Scale-space and Image Restoration
Signal and Image Processing

February 23, 2018

Feature detectors

1. Use the `skimage.feature.canny` function on the `hand.tiff` image. Try different settings for the parameters `sigma`, `low_threshold`, and `high_threshold` of the `canny` function. Create an illustration showing the results of the different settings and explain what the effect is of each of the parameters based on these results.

2. Use the `skimage.feature.corner_harris` function on the `modelhouses.png` image to compute a Harris corner response image (a.k.a. feature map). Try different settings for the parameters `sigma` and `k` for `method='k'` of the `corner_harris` function. Also try to fix `sigma` and change method to `method='eps'` and try different values of the `eps` parameter. Create an illustration showing the results of the different settings and explain what the effect is of each of the parameters based on these results.

3. Write a Python function that finds local maxima in the feature map generated by the `skimage.feature.corner_harris` function by using the function `skimage.feature.corner_peaks`. Apply this function to the `modelhouses.png` image and create a figure of the resulting corner points overlaid on the `modelhouses.png` image. Remember to indicate your choice of parameter settings in the caption of the figure.

Scale-space operators

1. Consider a Gaussian kernel

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}. \tag{1}$$

The convolution of a Gaussian with itself is also a Gaussian, i.e.,

$$G(x, y, \sigma) * G(x, y, \tau) = G(x, y, \sqrt{\sigma^2 + \tau^2}) \tag{2}$$

Make an image from the analytical expression,

$$I(x, y) = G(x, y, \sigma), \tag{3}$$

for some fixed σ, and visually confirm the above by calculating images from its scale-space,

$$I(x, y, \tau) = I(x, y) * G(x, y, \tau), \tag{4}$$

using Python and, e.g., using the `scale` function written previously.

2. Consider the 2-dimensional scale normalized derivatives at scale τ,

$$I_{x^i y^j}(x, y, \tau) = \tau^{\gamma(i+j)} \frac{\partial^{i+j} I(x, y, \tau)}{\partial x^i \partial y^j}, \tag{5}$$

where $\gamma \in \mathbb{R}$ is a parameter of the scale normalization and $I(x, y, \tau)$ is the scale space of the image defined in (3). Now consider the scale normalized image of the Laplacian,

$$H(x, y, \tau) = I_{xx}(x, y, \tau) + I_{yy}(x, y, \tau), \tag{6}$$

Using $\gamma = 1$, solve the following:

(a) Write the closed form expression for $H(x, y, \tau)$.

(b) Consider the point $(x, y) = (0, 0)$ and derive analytically the scale(s), τ, for which $H(0, 0, \tau)$ is extremal. Maple (or Mathematica) may be helpful. Characterize these extremal point(s) in terms of maximum, saddle, and minimum in (x, y, τ).

(c) Confirm your result in Python.

(d) Locating the maxima and minima (x, y, τ) in the scale-space of (6) applied to images $I(x, y)$ in general is called blob detection. Detect the 20 largest maxima and minima in the `sunflower.tiff` image, and indicate each detected scale τ with a circle centered on the point of detection and with a radius of τ. Choose different colors for the circle and point so you can distinguish maxima from minima. What image structure does maxima of (6) represent, and what image structure does minima represent?

3. Consider a soft edge,

$$J(x, y) = \int_{-\infty}^{x} G(x', 0, \sigma)\, dx' \tag{7}$$

for some constant σ and Gaussian function G defined as in (1). Consider also its scale-space,

$$J(x, y, \tau) = J(x, y) * G(x, y, \tau), \tag{8}$$

and the scale-normalized spatial squared gradient magnitude operator

$$\|\nabla J(x, y, \tau)\|^2 = J_x^2(x, y, \tau) + J_y^2(x, y, \tau). \tag{9}$$

Using (5) and $\gamma = \frac{1}{2}$, solve the following:

(a) Write the closed form expression for $\|\nabla J\|^2$.

(b) Derive analytically the scale, τ, for which $\|\nabla J\|^2$ is maximal in the point $(x, y) = (0, 0)$. Is this a maximum in (x, y, τ)?

(c) Confirm your result in Python.

(d) The maxima in (x, y, τ) of (9) is edge detection with scale-selection. Detect the 100 largest maxima in the `hand.tiff` image, and indicate the point of detection and scale by circles.