



HEIDELBERG UNIVERSITY

FP 13

**Measurement of Muon Properties in the
Advanced Students Laboratory**

Version 1.3

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Chapter 1

Physics background

The basis of particle physics is the so-called Standard Model. It was made about 30 years ago as a summary of all previous discoveries in particle physics. Up to date it is neither disproved in any important point, nor it has been yet experimentally confirmed in all fields. It is built on three pillars: the particles, the interactions and the Higgs mechanism. Particles and interactions are closely related, since the interactions are accomplished by and between particles. The Higgs mechanism gives the particles their mass and comes with an additional particle, the Higgs boson.

1.1 The particles

In the Standard Model, elementary particles belong to three groups, which are Quarks, Leptons and the mediator particles, which are responsible for the interactions between different particles.

Flavor	Mass [MeV/c ²]	el. charge	Flavor	Mass [MeV/c ²]	el. charge
up	$2.2^{+0.6}_{-0.4}$	2/3 (e)	down	$4.7^{+0.5}_{-0.4}$	- 1/3
charm	1270 ± 30	2/3 (e)	strange	96^{+8}_{-4}	- 1/3
top	173210 ± 850	2/3 (e)	bottom	4180^{+40}_{-30}	- 1/3

Table 1.1: Quarks in the Standard Model [1].

Quarks and leptons have spin 1/2 and therefore are fermions. They are the elementary constituents of matter. Quarks carry colour charge, and they can experience - in contrast to leptons - the strong force. They also have a fractional electrical charge (see Table 1.1). Compositions of quarks are called

hadrons. The combination of a quark and an anti-quark is called meson. In contrast, a particle is called baryon if it consists of three quarks. The masses of the quarks are very different and also very difficult to determine. They range from about $1.5 \text{ MeV}/c^2$ (up quark) up to about $173 \text{ GeV}/c^2$ (top quark).

Flavour	Mass	El. Charge	Flavour	Mass	El. Charge
e	0.511	-1e	ν_e	$< 2 \cdot 10^{-6}$	0
μ	105.66	-1e	ν_μ	$< 2 \cdot 10^{-6}$	0
τ	1776.86	-1e	ν_τ	$< 2 \cdot 10^{-6}$	0

Table 1.2: Leptons in the Standard Model [1]. Masses are stated in units of MeV/c^2

Like quarks, also leptons come in three families: electron, muon and tauon, each with the corresponding neutrino (see Table 1.2). The electron is stable, while muons and tauons usually decay in a lower mass lepton, thus an electron or a muon. In contrast to neutrinos, electron, muon and tauon have a negative electrical charge, while the corresponding anti-particle has a positive electrical charge. They are subject to gravitational, electromagnetic and weak interaction. Their mass ranges from $511 \text{ keV}/c^2$ to $1777 \text{ MeV}/c^2$. Neutrinos are not only electrically neutral, but their mass is also close to zero.

1.2 The Gauge bosons / interactions

There are four fundamental interactions: the gravitational, the electromagnetic, the strong and the weak interaction. Each of them has one or more gauge bosons associated, since in quantum field theory the interactions are described as the exchange of particles. The Standard Model describes all of them except the gravitation.

Interaction	Mediator	acts on
Electromagnetic	Photon	Quarks, leptons (not neutrinos)
Strong	8 Gluons	Quarks
Weak	W^\pm, Z^0	Quarks, leptons

Table 1.3: Gauge bosons in the Standard Model.

Gravity is the weakest of the four interactions. Even if, because of its long

range and because there is only positive charge (mass), it seems macroscopically very strong, it is small in relation to the mass of elementary particles. It is 10^{38} -times weaker than the strong interaction and therefore negligible in particle physics. As the topic of this experiment is the μ -decay, which takes place via the weak interaction, the discussions in the following are limited to this interaction.

When radioactivity and hence the β -decay of the neutron ($n \rightarrow p e \bar{\nu}_e$) was discovered by Becquerel in 1896, it soon became clear that, due to the long decay time, the underlying process could not be explained by the strong force. Moreover, such an electromagnetic decay was extremely unlikely. This was a hint of a new interaction, namely the weak interaction. In order to preserve the conservation of energy in this process, Pauli postulated the neutrino in 1930. In general, the weak force couples both to quarks and to leptons. In this processes particles can change the electrical charge - then the exchange particle is a W^+ or a W^- - or leave the same electrical charge - through the exchange of a Z^0 . The exchange particles couple to the weak charge g , which is carried by both quarks and leptons. A special feature of the weak interaction is that it is parity violating. This is for example visible in the muon decay.

The strong interaction is responsible for the cohesion between the quarks. The more they are separated from each other, the stronger is the attractive force between them. The exchange particles of the strong interaction are 8 gluons, which carry colour charge, the charge of the strong force. Thus the gluons can interact also with each other. The introduction of colour charge was necessary after the discovery of the Ω^- particle. Since it is made out of three s-quarks, they are subject to the Pauli principle and in order to be allowed there must be an additional quantum number.

1.3 The muon

The muon is an elementary particle. It is associated with the muon neutrino and belongs to the second generation of leptons. The production of a muon is always accompanied by the production of a muon anti-neutrino ($\bar{\nu}_\mu$). In the

case of an anti-muon, a muon neutrino (ν_μ) is necessary. Analogously, in the decay of a muon, a muon neutrino has to occur. The muon is roughly 207 times heavier than the electron ($m_e = 511\text{keV}/c^2$). So the muon is not the lightest particle and it can decay in an electron. Within today's knowledge the muon is a point-like particle without internal structure.

1.4 The muon from cosmic rays

The experiment is carried out with cosmic muons, whose rate at the sea level is about $70 \frac{\text{particles}}{\text{m}^2 \cdot \text{s} \cdot \text{sterad}}$ [2]. Primary cosmic rays consist of energetic protons. In the upper atmosphere layers they interact with molecules. This results mainly in the production of charged pions and kaons via the reactions

$$p + p \rightarrow p + n + \pi^+, \quad (1.1)$$

$$p + n \rightarrow p + p + \pi^-, \quad (1.2)$$

$$p + p \rightarrow p + \Lambda (\rightarrow p + \pi^- / n + \pi^0) + K^+. \quad (1.3)$$

The resulting pions and kaons are not stable, so they decay, among others, in

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad (1.4)$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad (1.5)$$

$$K^+ \rightarrow \mu^+ + \nu_\mu. \quad (1.6)$$

These decays happen via the weak interaction and are mediated by the exchange of the W^\pm bosons. An example is the Feynman diagram for the pion decay, which is shown in Figure 1.1. Because of the positive charge of the primary radiation and of the Earth's surface, there is a surplus of positive particles in the produced muons. The ratio is

$$\frac{\#\mu^+}{\#\mu^-} \approx 1.275 \pm 0.05 \quad [1] \quad (1.7)$$

The muons are generated at about 10 km altitude. Only because they are highly relativistic and there is time dilation they can reach the Earth's surface before decaying. The maximum of the energy distribution of the incoming muons at ground level is about 1 GeV (see Figure 1.2). Their average kinetic energy is about 2 GeV.

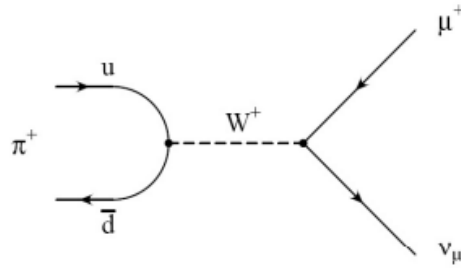


Figure 1.1: Feynman diagram of the pion decay as an example for the generation of muons.

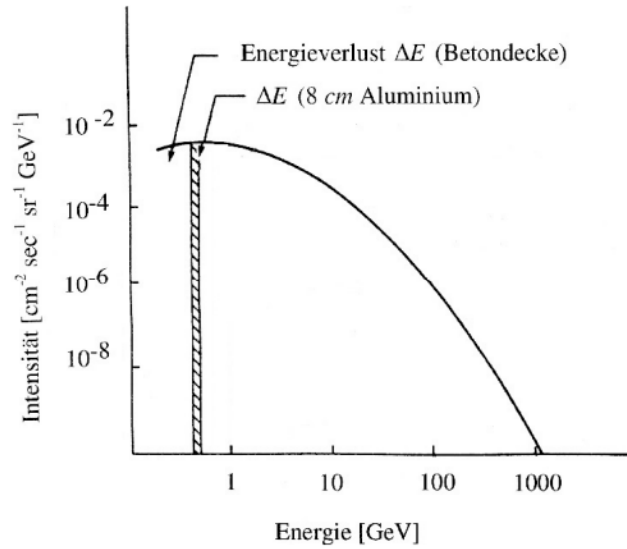


Figure 1.2: Energy spectrum of muons produced in cosmic radiation, measured at the Earth's surface.

Since muons are not subject to strong interactions, they lose their kinetic energy only via electromagnetic interactions, i.e. primarily by ionisation of atoms that are present in the air. The energy loss by bremsstrahlung is small compared to electrons because of the larger muon mass. In total, the energy loss amounts, at high kinetic energies, to $\sim 2 \text{ MeV/g} \cdot \text{cm}^2$ [1]. This corresponds to a loss of energy of about 1 MeV in 4 m of air near the Earth's surface.

1.5 The muon decay

Muons are decelerated by interactions with matter until they decay or come to rest. Positive muons decay either as free particles, or by capturing an electron and the formation of a "Muon Atom". Negative muons have access to a further reaction channel. They can be captured by atomic nuclei, form muonic atoms and then decay via weak interactions. Same as the muon production, the muon decay happens via the weak interaction and is mediated by the exchange of the W^\pm bosons.

Muons decay with an average lifetime of $2.19 \mu\text{s}$ via the reactions

$$\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e, \quad (1.8)$$

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \quad (1.9)$$

and according to the exponential decay law

$$N(t) = N(t_0) \cdot e^{-\frac{t-t_0}{\tau}}. \quad (1.10)$$

The Feynman diagram of the decay of the positive muon is shown in Figure 1.3. When the muon decays, its rest mass is transferred to the rest mass of the electron and the kinetic energy of the electron and the neutrinos. In the limit of $m_e \ll m_\mu$, the maximum kinetic energy of the electron is $E_{\text{max}} = \frac{1}{2} m_\mu c^2$.

From the mass of the muon and its lifetime, it is possible to determine the Fermi weak interaction coupling constant G_F . It is (in natural units) given by

$$G_F^2 = \frac{192 \cdot \pi^3 \cdot \hbar}{\tau_0 \cdot (m_\mu \cdot c^2)^5}. \quad (1.11)$$

Negative muons can decay also via another reaction channel, the μ -capture. If they are at rest and get into the electromagnetic field of an atom, they are captured by it and they fall down in the K-shell in less than 10^{-12} s. Thus the capture of the muon

$$\mu^- + p \rightarrow n + \nu_\mu \quad (1.12)$$

competes with the muon decay. The capturing process reduces the effective lifetime of negative muons to

$$\frac{1}{\tau} = \frac{1}{\tau_0} + \frac{1}{\tau_c}, \quad (1.13)$$

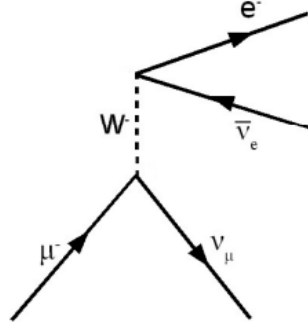


Figure 1.3: Feynman diagram of the muon decay.

where τ_c is the lifetime in case of pure muon capture [3]. As in this experiment we detect radiation from the cosmic rays, which is a mixture of positive and negative muons, we expect the following time dependence for the number of decaying muons:

$$N(t) = N(\mu^-, t_0) \cdot e^{-\frac{t-t_0}{\tau_0}} \cdot e^{-\frac{t-t_0}{\tau_c}} + N(\mu^+, t_0) \cdot e^{-\frac{t-t_0}{\tau_0}}. \quad (1.14)$$

The first part of this lab course is about the measurement of the muon lifetime τ_0 . Using equation 1.11, this easily measured quantity can be used to determine G_F , which is one of the few fundamental parameters of the electroweak sector of the SM.

1.6 Parity violation

A historically significant finding related to the muon decay is that it was one of the first reactions which was found to violate parity in the weak interaction. The same is true for the production of muons by cosmic radiation. One aim of this lab course is to prove the existence of parity violation in these two processes. This section is divided into three parts. First, the general concept of parity and its violation is described. Then, this concept is discussed in the context of the muon production. Finally, the same is done in the case of the muon decay.

1.6.1 General concept

The parity transformation \hat{P} flips the sign of the spacial coordinates. In the context of this lab course, the effect of a parity transformation on a elementary

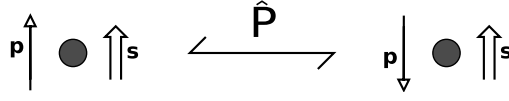


Figure 1.4: Impact of a parity transformation, \hat{P} , on a particle with spin \mathbf{s} and momentum \mathbf{p} .

particle with spin is crucial. In Figure 1.4, a particle with momentum \mathbf{p} and spin \mathbf{s} , together with the \hat{P} -conjugated state is shown. Due to the flip in the spacial coordinates, the momentum changes sign. The spin remains unchanged (recall that the classical analogue of the spin, the angular momentum, is invariant under parity transformation).

The relative sign of the directions of the spin of a particle and its momentum is called helicity. If these both quantities have the same direction, the particle has helicity $+1$ and is called right-handed. In the case of opposite direction of spin and momentum, the particle has helicity -1 and is called left-handed. This means that the parity transformation flips the helicity of a particle.

If the realisation of a process in nature differs from the realisation of the parity transformed process, this is referred to as the violation of parity. This also means that if a process distinguishes between left- and right-handed particles, it violates parity.

A crucial aspect of the weak interaction in the Standard Model is the different coupling to left- and right-handed particles. It is important to note that this handedness is not the handedness discussed before in the context of helicity but is related to the chirality of the particle. In contrast to the helicity, which describes the relation between spin and momentum, the chirality is a more abstract concept describing a orthogonal decomposition of an elementary particle. This means that a particle can be composed out of a left- and a right-handed chiral part. The weak interaction couples only to the left-handed part of particles and to the right-handed part of antiparticles.

In the case of massless particles, helicity and chirality coincide, meaning that a right-handed (helicity) particles has only a right-handed and no left-handed (chiral) part. This is not the case for particles with mass. In fact, a right-handed (helicity) particle with velocity $\beta = v/c$ is a superposition of a right-handed (chiral) state (fraction of $\sqrt{(1+\beta)}/2$) and a left-handed (chiral) state (fraction

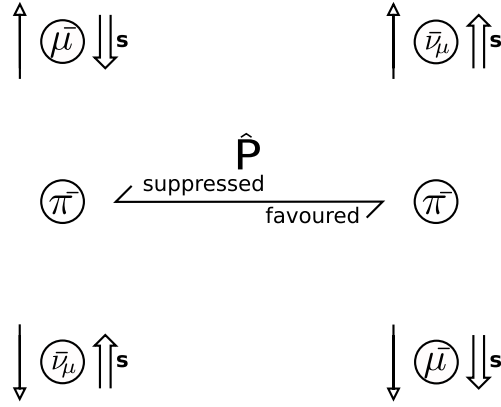


Figure 1.5: Parity violation in pion decay. The two possible configurations of the decay products in the pion decay. They are related by a parity transformation.

of $\sqrt{(1 - \beta)/2}$. This means that the coupling of the weak interaction to a left-handed (helicity) particle or right-handed (helicity) anti-particle is proportional to $(1 + \beta)/2$, while the coupling to a right-handed (helicity) particle or left-handed (helicity) anti-particle is proportional to $(1 - \beta)/2$. Therefore, the weak interaction violates parity.

1.6.2 Parity violation in the muon production

The considered muons in this lab course originate from the decay of pions and kaons. We focus here on the pion decay, but the kaon decay can be discussed analogously.

Since the pion has spin 0 and the spin should be conserved in its decay, the decay products, the muon and the anti-muon neutrino, are required to have opposite sign. This allows two possible configurations which are shown in Figure 1.5. In the first case, the two decay products are left-handed (helicity) and in the second case, which is the parity-transformed state, they are both right-handed (helicity).

To determine which of these two configurations is favoured, one has to remember that the pion decays via the weak interaction which couples differently to left- and right-handed particles. The muon is a particle and the weak interaction couples to the left-handed (chiral) part of it. The anti-muon neutrino is an anti-particle and the weak interactions couples therefore only to the right-

handed (chiral) part of it. As discussed in the previous section, the suppression of the coupling to suppressed handedness (helicity) depends on the velocity of the respective particle. Since the neutrino is nearly massless, this leads to a strong suppression of the configuration where both particles are left-handed. This means that parity is violated in the decay of the pion.

As a result of this, the spin of muons arriving at the experiment is preferably oriented downwards. For anti-muons, the opposite is the case. The exact polarisation P , given by the asymmetry between left and right handed muons $P = (N_R - N_L)/(N_R + N_L)$, depends on the shape of the energy spectrum and on the energy which is available in the decay of the primary component. If we take pions as primary particles the polarisation is $P = 0.33$, while for kaons it is $P = 0.54$.

The aim of the experimental setup is now to measure this polarisation, which is a direct proof of the parity violation in the weak interaction.

1.6.3 Parity violation in the muon decay

Similar considerations for parity violation can also be applied for the decay of the muon. In Figure 1.5, the two possible configurations for the case where the electron has the maximal possible energy are shown. They are linked through a parity transformation. Using the concepts discussed previously, it should be clear to you, why the electron emittance opposite to the muon spin direction is favoured. For the anti-muons the opposite is the case.

This is also a consequence of parity violation in the weak interaction and allows to translate the polarisation of the incoming muons in a preferred direction of the emitted electrons, which allows us to measure this polarisation.

If we consider the decay of positive muons we find that the spatial distribution of the produced electrons is

$$N(\theta) \propto 1 + A \cdot \cos\theta, \quad (1.15)$$

where θ is the angle of the e^+ with respect to the spin of the μ^+ . The asymmetry parameter A depends on the energy of the positrons. $A = 1$ corresponds to the maximum decay energy of the positron when neglecting the electron mass, $A = 1/3$ is the value that is obtained when averaging over all positron energies. Considering also multiple scattering in aluminium and further effects, we expect

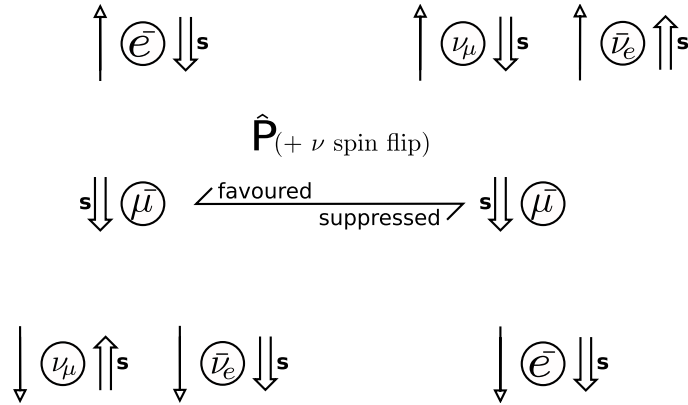


Figure 1.6: Parity violation in muon decay. The electron is emitted preferably opposite to direction of spin of the muon. The sketch shows the extreme case where the neutrinos are emitted in parallel which corresponds to the electron having the maximal possible energy.

an asymmetry of $A = 0.23$. Figure 1.7 illustrates the angular distribution of positive muon decays.

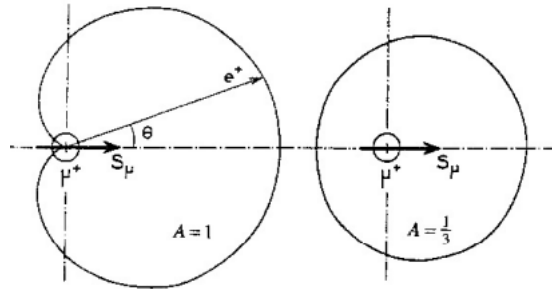


Figure 1.7: Angular distribution of positrons after the decay $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$.

1.7 The magnetic moment of the muon

The charged leptons carry a magnetic moment $\vec{\mu}$, which is proportional to the spin \vec{s} of the particle. The magnitude of the magnetic moment is proportional to the Bohr magneton of the particle and is inversely proportional to the mass of the particle. For a muon, the resulting magnetic moment is given by

$$\vec{\mu}_\mu = g_\mu \cdot \mu_\mu^{\text{Bohr}} \cdot \vec{s} \quad \text{with} \quad \mu_\mu^{\text{Bohr}} = \frac{e\hbar}{2m_\mu}. \quad (1.16)$$

The proportionality between the magnetic moment and the magneton is the g-factor, which is calculated within the Dirac theory for leptons to be exactly 2 [1]. Any difference from this exact value is called a magnetic moment anomaly

$$a_\mu = \frac{g_\mu - 2}{2}, \quad (1.17)$$

which is of the order of α/π , where $\alpha \approx 1/137$ is the Sommerfeld fine-structure constant.

When a muon is in an external magnetic field B and its spin \vec{s} is perpendicular to the field, its magnetic moment performs a Larmor precession at the frequency

$$\omega_{\text{Larmor}} = \frac{g \cdot \mu_\mu^{\text{Bohr}} \cdot B}{\hbar} \quad (1.18)$$

The positron coming from the decay of the positive muon is favoured to be emitted into the direction of the muon spin. Thus the muon spin precession can be measured by observing the time evolution of the decay asymmetry in an ensemble of muons.

1.8 Current researches about muons

For modern precision experiments it is not sufficient to use only muons coming from the cosmic rays. Therefore there are several accelerator facilities in the whole world, where intense beams of muons are produced. The properties of these beams are generally different, so it is possible to choose the most adequate one out of a large variety of beams with energies and properties depending on the experimental conditions [15]. In these so-called meson factories, muons are produced from protons. The protons are accelerated up to typical energies between 600 MeV and 1000 MeV, then they hit a beryllium or a carbon target. Via nuclear excitations, pions are produced and afterwards decay into muons. These muons are then collected in electromagnetic guiding systems, consisting of complex magnetic dipole, quadrupole, and some solenoid fields, arranged one after another, and then are directed into the experiment. In this way it is possible to reach a typical muon flux of 10^6 per second and cm^2 .

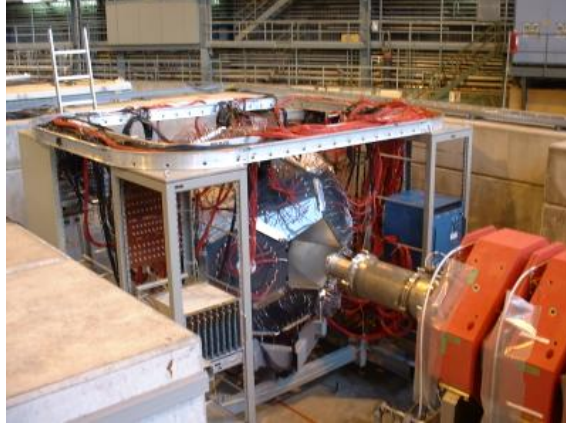


Figure 1.8: Two μ LAN detector for a precision measurement of the muon lifetime at the Paul Scherrer Institute in Villingen (Switzerland)

Currently the most precise measurement of the muon lifetime with a single experiment is given by the value obtained by the μ LAN experiment [9]

$$\tau = 2196.9803(22) \text{ ns } (1.0\text{ppm}) . \quad (1.19)$$

The μ LAN experiment (detector for Muon Lifetime Analysis) at PSI in Villingen (Switzerland) used a football-like detector. With this special form it was intended to minimise polarisation effects.

This measurement is a crucial input to the most precise calculation of the Fermi constant G_F [1]:

$$\frac{G_F}{(\hbar c)^3} = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} . \quad (1.20)$$

Chapter 2

The experiment

2.1 Experimental setup

The experiment is carried out with cosmic muons that arrive at sea level with a rate of approximately $70 \frac{\text{particles}}{\text{m}^2 \cdot \text{s} \cdot \text{sterad}}$ [2].

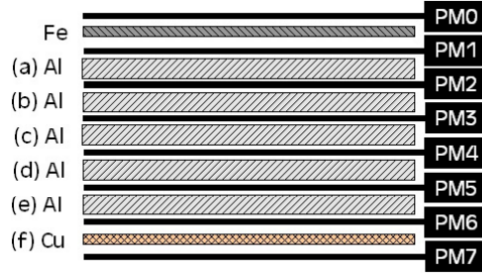


Figure 2.1: The detector setup.

The detector consists of layers of scintillators and metal plates assembled one after the other. The upper plate consists of 2.5 cm of iron, followed by 5 aluminium plates with a thickness of 8 cm. At the bottom, there is a 4 cm thick copper plate. The scintillators are each 1 cm thick. Some of the muons will completely be stopped when passing the metal and will then remain in one of the plates until they decay. Both the passing muon and the electron from the muon decay can create a light-signal in the scintillators.

The eight scintillators are connected to photomultipliers, which are in turn connected to the inputs of eight discriminators. These generate an output pulse when the input voltage exceeds a certain threshold. These thresholds need to be optimized before starting the measurements. The pulses are passed to a TDC

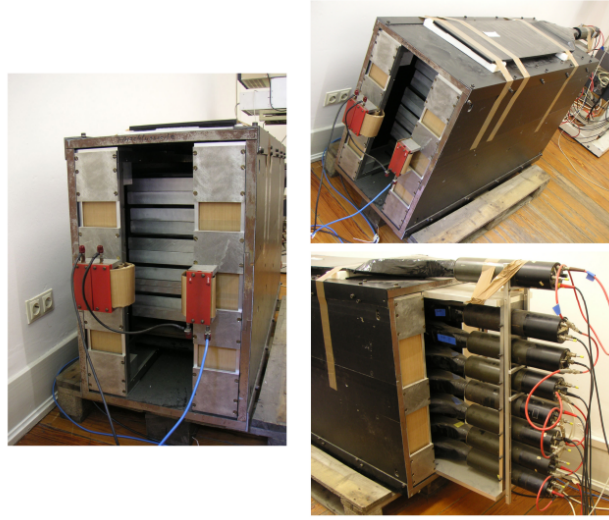


Figure 2.2: Different views of the detector.

(time to digital converter), which digitises the timing information for further processing with the computer (see Figure 2.4).

The cosmic muons pass through the detector with a velocity close to the speed of light c . When the muon is stopped in one of the metal plates, the electron/positron from its decay can be detected by the scintillators above ('upwards' decay) or below ('downwards' decay) the position of the stopped muon (see Figure 2.5). The thickness of the metal plates is chosen such that it matches approximately the average range of the high energetic electrons or positrons of the muon decay.

To have the opportunity to observe the Larmor precession of the muons, the metal plates and most of the scintillators are placed inside a magnet. It consists of two rectangular coils with 41 turns along the long side of the detector (see Figure 2.3). For this experiment a field of $4 \cdot 10^{-3}$ T is used.

2.2 Performed measurements

In the scope of this lab course you will perform two key measurements. The first is the measurement of the muon lifetime, which can be used to determine the Fermi constant G_F . After that, you will try to measure the polarisation of the incoming muons to show that the weak interaction violates parity. This sec-

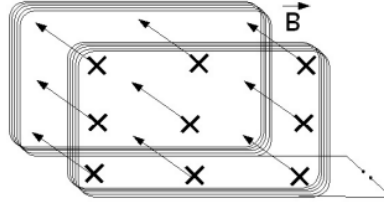


Figure 2.3: Arrangement of the two coils around the detector generating an homogeneous magnetic field.

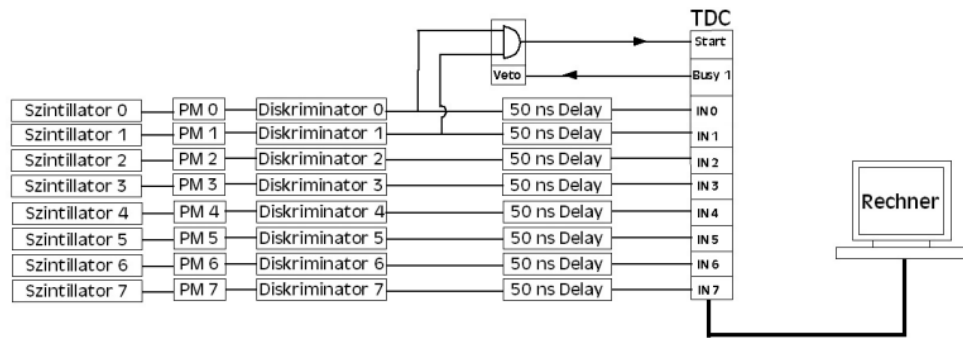


Figure 2.4: Schematic representation of the hardware.

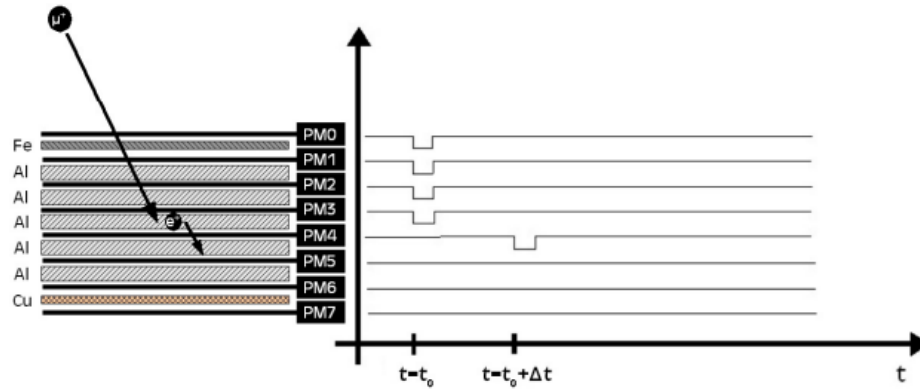


Figure 2.5: Downwards decay outlined in the detector (left) and the corresponding signal pattern of the discriminators (right).

tion will provide a short overview over the preparations and the measurements themselves. Afterwards, there will be more detailed instructions for each step. Please document your measurements immediately in the lab book and do the analysis (with the exception of the final adjustments of the theoretical curves

to the data) already during the execution of the experiment. On the last day the lab book is checked and signed by the assistant.

2.2.1 Preparation of the setup

1. Verification of the hardware

In this step you should become familiar with the hardware (photomultiplier, discriminators, trigger and readout chain) and check their functioning. The readout electronics have to be wired and the signals have to be checked with the oscilloscope.

2. Determination of optimal operating settings

The aim of this step is to set up the system in order to operate in the most efficient way without counting too many noise pulses. The time delays of the individual signals have to be checked, the discriminator thresholds have to be optimized and afterwards the efficiency of each layer has to be determined. For the final setup the individual and coincidence count rates have to be measured.

2.2.2 Lifetime measurement

The principle of the lifetime measurement is simple: The difference between the detection times of the muon and the electron/positron is measured. With a large amount of measurements it is then possible to determine the mean lifetime of muons by using the exponential decay law.

1. Data acquisition and online verification of data

At this step the apparatus is set up for data taking and one has to become familiar with the readout program and the online histograms. After checking the online histograms and the data acquisition rate, data is taken for two nights.

2. Filtering and checking the recorded data

Here you can familiarise yourself with the filter program, which allows to select good muon candidates and create histograms. The program is written in C++ and includes already a routine to identify upwards decays. You should modify it to work also for downwards-decays. After that, the

quality of the data and their consistency has to be investigated to determine which data sets are suitable for the analysis. At this stage the histograms coming from the filter program can be viewed and manipulated with the software package ROOT¹ [5]. This requires no prior knowledge of ROOT. The few necessary commands are described in the instructions. First, one should analyse the time-shifted pulses, which are either coming from the electron of the muon decay, or from after pulses of the photomultipliers or dark-noise pulses. The spectrum of afterpulses is analysed for all photomultipliers and the rate of afterpulses relative to a detected muon decay is determined.

Then, the measured decay spectra for all tubes for Up- and Downwards decays are corrected in order to take into account the afterpulses. Afterwards, it can be decided which data sets are appropriate for the determination of the mean lifetime and in which range of the decay times the mean lifetime should be determined.

3. Final analysis: Determination of the lifetime of the muon and evaluation of systematic errors

A script in ROOT is delivered for the measurement of the lifetime distribution from time delayed pulses. The fit method, script and the commands are discussed with the supervisor. After that, it should be easy for you to fit the data sets in different time intervals. A first fit has to be made during the execution of the lab course (last day). Then it is expected that you are able to decide which data sets are best suited to determine the lifetime and also to check the systematic consistency of the measurement and to estimate the systematic errors.

2.2.3 Polarisation measurement

For the measurement of the muon polarisation, there is also data recorded with a magnetic field. If the incoming muons have a net polarisation, the spin and the magnetic moment have, at the time when the muon stops, a preferred direction

¹In general, the data will be transferred to a computer of the CIP Pool and can be analysed with ROOT from any workstation of the physics department CIP Pool. Anyone who has a lot of experience can also save the data as a text file and analyse them with a program of his choice.

relative to the muon momentum. Since the direction of the emitted electron is correlated with the spin direction, also this quantity is not uniformly distributed. This means that the Larmor precession of the magnetic moment (and therefore the spin) of the muon in the magnetic field leads to a periodic change in the count rate for 'upwards' and 'downwards' decays as a function of the decay time.

For a given scintillator, the muon lifetime τ , a Larmor frequency ω_{Larmor} , a polarisation P and an asymmetry A (see Figure 1.7), one can calculate a time dependent counting rate of

$$Z(t) = Z_0 \cdot e^{-\frac{t}{\tau}} \cdot (1 + P \cdot A \cdot \cos(\omega_{\text{Larmor}} \cdot t + \phi)), \quad (2.1)$$

where ϕ is the angle between the direction of the electron and the muon-spin orientation.

The rate of delayed pulses of each layer as a function of decay times with and without the applied magnetic field is measured. After the afterpulse correction, the asymmetry $A = (N_{\text{without B}} - N_{\text{with B}}) / (N_{\text{without B}} + N_{\text{with B}})$ as a function of time can be calculated. The amplitude of this oscillating asymmetry is proportional to the polarisation of the muon. **The presence of a polarisation of the incoming muons shows that parity in the decay of the pion into a muon is violated!** The periodicity allows the determination of the Larmor frequency, which further allows, combined with the magnetic field strength, to calculate the magnetic moment of the muon. The data taking takes place over the weekend, after completing the lifetime measurement.

2.3 Detailed instructions for each point

2.3.1 Verification of the hardware

In the first part of the experiment, you become familiar with typical instruments of high energy physics such as scintillators, photomultipliers, discriminators, delays and logical elements. For this purpose, the circuits have to be wired and one should check how the pulses look in the oscilloscope and measure the counting rates. In Figure 2.6 and Figure 2.7 you can see the readout electronics. The core are two FPGA (Field Programmable Gate Array) based Logic boxes visible in Figure 2.7. They are controlled via LabView. In Box 1, there are the discriminators, while in Box 2, there is the TDC (Time to Digital Converter).

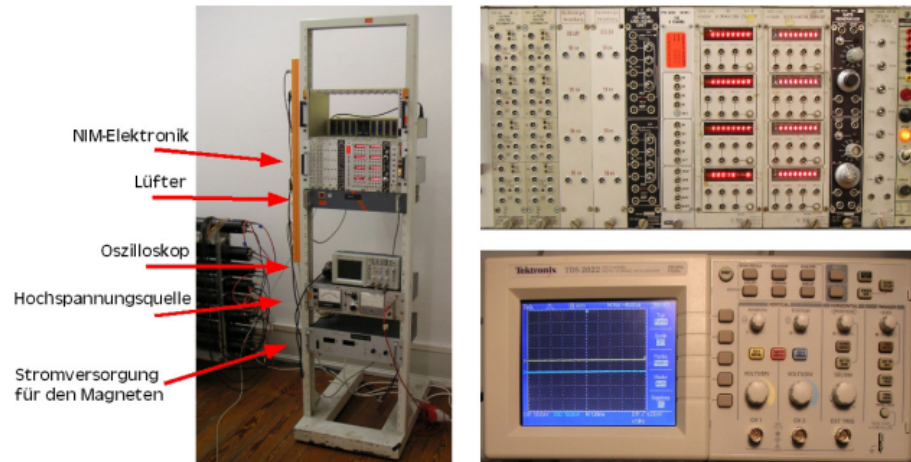


Figure 2.6: NIM-Electronics and oscilloscope

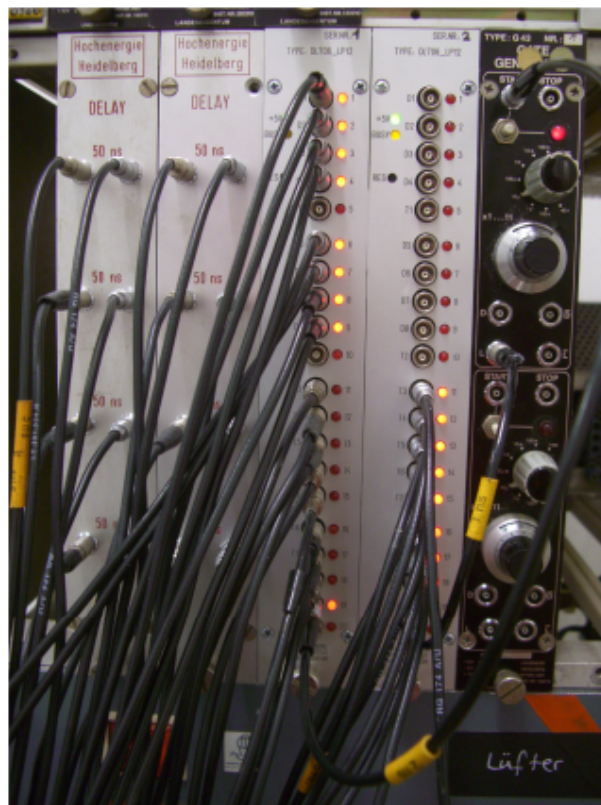


Figure 2.7: from left: 2 delays, 2 Logic boxes, 1 pulse generator (black)

- The first exercise is to check the output pulses of the photomultipliers with the oscilloscope. The value for the high voltage is set by the assistant and you don't have to change it. You should check the output of all eight photomultipliers.

Look at the effect of using a terminating resistor of 50Ω compared to the internal termination of the oscilloscope which has $1M\Omega$.

Sketch in the lab book the observed shape of the pulses from the photomultipliers. Take note of the pulse heights.

Then answer to these questions:

1. How are photons detected in a photomultiplier?
 2. How are the photons generated in the scintillator?
 3. How much energy deposit relativistic particles such as muons from cosmic radiation in a scintillator?
 4. Which impact has the terminator on the waveform?
 5. How do pulses from photomultiplier tubes look like?
 6. What determines the height of these pulses?
- Now look at the signals passing through the discriminators. Start the LabView program `FP13neu.vi`. By clicking on the button "Einstellen der Schwellen", you can start the program `Threshold.vi` with which the thresholds can be adjusted. The DIO (Digital Input Output) connectors D1, D2, ..., D8 on Logic Box 1 can be used as input for the 8 discriminators. The outputs of the discriminators are on the DIO connectors T3, T4, ..., T10 (see Figure 2.8). Modify the thresholds of individual discriminators and observe the effect on the signals that are accepted by the discriminator in the oscilloscope. Trigger the oscilloscope with the output signal of the discriminator.

Sketch also the observed pulse shapes and record the pulse heights.

Then answer to this questions:

1. What is the functionality of the discriminators?
2. Which impact has changing the thresholds?

- Now look at the signal of one photomultiplier before the discriminator on the oscilloscope channel 1 and the output signal of the same discriminator on the oscilloscope channel 2. Trigger on channel 1 and sketch the signal variation. Then answer to this questions:
 1. What do you observe?
 2. How can it be explained?
- Evaluate the value of the average pulse width (Δt) after the discriminators, and write it to your lab book.
- Measure the dimensions of the detector and determine at home the solid angle of the observed firmament by using the coincidence of two layers. Calculate from this the absolute rates of cosmic particles you expect in the detector.

2.3.2 Determination of optimal operating settings

The aim is to operate as efficiently as possible without too many noise pulses. One has to check and adjust the time delays, the time of the individual signals and also the discriminator thresholds. Then determine the efficiency of each layer. Under certain circumstances it can be needed to optimise the photomultiplier high voltages, but for this you must contact your supervisor. With your operational settings you have to measure the single and coincidence count rates.

- At the beginning you have to set the thresholds of the discriminators. To do this, measure the dark-pulse rates with the selected discriminator thresholds.

To measure the individual rates and to prepare the circuit for the following measurements, you have to connect the different components as shown in Figure 2.8. The 8 outputs of the individual discriminators from Logic Box 1 can be lead with a 50 ns delay into the lower inputs of the Logic Box 2. The trigger signal, which is a logical AND operation between discriminators 1 and 2, is in the T11 output. This signal is fed into a pulse generator which generates a trigger pulse of 75 μ s. This trigger pulse is also set in Logic Box 2 on T11 input and it will trigger the TDC.

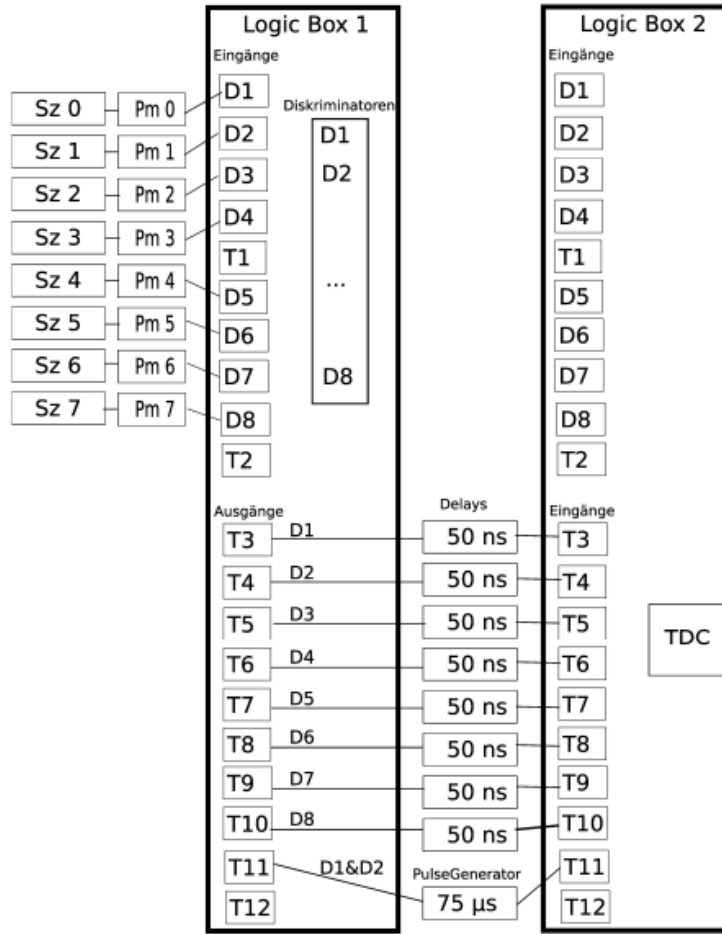


Figure 2.8: Complete circuit for lifetime measurement.

The internal logic of the FPGA is clocked with a 100MHz clock, which corresponds to a time resolution of 10 ns. To generate the 75 μs pulse for the trigger, the external pulse generator requires approximately 30 ns. The eight individual signals from the discriminators are therefore delayed by 50 ns, so that the trigger pulse for the TDC arrive before the remaining signals. To ensure that the relative delay is provided, do not use cables with latencies larger than 5 ns for all connections.

Start the LabView program `Einzelraten.vi` by clicking on "Messung Einzelraten" (you cannot use two LabVIEW programs simultaneously, so stop `Threshold.vi`, if it was running, before you start `Einzelraten.vi`).

The rates should be around 100 Hz. If not, you should vary the discriminator threshold in `Diskriminatoren.vi` until they receive an adequate number of pulses (if you encounter any strange results restart LabVIEW completely!).

- Then the detection efficiencies of the scintillators must be measured. Assume that a coincidence between two scintillators clearly indicates a particle (see next point). It can be assumed that if one detects a signal in the scintillator located between two other scintillators, the signal was due to the same particle. The efficiency measurement is carried out using the program `Effizienz-Rate.vi` which can be started by the button "Messung Effizienz".

If the efficiency of a scintillator is below 95%, consider to turn down the threshold of this scintillator and then determine the efficiency again. After measuring the efficiency for one layer restart `Effizienz-Rate.vi`. Otherwise a change of the selected layers will not be accepted. **Take notes of the efficiencies of all layers and the corresponding discriminator thresholds in a table in your lab book.**

- To get an estimate of the rate of random coincidences you can use the individual rates of the layers. The rate R_{12} of random coincidences, given the individual rates R_1 and R_2 and with a coincidence time of Δt (Pulse width from the prior section), is given by

$$R_{12} = R_1 \cdot R_2 \cdot \Delta t. \quad (2.2)$$

This formula is only valid if the rate of real coincidences is much smaller than the rate of a single layer. In either case the obtained rate is an upper bound on the rate of random coincidences.

Calculate this rate with the formula.

- The TDC contains an internal clock which is started by the trigger signal and stops after 75 μs . If there is a signal at one of the 8 signal channels, the time and the pattern of the signal is written to the internal memory of the TDC. The signal pattern needs 1 byte of storage, the time needs 2 bytes (see Figure 2.9).

Zeit – 2Byte	Detektormuster – 1Byte
40	00001111
2250	00010000




Figure 2.9: Scheme of the internal memory of the TDC.

2.3.3 Data acquisition and on-line review of the data

The TDC data are initially temporarily stored in a FIFO memory (First In First Out) and then read via a USB port using the LabView program `Lebensdauer.vi` which is started by the button "Messung der Lebensdauer". The data are both displayed in digital form, in the same way as they come from the TDC, and with an online event display. For a better understanding, in addition two histograms are automatically filled. One displaying all hits in respective detector layer, the other showing the slices which are hit by the muons. Slices are the number of different points in times which are written per event in the TDC.

Using the histogram displaying all hits in the detector, you can once again check the quality of your settings for the thresholds in the discriminators. In the first two layers there should be about the same number of hits, since we are asking for the coincidence between these two signals to start the TDC. They must not be exactly the same since after the start of the measurement they register also dark pulses, after pulses or decaying muons, which are taken into account in the histogram. For all other layers you should approximately expect an exponential decrease in the number of hits, since on the way down more and more muons get stopped by the metal plates.

After checking the online histograms and the data acquisition, data are taken for 2-3 days.

- Figure 2.10 shows the user interface of the selection program. In the bottom left box you can check the *trigger frequency* from the coincidence rate from the scintillator 0 and 1. The readout of the TDCs begins when

you click *start counting!*. The name and location of the output file is displayed in the field: *output file*. With the correct design of the circuit, the event rates are approximately 14-16 Hz. In order to understand what the program does after you press *start counting!*, you can also press the button *online display*. The online display will close automatically after displaying a few events as long as the stop button is enabled. You can stop the counting by disabling "start counting". If you want to readjust the discriminators thresholds you have to take care that the current program is closed. You can do this by starting again the program and then clicking on the normal stop button. Otherwise you will encounter a Read&Write error and you must restart LabVIEW completely. In this case, if you are asked if you want to save changes of "anything.vi", do not do it!

- Familiarise yourself with the program. Answer to the following questions:
 - Which quantities show each element on the user interface?
 - How is a muon decay depicted on the event display and the digital display?
 - Take a look at some events in the *display mode* and try to interpret them.
- The lifetime of the muons will be measured with an accuracy of about 5%. For this purpose, the measurement should run for at least two days. Start the measurement and watch the histograms.
- In addition to the lifetime of the muons, you can also measure from the same data also the much shorter lifetime of the negative muons, being captured by the nuclei of the aluminium or copper, the so called capturing time. This is about 880 ns for aluminium.
- For these measurements, after-pulses must be taken into account statistically. After-pulses occurs when a detector had actually only one hit, but it generates two signals in the photomultiplier. This happens when the primary signal is able to ionise the residual gas in the photomultiplier, then the positive ions run back to the cathode where a second pulse is

generated. Since such events go into the calculation, it is important to remove them later in order not to distort the lifetime measurement.

Once the measurement is done, the data, which during the measurement are stored locally on the disk of the computer as a text file, are transferred to one of the CIP Pool computers. Because the files are too big for your account, they will be saved in a temporary directory that is automatically deleted after 50 days. Save them in the directory `/local/mydirectory/myfile.txt`, which you can create with `mkdir /local/mydirectory`

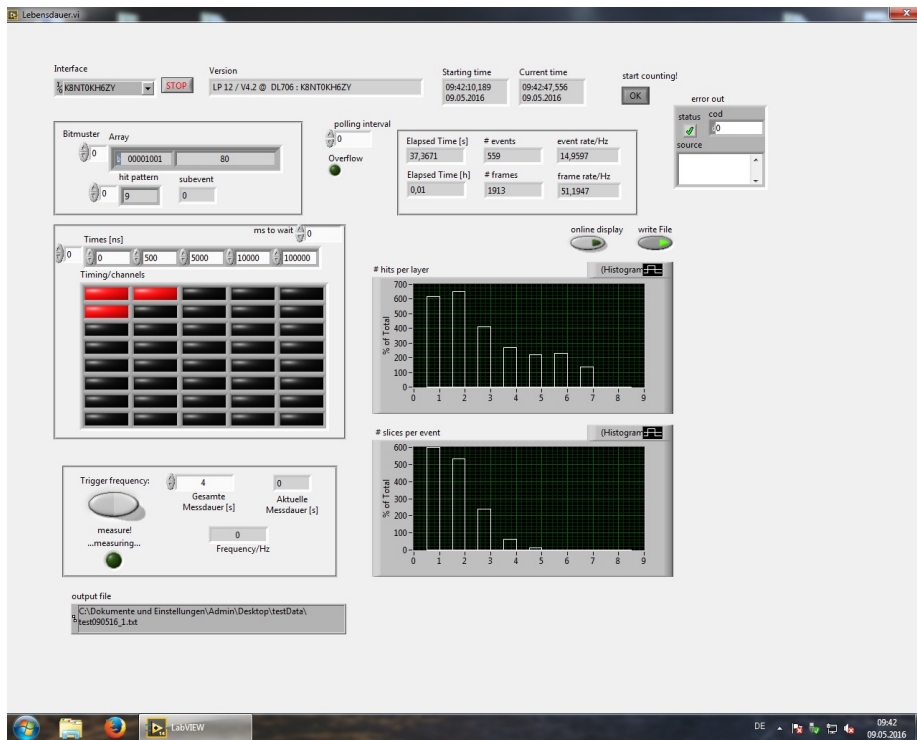


Figure 2.10: User interface of the selection program.

2.4 Filtering and verification of the data registered

To select from the data the events in which a decay occurred, a program written in C++ is used. This program selects those events, where the muon creates a signal simultaneously in all layers it is passing through and where a delayed signal is detected in either the last simultaneously hit (decay upwards) or the next

(decay downwards) scintillator. In general, delayed pulses are separately sorted for each layer into four different categories: decays upwards, decays downwards, after-pulses and all remaining events. In addition, several histograms are filled. Their contents are stored in a ROOT and a text file for further processing and to check the quality of the data.

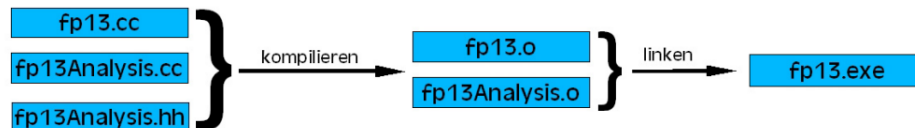


Figure 2.11: Compile the analysis program with `make`.

The C++ program consists of four files, three of them are shown in Figure 2.11, while the last one is the `Makefile` needed to compile the program. `fp13Analysis.hh` is the file where the functions and variables needed in the other files are declared. In the file `fp13.cc` there is the function `int main()` that starts the program. Here, the analysis is performed, using the class `fp13Analysis` which is called inside the function. In `fp13Analysis.cc` the different functions needed for the analysis are defined. They are called in the main function and will select the data. Only this file (`fp13Analysis.cc`) will be edited by you and should therefore be analysed and understood enough to implement your changes.

You can find the relevant files on the CERN GitLab:

<https://gitlab.cern.ch/HD-FP13/Students>

Enter the previously created directory with `cd /local/mydirectory` and download the files via

```
git clone https://gitlab.cern.ch/HD-FP13/Students.git
```

Then you have to answer/follow the following questions/steps:

- Which bit patterns correspond to the four categories (upwards decays, downwards decays, after-pulses and all remaining event)?
- Familiarise yourself with the structure of the program `fp13Analysis.cc`!
- Functions for classification of events into upwards decays and after-pulses

categories are already available. Write yourself a function for downwards decays and a more general function for the after-pulses.

- After calling these methods the program will fill several histograms. Familiarise yourself with this function and create, if necessary, histograms yourself for the downward decays and the after-pulses under the appropriate name.
- Now you have to compile the different files with the help of the `Makefile` in order to run your program on your data. For this purpose, you must first enter the command

```
source /janus/software/root/root-6.10.08/bin/thisroot.sh
```

to let your shell environment know where to find the `ROOT Data Analysis Framework` that we will use throughout the analysis of the recorded data. If you want to learn more about `ROOT`, visit <https://root.cern.ch>. In particular useful to get more information about the available commands and the functions that we are going to use is the documentation that you can find on <https://root.cern.ch/doc/v610>.

Now, to compile the program to process the raw data, type `make`. In this way the executable program `fp13` is generated. You can analyse the data-file named `myfile.txt` using the `-i` option: `./fp13 -i myfile.txt`.

If the program `fp13Analysis.cc` is changed, you have to repeat the make step each time, otherwise your changes will not be effective!

The histograms of the data are stored in the file `fp13.root` in your current directory. You will use this file for further analysis using the program of your choice. Further discussion is based on an analysis using `ROOT`. If you have big problems with the standalone program, you can ask your supervisor for advice.

2.5 Accessing the data and initial data analysis

Now the quality of the data and their consistency must be examined in order to determine which data sets are suitable for the analysis. Using the program

package ROOT, you can manipulate the histograms filled from the data acquisition program. (This does not require prior knowledge of ROOT, since the few necessary commands are described in the instructions.) In general, the data will be transferred to a computer of the CIP Pool and can be analysed with ROOT from any workstation of the physics department CIP Pool. But anyone who has a lot of experience can save the data as a text file and analyse them with a program of his choice.

ROOT allows you to fit the stored histograms and to work with the parameters of the fit function. The different steps can be directly executed in a command line one after the other, but they can also be written in macros, which can then be executed.

Getting the ROOT - Analysis:

First you must set the path to the ROOT library (if not already done for the previous step). Type

```
source /janus/software/root/root-6.10.08/bin/thisroot.sh
```

and then the command `root`. Your supervisor informs you about the current conditions. Initially familiarise yourself with the program and try to open some histograms from the command line. The histograms `h1` and `h2` you should already know from the GUI of the data taking.

- Load the file with the stored histograms: `TFile f("fp13.root")`.
- For a first overview over the generated histograms, the content of `fp13.root` can be examined with the `TBrowser` which is a graphical program which allows to quickly *browse* through the file.
To open it, type `new TBrowser()`.
- Draw one of the histograms that you have in your file. This is possible for example for the histogram `h1` by the command `h1->Draw()`. You can change the presentation of the histograms by specifying some options. Try the instructions `h1->Draw("hist")` and `h1->Draw("e")`. (You can use the arrow keys to get back to previously executed commands in the command line).

- Repeat the process for `h7` (number of slices) with `h7->Draw("hist")`. Here it will be very useful to have a logarithmic scale in the y axis! This option can be chosen in the figure: right click to the left of the y-axis, and then select `SetLogy`. Take a look at the other useful options!

- Watch also at the next hit distributions of each layer and at the corresponding distributions for after-pulses and decays up or downwards. For example:

`a2->Draw("e")` hit distribution in layer 2 for upwards decays

`w2->Draw("samehist")` after-pulses distribution of completely passing through muons. `x2->Draw("samehist")` after-pulses distribution obtained by the improved method.

- A list of all available histograms is given in Table 4.1 in Appendix A.
- Repeat the same procedure for another layer, which has a particularly pronounced after-pulses spectrum. Obviously you have to correct the spectrum for the after-pulses, i.e. you have to remove them.
- Correction for after-pulses for one layer (example). For a correct determination of the lifetime of muons from the decay spectrum, it is necessary to eliminate these after-pulses. This is done via the subtraction of the measured after-pulses spectrum from the measured decay spectrum. One must consider that the number of events which have the right start pattern to measure a after pulse do not correspond to the number of events having the right start pattern to measure a real muon decay. Therefore, before the after-pulses histogram is subtracted from the decay histogram, it must be multiplied by a scaling factor s . If the number of entries in the histogram `a2` is N_{tot} (uncorrected upwards decays in layer 2) and the number of entries in the histogram `x2` (number of after-pulses in layer 2) is denoted by N_{NP} , then the number of actual muons which decay upwards N_{μ}^0 is given by

$$N_{\mu}^0 = N_{\text{tot}} - s \cdot N_{\text{NP}} = N_{\text{tot}} - \frac{N_{2\text{stop}}}{N_{\text{NPstart}}} \cdot N_{\text{NP}}. \quad (2.3)$$

The quantity $N_{2\text{stop}}$ corresponds to the number of signal events which only give signal until layer 2 and thus have the right start pattern for

after pulses imitating decays upwards. $N_{NPstart}$ is the number of events from which N_{NP} , the number of after-pulses, was determined with your improved method. The ratio $\frac{N_{NP}}{N_{NPstart}}$ corresponds to the probability that after the passage of a particle, an after-pulse was produced by the scintillator. This probability must be multiplied by the number of events which have the right start pattern for an after pulse imitating a decay upwards. The result is the number of after-pulses which is expected in N_{tot} and therefore it can be subtracted from N_{tot} .

You can get the values of $N_{NPstart}$ and N_{2stop} from the histogram h8 in which the last simultaneously hit layers were filled.

The contents of a bin can be displayed directly in the histogram. Go with the mouse to the upper end of the bin or to the data point. Selecting the function **EventStatus** under **View**, you see the contents displayed (binc) under the histogram.

By the way: You can make calculations also in ROOT! Simply type in the mathematical formula e.g. `34000./15000.` and hit **Return**.

Now, the scaled after-pulses are subtracted:

```
a2->Draw("e")
x2->Scale(2.27)
x2->Draw("esame") the after-pulses are now rescaled
TH1D *x2s = new TH1D(*x2)
a2->Add(x2s,-1.)
```

If you did it correctly, you should see that the records for decays with long times are approximately zero (why?). Save the histograms for a layer and print and stick it to the lab book! To do so, click on **File** in the figure above and select the **SaveAs** option.

- Fit the lifetime after the correction for the after-pulses. Proposal: take layer 2 since this is the layer where most of the particles decay and consider decays upwards.

Load the macro **Lebensdauer.C** with `.L Lebensdauer.C`; This is the macro where the fit function **FitFuncF** is defined.

Then type the following commands:


```
TF1 *f1 = new TF1("f1", fitFuncF, 0., 20000., 3);
```

This defines a fit in the range $0 < x < 20000$ with 3 parameters.

```
f1->SetParameters(10., 5000., 2000.);
```

Initial values for the number of background events, the number of muon decays and the lifetime parameter [ns].

```
a2->Fit(f1, "", "e", 1000., 20000.);
```

Fits in the range $1000 < t \text{ [ns]} < 20000$ and draws a graphical representation of the fit function.

Try different ranges for the fit. **Print and put these results in the lab book.**

2.6 Final evaluations

In the last chapters you have seen how to correct the data sample and how to make a fit. Now you have to extend the whole procedure for all the layers and also for decays downwards. Similar macros are used for this purpose, which allow to execute command sequences automatically. The first step should be to get an overview over the data. Look at the macro `Weiteres.C` and familiarise yourself with the structure. Execute `Weiteres.C` as follows:

- Load the macro: `.L Weiteres.C`
- Execute a function defined in the macro using the data from `fp13.root`: `canvas3("fp13.root")`. You can check in Table 4.1 in Appendix A which histograms are shown by the individual functions (here: `canvas3()`).
- Again, here you will find the already known histograms `h1` and `h2`.
- Look also at the other functions of the macro `Weiteres.C`.

At the end of this guide you will find an overview about all existing histograms and their usage in the different macros. Check where those functions are located exactly and familiarise with them. Watch in particular the rate and the fluctuations of after-pulses in the time distributions of the individual layers. Which data sets and which time window can be used for the lifetime determination?

2.6.1 Determining the lifetime

- Open the macro `Lebensdauer.C` with a text editor and try to get an overview of it.
 - Where do you find again the already familiar commands?
 - What happens in the other sections?
 - Which function is used to fit the data?

A histogram, for example, should look like the ones in Figure 2.12. Both histograms contain the same data, but in the right plot it is shown with a logarithmic scale in the y-axis, and with error bars.

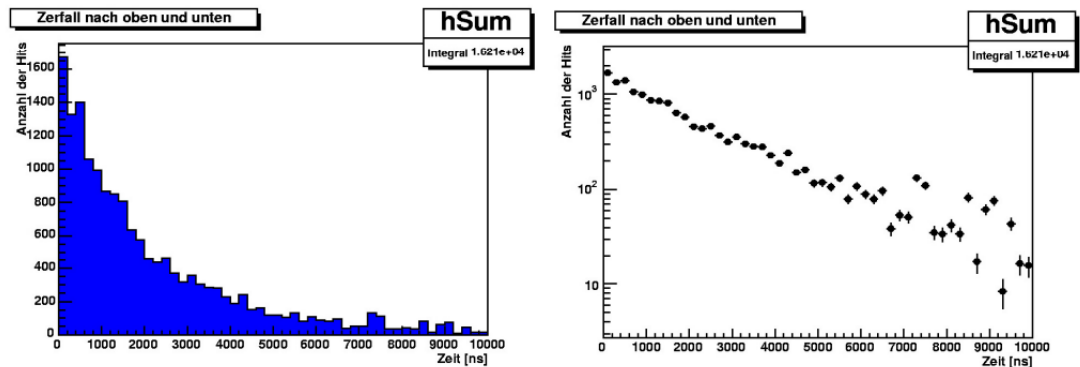


Figure 2.12: Histogram of all decays, where is plotted the number of events versus the lifetime.

Load the macro `.L Lebensdauer.C` to determine the lifetime and then execute the function `Lebensdauer(start, stop, "fp13.root")` where `start` and `stop` indicate the time window in which the fit is performed. They are given in ns. Initially look only at the three windows that open first (the windows can overlap each other).

- Take your time to look at all the histograms. Then answer to these questions:
 - Where can you find similarities in the decays in different layers?
 - Are there histograms particularly different from other histograms?
 - How do the after-pulses affect the spectra? What are the consequences for your next step?

- Which patterns have to be present in the decays upwards or downwards, so that after-pulses affect the spectrum? How does the efficiency of the layers influence it?
- As described in Section 2.5, it is necessary to eliminate the after-pulses from the decay spectrum. For this step it is very important that the determination of the scaling factor for the after-pulses histogram is done correctly for your improved method to measure the after-pulses. For your calculation Table 2.1 is useful. All the necessary information can be obtained from histogram h8.

Answer to this questions:

- Why is the specific inefficiency factor (see Table 2.1) necessary when calculating the number of start patterns for after-pulses in downward decays? Remember that after-pulses occur only if the scintillator was passed by a muon previously.

In the first column you have to register the starting pattern for decays upwards. For upwards-decays in layer 3, for example, you should put all events in which the muon was passing through until the third layer. In the second column you have to write down all possible starting patterns for your improved method to classify a delayed hit for the different layers as an after-pulse. To classify a delayed hit in layer 3 as an after-pulse, possible start patterns are muons passing through up to layer 4,5,6 or 7. You can get this information also from histogram h8. From these numbers the sum is formed.

The scaling factor is computed by dividing the respective entries in the first and the second column.

- Now modify the macro `Lebensdauer.C` to subtract the after-pulses using your scaling factors. How does the appearance of the spectra change? Plot both the scaled and the unscaled histogram in order to understand the contribution and intensity of the after-pulses. Decide which spectra and layers to use for the rest of the analysis in order to fit the lifetime. Modify the macro accordingly.

Location (Layer)	Number of start pattern for decays upwards	Number of start pattern for after-pulses	Scaling factor
1			
2			
3			
4			
5			
6			
Location (Layer)	Number of start pattern for decays upwards in next layer · Ineff/(1-Ineff)	Number of start pattern for after-pulses	Scaling factor
2			
3			
4			
5			
6			

Table 2.1: Scaling factor for the correction for the after-pulses histogram.

- In the following, fit using the function:

$$N(t) = N(\mu) e^{-\frac{t}{\tau_0}} + N_{BG} \quad (2.4)$$

Set the initial values of the parameters to sensible numbers by looking at the figures. Set also the range for the parameters of your fits.

Answer to this questions:

- What appears to be useful?
- At what time the capture of negative muons gives no contribution to the spectrum anymore?
- What does this tell you for your fit?

2.7 Advanced method for lifetime determination

The above-described method for the determination of the muon lifetime includes some systematic problems. The measured decay spectra include both contributions from the decay of free muons and muons which were captured by atoms. To take into account this contribution we will use equation 2.4 modified as follows:

$$N(t) = N_{\mu^+} \cdot e^{-\frac{t}{\tau_0}} \left(\frac{1}{f} e^{-\frac{t}{\tau_c}} + 1 \right) + N_{BG} , \quad (2.5)$$

where f is the ratio $\frac{N_{\mu^+}}{N_{\mu^-}}$.

The fit of this function is done with the macro `Einfangzeiten.C`. If you have a look to this macro, you will see that it is practically identical to the macro `Lebensdauer.C`, except that the function corresponding to the above fit function (`fitFuncF`) is defined differently. Thus, also the parameters used for fitting are different. Plug the scaling factors that you calculated in Section 2.6.1 into the macro and load it with the command `.L Einfangzeiten.C`. Now you can use the macro by executing `Einfangzeiten(start, stop, "fp13.root")`.

Check the results of the fit and in particular compare the results for the decays upwards and downwards. Discuss which of the parameters should be similar for both decay spectra and in which parameters it is possible to have differences. Does the result match your expectations?

It is possible that the results of the previously described fit seem to be nonsense. This can be due to the fact that, for example, the fit errors are very large or there are unphysical parameters. The cause is here the correlations between the different parameters of the fit which makes it impossible to measure all parameters simultaneously. Furthermore, the experimental arrangement is not suitable for the determination of the parameter f , the ratio of positive to negative muons, since the charge of the muons is not directly measured. A summary of such measurements can be found in [1]. In the relevant experiments for this energy range, the ratio was determined to be $f \approx 1.275 \pm 0.05$ [1]. You can now specify this measured value for the ratio when you make the fit. This usually leads to the fact that the fit method described above provides meaningful results.

2.8 Determination of the systematic errors of the lifetime

In addition to the purely statistical error, the error specified by ROOT for the fit parameters is also subject to systematic errors. For this lab course you should study, besides the **influence of the fit range**, quantitatively two systematic error sources:

- **Inaccuracy of the correction for the after-pulses.** The measurement of the after-pulses spectra is subject to statistical fluctuations. This leads

to a possibly wrong correction. To take that into consideration you should vary the correction factors for the afterpulses (in which range do you think it makes sense?). How much does the fit result change?

- **Influence of the ratio f .** In the last step, the parameter f was fixed to the measured value. This value itself has of course an experimental uncertainty. Check the changes of the lifetime of the muon when f is varied up to $\pm 1\sigma$ and $\pm 2\sigma$ of the current measured value.

Think of how to estimate the bias arising from the fit ranges and combine all different systematic uncertainties.

2.9 Measurement of the muon polarisation

The polarisation of the muons can be measured by applying a magnetic field. The magnet consists of two rectangular coils with 41 turns along the long side of the detector (see Figure 2.13). There is a local power supply which can operate up to 25 A. The magnetic field can be calculated as

$$B = I \cdot 2.0 \cdot 10^{-4} \cdot \frac{\text{T}}{\text{A}}. \quad (2.6)$$

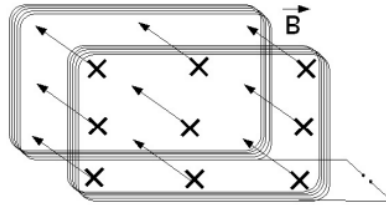


Figure 2.13: Arrangement of the two coils around the detector generating an homogeneous magnetic field.

Adjust the amperage to 20 A. How big will be the field strength? Let's make these measurements with statistics as large as possible (until beginning of the following week).

Answer to the following question:

- Why are the muons polarised?

To determine the polarisation you have to compare the measurement with and without the magnetic field (including the scaling factor correction). As

discussed in Section 1.7, for a fixed scintillator we obtain, due to the Larmor frequency ω_{Larmor} , a time dependent count rate of

$$Z^{\text{with}}(t) = Z_0 \cdot e^{-\frac{t}{\tau}} \cdot (1 + P \cdot A \cdot \cos(\omega_{\text{Larmor}} \cdot t + \phi)) \quad (2.7)$$

if a magnetic field is present.

For the measurement without magnetic field the count rate is given by

$$Z^{\text{without}}(t) = Z_0 \cdot e^{-\frac{t}{\tau}} \cdot (1 + P \cdot A \cdot \cos(\phi)). \quad (2.8)$$

This results, in good approximation, in the asymmetry function

$$\frac{Z^{\text{with}}(t) - Z^{\text{without}}(t)}{Z^{\text{with}}(t) + Z^{\text{without}}(t)} \sim \frac{P \cdot A}{2} \cdot \cos(\omega_{\text{Larmor}} \cdot t + \phi) + c. \quad (2.9)$$

Here we have an oscillating function, and the exponential decay is eliminated. Think about it and try to derive it on your own. You can use the macro `Asymmetrie.C` to fit the measured data. The following steps are necessary to extract the asymmetry:

- First, you have to use the program `fp13` to create the corresponding ROOT file for the data with magnetic field. You can specify the name of the output file with:

```
./fp13 -i myfile.txt -o outputfile.txt
```

- As before, you have to correct the spectra for after-pulses. To do this, take from the previous section the correction factors and put them in the appropriate place in `Asymmetrie.C` for the histograms of the measurement without a magnetic field. For the measurement with a magnetic field you have to determine again the factors and put them again at the appropriate point in `Asymmetrie.C`.
- Now one must consider that the duration of the measurements with and without the magnetic field is different and therefore the number of entries in the histograms are different. To do so, one has to scale in the subtraction or addition the number of entries in the histogram with the magnetic field by using the scaling-factor $s = Z_0^{\text{without}}/Z_0^{\text{with}}$.

This factor is obtained automatically. Load the macro `.L Asymmetrie.C` and execute

```
Asymmetrie(start, stop, "file.root", "fileWithB.root")
```

Look for the canvas with the decay spectra with and without the magnetic field. The scaling factors are derived from the integrals of the respective histograms listed there. Note that the asymmetry functions for the decays upwards and downwards decays are determined separately, and hence also the scaling factor for both asymmetry functions has to be determined separately.

In the macro, the histogram

$$\frac{Z^{\text{without}}(t) - Z^{\text{with}}(t) \cdot s}{Z^{\text{without}}(t) + Z^{\text{with}}(t) \cdot s} \quad (2.10)$$

is calculated and the function

$$\frac{Z^{\text{without}}(t) - Z^{\text{with}}(t) \cdot s}{Z^{\text{without}}(t) + Z^{\text{with}}(t) \cdot s} = \frac{P \cdot A}{2} \cdot \cos(\omega_{\text{Larmor}} \cdot t + \phi) + c \quad (2.11)$$

is fitted.

Understand these steps in the macro `Asymmetrie.C`! Now look at the data from each layer. Do they differ from those without a magnetic field? Choose again the best data set. Compare again the results for the decays downwards and upwards. Which of them seems to have less systematic problems? Try also to vary the initial values of the parameters until the fit function describes well the data. Test also how much the fit result depends on the fit range.

Answer to the questions:

- What values do you expect for the parameters? (Concerning ϕ : think about with which polarisation the muons are created and with which polarisation they reach the Earth's surface).
- Are the expected values close to those calculated by the program?
- Which parameter shows the parity violation in the weak interaction?
- Calculate the magnetic moment of the muon from the measured Larmor frequency.

- Discuss the results critically. In particular, think about whether your measurements show significant polarisation of the cosmic muons. How can you show this quantitatively?

Measure the polarisation with an increased data set.

The measurement of the muon polarisation is experimentally difficult. In this lab experiment it is limited by the measuring time, so by the limited statistical accuracy. In order to analyse the polarisation of the muons with a higher statistics, your own dataset can be combined with those of other groups. You need the files `ResultsAll.root` and `AccumulatedAsymmetrie.C` which you have already downloaded with the other files. Then execute the macro `AccumulatedAsymmetrie.C` in ROOT. If necessary, you have to adjust the file-names in the macro. Here, too, it is very important that you choose the best fit range. Now, the measurement is performed on a dataset which is about a factor of 8 greater than the data collected by you during one week.

Chapter 3

Fitting Data

In high energy physics, but also in the experiment that you are doing, physical quantities are determined by making use of a lot of data, which are taken and used to fill one or more histograms. These histograms can then be fitted to extract the physical quantities. For this purpose there are generally two methods: the minimum- χ^2 and the maximum likelihood method.

- **Minimum- χ^2 .** The minimum- χ^2 fit method is an algorithm that tries to minimise the quantity:

$$\chi^2 = \sum_{i=1}^N \frac{(y_i(\text{meas}) - y_i(\text{fit}))^2}{(\Delta y_i)^2} \quad (3.1)$$

For each entry in the histogram the squared difference between the entry and the value of the fit function is calculated. This quantity is divided by the square of the error on the entry. This is done for all the entries, then the resulting function is minimised. If the fit function describes the data well, it is expected that for many entries the deviation of the data from the fit function on average is as large as the error of the individual entries $y(\text{meas})$. So one expects that $\chi^2 = \text{NDF}$, where NDF is the number of degrees of freedom in the fit. The software package ROOT assumes that the error on $y(\text{meas})$ is equal to $\sqrt{y(\text{meas})}$. But this is valid only for sufficiently high count rates. Therefore, the χ^2 -criteria is not a good criteria in case of small statistics. In such cases, the maximum likelihood method is much more useful.

- **Maximum Likelihood.** In order to determine the parameters \mathbf{a} of the

fit function with the maximum likelihood method, starting from N independent measurements x_i one has to calculate the probability density $P(x|a)$. Then the parameters are determined maximising the product of individual probabilities, which is called the likelihood function:

$$L(x_1, \dots, x_N|a) = \prod_{i=1}^N P(x_i|a) \quad (3.2)$$

The fit does not always converge, but even if it converges, you should apply some criteria to check its goodness:

1. Check your fit by eye. Are the data distributed around the customised fit function or are there systematic differences?
2. If your starting values and your bin width was correctly chosen than you should see that $\chi^2 \sim \text{NDF}$ and $\chi^2/\text{NDF} \sim 1$.
3. Check also the information that you get from the fit routine. When it is written that a parameter is ****at limit****, this means that the "true" value of the parameter is above/under the edges of its allowed range. In this case, increase the allowed range until no more ****at limit**** messages are printed.

Chapter 4

Notes on the protocol of the experiment

When you write the protocol you should focus on a critical evaluation and discussion of your results. It should be clear what problems (can) occur in the data analysis and what methods are used to solve them. Furthermore you should discuss in which way the methods systematically influence your results. The aim of the experiment, the methods used and the results are the topics on which you have to focus on. However, you should also mention the equations and parameters used for data analysis. In particular, the experimental protocol should include the following:

- The efficiency of the individual scintillators. Please put your values (threshold, individual counting and coincidence rate and efficiency) for all layers together in a table.
- Quote the single rates of each scintillator with your discriminator thresholds. With this you can estimate the rate of random coincidences for a known length of the standard pulses using the formula explained in the instruction.
- Describe the major steps of data analysis.
- Determine the lifetime of the muon. Discuss the results critically. How can the measurement be systematically biased?
- Determine the systematic error in the lifetime as described in Section 2.8.

- Calculate the number of expected muon decays in the time range $> 10 \mu\text{s}$.

Answer to this questions:

- Can you explain why for periods over $10 \mu\text{s}$ a lot of background is observed?
 - Is $10 \mu\text{s}$ a particularly characteristic time?
- Determine from your measurement the Fermi coupling constant G_F of the weak interaction.
- From the spin of the muon you can determined the magnetic moment precession. What is the original spin-direction with respect to the moving direction of the muons?
- Determine the polarisation of cosmic muons. Which conclusions can be drawn about the genesis of cosmic muons?

Appendix A

4.1 Overview of the histograms

h1	Total hits in the layers
h2	Detector after-pulses (improved)
h3	Position of the decays upwards
h4	Position of the decays downwards
h5	Detector after-pulses 2 (default)
h6	Number of slices
h7	Number of slices taken
h8	Start pattern
h21(z1...z7)	Time of the first hit for layer i (zi)
h22(x1...x7)	After-pulses for layer i (xi) (improved)
h23(a1...a7)	Position for the decay upwards i (ai)
h24(b1...b7)	Position for the decay downwards i (bi)
h25(w1...w7)	After-pulses 2 for layer i (wi) (default)

Table 4.1: Histograms, which are defined and filled in `fp13Analysis.cc`.

Lebensdauer.C with Lebensdauer()
⇒ Scaled spectra of decays upwards and downwards for all layers
⇒ Spectra for all layers
⇒ All decays upwards and downwards
⇒ Fit of the lifetime
Einfangzeiten.C with Einfangzeiten()
⇒ All decays upwards and downwards
⇒ Spectra for all layers
Asymmetrie.C with Asymmetrie()
⇒ Scaled spectra of decays upwards and downwards for all layers
⇒ Spectra for all layers
⇒ All decays upwards and downwards
⇒ Asymmetry fit
Weiteres.C
⇒ canvas1()
Time of the first hit for all layers (z0-z7)
⇒ canvas2()
After-pulses per layer (x0-x7)
⇒ canvas3()
Hits in layers total, number of all slices,
Detector after-pulses, start pattern (h1, h6, h2, h8)
⇒ canvas4()
Decays upwards per layer (a1-a6)
⇒ canvas5()
Decay downwards per layer (b1-b6)
⇒ canvas6()
Layer of the upwards decays and downwards decays (h3, h4)
⇒ canvas7()
After-pulses2 per layer (w0-w7)
⇒ canvas8()
Time of the second hit for all layers (o0-o7)

Table 4.2: Available Macros for the analysis.

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