Name: ________________________

Inductor Network Worksheet

Introduction

In this worksheet/lab, we will examine the properties of inductor networks in series and parallel configurations. We will use the formulae for determining the equivalent inductance of these networks to calculate the inductance of inductors connected in parallel and those connected in series.

The second part of this exercise will ask us to build parallel and series inductor networks and examine circuit properties for these networks. We will also use the charge time profile to measure the actual equivalent inductances to compare to the calculated values.

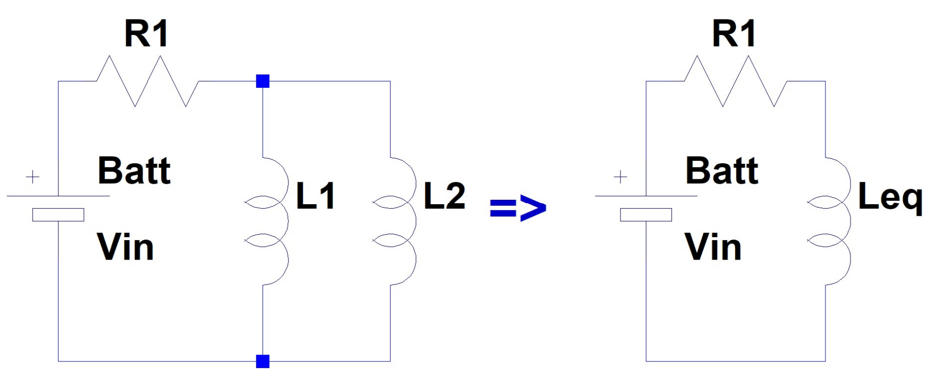
Discussion Overview

Parallel and Series Inductor Networks

Parallel Networks

When components are connected in parallel, the voltage across each component in the parallel configuration is the same. This was true for resistors and capacitors, and it is true for inductors as well. In the case of inductors, this means that, as the magnetic flux is increasing around the inductors, the varying voltages across the inductors are the same. Therefore,

$V_1 = V_2 = V_{eq}$ **Eq. 1**



The sum of currents flowing inductor L_1 & L_2, however, is equal to the total current flowing through L_{eq}.

$I_{eq} = I_1 + I_2$ **Eq. 2**

From the “constitutive relation” for inductors, however, we know that

$$\Phi = LI_L \Rightarrow V_L = L\frac{dI_L}{dt} \Longrightarrow \frac{V_L}{L} = \frac{dI_L}{dt}$$ **Eq. 3**

Therefore, from Eq. 2

$$\frac{dI_{eq}}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} \Longrightarrow \frac{V_{eq}}{L_{eq}} = \frac{V_1}{L_1} + \frac{V_2}{L_2}$$ **Eq. 4**

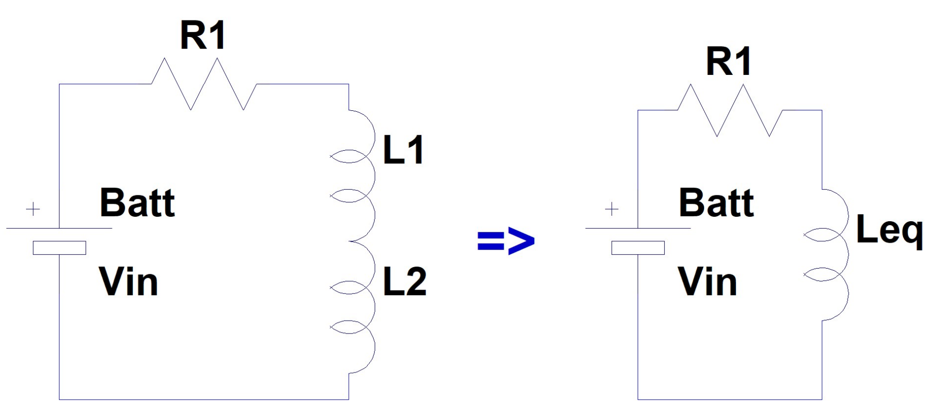
However, since $V_1 = V_2 = V_{eq}$,

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$ **Eq. 5**

Series Networks

In series networks, we know from Kirchhoff’s Voltage law that the sum of the voltages across individual components is equal to the total voltage across the full network. Therefore, for two inductors in a series configuration, we have

$$V_{eq} = V_1 + V_2$$ **Eq. 6**



The current flowing through each inductor L_1 & L_2, on the other hand, is the same and equal to the current flowing through the equivalent inductor. Therefore,

$$I_{eq} = I_1 = I_2 \Longrightarrow \frac{dI_{eq}}{dt} = \frac{dI_1}{dt} = \frac{dI_2}{dt}$$ **Eq. 7**

Using the “constitutive relation” for inductors again, we have

Name: ________________________

$$V_L = L\frac{dI_L}{dt}$$ **Eq. 8**

Plugging Eq. 8 above into the KVL for series circuits Eq. 6, we have

$$L_{eq}\frac{dI_{eq}}{dt} = L_1\frac{dI_1}{dt} + L_2\frac{dI_2}{dt}$$ **Eq. 9**

From Eq. 7, however, we have $\frac{dI_{eq}}{dt} = \frac{dI_1}{dt} = \frac{dI_2}{dt}$. Therefore,

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$ **Eq. 10**

Name: ________________________

Procedure

Calculations

Consider the two circuits below and answer the following questions.

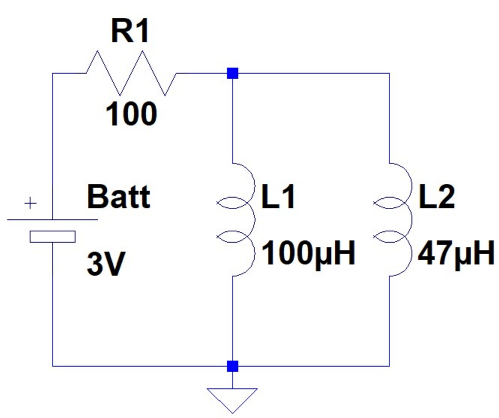


Figure 1 – Parallel Inductor Network

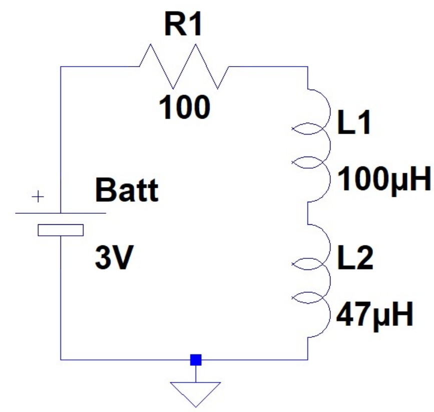


Figure 2 - Series Inductor Network

1. For the parallel network in Figure 1, do you believe the equivalent inductance will be

    a. $L_{eq} > 100\mu H$
    b. $L_{eq} < 47\mu H$
    c. $47\mu H < L_{eq} < 100\mu H$

2. Calculate the equivalent inductance for the circuit in Figure 1:

    $L_{eq} =$ ________________ μH

3. Extra credit: When fully "charged", determine the currents through L1 and L2:

    $I_1 =$ ________________ A

    $I_2 =$ ________________ A

4. For the series network in Figure 2, do you believe the equivalent inductance will be

    a. $L_{eq} > 100\mu H$
    b. $L_{eq} < 47\mu H$
    c. $47\mu H < L_{eq} < 100\mu H$

Name: ________________________

5. Calculate the equivalent inductance for the circuit in Figure 2:

   $L_{eq} =$ ________________ μH

6. When fully charged, determine the currents through L1 and L2:

   $L_1 =$ ________________ A

   $L_2 =$ ________________ A

Measurements

1. Build the circuit in Figure 1 on a breadboard.
   a. Instead of the battery, connect the circuit to the waveform generator.
   b. Set the output of the waveform generator to a square wave with
      i. A peak to peak voltage of 3V (amplitude of 1.5V).
      ii. An offset of 1.5V. (Note that the waveform should go from a minimum of 0V to a maximum of 3V.)
   c. Connect the oscilloscope's probe to across the inductor network in your circuit.
   d. Adjust the horizontal setting and waveform's frequency to clearly capture the charge profile for the inductor network.
   e. What is the initial voltage across the inductors?

      $V_0 =$ ________________ V

   f. What is the voltage across the inductor network when it has fallen by 95%, or in other words, for the inductor to reach its 95% charge?

      $V_{95\%} = 0.05V_0 =$ ________________ V

   g. How long does it take the voltage across the inductor network to drop by 95%?

      $t_{95\%} =$ ___________________ sec

Name: ________________________

h. The 95% "charge" time is equal to 3 times the $\frac{L}{R}$ value. What is, therefore, the equivalent inductance of your circuit?

$$t_{95\%} = 3\frac{L}{R} \Rightarrow L = \frac{t_{95\%}R}{3} \Rightarrow L = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \mu H$$

i. How does this value compare to your calculated value in step 2 of the previous section?

2. Repeat the steps 1a-1e above for the circuit in Figure 2.
   a. What is the initial voltage across the inductors?

   $V_0 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ V$

   b. What is the voltage across the inductor network when it has fallen by 95%?

   $V_{95\%} = 0.05V_0 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ V$

   c. How long does it take the voltage across the inductor network to drop by 95%, or in other words, for the inductor to reach its 95% charge?

   $t_{95\%} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ sec$

   d. The 95% "charge" time is equal to 3 times the $\frac{L}{R}$ value. What is, therefore, the equivalent inductance of your circuit?

   $$t_{95\%} = 3\frac{L}{R} \Rightarrow L = \frac{t_{95\%}R}{3} \Rightarrow L = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \mu H$$

   e. How does this value compare to your calculated value in step 5 of the previous section?