# *FIRST ORDER DE*

- ***Separable***
  - $\frac{dx}{dt} = g(t) \cdot f\big(x(t)\big)$

    $\Rightarrow \dfrac{1}{f(x(t))} dx = g(t)dt$
- ***Homogenous***
  - $\frac{dx}{dt} = f\left(\frac{x(t)}{t}\right)$

    $-substitution: z(t) = \dfrac{x(t)}{t} \Rightarrow x(t) = t \cdot z(t) \Rightarrow \dfrac{dx}{dt} = \dfrac{dz}{dt} \cdot t + z(t)$

    $\Rightarrow \displaystyle\int \frac{1}{f\big(z(t)\big) - z(t)} dz = \ln|x| + c$
- ***Exact***

  $P\big(t, x(t)\big)dt + Q\big(t, x(t)\big)dx = 0$

  $*\ exactness\ cond.: \dfrac{dP}{dx} = \dfrac{dQ}{dt}$

  $\Rightarrow \displaystyle\int_{t_0}^{t} P(t, x(t)dt + \int_{x_0}^{x} Q(t_0, x(t))dx = c$

  $*\ else:$
  - $\dfrac{\frac{dP}{dx} - \frac{dQ}{dt}}{Q} = f(t)$
  - $\dfrac{\frac{dQ}{dt} - \frac{dP}{dx}}{P} = g(x)$

    $\Rightarrow \dfrac{1}{\mu} d\mu = f(t)dt\ \ OR\ g(x)dx$
- ***Linear***
  - $\frac{dx}{dt} = f(t) \cdot x + g(t)$

    $\Rightarrow x(t) = x_h(t) + x_0(t)$

    $x_h(t) = c \cdot e^{\int f(t)dt}$

    $x_0(t) = e^{\int f(t)dt} \cdot \displaystyle\int g(t) \cdot e^{-\int f(t)dt} dt$

    $\Rightarrow x(t) = e^{\int f(t)dt}(C + \displaystyle\int g(t) \cdot e^{-\int f(t)dt} dt)$
- ***Bernoulli***
  - $\frac{dx}{dt} = f(t) \cdot x + g(t) \cdot x^{\alpha}$

    $-substitution: z(t) = [x(t)]^{1-\alpha} \Rightarrow \dfrac{dz}{dt} = (1-\alpha) \cdot x^{-\alpha} \cdot \dfrac{dx}{dt}$

    $\Rightarrow \dfrac{dz}{dt} = (1-\alpha)f(t)z + (1-\alpha)g(t)$

### *HIGHER ORDER LINEAR DE*

- $y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1} y^I + a_n y = f(x)$
  $\Rightarrow y(x) = \; y_h(x) + y_0(x)$

### *SYSTEMS OF DEs*

- $\begin{cases} x_1^I = a \cdot x_1 + b \cdot x_2 + f_1(t) \\ x_2^I = c \cdot x_1 + d \cdot x_2 + f_2(t) \end{cases} \Rightarrow X^I = A \cdot X + F$

  $\Rightarrow X(t) = X_h(t) + X_0(t)$

  $\Rightarrow X(t) = \Phi(t)[C + \int_0^t \Phi^{-1}(t) \cdot F(t)dt]$

  $X_h(t) = c_1 X_1(t) + c_2 X_2(t) + \cdots = \Phi(t) \cdot C$

  $X_0(t) = \Phi(t) \cdot \int_0^t \Phi^{-1}(t) \cdot F(t)dt$

  $X_i = V_i \cdot e^{\lambda_i \cdot t}$

  $\lambda_i = \lambda_j \Rightarrow X_j = (t \cdot V_i + W_j) \cdot e^{\lambda_j \cdot t}$

  $\left(A - \lambda_j \cdot I\right) \cdot W_j = V_i$

  $\Phi(t) = [X_1(t) | X_2(t)]$