_3 = x_3$