F13 Measurement of Muon Properties in the Advanced Student Laboratory
Short report

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Abstract

In this experiment we measured and analyzed the decay of cosmic muons. The main aim was to determine their lifetime τ, but we were also interested in information about their polarization when entering the atmosphere. The measurement was done using a set of eight scintillators connected to an electronic setup converting the data to digital signals, that were analyzed using a root program. In this experiment the results matched the literature values well. The results are shown in table 1.

lifetime	τ_0	$(2106 \pm 46_{stat} \pm 89_{sys})ns$
coupling constant	G_F	$(1.181 \pm 0.056)10^{-11}MeV^{-2}$
Larmor frequency	ω_{Larmor}	$(3.37 \pm 0.25_{stat} \pm 0.55_{sys})MHz$
magnetic moment	μ_{muon}	$(2.77 \pm 0.50) \cdot 10^{-7}\frac{eV}{T}$

Table 1: Results

1 Theoretical Background

1.1 Muon Production and Decay

Muons are elementary particles, they belong to the second generation of leptons. Since they are heavier than electrons (207 times) they can decay into these.

In the experiment we measured cosmic muons. These originate from cosmic rays. Cosmic rays mainly consist of energetic protons. The following reactions occur do to interaction of these protons with molecules in the upper atmospheric layers.

$$p + p \rightarrow p + n + \pi^+ \tag{1}$$

$$p + n \rightarrow p + p + \pi^- \tag{2}$$

$$p + p \rightarrow p + \Lambda + K^+ \tag{3}$$

Where p and n are protons and neutrons, π^+, π^- and K^+ are pions and Kaons and Λ is the lambda baryon.

Since kaons and pions are not stable they then decay as follows:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{4}$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad (5)$$

$$K^+ \rightarrow \mu^+ + \nu_\mu \quad (6)$$

μ^+ and μ^- are the positive and negative muons we want to measure in the experiment. ν_μ and $\bar{\nu_\mu}$ are their corresponding neutrinos.

Muons decay with an average lifetime of 2.19μs according to the following reactions:

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \quad (7)$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \quad (8)$$

By knowing the mass(m_μ) and lifetime (τ_0) of the muon, one can determine the Fermi weak interaction coupling constant G_F using the speed of light c and the dirac constant $\hbar$:

$$G_F^2 = \frac{192 \cdot \pi^3 \cdot \hbar}{\tau_0 \cdot (m_\mu \cdot c^2)^5} \quad (9)$$

In addition to this, negative muons can also decay via μ-capture. Negative muons at rest get caught in the electric field of an atom and following reaction occurs:

$$\mu^- + p \rightarrow n + \nu_\mu \quad (10)$$

This happens in less than 10^{-12}s, so the effective lifetime of negative muons is reduced by:

$$\frac{1}{\tau} = \frac{1}{\tau_0} + \frac{1}{\tau_c} \quad (11)$$

Where τ_c is the lifetime of pure muon capture.

For the time dependence of the number of decaying (positive and negative) muons we expect:

$$N(t) = N(\mu^-, t_0) \cdot \exp(\frac{t - t_0}{\tau_0}) \cdot \exp(\frac{t - t_0}{\tau_c}) + N(\mu^+, t_0) \cdot \exp(\frac{t - t_0}{\tau_0}) \quad (12)$$

1.2 Parity Violation

If parity were conserved one would expect the same amount of postitrons emitted opposite and along the direction of spin of the muon. This is not the case. The spatial distribution of the positrons from the positive muon decay is:

$$N(\theta) \propto 1 + A \cdot \cos(\phi_0) \quad (13)$$

A is the asymmetry parameter and depends on the energy of the positrons. In the experiment we expect it to be $A = 0.23$. ϕ_0 is the angle of the positron with respect to the spin of the muon. The spin and the magnetic moment (μ^{Bohr}) of a muon in an external magnetic field, which is perpendicular to the direction of the spin, perform a Larmor precession with the frequency

$$\omega_{Larmor} = \frac{g \cdot \mu_\mu^{Bohr} \cdot B}{\hbar} \quad (14)$$

where g is the Land$\acute{e}$ factor and B is the magnetic field strength.

Since the positron is preferably emitted along the direction of the spin of the muon, the Larmor frequency can be measured by observing the asymmetry of decay with and without magnetic field. The asymmetry function is given by the following equation. It quantifies the time dependent difference in amount of decays in a certain direction (upwards/ downwards). This changes in time due to the Larmor precission.

$$\frac{Z^{with}(t) - Z^{without}(t)}{Z^{with}(t) + Z^{without}(t)} \propto \frac{P \cdot A}{2} \cdot cos(\omega_{Larmor} \cdot t + \phi_0) + BG \quad (15)$$

Where Z is the time dependent counting rate of a scintillator. The counting rate is defined as:

$$Z(t) = Z_0 \cdot e^{-\frac{t}{\tau}} \cdot (1 + P \cdot A \cdot \cos(\omega_{Larmor} \cdot t + \phi_0)) \quad (16)$$

And P is the polarization of the muon due to the decay of the pions and kaons. Considering parity violation in these decays one will see that negative cosmic muons will be right-handed and positive cosmic muons will be left-handed. Considering this, upwards decay should dominate over downwards decay in the experiment. The meaning of upwards and downwards decay will be explained in section 2.2.

2 Experimental Methods

2.1 Setup

The detector used in the experiment consists of layers of scintillators and metal plates assembled as shown in figure 1. The black lines are the eight 1cm thick scintillators that are connected to eight photomultipliers PM0-PM7.

Figure 1: Detector Setup. Modified from [1, Figure 2.1]

The photomultipliers are connected to eight discriminators which are connected to a TDC (time to digital converter). The discriminators generate a Boolean signal, when the input voltage reaches a certain threshold. This is necessary to prevent the measurement of background pulses. The TDC converts the signal so that the information where(which scintillator) and when(which time slot) the muons fly through the detector can be saved by a computer. The time resolution of the setup is given by the length of the time slot. $\Delta t = 10ns$

Additionally the whole setup is placed inside a magnet composed of two coils around the detector. Switching on the homogenous magnetic field gives us the opportunity to observe the Larmor precession of the muons.

2.2 Experimental Procedure

The muons reach the earths surface with a relativistic velocity. Therefore, if they pass through all the scintillators, they are detected in one time slot, since the length of a time slot is larger than the time needed for the muons to pass through the setup. If a muon is stopped in one of the metal plates it decays. Since scintillators can detect charged particles, the electrons/positrons from the decay can also be measured. As we have discussed before, these produced electrons/positrons can decay upwards or downwards from the metal plate where the muon was stopped. They therefore show up either in the same scintillator where the muon was detected (upwards) or in the one below(downwards). This will mostly happen in a new time slot and from the time difference the lifetime of muons can be determined using the exponential decay law and a large amount of measurements.

To set up the detector arrangement in the way that it operates most efficiently we checked the individual discriminator threshholds and afterwards took the efficiencies of each scintillator.

After this was done we started our first mea-

Scintillator	Inefficiency [%]
1	3
2	4
3	5
4	7
5	5
6	5

Table 2: Scintillator Inefficiencies

surement and took data for the two following days. Right after this measurement we switched on the magnetic field and started a second measurement for another four days.

3 Evaluation of Results

3.1 Correction of systematic errors

Before the muon lifetime can be determined we have to correct the systematic errors that occur. One of them is due to the effect of afterpulses in the photomultipliers. This means that we get a second signal after the actual detection and therefore in a different time slot.We won't explain how these arise but only their effect on our measurement. We want to be sure that we only measured downwards or upwards decays. An afterpuls in the last scintillator layer can be mistaken as a decay upwards. To calculate the probability that the decays that seem to be upwards decays are due to afterpulses we check for the events where only a muon was detected(i.e. the first n layers were hit in one time slot but there is no hit in layer n or n+1 in the next time slot) if there was an afterpuls in the layers 1 to n-1. Substracting these possibilities leaves us with a better measurement of real upwards decays.

The second systematic error we fixed is if an afterpuls in layer n+1 seems to be a decay downwards. This only occurs if the scintillator in layer n+1 did not detect the muon due to its inefficiency. This is corrected in the same way as the other afterpuls events only this time the afterpuls rate is multiplied by the inefficiency of the corresponding scintillator.

3.2 Determination of muon lifetime

For each scintillator the number of decays are measured over time. The data of all scintillators is combined in one histogram as can be seen in figure 2. In order to take into account the contribution from the decay of free muons and those captured by atoms, we used following fit function to fit the measured data:

$$N(t) = N_{\mu^+} \cdot e^{-\frac{t}{\tau_0}} (\frac{1}{f} e^{-\frac{t}{\tau_c}} + 1) + N_{BG} \qquad (17)$$

Where N_{BG} is the background value and $f = \frac{N_{\mu+}}{N_{\mu^-}}$ and was given as $f = 1.3$. The value for the lifetime is: $\tau_0 = 2106 \pm 46$. This can also be seen in the same figure.

From the fit we get a purely statistical error for the fit parameters but the muon lifetime also has systematic errors. To take this into account we considered four systematic error sources. First we varied the value of f by $\pm 1\sigma$ to $\pm 2\sigma$ ($f = 1.3 \pm 0.2$) and estimated the error of the lifetime due to this to about $\Delta\tau_{0,1} = 30ns$. Then varied the correction factors for the afterpulses by about 5% and estimated this systematic error of the lifetime to about $\Delta\tau_{0,2} = 20ns$. We also tried different fit ranges and chose a range where the fit matched our values best (smallest χ^2). This can be taken into account as an error of $\Delta\tau_{0,3}$ =

$15ns$. A fourth systematic error we took into account is due to the different fit results we got using decays upwards or downwards. The error is about $\Delta\tau_{0,4} = 80ns$. This error is important because of the difference in the quality of the measurement of upwards and downwards decay. The difference in quality is due to the afterpuls correction. In comparison the possible error of the afterpuls correction for downwards decay is small, because the inefficiencies of the scintillators are small and their values known. Whereas for upwards decay we first had to find the probability for an afterpuls to occur from our data. Therefore the total systematic error of the lifetime is $\Delta\tau_{0,sys} = 89ns$. In the end our measured lifetime of the muons is: $(\tau_0 = 2106 \pm 46_{stat} \pm 89_{sys})ns$.

Einfangzeit.png

Figure 2: Muon Lifetime

Using equation 9 we can now calculate the Fermi weak interaction coupling constant G_F:

$$G_F = \sqrt{\frac{192 \cdot \pi^3 \cdot \hbar}{\tau_0 \cdot (m_\mu \cdot c^2)^5}} = (1.181 \pm 0.056)10^{-11}MeV^{-2} \tag{18}$$

We used the known muon mass $m_\mu = 1.883 \cdot 10^{-28}kg$, the dirac constant $\hbar = 6.582 \cdot 10^{-16}eVs$ and the speed of light $c = 3 \cdot 10^8 \frac{m}{s}$. The literature value is $G_F^{th} = 1.16637(1) \cdot 10^{-11}MeV^{-2}$.

3.3 Verification of parity violation

To determine the polarization of the muons we compare the measurement with and without a magnetic field. Since our measurement was very short (only three days), we use the accumulated data set of all experiments including ours. We fit out data with the asymmetry function (equation 15).

We determined a Larmor frequency of $\omega_{Larmor} = (3.37 \pm 0.25)MHz$. Again, this only takes into account the statistical error of the fit, but we can also estimate a systematical error comparing the derived Larmor frequency using only decays upwards and downwards. The error is $\Delta\omega_{Larmor,sys} = 0.55MHz$. We again chose the best fit range. The theoretical value is: $\omega_{Larmor}^{th} = 3.4111Mhz$. Calculated with equation 14 and the theoretical value for the magnetic moment of the muon.

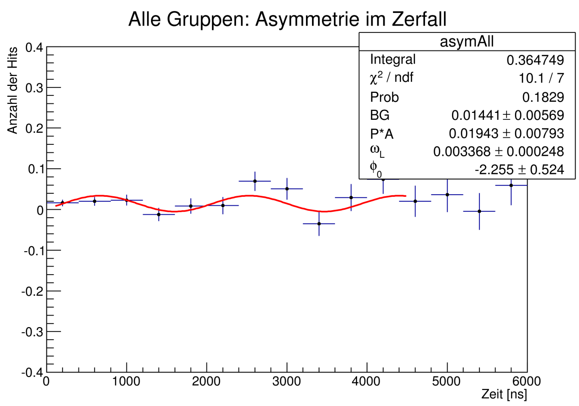

Figure 3: Larmor Frequency

Now we can calculate the magnetic moment of the muons:

$$\mu_{Muon} = \frac{\hbar\omega_{Larmor}}{2B} = (2.77 \pm 0.50) \cdot 10^{-7} \frac{eV}{T} \tag{19}$$

Where we used $B = 4 \cdot 10^{-3} T$ and the dirac constant as before. The theoretical value is $\mu^{th}_{Muon} = 2.8065 \cdot 10^{-7} \frac{eV}{T}$

From the found value for the product of asymmetry and polarization $P \cdot A$ we can determine the polarization using the given value of A=0.23:

$$P = \frac{0.0194}{0.23} = (0.084 \pm 0.034) \tag{20}$$

4 Discussion

In section 3.2 we determined the lifetime of muons from our measured data and calculated the Fermi weak interaction coupling constant. Our results compared to the theoretical values are shown in table 3:

	lifetime τ_0	coupling constant G_F
exp. value	$(2106(46)_{stat}(89)_{sys})ns$	$(1.181(56))\frac{10^{-11}}{MeV^2}$
theo. value	$2190ns$	$1.166 \cdot \frac{10^{-11}}{MeV^2}$
difference	1σ	1σ

Table 3: Muon Lifetime and Coupling constant

Both the muon lifetime and the experimental coupling constant differ by less than 1σ from the theoretical values. All in all the results are in good agreement with the theoretical values, taking into account that our measurement was only done for 2 consecutive days and not a longer period of time.

In section 3.3 we calculated the magnetic moment and Larmor frequency of the muon. The results are displayed in table 4.

	ω_{Larmor}	μ_{Muon}
exp. value	$3.37(25)_{stat}(55)_{sys} MHz$	$2.77(50)\frac{10^{-7}eV}{T}$
theo. value	$3.4111 MHz$	$2.8065\frac{10^{-7}eV}{T}$
difference	1σ	1σ

Table 4: Larmor frequency and Magnetic Moment

These results are very satisfying since they both differ less than 1σ from their theoretical values.

In this section we also get a value for ϕ_0. It is $(-2.26 \pm 0.52)^\circ$, therefore it is close to 0. This was to be expected as positrons prefer to be right-handed. So their direction of flight matches their spin, which has to match the spin of the muon.

From the fit in figure 3 we got a value for $P \cdot A$, from which we calculated the value for P. The value we obtained is close to zero, its within a 3σ range of zero. This is not what we expected, since theoretically cosmic muons should be polarized. We expected values at about $P = 0.33$ (from pion decay) and $P = 0.54$ (from kaon decay).

References

[1] F13 - Measurement of Muon Properties in the Advanced Student Laboratory, URL http://www.physi.uni-heidelberg.de/~bachmann/Lehre/F13_Instruction.pdf, version 1.0 (21.3.2013)