PDF & CDF

- ***Cumulative Distribution Function***

$$F_X(x) = P(X \leq x)$$

- ***Probability Density Function***

$$f_X(x) - given(family\ of\ CRV)$$

	Probability Density Function	*Cumulative Distribution Function*
interchange	$f_X(x) = F_X'(x)$	$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$
$P(a < X \leq b)$	$\int_a^b f_X(x)\,dx$	$F_X(b) - F_X(a)$
	$f_X(x) \geq 0$	$\lim_{x \to -\infty} F_X(x) = 0$
	$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$	$\lim_{x \to \infty} F_X(x) = 1$

  - *Probability of any one outcome:* $P(X = x) = 0$

- ***Expected Values***

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$$

  - *Function* $g(X)$*:*

$$E\big(g(X)\big) = \int_{-\infty}^{\infty} g(x) \cdot f_X(x)\,dx$$

Families of CRV

- ***Uniform (a, b)***

$$f_X(x) = \begin{cases} \frac{1}{b-a}, a \leq x \leq b \\ 0, otherwise \end{cases}$$

$$F_X(x) = \begin{cases} 0, x \leq a \\ \frac{x-a}{b-a}, a \leq x < b \\ 1, x > b \end{cases}$$

  - $E(X) = \frac{a+b}{2}$
  - $Var(X) = \frac{(a-b)^2}{12}$

- ***Exponential (λ)***

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0 \\ 0, otherwise \end{cases}$$

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, x \geq 0 \\ 0, otherwise \end{cases}$$

  - $E(X) = \frac{1}{\lambda}$
  - $Var(X) = \frac{1}{\lambda^2}$

- ***Gaussian (μ, σ)***

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$F_X(x) = \Phi(\frac{x-\mu}{\sigma})$$

$$\Phi(z) = \frac{1}{2\pi} \int_{-\infty}^{z} e^{-\frac{t^2}{2}} dt$$

  - $E(X) = \mu$
  - $Var(X) = \sigma^2$

$$P(a < X \leq b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi(\frac{a-\mu}{\sigma})$$

$$\Phi(-z) = 1 - \Phi(z)$$

- ***Pareto (α, β)***

$$f_X(x) = \begin{cases} \frac{\alpha}{\beta} \cdot (\frac{\beta}{x})^{\alpha+1}, x \geq \beta \\ 0, x < \beta \end{cases}$$

$$F_X(x) = \begin{cases} 1 - (\frac{\beta}{x})^{\alpha}, x \geq \beta \\ 0, x < \beta \end{cases}$$

  - $E(X) = \frac{\alpha\beta}{\alpha-1}$
  - $Var(X) = \frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$

Conditional CRV

$$f_{X|B}(x) = \begin{cases} \frac{f_X(x)}{P(B)}, x \in B \\ 0, otherwise \end{cases}$$

$$f_X(x) = \sum_{i=0}^{n} f_{X|B}(x) \cdot P(B_i)$$

  - $E(X|B) = \int_{-\infty}^{\infty} x \cdot f_{X|B}(x)\, dx$

Derived CRV

- $\boldsymbol{Y = c \cdot X}$
  - $Uniform\ (a, b) \Longrightarrow Uniform\ (c \cdot a, c \cdot b)$
  - $Exponential\ (\lambda) \Longrightarrow Exponential\ (\frac{\lambda}{c})$
  - $Gaussian\ (\mu, \sigma) \Longrightarrow Gaussian\ (c \cdot \mu, c \cdot \sigma)$

	$\boldsymbol{Y = c \cdot X}$	$\boldsymbol{Y = X + c}$
$\boldsymbol{CDF}$	$F_Y(y) = F_X(\frac{y}{c})$	$F_Y(y) = F_X(y - c)$
$\boldsymbol{PDF}$	$f_Y(y) = \frac{1}{c} f_X(\frac{y}{c})$	$f_Y(y) = f_X(y - c)$