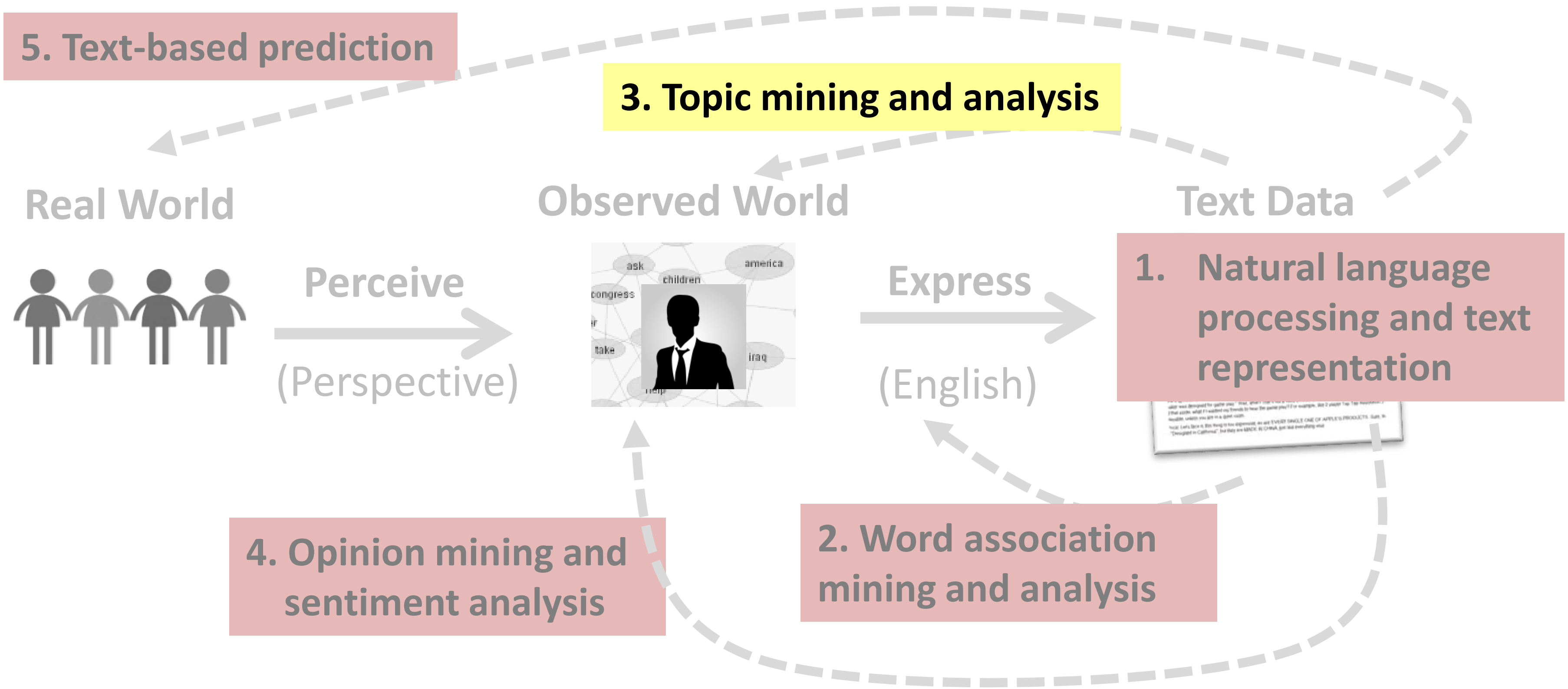
Probabilistic Topic Models: Mixture of Unigram Language Models

Slide adapted from ChengXiang Zhai's presentation

Probabilistic Topic Models: Mixture of Unigram LMs



Slide adapted from ChengXiang Zhai's presentation

Factoring out Background Words

d

Text mining paper

$p(w|\theta)$

the 0.031
a 0.018
…
text 0.04
mining 0.035
association 0.03
clustering 0.005
computer 0.0009
…
food 0.000001
…

How can we get rid of these common words?

Generate d Using Two Word Distributions

d

Text mining paper

$P(w|\ \theta_d)$

$p(w|\ \theta_B)$

Topic: θ_d

text 0.04
mining 0.035
association 0.03
clustering 0.005
…
the 0.000001

the 0.03
a 0.02
is 0.015
we 0.01
food 0.003
…
text 0.000006
…

Background (topic) θ_B

$p(\theta_d)+(\theta_B)=1$

$P(\theta_d)=0.5$

Topic Choice

$P(\theta_B)=0.5$

What’s the probability of observing a word w?

Topic: θ_d

text 0.04
mining 0.035

the 0.000001

$p(\theta_d)+(\theta_B)=1$

=0.5

Topic
hoice

.5

d

“the”?

“text”?

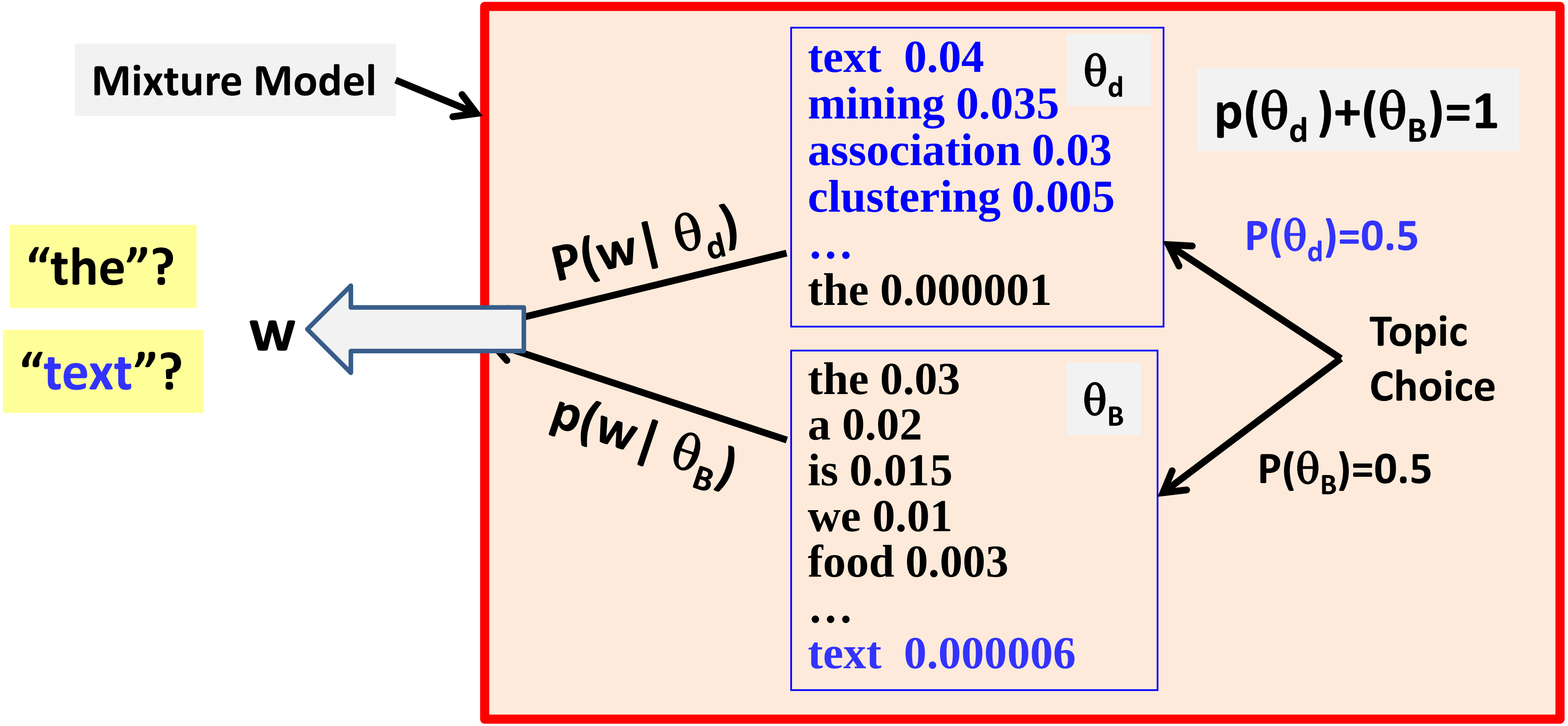
$P(\text{“the”}) = p(\theta_d)p(\text{“the”}|\theta_d) + p(\theta_B)p(\text{“the”}|\theta_B)$
$= 0.5*0.000001+0.5*0.03$

$P(\text{“text”}) = p(\theta_d)p(\text{“text”}|\theta_d) + p(\theta_B)p(\text{“text”}|\theta_B)$
$= 0.5*0.04+0.5*0.000006$

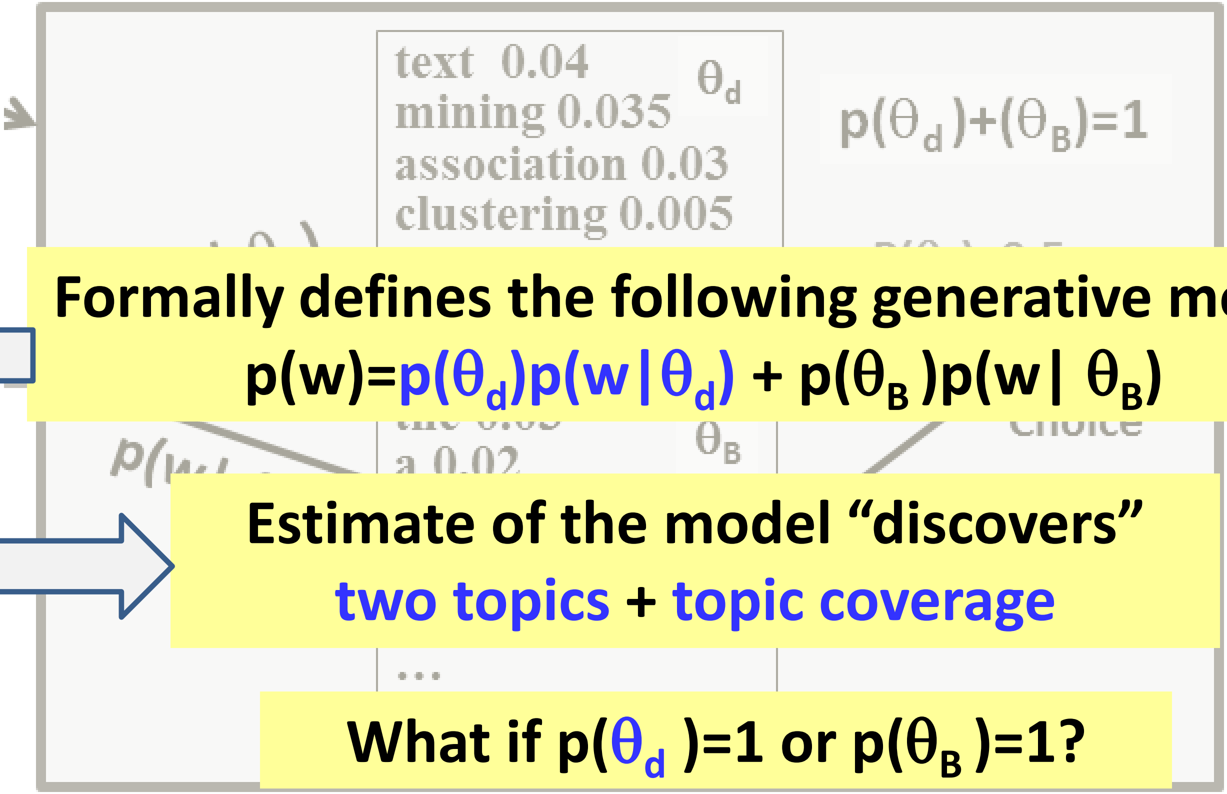
we 0.01
food 0.003
…
text 0.000006
…

Background (topic) θ_B

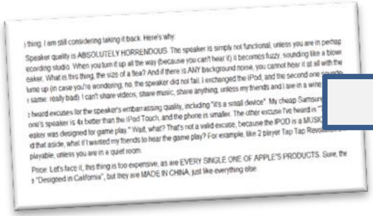
The Idea of a Mixture Model



As a Generative Model…



w



Formally defines the following generative model:

$$p(w)=p(\theta_d)p(w|\theta_d) + p(\theta_B)p(w|\theta_B)$$

Estimate of the model "discovers"

two topics + topic coverage

What if $p(\theta_d)=1$ or $p(\theta_B)=1$?

Mixture of Two Unigram Language Models

- **Data**: Document d
- Mixture **Model**: **parameters** Λ=({p(w|θ_d)}, {p(w|θ_B)}, p(θ_B), p(θ_d))
  - Two unigram LMs: θ_d **(the topic of d); θ_B (background topic)**
  - Mixing weight (topic choice): **p(θ_d)+p(θ_B)=1**
- **Likelihood** function:

$$p(d|\Lambda) = \prod_{i=1}^{|d|} p(x_i|\Lambda) = \prod_{i=1}^{|d|} [p(\theta_d)p(x_i|\theta_d) + p(\theta_B)p(x_i|\theta_B)]$$

$$= \prod_{i=1}^{M} [p(\theta_d)p(w_i|\theta_d) + p(\theta_B)p(w_i|\theta_B)]^{c(w,d)}$$

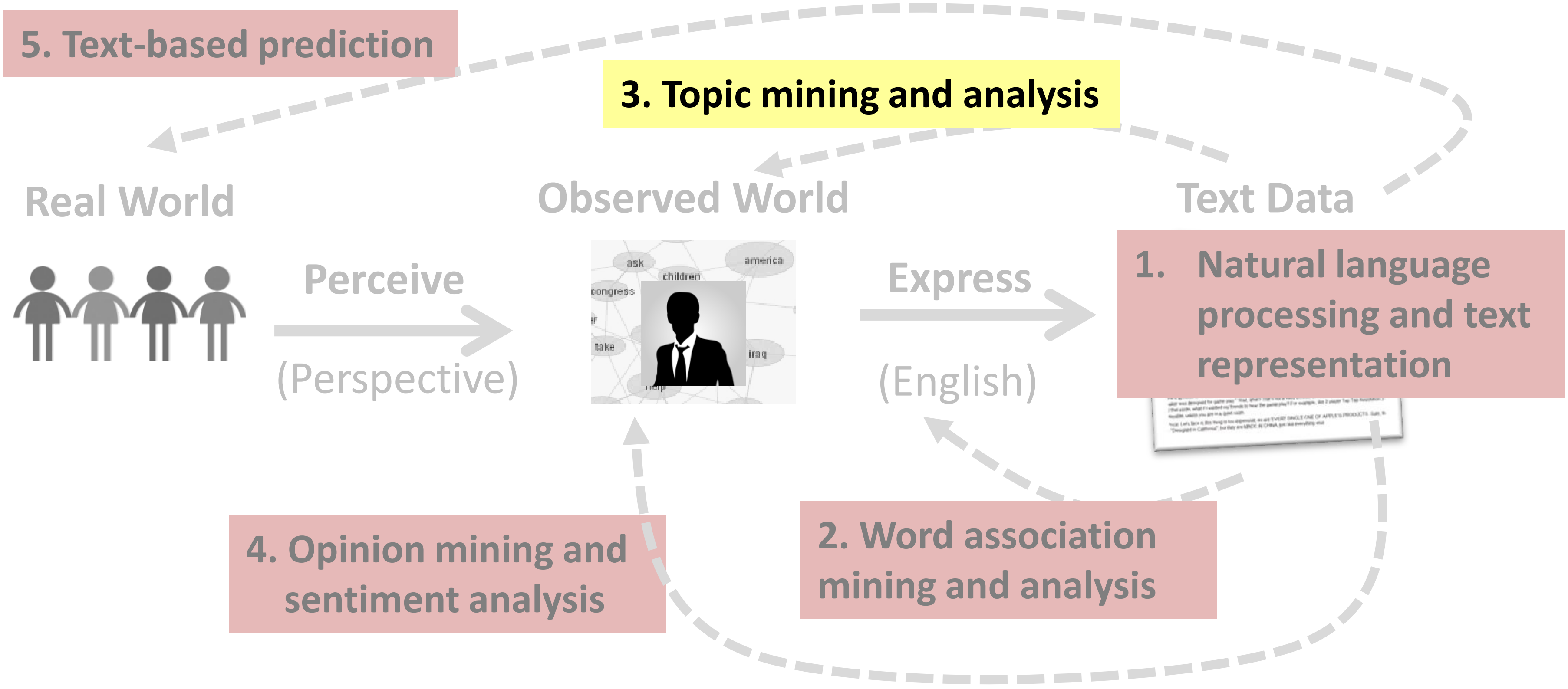
- ML **Estimate:** $\Lambda^* = \arg\max_{\Lambda} p(d|\Lambda)$

Subject to $\sum_{i=1}^{M} p(w_i|\theta_d) = \sum_{i=1}^{M} p(w_i|\theta_B) = 1 \qquad p(\theta_d) + p(\theta_B) = 1$

Slide adapted from ChengXiang Zhai's presentation

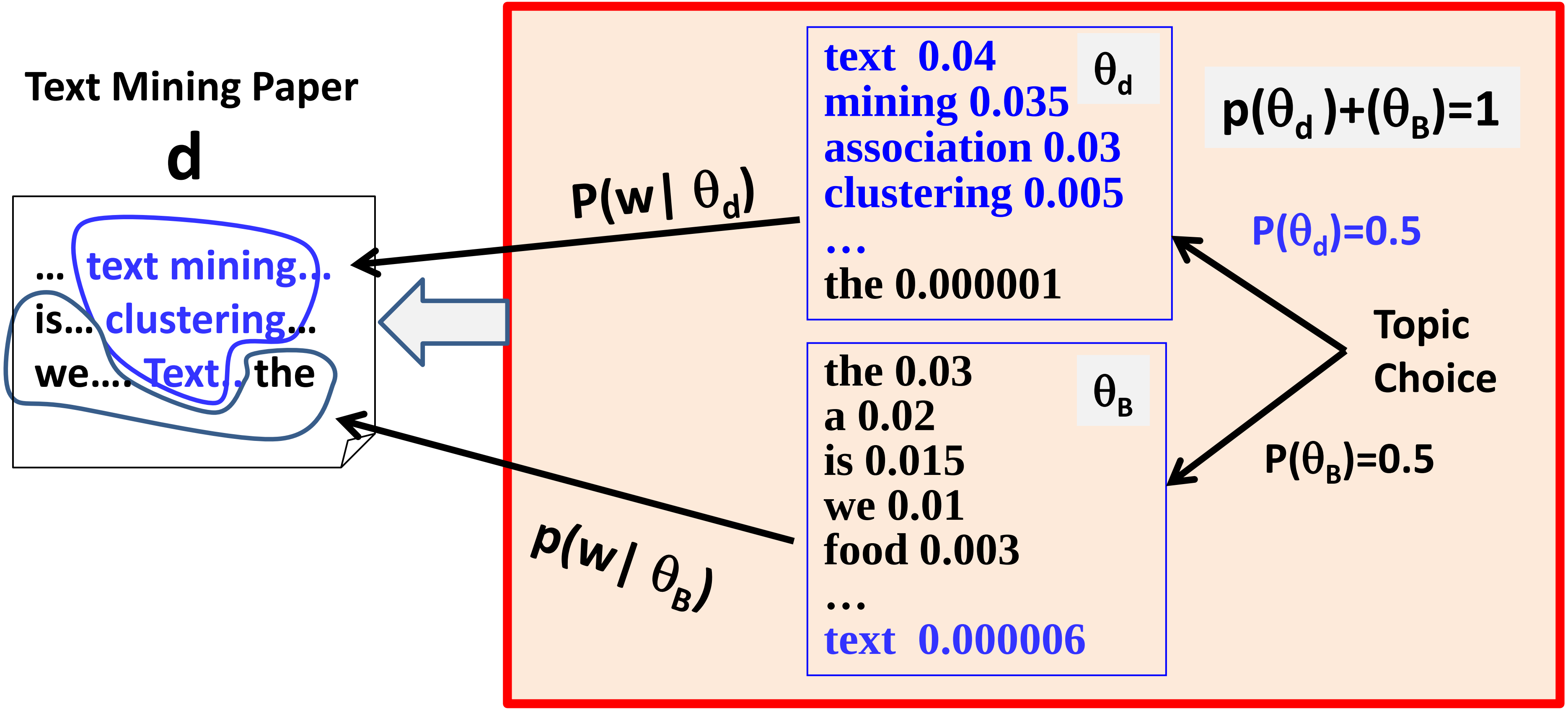
Probabilistic Topic Models: Mixture Model Estimation

Probabilistic Topic Models: Mixture Model Estimation



Slide adapted from ChengXiang Zhai's presentation

Back to Factoring out Background Words



Slide adapted from ChengXiang Zhai's presentation

Estimation of One Topic: **$P(w|\theta_d)$**

Adjust θ_d to maximize $p(d|\Lambda)$
(all other parameters are known)

Would the ML estimate <u>demote</u> background words in θ_d ?

d

… text mining…
is… clustering…
we…. Text.. the

θ_d

text ?
mining ?
association ?
clustering ?
…
the ?

θ_B

the 0.03
a 0.02
is 0.015
we 0.01
food 0.003
…
text 0.000006

$p(\theta_d)+(\theta_B)=1$

$P(\theta_d)=0.5$

Topic Choice

$P(\theta_B)=0.5$

Slide adapted from ChengXiang Zhai's presentation

Behavior of a Mixture Model

d = text the

Likelihood:

$$P(\text{"text"}) = p(\theta_d)p(\text{"text"}|\theta_d) + p(\theta_B)p(\text{"text"}|\theta_B)$$
$$= 0.5 * p(\text{"text"}|\theta_d) + 0.5*0.1$$

$$P(\text{"the"}) = 0.5 * p(\text{"the"}|\theta_d) + 0.5*0.9$$

$$p(d|\Lambda) = p(\text{"text"}|\Lambda)\, p(\text{"the"}|\Lambda)$$

$$= [0.5 * p(\text{"text"}|\theta_d) + 0.5*0.1] \times [0.5 * p(\text{"the"}|\theta_d) + 0.5*0.9]$$

θ_d:
text ?
the ?

$P(\theta_d)=0.5$

$P(\theta_B)=0.5$

θ_B:
the 0.9
text 0.1

How can we set $p(\text{"text"}|\theta_d)$ & $p(\text{"text"}|\theta_d)$ to maximize it?

Note that $p(\text{"text"}|\theta_d) + p(\text{"the"}|\theta_d) = 1$

Slide adapted from ChengXiang Zhai's presentation

“Collaboration” and “Competition” of θ_d and θ_B

$p(d|\Lambda)=p(\text{“text”}|\Lambda)\, p(\text{“the”}|\Lambda)$

$= [0.5 * p(\text{“text”}|\theta_d) + 0.5*0.1] \times$

$[0.5 * p(\text{“the”}|\theta_d) + 0.5*0.9]$

d = text the

text ?
the ?
θ_d

$P(\theta_d)=0.5$

$P(\theta_B)=0.5$

the 0.9
text 0.1
θ_B

Note that $p(\text{“text”}|\theta_d) + p(\text{“the”}|\theta_d) = 1$

If $x + y = constant$, then xy reaches maximum when $x = y$.

$0.5 * p(\text{“text”}|\theta_d) + 0.5*0.1 = 0.5 * p(\text{“the”}|\theta_d) + 0.5*0.9$

➔ $p(\text{“text”}|\theta_d)=0.9 \quad >> \quad p(\text{“the”}|\theta_d) = 0.1$!

Behavior 1: if $p(w1|\theta_B) > p(w2|\theta_B)$, then $p(w1|\theta_d) < p(w2|\theta_d)$

Slide adapted from ChengXiang Zhai's presentation

Response to Data Frequency

d = text the

$$p(d|\Lambda) = [0.5*p(\text{"text"}|\theta_d) + 0.5*0.1] \times [0.5*p(\text{"the"}|\theta_d) + 0.5*0.9]$$

➔ $p(\text{"text"}|\theta_d)=0.9$ >> $p(\text{"the"}|\theta_d) = 0.1$!

d' = text the the the the …the

$$p(d'|\Lambda) = [0.5*p(\text{"text"}|\theta_d) + 0.5*0.1] \times [0.5*p(\text{"the"}|\theta_d) + 0.5*0.9] \times [0.5*p(\text{"the"}|\theta_d) + 0.5*0.9] \times [0.5*p(\text{"the"}|\theta_d) + 0.5*0.9] \cdots \times [0.5*p(\text{"the"}|\theta_d) + 0.5*0.9]$$

What if we increase $p(\theta_B)$?

What's the optimal solution now? $p(\text{"the"}|\theta_d) > 0.1$? or $p(\text{"the"}|\theta_d) < 0.1$?

Behavior 2: high frequency words get higher $p(w|\theta_d)$

Slide adapted from ChengXiang Zhai's presentation

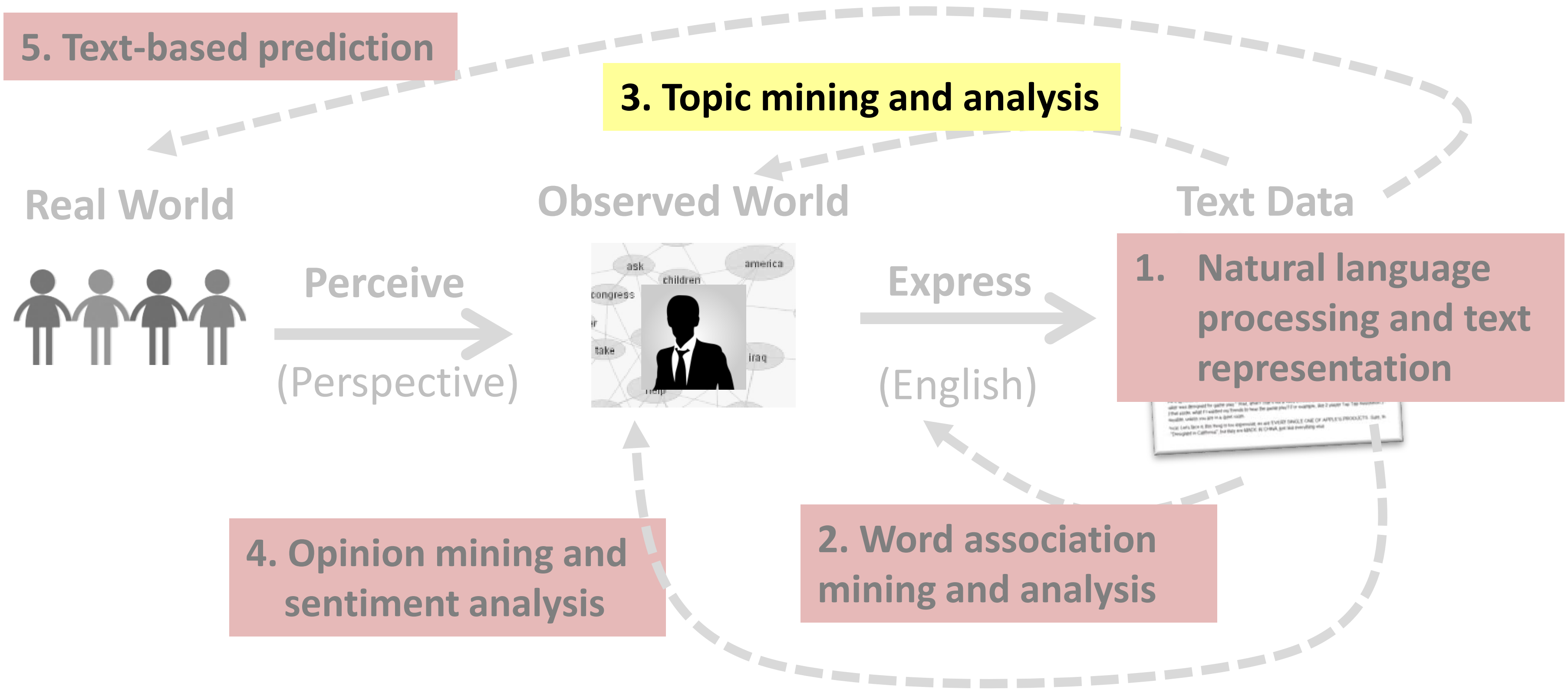
Summary

- General behavior of a mixture model:
  - Every component model attempts to assign high probabilities to highly frequent words in the data (to "collaboratively maximize likelihood")
  - Different component models tend to "bet" high probabilities on different words (to avoid "competition" or "waste of probability")
  - The probability of choosing each component "regulates" the collaboration/competition between the component models
- Fixing one component to a background word distribution (i.e., background language model):
  - Helps "get rid of background words" in other component
  - Is an example of imposing a prior on the model parameters (prior = one model must be exactly the same as the background LM)

Slide adapted from ChengXiang Zhai's presentation

Probabilistic Topic Models: Expectation-Maximization Algorithm

Probabilistic Topic Models: Expectation-Maximization (EM) Algorithm



Slide adapted from ChengXiang Zhai's presentation

Estimation of One Topic: $P(w|\theta_d)$

How to set θ_d to maximize $p(d|\Lambda)$?
(all other parameters are known)

d

... text mining...
is... clustering...
we.... Text.. the

θ_d

text ?
mining ?
association ?
clustering ?
…
the ?

θ_B

the 0.03
a 0.02
is 0.015
we 0.01
food 0.003
…
text 0.000006

$p(\theta_d)+(\theta_B)=1$

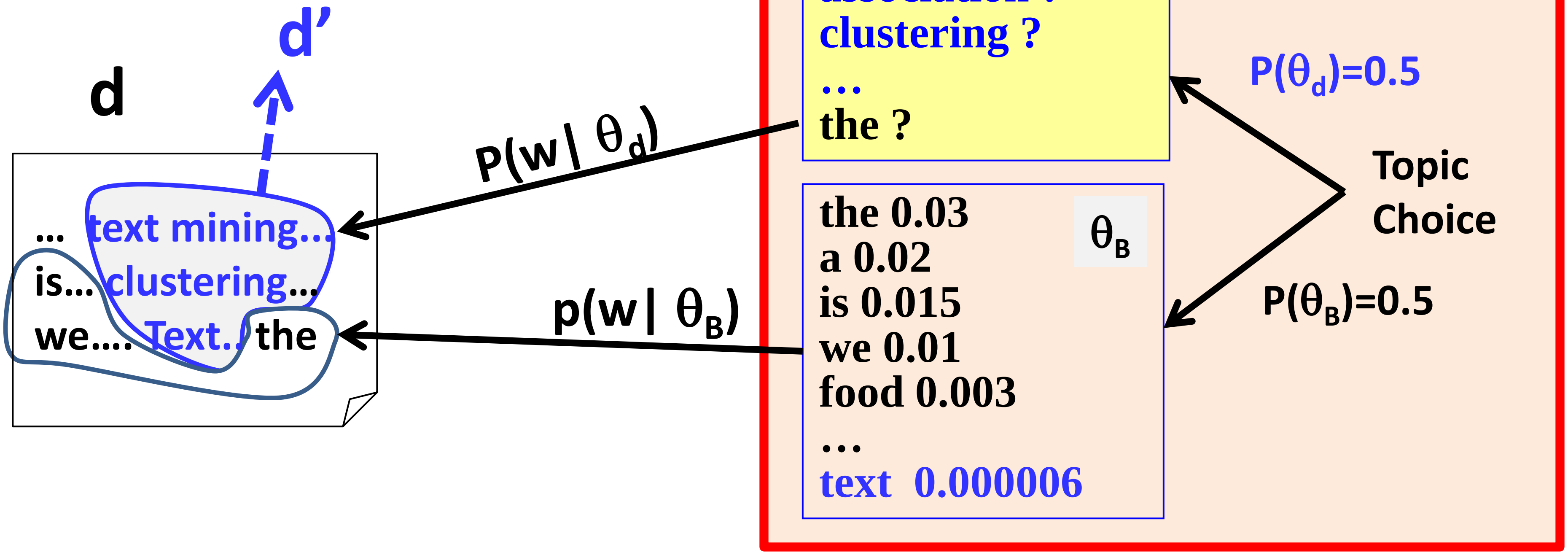
$P(\theta_d)=0.5$

Topic Choice

$P(\theta_B)=0.5$

If we know which word is from which distribution…

$$p(w_i \mid \theta_d) = \frac{c(w_i, d')}{\sum_{w' \in V} c(w', d')}$$



Given all the parameters, infer the distribution a word is from…

Is "text" more likely from θ_d or θ_B ?

From θ_d (Z=0)?

$p(\theta_d)p(\text{“text”}|\theta_d)$

From θ_B (Z=1)?

$p(\theta_B)p(\text{“text”}|\theta_B)$

$$p(z=0 \mid w=\text{"text"}) = \frac{p(\theta_d)p(\text{"text"} \mid \theta_d)}{p(\theta_d)p(\text{"text"} \mid \theta_d) + p(\theta_B)p(\text{"text"} \mid \theta_B)}$$

$P(w|\ \theta_d)$

θ_d

text 0.04
mining 0.035
association 0.03
clustering 0.005
…
the 0.000001

$p(w|\ \theta_B)$

θ_B

the 0.03
a 0.02
is 0.015
we 0.01
food 0.003
…
text 0.000006

$p(\theta_d)+p(\theta_B)=1$

$P(\theta_d)=0.5$

Topic Choice

$P(\theta_B)=0.5$

Slide adapted from ChengXiang Zhai's presentation

The Expectation-Maximization (EM) Algorithm

Hidden Variable: $z \in \{0, 1\}$

	z
the	**1**
paper	**1**
presents	**1**
a	**1**
text	**0**
mining	**0**
algorithm	**0**
for	**1**
clustering	**0**
...	**...**

Initialize $p(w|\theta_d)$ with random values.
Then iteratively improve it using E-step & M-step.
Stop when likelihood doesn't change.

$$p^{(n)}(z=0|w) = \frac{p(\theta_d)p^{(n)}(w|\theta_d)}{p(\theta_d)p^{(n)}(w|\theta_d) + p(\theta_B)p(w|\theta_B)}$$

E-step

How likely w is from θ_d

$$p^{(n+1)}(w|\theta_d) = \frac{c(w,d)p^{(n)}(z=0|w)}{\sum_{w' \in V} c(w',d)p^{(n)}(z=0|w')}$$

M-step

Slide adapted from ChengXiang Zhai's presentation

EM Computation in Action

E-step $$p^{(n)}(z=0|w)=\frac{p(\theta_d)p^{(n)}(w|\theta_d)}{p(\theta_d)p^{(n)}(w|\theta_d)+p(\theta_B)p(w|\theta_B)}$$

M-step $$p^{(n+1)}(w|\theta_d)=\frac{c(w,d)p^{(n)}(z=0|w)}{\sum_{w'\in V}c(w',d)p^{(n)}(z=0|w')}$$

Assume
$p(\theta_d)=p(\theta_B)=0.5$
and $p(w|\theta_B)$ is known

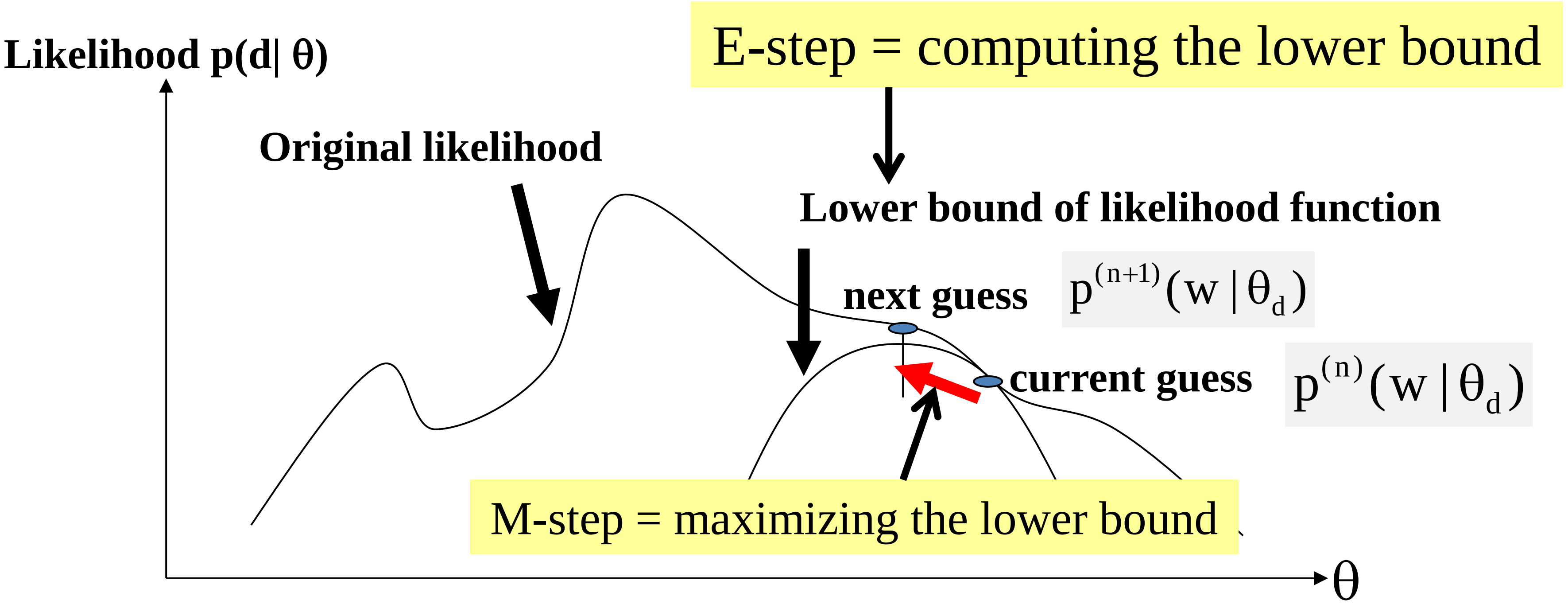
Word	#	$p(w\|\theta_B)$	Iteration 1		Iteration 2		Iteration 3	
			$P(w\|\theta)$	$p(z=0\|w)$	$P(w\|\theta)$	$P(z=0\|w)$	$P(w\|\theta)$	$P(z=0\|w)$
The	4	0.5	**0.25**	0.33	**0.20**	0.29	**0.18**	0.26
Paper	2	0.3	**0.25**	0.45	**0.14**	0.32	**0.10**	0.25
Text	4	0.1	**0.25**	0.71	**0.44**	0.81	**0.50**	0.93
Mining	2	0.1	**0.25**	0.71	**0.22**	0.69	**0.22**	0.69
Log-Likelihood			-16.96		-16.13		-16.02	

Likelihood increasing

"By products": Are they also useful?

Slide adapted from ChengXiang Zhai's presentation

EM As Hill-Climbing ➔ Converge to Local Maximum

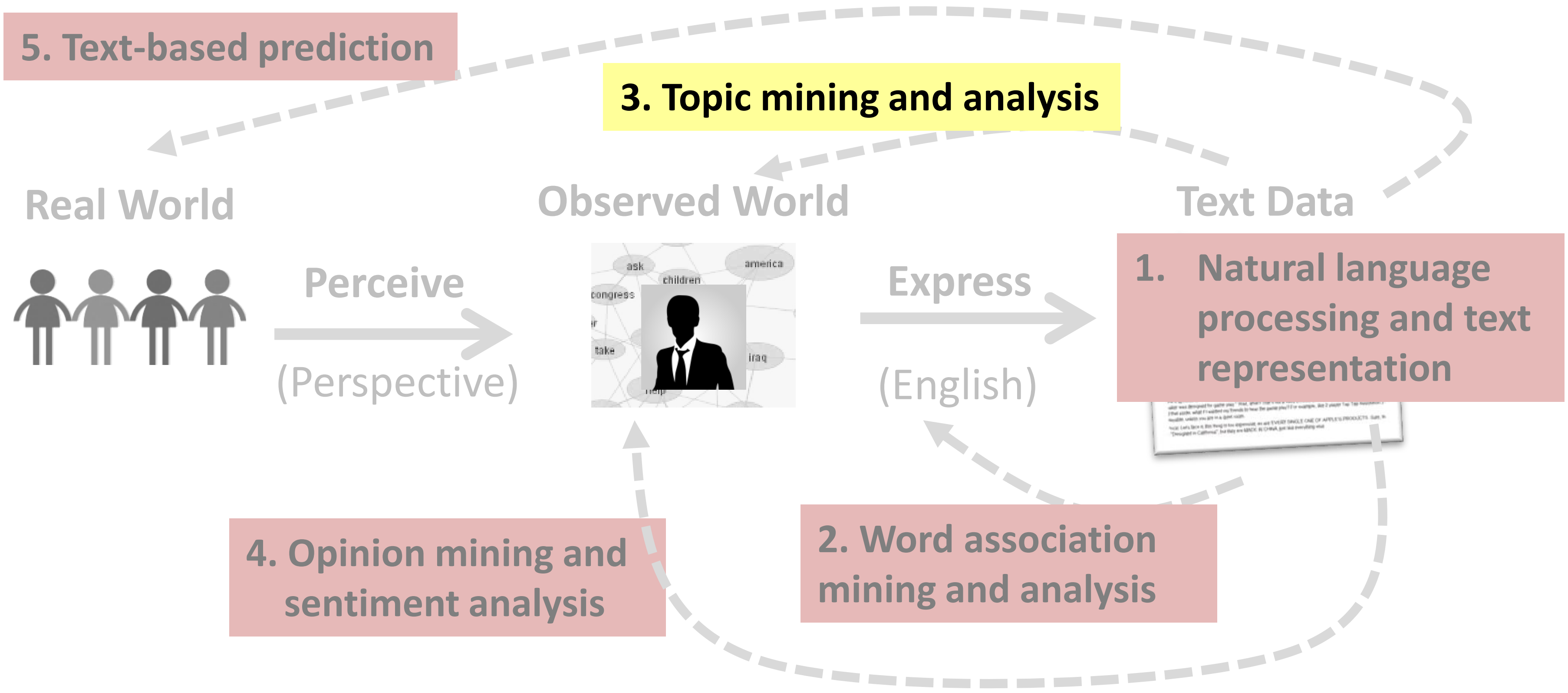


Summary

- Expectation-Maximization (EM) algorithm
  - General algorithm for computing ML estimate of mixture models
  - Hill-climbing, so can only converge to a local maximum (depending on initial points)
- E-step: “augment” data by predicting values of useful hidden variables
- M-step: exploit the “augmented data” to improve estimate of parameters (“improve” is guaranteed in terms of likelihood)
- “Data augmentation” is probabilistic ➔ Split counts of events probabilistically

Probabilistic Latent Semantic Analysis (PLSA)

Probabilistic Latent Semantic Analysis (PLSA)



Slide adapted from ChengXiang Zhai's presentation

Document as a Sample of Mixed Topics

Topic θ_1

government 0.3
response 0.2
...

Topic θ_2

city 0.2
new 0.1
orleans 0.05
...

. . .

Topic θ_k

donate 0.1
relief 0.05
help 0.02
...

Background θ_B

the 0.04
a 0.03
...

Blog article about "Hurricane Katrina"

[Criticism of government response to the hurricane primarily consisted of criticism of its response to the approach of the storm and its aftermath, specifically in the delayed response] to the [flooding of New Orleans. … 80% of the 1.3 million residents of the greater New Orleans metropolitan area evacuated] …[Over seventy countries pledged monetary donations or other assistance]. …

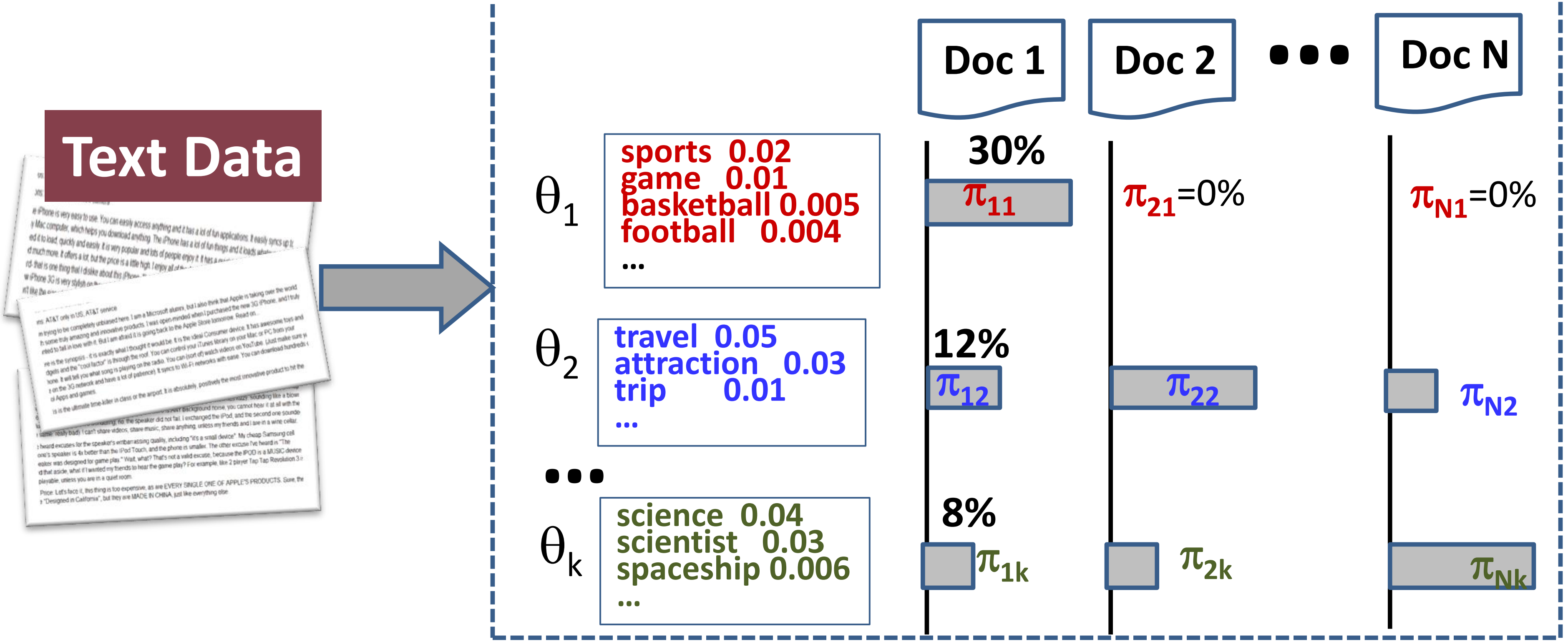
Many applications are possible if we can "decode" the topics in text…

Slide adapted from ChengXiang Zhai's presentation

Mining Multiple Topics from Text

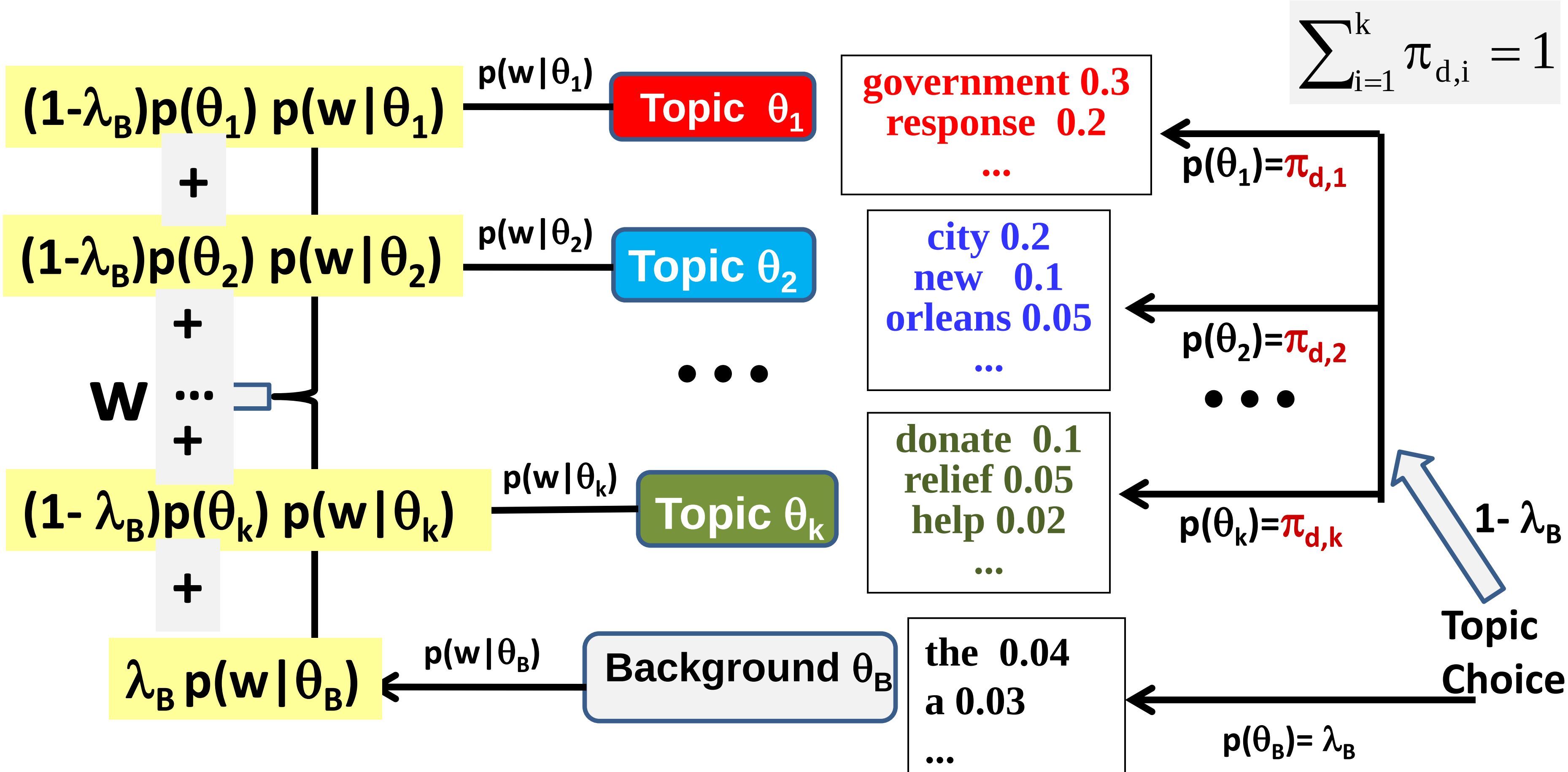
INPUT: C, k, V

OUTPUT: $\{ \theta_1, \ldots, \theta_k \}, \{ \pi_{i1}, \ldots, \pi_{ik} \}$



Slide adapted from ChengXiang Zhai's presentation

Generating Text with Multiple Topics: p(w)=?



Slide adapted from ChengXiang Zhai's presentation

Probabilistic Latent Semantic Analysis (PLSA)

Percentage of background words (known)

Background LM (known)

Coverage of topic θ_j in doc d

Prob. of word w in topic θ_j

$$p_d(w) = \lambda_B p(w \mid \theta_B) + (1-\lambda_B)\sum_{j=1}^{k} \pi_{d,j}\, p(w \mid \theta_j)$$

$$\log p(d) = \sum_{w \in V} c(w,d) \log[\lambda_B p(w \mid \theta_B) + (1-\lambda_B)\sum_{j=1}^{k} \pi_{d,j}\, p(w \mid \theta_j)]$$

$$\log p(C \mid \Lambda) = \sum_{d \in C} \sum_{w \in V} c(w,d) \log[\lambda_B p(w \mid \theta_B) + (1-\lambda_B)\sum_{j=1}^{k} \pi_{d,j}\, p(w \mid \theta_j)]$$

Unknown Parameters: $\Lambda=(\{\pi_{d,j}\}, \{\theta_j\})$, j=1, …, k

How many unknown parameters are there in total?

ML Parameter Estimation

$$p_d(w) = \lambda_B p(w \mid \theta_B) + (1-\lambda_B)\sum_{j=1}^{k} \pi_{d,j}\, p(w \mid \theta_j)$$

$$\log p(d) = \sum_{w \in V} c(w,d) \log[\lambda_B p(w \mid \theta_B) + (1-\lambda_B)\sum_{j=1}^{k} \pi_{d,j}\, p(w \mid \theta_j)]$$

$$\log p(C \mid \Lambda) = \sum_{d \in C}\sum_{w \in V} c(w,d) \log[\lambda_B p(w \mid \theta_B) + (1-\lambda_B)\sum_{j=1}^{k} \pi_{d,j}\, p(w \mid \theta_j)]$$

Constrained Optimization: $\Lambda^* = \arg\max_{\Lambda} p(C \mid \Lambda)$

$$\forall j \in [1,k], \sum_{i=1}^{M} p(w_i \mid \theta_j) = 1 \qquad \forall d \in C, \sum_{j=1}^{k} \pi_{d,j} = 1$$

EM Algorithm for PLSA: E-Step

Hidden Variable (=topic indicator): $z_{d,w} \in \{B, 1, 2, \ldots, k\}$

Probability that **w in doc d** is generated from **topic** θ_j

Use of Bayes Rule

$$p(z_{d,w} = j) = \frac{\pi_{d,j}^{(n)} p^{(n)}(w \mid \theta_j)}{\sum_{j'=1}^{k} \pi_{d,j'}^{(n)} p^{(n)}(w \mid \theta_{j'})}$$

$$p(z_{d,w} = B) = \frac{\lambda_B p(w \mid \theta_B)}{\lambda_B p(w \mid \theta_B) + (1 - \lambda_B) \sum_{j=1}^{k} \pi_{d,j}^{(n)} p^{(n)}(w \mid \theta_j)}$$

Probability that **w in doc d** is generated from **background** θ_B

Slide adapted from ChengXiang Zhai's presentation

EM Algorithm for PLSA: M-Step

Hidden Variable (=topic indicator): $z_{d,w} \in \{B, 1, 2, \ldots, k\}$

ML Estimate based on "allocated" word counts to topic θ_j

Re-estimated **probability** of **doc d** covering **topic** θ_j

$$\pi_{d,j}^{(n+1)} = \frac{\sum_{w \in V} \boxed{c(w,d)(1-p(z_{d,w}=B))p(z_{d,w}=j)}}{\sum_{j'} \sum_{w \in V} c(w,d)(1-p(z_{d,w}=B))p(z_{d,w}=j')}$$

$$p^{(n+1)}(w \mid \theta_j) = \frac{\sum_{d \in C} \boxed{c(w,d)(1-p(z_{d,w}=B))p(z_{d,w}=j)}}{\sum_{w' \in V} \sum_{d \in C} c(w',d)(1-p(z_{d,w'}=B))p(z_{d,w'}=j)}$$

Re-estimated **probability** of **word w** for **topic** θ_j

Computation of the EM Algorithm

- Initialize all unknown parameters randomly
- Repeat until likelihood converges
  - E-step $p(z_{d,w} = j) \propto \pi_{d,j}^{(n)} p^{(n)}(w \mid \theta_j)$ $\qquad \sum_{j=1}^{k} p(z_{d,w} = j) = 1$

    $p(z_{d,w} = B) \propto \lambda_B p(w \mid \theta_B)$ ← What's the normalizer for this one?
  - M-step

$$\pi_{d,j}^{(n+1)} \propto \sum_{w \in V} c(w,d)(1 - p(z_{d,w} = B)) p(z_{d,w} = j) \qquad \forall d \in C, \sum_{j=1}^{k} \pi_{d,j} = 1$$

$$p^{(n+1)}(w \mid \theta_j) \propto \sum_{d \in C} c(w,d)(1 - p(z_{d,w} = B)) p(z_{d,w} = j) \qquad \forall j \in [1,k], \sum_{w \in V} p(w \mid \theta_j) = 1$$

In general, accumulate counts, and then normalize

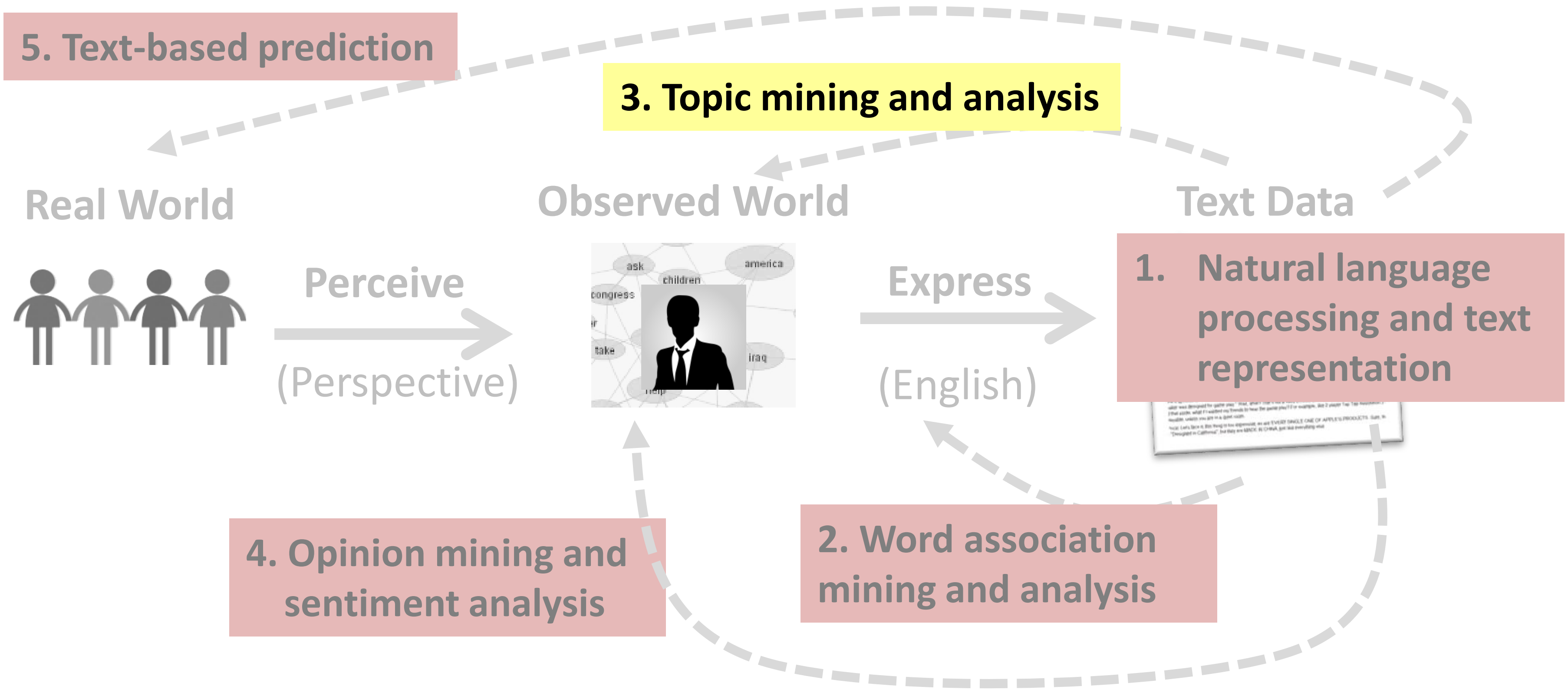
Summary

- PLSA = mixture model with k unigram LMs (k topics)
- Adding a pre-determined background LM helps discover discriminative topics
- ML estimate “discovers” topical knowledge from text data
  - k word distributions (k topics)
  - proportion of each topic in each document
- The output can enable many applications!
  - Clustering of terms and docs (treat each topic as a cluster)
  - Further associate topics with different contexts (e.g., time periods, locations, authors, sources, etc.)

Latent Dirichlet Allocation (LDA)

Slide adapted from ChengXiang Zhai's presentation

Latent Dirichlet Allocation (LDA)



Slide adapted from ChengXiang Zhai's presentation

Extensions of PLSA

- PLSA with prior knowledge ➔ User-controlled PLSA
- PLSA as a generative model ➔ Latent Dirichlet Allocation

PLSA with Prior Knowledge

- Users may have expectations about which topics to analyze:
  - We expect to see “retrieval models” as a topic in IR
  - We want to see aspects such as “battery” and “memory” for opinions about a laptop
- Users may have knowledge about what topics are (or are NOT) covered in a document
  - Tags = topics ➔ A doc can only be generated using topics corresponding to the tags assigned to the document
- We can incorporate such knowledge as priors of PLSA model

Maximum a Posteriori (MAP) Estimate

$$\Lambda^* = \arg\max_{\Lambda} p(\Lambda) p(Data \mid \Lambda)$$

- We may use $p(\Lambda)$ to encode all kinds of preferences and constraints, e.g.,
  - $p(\Lambda)>0$ if and only if one topic is precisely "background": $p(w|\theta_B)$
  - $p(\Lambda)>0$ if and only if for a particular doc d, $\pi_{d,3}=0$ and $\pi_{d,1}=1/2$
  - $p(\Lambda)$ favors a Λ with topics that assign high probabilities to some particular words
- The MAP estimate (with conjugate prior) can be computed using a similar EM algorithm to the ML estimate with smoothing to reflect prior preferences

EM Algorithm with Conjugate Prior on p(w| θ_i)

$$p(z_{d,w}=j)=\frac{\pi_{d,j}^{(n)}p^{(n)}(w|\theta_j)}{\sum_{j'=1}^{k}\pi_{d,j'}^{(n)}p^{(n)}(w|\theta_{j'})}$$

$$p(z_{d,w}=B)=\frac{\lambda_B p(w|\theta_B)}{\lambda_B p(w|\theta_B)+(1-\lambda_B)\sum_{j=1}^{k}\pi_{d,j}^{(n)}p^{(n)}(w|\theta_j)}$$

$$\pi_{d,j}^{(n+1)}=\frac{\sum_{w\in V}c(w,d)(1-p(z_{d,w}=B))p(z_{d,w}=j)}{\sum_{j'}\sum_{w\in V}c(w,d)(1-p(z_{d,w}=B))p(z_{d,w}=j')}$$

$$p^{(n+1)}(w|\theta_j)=\frac{\sum_{d\in C}c(w,d)(1-p(z_{d,w}=B))p(z_{d,w}=j)\ +\mu p(w|\theta'_j)}{\sum_{w'\in V}\sum_{d\in C}c(w',d)(1-p(z_{d,w'}=B))p(z_{d,w'}=j)\ +\mu}$$

Prior: p(w|θ'_j)

battery 0.5
life 0.5

Pseudo counts of w from prior θ'

Sum of all pseudo counts

What if μ=0? What if μ=+∞?

We may also set any parameter to a constant (including 0) as needed

Slide adapted from ChengXiang Zhai's presentation

Deficiency of PLSA

- Not a generative model
  - Can't compute probability of a new document
  - Heuristic workaround is possible, though
- Many parameters ➔ high complexity of models
  - Many local maxima
  - Prone to overfitting
- Not necessarily a problem for text mining (only interested in fitting the "training" documents)

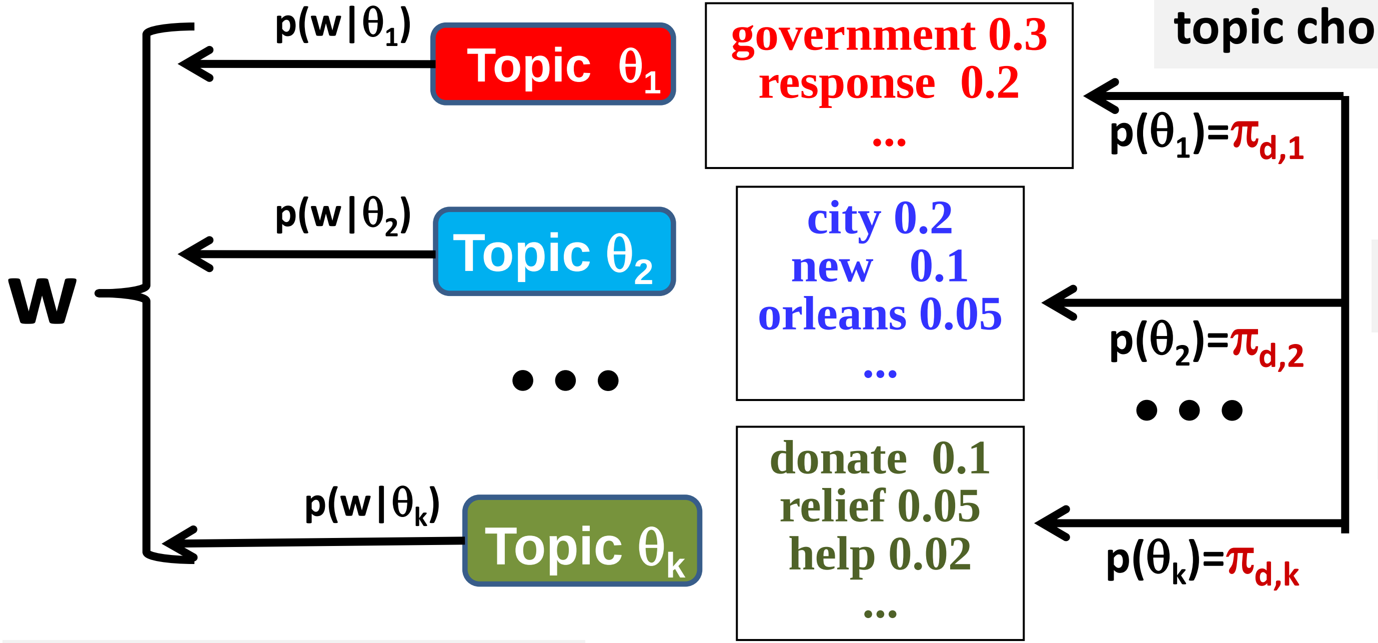
Slide adapted from ChengXiang Zhai's presentation

Latent Dirichlet Allocation (LDA)

- Make PLSA a generative model by imposing a Dirichlet prior on the model parameters ➔
  - LDA = Bayesian version of PLSA
  - Parameters are regularized
- Can achieve the same goal as PLSA for text mining purposes
  - Topic coverage and topic word distributions can be inferred using Bayesian inference

PLSA → LDA

Both word distributions and topic choices are free in PLSA



$$\vec{\pi}_d = (\pi_{d,1}, \ldots, \pi_{d,k})$$

$$p(\vec{\pi}_d) = \text{Dirichlet}(\vec{\alpha})$$

$$\vec{\alpha} = (\alpha_1, \ldots, \alpha_k), \alpha_i > 0$$

$$\vec{\theta}_i = (p(w_1 | \theta_i), \ldots, p(w_M | \theta_i))$$

$$p(\vec{\theta}_i) = \text{Dirichlet}(\vec{\beta})$$

$$\vec{\beta} = (\beta_1, \ldots, \beta_M), \beta_i > 0$$

LDA imposes a prior on both

Slide adapted from ChengXiang Zhai's presentation

Likelihood Functions for PLSA vs. LDA

PLSA

$$p_d(w|\{\theta_j\},\{\pi_{d,j}\}) = \underline{\sum_{j=1}^{k} \pi_{d,j}\, p(w|\theta_j)}$$

$$\log p(d|\{\theta_j\},\{\pi_{d,j}\}) = \sum_{w \in V} c(w,d) \log[\sum_{j=1}^{k} \pi_{d,j}\, p(w|\theta_j)]$$

$$\log p(C|\{\theta_j\},\{\pi_{d,j}\}) = \sum_{d \in C} \log p(d|\{\theta_j\},\{\pi_{d,j}\})$$

Core assumption in all topic models

LDA

$$p_d(w|\{\theta_j\},\{\pi_{d,j}\}) = \underline{\sum_{j=1}^{k} \pi_{d,j}\, p(w|\theta_j)}$$

$$\log p(d|\vec{\alpha},\{\theta_j\}) = \int \boxed{\sum_{w \in V} c(w,d) \log[\sum_{j=1}^{k} \pi_{d,j}\, p(w|\theta_j)]}\, \underline{p(\vec{\pi}_d|\vec{\alpha})d\vec{\pi}_d}$$

$$\log p(C|\vec{\alpha},\vec{\beta}) = \int \sum_{d \in C} \log p(d|\vec{\alpha},\{\theta_j\}) \underline{\prod_{j=1}^{k} p(\theta_j|\vec{\beta})d\theta_1 \ldots d\theta_k}$$

PLSA component

Added by LDA

Parameter Estimation and Inferences in LDA

- Parameters can be estimated using ML estimator

$$(\hat{\vec{\alpha}}, \hat{\vec{\beta}}) = \arg\max_{\vec{\alpha}, \vec{\beta}} \log p(C \mid \vec{\alpha}, \vec{\beta})$$

How many parameters in LDA vs. PLSA?

- However, $\{\theta_j\}$ **and** $\{\pi_{d,j}\}$ must now be computed using posterior inference
  - Computationally intractable
  - Must resort to approximate inference
  - Many different inference methods are available

Slide adapted from ChengXiang Zhai's presentation

Summary of Probabilistic Topic Models

- Probabilistic topic models provide a general principled way of mining and analyzing topics in text with many applications
- Basic task setup:
  - Input: Text data
  - Output: k topics + proportions of these topics covered in each document
- PLSA is the basic topic model, often adequate for most applications
- LDA improves over PLSA by imposing priors
  - Theoretically more appealing
  - Practically, LDA and PLSA perform similarly for many tasks

Suggested Readings

- Blei, D. 2012. “Probabilistic Topic Models.” *Communications of the ACM* 55 (4): 77–84. doi: 10.1145/2133806.2133826.
- Qiaozhu Mei, Xuehua Shen, and ChengXiang Zhai. “Automatic Labeling of Multinomial Topic Models.” *Proceedings of ACM KDD 2007*, pp. 490-499, DOI=10.1145/1281192.1281246.
- Yue Lu, Qiaozhu Mei, and Chengxiang Zhai. 2011. Investigating task performance of probabilistic topic models: an empirical study of PLSA and LDA. *Information Retrieval*, 14, 2 (April 2011), 178-203. DOI=10.1007/s10791-010-9141-9.