# Conservative force, Momentum and more!

## Momentum

$$\vec{p}_i = m_i \vec{v}_i$$

$$\vec{p}_1 = m_1 \vec{v}_1 \quad \wedge \quad \vec{p}_2 = m_2 \vec{v}_2$$

$$\vec{P} = \sum_{i=1}^{N=2} \vec{p}_i = \vec{p}_1 + \vec{p}_2$$

Force on 1

$$\vec{F}_1 = \vec{F}_1^{ext} + \underbrace{\vec{F}_{12}}_{\text{internal force}}$$

$$\vec{F}_2 = \vec{F}_2^{ext} + \vec{F}_{21}$$

Newton's 3rd law

$$\vec{F}_{21} = -\vec{F}_{12}$$

$$\vec{F} = \sum_{i=1}^{N=2} \vec{F}_i^{ext} + \sum_{\substack{i,j \\ i \neq j}}^{N=2} \vec{F}_{ij}$$

$$\sum_{\substack{i,j \\ i \neq j}} \vec{F}_{ij} = \sum_{i}^{N} \sum_{j>i}^{N} \left( \vec{F}_{ij} + \vec{F}_{ji} \right) = 0$$

$$\boxed{\vec{F}_{ij} = -\vec{F}_{ji}}$$

$N=2$ : $\vec{F}_{12} + \vec{F}_{21} = 0$

$$\vec{F}^{net} = \sum_{i=1}^{N} \vec{F}_i^{ext}$$

$$\vec{P} = \sum_{i=1}^{N} m_i \vec{v}_i = \sum_{i=1}^{N} \vec{P}_i$$

$$\vec{F}_i^{net} = m_i \vec{a}_i = m_i \frac{d\vec{v}_i}{dt}$$

$m_i$ is time independent

$$\frac{d\vec{P}_i}{dt} = m_i \frac{d\vec{v}_i}{dt} = \vec{F}_i^{net}$$

$\Rightarrow$

$$\frac{d\vec{P}}{dt} = \sum_{i=1}^{N} \vec{F}_i^{net} = \vec{F}^{net}$$

$$= \vec{F}^{ext}$$

if $\vec{F}^{ext} = 0$ (sum over all $N$)

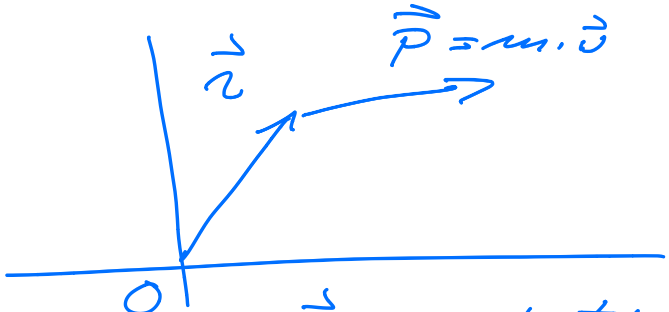
$$\frac{dP}{dt} = 0$$ what does it mean?

if net external forces sum to zero, then linear Momentum is conserved. Momentum is a constant of motion.

- angular momentum

1) single object

$$\vec{\ell} = \vec{r} \times \vec{p} \qquad \vec{p} = m\vec{v}$$




$\vec{\ell}$ is relative to -O-

$$\frac{d\vec{\ell}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

Ex 4 from hw 1

$$= \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\underbrace{\vec{v} \times m\vec{v}}_{=0}$$

$$= \vec{r} \times \underbrace{\frac{d\vec{p}}{dt}}_{= \vec{F}} = \boxed{\vec{r} \times \vec{F}}$$

$$= \vec{\tau} = \text{torque}$$

Many objects

tot angular momentum

$$\vec{L} = \sum_{i=1}^{N} \vec{l}_i = \sum_{i=1}^{N} \vec{r}_i \times \underbrace{m_i \vec{v}_i}_{\vec{p}_i}$$

$$\frac{d\vec{L}}{dt} = \underbrace{\sum_{i=1}^{N} \underbrace{\frac{d\vec{r}_i}{dt}}_{\vec{v}_i} \times m_i \vec{v}_i}_{=0}$$

$$+ \sum_{i=1}^{N} \vec{r}_i \times \frac{d\vec{v}_i}{dt} m_i$$

$$= \sum_{i=1}^{N} \vec{r}_i \times \vec{F}_i^{net}$$