PHY 321 FEB 12

$$\vec{L} = \sum_{i=1}^{N} \vec{l}_i$$

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \sum_{i=1}^{N} \vec{\tau}_i = \sum_{i=1}^{N} \frac{d\vec{l}_i}{dt}$$

$$\vec{L} = \sum_{i=1}^{N} \vec{r}_i \times m_i \vec{v}_i$$

Ex 4 ta 1

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{N} \underbrace{\frac{d\vec{r}_i}{dt} \times m_i \vec{v}_i}_{=0}$$

$$+ \sum_{i=1}^{N} \vec{r}_i \times \underbrace{m_i \frac{d\vec{v}_i}{dt}}_{\vec{F}_i^{net}}$$

$$\vec{F}_i^{net} = \vec{F}_i^{ext} + \sum_{j\neq i}^{N} \vec{F}_{ij}$$

$$= \sum_{i=1}^{N} \left[\underline{\vec{r}_i \times \vec{F}_i^{ext}} + \sum_{j\neq i} \vec{r}_i \times \vec{F}_{ij} \right]$$

$$\sum_{i=1}^{N} \sum_{j\neq i}^{N} \vec{r}_i \times \vec{F}_{ij}$$

$$= \sum_{i=1}^{N} \sum_{j>i}^{N} \left[(\vec{r}_i \times \vec{F}_{ij}) + (\vec{r}_j \times \vec{F}_{ji}) \right]$$

$N = 2$:

$$\vec{r}_1 \times \vec{F}_{12} + \vec{r}_2 \times \vec{F}_{21} \qquad \vec{F}_{12} = -\vec{F}_{21}$$

$$= (\vec{r}_1 - \vec{r}_2)\, \vec{F}_{12}$$

$$= \sum_{i=1}^{N} \sum_{j>i}^{N} (\vec{r}_i - \vec{r}_j)\, \vec{F}_{ij}$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{N} (\vec{r}_i \times \vec{F}_i^{\,ext}) + \sum_{i=1}^{N} \sum_{j>i}^{N} (\vec{r}_i - \vec{r}_j)\, \vec{F}_{ij}$$

Earth–Sun :

$$\vec{F}_{E\odot} = -\frac{G M_\odot M_E}{|\vec{r}_\odot - \vec{r}_E|^3} (\vec{r}_\odot - \vec{r}_E)$$

$|r_\odot - r_E|$

$\vec{F}_i^{ext} = 0$

$N = 2$

$$\frac{dL}{dt} = (\vec{r}_\odot - \vec{r}_E) \times \vec{F}_{E\odot}$$

$$= 0$$

if the force is given by $\vec{F}_{ij} = c_{ij}(\vec{r}_i - \vec{r}_j)$ (central force), then $\vec{L}$ is conserved!

(Note : no external forces)

Conservation of Energy & Conservative forces.

(i) $\vec{F} = \vec{F}(\vec{r})$

(ii)

(ii) for any points 1, 2 the work done between 1 and 2, is independent of the path:

$$W_{12} = \int_1^2 \vec{F}(\vec{r})\, d\vec{r}$$

(iii) $\vec{\nabla} \times \vec{F} = 0$

Definition of potential energy ($E = K + V$)

Potential energy at $\vec{r}$

$$V(\vec{r}) = -W(\vec{r}_0 \to \vec{r})$$

$$= -\int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r}\,')\, d\vec{r}\,'$$

$$W(\vec{r} \Rightarrow \vec{r} + d\vec{r}) = \vec{F}(\vec{r})\, d\vec{r}$$

$$= F_x \cdot dx + F_y\, dy + F_z\, dz$$

$$W(\vec{r} \Rightarrow \vec{r} + d\vec{r}) =$$

$$-\left[V(\vec{r} + d\vec{r}) - V(\vec{r})\right]$$

$$= -\left[V(x+dx, y+dy, z+dz) - V(x,y,z) \right]$$

$$\boxed{\begin{aligned} df &= f(x+dx) - f(x) \\ &= \frac{df}{dx}\, dx \end{aligned}}$$

$$\begin{aligned} dV &= V(x+dx, y+dy, z+dz) - V(x,y,z) \\ &= \underbrace{\frac{\partial V}{\partial x}}_{F_x} dx + \underbrace{\frac{\partial V}{\partial y}}_{F_y} dy + \underbrace{\frac{\partial V}{\partial z}}_{F_z} dz \end{aligned}$$

$$\vec{F} = -\frac{\partial V}{\partial x}\vec{e}_1 - \frac{\partial U}{\partial y}\vec{e}_2 - \frac{\partial U}{\partial z}\vec{e}_z$$

$$= -\vec{\nabla} V(\vec{r})$$

Example 1

$$V(x,y,z) = \underline{\gamma}\, xy^2 + \delta \sin z$$

$$\vec{F} = -\left(\gamma y^2 \vec{e}_1 + \gamma 2xy\,\vec{e}_2 + \delta \cos z\, \vec{e}_3\right)$$

Example 2

$v = \beta / x$ (1-Dim)

Assume at t_0 we have

$$E_0 = \frac{1}{2} m v^2 + V(x)$$

$$= \frac{1}{2} m \beta^2 / x^2 + V(x)$$

How do we find $V(x)$?

$$\frac{dE_0}{dx} = 0 = -m\beta^2/x^3 + \frac{dV}{dx}$$

$$\Rightarrow \frac{dV}{dx} = -F(x) = m\beta^2/x^3$$

$$F(x) = -m\beta^2/x^3$$

(Exercise 5 in HW 4)

$$\vec{F} = -\vec{\nabla} V(\vec{r})$$

Example 3

$$V(x, y, z) = A e^{-(x+z)}$$

$$F_x = -\frac{\partial V}{\partial x} = A e^{-(x+z)}$$

$$F_y = -\frac{\partial V}{\partial y} = 0$$

$$F_z = -\frac{\partial U}{\partial z} = A e^{-(x+z)}$$

is linear Momentum $\vec{p}$ conserved?

$$\frac{d\vec{p}}{dt} = \vec{F}$$

$$\frac{dp_x}{dt} = \underline{F_x} \neq 0 \qquad \frac{dp_y}{dt} = 0 \qquad \frac{dp_z}{dt} = F_z \neq 0$$

$$\boxed{\begin{aligned} U &= A e^{-(x+z)} \\ \frac{dU}{dx} &= -A e^{-(x+z)} \\ F_x &= -\frac{dU}{dx} = A e \\ U &= A e^{-\left(\frac{x^2+z^2}{a}\right)} \\ F_x &= -\frac{dU}{dx} = +\frac{2x}{a} A e^{-\left(\frac{x^2+z^2}{a}\right)} \end{aligned}}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{} = \vec{r} \times \vec{F}^{net} = \vec{r} \times \vec{F}.$$

$\overline{dt}$ $\qquad \vec{r} = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$

$$= (yF_z - z\underbrace{F_y}_{=0})\vec{e}_1 \qquad F_x \neq 0$$

$$+ (xF_z - zF_x)\vec{e}_2 \qquad F_y = 0$$

$$F_z \neq 0$$

$$+ (yF_z - zF_y)\vec{e}_3$$

$$yF_z\vec{e}_1 + (xF_z - zF_x)\vec{e}_2$$

$$+ yF_z\vec{e}_3$$

no conservation of

$\vec{L}$