Name: ______________________

Inductor Reactance Analysis

Introduction

In this lesson, we continue examining the effect of complex impedances on performing AC circuit analysis. As seen in the previous lesson on capacitor reactance, the presence of reactive components such as capacitors and inductors in an AC circuit necessitates performing circuit analysis in complex domain which will involve not only the magnitude but also the phase of the signals. In this lesson, we will look at circuit analysis for a simple RL circuit in AC domain.

Discussion Overview

Similar to the analysis we performed for an RC circuit, in this project, we'd like to examine the process of determining the inductance of an inductor using the simple circuit depicted in Figure 1.

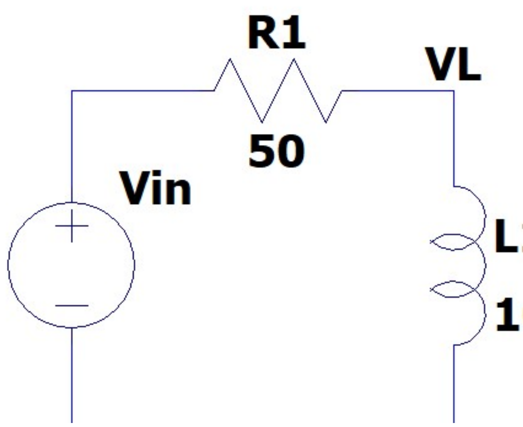

Figure 1 – Simple RL Circuit

As we know, however, the impedance of L1 is complex; and therefore, we will need to pay close attention to the phase relationship between the current flowing through the inductor and the voltage across it.

In order to gain a better understanding of this phase relationship, we will first use a current source, as shown in the circuit in Figure 2, to examine the voltage developed across the capacitor as a function of the current supplied by the current source.

$V_L = I \times j\omega L$

Revision Date: June 22, 2019

Name: ______________________

Procedure

Simple Inductor and Current Source Circuit

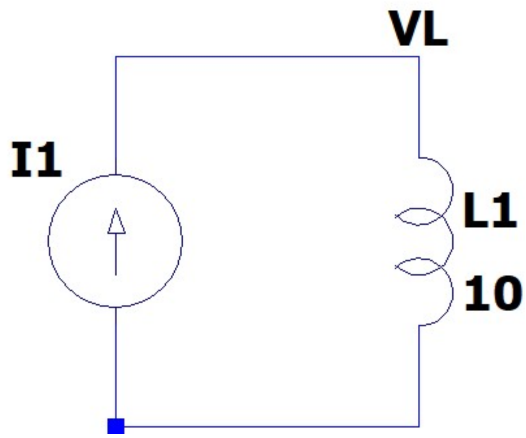

Figure 2 - Inductor and Current Source

A. Build a circuit consisting of a current source and an inductive load as shown above.
    a. Select a value of 100µH for the inductor, and leave all the other fields blank.
B. Configure the current source as a sine wave with the following settings:
    a. DC offset.................. 0
    b. Amplitude................. 100mA
    c. Frequency................ 1000Hz
    d. All the remaining fields should be left blank

C. Setup the model to run a “transient” simulation with the following parameters
    a. Starting the print at time 0,
    b. Ending at time 2ms,
    c. Starting capture at time 0ms,
    d. With a maximum simulation step size of 1us

    Below is the syntax for your reference:

    .tran <Tprint> <Tstop> [<Tstart> [<Tmaxstep.]] [<options>]

D. Run the simulation and display the waveforms for the voltage and current associated with the inductor.

Name: ________________________

E. Record the period and time for max current and max voltage below?

$T =$ ________________________ s

$t_{Imax} =$ ________________________ s

$t_{Vmax} =$ ________________________ s

F. What is the difference between these two time values?

$t_{Vmax} - t_{Imax} =$ ________________ s

G. What is the ratio of this difference to the period?

$$\frac{t_{Vmax} - t_{Imax}}{T} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ s$$

H. What is the phase difference between the voltage and current?

$$\frac{t_{Vmax} - t_{Imax}}{T} \times 360 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_^{o}$$

I. Is the voltage leading or lagging the current?

As seen from the above experiment, maximum voltage leads them maximum current by $\frac{T}{4}$ or 90^{o}. Intuitively, we note that

- As the current is starting from 0 and increasing, it goes through its maximum change. This abrupt change in current results in a maximum reactance by the inductor opposing the change in current, which in turn, results in the maximum voltage to appear across the inductor.
- As the current gradually increases, and the magnetic field around the inductor is built up, the voltage across the inductor decreases.
- As the current through the inductor levels off at its max, the voltage across the inductor is drops to zero.
- As the current starts to decrease, the inductor opposes this change by collapsing its magnetic field to provide a negative voltage to keep the current flowing.

- And finally, when the current is crossing zero, going through another maximal change in the negative direction, the inductor is opposing this change by providing the maximum negative voltage across itself.

We will use our understanding of this phase relationship between current and voltage to determine the value of an unknown inductor in a simple RL circuit.

Measuring Inductance

An Ohm meter measures the resistance of a resistor by placing a small amount of current through the resistor and measuring the voltage across it. Once the voltage is measure and given the known current, using Ohm's law, the resistance is simply

$$R = \frac{V}{I}$$

The same simple procedure can be applied to a reactive component, such as an inductor, to determine its inductance. The only difference is that the impedance of a inductor is complex, and therefore, one needs to be cognizant of the phase relationships.

The impedance of a inductor is given by $Z_L = j\omega L$. As seen, this impedance is purely imaginary (a complex number with only the imaginary component). The voltage across such an inductor is, therefore, given by

$$V_L = I_L Z_L = I_L \times j\omega L$$

Therefore, the magnitude and phase of the voltage are given by

$$|V_L| = |I_L|\omega L = |I_L| 2\pi f L$$

And

$$\angle V_L = +90^o$$

This is exactly what we observed in the experiment of the previous section. Consequently, if we can determine the current flowing through our inductor and measure the voltage across it $\frac{T}{4}$ earlier (leading by 90^o) , we can use above equations to determine the value of our inductor.

A. Build the circuit shown in Figure 3

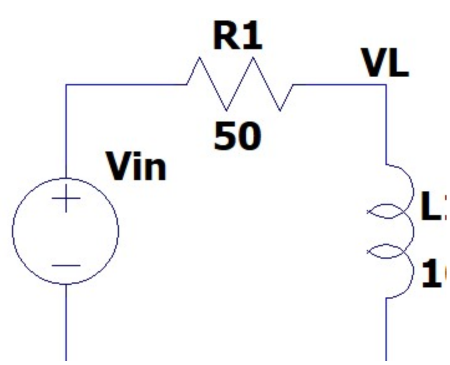

Figure 3 - Simple RL Circuit

Name: ______________________

B. Use the function generator as your Vin source. Configure the function generator as follows
   a. Amplitude = 3V ($V_{pk-pk} = 6V$)
   b. Offset = 0V
   c. Frequency = 10KHz

   Note: Real inductors contain some amount of series resistance. This resistance affects the overall impedance of the inductor, and consequently, the voltage-current phase relationship.

   $Z_L = R_L + j\omega L$

   Therefore,

   $|Z_L| = \sqrt{R_L^2 + \omega^2 L^2}$ *and* $\angle Z_L = \frac{\omega L}{R_L}$

   In order to minimize the effect of the series resistance on our measurements, we choose a high enough frequency that would make the imaginary component, ωL*, of the impedance much larger than the real component,* R_L*.*

C. Connect probe one of your oscilloscope to Vin.
D. Connect probe two to VL.
E. Use the "Math" function to get the waveform Vin-VL.
F. Measure max(Vin-VL) and max(VL) and record them for the different frequencies specified in Table 1.
G. Determine the current through R1 and L1 and record them in Table 1 below.
H. Using the current found in step G above, determine the value of L for the different frequencies.

   Hint: You can use Google Sheets to make these repetitive calculations easier.

I. Find the average value of L.

Table 1 - RC Circuit Measurements

Frequency	$max(V_{in} - V_L)$	$max(V_L)$	$I_{max} = \frac{max(V_{in} - V_L)}{R_1}$	$L = \frac{max(V_L)}{I_{max}} \frac{1}{2\pi f}$
10K				
20K				
40K				
60K				

Name: ________________________

80K				
100K				
			$avg(L) =$	

J. How do you think the values for $max(V_L)$ would change if the frequency is decreased beyond 10KHz?

K. How do you think your measurements above would change if a smaller inductor was used in the circuit?