TATU BOGDAN
CTI - EN 3.1./2.3 EXAM

$x^3 - x - 2 = 0 \quad x \in [1,2]$

$\varepsilon_x = 10^{-3}$

$\varepsilon_f = 10^{-2}$

• Bisection

$r^0 = 1 \quad s^0 = 2 \quad |r^0 - s^0| = 1$

$k=1 \quad x^1 = \frac{r^0 + s^0}{2} = 1.5$

$f(x^1) = 1.5^3 - 1.5 - 2 = -0.125$

$f(r^0) = 1 - 1 - 2 = -2$

$f(x^1) \cdot f(r^0) > 0 \Rightarrow r^1 = x^1 = 1.5 \quad s^1 = s^0 = 2$

$|r^1 - s^1| = 0.5 > \varepsilon_x$

$|f(x^1)| = 0.125 > \varepsilon_f$

$k=2 \quad x^2 = \frac{r^1 + s^1}{2} = 1.75$

$f(x^2) = 1.75^3 - 1.75 - 2 = 1.609$

$f(r^1) = -0.125$

$f(x^2) \cdot f(r^1) < 0 \Rightarrow r^2 = r^1 = 1.5 \; s^2 = x^2 = 1.75$

$|r^2 - s^2| = 0.25 > \varepsilon_x$

$|f(x^2)| = 1.609 > \varepsilon_f$

$k=3 \quad x^3 = \frac{r^2 + s^2}{2} = 1.625 \qquad |r^3 - s^3| = 0.125 > \varepsilon_x$

$f(x^3) = 0.666 \qquad |f(x^3)| = 0.666 > \varepsilon_f$

$f(r^2) = -0.125 \quad \cdot < 0 \Rightarrow r^3 = r^2 \quad s^3 = x^3 = 1.625$

1° further iterations should be performed

- False Position

$x^3 - x - 2 = 0 \qquad x \in [1;2]$

$r^0 = 1 \qquad s^0 = 2$

$f(r^0) = -2 \qquad f(s^0) = 4$

$k=1: \quad x' = \dfrac{r^0 \cdot f(s^0) - s^0 \cdot f(r^0)}{f(s^0) - f(r^0)} = \dfrac{1 \cdot 4 - 2 \cdot (-2)}{4+2} = 1.333$

$f(x') = 1.333^3 - 1.333 - 2 = -0.963$

$f(x') \cdot f(r^0) > 0 \Rightarrow r' = x' = 1.333 \quad s' = s^0 = 2$

$|r' - s'| = 0.667 > \varepsilon_x \quad |f(x')| = 0.963 > \varepsilon_x$

$k=2 \quad x^2 = \dfrac{r' \cdot f(s') - s' \cdot f(r')}{f(s') - f(r')} = 1.463$

$f(x^2) = -0.332$

$f(x^2) \cdot f(r') > 0 \Rightarrow r^2 = x^2 = 1.463 \quad s^2 = s' = 2$

$|r^2 - s^2| = 0.537 > \varepsilon_x \quad |f(x^2)| = 0.332 > \varepsilon_f$

$k=3 \quad x^3 = \dfrac{r^2 \cdot f(s^2) - s^2 \cdot f(r^2)}{f(s^2) - f(r^2)} = 1.516$

$f(x^3) = -0.03$

$f(x^3) \cdot f(r^2) > 0 \Rightarrow r^3 = x^3 = 1.516 \quad s^3 = s^2 = 2$

$|r^3 - s^3| = 0.484 > \varepsilon_x \quad |f(x^3)| = 0.03 > \varepsilon_f$

- the false position method alway converged closer to 0, while the bisection method converged, but slower, ε_x got smaller and smaller for false position
- for both further iterations should be performed