Propositional Stability[1]

Adam InTae Gerard[2]

0.0 Introduction

This short article adumbrates a new and useful notion relevant to so-called *combined modal logics*, *Markov Logic Networks*, and *Transactional Logic* (forthcoming). Specifically, we seek to define and identify the conditions under which truth-values remain stable when interacted with by ***more than one logic***.

"Under what conditions", we might ask, "do propositions remain unchanged in their truth-determinations?" Furthermore, "how might we proceed to calculate that and track such changes?" *Propositional Stability* is introduced to that end.

1.0 Overview and Motivation

Post-truth, subjectivism, post-modernity, anti-rationalism, anti-intellectualism, memetics, black-boxed artificial intelligence, iterative logics (logics that fail to exhibit *eternalism*), hyper-dimensional logics (forth-coming), logical pluralism, substructural logics, logics of contradiction and paradox, declassified UFO's, and constructive mathematics.

Formally, *Propositional Stability* ensures that when a proposition is *transacted* between two logics (more on this later) - it never acquires a new truth-value *beyond those it could have already acquired under the first logic under which it is evaluated*.

2.0 Conventions

Where:

1. ⊶ $\in \mathbb{N}$
2. ⁕ $\in \{a, \ldots, z, \ldots\}$
3. $\{a, \ldots, z, \ldots\} = \mathbb{N}$

We write (quotes[3] are dropped):

1. *ML*⊶⁕ to denote a semantics (model or truth-assignment *M*) for a language *L*⊶ with ⁕-many truth values.

---

[1] Work in Progress – Under Heavy Revision
[2] **Revision 0.0.3 – 10.13.18** - https://www.linkedin.com/in/adamintaegerard/
[3] https://plato.stanford.edu/entries/quotation/#2.2

2. $VML⊶⁎(p)$ to denote a truth-evaluation of p under semantics (model or truth-assignment M) for a language $L⊶$ with ⁎-many truth values.

3. $VML1aVML2b(p)^*$ to denote any possible truth-evaluation of p to a truth-value t in semantics $ML2b$ such that: $t \in ML2b$ and $t \notin ML1a$.

3.0 Definitions

Definition 1. *Instruction set*.

An ***instruction set*** is a finite procedure or algorithm mapping one input to one output.

Definition 2. *Strong propositional stability*.

1. A proposition or sentence p evaluated under semantics $ML1a$ will preserve its exact truth-value under semantics $ML2b$ whenever $a \subseteq b$ and no *instruction set* exists to map $VML1a(p)$ to any other truth-value. p is then said to exhibit ***strong propositional stability***.

2. A proposition p exhibits ***strong propositional stability*** *when and only when*:

    a. $VML1a(p) = VML2b(p)$
    b. $t \in VML1a \cup VML2b$
    c. $VML1a(p) \neq t$
    d. No *instruction set* exists to map $VML1a(p)$ to t

Definition 3. *Weak propositional stability*.

1. A proposition or sentence p evaluated under semantics $ML1a$ will preserve its range of truth-values under semantics $ML2b$ whenever $a \subseteq b$ and no *instruction set* exists to map $VML1a(p)$ to any $VML1aVML2b(p)^*$. p is then said to exhibit ***weak propositional stability***.

2. A proposition p exhibits ***weak propositional stability*** *when and only when*:

    a. $VML1a(p) \subseteq VML2b(p)$
    b. No *instruction set* exists to map $VML1a(p)$ to any $VML1aVML2b(p)^*$.

Definition 4. *Truth stability*.

1. A proposition or sentence p evaluated under semantics $ML1a$ will preserve its exact truth-value under semantics $ML2b$ whenever $a \subseteq b$. p is then said to exhibit ***truth stability***.

2. A proposition p exhibits ***truth stability*** *when and only when* $VML1a(p) = VML2b(p)$.

Definition 5. *Propositional instability.*

A proposition p exhibits ***propositional instability*** whenever it does not exhibit ***weak propositional stability***.

Definition 6. *Truth instability.*

A proposition p exhibits ***truth instability*** whenever it does not exhibit ***truth stability***.

4.0 Discussion

Remark 1. ***Strong propositional stability*** entails ***weak propositional stability*** and **truth stability**.

Discussion: ***Strong propositional stability*** requires that a proposition retains its exact truth-value under two logics and that no method exists for that truth-value to vary. Thus, it is constrained by the same range of truth-values.

Remark 2. Truth stability guarantees only incidental sameness of truth-assignment. In some cases, truth stability will converge with ***strong propositional stability***, in others it will not.

5.0 Results

Fact 1. Any proposition truth-evaluated under a Boolean logic will exhibit ***strong propositional stability*** when truth-evaluated under a Kleene 3-Value Algebra.

Proof: Obvious. No single proposition already assigned a truth-value of 'true' or 'false' can receive a truth-value of 'indeterminate' or 'true and false'. ■

Fact 2. Given:

1. Monotonic axiom systems Ω_1, Ω_2
2. $\Omega_1 \subset \Omega_2$

If $\Omega_1 \vdash A$, A will exhibit ***strong propositional stability*** under Ω_2.

> **Proof:** Obvious. If $\Omega_1 \vdash A$, then $\Omega_2 \vdash A$. A will remain a derived tautology under Ω_2. ■

Fact 3. Given:

1. Monotonic axiom systems Ω_1, Ω_2
2. $\Omega_1 \subset \Omega_2$
3. $\Gamma \vdash A$

If $\Omega_2 \vdash A$ and $\Gamma \subseteq \Omega_2$, A will exhibit:

1. ***Strong propositional stability*** under Ω_1 only when $\Gamma \subseteq \Omega_1$
2. ***Propositionally instability*** otherwise.

6.0 Modal Logic and Axioms Systems

... TBD about combine modal logics.

7.0 Conclusion

Here and elsewhere, I have asserted that the fundamental concepts currently in wide-spread use throughout mathematics, philosophy, science, finance, ethics, law, and so on all largely rely on *ontological dogmas* including truth-monism, classicality, the T-Schema, and objecthood. While I will not argue on the subject here, *propositional stability* remains of interest to all such considerations.

A.0 Appendix

Originally Posted at: http://www.postlib.com/propositional-stability/