$$\mathcal{L}\{2t^2\} = \int_0^\infty e^{-st} \cdot 2t^2 \, dt$$

$$f = t^2 \Rightarrow f' = 2t$$
$$g' = e^{-st} \Rightarrow g = -\frac{1}{s} \cdot e^{-st}$$

$$= -\frac{1}{s} \cdot e^{-st} \cdot 2t^2 \Big|_0^\infty + \int_0^\infty \frac{4t \cdot e^{-st}}{s} \, dt$$

$$= \frac{-2t^2 \cdot e^{-st}}{s}\Big|_0^\infty + \frac{4}{s} \int_0^\infty t \cdot e^{-st} \, dt$$

$$f = t \Rightarrow f' = 1$$
$$g' = e^{-st} \Rightarrow g = -\frac{1}{s} \cdot e^{-st}$$

$$= \frac{-2t^2 \cdot e^{-st}}{s}\Big|_0^\infty + \frac{4}{s}\left(-\frac{t \cdot e^{-st}}{s}\Big|_0^\infty - \frac{1}{s}\int_0^\infty e^{-st} \, dt\right)$$

$$= \frac{-2t^2 \cdot e^{-st}}{s}\Big|_0^\infty + \frac{4}{s}\left(\frac{-t \cdot e^{-st}}{s}\Big|_0^\infty + \frac{1}{s^2} \cdot e^{-st}\Big|_0^\infty\right)$$

$$= -\frac{2t^2 \cdot e^{-st}}{s}\Big|_0^\infty - \frac{4te^{-st}}{s^2}\Big|_0^\infty - \frac{4e^{-st}}{s^3}\Big|_0^\infty$$

$$= (0-0) - (0-0) - \left(0 - \frac{4}{s^3}\right)$$

$$\mathcal{L}\{3e^{-t}\} = \int_0^\infty e^{-st}\, 3e^{-t}\, dt = 3\int_0^\infty e^{-t(s+1)}\, dt$$

$$= \frac{1}{-1-s} e^{t(-1-s)}\Big|_0^\infty = \left(\lim_{t \to \infty} e^{t(-1-s)} - \lim_{t \to 0} e^{t(-1-s)}\right) \cdot \frac{1}{-1-s} = \frac{1}{-1-s}(0-1) = \frac{-1}{-1-s} = \frac{1}{1+s}$$

2. b) $\mathcal{L}^{-1}\left\{\frac{2s+4}{(s+1)(s^2-2)}\right\}$

$$\Rightarrow \frac{2s+4}{(s+1)(s^2-2)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-2} = \frac{A(s^2-2)+(Bs+C)(s+1)}{(s+1)(s^2-2)}$$

let $s=-1 \Rightarrow 2 = A(-1) + (Bs+C)\cdot 0 \Rightarrow A=-2$

let $s=0 \Rightarrow 4 = -2(-2) + C \Rightarrow C=0$

let $s=1 \Rightarrow 6 = -2\cdot(-1) + 2B \Rightarrow 4 = 2B \Rightarrow B=2$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{2s+4}{(s+1)(s^2-2)}\right\} = -2\cdot\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 2\cdot\mathcal{L}^{-1}\left\{\frac{s}{s^2-2}\right\}$$

$$= -2\cdot e^{-t} + 2\cdot\cos(i\sqrt{2}\,t)$$

a) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+2s+1}\right\}$

$$\frac{1}{s^2+2s+1} = \frac{1}{(s+1)^2}$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s^2+2s+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = t\cdot e^{-t}$$