# PHY321: Classical Mechanics 1

**Homework 4, due Monday February 15**

Feb 6, 2021

**Practicalities about homeworks and projects.**

1. You can work in groups (optimal groups are often 2-3 people) or by yourself. If you work as a group you can hand in one answer only if you wish. **Remember to write your name(s)!**
2. Homeworks are available ten days before the deadline.
3. How do I(we) hand in? You can hand in the paper and pencil exercises as a scanned document. For this homework this applies to exercises 1-5. Alternatively, you can hand in everyhting (if you are ok with typing mathematical formulae using say Latex) as a jupyter notebook at D2L. The numerical exercise(s) (exercise 6 here) should always be handed in as a jupyter notebook by the deadline at D2L.

**Introduction to homework 4.** This week's sets of classical pen and paper and computational exercises deal with simple motion problems and conservation laws; energy, momentum and angular momentum. These conservation laws are central in Physics and understanding them properly lays the foundation for understanding and analyzing more complicated physics problems.

The relevant reading background is

1. chapters 3, 4.1, 4.2 and 4.3 of Taylor (there are many good examples there) and
2. chapters 10-13 of Malthe-Sørenssen.

In both textbooks there are many nice worked out examples. Malthe-Sørenssen's text contains also several coding examples you may find useful.

The numerical homework focuses on another motion problem where you can use the code you developed in homework 3, almost entirely. Please take a look at the posted solution (jupyter-notebook) for homework 3. You need only to change the forces at play. The numerical problem this time is based on your code from homework 3 and we will try to make the motion of a falling object in two dimensions more realistic by allowing to bounce up again due to a normal force from the floor.

**Exercise 1 (10 pt), Conservation laws, Energy and momentum.**

- 1a (2pt) How do we define a conservative force?
- 1b (4pt) Use the work-energy theorem to show that energy is conserved with a conservative force.
- 1c (4pt) Assume that you have only internal two-body forces acting on $N$ objects in an isolated system. The force from object $i$ on object $j$ is $\boldsymbol{F}_{ij}$. Show that the linear momentum is conserved.

**Exercise 2 (10 pt), Conservation of angular momentum.**

- 2a (2pt) Define angular momentum and the torque for a single object with external forces only.
- 2b (4pt) Define angular momentum and the torque for a system with $N$ objects/particles with external and internal forces. The force from object $i$ on object $j$ is $\boldsymbol{F}_{ij}$.
- 2c (4pt) With internal forces only, what is the mathematical form of the forces that allows for angular momentum to be conserved?

**Exsercise 3 (10pt), Example of potential.** Consider a particle of mass $m$ moving according to the potential

$$V(x,y,z) = A\exp\left\{-\frac{x^2+z^2}{2a^2}\right\}.$$

- 3a (2pt) Is energy conserved? If so, why?
- 3b (4pt) Which of the quantities, $p_x, p_y, p_z$ are conserved?
- 3c (4pt) Which of the quantities, $L_x, L_y, L_z$ are conserved?

**Exercise 4 (15pt), Angular momentum case.** At $t=0$ we have a single object with position $\boldsymbol{r}_0 = x_0\boldsymbol{e}_x + y_0\boldsymbol{e}_y$.We add also a force in the $x$-direction at $t=0$. We assume that the object is at rest at $t=0$.

$$\boldsymbol{F} = F\boldsymbol{e}_x.$$

- 4a (5pt) Find the velocity and momentum at a given time $t$ by integrating over time with the above initial conditions.
- 4b (5pt) Find also the position at a time $t$.
- 4c (5pt) Use the position and the momentum to find the angular momentum and the torque. Is angular momentum conserved?

**Exercise 5 (15pt), forces and potentials.** A particle of mass $m$ has velocity $v = \alpha/x$, where $x$ is its displacement.

- 5a (5pt) Find the force $F(x)$ responsible for the motion.

A particle is thereafter under the influence of a force $F = -kx + kx^3/\alpha^2$, where $k$ and $\alpha$ are constants and $k$ is positive.

- 5b (5pt) Determine $U(x)$ and discuss the motion. It can be convenient here to make a sketch/plot of the potential as function of $x$.
- 5c (5pt) What happens when the energy of the particle is $E = (1/4)k\alpha^2$? Hint: what is the maximum value of the potential energy?

**Exercise 6 (40pt), Bouncing object.** This exercise builds on the code you wrote for solving homework 3. There we introduced gravity and air resistance and studied their effects via a constant acceleration due to gravity and the force arising from air resistance. But what happens when the ball hits the floor? What if we would like to simulate the normal force from the floor acting on the ball? This exercise shows how we can include more complicated forces with no pain! And the force we include here is an example of a case where analytical solutions may either be difficult to find or we cannot find an analytical solution at all.

We need then to include a force model for the normal force from the floor on the ball. The simplest approach to such a system is to introduce a contact force model represented by a spring model. We model the interaction between the floor and the ball as a single spring. But the normal force is zero when there is no contact. Here we define a simple model that allows us to include such effects in our models.

The normal force from the floor on the ball is represented by a spring force. This is a strong simplification of the actual deformation process occurring at the contact between the ball and the floor due to the deformation of both the ball and the floor.

The deformed region corresponds roughly to the region of **overlap** between the ball and the floor. The depth of this region is $\Delta y = R - y(t)$, where $R$ is the radius of the ball. This is supposed to represent the compression of the spring. Our model for the normal force acting on the ball is then

$$\boldsymbol{N} = -k(R - y(t))\boldsymbol{e}_y.$$

The normal force must act upward when $y < R$, hence the sign must be negative. However, we must also ensure that the normal force only acts when the ball is in contact with the floor, otherwise the normal force is zero. The full formation of the normal force is therefore

$$\boldsymbol{N} = -k(R - y(t))\boldsymbol{e}_y,$$

when $y(t) < R$ and zero when $y(t) \leq R$. In the numerical calculations you can choose $R = 0.1$ m and the spring constant $k = 1000$ N/m.

- 6a (10pt) Identify the forces acting on the ball and set up a diagram with the forces acting on the ball. Find the acceleration of the falling ball now with the normal force as well.
- 6b (30pt) Choose a large enough final time so you can study the ball bouncing up and down several times. Add the normal force and compute the height of the ball as function of time with and without air resistance. Comment your results.