Name: ________________________

Square Wave Spectrum

Introduction

Understanding how sum of sinusoids can be used to represent any signal is one of the fundamental topics in electrical engineering. However, the math skills required to derive the general equations for decomposing signals into their sinusoidal components is beyond the scope of this class. Instead, we will use a square wave to demonstrate the principles of this technique and its usefulness.

Discussion Overview

It can be shown mathematically that any signal can be broken into sum of sinusoidal signals. A plot of the sinusoidal components of a signal in the frequency domain is called the spectrum of the signal. For non-periodic signals (signals that do not repeat), there could be an infinite number of sinusoidal components in the sum. In this case, the spectrum of the signal is a continuous curve. For example, a square pulse in the time domain has an infinite continuous frequency spectrum as shown below.

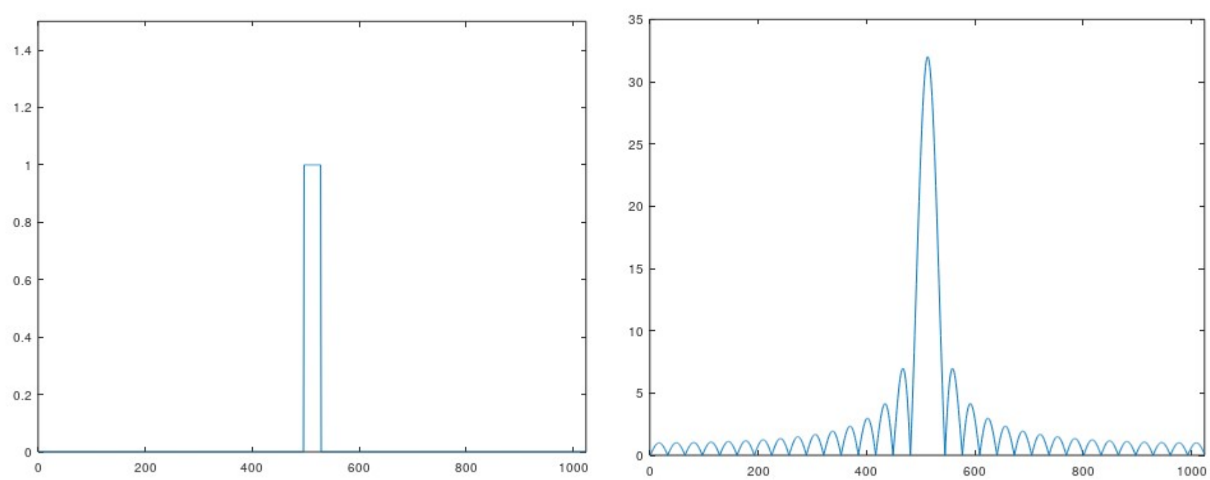

Figure 1 - Square Pulse Time and Frequency Plots

For periodic signals, on the other hand, the spectrum consists of individual discrete frequencies. The sinusoidal sum of periodic signals could still contain an infinite number of frequencies, but the frequencies are discrete. A square wave is an example of a periodic signal with an infinite number of discrete frequencies. A square wave with a fundamental frequency of f_o can be synthesized by the infinite sum below

$$V_{sq} = \frac{4}{\pi}\sum_{i=1}^{\infty}\frac{1}{(2i-1)}sin\big((2i-1)2\pi f_o t\big)$$

© 2019 Ramz Academy LLC
Revision Date: May 17, 2019

Name: ________________________

A spectrum plot showing the first four frequency components of a square wave with $f_o = 100Hz$ is shown below.

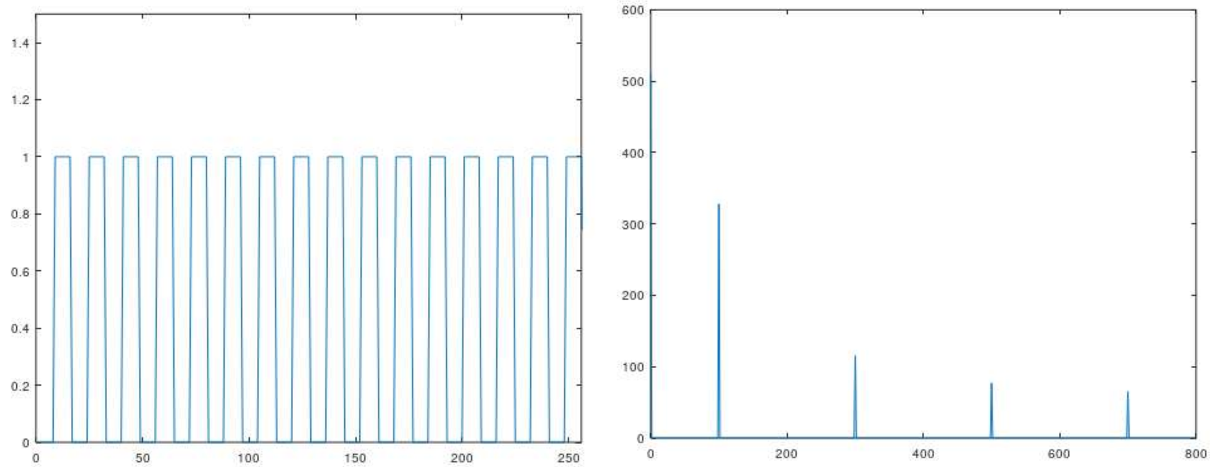

Figure 2 - Square Wave Time and Frequency Plots

We can examine how each frequency component contributes to forming the square wave by adding one component at a time. The plot for the first component $V_{sq} = \frac{4}{\pi} sin(2\pi f_o t)$ is shown below

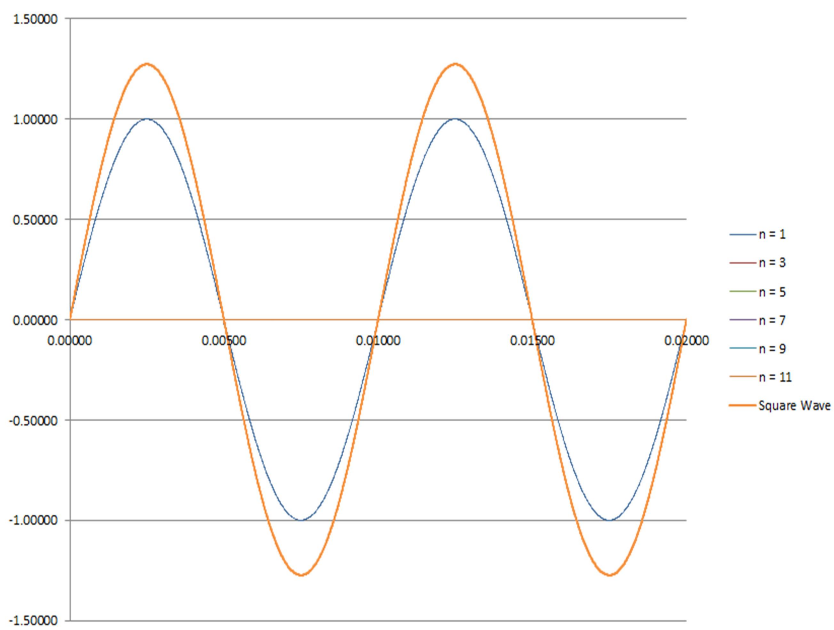

Figure 3 - Sum of 1 Term (Fundamental Frequency)

© 2019 Ramz Academy LLC

The following plot shows the resulting plot after adding the second component:

$$V_{sq} = \frac{4}{\pi}\left(sin(2\pi f_o t) + \frac{1}{3}sin(3 \times 2\pi f_o t)\right)$$

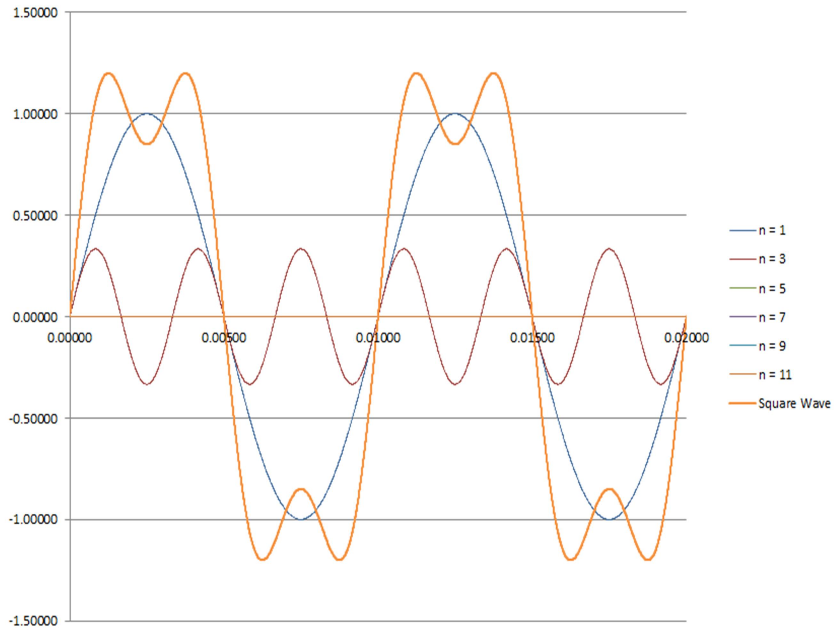

Figure 4 - Sum of 2 Terms

Adding the third component will result in the following plot:

$$V_{sq} = \frac{4}{\pi}\left(sin(2\pi f_o t) + \frac{1}{3}sin(3 \times 2\pi f_o t) + \frac{1}{5}sin(5 \times 2\pi f_o t)\right)$$

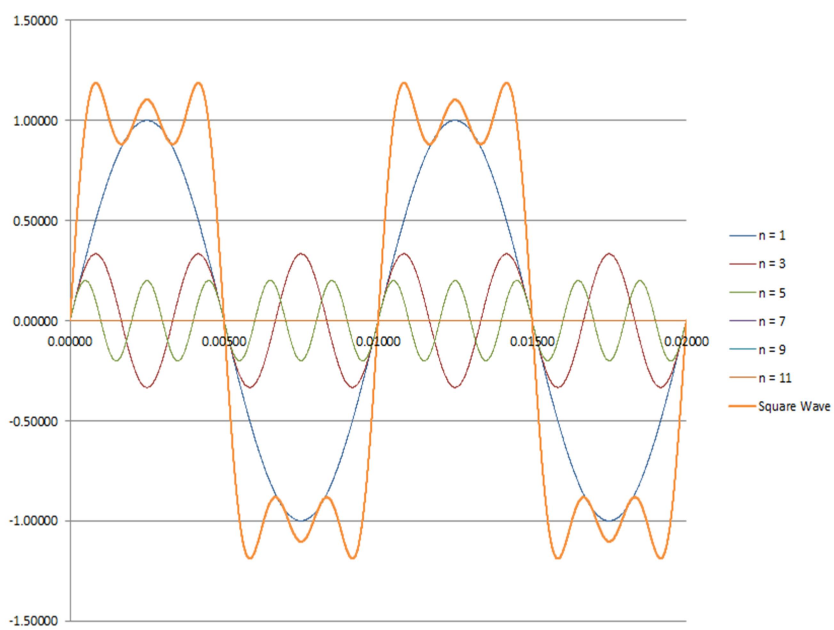

Figure 5 - Sum of 3 Terms

© 2019 Ramz Academy LLC

As it can be seen, adding each new frequency component makes the sum get closer to a square wave. Below is the plot of the sum with 25 components.

$$V_{sq} = \frac{4}{\pi}\sum_{i=1}^{25}\frac{1}{(2i-1)}sin\big((2i-1)2\pi f_o t\big)$$

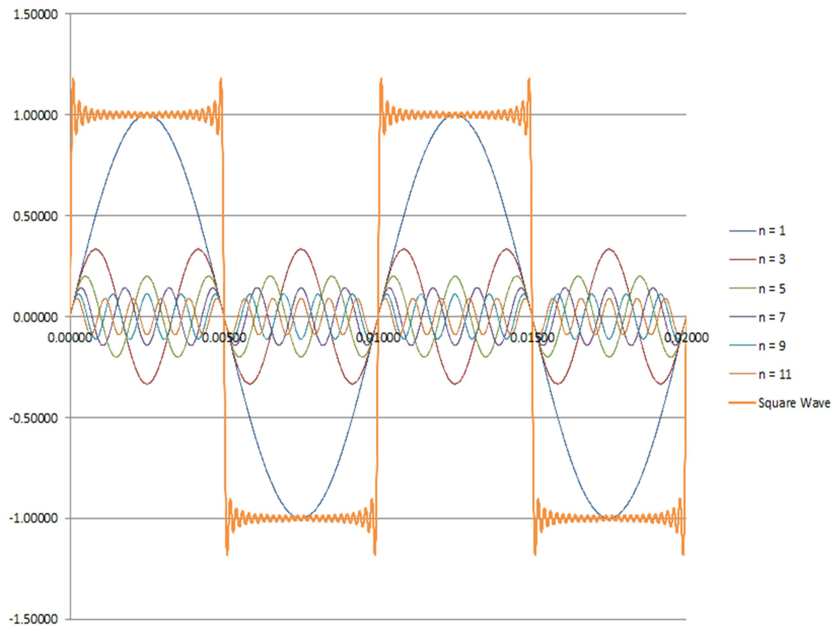

Figure 6 - Sum of 25 Terms

Procedure

In this section, you will first build a simple circuit in SPICE to observe the spectrum of a square wave using the FFT view function. Next, you will repeat the experiment using a function generator and an oscilloscope. You will use the “Math” function of the oscilloscope to examine the spectrum of the square wave generated by the function generator.

Simple SPICE Circuit

A. Build a circuit consisting of a voltage source and a resistive load. Set the resistive load to 1KΩ, and configure the voltage source as a pulse with the following settings:
   a. Initial Voltage .... -1V
   b. On Voltage................ 1V
   c. Time Delay................ 5ms
   d. Rise Time 1us
   e. Fall Time................... 1us
   f. On Time.................... 5ms
   g. Period 10ms

© 2019 Ramz Academy LLC

Name: ________________________

B. Setup the model to run a "transient" simulation with the following parameters
    a. Starting the print at time 0,
    b. Ending at time 100ms,
    c. Starting capture at time 0ms,
    d. With a maximum simulation step size of 1ms

    Below is the syntax for your reference:

    .tran <Tprint> <Tstop> [<Tstart> [<Tmaxstep.]] [<options>]

C. Run the simulation and display the waveform for the voltage across the resistive load.

D. What are the period and frequency of the square waveform?

$T_o =$ ________________s

$f_o =$ ________________Hz

E. Right click on the waveform window and select "View → FFT".
    a. Set the Window Function to "(none)" and click "OK".

F. Once the spectrum is displayed, right click on the vertical axis and select Linear".

G. Right click on the Horizontal axis and set the "Right" limit to 5Khz.

H. What is the value of the fundamental frequency?

$f_o =$ ________________Hz

I. How does this value compare to the one you measured in step C?

J. What are the frequencies of the 2^{nd}, 3^{rd} and 4^{th} components?

$f_1 =$ ________________Hz

$f_2 =$ ________________Hz

© 2019 Ramz Academy LLC

$f_3 =$ ________________ Hz

Lab Build and Measurements

In this section, you will examine the spectrum of the square wave generated by the function generator on the oscilloscope.

A. Set the waveform generator on your oscilloscope to a square wave with the following parameters:
   a. Amplitude = 1V ($V_{pk-pk} = 2V$)
   b. Offset = 0V
   c. Frequency = 100Hz

B. Connect the probe on channel one of the oscilloscope to the output of the signal generator.

C. What are the period and frequency of the square waveform?

$T_o =$ ________________ s

$f_o =$ ________________ Hz

D. Press the “Math” button and select “FFT”.

E. From the spectrum displayed, measure the following parameters.
   a. What is the value of the fundamental frequency?

   $f_o =$ ________________ Hz

   b. What are the frequencies of the 2^{nd}, 3^{rd} and 4^{th} components?

   $f_1 =$ ________________ Hz

   $f_2 =$ ________________ Hz

   $f_3 =$ ________________ Hz

© 2019 Ramz Academy LLC
Revision Date: May 17, 2019