Name: ______________________

RC Circuit in AC Domain

Introduction

Presence of reactive components such as capacitors and inductors in an AC circuit results in complex impedance values. Therefore, the circuit analysis for such circuits will need to be performed in complex domain which will involve not only the magnitude but also the phase of the signals. In this lesson, using hands-on experiments, we will learn how to perform circuit analysis for simple RC circuits in complex domain.

Discussion Overview

Circuit analysis for simple resistive circuits, such as the one shown in Figure 1, has involved simple "real" math.

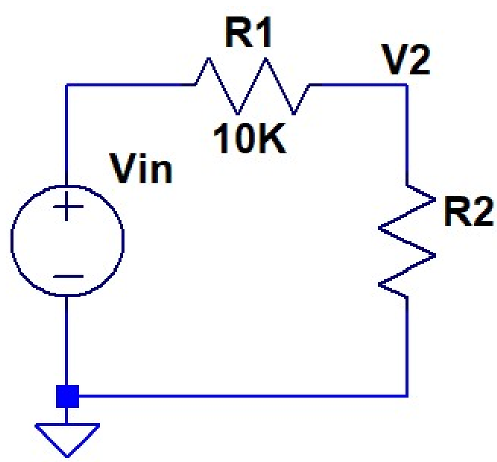

Figure 1 - Simple Resistive Circuit

Working with "real" math has meant that the phase relationship between the current flowing through such circuit and the voltages associated with that current have always been in phase. For an alternating voltage source (like a sine wave), the current through the circuit is at its max when the voltages are at their max.

Performing circuit analysis to determine the value of R_2 in the circuit above, for example, can simply be done by measuring the voltage at V_2. One can then determine the current flowing through R_1 and R_2 using the equation $I = \frac{V_{in} - V_2}{R_1}$, and finally, one can determine the value of $R_2 = \frac{V_2}{I}$.

Revision Date: May 17, 2019

This task, however, becomes more "complex" when there are reactive components present in a circuit such as the one shown in Figure 2.

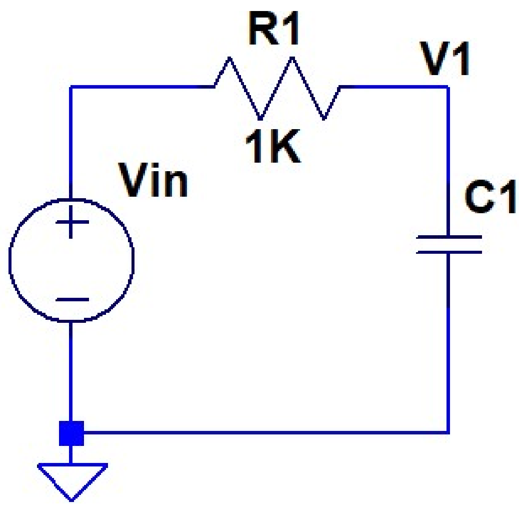

Figure 2 - RC Circuit in AC Domain

In this case, because the impedance of C1is complex, one will need to pay close attention to the phase relationship between the current flowing through the capacitor and the voltage across it.

In order to gain a better understanding of this phase relationship, we will use the circuit shown in to make some simple observations.

Procedure

Simple Capacitor and Current Source Circuit

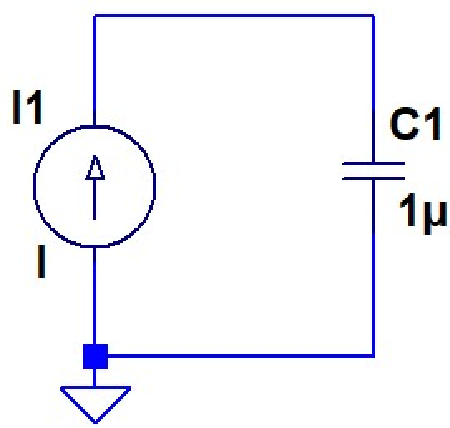

Figure 3 - Capacitor and Current Source

A. Build a circuit consisting of a current source and a capacitive load as shown above.

Name: ________________________

B. Configure the current source as a sine wave with the following settings:
   a. DC offset.................. 0
   b. Amplitude................. 1mA
   c. Frequency................. 100Hz
   d. All the remaining fields should be left blank

C. Setup the model to run a “transient” simulation with the following parameters
   a. Starting the print at time 0,
   b. Ending at time 50ms,
   c. Starting capture at time 30ms,
   d. With a maximum simulation step size of 1us

   Below is the syntax for your reference:

   .tran <Tprint> <Tstop> [<Tstart> [<Tmaxstep.]] [<options>]

D. Run the simulation and display the waveforms for the voltage and current associated with the capacitor.

E. Record the period and time for max current and max voltage below?

$T =$ ________________________ s

$t_{Imax} =$ ______________________ s

$t_{Vmax} =$ ______________________ s

F. What is the difference between these two time values?

$t_{Vmax} - t_{Imax} =$ _______________ s

G. What is the ratio of this difference to the period?

$$\frac{t_{Vma} \;\; - t_{Imax}}{T} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ s$$

H. What is the phase difference between the voltage and current?

$$\frac{t_{Vmax} - t_{Imax}}{T} \times 360 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ^{o}$$

I. Is the voltage leading or lagging the current?

As seen from the above experiment, maximum voltage lags behind maximum current by $\frac{T}{4}$ or 90^o. Intuitively, we note that

- As the current is increasing, the capacitor is continuing to charge.
- When the current is at its max, the capacitor is charging at the highest rate, which means that the voltage across the capacitor has not reached its max.
- However, when the current reaches zero, the capacitor is not charging any longer, and the charge accumulated across the capacitor is at its max. Therefore, the voltage across the capacitor is at its max.

We will use our understanding of this phase relationship between current and voltage to determine the value of an unknown capacitor in a simple RC circuit.

Measuring Capacitance

An Ohm meter measures the resistance of a resistor by placing a small amount of current through the resistor and measuring the voltage across it. Once the voltage is measure and given the known current, using Ohm's law, the resistance is simply

$$R = \frac{V}{I}$$

The same simple procedure can be applied to a reactive component, such as a capacitor, to determine its capacitance. The only difference is that the impedance of a capacitor is complex, and therefore, one needs to be cognizant of the phase relationships.

The impedance of a capacitor is given by $Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$. As seen, the impedance is purely imaginary (a complex number with only the imaginary component). The voltage across such capacitor is, therefore, given by

$$V_C = I_C Z_C = I_C \frac{-j}{\omega C}$$

Therefore, the magnitude and phase of the voltage are given by

$$|V_C| = |I_C| \frac{1}{\omega C} = |I_C| \frac{1}{2\pi f C}$$

And

$$\angle V_C = -90^o$$

This is exactly what we observed in the experiment of the previous section. Consequently, if we can determine the current flowing through our capacitor and measure the voltage across it $\frac{T}{4}$ later (lagging by 90^o) , we can use above equations to determine the value of our capacitor.

A. Build the circuit shown in

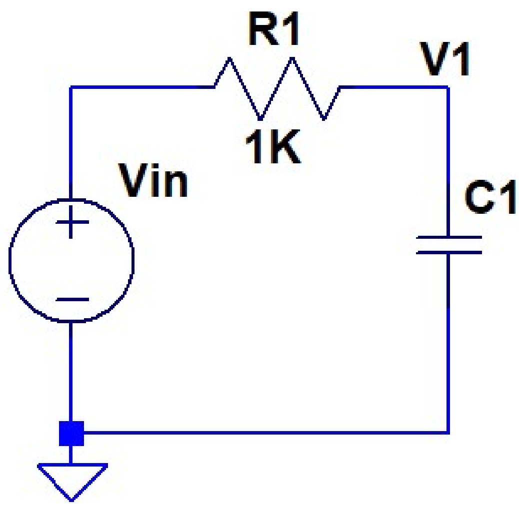

Figure 4 - Simple RC Circuit in AC Domain

B. Use the function generator as your Vin source. Configure the function generator as follows
   a. Amplitude = 3V ($V_{pk-pk} = 6V$)
   b. Offset = 0V
   c. Frequency = 200Hz

C. Connect probe one of your oscilloscope to Vin.
D. Connect probe two to V2.
E. Use the “Math” function to get the waveform Vin-V1.
F. Measure max(Vin-V1) and max(V1) and record them for the different frequencies specified in .
G. Determine the current through R1 and C1 and record them in .
H. Using the current found in step G above to determine the value of C for the different frequencies.

   Hint: You can use Google Sheets to make these repetitive calculations easier.

I. Find the average value of C.

Name: ______________________

Table 1 - RC Circuit Measurements

Frequency	$max(V_{in} - V_1)$	$max(V_1)$	$I_{max} = \frac{max(V_{in} - V_1)}{R_1}$	$C = \frac{I_{max}}{max(V_1)} \frac{1}{2\pi f}$
100				
200				
400				
600				
800				
1000				
			$avg(C) =$	

J. How do you think the values for $max(V_1)$ would change if the frequency is increased beyond 1000Hz?

K. How do you think your measurements above would change if a larger capacitor was used in the circuit?

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