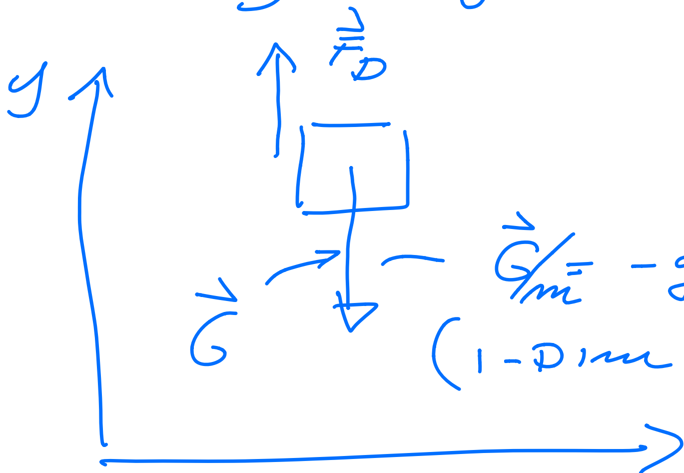
Lecture January 29

Falling object



$\vec G/m = -g\,\vec e_x = -g\,\vec e_z$

(1-Dim $G = -gm$)

$$\vec F_{net} = \vec F_D + \vec G$$

$$\vec F_D = \begin{cases} \boxed{-D\,\vec v\,|\vec v(t)|} & \text{large } v \\ -k_v\,\vec v(t) & \text{small velocity} \end{cases}$$

Dimensionality of v = length/time = $[v]$

$[\vec F]$ = mass · length/time^2

1-Dim:

$$F_D = -D\,\underbrace{v^2(t)}_{[v^2] = \text{length}^2/\text{time}^2}$$

$[D]$ = mass/length

$$[kv] = \text{mass}/\text{time}$$

$$F_{net} = -mg + Dv^2(t)$$

$$= m \cdot \frac{d^2 y}{dt^2} = m \cdot \frac{dv}{dt} = ma$$

$$a = -g + \frac{D}{m} v^2(t) = \frac{dv}{dt}$$

$$\frac{dv}{dt} = -g + \underbrace{\frac{D}{m}}_{\gamma} v^2(t)$$

$$= -g + \gamma v^2(t)$$

$$\frac{dv}{g - \gamma v^2(t)} = -dt$$

v_0 $\quad v(t_0) = 0 \ m/s \qquad t_0 = 0$

$$\int_{v_0}^{v} \frac{dv'}{\underline{g} - \underline{\gamma} v^2(t)} = -\int_0^t \underline{dt'}$$

$$v_T = \sqrt{g/\gamma} \qquad \gamma = D/m$$

$$[v_T] = \text{length}/\text{time} \qquad [\gamma] = \frac{1}{\text{length}}$$

↪ natural velocity

$$[g] = \text{length}/\text{time}^2$$

$$v_T = \sqrt{\left(\text{length}/\text{time}^2\right)/\text{length}}$$

$$= \text{length}/\text{time}$$

$$\frac{1}{\gamma}\int_0^v \frac{dv'}{v_T^2 - v'^2} = -\int_0^t dt'$$

$$\left[\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{a}\,\text{arctanh}\,\frac{x}{a} + C\right]$$

$$\frac{1}{\gamma}\,\frac{1}{v_T}\,\text{arctanh}\left(\frac{v}{v_T}\right)\Bigg|_0^v = -t$$

$$\frac{v_T}{g}\,\text{arctanh}\left(\frac{v}{v_T}\right) = -t$$

$$v(t) = v_T \tanh\left(-\frac{g\,t}{v_T}\right)$$

Dimless

$$\frac{\cancel{\text{length}}}{\cancel{\text{time}^2}}\;\cancel{\text{time}}$$

length/time

$$\frac{dy}{dt} = v(t) = \underline{v_T \tanh\left(\frac{-gt}{v_T}\right)}$$

$$\int_0^t \frac{dy}{dt'} \cdot dt' = \int_0^t v(t')\, dt'$$

$$y(t) - y_0 = -\frac{v_T^2}{g} \ln\left[\cosh\left(\frac{gt}{v_T}\right)\right]$$

$$y(t) = y_0 - \frac{v_T^2}{g} \ln\left[\cosh\left(\frac{gt}{v_T}\right)\right]$$