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Sinusoidal Waveforms Worksheet

Introduction

In this worksheet, we will learn how sinusoidal waveforms are used as the basic mathematical representation of AC electrical waveforms. As we transition from DC circuits to studying AC circuits, we develop the mathematical understanding and tools to analyze such circuits. We will learn that the electricity generated and used at homes are represented by sine waves, and as we learn about other forms of periodic waveforms (e.g. square or triangular waves), we learn that any periodic signal can be written in terms of sum of sinusoidal waves.

In order to gain a better understanding of sinusoidal waves, we start with an introduction to polar representation of angles and trigonometric functions such as sine and cosine. We will learn about the unit circle and how sine and cosine are represented on a unit circle.

Discussion Overview

Polar Representation of Angles

You might recall that the length of an arc is found by multiplying the radius of the arc by the angle of the arc:

$$l_{arc} = \theta r$$

If we extend the arc to a full circle, then the length of the arc is the circumference of the circle:

$$C = 2\pi r$$

Comparing the two equations above, one can see that the angle represented by a full circle is $\theta = 2\pi$. This is called the polar representation of an angle where $360^o=2\pi$, $180^o=\pi$, $90^o=\pi/2$, $45^o=\pi/4$, $30^o=\pi/6$...

Trigonometric Functions and Unit Circle

In trigonometry, *sin(θ)* is defined as the ratio of the side the opposite of the angle θ to the hypotenuse in a right triangle. *cos(θ)*, on the other hand, is defined as the ratio of the side adjacent to the angle θ to the hypotenuse in a right triangle.

One can draw a unit circle where the y and x coordinates of any point on the unit circle would represent the sine and cosine of the angle that the line connecting the point on the unit circle to the origin would make with the x axis.

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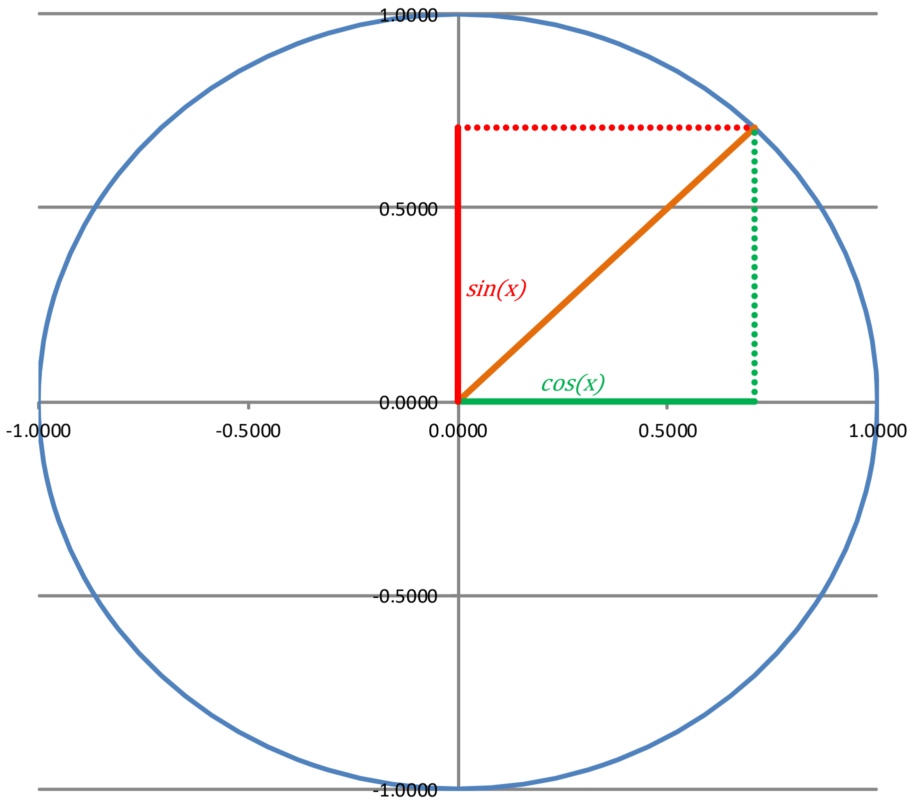
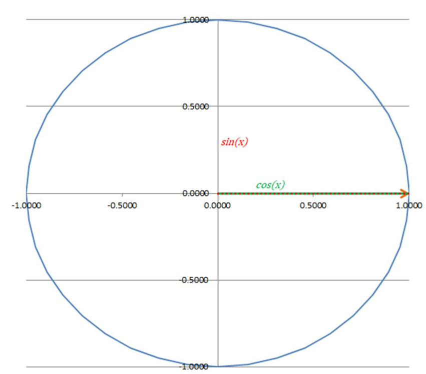


Figure 1 - Unit Circle

Table below shows the values of sin(θ) & cos(θ) for a few different values of angle θ.

Table 1 - Examples of sin(θ) and cos (θ)

θ	**sin(θ)**	**cos(θ)**	
0	0	1	

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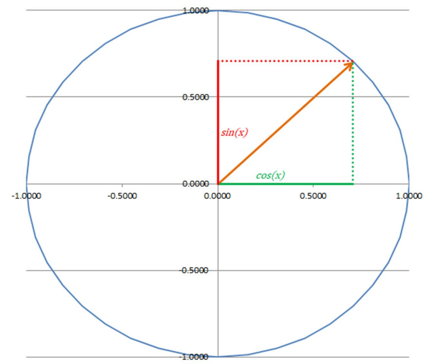
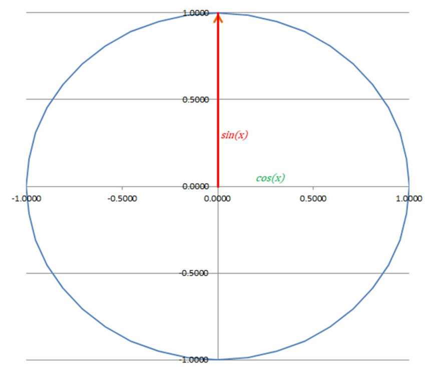
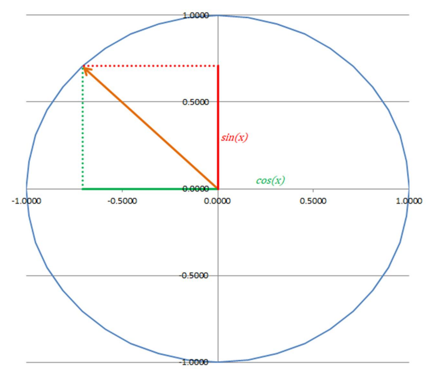
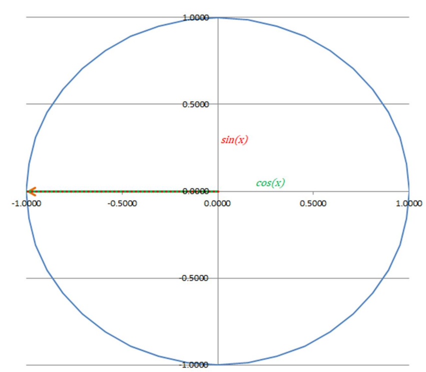
θ	sin(θ)	cos(θ)	
$\pi/4$	$\sqrt{2/2}$	$\sqrt{2/2}$	
$\pi/2$	1	0	
$3\pi/4$	$\sqrt{2/2}$	$-\sqrt{2/2}$	
π	0	-1	

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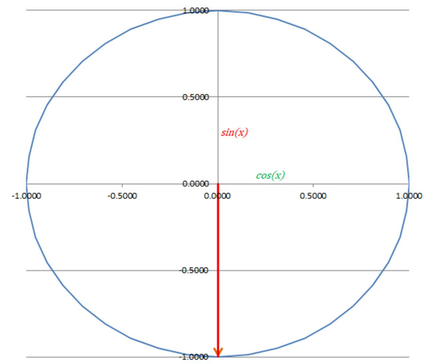
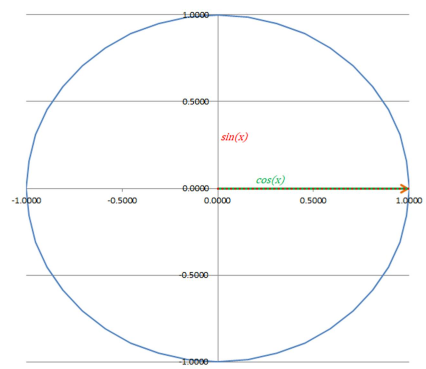
θ	sin(θ)	cos(θ)	
3π/2	-1	0	
2π	0	1	

Sinusoidal Waveforms

A sine function is a representation of a wave with a frequency and amplitude:

$v(t) = Asin(2\pi f t) = Asin\left(\frac{2\pi}{T}t\right)$, where

A = Amplitude of the signal
$f = 1/T =$ Frequency of the signal
$T =$ Period of the signal
$\omega = 2\pi f$ = Angular frequency
$t =$ Time

A sine wave with a higher frequency (cycles per second) has a shorter period and more number of cycles per second. A sine wave with a lower frequency, on the other hand, has a longer period and a fewer number of cycles per second. One can plot a sine wave in the time domain as a repeating wave with certain amplitude. The figure below is a plot of the function $v(t) = sin(2\pi \cdot 100 \cdot t)$

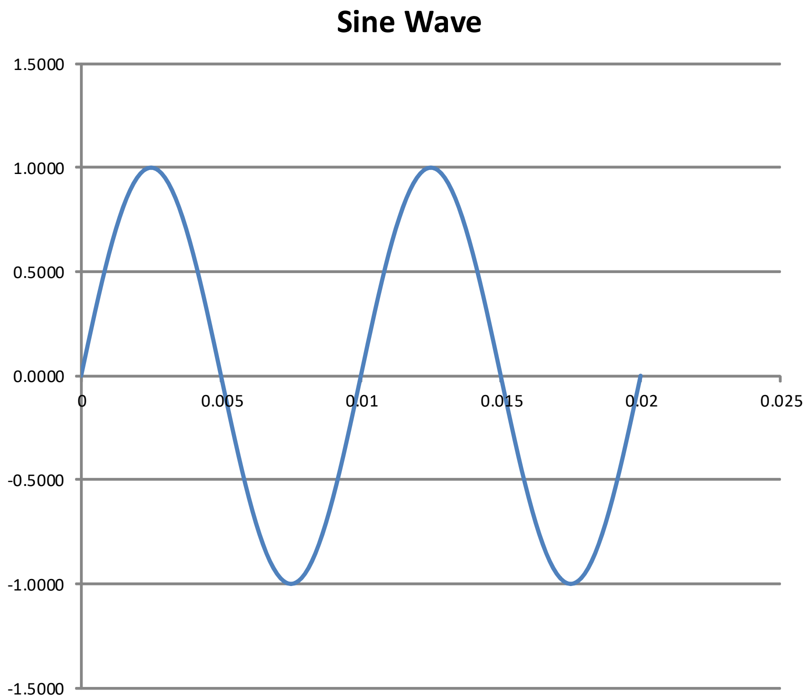


Figure 2 - $v(t) = sin(200\pi t)$

A sine wave can also be represented in the frequency domain as a single “spike” with an amplitude as shown below.

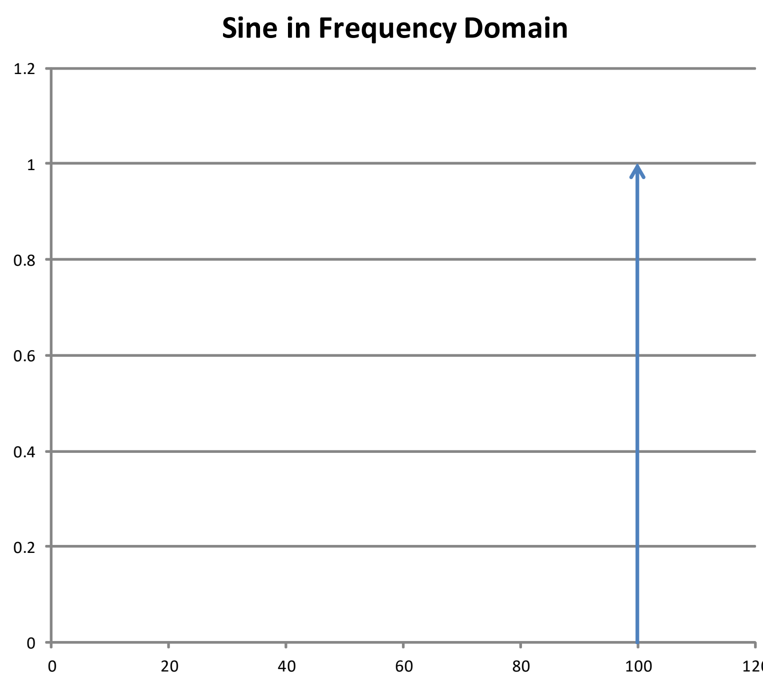


Figure 3 - Frequency Domain Representation of $v(t) = sin(200\pi t)$

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Procedure

A. In the table below, give the amplitude, frequency and period with the proper unit for the sinusoidal waveforms given in each row.

Table 2 - Sinusoidal Waveforms Exercise

Waveform	Amplitude	Frequency	Period
$v(t) = 120\, sin(2\pi \cdot 60 \cdot t)$			
$i(t) = 0.01\, sin(120\pi t)$			
$v(t) = 5sin\left(\frac{2\pi}{0.01}t\right)$			
$i(t) = sin\left(\frac{2\pi}{1e6}t\right)$			

B. In the table below, give the equation for the sinusoidal waveforms given the amplitude, frequency or period specified in each row.

Table 3 - Sinusoidal Wavforms Exercise

Amplitude	Frequency	Period	Waveform
10V	*150Hz*		
5ma	*500Hz*		
1000V		*1ms*	
15μa		*5μs*	