

# Homework 7

## 1 Coin changing (50')

Consider the problem of making change for  $n$  cents using the fewest number of coins. Assume that each coin's value is an integer. We recommend to read the chapter 6 of book *"Introduction to Algorithms"* (3rd Edition).

- **a.** Describe a greedy algorithm to make change consisting of quarters, dimes, nickels and pennies. Please give the pseudocode, and prove that your algorithm yields an optimal solution.
- **b.** Suppose that the available coins are in the denominations that are powers of  $c$ , i.e., the denominations are  $c^0; c^1; \dots; c^k$  for some integers  $c > 1$  and  $k \geq 1$ . Show that the greedy algorithm always yields an optimal solution.
- **c.** Give a set of coin denominations for which the greedy algorithm does not yield an optimal solution. Your set should include a penny so that there is a solution for every value of  $n$ .

## 2 Genetic Algorithm (50')

Use genetic algorithm to find  $x^*$  that

$$x^* = \arg \min_x x \cdot \sin(x)$$
$$x \in (-1, 15)$$

Use 16-bit binary digits to represent the chromosome of an individual.

- Implement your code in a `.cpp` file. Print the individual  $x$  with the best fitness score in each generation.
- Describe the implementation details, such as the stopping criterion, the selection method, the crossover method, the mutation method and so on, in a README file.

Hint:  $x^* = \arg \min_x x \cdot \sin(x) \approx 11.0857$

## 3 Bonus (Voluntary) (50')

Answer the question 15-5 (a) in book *Introduction to Algorithm (3rd Edition)*, chapter 15. Write down the dynamic programming pseudo-code.

## 4 Due

1. Due: Nov. 21st, 23:59.
2. Please write all your answers in a PDF file. Submit on canvas a ZIP file containing the PDF, the code and README.