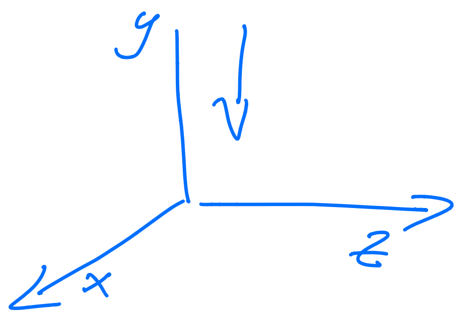
Lecture Jan 27

From Monday

- Falling object (1-Dim)



$$\vec{a} = -g\,\vec{e}_2$$

$$\vec{e}_y = \vec{j} = \vec{e}_2$$

$$a_y = -g$$

$$a = -g \qquad \left(\vec{a} = \overset{a_x}{a_1}\,\vec{e}_1 + \underset{a_y}{\underset{\shortparallel}{a_2}}\,\vec{e}_2 + \underset{a_z}{\underset{\shortparallel}{a_3}}\,\vec{e}_3\right)$$

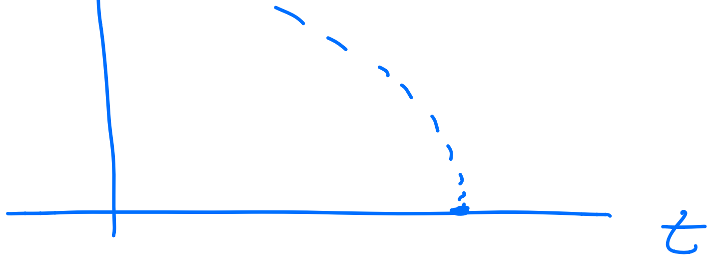
- Define initial conditions

  initial height $y_0 = y(t_0)$

  t_0 = initial time

  initial velocity $v_0 = v(t_0)$

  $v_0 = 0\ m/s$



$$m \cdot a = F = F(t, x, v)$$

$$a = a(t, x, v)$$

$$v = v(t, x)$$

$$x = x(t)$$

$$\cancel{m} \cdot a = \cancel{m} \cdot \frac{dv}{dt} = \cancel{m} \frac{d^2x}{dt^2}$$

$$\frac{dv}{dt} = a \quad \wedge \quad \frac{dx}{dt} = v$$

$$\frac{dv}{dt} = -g \qquad v_0 = 0\ m/s$$

$$\int_{t_0=0}^{t} \frac{dv}{dt'} dt' = -\int_{t_0=0}^{t} (g)\, dt'$$

$$v(t) = -gt$$

$$\frac{dx}{dt} = v \qquad \text{integrate again}$$

$$y(t) = y_0 - \frac{1}{2} g t^2$$

Numerically

$$\boxed{\frac{dx}{dt} = v \qquad \frac{dv}{dt} = a}$$

$$\Delta t = \frac{\underline{t_f} - \underline{t_0}}{n}$$

$$\Delta t = 0.01$$

t = np.arange(t_0, t_f, Δt)

n = np.size(t)

$0, 1, 2, \ldots n-1$

$$\boxed{t_i = t_0 + i \Delta t} \qquad i = 0, 1, 2, \ldots \underline{n}$$

$$x(t) \to x(t_i) = x_i$$

initial conditions

$$x(t_0) = x_0 \qquad v(t_0) = v_0$$

$$a(t) \to a(t_i) = a_i$$

$$v(t) \to v(t_i) = v_i$$

Euler's method:

$$x_{i+1} = x_i + \Delta t \, v_i$$

$$v_{i+1} = v_i + \Delta t \cdot a_i$$

x = np.zeros(n+1)

x[0], x[1], ..., x[n]

<u>integrate</u>

$-g$

for i in range(0, n-1):

$v[i+1] = v[i] + \Delta t \, a[i]$

$x[i+1] = x[i] + \Delta t \, v[i]$

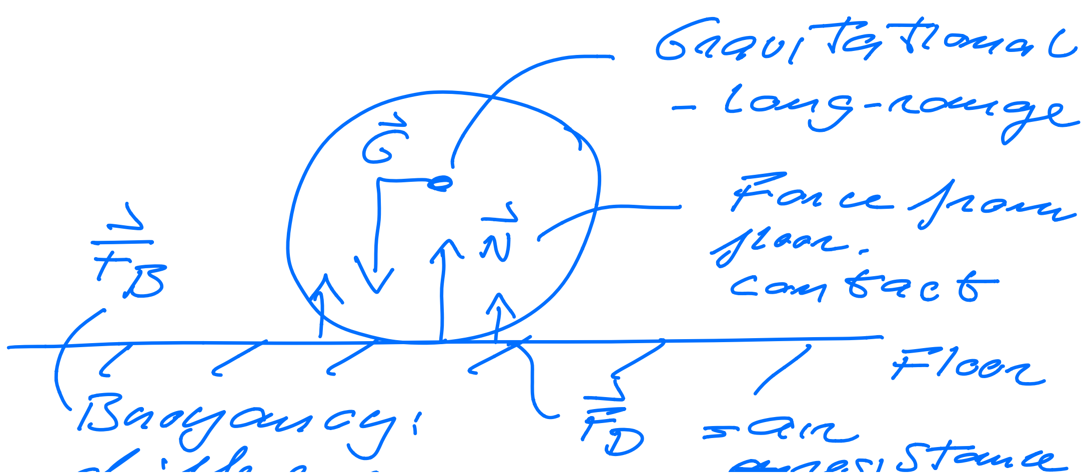
time

new physics case (1-Dim), just change a !

Drag Force

$$a = -g + \begin{cases} D v^2(t) \\ D v(t) \end{cases}$$

analytically on Friday.



difference
in air
pressure

$$\vec{F}_{net} = \vec{G} + \vec{N} + \vec{F}_D + \vec{F}_B$$

$$= \sum_{i=1}^{\text{Next F}} F_i' = \boxed{m \cdot \vec{a}}$$