PHY 321 February 17

Earth-Sun system

$$\vec{F}_i^{\,net} = \underbrace{\vec{F}_i^{\,ext}(\vec{r}_i)}_{=0} + \sum_{j\neq i}^{N} \vec{F}_{ij}$$

Two-body problem

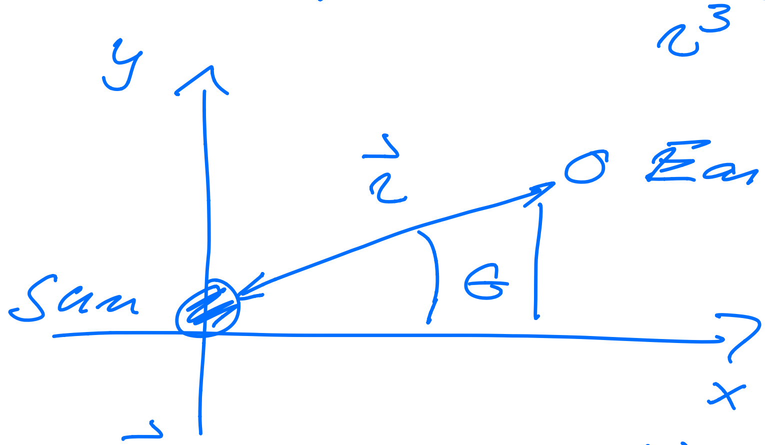
$$\vec{F}_{Earth} = -\frac{G M_\odot M_E}{|\vec{r}_\odot - \vec{r}_E|^3}(\vec{r}_\odot - \vec{r}_E)$$

relative distance

$$\vec{r} = \vec{r}_\odot - \vec{r}_E$$

$$|\vec{r}| = 1\,AU$$

$$\vec{F}_{Earth} = -\frac{G M_\odot M_E}{r^3}\,\vec{r}$$



$$\vec{F}_{Earth} = f(\vec{r})\,\vec{r}$$

central is directed toward

or away from a chosen force center".

$F_i^{ext} = 0$, only two-body forces

$\vec{F}_{Earth}$ is a conservative force

- Energy is conservation
- depends only on position
- $\underline{\vec{\nabla} \times \vec{F} = 0}$
- with only internal forces $\Rightarrow$ linear momentum
- $\dfrac{d\vec{L}}{dt} = \sum_{i=1}^{N} \sum_{j>i}^{N} (\vec{r}_i - \vec{r}_j) \times \vec{F}_{ij}$

$F_{ij}(\vec{r}) = f(\vec{r})\,\vec{r}$

$\vec{r} = \vec{r}_i - \vec{r}_j \Rightarrow$

angular momentum is conserved.

- translational invariance

$\vec{F}(\vec{r}) = f(\vec{r})\,\vec{r}$

$f(r)$ | $f(r) = \dfrac{GM_{\odot}M_E}{r^3}$

$\vec{r} = \vec{r}_i - \vec{r}_j$

$\vec{r}_i \to \vec{r}_i + \vec{a}$

$r_j \to \vec{r}_j + \vec{a}$

$\vec{r}_i - \vec{r}_j = \vec{r}_i + \vec{a} - \vec{r}_j - \vec{a} = \vec{r}_i - \vec{r}_j$

$(\vec{r})$ is the same.

- spherical symmetry. Taylor chapter 4.8

- Energy conservation

$V(\vec{r}) \to \vec{F}(\vec{r}) = -\vec{\nabla} V(\vec{r})$

$E = K + V(\vec{r})$

1-body problem

$K = \frac{1}{2} m v^2$

$$\frac{dE}{dt} = 0 = \frac{d}{dt}\left[\frac{1}{2} m v_x^2 + \frac{1}{2} m \times v_y^2 + \frac{1}{2} m v_z^2\right] + \frac{d}{dt} V(\vec{r})$$

$$= m v_x \frac{dv_x}{dt} + m v_y \frac{dv_y}{dt} + m v_z \frac{dv_z}{dt} + \frac{d}{d\vec{r}} V(\vec{r}) \frac{d\vec{r}}{dt}$$

$$\cancel{v_x} F_x + \cancel{v_y} F_y + \cancel{v_z} F_z$$

$$\underbrace{\frac{dV}{dx}}_{-F_x} \cdot \underbrace{\frac{dx}{dt}}_{v_x} + \underbrace{\frac{\partial V}{\partial y}}_{-F_y} \underbrace{\frac{dy}{dt}}_{v_y} + \underbrace{\frac{dV}{dz}}_{-F_z} \underbrace{\frac{dz}{dt}}_{v_z}$$

$$\left(\vec{F} = -\nabla V(\vec{r}) \right)$$

$$= 0 = \frac{dE}{dt}$$

$$\boxed{\frac{d}{dt} = \frac{d}{dx} \frac{dx}{dt}}$$

Earth - Sun case

2-Dim

$$r = \sqrt{x^2+y^2}$$

$$x = r \cdot \cos\theta \quad \wedge \quad y = r \sin\theta$$

$$\frac{F_{EARTH}}{M_E} = -\frac{G M_\odot}{r^3} \vec{r}$$

$$= \vec{a} = \frac{d^2 \vec{r}}{dt^2}$$

$$a_x = \frac{d^2x}{dt^2} = -\frac{GM_B}{r^3}x$$

$$a_y = \frac{d^2y}{dt^2} = -\frac{GM_B\,y}{r^3}$$

$$v_x = \frac{dx}{dt}$$

$$a_x = \frac{dv_x}{dt} = -\frac{GM_B\,x}{r^3}$$

$$v_y = \frac{dy}{dt}$$

$$a_y = \frac{dv_y}{dt} = -\frac{GM_B \cdot y}{r^3}$$

$$r = \sqrt{x^2+y^2}$$

Euler's method:

$$x_{i+1} = x_i + \Delta t \cdot v_{x_i}$$

$$v_{x_{i+1}} = v_{x_{i+1}} + \Delta t \cdot a_{x_i}$$

$$-\frac{GM_B\,x_i}{r_i^3}$$

$$\left(\sqrt{x_i^2+y_i^2}\right)$$

$$y_{i+1} = y_i + \Delta t \, v_{y_i}$$

$$v_{y_{i+1}} = v_{y_i} + \Delta t \, a_{y_i}$$

$$-\frac{GM_\odot \, y_i}{\left(\sqrt{x_i^2+y_i^2}\right)^3}$$

$$r = 1\,AU = 1{,}5 \cdot 10^{11}\,m$$

$$x_0 = 1\,AU \qquad y_0 = 0$$

Circular motion

$$\frac{M_E v^2}{r} = F = \frac{GM_\odot M_E}{r^2}$$

$$GM_\odot = v^2 r$$

$$v = 2\pi \cdot r / 1\,yr$$

$$= 2\pi \, 1\,AU / 1\,yr$$

$$GM_\odot = v^2 r = 4\pi^2 \frac{(AU)^3}{yr^2}$$

— 2 .

$$v_{y_{i+1}} = v_{y_i} - \Delta t \cdot \frac{4\pi \, y_i}{r^3}$$