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CTI-EN 3.1.

MS 1.13

1. $\begin{cases} xy' + 4y = x^4y^2 \\ y(2) = -1 \end{cases}$

$xy' + 4y = x^4y^2 \Rightarrow y' = \cancel{x^4 4y} \; \frac{x^4y^2 - 4y}{x}$

$\Rightarrow y' = x^3y^2 - \frac{4}{x} \cdot y$ (1) bernoullie D.E.

- substitution: $z(x) = [y(x)]^{1-2} = [y(x)]^{-1} \Rightarrow z' = (-1)y^{-2} \cdot y'$

$\overset{(1) \cdot y^{-2}}{\Rightarrow} \quad y^{-2} \cdot y' = -\frac{4}{x} \cdot y^{-1} + x^3$

$\Rightarrow z' = (-1) \cdot \frac{-4}{x} \cdot z + (-1) \cdot x^3$

$= \frac{4}{x} \cdot z \cancel{\,x\,} - x^3$ - non-homog linear DE

$\Rightarrow z(x) = e^{\int \frac{4}{x} dx} \left(c + \int -x^3 \cdot e^{-\int \frac{4}{x} dx} dx\right)$

$= e^{4\ln x}\left(c + \int -x^3 e^{-4\ln x} dx\right)$

$= x^4\left(c + \int -x^3 \cdot x^{-4} dx\right)$

$= x^4\left(c + (-)\int \frac{1}{x} dx\right)$

$= x^4(c - \ln x)$

$\overset{z(x) = y^{-1}(x)}{\Rightarrow} y(x) = \frac{1}{x^4(c - \ln x)}$

$y(2) = -1 \Rightarrow \frac{1}{16(c - \ln 2)} = -1$

$\Rightarrow 1 = -16(c - \ln 2)$

$\Rightarrow -\frac{1}{16} = c - \ln 2 \Rightarrow c = -\frac{1}{16} + \ln 2$

$\Rightarrow y(x) = \frac{1}{x^4(\ln 2 + \ln x - \frac{1}{16})} = \frac{1}{x^4(\ln 2x - \frac{1}{16})}$

2. Let D - "defective boards"

$P(D) = 3\% = 0.03 \overset{not}{=} p$

$n = 120$ boards

a) $\Rightarrow$ Pascal (k, x) r.v:

$P_X(x) = C_{x-1}^{k-1} \cdot p^k (1-p)^{x-k}$

$k = 2$

$P_X^{k=2}(120) = C_{120}^{2} \cdot (0.03)^2 (0.97)^{118}$

$= \frac{120!}{118!} \cdot (0.03)^2 (0.97)^{118}$

$= 119 \cdot (0.0009) \cdot (0.97)^{118} \cdot 120$

b) at least 2 defective

$P = 1 - [P_X^{k=1}(120) + P_X^{k=0}(120)]$

$= 1 - [C_{120}^{1} \cdot (0.03) \cdot (0.97)^{119} + C_{120}^{0} (0.03)^0 \cdot (0.97)^{120}$

$= 1 - [120! \cdot (0.03) \cdot (0.97)^{119} + 1 \cdot 1 \cdot (0.97)^{120}]$

opposite of at most 1 deffective

3. i) a) INGREDIENT

I − 2
N − 2
G − 1
R − 1
E − 2
D − 1
T − 1

$\Rightarrow$ 10 total $\Rightarrow$ anagrams $= \dfrac{10!}{2!\,2!\,1!\,1!\,2!\,1!\,1!}$

$= \dfrac{10!}{8} = 453600$ anagrams

b) 1st vowel − 4 choices
~~and~~ last vowel − 3 choices

ii) IE EI EE

$\Rightarrow$ same vowel + diff vowel $= \frac{1}{2} \cdot 4 \cdot 3 \cdot \frac{8!}{2!\,2!} + \frac{1}{2} \cdot 4 \cdot 3 \cdot \frac{8!}{2!}$

$p = \frac{1}{2}$ $\quad$ $p = \frac{1}{2}$

ii) a) $\Rightarrow$ Geometric(p) r.v.

$$P_X(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, 3 \ldots \\ 0, & \text{otherwise} \end{cases}$$

b) $p = 0.2 \Rightarrow \begin{cases} P_X(x) = 0.2 \cdot 0.8^{x-1} & x = 1, 2, 3 \ldots \\ 0, & \text{otherwise} \end{cases}$

$$\begin{aligned} P(X>4) &= 1 - P(X \leq 4) \\ &= 1 - [P(1) + P(2) + P(3) + P(4)] \\ &= 1 - [0.2 + 0.16 + 0.128 + 0.1024] \\ &= 1 - [0.5904] \\ &= 0.4096 = 40.96\% \end{aligned}$$

c) $P_{X|X>2}(x) = \begin{cases} \dfrac{P_X(x)}{P(X>2)}, & x > 2 \\ 0, & \text{otherwise} \end{cases}$

$$\begin{aligned} P(X>2) &= 1 - P(X \leq 2) \\ &= 1 - [P(1) + P(2)] \\ &= 1 - 0.36 = 0.64 \end{aligned}$$

$\Rightarrow P_{X|X>2}(x) = \begin{cases} \dfrac{p(1-p)^{x-1}}{0.64}, & x > 2 \\ 0, & \text{otherwise} \end{cases}$