Homework 8

Problem 1. *What is the probability that a random graph in $\mathcal{G}(n,p)$ has exactly m edges, for $0 \le m \le \binom{n}{2}$ fixed?*

Solution. The probability is $\binom{n}{m}p^m(1-p)^{\binom{n}{2}-m}$. □

Problem 2. *What is the expected number of edges in $G \in \mathcal{G}(n,p)$?*

Solution. $\binom{n}{2}p$. □

Problem 3. *What is the expected number of trees with k vertices in $G \in \mathcal{G}(n,p)$?*

Solution. By *Cayley's formula* and the linearity of expectation, it is $\binom{n}{k}k^{k-2}p^{k-1}$ □

Problem 4. *Show that if almost all $G \in \mathcal{G}(n,p)$ have a graph property $\mathcal{P}_1$ and almost all $G \in \mathcal{G}(n,p)$ have a graph property $\mathcal{P}_2$, then almost all $G \in \mathcal{G}(n,p)$ have both properties.*

Solution. The portion of the graphs have both properties equals 1 minus the portion of the graphs which does not have property $\mathcal{P}_1$ or $\mathcal{P}_2$. However the portion of the graph does not have property $\mathcal{P}_1$ or $\mathcal{P}_2$ is bounded by the sum of the portion of the graphs does not have property $\mathcal{P}_1$ and the the portion of the graphs does not have property $\mathcal{P}_2$, which both tend to 0 as n approaches ∞. The claim in the question then follows. □