

# Building Models

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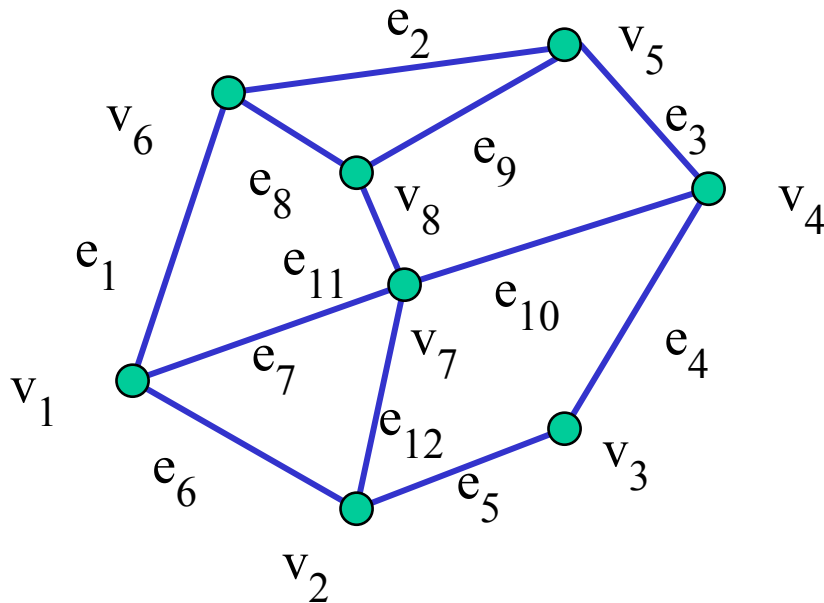
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# Objectives

- Introduce simple data structures for building polygonal models
  - Vertex lists
  - Edge lists

# Representing a Mesh

- Consider a mesh
- There are 8 nodes and 12 edges
  - 5 interior polygons
  - 6 interior (shared) edges
- Each vertex has a location  $v_i = (x_i \ y_i \ z_i)$



# Simple Representation

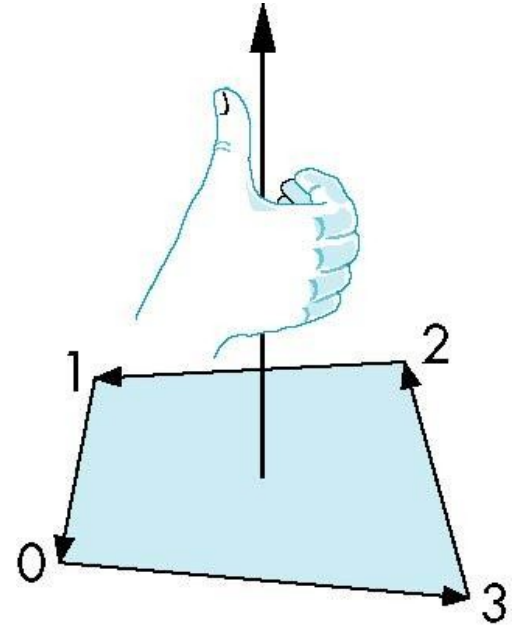
- Define each polygon by the geometric locations of its vertices
- Leads to WebGL code such as

```
vertex.push(vec3(x1, y1, z1));  
vertex.push(vec3(x6, y6, z6));  
vertex.push(vec3(x7, y7, z7));
```

- Inefficient and unstructured
  - Consider moving a vertex to a new location
  - Must search for all occurrences

# Inward and Outward Facing Polygons

- The order  $\{v_1, v_2, v_7\}$  and  $\{v_7, v_2, v_1\}$  are equivalent in that the same polygon will be rendered by OpenGL but the order  $\{v_1, v_2, v_7\}$  is different
- The first two describe *outwardly facing* polygons
- Use the *right-hand rule* = counter-clockwise encirclement of outward-pointing normal
- OpenGL can treat inward and outward facing polygons differently

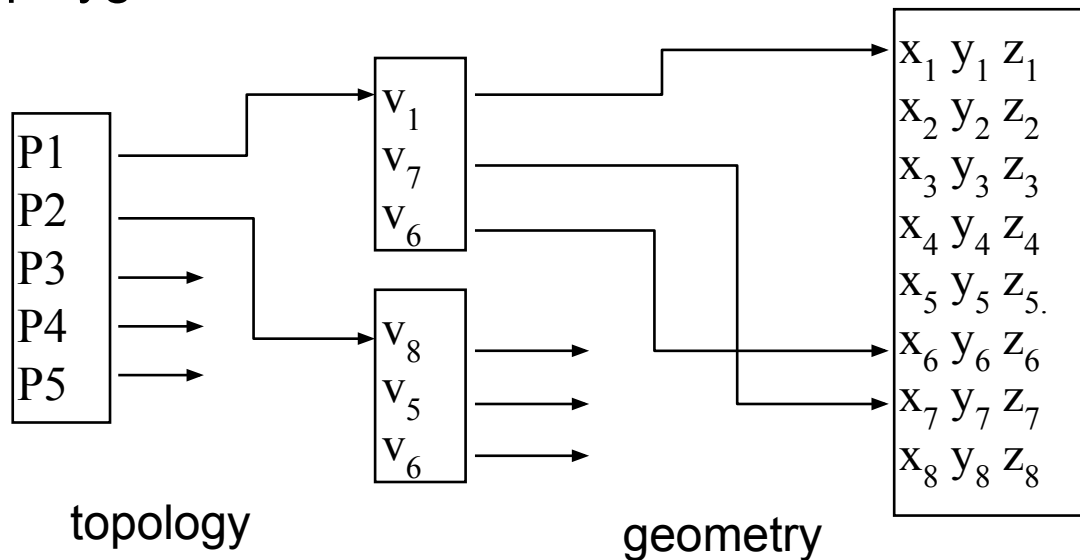


# Geometry vs Topology

- Generally it is a good idea to look for data structures that separate the geometry from the topology
  - Geometry: locations of the vertices
  - Topology: organization of the vertices and edges
  - Example: a polygon is an ordered list of vertices with an edge connecting successive pairs of vertices and the last to the first
  - Topology holds even if geometry changes

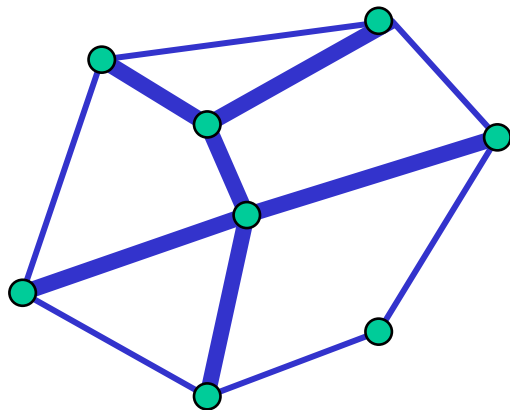
# Vertex Lists

- Put the geometry in an array
- Use pointers from the vertices into this array
- Introduce a polygon list



# Shared Edges

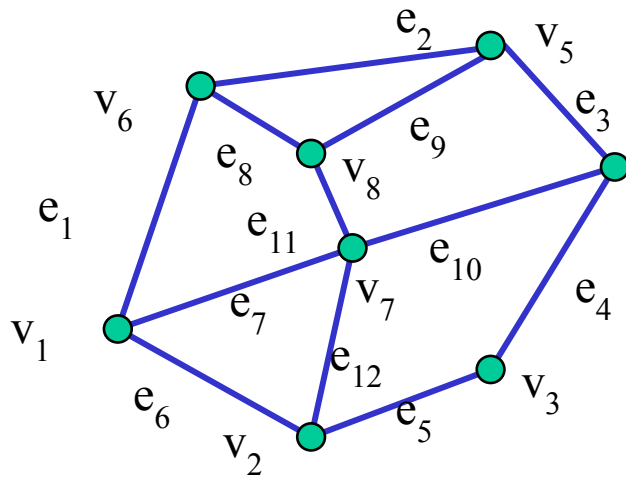
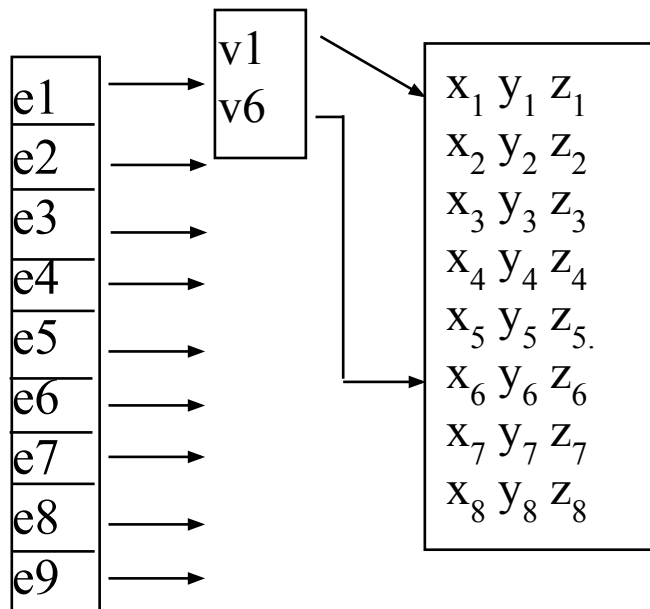
- Vertex lists will draw filled polygons correctly but if we draw the polygon by its edges, shared edges are drawn twice



- Can store mesh by *edge list*



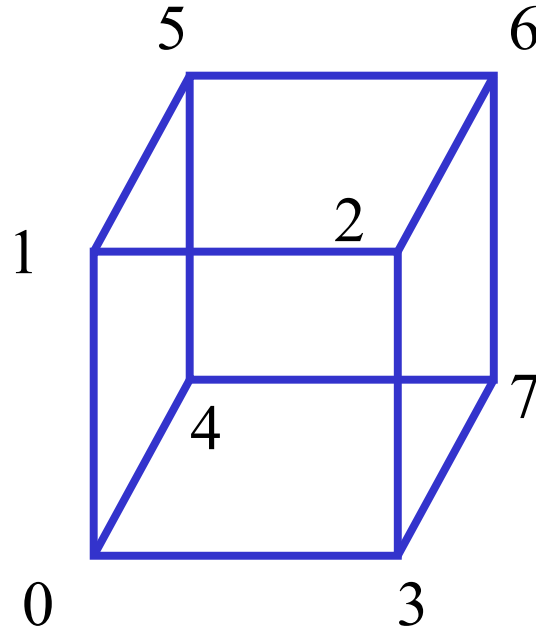
# Edge List



Note polygons are  
not represented

# Draw cube from faces

```
var colorCube( )  
{  
    quad(0,3,2,1) ;  
    quad(2,3,7,6) ;  
    quad(0,4,7,3) ;  
    quad(1,2,6,5) ;  
    quad(4,5,6,7) ;  
    quad(0,1,5,4) ;  
}
```



# The Rotating Square

# Objectives

- Put everything together to display rotating cube
- Two methods of display
  - by arrays
  - by elements

## Video:

[webgl/Code/w05/cube.html](http://webgl/Code/w05/cube.html)

# Modeling a Cube

Define global array for vertices

```
var vertices = [  
    vec3( -0.5, -0.5,  0.5 ),  
    vec3( -0.5,  0.5,  0.5 ),  
    vec3(  0.5,  0.5,  0.5 ),  
    vec3(  0.5, -0.5,  0.5 ),  
    vec3( -0.5, -0.5, -0.5 ),  
    vec3( -0.5,  0.5, -0.5 ),  
    vec3(  0.5,  0.5, -0.5 ),  
    vec3(  0.5, -0.5, -0.5 )  
];
```

# Colors

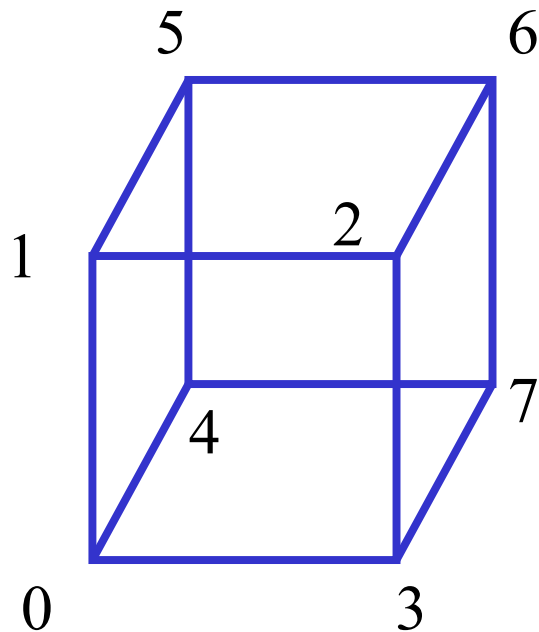
Define global array for colors

```
var vertexColors = [  
    [ 0.0, 0.0, 0.0, 1.0 ], // black  
    [ 1.0, 0.0, 0.0, 1.0 ], // red  
    [ 1.0, 1.0, 0.0, 1.0 ], // yellow  
    [ 0.0, 1.0, 0.0, 1.0 ], // green  
    [ 0.0, 0.0, 1.0, 1.0 ], // blue  
    [ 1.0, 0.0, 1.0, 1.0 ], // magenta  
    [ 0.0, 1.0, 1.0, 1.0 ], // cyan  
    [ 1.0, 1.0, 1.0, 1.0 ] // white  
];
```

# Draw cube from faces

```
function colorCube( )  
{  
    quad(0,3,2,1) ;  
    quad(2,3,7,6) ;  
    quad(0,4,7,3) ;  
    quad(1,2,6,5) ;  
    quad(4,5,6,7) ;  
    quad(0,1,5,4) ;  
}
```

Note that vertices are ordered so that we  
obtain correct outward facing normals  
Each quad generates two triangles





# Initialization

```
var canvas, gl;
var numVertices = 36;
var points = [];
var colors = [];

window.onload = function init(){
    canvas = document.getElementById( "gl-canvas" );
    gl = WebGLUtils.setupWebGL( canvas );

    colorCube();

    gl.viewport( 0, 0, canvas.width, canvas.height );
    gl.clearColor( 1.0, 1.0, 1.0, 1.0 );
    gl.enable(gl.DEPTH_TEST);

    // rest of initialization and html file
    // same as previous examples
```

# The quad Function

Put position and color data for two triangles from a list of indices into the array **vertices**

```
var quad(a, b, c, d)
{
    var indices = [ a, b, c, a, c, d ];
    for ( var i = 0; i < indices.length; ++i ) {
        points.push( vertices[indices[i]] );
        colors.push( vertexColors[indices[i]] );
    }
    // for solid colored faces use
    // colors.push(vertexColors[a]);
}
```

# Render Function

```
function render(){  
    gl.clear( gl.COLOR_BUFFER_BIT |gl.DEPTH_BUFFER_BIT);  
    gl.drawArrays( gl.TRIANGLES, 0, numVertices );  
    requestAnimationFrame( render );  
}
```

# Mapping indices to faces

```
var indices = [  
  1,0,3,  
  3,2,1,  
  2,3,7,  
  7,6,2,  
  3,0,4,  
  4,7,3,  
  6,5,1,  
  1,2,6,  
  4,5,6,  
  6,7,4,  
  5,4,0,  
  0,1,5  
];
```

# Rendering by Elements

- Send indices to GPU

```
var iBuffer = gl.createBuffer();  
gl.bindBuffer(gl.ELEMENT_ARRAY_BUFFER, iBuffer);  
gl.bufferData(gl.ELEMENT_ARRAY_BUFFER,  
              new Uint8Array(indices), gl.STATIC_DRAW);
```

- Render by elements

```
gl.drawElements( gl.TRIANGLES, numVertices,  
                 gl.UNSIGNED_BYTE, 0 );
```

- Even more efficient if we use triangle strips or triangle fans

# Adding Buttons for Rotation

```
var xAxis = 0;
var yAxis = 1;
var zAxis = 2;
var axis = 0;
var theta = [ 0, 0, 0 ];
var thetaLoc;
```

```
document.getElementById( "xButton" ).onclick =
function () {          axis = xAxis;          };
document.getElementById( "yButton" ).onclick =
function () {          axis = yAxis;          };
document.getElementById( "zButton" ).onclick =
function () {          axis = zAxis;          };
```

# Render Function

```
function render() {  
    gl.clear( gl.COLOR_BUFFER_BIT | gl.DEPTH_BUFFER_BIT );  
    theta[axis] += 2.0;  
    gl.uniform3fv(thetaLoc, theta);  
    gl.drawArrays( gl.TRIANGLES, 0, numVertices );  
    requestAnimationFrame( render );  
}
```

# Classical Viewing



# Objectives

- Introduce the classical views
- Compare and contrast image formation by computer with how images have been formed by architects, artists, and engineers
- Learn the benefits and drawbacks of each type of view

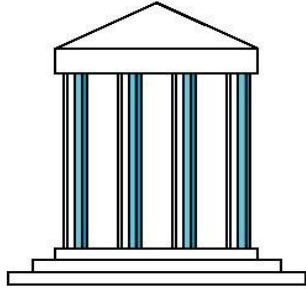
# Classical Viewing

- Viewing requires three basic elements
  - One or more objects
  - A viewer with a projection surface
  - Projectors that go from the object(s) to the projection surface
- Classical views are based on the relationship among these elements
  - The viewer picks up the object and orients it how she would like to see it
- Each object is assumed to be constructed from flat *principal faces*
  - Buildings, polyhedra, manufactured objects

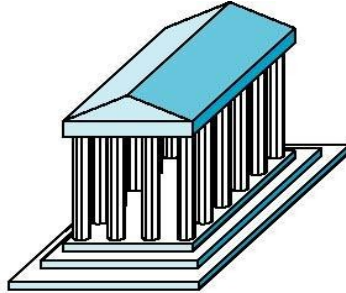
# Planar Geometric Projections

- Standard projections project onto a plane
- Projectors are lines that either
  - converge at a center of projection
  - are parallel
- Such projections preserve lines  
but not necessarily angles
- Nonplanar projections are needed for applications such as map construction

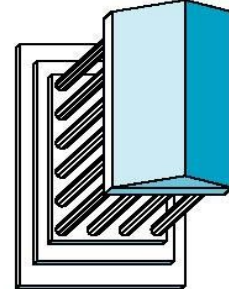
# Classical Projections



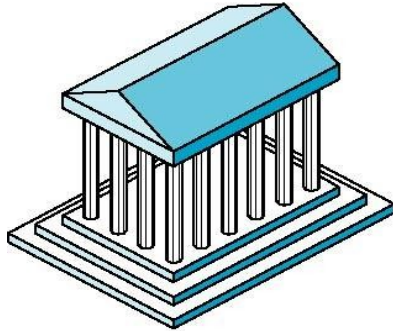
Front elevation



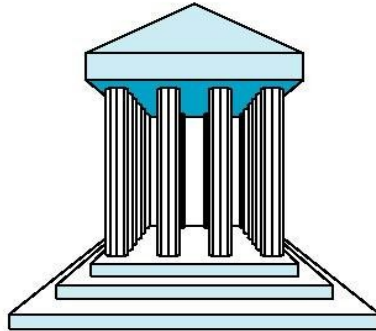
Elevation oblique



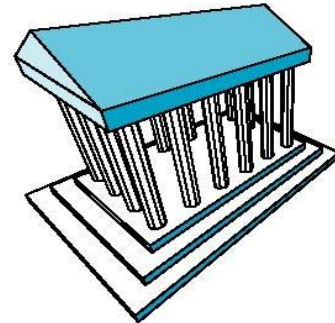
Plan oblique



Isometric



One-point perspective

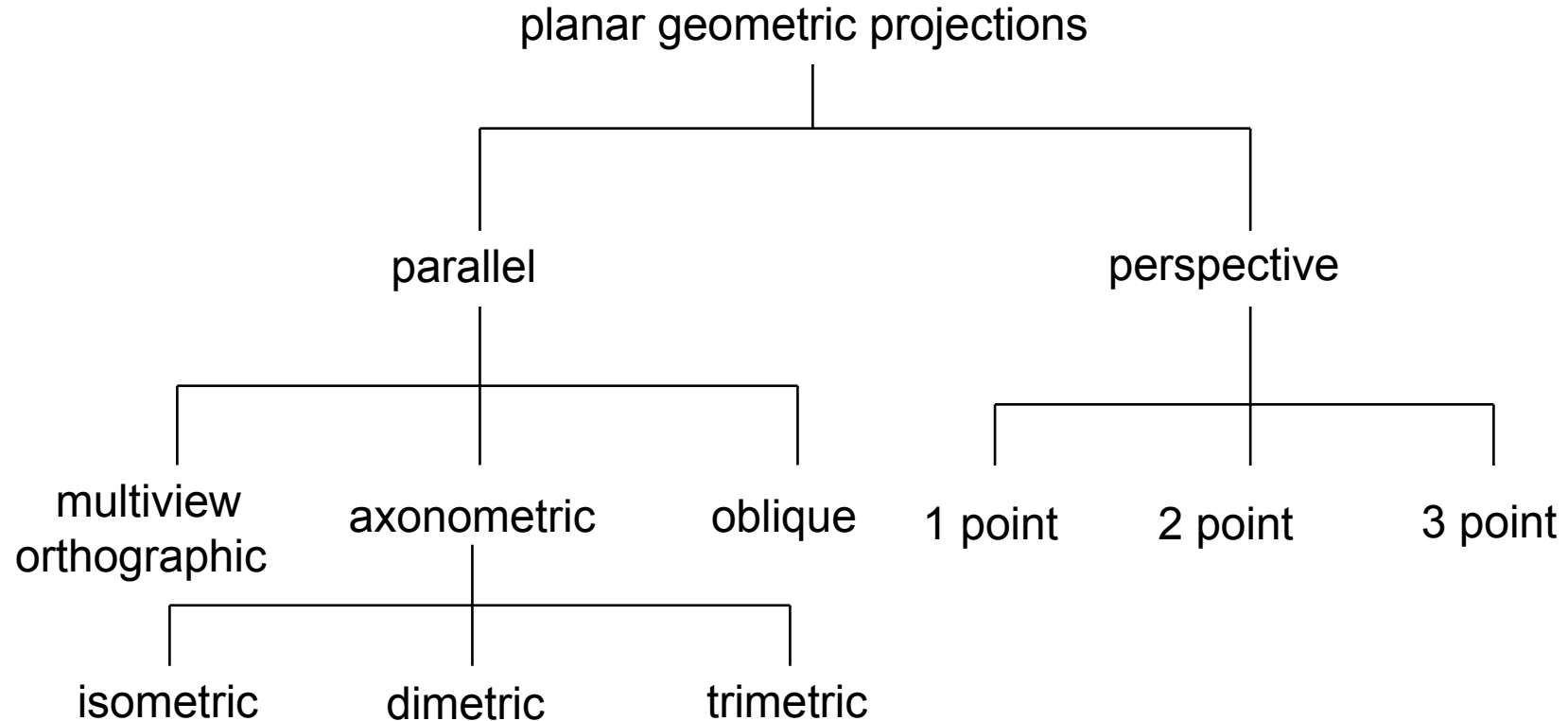


Three-point perspective

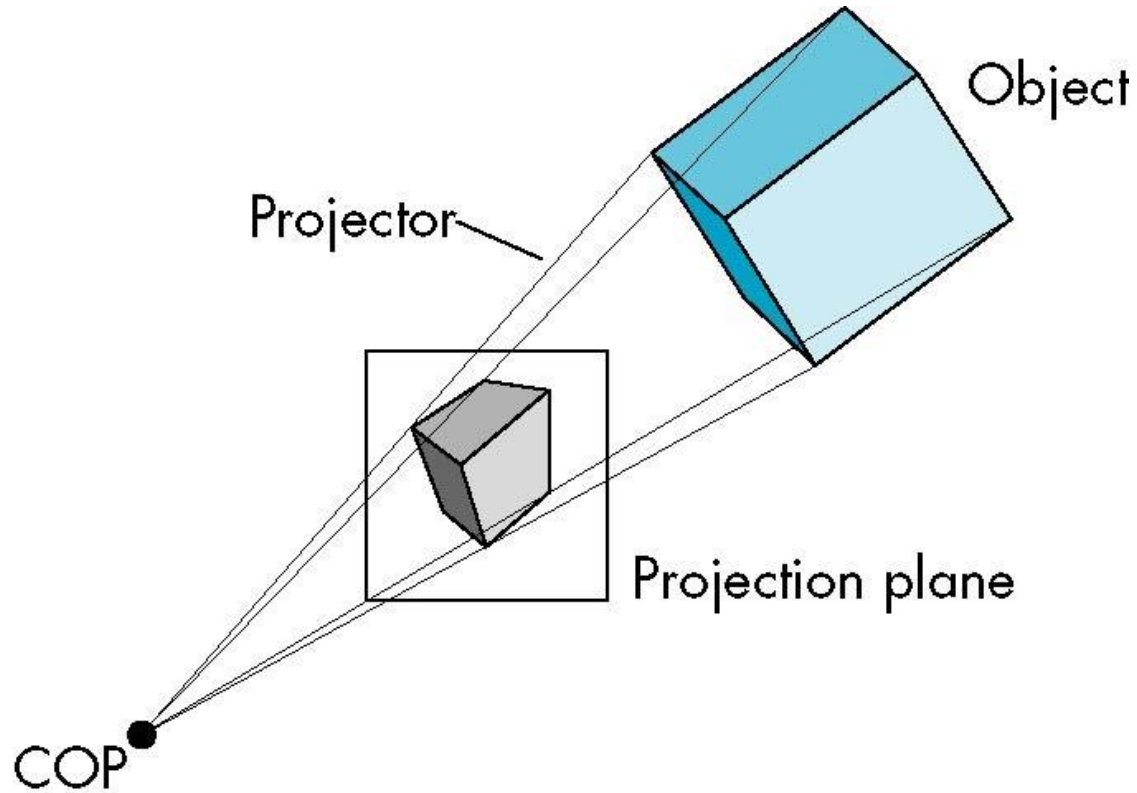
# Perspective vs Parallel

- Computer graphics treats all projections the same and implements them with a single pipeline
- Classical viewing developed different techniques for drawing each type of projection
- Fundamental distinction is between parallel and perspective viewing even though mathematically parallel viewing is the limit of perspective viewing

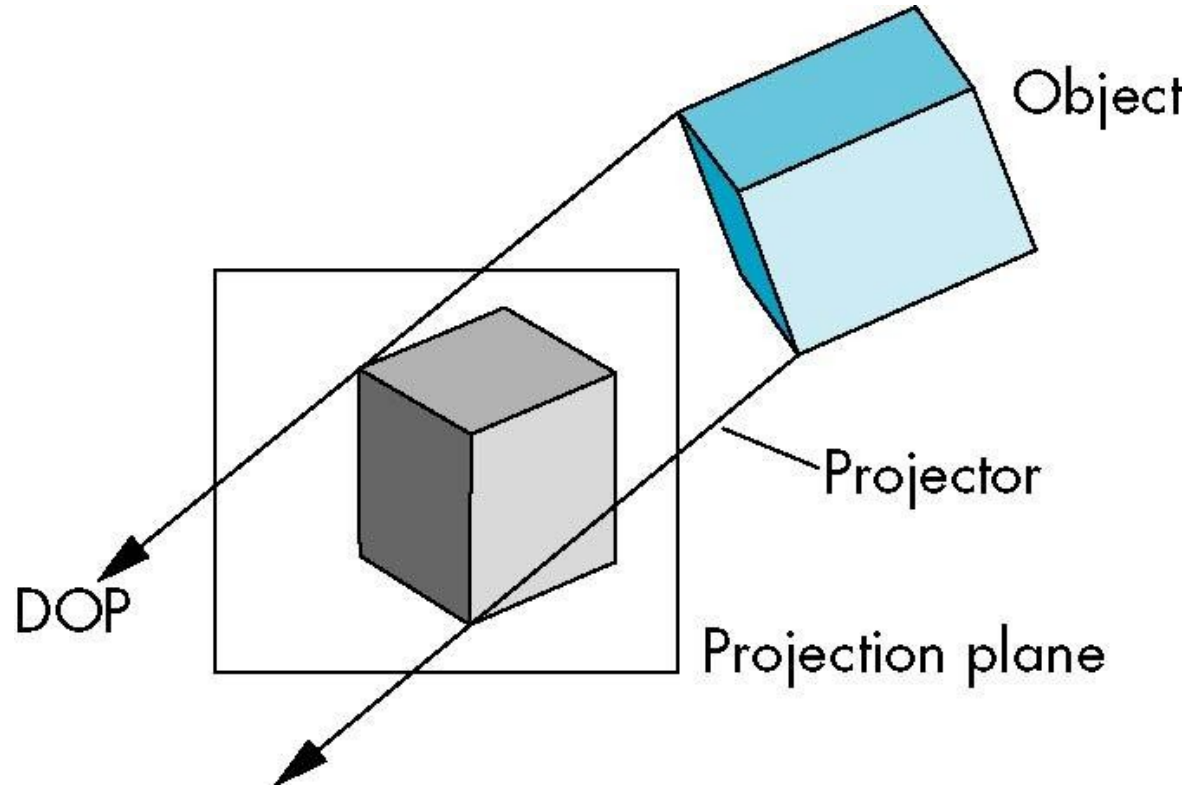
# Taxonomy of Planar Geometric Projections



# Perspective Projection



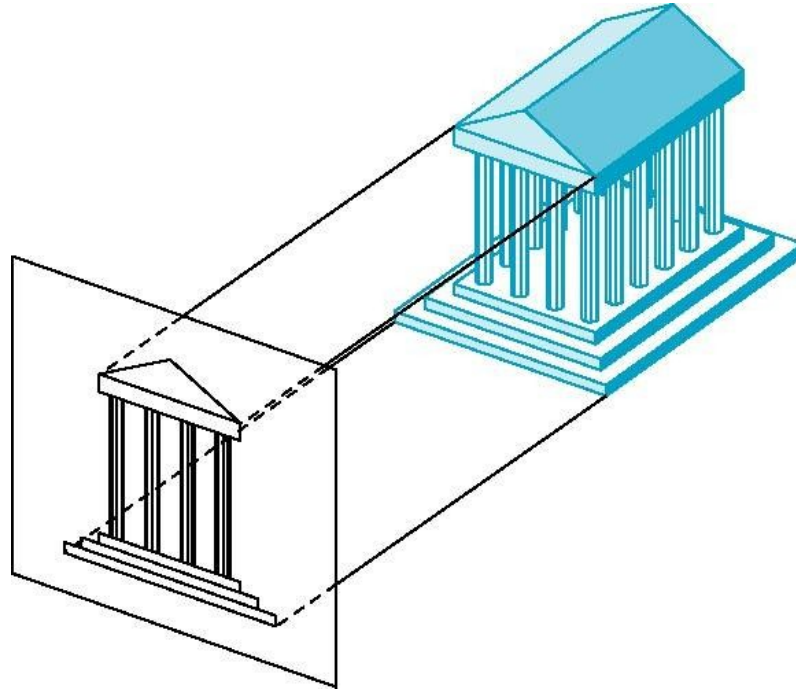
# Parallel Projection





# Orthographic Projection

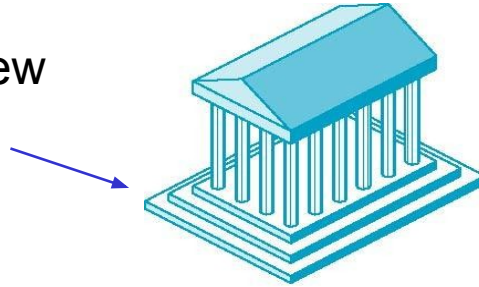
Projectors are orthogonal to projection surface



# Multiview Orthographic Projection

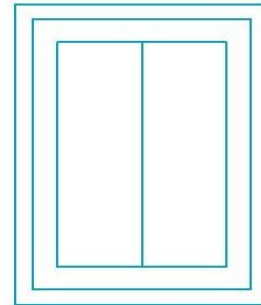
- Projection plane parallel to principal face
- Usually form front, top, side views

isometric (not multiview  
orthographic view)

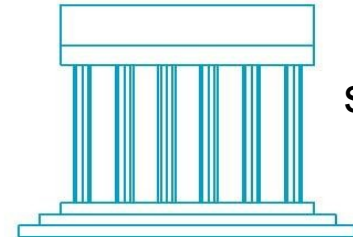


front

in CAD and architecture,  
we often display three  
multiviews plus isometric



top



side

# Advantages and Disadvantages

- Preserves both distances and angles
  - Shapes preserved
  - Can be used for measurements
    - Building plans
    - Manuals
- Cannot see what object really looks like because many surfaces hidden from view
  - Often we add the isometric

# Axonometric Projections

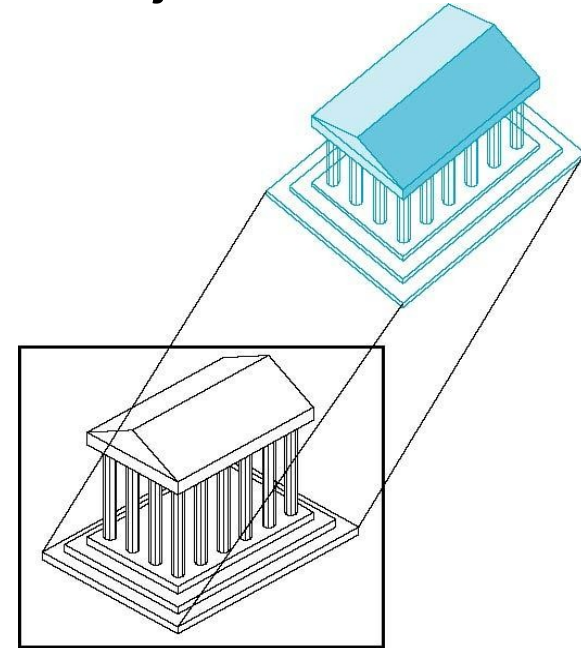
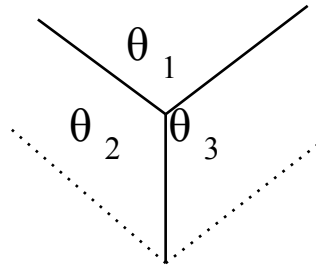
Allow projection plane to move relative to object

classify by how many angles of  
a corner of a projected cube are  
the same

none: trimetric

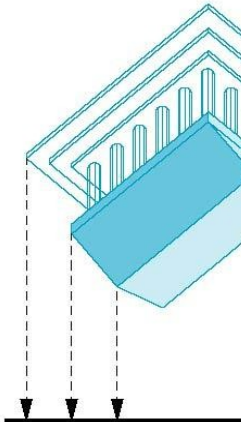
two: dimetric

three: isometric



Projection plane

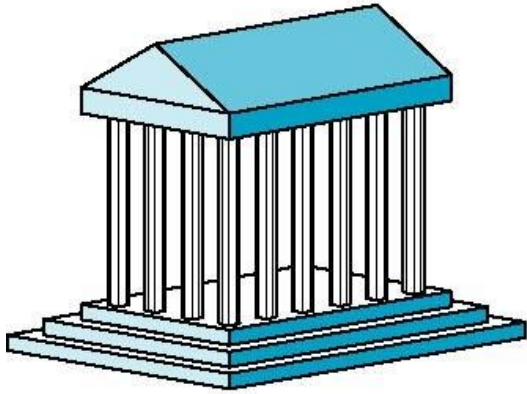
(a)



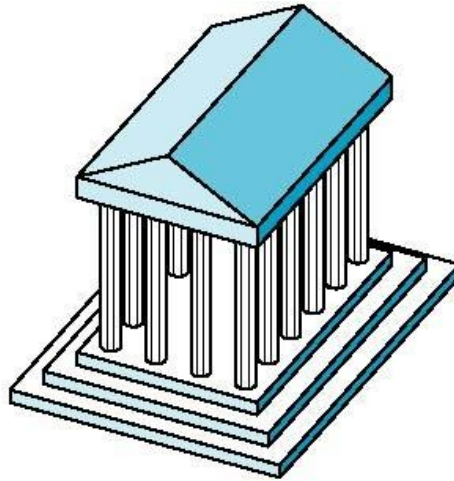
Projection plane

(b)

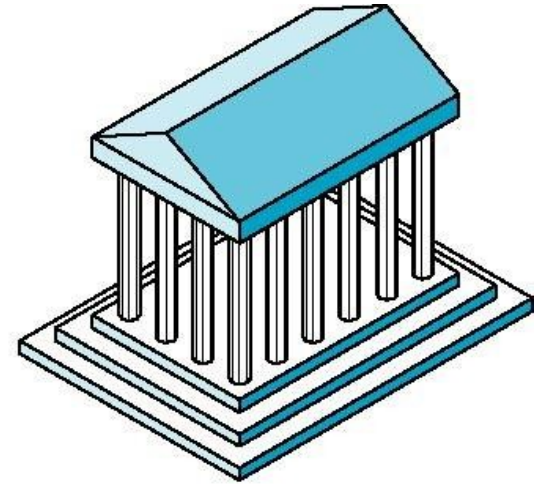
# Types of Axonometric Projections



Dimetric



Trimetric



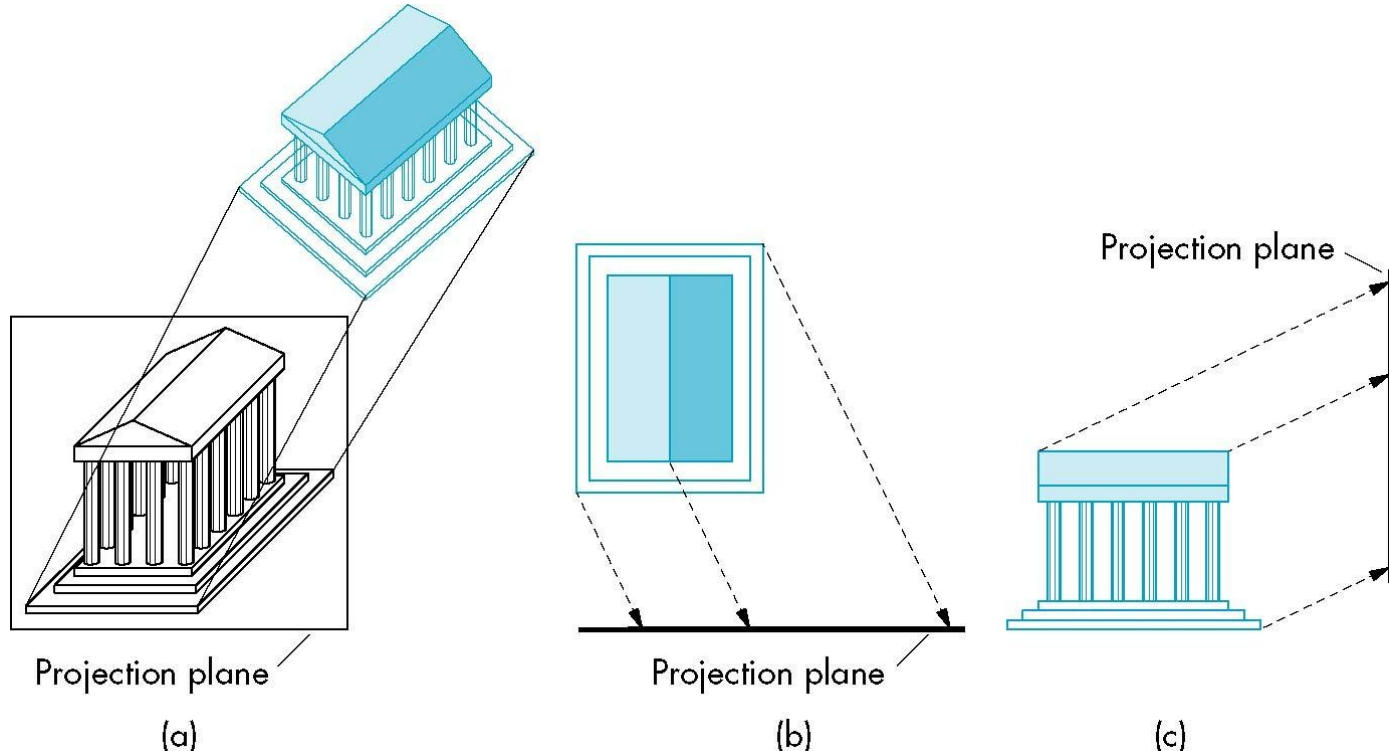
Isometric

# Advantages and Disadvantages

- Lines are scaled but can find scaling factors
- Lines preserved but angles are not
  - Projection of a circle in a plane not parallel to the projection plane is an ellipse
- Can see three principal faces of a box-like object
- Some optical illusions possible
  - Parallel lines appear to diverge
- Does not look real because far objects are scaled the same as near objects
- Used in CAD applications

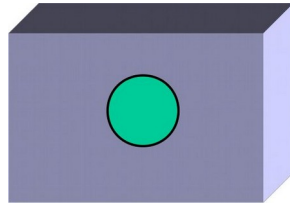
# Oblique Projection

Arbitrary relationship between projectors and projection plane



# Advantages and Disadvantages

- Can pick the angles to emphasize a particular face  
Architecture: plan oblique, elevation oblique
- Angles in faces parallel to projection plane are preserved while we can still see “around” side

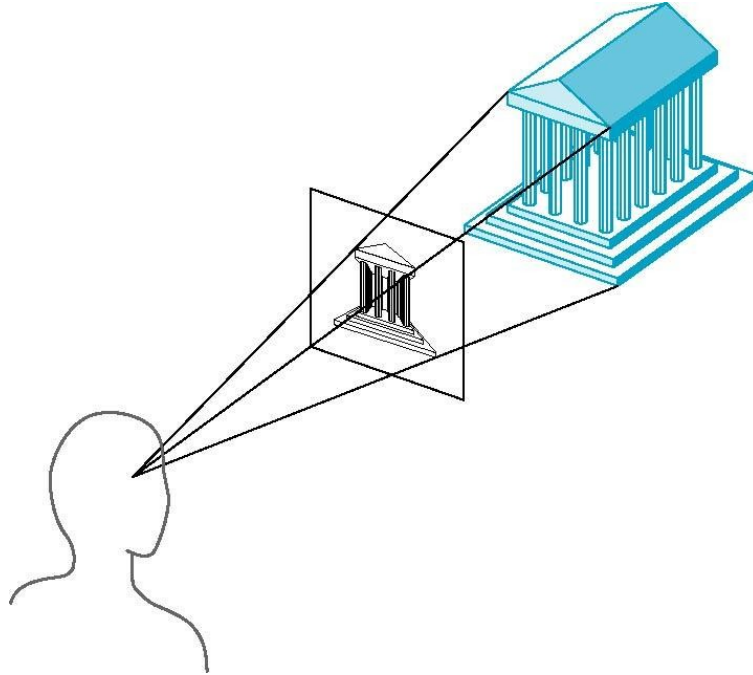


- In physical world, cannot create with simple camera; possible with Bellows camera or special lens (architectural)



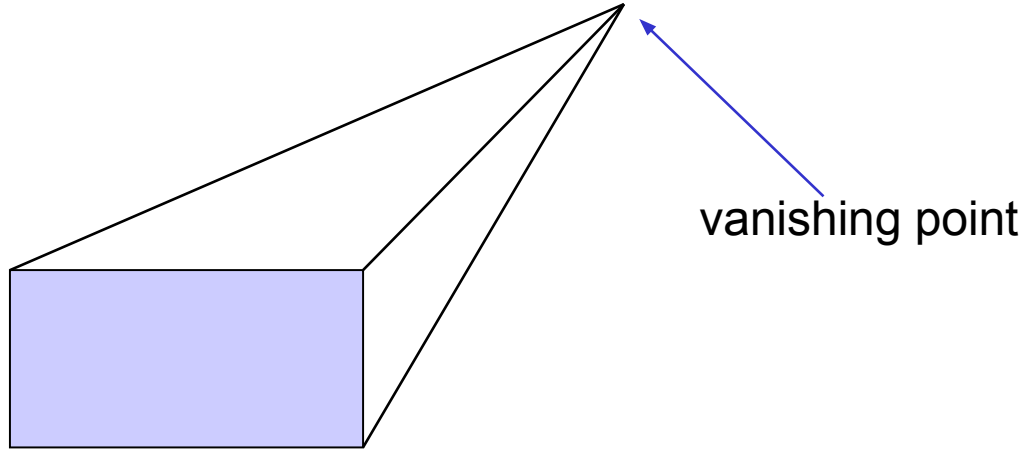
# Perspective Projection

Projectors converge at center of projection



# Vanishing Points

- Parallel lines (not parallel to the projection plan) on the object converge at a single point in the projection (the *vanishing point*)
- Drawing simple perspectives by hand uses these vanishing point(s)



# Three-Point Perspective

- No principal face parallel to projection plane
- Three vanishing points for cube



# Two-Point Perspective

- On principal direction parallel to projection plane
- Two vanishing points for cube



# One-Point Perspective

- One principal face parallel to projection plane
- One vanishing point for cube



# Advantages and Disadvantages

- Objects further from viewer are projected smaller than the same sized objects closer to the viewer (*diminution*)  
Looks realistic
- Equal distances along a line are not projected into equal distances (*nonuniform foreshortening*)
- Angles preserved only in planes parallel to the projection plane
- More difficult to construct by hand than parallel projections (but not more difficult by computer)

# **Computer Viewing: Positioning the Camera**

# Objectives

- Introduce the mathematics of projection
- Introduce WebGL viewing functions in MV.js
- Look at alternate viewing APIs



# From the Beginning

- In the beginning:
  - fixed function pipeline
  - Model-View and Projection Transformation
  - Predefined frames: model, object, camera, clip, ndc, window
- After deprecation
  - pipeline with programmable shaders
  - no transformations
  - clip, ndc window frames
- MV.js reintroduces original capabilities

# Computer Viewing

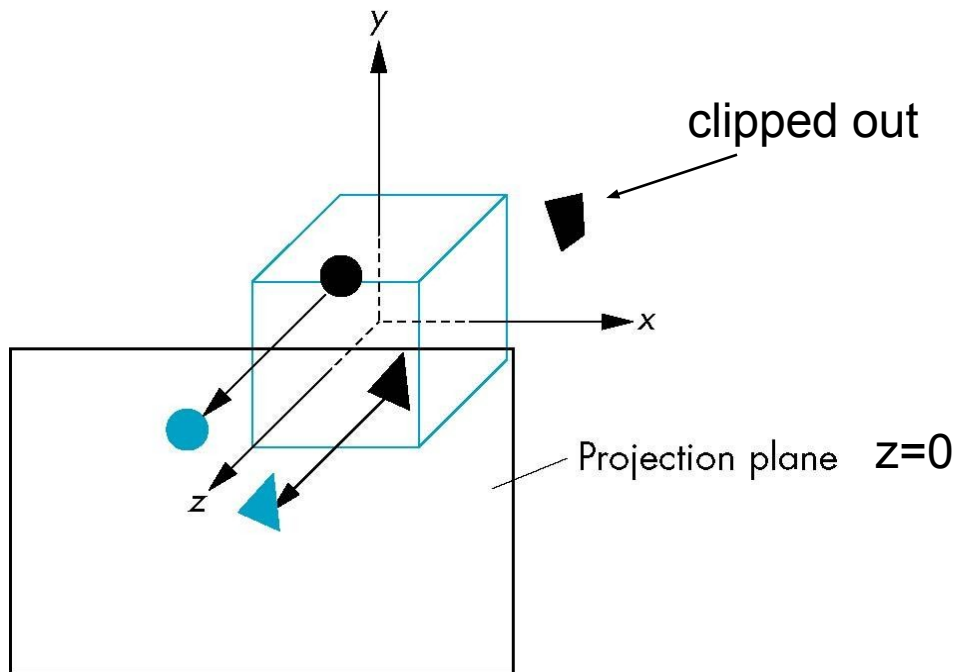
- There are three aspects of the viewing process, all of which are implemented in the pipeline,
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix
  - Clipping
    - Setting the view volume

# The WebGL Camera

- In WebGL, initially the object and camera frames are the same
  - Default model-view matrix is an identity
- The camera is located at origin and points in the negative z direction
- WebGL also specifies a *default view volume* that is a cube with sides of length 2 centered at the origin
  - Default projection matrix is an identity

## Default Projection

## Default projection is orthogonal



# Moving the Camera Frame

- If we want to visualize objects with both positive and negative z values we can either

Move the camera in the positive z direction

- Translate the camera frame

Move the objects in the negative z direction

- Translate the world frame

- Both of these views are equivalent and are determined by the model-view matrix

Want a translation (`translate(0.0,0.0,-d) ;`)

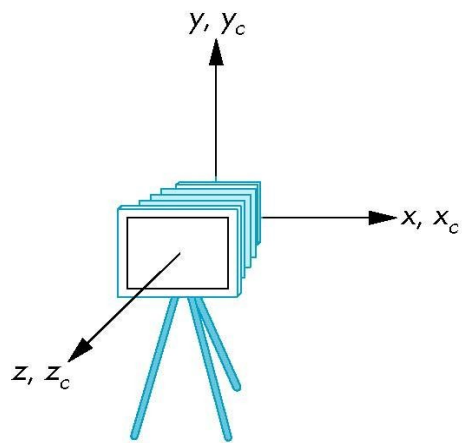
$d > 0$

# Moving Camera back from Origin

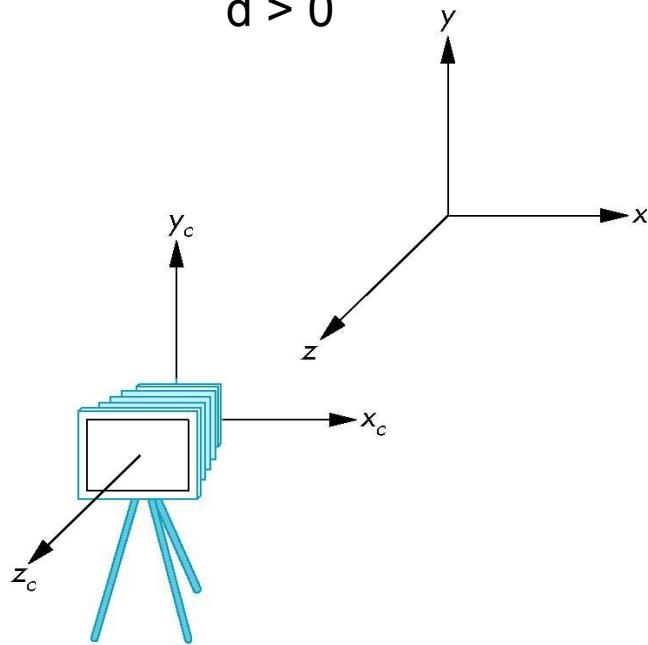
frames after translation by  $-d$

$d > 0$

default frames



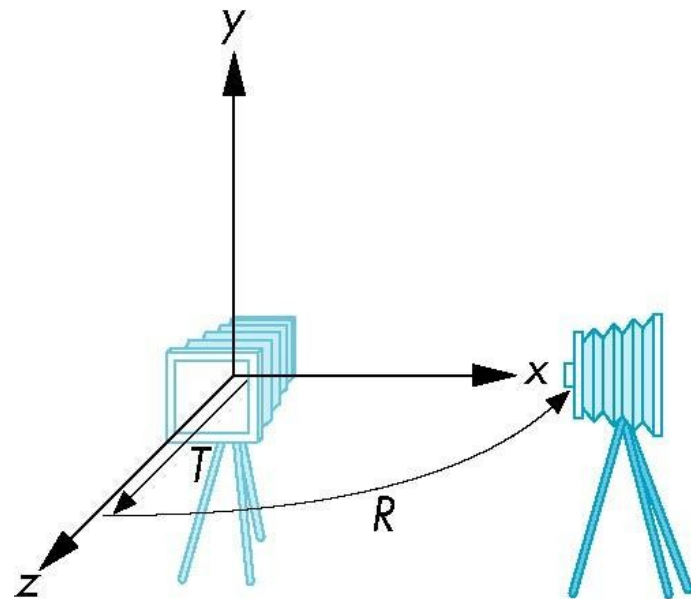
(a)



(b)

# Moving the Camera

- We can move the camera to any desired position by a sequence of rotations and translations
- Example: side view
  - Rotate the camera
  - Move it away from origin
  - Model-view matrix  $C = TR$



# WebGL code

Remember that last transformation specified is first to be applied

```
// Using MV.js
```

```
var t = translate (0.0, 0.0, -d);  
var ry = rotateY(90.0);  
var m = mult(t, ry);
```

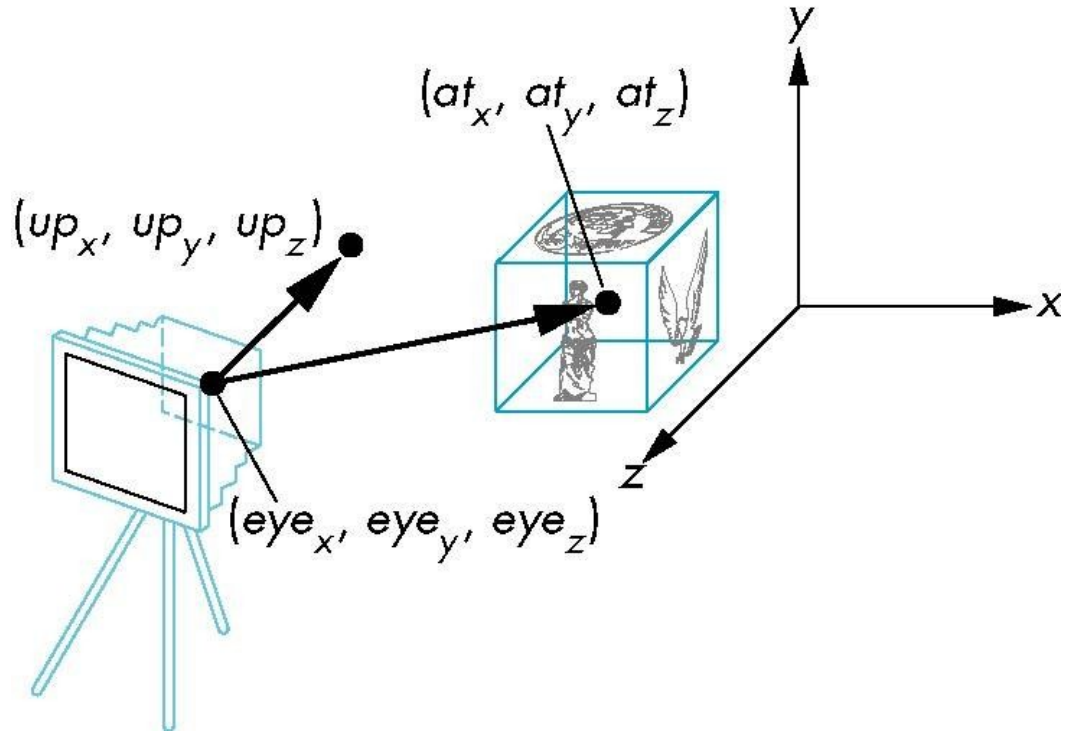
or

```
var m = mult(translate (0.0, 0.0, -d),  
             rotateY(90.0));
```



# lookAt

`LookAt(eye, at, up)`



# The lookAt Function

- The GLU library contained the function `gluLookAt` to form the required modelview matrix through a simple interface
- Note the need for setting an up direction
- Replaced by `lookAt()` in MV.js
  - Can concatenate with modeling transformations
- Example: isometric view of cube aligned with axes

```
var eye = vec3(1.0, 1.0, 1.0);  
var at = vec3(0.0, 0.0, 0.0);  
var up = vec3(0.0, 1.0, 0.0);  
  
var mv = LookAt(eye, at, up);
```

## Other Viewing APIs

- The LookAt function is only one possible API for positioning the camera
- Others include
  - View reference point, view plane normal, view up (PHIGS, GKS-3D)
  - Yaw, pitch, roll
  - Elevation, azimuth, twist
  - Direction angles

# **Computer Viewing: Projection**

# Objectives

- Introduce the mathematics of projection
- Add WebGL projection functions in MV.js

# Projections and Normalization

- The default projection in the eye (camera) frame is orthogonal
- For points within the default view volume

$$x_p = x$$

$$y_p = y$$

$$z_p = 0$$

- Most graphics systems use *view normalization*
  - All other views are converted to the default view by transformations that determine the projection matrix
  - Allows use of the same pipeline for all views

# Homogeneous Coordinate Representation

default orthographic projection

$$\begin{aligned}x_p &= x \\y_p &= y \\z_p &= 0 \\w_p &= 1\end{aligned}$$

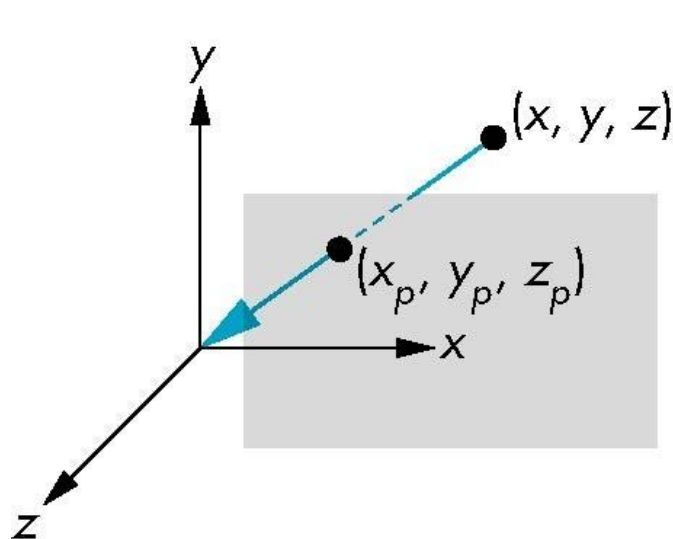
$$\mathbf{p}_p = \mathbf{M}\mathbf{p}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

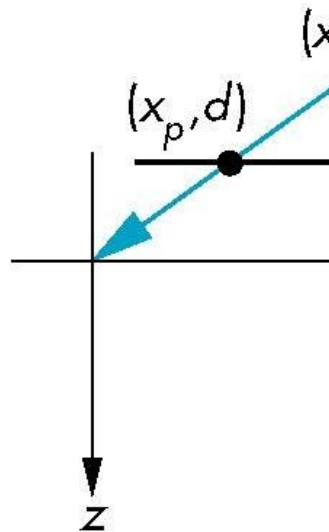
In practice, we can let  $\mathbf{M} = \mathbf{I}$  and set the  $z$  term to zero later

# Simple Perspective

- Center of projection at the origin
- Projection plane  $z = d$ ,  $d < 0$



(a)

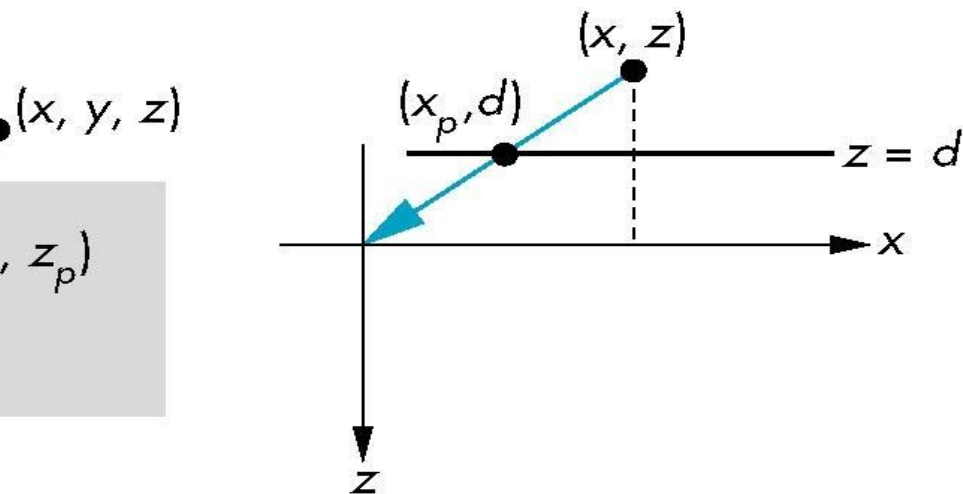


(b)



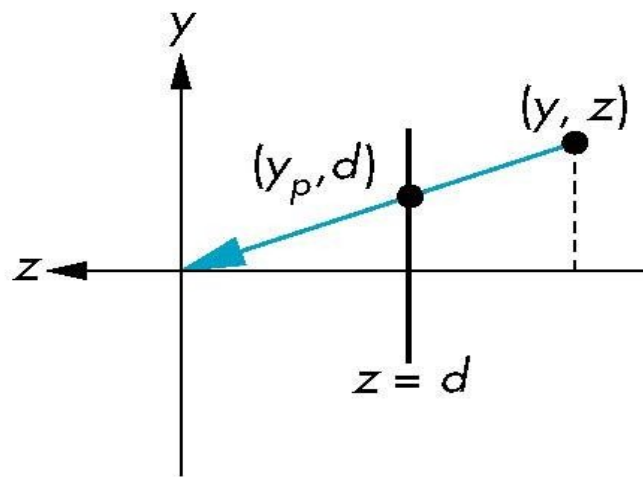
# Perspective Equations

Consider top and side views



$$x_p = \frac{x}{z/d} \quad (b)$$

$$y_p = \frac{y}{z/d}$$



$$z_p = d \quad (c)$$

# Homogeneous Coordinate Form

consider  $\mathbf{p} = \mathbf{M}\mathbf{q}$  where  $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

# Perspective Division

- However  $w \neq 1$ , so we must divide by  $w$  to return from homogeneous coordinates
- This *perspective division* yields

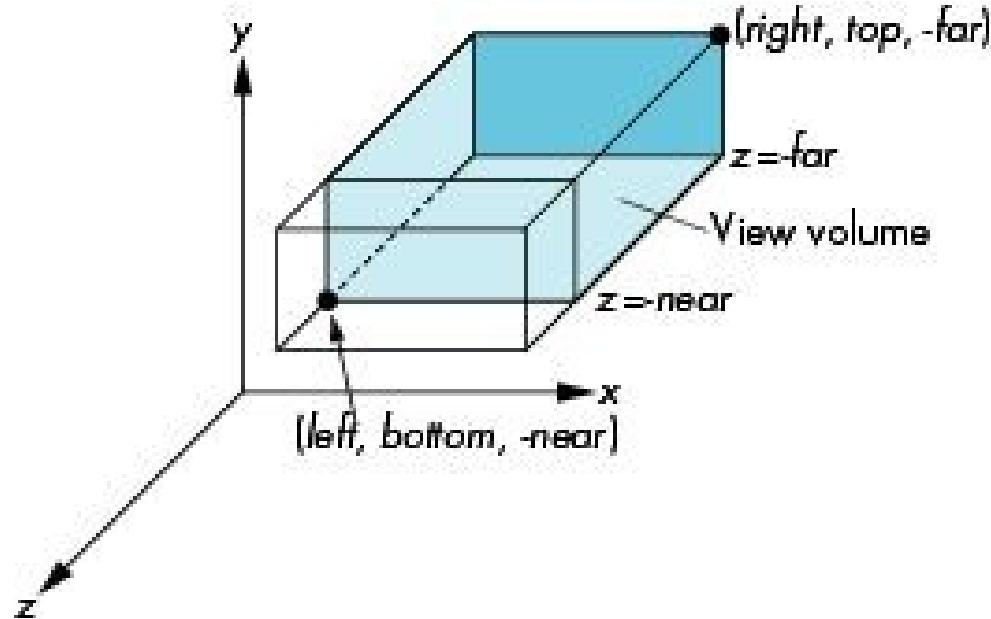
$$x_p = \frac{x}{z/d} \quad y_p = \frac{y}{z/d} \quad z_p = d$$

the desired perspective equations

- We will consider the corresponding clipping volume with `mat.h` functions that are equivalent to deprecated OpenGL functions

# WebGL Orthogonal Viewing

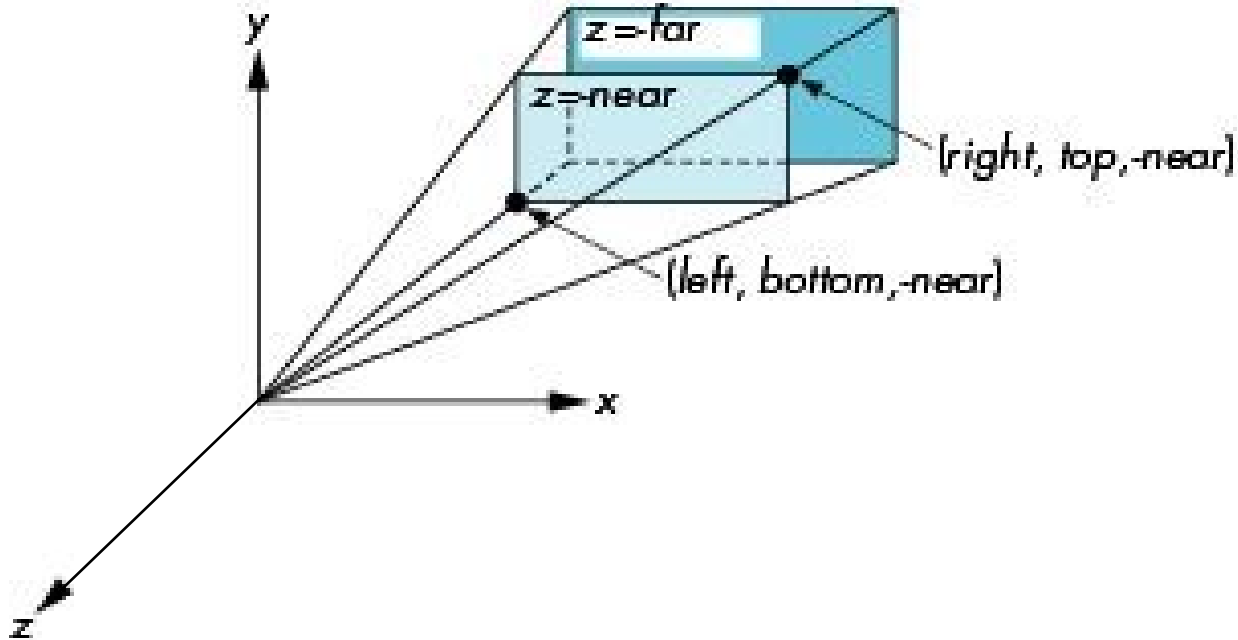
`ortho(left, right, bottom, top, near, far)`



**near** and **far** measured from camera

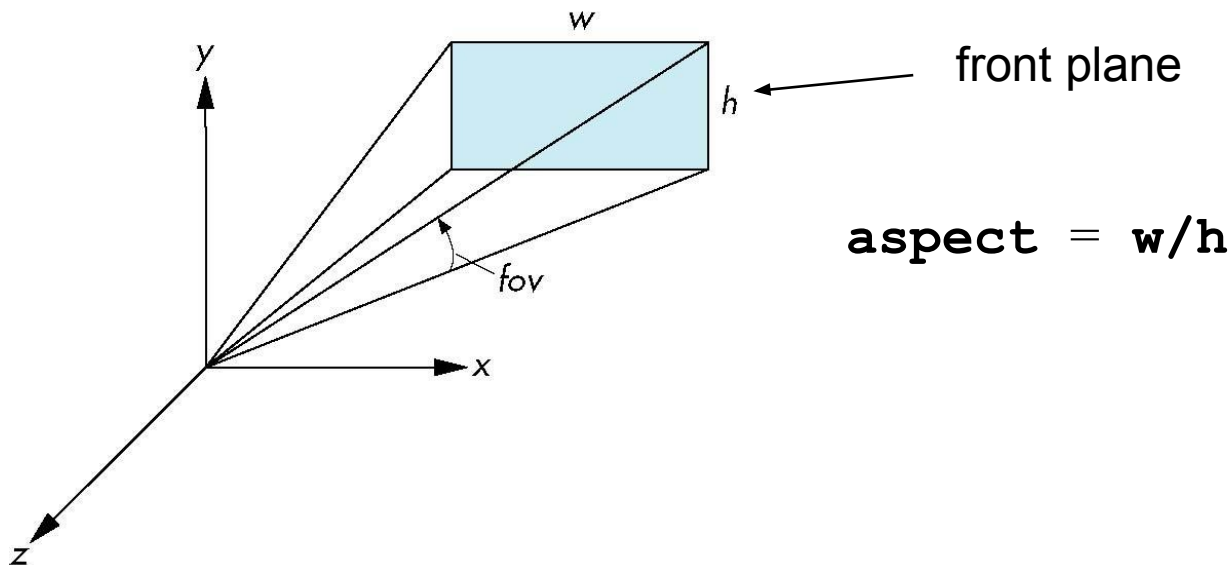
# WebGL Perspective

`frustum(left, right, bottom, top, near, far)`



# Using Field of View

- With **frustum** it is often difficult to get the desired view
- **perspective(fovy, aspect, near, far)** often provides a better interface

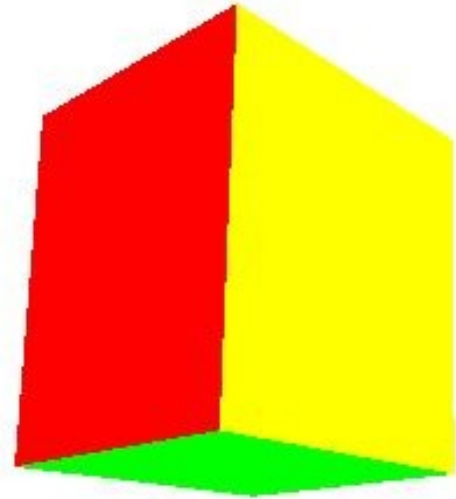


fov 10  120  
aspect 0.5  2

# Computing Matrices

- Compute in JS file, send to vertex shader with `gl.uniformMatrix4fv`
- Dynamic: update in `render()` or shader

zNear .01  3  
zFar 3  10  
radius 0.05  10  
theta -90  90  
phi -90  90  
fov 10  120  
aspect 0.5  2



# Video

[webgl/Code/w06/perspective2.html](http://webgl/Code/w06/perspective2.html)



## perspective2.js

```
var render = function() {  
    gl.clear( gl.COLOR_BUFFER_BIT | gl.DEPTH_BUFFER_BIT );  
    eye = vec3(radius*Math.sin(theta)*Math.cos(phi),  
        radius*Math.sin(theta)*Math.sin(phi),  
        radius*Math.cos(theta));  
    modelViewMatrix = lookAt(eye, at, up);  
    projectionMatrix = perspective(fovy, aspect, near, far);  
    gl.uniformMatrix4fv( modelViewMatrixLoc, false,  
        flatten(modelViewMatrix) );  
    gl.uniformMatrix4fv( projectionMatrixLoc, false,  
        flatten(projectionMatrix) );  
    gl.drawArrays( gl.TRIANGLES, 0, NumVertices );  
    requestAnimationFrame(render);  
}
```

# vertex shader

```
attribute  vec4 vPosition;  
attribute  vec4 vColor;  
varying vec4 fColor;  
uniform mat4 modelViewMatrix;  
uniform mat4 projectionMatrix;  
  
void main() {  
    gl_Position = projectionMatrix*modelViewMatrix*vPosition;  
    fColor = vColor;  
}
```