Expert Systems

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S·I·M·T = Signal, Image and Machine Learning Team


Is there a cat in this image?

Suppose we have a ‘cat-filter’ W

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C-S Inequality

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$$X \cdot W \leq (X \cdot X)^{1/2} (W \cdot W)^{1/2}$$

with equality if and only if X is parallel to a cat.

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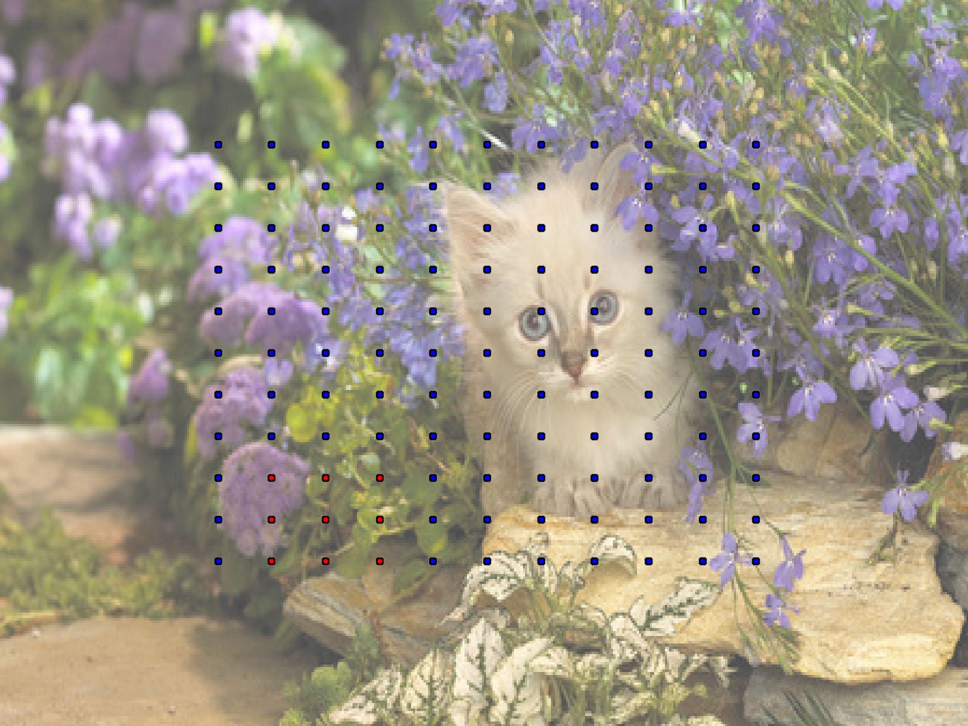
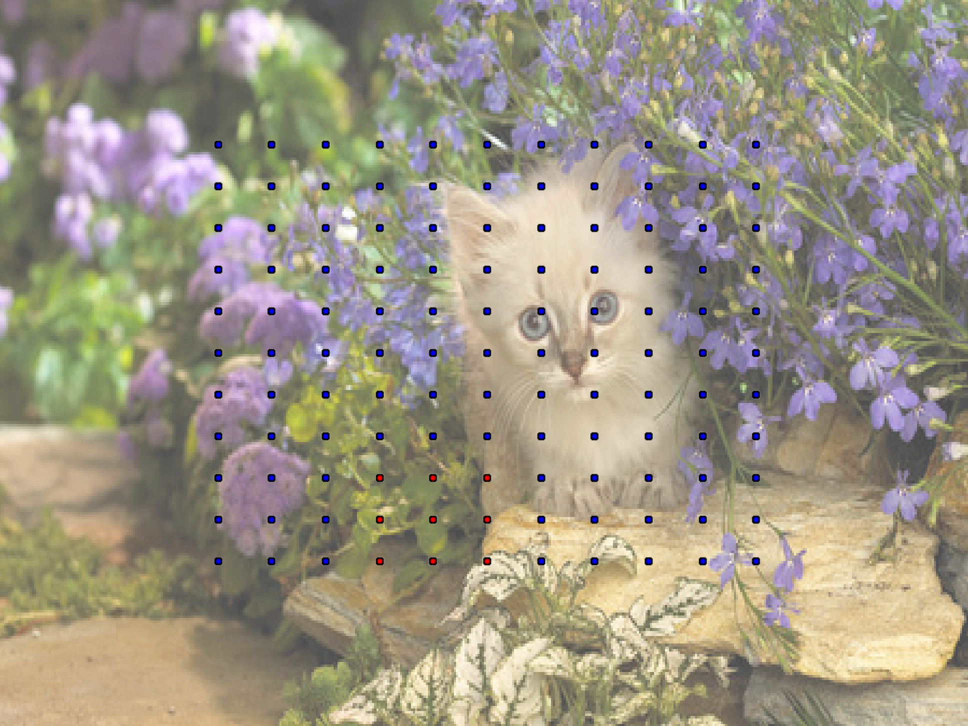
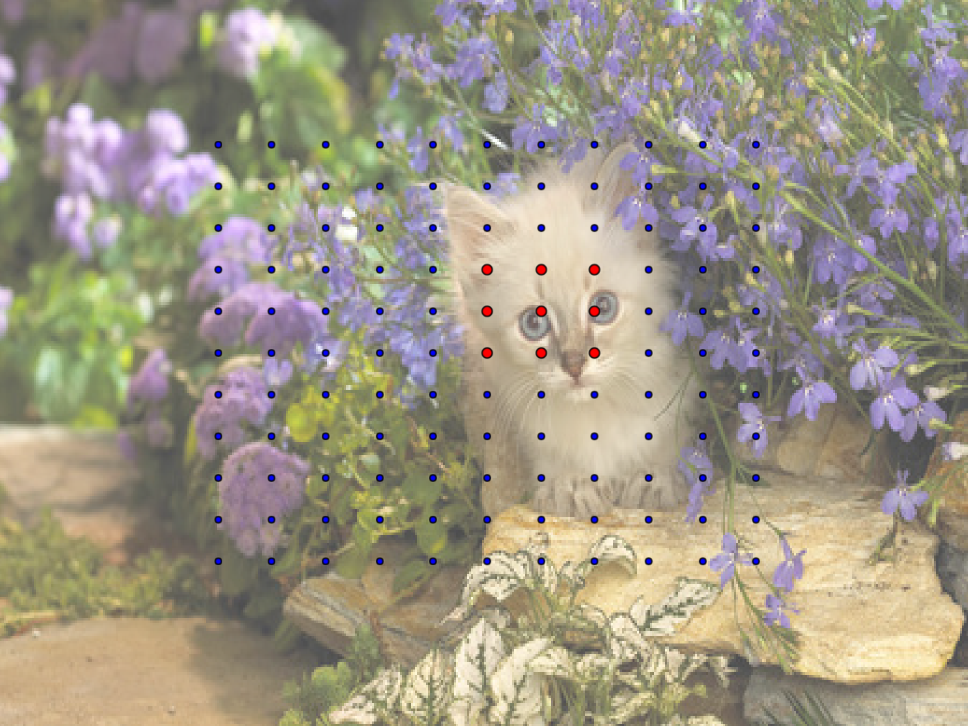
perform cat-test on all 3×3 image subpatches!







Convolution

Definition

Suppose that $X, Y \in \mathbb{R}^{n\times n}$. Then $Z = X * Y \in \mathbb{R}^{n\times n}$ is defined as

$$Z[i,j] = \sum_{k,l=0}^{n-1} X[i-k, j-l]Y[k,l],$$

where periodization or zero-padding of X, Y is used if $i-k$ or $j-l$ is not in $\{0,\ldots,n-1\}$.

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Efficient computation possible via FFT (or directly if X or Y are sparse)!

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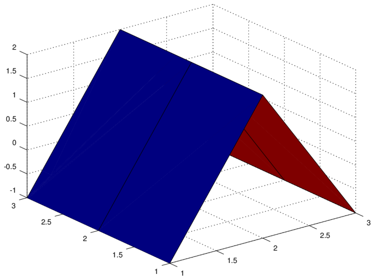

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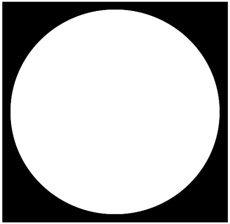

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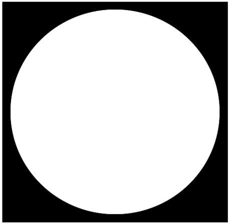
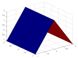
 $*$ 

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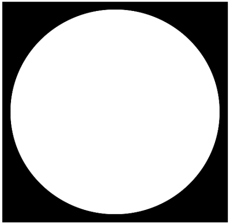

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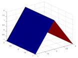

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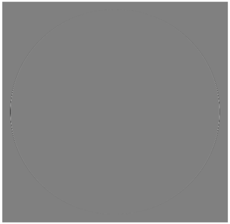

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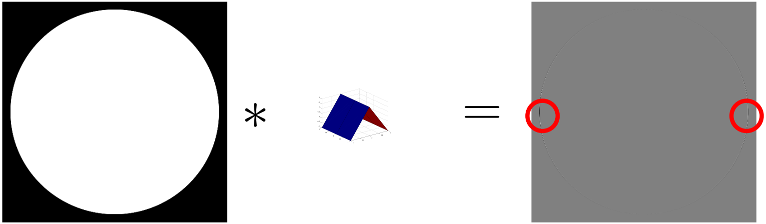

Introducing Convolutional Nodes

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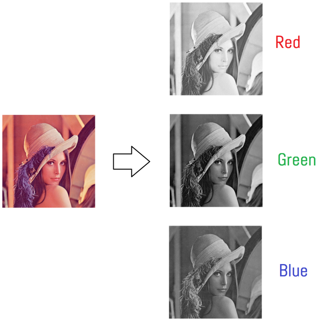

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- Given a filter $W \in \mathbb{R}^{F \times F \times S}$, where F is the *spatial extent* and a *bias* $b \in \mathbb{R}$, it computes a matrix

$$Z = W *_{12} X := \sum_{i=1}^{S} X[:,:,i] * W[:,:,i] + b.$$

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- A convolutional layer consists of K convolutional nodes $((W_i, b_i))_{i=1}^{K} \subset \mathbb{R}^{F \times F \times S} \times \mathbb{R}$ and produces as output a stack $Z \in \mathbb{R}^{n_1 \times n_2 \times K}$ via

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A convolutional layer can be written as a conventional neural network layer!

Activation Layers

The activation layer is defined in the same way as before, e.g., $Z \in \mathbb{R}^{n_1 \times n_2 \times K}$ is mapped to

$$A = \mathsf{ReLU}(Z)$$

where ReLU is applied component-wise.

Pooling Layers

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Reduce dimensionality after filtering.

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A *pooling operator* $\mathbf{R}$ acts layer-wise on a tensor $X \in \mathbb{R}^{n_1 \times n_2 \times S}$ to result in a tensor $\mathbf{R}(X) \in \mathbb{R}^{m_1 \times m_2 \times S}$, where $m_1 < n_1$ and $m_2 < n_2$.

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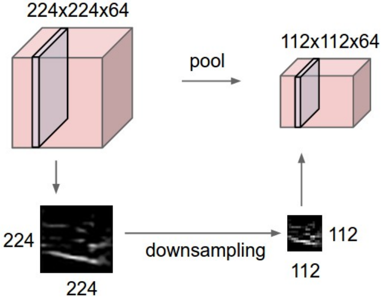

Downsampling

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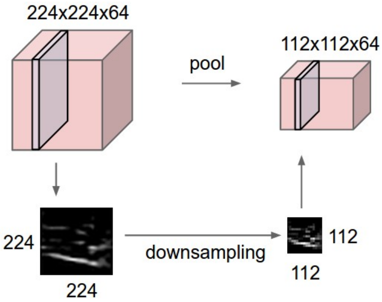

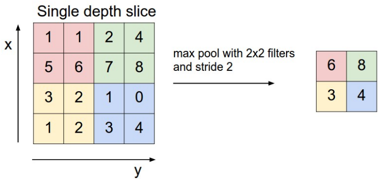

Max-pooling

Downsampling

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A CNN with L layers consists of L iterative applications of a convolutional layer, followed by an activation layer, (possibly) followed by a pooling layer.

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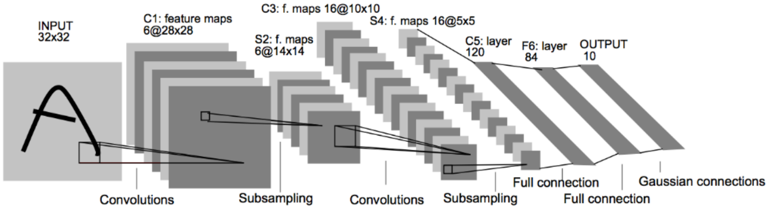

Figure: LeNet (1998, LeCun etal): the first successful CNN architecture, used for reading handwritten digits

Feature Extractor vs. Classifier












Feature Extractor vs. Classifier

Feature Extractor vs. Classifier

D. Trump

B. Sanders

A. Merkel

B. Johnson

```
x_image = tf.reshape(x, [-1, 28, 28, 1])
# First convolutional layer - maps one grayscale image to 32 feature maps.
W_conv1 = weight_variable([5, 5, 1, 32])
b_conv1 = bias_variable([32])
h_conv1 = tf.nn.relu(conv2d(x_image, W_conv1) + b_conv1)
# Pooling layer - downsamples by 2X.
h_pool1 = max_pool_2x2(h_conv1)
# Second convolutional layer -- maps 32 feature maps to 64.
W_conv2 = weight_variable([5, 5, 32, 64])
b_conv2 = bias_variable([64])
h_conv2 = tf.nn.relu(conv2d(h_pool1, W_conv2) + b_conv2)
# Second pooling layer.
h_pool2 = max_pool_2x2(h_conv2)
# Fully connected layer 1 -- after 2 round of downsampling, our 28x28 image
W_fc1 = weight_variable([7 * 7 * 64, 1024])
b_fc1 = bias_variable([1024])
h_pool2_flat = tf.reshape(h_pool2, [-1, 7*7*64])
h_fc1 = tf.nn.relu(tf.matmul(h_pool2_flat, W_fc1) + b_fc1)
# Map the 1024 features to 10 classes, one for each digit
W_fc2 = weight_variable([1024, 10])
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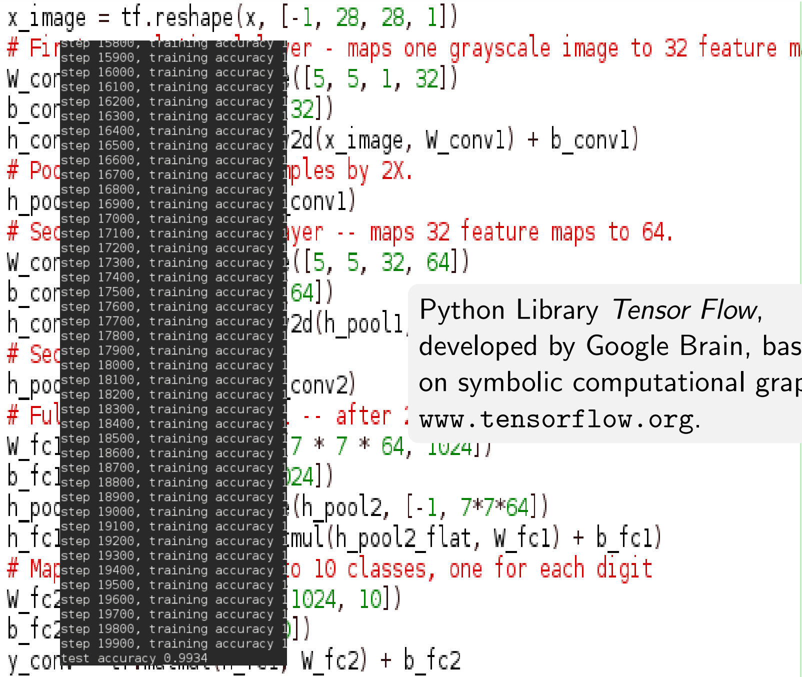

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