7 $\begin{cases} x(k+1) - y(k) = 0 \\ y(k+1) + x(k) = 0 \end{cases}$ $x(0) = 1,\ y(0) = 0$

$$\Rightarrow \begin{cases} \mathcal{Z}\{x(k+1)\} - \mathcal{Z}\{y(k)\} = 0 \\ \mathcal{Z}\{y(k+1)\} + \mathcal{Z}\{x(k)\} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} z(x(z) - x(0)) - y(z) = 0 \\ z(y(z) - y(0)) + x(z) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} z(x(z) - 1) - y(z) = 0 \\ z(y(z)) + x(z) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} z \cdot x(z) - y(z) = \cancel{0} + z \\ x(z) + z \cdot y(z) = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} z & -1 \\ 1 & z \end{vmatrix} = z^2 + 1$$

$$\Delta x = \begin{vmatrix} z & -1 \\ 0 & z \end{vmatrix} = z^2$$

$$\Delta y = \begin{vmatrix} z & z \\ 1 & 0 \end{vmatrix} = -z$$

$$x(z) = \frac{\Delta x}{\Delta} = \frac{z^2}{z^2+1}$$

$$y(z) = \frac{\Delta y}{\Delta} = \frac{-z}{z^2+1}$$

$$\Rightarrow x(k) = \mathcal{Z}^{-1}\left\{\frac{z^2}{z^2+1}\right\} = \mathcal{Z}^{-1}\left\{\frac{1}{1+z^{-2}}\right\}$$

$$= \mathcal{Z}^{-1}\left\{\frac{1 - z^{-1}\cos\frac{\pi}{2}T}{1 - 2z^{-1}\cos\frac{\pi}{2}T + z^{-2}}\right\} = \cos\frac{\pi}{2}k \cdot T$$

$$\Rightarrow y(k) = \mathcal{Z}^{-1}\left\{\frac{-z}{z^2+1}\right\} = \mathcal{Z}^{-1}\left\{-\frac{z^{-1}}{1+z^{-2}}\right\}$$

$$= \mathcal{Z}^{-1}\left\{\frac{z^{-1} \cdot \sin\frac{3\pi}{2}T}{1 - 2z^{-1}\cos\frac{3\pi}{2}T + z^{-2}}\right\} = \sin\frac{3\pi}{2}k \cdot T$$

4.d) $x(k) = \begin{cases} 2, & k \geq 3 \\ 0, & k < 3 \end{cases}$

$Z\{x(k)\} = x(0) \cdot z^0 + x(1) \cdot z^{-1} + x(2) \cdot z^{-2} + \dots$

$= 0 \cdot 1 + 0 \cdot z^{-1} + 0 \cdot z^{-2} + 2 \cdot z^{-3} + 2 \cdot z^{-4} + \dots$

$= 2 \cdot z^{-3} + 2 \cdot z^{-4} + 2 \cdot z^{-5} + \dots$

$= \sum_{k=3}^{\infty} 2 \cdot z^{-k}$

geometric series when $k \to \infty$, $|r| < 1$:

$z^{-3}(a + a \cdot r + a \cdot r^2 + \dots) = z^{-3} \sum_{k=0}^{\infty} a \cdot r^k = z^{-3} \cdot \frac{a}{1-r}$

$a = 2$; $r = z^{-1} = \frac{1}{z}$

$= z^{-3} \cdot \frac{2}{1-z^{-1}} = \frac{1}{z^{\cancel{3}2}} \cdot 2 \cdot \frac{\cancel{z}}{z-1} = \frac{2}{z^2(z-1)}$