cit CTS

2.3.4 Conditional Sum Adder (CSuA)

- principle of sum conditioned by carry
- generalization of CSeA on multiple levels
- on multiple levels: → first: generates → z_i^1, c_{i+1}^1 ; → z_i^0, c_{i+1}^0 ; → subsequent stages: select the correct variant

- $c_0 = 0$!!!

$a = a(\underbrace{b+\bar{b}}_{1})$

$$c_{i+1} = x_i \cdot y_i + x_i \cdot c_i + y_i \cdot c_i$$
$$= x_i \cdot y_i(c_i + \bar{c_i}) + x_i \cdot c_i + y_i c_i = x_i \cdot y_i \cdot \bar{c_i} + \underbrace{x_i \cdot y_i \cdot c_i + x_i c_i}+ y_i c_i$$
$$= x_i \cdot y_i \cdot \bar{c_i} + x_i \cdot c_i(\underbrace{\cancel{y_i + 1}}_{1}) + y_i \cdot c_i \Rightarrow$$
$$= x_i \cdot y_i \cdot \bar{c_i} + x_i \cdot c_i + y_i \cdot c_i =$$
$$= x_i \cdot y_i \cdot \bar{c_i} + (x_i + y_i) \cdot c_i$$

$g_i = x_i \cdot y_i$

$p_i = x_i + y_i$

$$\boxed{c_{i+1} = g_i \cdot \bar{c_i} + p_i \cdot c_i}$$

$$\overline{c_{i+1}} = \overline{g_i \cdot \bar{c_i} + p_i \cdot c_i} = \overline{g_i \cdot \bar{c_i}} \cdot \overline{p_i \cdot c_i} =$$
$$= (\bar{g_i} + c_i)(\bar{p_i} + \bar{c_i}) = \underbrace{\bar{g_i}\bar{p_i}}_{\to \text{ absorbed}} + \bar{g_i} \cdot \bar{c_i} + \bar{p_i} \cdot c_i + \underbrace{\cancel{c_i \cdot \bar{c_i}}}_{0}$$

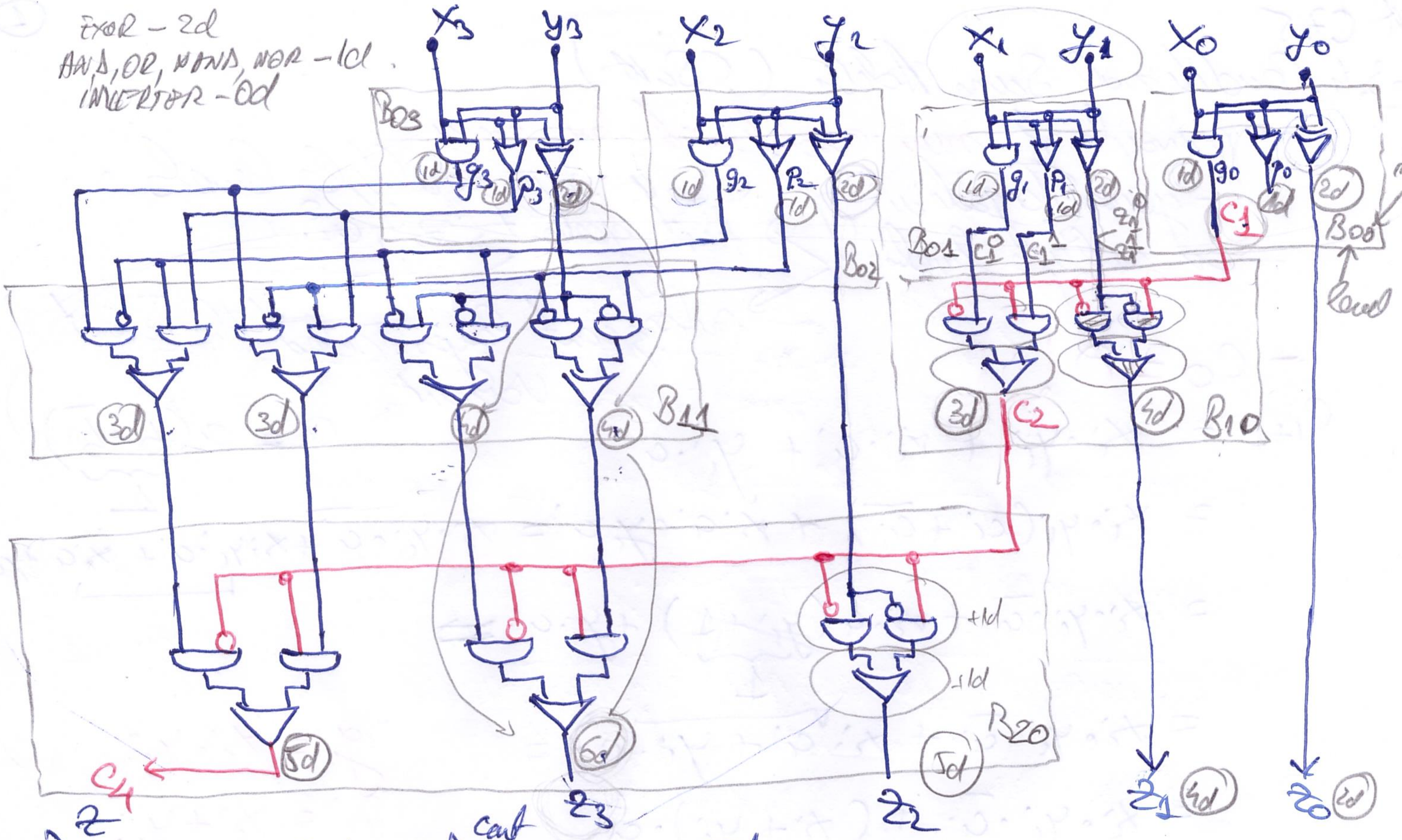
$$\boxed{\overline{c_{i+1}} = \bar{g_i} \cdot \bar{c_i} + \bar{p_i} \cdot c_i}$$

$a \oplus b = \bar{a} \cdot b + a \cdot \bar{b}$

Let X, Y - 4-bit operands

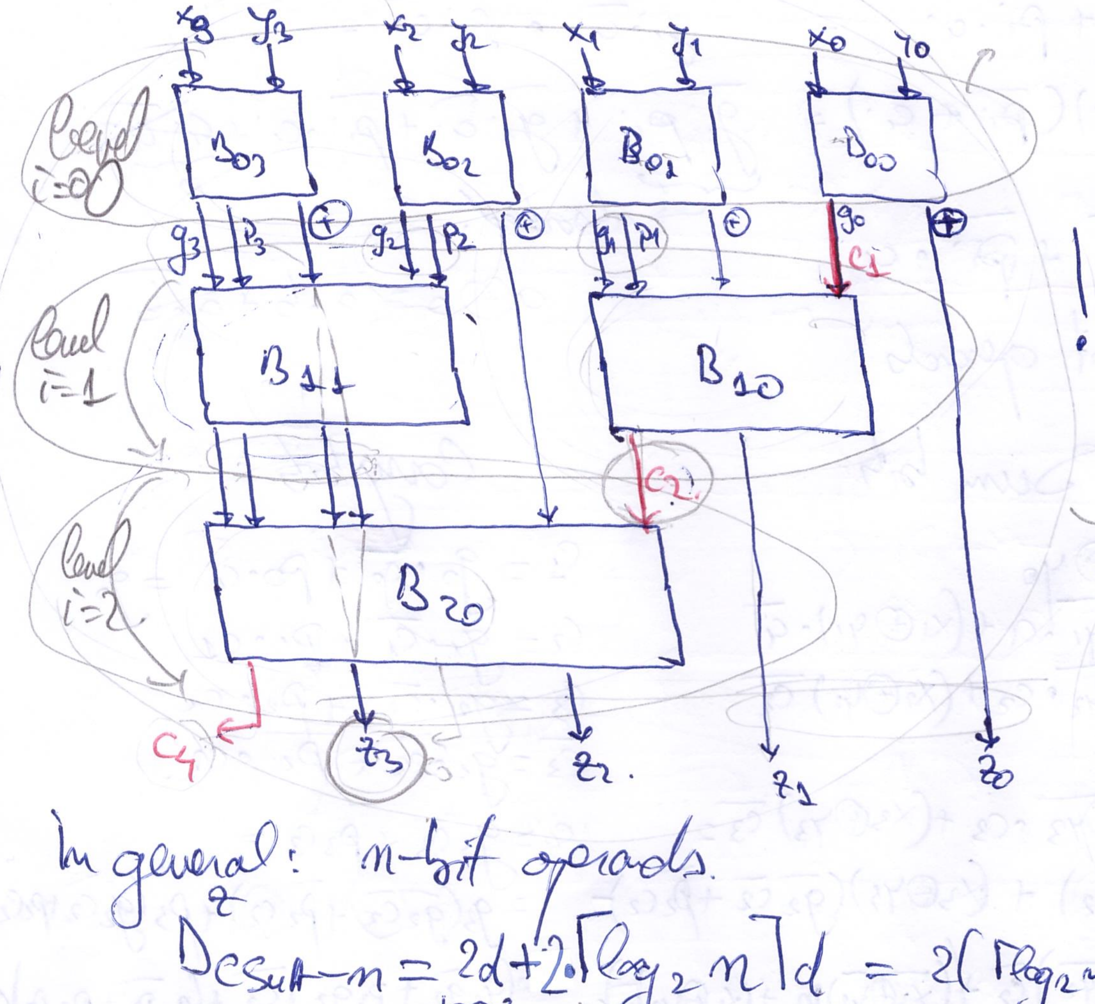
Sum bits	Carry bits
$z_0 = x_0 \oplus y_0 \oplus \overset{=0}{c_0} = x_0 \oplus y_0$	$c_1 = g_0 \cdot \overset{=1}{\bar{c_0}} + p_0 \cdot \overset{=0}{c_0} = g_0$
$z_1 = x_1 \oplus y_1 \oplus c_1 = \overline{x_1 \oplus y_1} \cdot c_1 + (x_1 \oplus y_1) \cdot \bar{c_1}$	$c_2 = g_1 \cdot \bar{c_1} + p_1 \cdot c_1$
$z_2 = x_2 \oplus y_2 \oplus c_2 = \overline{x_2 \oplus y_2} \cdot c_2 + (x_2 \oplus y_2) \bar{c_2}$	$c_3 = g_2 \cdot \bar{c_2} + p_2 \cdot c_2$
	$\bar{c_3} = \bar{g_2} \cdot \bar{c_2} + \bar{p_2} \cdot c_2$
$z_3 = x_3 \oplus y_3 \oplus c_3 = \overline{x_3 \oplus y_3} \cdot c_3 + (x_3 \oplus y_3) \bar{c_3} =$	$c_4 = g_3 \bar{c_3} + p_3 c_3 =$
$\overline{x_3 \oplus y_3}(g_2 \bar{c_2} + p_2 c_2) + (x_3 \oplus y_3)(\bar{g_2} \bar{c_2} + \bar{p_2} c_2) =$	$= g_3(\bar{g_2}\bar{c_2} + \bar{p_2} c_2) + p_3(g_2 \bar{c_2} + p_2 c_2)$
$((\overline{x_3 \oplus y_3}) g_2 + (x_3 \oplus y_3) \bar{g_2}) \bar{c_2} + ((\overline{x_3 \oplus y_3}) p_2 + (x_3 \oplus y_3) \bar{p_2}) c_2$	$= (g_3 \bar{g_2} + p_3 g_2) \bar{c_2} + (g_3 \bar{p_2} + p_3 p_2) c_2$

EXOR – 2d
AND, OR, NAND, NOR – 1d
INVERTER – 0d



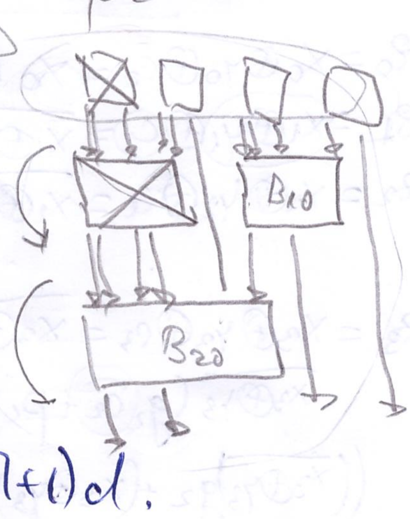
$D^{z}_{CSuA-4} = 6d \qquad \Delta^{cout}_{CSuA-4} = 5d.$

for 2-bit op



textbook
8-bit CSuA –
block diagram

for 3-bit



In general: n-bit operands.

$$D^{z}_{CSuA-n} = \underbrace{2d}_{\text{EXOR gate}} + 2\cdot\underbrace{\lceil \log_2 n \rceil d}_{\text{block levels besides the first level}} = 2(\lceil \log_2 n \rceil + 1)d.$$

$$\Delta^{cout}_{CSuA-n} = \underbrace{1d}_{g_i,p_i} + 2\lceil \log_2 n\rceil d = (2\lceil \log_2 n\rceil + 1)d.$$

Comparison: $n = 64$ bits

CSuA	CSeA	ML-CLA	RCA
$\Delta^{\Sigma} = 14d$	$\Delta^{\Sigma} = (n+2)d = 66d$.	$\Delta^{\Sigma} = (4\lceil\log_2 n\rceil + 1)d = 25d$.	$\Delta^{\Sigma} = 2nd = 128d$.
$\Delta^{cout} = 13d$	$\Delta^{cout} = (n+2)d = 66d$.	$\Delta^{cout} = (2\lceil\log_2 n\rceil + 3)d = 15d$.	$\Delta^{cout} = 2nd = 128d$.

Example: X, Y – 8-bit.

X = 1 0 0 0 1 1 0 1 = 128 + 13 = 141

Y = 0 1 1 1 0 1 0 1 = 112 + 5 = 117

$C_0 = 0$!!!

Operand \ Rank		7		6		5		4		3		2		1		0	
X		1		0		0		0		1		1		0		1	
Y		0		1		1		1		0		1		0		1	
Block level	Carry in	C	S	C	S	C	S	C	S	C	S	C	S	C	S	C	S
i=0	Cin=0	0	1	0	1	0	1	0	1	0	1	1	0	0	0	1	0
	Cin=1	1	0	1	0	1	0	1	0	1	0	1	1	0	1		
i=1	Cin=0	0	1		1	0	1		1	1	0		0	0	1		0
	Cin=1	1	0		0	1	0		0	1	0		1				
i=2	Cin=0	0	1		1		1		1	1	0		0		1		0
	Cin=1	1	0		0		0		0								
i=3	Cin=0	1	0		0		0		0		0		0		1		0
	Cin=1																

i=1 i=2 i=1 i=3 i=1 i=2 i=1

Verify:

```
 Cout                 0=c0
       1 0 0 0 1 1 0 1        141+
       0 1 1 1 0 1 0 1        117
     ------------------      -----
     1 0 0 0 0 0 0 1 0 = 256+2 = 258     258
```