# PHY321 Lecture Feb 5

Work-Energy theorem

kinetic energy

$$K = \frac{1}{2} m v^2$$

$$\vec{F} = F(\vec{r}, \vec{v}, t)$$

$$\vec{a} = \vec{F}/m$$

$$v^2 = \vec{v} \cdot \vec{v}$$

$$\frac{dK}{dt} = \frac{1}{2} m \frac{d(\vec{v}\cdot\vec{v})}{dt}$$

$$= \frac{1}{2} m \left[ \frac{d\vec{v}}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d\vec{v}}{dt} \right]$$

$$= \underbrace{m \frac{d\vec{v}}{dt}}_{\vec{F}} \vec{v} = m \dot{\vec{v}} v$$

$$\frac{dK}{dt} = \lim_{t \to 0} \frac{K_2 - K_1}{t_2 - t_1}$$

Discrete version $\Delta t = t_2 - t_1$

$$= \frac{\Delta K}{\Delta t}$$

$$= m \frac{\Delta \vec{v}}{\Delta t} \cdot \vec{v} = F \cdot \vec{v}$$

$$= \vec{F} \cdot \underbrace{\vec{v}}_{\frac{\Delta \vec{r}}{\Delta t}} = \frac{\Delta K}{\Delta t} \qquad \Bigg| \quad \vec{v} = \frac{d\vec{r}}{dt}$$

$$\Delta K = \vec{F} \cdot \Delta \vec{r}$$

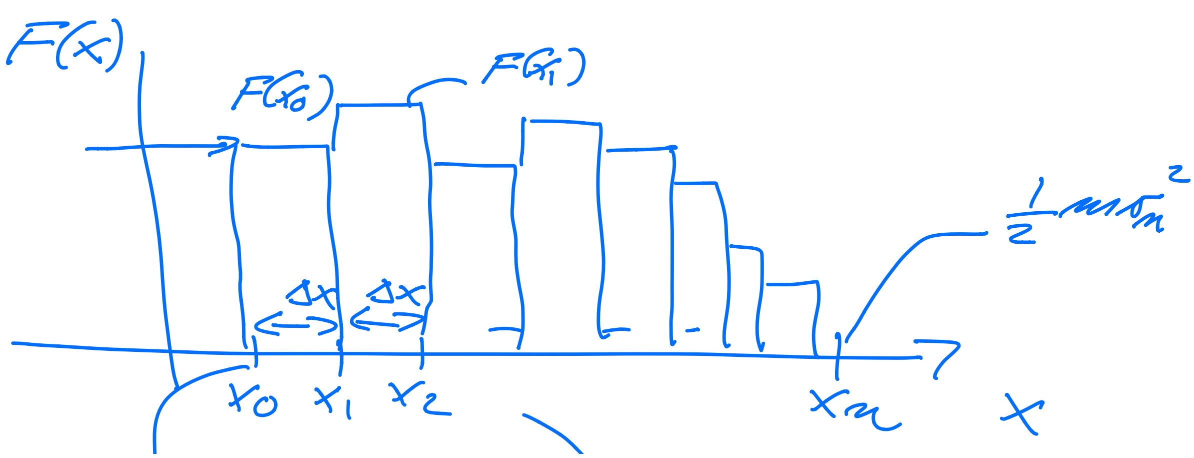
$$\Delta K = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2$$

$\vec{F} \cdot \Delta \vec{r}$ Defines work done by the force $\vec{F}$ in the displacement $\Delta \vec{r}$

$[\Delta \vec{r}] =$ length

$[\vec{F}] =$ mass · length/time$^2$

<u>1-DIM</u>



$\frac{1}{2} m v_0^2$ area $= F_i \Delta x$

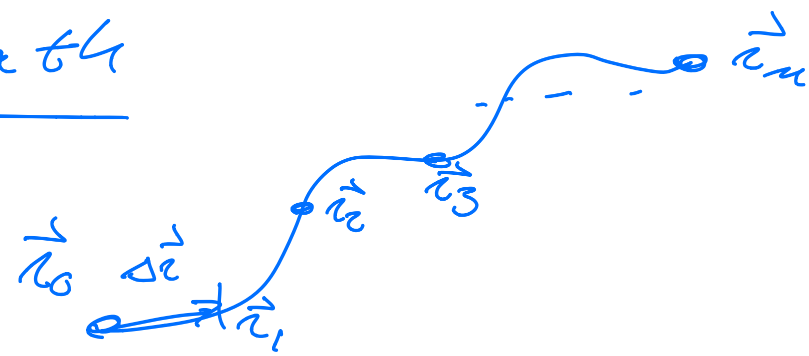
$$\frac{1}{2} m v_n^2 - \frac{1}{2} m v_0^2 = \sum_{i=0}^{n} F_i \Delta x$$

$$\lim_{\Delta x \to 0} \sum_i F_i \Delta x = \int_{x_0}^{x_n} F(x)\, dx$$

$$\frac{1}{2} m v_n^2 - \frac{1}{2} m v_0^2 = \int_{x_0}^{x_n} F(x)\, dx$$

Work-Energy theorem

<u>path</u>




$$\int_{x_0}^{x_n} F(x)\, dx \to \int_C F(\vec r)\, d\vec r$$

$$= W_C$$

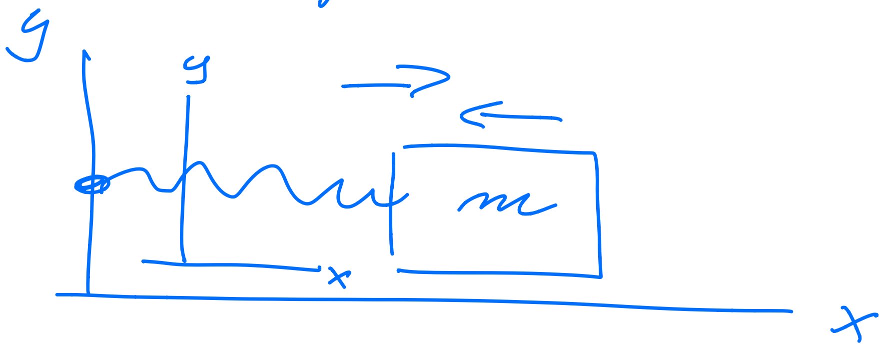
1) $\vec F \cdot \Delta \vec r$ is negative. what does that mean?

Reduced kinetic energy.

2) $\vec{F} \perp \Delta \vec{r} = 0$ ?

Force perpendicular to displacement does not change KE.

Example



$F = -kx$

we move from $x_0$ to $x_1$

$$\boxed{W_{10}} = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2$$

Dimension energy

$$= -k \int_{x_0}^{x_1} x \, dx$$

$$= -\frac{k}{2} x^2 \Big|_{x_0}^{x_1}$$

$$= -\frac{k}{2} x_1^2 + \frac{k}{2} x_0^2$$

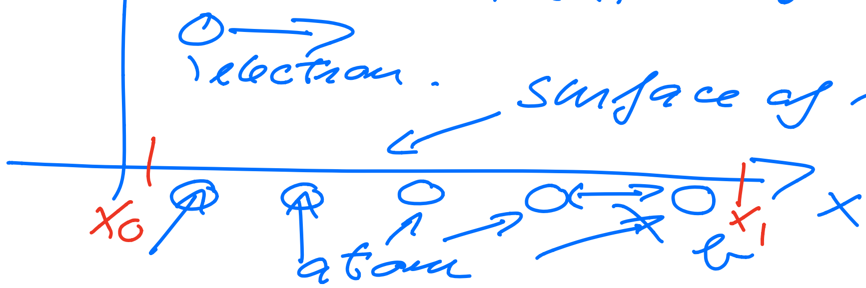
$$\boxed{\frac{1}{2} m v_1^2 + \frac{k}{2} x_1^2 = \frac{1}{2} m v_0^2 + \frac{k}{2} x_0^2}$$

kin energy at $x_1$ — potential energy at $x_1$ — at $x_0$

**Total Energy stays constant (⇒ Energy conservation)**

Example 2

$$F(x) = -F_0 \sin\frac{2\pi x}{b}$$




$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 = -\int_{x_0}^{x_1} F_0 \sin\left(\frac{2\pi x}{b}\right) dx$$

$$= \frac{F_0 b}{2\pi}\left[\cos\frac{2\pi x_1}{b} - \underbrace{\cos\frac{2\pi x_0}{b}}_{1}\right]$$

$x_0 = 0 \qquad v_0 = 0$

$$\times \frac{2}{m}: \quad v_1^2 = \frac{F_0 b}{m\pi}\left[\cos\frac{2\pi x_1}{b} - 1\right]$$