Name: ______________________

Complex Numbers Worksheet

Introduction

Imaginary and complex number representations are useful mathematical constructs in describing and examining the behavior of time varying (AC) circuits. Mastering complex number math is crucial in analyzing filters, oscillators and many more useful electronic circuits. In this lesson, we will provide an introduction to imaginary numbers, complex numbers and math operations with complex numbers.

Discussion Overview

Function Domains

From your high school math, you might recall that functions have domains. The domain of a function is the set of valid values on the real number axis that can be plugged into the function. For examples,

The domain of the function $f(x) = \frac{1}{x}$ is the set of all the real values except for $x=0$.

The domain of the function $f(x) = \frac{1}{x-10}$ is the set of all the real values except for $x=10$

The domain of the function $f(x) = \sqrt{x}$ is the set of all positive real values and 0.

Therefore, $f(-1) = \sqrt{-1}$ is undefined.

Imaginary numbers

In math, there is a special symbol for $\sqrt{-1}$; it is represented with either i or j. This gives us a new class of numbers called imaginary numbers. Imaginary numbers are written as a real number multiplied by i or j. For example,

$23i$, $-1.5j$, $2.45e3i$ or $-1.899 \times 10^{-5}j$

Complex Numbers

Similar to the way one can represent a point on the x, y plane with an ordered pair (x, y) or equivalently in vector notation $p = ax + by$, one can represent a number on a complex plane where the x axis represents the real part and the y axis represents the imaginary part of the ordered pair $s = a + bj$ where a = real part, b = imaginary part and $j = \sqrt{-1}$).

Name: ______________________

Phasor Notation

In addition to the vector notation discussed above, complex numbers can be represented by their amplitude and angle. This is similar to the polar notation, (r, θ), of a point in the x, y plane where r is the length of the line connecting the point to the origin, and θ is the angle that the line makes with the x axis.

Similarly, as seen in Figure 1, a complex number can be represented as $a + bj = re^{j\theta}$where $r = \sqrt{a^2 + b^2}$ is the length of the line connecting the origin to the point (a, b) on the complex plane, and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ is the angle that the line makes with the real axis.

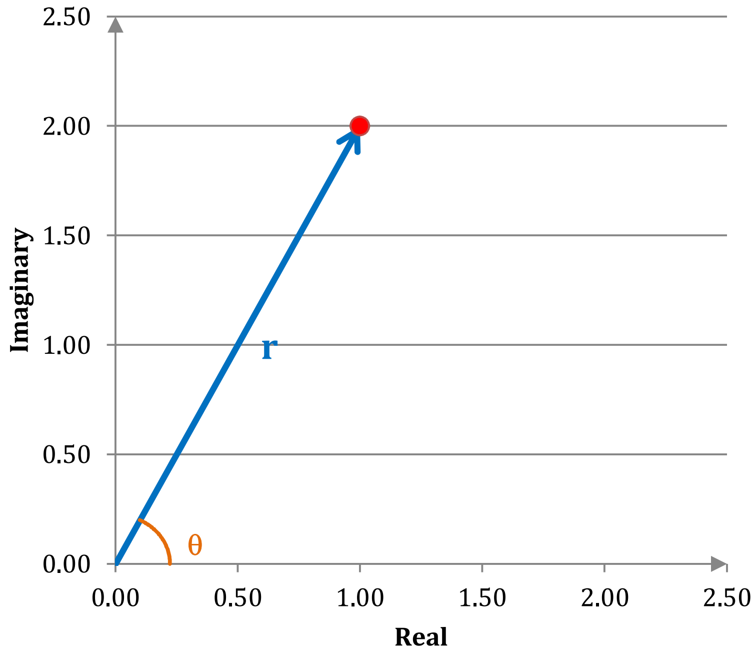

Figure 1 - Phasor Representation of Complex Numbers

Conversely, if a complex number is given in phasor notation, one can determine its real and imaginary parts by using the following formulae:

$a = r\cos\theta$ and $b = r\sin\theta$

In other words,

$re^{j\theta} = (r\cos\theta) + (r\sin\theta)j$

Name: ________________________

Complex number math

Math operations in complex domain follow the same rules as vector operations. For each operation, the final result should be written in terms of a real part and an imaginary part. Simple commutative and distributive properties are used to simplify operations. Below, complex addition, subtraction, multiplication and division operations are defined.

$$(a + bj) + (c + dj) = (a + c) + (b + d)j$$

$$(a + bj) - (c + dj) = (a - c) + (b - d)j$$

$$(a + bj) \times (c + d)j = (ac - bd) + (ad + bc)j$$

$$\frac{a + bj}{c + dj} = \frac{(a + bj)}{(c + dj)}\frac{(c - dj)}{(c - dj)} = \frac{(ac + bd) + (bc - ad)j}{c^2 - d^2}$$

Procedure

A. Write the following complex numbers in (real, imaginary) notation and draw the phasor diagram.

$1e^{j\frac{\pi}{4}} =$ ____________________

$-2e^{j\frac{\pi}{2}} =$ ____________________

$2.5e^{j\pi} =$ ____________________

$120e^{j0} =$ ____________________

B. Write the following complex numbers in phasor notation and mark them on the complex plane

$1 + 1j =$ ____________________

$-1 + 1j =$ ____________________

Name: ________________________

$10j =$ ________________________

$2.5 =$ ________________________

C. Perform the following math operations.

$(3 + 5j) + (2 - 4j) =$ ________________________

$(-3 + 5j) + (3 + 4j) =$ ________________________

$(24 - 3j) - (20 - 3j) =$ ________________________

$(-6 - 9j) - (4 - 4j) =$ ________________________

$(2 - 1j) \times (15 - 2j) =$ ________________________

$(4 + 3j) \times (4 - 3j) =$ ________________________

$10e^{j\frac{2\pi}{3}} \times 5e^{j\frac{\pi}{3}} =$ ________________________

$2e^{-j\frac{2\pi}{3}} \times 1.5e^{j\frac{\pi}{3}} =$ ________________________

$\frac{(4 + 3j)}{(4 + 3j)} =$ ________________________

$\frac{(4 + 3j)}{(4 - 3j)} =$ ________________________

$$\frac{(4+3j)}{(5+2j)} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\frac{10e^{j\frac{2\pi}{3}}}{5e^{j\frac{\pi}{3}}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\frac{5e^{-j\frac{2\pi}{3}}}{2e^{j\frac{\pi}{3}}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$