First order perturbative treatment of the cosmic density-fluctuation power spectrum in the Zel'dovich approximation

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Central question

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Linear evolution of the density power spectrum

$$G^{\text{Linear}}_{\rho\rho,\,\text{SPT}}(k,\tau) = G^{(0),\text{Linear}}_{\rho\rho} \tag{1}$$

$$\propto D_+^2(\tau) P_\delta^{(\text{i})}(k).$$

Full density power spectrum to first order in the perturbations

$$G^{(1)}_{\rho_1\rho_2} = G^{(0)}_{\rho_1\rho_2} + \delta\, G_{\rho_1\rho_2}. \tag{2}$$

Kinetic Field Theory

Kinetic Field Theory in a nutshell

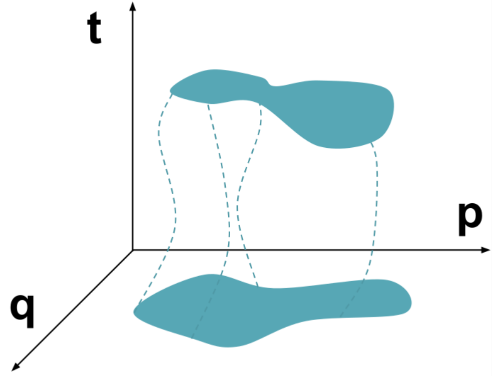


Figure 1: The phase-space evolution of a statistical system made up of N particles.

Encoded in the canonical generating functional

$$Z_C = \int \mathrm{d}\boldsymbol{x}^{(\mathrm{i})}\, \mathcal{P}(\boldsymbol{x}^{(\mathrm{i})}) \int_{\boldsymbol{x}^{(\mathrm{i})}} \mathcal{D}\boldsymbol{x}(t)\; \delta_D \left[\boldsymbol{x} - \boldsymbol{x}_{\mathrm{cl}}(\boldsymbol{x}^{(\mathrm{i})})\right].$$

The Zel'dovich approximation

Equations of motion

$$\begin{aligned} \vec{q}\,' &= \vec{p}_{\mathrm{zel}} \\ \vec{p}\,'_{\mathrm{zel}} &= -\frac{1}{g(a)}\,\vec{\nabla}_q V \underbrace{-\frac{g'(a)}{g(a)}\,\vec{p}_{\mathrm{zel}}}_{\text{drag force}}, \end{aligned} \tag{3}$$

Account for drag force in particle interactions

$$S_{\mathrm{I}} = S_{\mathrm{V}} + S_{\mathrm{D}}, \tag{4}$$

The Zel'dovich approximation within Kinetic Field Theory

The canonical generating functional

$$Z_C[\boldsymbol{J}, \boldsymbol{K}, \mathrm{H}] = e^{i\hat{S}_I} \, e^{i\,\mathrm{H}\cdot\hat{\Phi}} \, Z_{C,0}[\boldsymbol{J}, \boldsymbol{K}]. \tag{5}$$

Perturbative corrections to the density power spectrum

$$\delta G_{\rho_1\rho_2} = \delta G^{\mathrm{V}}_{\rho_1\rho_2} + \delta G^{\mathrm{D}}_{\rho_1\rho_2}. \tag{6}$$

The density power spectrum to first order in the perturbations

The drag cumulant

$$\delta G^{\mathrm{D}}_{\rho_1\rho_2} = \delta G^{\mathrm{D},1}_{\rho_1\rho_2} + \delta G^{\mathrm{D},2}_{\rho_1\rho_2}.$$

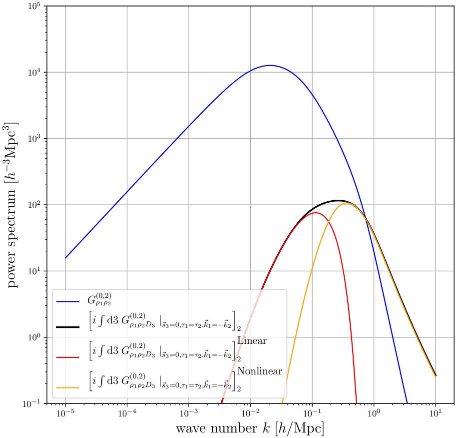


Figure 2: Comparison of the first contribution to the drag cumulant with the free density cumulant.

The drag cumulant

$$\delta G^{\mathrm{D}}_{\rho_1\rho_2} = \delta G^{\mathrm{D},1}_{\rho_1\rho_2} + \delta G^{\mathrm{D},2}_{\rho_1\rho_2}.$$

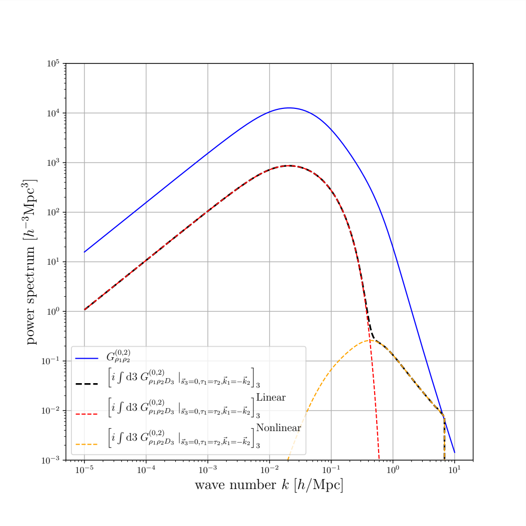


Figure 3: Comparison of the second contribution to the drag cumulant with the free density cumulant.

Comparison of drag and interaction cumulant

$$- \delta G^{\mathrm{D,Linear}}_{\rho_1 \rho_2} \approx \delta G^{\mathrm{V,Linear}}_{\rho_1 \rho_2} .$$

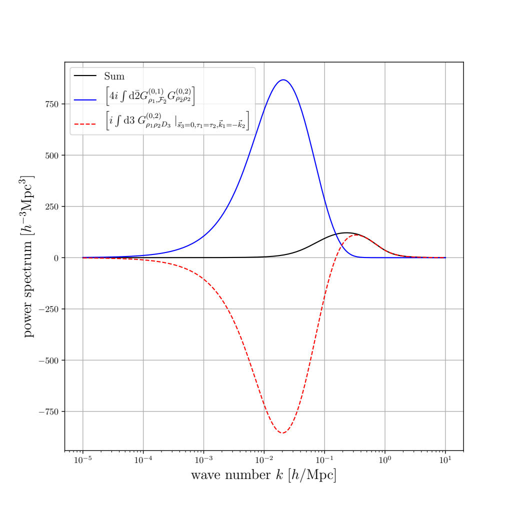


Figure 4: Comparison of the interaction and the full drag cumulant.

The full first order perturbed density power spectrum

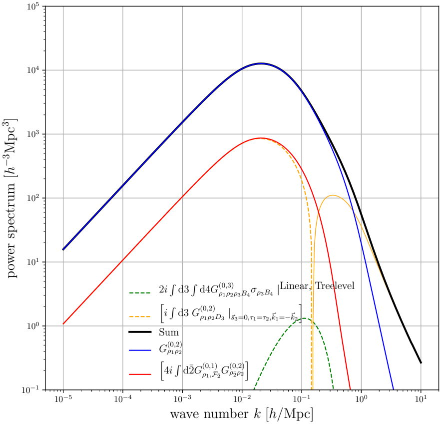


Figure 5: Free and first order perturbative contributions to the density-fluctuation power spectrum.

Conclusion

Conclusion

1. KFT employing a Zel'dovich approximation reproduces the growth behaviour obtained from conventional approaches in the **linear** regime.

2. Qualitatively, the behaviour in the **non-linear** regime of the ΛCDM density power spectrum is recreated.

3. It remains to be seen whether this result holds true in next-to-leading order perturbation theory.

Final density power spectrum

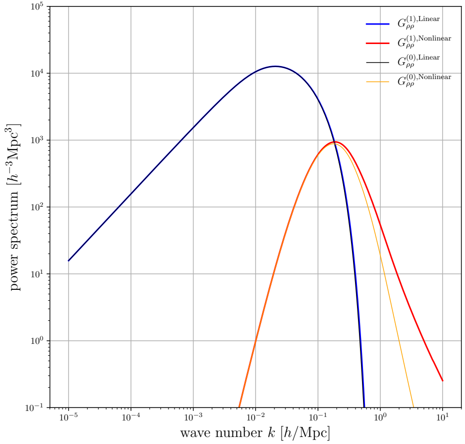


Figure 6: Linear and non-linear contributions to the density power spectrum to zeroth and first order in the perturbations.

The drag field

Drag field in Fourier space

$$S_D = \int \mathrm{d}1 \ \sum_{j=1}^{N} \vec{\chi}_{p_j}(\tau_1)\delta_D(\vec{x}_1 - \vec{x}_j(\tau_1)) \ \underbrace{\frac{g'}{g}(\tau_1)}_{:=A_D(\tau_1)} \ \vec{p}_1 =: \int \mathrm{d}1 \ \Phi_D(1)$$

$$\begin{aligned}\hat{\Phi}_D(1) &= \sum_{j=1}^{N} A_D(\tau_1)(2\pi)^6 \delta_D(\vec{k}_1) \left(\frac{\partial}{i\partial \vec{l}_1} \delta_D(\vec{l}_1) \right) \hat{\vec{\chi}}_{p_j}(\tau_1) \hat{\Phi}_{f_j}(1) \\ &=: \sum_{j=1}^{N} \hat{d}_j(1) \hat{\Phi}_{f_j}(1),\end{aligned}$$

The interaction or non-mode-coupling cumulant

Symmetric contribution of the non-mode-coupling cumulants

$$4i \int \mathrm{d}\bar{2}\ G^{(0,1)}_{\rho_{\bar{1}}\mathcal{F}_{\bar{2}}} G^{(0,2)}_{\rho_{\bar{2}}\rho_2}$$

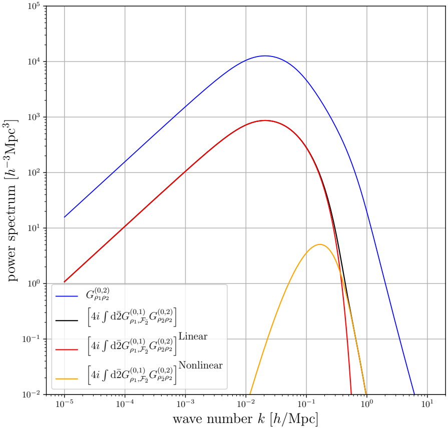
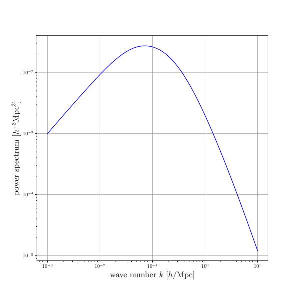


Figure 7: Linear and non-linear contributions in the initial power spectrum to the interacting cumulant.

The initial power spectrum

Bardeen, Bond, Kaiser and Szalay (BBKS)

$$P_\delta^{(\mathrm{i})}(k) := P_\delta(\vec{k}, t_i) \propto \begin{cases} k^{n_s} & (k \ll k_{eq}) \\ k^{n_s - 4} & (k \gg k_{eq}) \end{cases} \qquad \Rightarrow P_\delta^{(\mathrm{i})}(k) \propto k^{n_s}\, T^2(k),$$



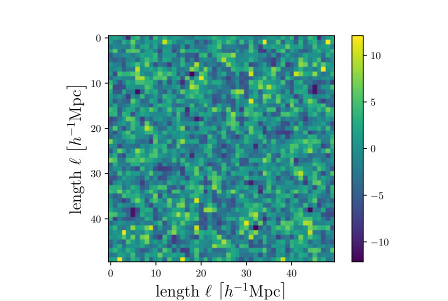


Figure 8: Left: Initial power spectrum by Bardeen et al., Right: Initial configuration in real space for EdS.

Perturbations in the radiation-dominated era

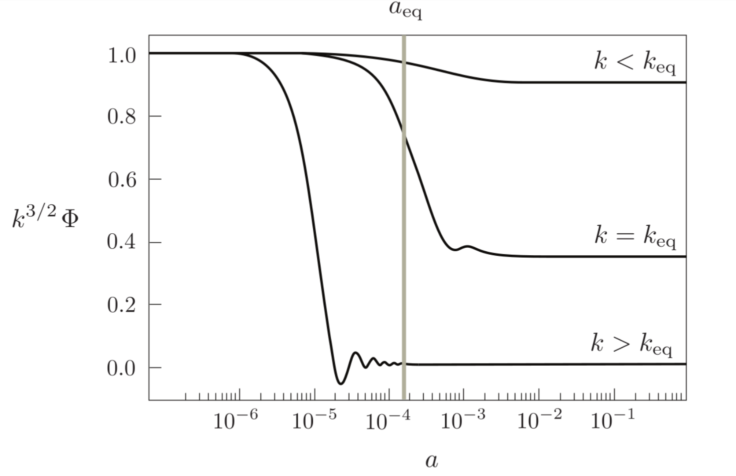


Figure 9: Numerical solutions for the linear evolution of the gravitational potential.

Source: Baumann Cambridge Lecture **Cosmology** Mathematical Tripos III

Measurements

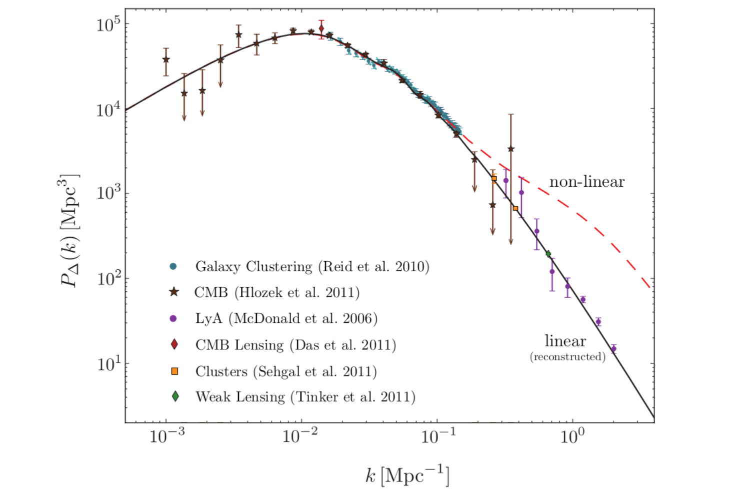


Figure 10: Collection of measurements of the ΛCDM matter power spectrum.

Source: Baumann Cambridge Lecture **Cosmology** Mathematical Tripos III

On the calculation of cumulants

$$G_{ab} = \hat{H}_a \hat{H}_b \ln Z_C[H, J, K]_0 = \hat{H}_a \left(\frac{1}{Z_C} e^{i\hat{S}_I} W_b^{(0)} e^{W^{(0)}} \right)_0 \tag{7}$$

$$= \frac{1}{Z_C[0]} e^{i\hat{S}_I} \left(W_{ab}^{(0)} + W_a^{(0)} W_b^{(0)} \right) e^{W^{(0)}} \ |_0$$
$$- \frac{1}{Z_C^2[0]} \left(e^{i\hat{S}_I} W_a^{(0)} e^{W^{(0)}} \right) \left(e^{\hat{S}_I} W_b^{(0)} e^{W^{(0)}} \right)_0 \tag{8}$$

Then use the following relation after applying all neccessary derivatives to identify cumulants:

$$W_{a_1 \ldots a_n}^{(0)} \ |_0 = G_{a_1 \ldots a_n}^{(0)} \tag{9}$$