Lecture January 25

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \left(\text{1-Dim} \quad a = \frac{dv}{dt}\right)$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Taylor expansion (1-Dim)

$$f(x+h) = f(x) + \frac{h}{1!} f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f^{(3)}(x) + O(h^4)$$

$$f(x) \Rightarrow v(t)$$

$$v(t+h) = v(t) + h v'(t) + \frac{h^2}{2!} v''(t) + O(h^3)$$

$$\frac{v(t+h) - v(t)}{h} = v'(t) + \frac{h}{2!} v'' + O(h^2)$$

$$\lim_{h \to 0} \frac{v(t+h) - v(t)}{h} = v'(t) = \frac{dv(t)}{dt}$$

$(h = \Delta t)$

$$\boxed{v'(t) \simeq \frac{v(t+h) - v(t)}{h}}$$

Differential equation

$$\boxed{\frac{dv(t)}{dt} = a(t)}$$

$$F(t) = m\,a(t)$$

$$\frac{dv(t)}{dt} = \frac{F(t)}{m} = \underline{\underline{a(t)}}$$

In this course we have often analytical expressions for $a(t)$.

$$a(t) \simeq \boxed{\frac{v(t+h) - v(t)}{h}}$$

$$v(t+h) = v(t) + h\,a(t)$$

(First order diff eq

$$\frac{dv}{dt} = a(t)$$

> Euler's equation for 1st Diff eq.

$$v(t+h) = v(t+\Delta t)$$

$$= v(t_i + \underline{\Delta t}) = v(t_i + \underline{h})$$

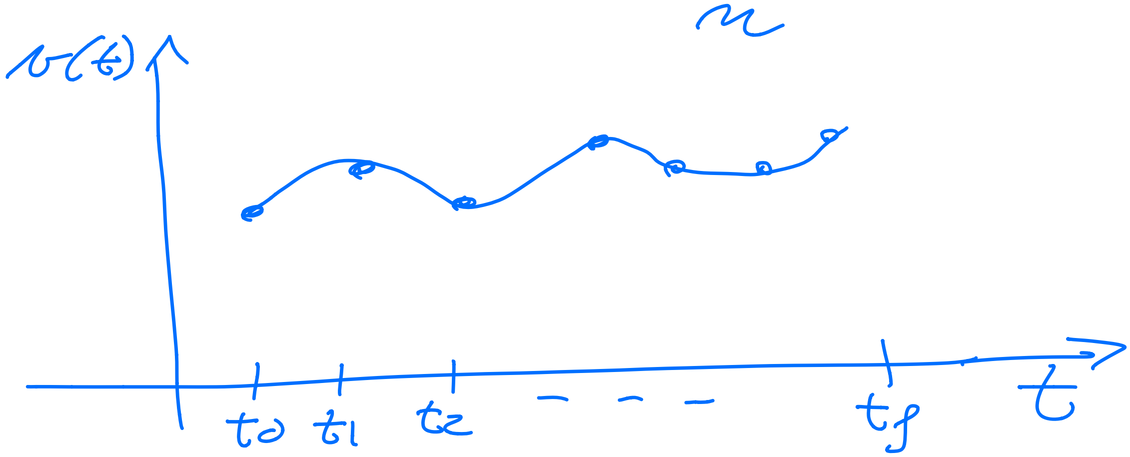
$$= v_{i+1}$$

initial time t_0

final time t_f

n – integration points

$$h = \Delta t = \frac{t_f - t_0}{n}$$



$$v(t) \to v(t_i) = v_i$$

$$t_i = t_0 + i\Delta t$$

Discretized equation:

$$v(t + \underbrace{h}_{\Delta t}) = v(t) + \underbrace{h}_{\Delta t} a(t)$$

$$v_{i+1} = v_i + \Delta t \, a_i$$

$$\boxed{v(t) = \frac{dx(t)}{dt}}$$

$$x(t+\Delta t) = x(t) + \Delta t\, \boxed{x}' + \frac{\Delta t^2}{2!} x'' + O(\Delta t^3)$$

Digression: 3-dim

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} \Rightarrow$$

$$\begin{cases} v_x(t) = \dfrac{dx(t)}{dt} \\ v_y(t) = \dfrac{dy(t)}{dt} \\ v_z(t) = \dfrac{dz(t)}{dt} \end{cases}$$

$$v(t) \simeq \frac{x(t+\Delta t) - x(t)}{\Delta t} \Rightarrow$$

$$x(t+\Delta t) = x(t) + \Delta t\, v(t)$$

$$F(t) = m a(t) \Rightarrow$$

$$\cancel{m} \frac{dv}{dt} = \cancel{m} a(t)$$

$$\frac{d^2 x}{dt^2} = \frac{d}{dt}\Big(\underbrace{\frac{d}{dt} x}_{v(t)}\Big) =$$

$$= \underbrace{\frac{d}{dt} v(t)}_{a(t)}$$

$$\left.\begin{array}{l}\dfrac{dv}{dt} = a(t) \\ \quad\text{and} \\ \dfrac{dx}{dt} = v(t)\end{array}\right\}$$

Discretize and use Euler's method

$$\boxed{\begin{array}{l} v_{i+1} = v_i + \Delta t\, a_i \\ x_{i+1} = x_i + \Delta t\, v_i \end{array}}$$

From model for the force

Falling ball in 1-Dim.

$a(t) = -g$ (no drag?)

$$\frac{dv(t)}{dt} = -g$$

integrate from t_0 to t_f

$t_0 = 0$

$$\int_0^{t_f} \frac{dv(t')}{dt'}\, dt' = \int_0^{t_f} (-g)\, dt'$$

$$= v(t_f) - \underbrace{v(t_0)}_{v_0} = -g \cdot t_f$$

$t_f \Rightarrow t$

$$v(t) = v_0 - g \cdot t$$

$$v(t) = \frac{dx}{dt}$$

integrate !

$$\boxed{x(t) = x_0 - \frac{1}{2} g t^2 + v_0 \cdot t}$$

$v_0 = 0$.