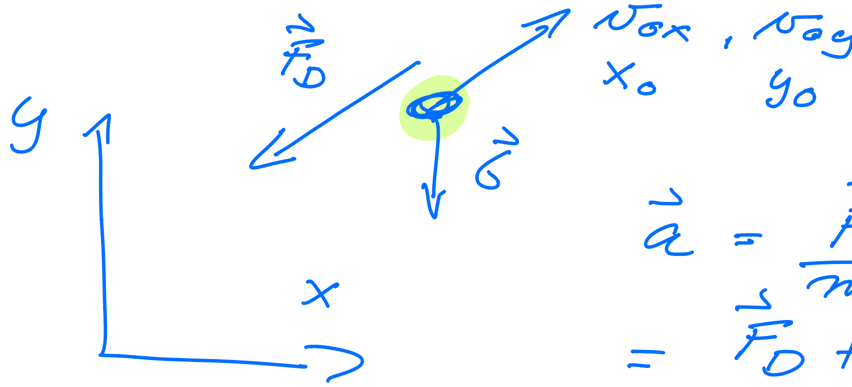
PHY 321 Lecture Feb 3



$$\vec{a} = \frac{\vec{F}^{net}}{m}$$

$$= \frac{\vec{F}_D + \vec{G}}{m}$$

$$\boxed{\begin{aligned} \frac{dv_x}{dt} &= a_x = -\gamma v_x(t) \\ \frac{dv_y}{dt} &= a_y = -g - \gamma \underline{v_y}(t) \end{aligned}}$$

$$\frac{\vec{F}_D}{m} = -\gamma \vec{v}$$

solve in at least 3 ways;

(i) $$\frac{dv_x}{v_x} = -\gamma\, dt$$

$$\int_{v_0}^{v_x} \frac{dv_x'}{v_x'} = -\gamma \int_{t_0}^{t} dt'$$

(ii) Solve the diff. eqs

$$\frac{dv_x}{dt} = -\gamma v_x$$

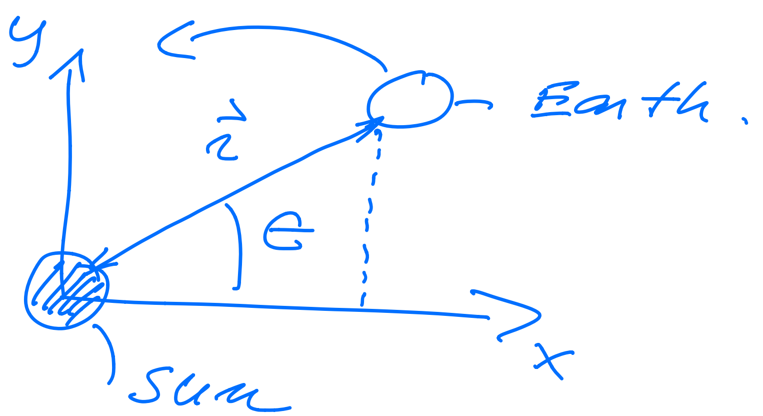
(see procedures in slides for this week)

(iii) Numerical.

$$\vec{F} = \frac{G M_{\odot} M_E}{r^3}\,\vec{r} \quad \left(\frac{\vec{r}}{|\vec{r}|}\right)$$

$$r = 1\,\text{AU} = 1.5 \cdot 10^{11}\,\text{m}$$

$$r = \sqrt{x^2+y^2} \qquad \text{2-DIM}$$



$$x = \vec{r} \cdot \cos\theta$$

$$y = \vec{r} \cdot \sin\theta$$

$$F_x = -\frac{G M_{\odot} M_E\, x}{r^3}$$

$$F_y = -\frac{G M_{\odot} M_E\, y}{r}$$

r^-

$$\frac{F_x}{M_E} = a_x = -\frac{G M_\odot}{r^3} x = \frac{dv_x}{dt}$$

$$a_y = \frac{dv_y}{dt} = -\frac{G M_\odot\, y}{r^3}$$

$$\boxed{\begin{aligned} \frac{dv_x}{dt} &= -\frac{G M_\odot\, x}{\left(\sqrt{x^2+y^2}\right)^3} \\ \frac{dv_y}{dt} &= -\frac{G M_\odot\, y}{\left(\sqrt{x^2+y^2}\right)^3} \\ \frac{dx}{dt} &= v_x \quad \wedge \quad \frac{dy}{dt} = v_y \end{aligned}}$$

in cartesian coordinates need to solve numerically?

Later: two-body problems with central forces.
Transform eqs from cartesian to polar coordinates => solve analytically

Next Example

1- Dim problem

$$F(x,v;t) = -kx$$

(Derive in harmonic oscillator)

$$F(x,v,t) = ma(x,v,t)$$

$$= ma = m \cdot \frac{d^2x}{dt^2} = -kx$$

$$= \boxed{\frac{d^2x}{dt^2} = -\underbrace{\frac{k}{m}}_{\omega_0^2} x}$$

$\omega_0 = \sqrt{k/m}$ = natural frequency

$$F = -kx$$

$[F]$ = mass·length/time^2

$$= -kx$$

$$[x] = \text{length}$$

$$[k] = \text{mass}/\text{time}^2$$

$$[\omega_0] = \left[\sqrt{\frac{\cancel{\text{mass}}}{\text{time}^2 \, \cancel{\text{mass}}}} \right]$$

$$= \frac{1}{\text{time}}$$

$$\frac{dv_x}{dt} = -kx$$

$$\frac{dx}{dt} = v_x$$ Defined by $v_{x0} \wedge x_0$

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$