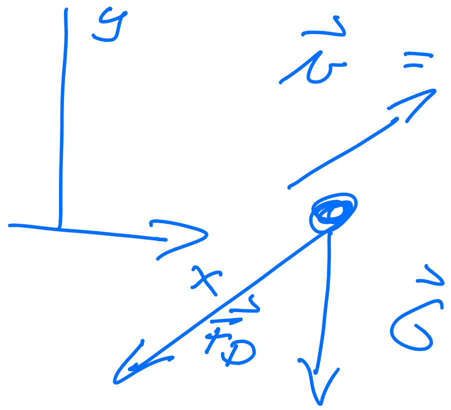
PHY321 Lecture Feb 1

$$\vec{v} = (v_x, v_y)$$



initial velocity

$$v_x(t_0) = v_{0x}$$

$$v_y(t_0) = v_{0y}$$

$$t_0 = 0$$

$$\vec{F}_D = -m\gamma\vec{v}(t)$$

$$\vec{G} = -\underline{m \cdot g\,\vec{e}_2} \quad (\vec{e}_y, \vec{j})$$

initial position

$$x(t_0) = x_0 \wedge y(t_0) = y_0$$

$$\vec{F}_{net} = \vec{F}_D + \vec{G}$$

$$\vec{a} = \vec{F}/m$$

$$a_x = \frac{dv_x}{dt} = -\gamma v_x(t) = a_x(t)$$

$$a_y = \frac{dv_y}{dt} = -\gamma v_y(t) - g$$

$$\frac{dx}{dt} = v_x(t) \wedge \frac{dy}{dt} = v_y(t)$$

dt dt

$$\vec{F}_{net} = m \frac{d^2\vec{r}(t)}{dt^2}$$

2-Dim $\Rightarrow$ 4 coupled differential equations where x & y (degrees of freedom) are decoupled.

$$\frac{dv_x}{dt} = -\gamma v_x$$

$$\frac{dv_x}{v_x} = -\gamma dt$$

$$\int_{v_{0x}}^{v_x(t)} \frac{dv_x'}{v_x'} = -\gamma \int_0^t dt'$$

$$\ln \frac{v_x(t)}{v_{0x}} = -\gamma \cdot t$$

$$\frac{v_x(t)}{v_{0x}} = e^{-\gamma \cdot t}$$

$$v_x(t) = v_{0x} e^{-\gamma \cdot t}$$

$$\frac{dx}{dt} = v_x(t)$$

$$\int_{x_0}^{x(t)} dx' = \int_{t_0=0}^{t} dt'\, v_x(t')$$

$$x(t) - x_0 = \int_0^t dt'\, \underline{v_{0x}}\, e^{-\gamma t}$$

$$\boxed{x(t) = x_0 + \frac{v_{0x}}{\gamma}\left(1 - e^{-\gamma t}\right)}$$

y-component (Taylor chapter 2.1-2.3)

$$\frac{dv_y}{-\gamma v_y(t) - g} = dt$$

$$-\gamma \qquad \frac{dv_y}{+v_y(t) + g/\gamma} = -\gamma\, dt$$

$$\int_{v_{0y}}^{v_y(t)} \frac{dv_y'}{v_y' + g/\gamma} = -\gamma \int_0^t dt'$$

$$\ln\left(\frac{v_y(t)+g/\gamma}{v_{0y}+g/\gamma}\right) = -\gamma t$$

$$v_y(t) = -\underline{g/\gamma} + \underline{e^{-\gamma t}(v_{0y}+g/\gamma)}$$

$v_y(t)$ decays exponentially to $-g/\gamma$

$$\frac{dy(t)}{dt} = v_y(t)$$

$$\int_{y_0}^{y(t)} dy'(t) = \int_0^t v_y(t')\,dt'$$

$$y(t) = y_0 - g\,t/\gamma + (v_{0y}+g/\gamma)(1-e^{-\gamma t})\frac{1}{\gamma}$$

python technicality

HW2 (and needed in hw3)

$\vec{r}(t)$ how do we write it in Python?

n - discrete time steps:

in 1-Dim

r = np.zeros(n)

in 2-Dim

r = np.zeros((n,2))

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                      x0     y0
r0 = np.array([0.0, 10.0])
                 x-value  y-value
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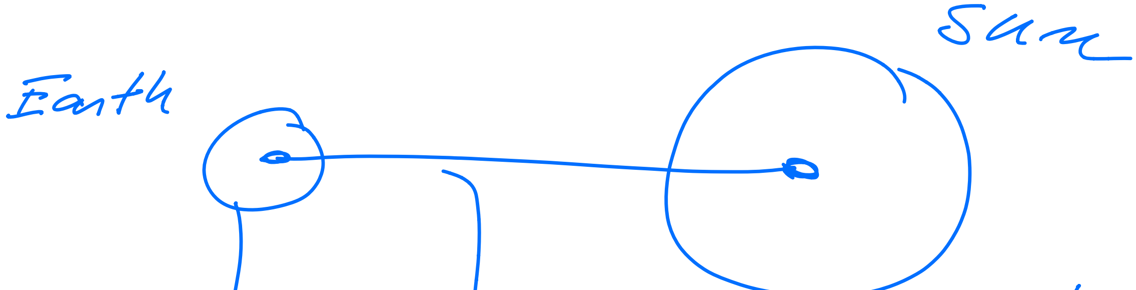
$$\boxed{r[0] = r_0}$$

x_0

Long-range Force:

Gravitational



$r = 1\,AU = 1.5 \cdot 10^{11}\,m$

$M_E = 6 \times 10^{24}\,kg$

$$M_\odot = 2 \cdot 10 \ kg$$

$$\vec{F} = G \cdot \frac{M_\odot M_E \cdot \vec{r}}{r^3}$$

2-DIM

$$r = \sqrt{x^2+y^2}$$