PHY321 FEB 15

$$V(\vec{r}) = -W(\vec{r}_0 \to \vec{r})$$

$$= -\int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r})\, d\vec{r}$$

Work-Energy theorem

$$\frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = \int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r})\, d\vec{r}$$

$$\vec{F}(\vec{r}) = -\vec{\nabla} V(\vec{r})$$

<u>Example</u> (1-DIM)

$$F(x) = -F_0 \sin\left(\frac{2\pi x}{b}\right)$$

Can we find $V(x)$

$$\left(F(x) = -\frac{d}{dx} V(x) \right)$$

- $F(x)$ is position dependent (may be conservative force)

- $\vec{\nabla} \times \vec{F} = 0$

$$(\nabla \times F)_x = \frac{\partial}{\partial y} F_z - \frac{\partial F_y}{\partial z} = 0$$

$$(\nabla \times F)_y = \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} = 0$$

$$(\nabla \times F)_z = \frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x} = 0$$

$\Rightarrow$ Conservative force.

$v(x) = ?$

$$\frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = \int_{x_0}^{x} F(x')\,dx'$$

x

$$= -\int_{x_0} F_0 \sin\left(\frac{2\pi x'}{b}\right) dx'$$

$$= \frac{F_0 b}{2\pi}\left[\cos\left(\frac{2\pi x}{b}\right) - \cos\left(\frac{2\pi x_0}{b}\right)\right]$$

$$v_0 = 0$$

$$\frac{1}{2} m v^2 = \frac{F_0 b}{2\pi}\left[\qquad\right]$$

$$v(x) = \pm\sqrt{\frac{\cancel{2}}{m}\frac{F_0 b}{\cancel{2}\pi}\left[\cos\left(\frac{2\pi x}{b}\right) - \cos\left(\frac{2\pi x_0}{b}\right)\right]}$$

$$V(x) - V(x_0) = \Delta V = -W$$

$$= \int_{x_0}^{x} \left(-F(x')\right) dx'$$

$$V(x) = V(x_0) - \int_{x_0}^{x} F(x')\,dx'$$

$$V(x) \quad F_0 b \left[\cos\left(2\pi x\right)\right.$$

$$= V(x_0) - \frac{F_0 b}{2\pi}\left[\cos\left(\frac{2\pi x}{b}\right) - \cos\left(\frac{2\pi x_0}{b}\right)\right]$$

Define

$$V(x_0) = -\frac{F_0 b}{2\pi}\cos\left(\frac{2\pi x_0}{b}\right)$$

$$V(x) = -\frac{F_0 b}{2\pi}\cos\left(\frac{2\pi x}{b}\right)$$

$$\frac{d}{dx}V(x) = F_0 \sin\left(\frac{2\pi x}{b}\right)$$

$$F(x) = -\frac{d}{dx}V(x)$$

$$F(x) = -F_0 \sin\left(\frac{2\pi x}{b}\right)$$

Example 2

$$\vec{F}(\vec{r}) = \frac{\gamma\,\vec{r}}{r^3} = \frac{\gamma}{r^3}\left(x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3\right)$$

3-Dim : $r = \sqrt{x^2 + y^2 + z^2}$

$$\gamma = G M_i M_j \,,$$ gravitational force

$$\gamma = \frac{q_i q_j}{4\pi \varepsilon_0}$$ electrostatic force.

$$r = |\vec{r}_i - \vec{r}_j| = r_{ij}$$

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$$

- position dependent only, no time-dependence and no velocity dependence.

$$\vec{F} = \vec{F}(\vec{r}, \vec{v}, t)$$

Here $\vec{F} = \vec{F}(\vec{r}_{ij})$

Two-body problem

$$\vec{F} = \vec{F}(\vec{r}_{12})$$

$$= \vec{F}(|\vec{r}_{12}|)$$

$\vec{\ }$ $\vec{\ }$

$$r_{ij} \rightarrow r$$

$$r = (x^2+y^2+z^2)^{1/2}$$

$$\vec{\nabla} \times \vec{F} = 0$$

$$(\nabla \times F)_x = \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}$$

$$= \frac{\partial}{\partial y}\left(\frac{\gamma z}{r^3}\right) - \frac{\partial}{\partial z}\left(\frac{\gamma y}{r^3}\right)$$

$$\gamma z \frac{\partial}{\partial y} r^{-3} - \gamma y \frac{\partial}{\partial z} r^{-3}$$

$$= \gamma z \left(-\frac{3}{r^4} \frac{y}{r}\right)$$

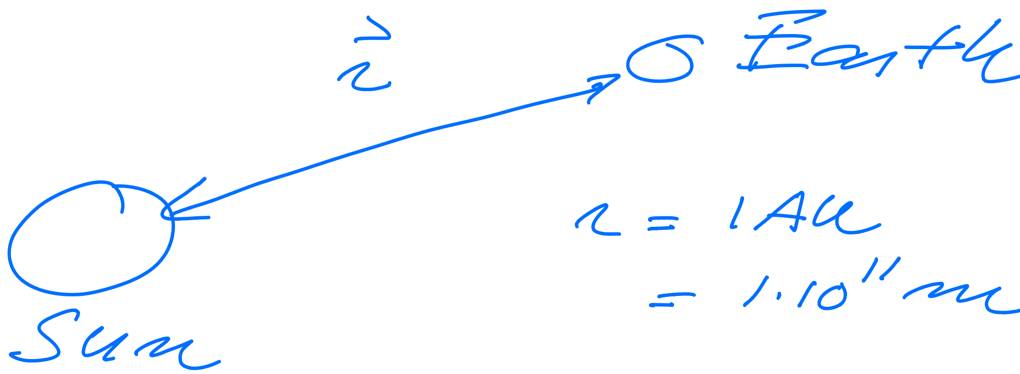
$$- \gamma y \left(-\frac{3}{r^4} \frac{z}{r}\right) = 0$$

$$(\nabla \times F)_y = \frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} = 0$$

$$(\nabla \times F)_z = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 0$$

Gravitational, electrostatic and all forces on the form $\gamma \frac{\vec{r}}{r^3}$ conserve energy?

Earth-Sun system



$$r = 1\,\mathrm{AU}$$
$$= 1 \cdot 10^{11}\,\mathrm{m}$$

$$F(\vec{r}) = -\frac{G M_{\odot} M_E \vec{r}}{r^3}$$