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Series Resistor Networks

Discussion Overview

Nodes, Branches and Series Circuits

In a circuit, the point where two or more components come together (are connected together) is called a node. The connection between two nodes via a component, on the other hand, is called a branch. Note that two nodes connected via a wire do not constitute a branch. A branch must contain one and only one component between two nodes.

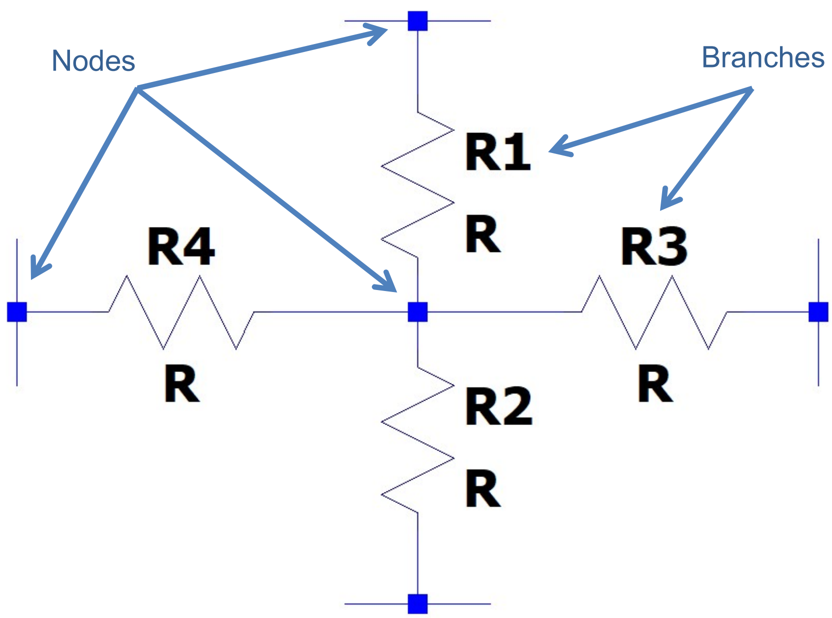

Figure 1 - Nodes and Branches

A series circuit is defined as a single loop in which all components are arranged in a daisy-chain fashion. The current in a series circuit is the same for all the branches in the loop. This property can be arrived at intuitively by observing that if the current exiting a branch is different than the current entering the next branch, it would mean that the current is either "pooling up" at the node between the branches, or it is leaking out. As this is clearly not the case, the currents in each branch of a series circuit must be the same.

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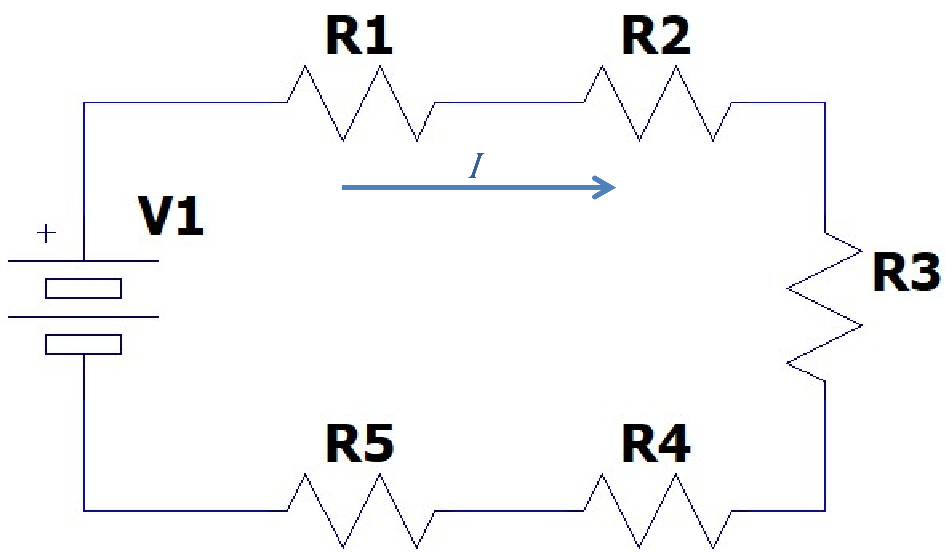

Figure 2 - Example of a series circuit

The current in the series circuit may be found by dividing the total voltage applied across the entire circuit by the total resistance of the circuit.

$$I = \frac{V_{source}}{R_T} \qquad \text{Eq. 1}$$

Equivalent Resistance of a Series Resistive Network

There are times that a designer needs to combine two or more resistors to create a resistance that is larger than the individual resistances used in the combination. In order to study how this can be achieve, let us recall the formula for determining the resistance of a wire

$$R = \rho \frac{l}{A}$$

One can see that one way of increasing resistance of a wire is by increasing its length. Therefore, If we have two pieces of long wires with equal cross section areas A and lengths l_1 and l_2, we can create a larger resistance by connecting the two lengths of wire in series. The total resistance of the resulting wire would then be given by

$$R_T = \rho \frac{l_1 + l_2}{A}$$

Working through the math, one can see that the result is the same as adding the resistances of the individual resistors:

$$R_T = \rho \frac{l_1 + l_2}{A} = \rho \frac{l_1}{A} + \rho \frac{l_2}{A} = R_1 + R_2$$

This formula can be easily extended to 3 or more resistors.

$$R_T = R_1 + R_2 + R_3 + \cdots + R_n = \sum_{i=1}^{n} R_i \qquad \textit{Eq. 2}$$

Kirchhoff's Voltage Law (KVL)

Gustav Robert Kirchhoff was a German Physicist who lived in 1824-1887 and contributed to the fundamental understanding of electrical circuits. One of the properties of electrical circuits studied by Kirchhoff was the relationship between all the voltage drops across the components in a series circuit and the total voltage applied to the circuit known as Kirchhoff's Voltage Law (KVL).

Kirchhoff's Voltage Law states that the sum of all the voltage "drops" across the components in a series circuit is equal to the total voltage difference across the entire circuit. This can be easily shown by making the following observations. Let's use the circuit in Figure 2 as our example.

- We would like to determine the sum of all voltage drops in the circuit:

  $$V_1 + V_2 + V_3 + V_4 + V_5$$

- We know that the current I flowing through each component in a series circuit is the same for all the components. Therefore, using Ohm's Law ($V = IR$), the sum of voltage drops can be written as

  $$V_1 + V_2 + V_3 + V_4 + V_5 = IR_1 + IR_2 + IR_3 + IR_4 + IR_5 = I(R_1 + R_2 + R_3 + R_4 + R_5)$$

- From Eq. 2, however, we know that the sum of the resistance in the series circuit is equal to the total or equivalent resistance R_T. Therefore,

  $$V_1 + V_2 + V_3 + V_4 + V_5 = I(R_1 + R_2 + R_3 + R_4 + R_5) = IR_T = V_T$$

- Therefore, the sum of all voltage drops across the individual components in a series circuit is equal to the total voltage drop across the equivalent resistance. The value of the total voltage drop is, however, equal to the value of the voltage source applied to the circuit. Therefore,

  $$V_T = V_1 + V_2 + V_3 + V_4 + V_5 = V_{source}$$

Another way of expressing KVL is in terms of voltage gains and voltage drops. Recall that voltage sources provide voltage gains while voltage drops happen through circuit components like resistors. Therefore, above equation can be re-written in a more general form for n components in the circuit as

$V_{gain} = V_{drop\_1} + V_{drop\_2} + V_{drop\_3} + \cdots + V_{drop\_n}$

Rearranging the equation above, we have

$V_{gain} - V_{dro\ \ 1} - V_{drop_2} - V_{drop_3} - \cdots - V_{drop_n} = 0$

This equation states that the sum of all voltage gains and voltage drops in a series circuit is equal to zero.

Finally, in a more generic form, treating all voltage gains as positive and all voltage drops as negative, KVL is expressed as

$V_1 + V_2 + V_3 + \cdots + V_n = \sum_{i=1}^{n} V_i = 0$ *Eq. 3*

Voltage Dividers

As discussed earlier, the total or equivalent resistance for a series resistive circuit is given by the sum of the individual resistances in that circuit (Eq. 2). Therefore, the current flowing in a series circuit can be found using Ohm's law as

$I = \frac{V_{source}}{R_T} = \frac{V_{source}}{\sum_{i=1}^{n} R_i}$ *Eq. 4*

Since in a series circuit, the current (I) flowing through the circuit is the same for all the components in that circuit, the voltage drop across each individual resistor may then be found using Ohm's Law. Multiplying the current I by the resistor's value, we have

$V_j = IR_j$

Substituting Eq. 4 for I above, we get

$$V_j = V_{source} \frac{R_j}{\sum_{i=1}^{n} R_i}$$

The technique used above to find the voltage drop across an individual resistor is called the voltage divider rule. The voltage divider rule states the voltage across any resistor (or combination of resistors) as the ratio of the resistance of interest to the total resistance.

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Schematics

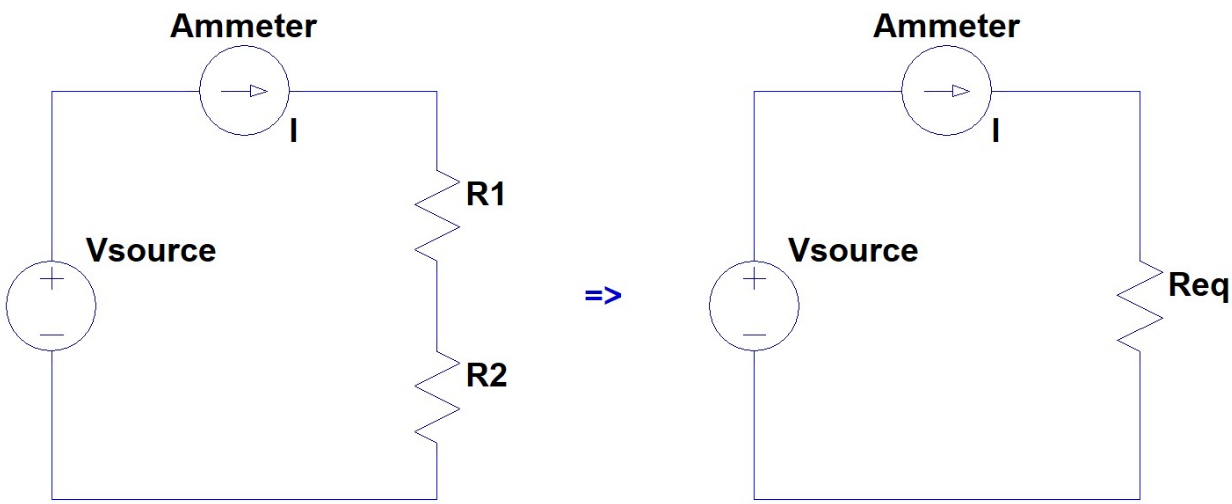

Figure 3 - Series Resistive Network

Procedure

A. Given the circuit shown in Figure 3, determine the equivalent resistance of the circuit for R_1 and R_2 with the values given in Table 1 below. (Note that for the cases where R_2 is either a photo-resistor or a thermistor, you will need to measure the resistance of R_2 under the conditions given in the table.)

B. Given the equivalent resistance, using Ohm's law ($I = \frac{V}{R}$) and V_{source} = 9V, determine the theoretical current in the circuit and record it in Table 1 below.

C. Given the calculated current in step B and using Ohm's law ($V = IR$), determine the voltage across R1 and R2. Record the values in Table 1.

D. Construct the resistor network on a breadboard. **Do not connect the voltage source at this point! Before** connecting the voltage source, measure the value of each resistor and that of the equivalent resistance. Record the equivalent resistance value in Table 1 and compare it to the theoretical value determined in step A.

E. Connect the source and multi-meter to measure the current. Record the value in Table 1 and compare it to the theoretical value determined in step B.

F. With another multi-meter measure the voltage across only R1 and then across only R2. Record the values in Table 1 and compare them to the theoretical values determined in step C.

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G. Record the sum of voltage drops measured in step F above in the Table.

Table 1 - Calculated and Measured Circuit Values

		Calculated				Measured						
R1 (Ω)	**R1 (Ω)**	**Req (Ω) = R1 + R2**	**I (A) = V_{source} / Req**	**V_{R1} (V) = I x R1**	**V_{R2} (V) = I x R2**	**R1 (Ω)**	**R2 (Ω)**	**Req (Ω) = R1 + R2**	**I (A)**	**V_{R1} (V)**	**V_{R2} (V)**	**V_{R1} + V_{R2} (V)**
1K	5.1K											
1K	Photo-resistor No Light											
1K	Photo-resistor Direct Light											
1K	Thermistor Ambient Temp											
1K	Thermistor Body Temp											

H. Answer the questions below

a. How do the measured currents compare to the calculated values and why?

b. How do the measured voltages compare to the calculated values and why?

c. How do the sum of the measured voltages compare to the sum of calculated voltages and why?