TATU BOGDAN – GR 3.1 – CLOCKED SYNCHRONOUS STATE MACHINES

1. **Design a modulo 11 cyclic counter which counts either in ascending, or in descending, binary counting order (not both of them). Implement the counter with:**
   a. **D flip-flops**

D flip-flops - ascending binary counting order								
Nr.	Q3	Q2	Q1	Q0	Q3*	Q2*	Q1*	Q0*
0	0	0	0	0	0	0	0	1
1	0	0	0	1	0	0	1	0
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	1	0	0
4	0	1	0	0	0	1	0	1
5	0	1	0	1	0	1	1	0
6	0	1	1	0	0	1	1	1
7	0	1	1	1	1	0	0	0
8	1	0	0	0	1	0	0	1
9	1	0	0	1	1	0	1	0
10	1	0	1	0	0	0	0	0
11	1	0	1	1	d/0	d/0	d/0	d/0
12	1	1	0	0	d/0	d/0	d/0	d/0
13	1	1	0	1	d/0	d/0	d/0	d/0
14	1	1	1	0	d/0	d/0	d/0	d/0
15	1	1	1	1	d/0	d/0	d/0	d/0
					D3	D2	D1	D0

- **Karnaugh Maps – minimal risk/cost implementations**

D3					
				Q3	
	Q3Q2 / Q1Q0	00	01	11	10
	00	0	0	d/0	1
	01	0	0	d/0	1
Q1	11	0	1	d/0	d/0
Q1	10	0	0	d/0	0
			Q2	Q2	

Q0 (rows 01, 11)

MINIMAL COST

D3 = Q1' * Q3 + Q0 * Q1 * Q2

MINIMAL RISK

D3 = Q1' * Q2' * Q3 + Q0 * Q1 * Q2 * Q3'

D2

Q3Q2 / Q1Q0	00	01	11	10
00	0	1	d/0	0
01	0	1	d/0	0
11	1	0	d/0	d/0
10	0	1	d/0	0

Q3 Q0 Q1 Q2

MINIMAL COST

D2 = Q1' * Q2 + Q0' * Q2 + Q0 * Q1 * Q2'

MINIMAL RISK

D2 = Q1' * Q2 * Q3' + Q0' * Q2 * Q3' + Q0 * Q1 * Q2' * Q3'

D1

Q3Q2 / Q1Q0	00	01	11	10
00	0	0	d/0	0
01	1	1	d/0	1
11	0	0	d/0	d/0
10	1	1	d/0	0

Q3 Q0 Q1 Q2

MINIMAL COST

D1 = Q0 * Q1' + Q0' * Q1 * Q3'

MINIMAL RISK

D1 = Q0 * Q1' * Q2' + Q0 *Q1' * Q3' + Q0' * Q1 * Q3'

D0				
			Q3	Q3
Q3Q2 / Q1Q0	00	01	11	10
00	1	1	d/0	1
01	0	0	d/0	0
11	0	0	d/0	d/0
10	1	1	d/0	0
		Q2	Q2	

Q0: rows 01, 11. Q1: rows 11, 10.

MINIMAL COST

D1 = Q0' * Q3' + Q0' * Q1'

MINIMAL RISK

D1 = Q0' * Q3' + Q0' * Q1' * Q2'

b. T flips-flops (enable)

T flip-flops (enable) - ascending binary counting order

Nr.	Q3	Q2	Q1	Q0	Q3*	Q2*	Q1*	Q0*	EN3	EN2	EN1	EN0
0	0	0	0	0	0	0	0	1	0	0	0	1
1	0	0	0	1	0	0	1	0	0	0	1	1
2	0	0	1	0	0	0	1	1	0	0	0	1
3	0	0	1	1	0	1	0	0	0	1	1	1
4	0	1	0	0	0	1	0	1	0	0	0	1
5	0	1	0	1	0	1	1	0	0	0	1	1
6	0	1	1	0	0	1	1	1	0	0	0	1
7	0	1	1	1	1	0	0	0	1	1	1	1
8	1	0	0	0	1	0	0	1	0	0	0	1
9	1	0	0	1	1	0	1	0	0	0	1	1
10	1	0	1	0	0	0	0	0	1	0	1	0
11	1	0	1	1	d/0	d/0	d/0	d/0	d/1	d/0	d/1	d/1
12	1	1	0	0	d/0	d/0	d/0	d/0	d/1	d/1	d/0	d/0
13	1	1	0	1	d/0	d/0	d/0	d/0	d/1	d/1	d/0	d/1
14	1	1	1	0	d/0	d/0	d/0	d/0	d/1	d/1	d/1	d/0
15	1	1	1	1	d/0	d/0	d/0	d/0	d/1	d/1	d/1	d/1

- **Karnaugh Maps – minimal risk/cost implementations**

EN3

Q3

Q3Q2 / Q1Q0	00	01	11	10
00	0	0	d/1	0
01	0	0	d/1	0
11	0	1	d/1	d/1
10	0	0	d/1	1

Q0

Q1

Q2

MINIMAL COST

EN3 = Q1 * Q3 + Q0 * Q1 * Q2

MINIMAL RISK

EN3 = Q1 * Q3 + Q0 * Q1 * Q2 + Q2 * Q3

EN2

Q3Q2 / Q1Q0	00	01	11	10
00	0	0	d/1	0
01	0	0	d/1	0
11	1	1	d/1	d/0
10	0	0	d/1	0

MINIMAL COST

EN2 = Q0 * Q1

MINIMAL RISK

EN2 = Q0 * Q1 * Q3' + Q2 * Q3

EN1

Q3Q2 / Q1Q0	00	01	11	10
00	0	0	d/0	0
01	1	1	d/0	1
11	1	1	d/1	d/1
10	0	0	d/1	1

MINIMAL COST

EN1 = Q0 + Q1 * Q3

MINIMAL RISK

EN1 = Q0 * Q3' + Q0 * Q2' + Q1 * Q3

EN0

Q3

Q3Q2 / Q1Q0	00	01	11	10
00	1	1	d/0	1
01	1	1	d/1	1
11	1	1	d/1	d/1
10	1	1	d/0	0

Q0

Q1

Q2

MINIMAL COST

EN0 = Q3' + Q1' * Q2'

MINIMAL RISK

EN0 = Q3' + Q1' * Q2' + Q0

2. **Design a zero-counting machine: a clocked synchronous state machine with one input X and one output Z. The output will be 1 if and only if the number of 0's (zero) received at the input X since reset is a multiple of 5, and the output will be 0 otherwise. Use D or T flip-flops for implementing the machine. Use either minimal risk or minimal cost approach.**

Zero-counting machine - modulo 5 (D flip-flops)										
				X = 0			X = 1			
Nr.	Q2	Q1	Q0	Q2*	Q1*	Q0*	Q2*	Q1*	Q0*	Z
0	0	0	0	0	0	1	0	0	0	1
1	0	0	1	0	1	0	0	0	1	0
2	0	1	0	0	1	1	0	1	0	0
3	0	1	1	1	0	0	0	1	1	0
4	1	0	0	0	0	0	1	0	0	0
5	1	0	1	d/0	d/0	d/0	d/0	d/0	d/0	d/0
6	1	1	0	d/0	d/0	d/0	d/0	d/0	d/0	d/0
7	1	1	1	d/0	d/0	d/0	d/0	d/0	d/0	d/0
				D2	D1	D0	D2	D1	D0	

- **Karnaugh Maps – minimal risk/cost implementations**

D2

X

X Q2 / Q1Q0	00	01	11	10
00	0	0	1	0
01	0	d/0	d/0	0
11	1	d/0	d/0	0
10	0	d/0	d/0	0

Q0

Q1

Q2

MINIMAL COST

D2 = X * Q2 + X' * Q0 * Q1

MINIMAL RISK

D2 = X * Q0' * Q1' * Q2 + X' * Q0 * Q1 * Q2'

D1

X

X Q2 / Q1Q0	00	01	11	10
00	0	0	0	0
01	1	d/0	d/0	0
11	0	d/0	d/0	1
10	1	d/0	d/0	1

Q0

Q1

Q2

MINIMAL COST

D1 = Q0' * Q1 + X' * Q0 * Q1' + X * Q1

MINIMAL RISK

D1 = X * Q1 * Q2' + Q0' * Q1 * Q2' + X' * Q0 * Q1' * Q2'

D0

X

X Q2 / Q1Q0	00	01	11	10
00	1	0	0	0
01	0	d/0	d/0	1
11	0	d/0	d/0	1
10	1	d/0	d/0	0

Q0

Q1

Q2

MINIMAL COST

D0 = X' * Q0' * Q2' + X * Q0

MINIMAL RISK

D0 = X' * Q0' * Q2' + X * Q0 * Q2'