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Parallel Resistor Networks

Introduction

The focus of this exercise is an examination of basic parallel DC circuits with resistors. A key element is Kirchhoff's Current Law which states that the sum of currents entering a node must equal the sum of the currents exiting that node. The current divider rule will also be investigated.

Discussion Overview

Parallel Networks

A parallel circuit is defined by the fact that all components in the circuit share two common nodes as shown in Figure 3.

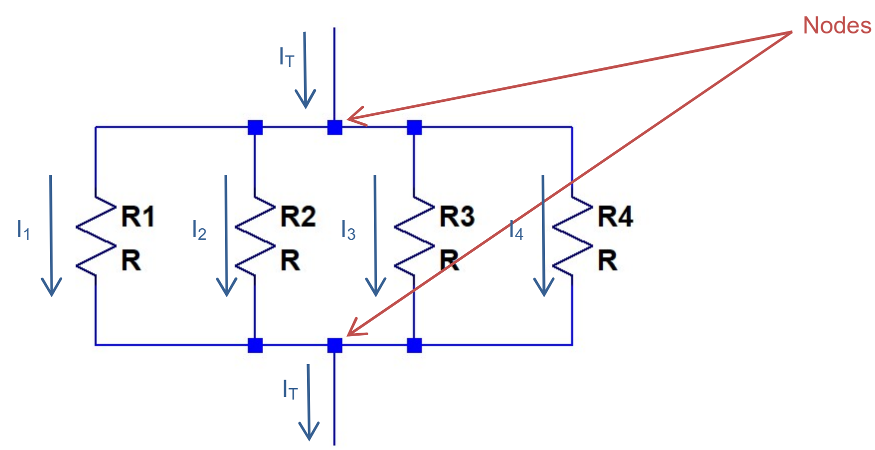

Figure 1 - Parallel Resistive Network

In a parallel circuit, the voltage across each individual component is the same as is equal to the voltage across the common nodes for the parallel network. For example, for the circuit shown in Figure 1,

$V_{R1} = V_{R2} = V_{R3} = V_{R4}$

Kirchhoff's Current Law

Another characteristic of a parallel network is that the sum of the currents in the individual branches in the network is equal to the total current entering or exiting the network. As shown in Figure 1,

$I_T = I_1 + I_2 + I_3 + I_4$

In general, the behavior of currents at a node is called Kirchhoff's Current Law (KCL) which states that the sum of all currents entering and exiting a node is equal to zero.

$$\sum_{i=1}^{n} I_i = 0$$

For example, if we denote all the currents entering a node as positive and all the currents exiting a node as negative, for the top node in the circuit in Figure 1,

$$I_T - I_1 - I_2 - I_3 - I_4 = 0$$

Equivalent Resistance of Parallel Resistive Networks

In order to determine the equivalent resistance of a parallel network, we examine the circuit in Figure 3.

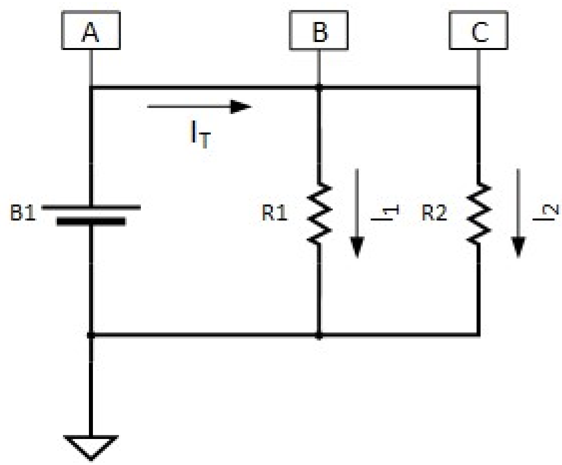

Figure 2 - A Two Resistor Parallel Circuit

We note that the voltage across both resistors is the same and is equal to the applied voltage source.

$$V_A = V_B = V_C \qquad \text{Eq. 1}$$

The total supplied current, on the other hand, may be found by dividing the voltage source by the equivalent or total parallel resistance.

$$I_T = \frac{V_A}{R_T} \qquad \text{Eq. 2}$$

The total current may also be found by summing the currents in all of the branches.

$$I_T = I_1 + I_2 \qquad \text{Eq. 3}$$

The individual currents through each resistive branch may be found by dividing the voltage across the resistor by the resistor value.

$I_1 = \frac{V_B}{R_1}$ and $I_2 = \frac{V_C}{R_2}$ *Eq. 4*

Combining Eq. 2 - Eq. 4, we have

$$\frac{V_A}{R_T} = \frac{V_B}{R_1} + \frac{V_C}{R_2}$$

However, from Eq. 1, we have $V_A = V_B = V_C$. Therefore,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

In general, for n resistors connected in parallel, the total or equivalent resistance is found by using the following equation.

$$\frac{1}{R_T} = \sum_{i=1}^{n} \frac{1}{R_i}$$

Current Dividers

Using Ohm's Law, we can find the total current by dividing the voltage across the parallel network by the total or equivalent resistance.

$$I_T = \frac{V_T}{R_T} = V_T \frac{1}{R_T} = V_T \sum_{i=1}^{n} \frac{1}{R_i} \qquad \textit{Eq. 5}$$

The voltage across the parallel network, on the other hand, can be found by multiplying the current in a single branch by the resistance of that branch. Therefore,

$$V_T = I_j R_j$$

Substituting above equation into Eq. 5 for V_T, we have

$$I_T = I_j R_j \sum_{i=1}^{n} \frac{1}{R_i}$$

Or,

$$I_j = \frac{I_T}{R_j \sum_{i=1}^{n} \frac{1}{R_i}} = \frac{I_T}{R_j \frac{1}{R_T}} = I_T \frac{R_T}{R_j} \qquad \textit{Eq. 6}$$

Therefore, the currents in a parallel circuit are inversely proportional to the associated resistances. More precisely, the current of each branch is equal to the total current times the ratio of the total resistance to the resistance of that branch.

For a two resistor parallel circuit, this states that the current through one resistor is equal to the total current times the ratio of the other resistor to the total resistance. To see this, recall that the total resistance for a two resistor parallel circuit is given by

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

Substituting this equation for R_T in Eq. 6, we have

$$I_1 = I_T \frac{1}{R_1} \frac{R_1 R_2}{R_1 + R_2} = I_T \frac{R_2}{R_1 + R_2}$$

Schematics

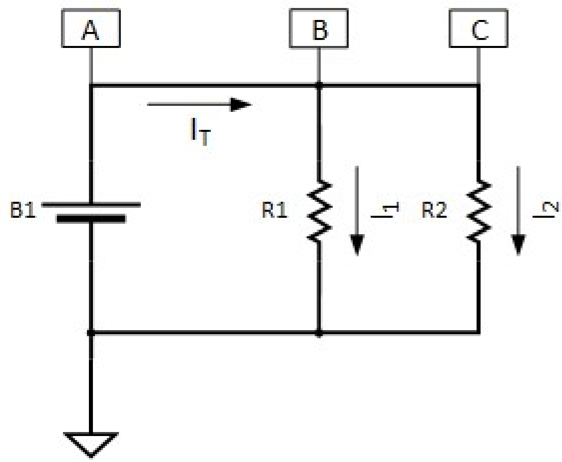

Figure 3 – Two Resistor Parallel Resistive Circuit

Procedure

A. Determine the equivalent resistance for the parallel resistive network shown in Figure 3 and for the values of R_1 and R_2 given in Table 1 below. (Note that for the case where R_2 is a thermistor, you will need to measure the resistance under the conditions given in the table.)

B. Given $V_{B1} = 9V$, determine the voltages at points A, B and C with respect to ground:

$V_A =$ ______________V

$V_B =$ ______________V

$V_C =$ ______________V

C. Given the equivalent resistance found in A and using Ohm's law ($I = \frac{V}{R}$), determine the theoretical total current (I_T), the current through R_1 (I_1) and the current through R_2 (I_2). Record the values in Table 1 below.

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D. Construct the resistor network on a breadboard. **<u>Do not connect the battery at this point!</u>** **<u>Before</u>** connecting the battery, measure the value of each resistor and the equivalent resistance. Record the equivalent resistance value in Table 1 and compare it to the theoretical value determined in step A.

E. Connect the battery and multi-meter to measure the voltages at points A, B and C with respect to ground. (Note that the black lead of the multi-meter should be connected to the ground reference point and the red lead should be connected to points A, B or C respectively.) Record the values below and compare to theoretical values in step B.

$V_A =$ ____________V

$V_B =$ ____________V

$V_C =$ ____________V

F. Using the multi-meter, measure the total current (between points A and B), the current through R_1 (between point B and R_1), and the current through R_2 (between point C and R_2). (Note that current is measured at a single point in the circuit. The multi-meter will need to be inserted between point B and R_1, for example, to measure I_1. The red lead of the multi-meter should be connected to where the current is entering the meter and the black lead connected to the point where the current is exiting.) Record the values in Table 1 and compare them to the theoretical values determined in step C.

Table 1 - Calculated and Measured Circuit Values

		Calculated				Measured		
R_1 (Ω)	R_2 (Ω)	R_T (Ω)	$I_T = \frac{V_{B1}}{R_T}$	I_1 (A)	I_2 (A)	I_T (A)	I_1 (A)	I_2 (A)
1K	2K							
100K	Thermistor Room Temp =							
100K	Thermistor Iced =							

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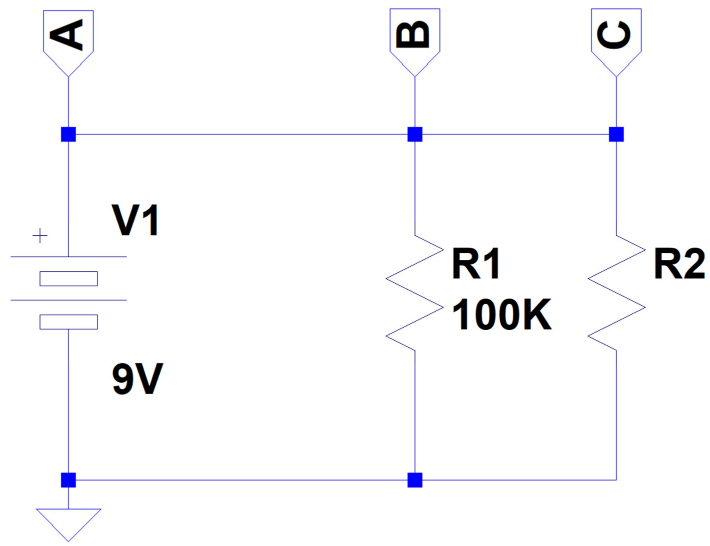
G. Enter the circuit below in LTSpice



a. Make the value of R2 a parameter.

b. Enter a SPICE directive to step the value of R2 from 80K to 480K in steps of 80K.

c. Enter another SPICE directive to run a transient (time) simulation from 0 to 1ms stepping at 1us. The initial voltage value should be set to 0. (Recall that this was done by adding the “startup” parameter to the .tran SPICE directive.)

d. Run the simulation, and record the values listed in Table 2 below for R2 = 80K and 480K.

Table 2 Parallel Resistor Network SPICE Simulation Results

R2	**V_A (V)**	**V_B (V)**	**V_C (V)**	**I_T (A)**	**I_1 (A)**	**I_2 (A)**
80K						
480K						