Stanford University ICPC Team Notebook (2015-16)

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1 Combinatorial optimization

1.1 Edmonds-Karp max-flow

```
#include <bits/stdc++.h>
using namespace std;

template<typename D>
struct EdmondsKarp {
    static constexpr D Inf = 11 << 60;

    struct Edge {
        size_t from, to; D cap;
    }:</pre>
```

```
size_t N;
     vector<Edge> edges;
     vector<vector<size_t>> outs;
     EdmondsKarp(size_t n) : N(n) {
          outs.resize(n);
     void add(size_t from, size_t to, D cap) {
          assert (from < N);
          assert (to < N);
          if (cap > 0) {
               edges.emplace_back(Edge { from, to, cap });
outs[from] push_back(edges.size() - 1);
edges.emplace_back(Edge { to, from, 0 });
               outs[to].push_back(edges.size() - 1);
     vector<D> amount;
     vector<size_t> route;
     vector<size_t> visiting;
     D solve(size_t from, size_t to) {
          amount.resize(N);
          route.resize(N):
          visiting.resize(N):
          D flow = 0;
               fill(amount.begin(), amount.end(), 0);
               size_t head = 0, tail = 0;
               amount[from] = Inf;
               visiting[tail++] = from;
               while (head < tail && !amount[to]) {
    size_t i = visiting[head++];</pre>
                    for (size_t j = 0; j < outs[i].size(); ++j) {
   Edge &e = edges[outs[i][j]];</pre>
                         if (!amount[e.to] && e.cap) {
   amount[e.to] = min(amount[i], e.cap);
   route[e.to] = outs[i][j];
                               visiting[tail++] = e.to;
               if (!amount[to]) break;
               for (size_t i = to; i != from; i = edges[route[i]].from) {
                    edges[route[i]].cap -= amount[to];
edges[route[i] ^ 1].cap += amount[to];
               flow += amount[to];
          return flow;
};
```

1.2 Dinic max-flow

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
// O(IV|^2 |E|)
// INPUT:
// - graph, constructed using AddEdge()
// - source and sink
//
// OUTPUT:
// - maximum flow value
// - To obtain actual flow values, look at edges with capacity > 0
// (zero capacity edges are residual edges).
#include<cstdio>
#include<cyuctor>
#include<queue>
```

```
using namespace std;
typedef long long LL;
struct Edge {
  int u, v;
  LL cap, flow;
  Edge () {}
 Edge(int u, int v, LL cap): u(u), v(v), cap(cap), flow(0) {}
};
struct Dinic {
 int N;
  vector<Edge> E;
  vector<vector<int>> g;
  vector<int> d, pt;
  Dinic(int N): N(N), E(0), g(N), d(N), pt(N) {}
  void AddEdge(int u, int v, LL cap) {
    if (u != v) {
      E.emplace_back(u, v, cap);
      g[u].emplace_back(E.size() - 1);
      E.emplace_back(v, u, 0);
      g[v].emplace_back(E.size() - 1);
  bool BFS(int S, int T) {
    queue<int> q({S});
    fill(d.begin(), d.end(), N + 1);
    d[S] = 0;
    while(!q.empty()) {
  int u = q.front(); q.pop();
      if (u == T) break;
      for (int k: g[u]) {
        Edge &e = E[k];
        if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
          d[e.v] = d[e.u] + 1;
          q.emplace(e.v);
    return d[T] != N + 1;
  LL DFS(int u, int T, LL flow = -1) {
    if (u == T || flow == 0) return flow;
    for (int &i = pt[u]; i < g[u].size(); ++i) {</pre>
      Edge &e = E[g[u][i]];
      Edge &oe = E[g[u][i]^1];
      if (d[e.v] == d[e.u] + 1) {
        LL amt = e.cap - e.flow;
if (flow != -1 && amt > flow) amt = flow;
        if (LL pushed = DFS(e.v, T, amt)) {
         e.flow += pushed;
oe.flow -= pushed;
          return pushed;
      }
    return 0;
  LL MaxFlow(int S, int T) {
    LL total = 0;
    while (BFS(S, T)) {
      fill(pt.begin(), pt.end(), 0);
      while (LL flow = DFS(S, T))
       total += flow;
    return total:
};
// The following code solves SPOJ problem #4110: Fast Maximum Flow (FASTFLOW)
  scanf("%d%d", &N, &E);
  Dinic dinic(N);
  for(int i = 0; i < E; i++)
    int u. v:
    LL cap;
    scanf("%d%d%lld", &u, &v, &cap);
    dinic AddEdge(u - 1, v - 1, cap);
    dinic.AddEdge(v - 1, u - 1, cap);
  printf("%lld\n", dinic.MaxFlow(0, N - 1));
```

return 0;

1.3 Min-cost max-flow

// END CUT

```
// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972). This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]). For a regular max flow, set all edge costs to 0.
// Running time, O(|V|^2) cost per augmentation
                         O(|V|^3) augmentations
      max flow:
       min cost max flow: O(|V|^4 * MAX_EDGE_COST) augmentations
// INPUT:
       - graph, constructed using AddEdge()
       - source
       - (maximum flow value, minimum cost value)
       - To obtain the actual flow, look at positive values only.
#include <bits/stdc++.h>
using namespace std:
typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;
const L INF = numeric_limits<L>::max() / 4;
struct MinCostMaxFlow (
    int N:
    VVL cap, flow, cost;
    VI found:
    VL dist, pi, width;
    VPII dad;
    MinCostMaxFlow(int N) :
        N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
        found(N), dist(N), pi(N), width(N), dad(N) {}
    void AddEdge(int from, int to, L cap, L cost) {
        this->cap[from][to] = cap;
        this->cost[from][to] = cost;
    void Relax(int s, int k, L cap, L cost, int dir) {
    L val = dist[s] + pi[s] - pi[k] + cost;
    if (cap && val < dist[k]) {</pre>
            dist[k] = val;
            dad[k] = make_pair(s, dir);
            width[k] = min(cap, width[s]);
    L Dijkstra(int s, int t) {
        fill(found.begin(), found.end(), false);
        fill(dist.begin(), dist.end(), INF);
        fill(width.begin(), width.end(), 0);
        dist[s] = 0:
        width[s] = INF:
        while (s != -1) {
            int best = -1;
             found[s] = true;
            for (int k = 0; k < N; k++) {
                if (found[k]) continue;
                 Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
                 Relax(s, k, flow[k][s], -cost[k][s], -1);
                 if (best == -1 || dist[k] < dist[best]) best = k;
            s = best;
        for (int k = 0; k < N; k++)
pi[k] = min(pi[k] + dist[k], INF);</pre>
        return width[t];
    pair<L, L> GetMaxFlow(int s, int t) {
```

```
L totflow = 0, totcost = 0;
        while (L amt = Dijkstra(s, t)) {
            totflow += amt;
            for (int x = t; x != s; x = dad[x].first) {
                if (dad[x].second == 1) {
                    flow[dad[x].first][x] += amt;
                    totcost += amt * cost[dad[x].first][x];
                    flow[x][dad[x].first] -= amt;
                    totcost -= amt * cost[x][dad[x].first];
        return make_pair(totflow, totcost);
};
// BEGIN CUT
// The following code solves UVA problem #10594: Data Flow
    int N, M;
    while (scanf("%d%d", &N, &M) == 2) {
        VVL v(M, VL(3));
        for (int i = 0; i < M; i++)
            scanf("%Ld%Ld%Ld", &v[i][0], &v[i][1], &v[i][2]);
        LD.K:
        scanf("%Ld%Ld", &D, &K);
        MinCostMaxFlow mcmf (N+1);
        for (int i = 0; i < M; i++) {
            mcmf.AddEdge(int(v[i][0]), int(v[i][1]), K, v[i][2]);
            mcmf.AddEdge(int(v[i][1]), int(v[i][0]), K, v[i][2]);
        mcmf.AddEdge(0, 1, D, 0);
        pair<L, L> res = mcmf.GetMaxFlow(0, N);
        if (res.first == D) {
            printf("%Ld\n", res.second);
        } else {
            printf("Impossible.\n");
    return 0;
// END CUT
```

1.4 Push-relabel max-flow

```
//\ {\tt Adjacency\ list\ implementation\ of\ FIFO\ push\ relabel\ maximum\ flow}
// with the gap relabeling heuristic. This implementation is // significantly faster than straight Ford-Fulkerson. It solves
// random problems with 10000 vertices and 1000000 edges in a few
// seconds, though it is possible to construct test cases that
// achieve the worst-case.
// Running time:
      0(|V|^3)
       - graph, constructed using AddEdge()
       - sink
        - maximum flow value
        - To obtain the actual flow values, look at all edges with
         capacity > 0 (zero capacity edges are residual edges).
#include <cmath>
#include <vector>
#include <iostream>
#include <queue>
using namespace std;
typedef long long LL;
struct Edge {
  int from, to, cap, flow, index;
  Edge(int from, int to, int cap, int flow, int index) :
    from(from), to(to), cap(cap), flow(flow), index(index) {}
```

```
struct PushRelabel {
  int N;
  vector<vector<Edge> > G;
  vector<LL> excess;
  vector<int> dist, active, count;
  queue<int> Q;
  PushRelabel(int N): N(N), G(N), excess(N), dist(N), active(N), count(2*N) {}
  void AddEdge(int from, int to, int cap)
    G[from].push\_back(Edge(from, to, cap, 0, G[to].size()));
    if (from == to) G[from].back().index++;
    G[to].push\_back(Edge(to, from, 0, 0, G[from].size() - 1));
  void Enqueue(int v) {
    if (!active[v] && excess[v] > 0) { active[v] = true; Q.push(v); }
  void Push (Edge &e) {
    int amt = int(min(excess[e.from], LL(e.cap - e.flow)));
    if (dist[e.from] <= dist[e.to] || amt == 0) return;</pre>
    e.flow += amt:
    G[e.to][e.index].flow -= amt;
    excess[e.to] += amt;
    excess[e.from] -= amt:
    Enqueue (e.to);
  void Gap(int k) {
    for (int v = 0; v < N; v++) {
      if (dist[v] < k) continue;</pre>
       count[dist[v]]--;
      dist[v] = max(dist[v], N+1);
      count[dist[v]]++;
      Enqueue (v);
  void Relabel(int v) {
    count[dist[v]]--;
    dist[v] = 2*N;
for (int i = 0; i < G[v].size(); i++)
  if (G[v][i].cap - G[v][i].flow > 0)
        dist[v] = min(dist[v], dist[G[v][i].to] + 1);
    count[dist[v]]++;
    Enqueue (v);
  void Discharge(int v) {
    for (int i = 0; excess[v] > 0 && i < G[v].size(); i++) Push(G[v][i]);</pre>
    if (excess[v] > 0) {
      if (count[dist[v]] == 1)
         Gap(dist[v]);
      else
         Relabel(v):
  LL GetMaxFlow(int s, int t) {
    count[0] = N-1;
    count[N] = 1;
    dist[s] = N;
dist[s] = N;
active[s] = active[t] = true;
for (int i = 0; i < G[s].size(); i++) {
    excess[s] += G[s][i].cap;</pre>
      Push (G[s][i]);
    while (!Q.empty()) {
  int v = Q.front();
      Q.pop();
      active[v] = false;
      Discharge(v);
    for (int i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;</pre>
    return totflow;
};
// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (FASTFLOW)
int main() {
  int n. m:
  scanf("%d%d", &n, &m);
  PushRelabel pr(n);
  for (int i = 0; i < m; i++) {
```

int a. b. c:

```
scanf("%d%d%d", &a, &b, &c);
if (a == b) continue;
pr.AddEdge(a-1, b-1, c);
pr.AddEdge(b-1, a-1, c);
}
printf("%Ld\n", pr.GetMaxFlow(0, n-1));
return 0;
}
// END CUT
```

1.5 Min-cost matching

```
// Min cost bipartite matching via shortest augmenting paths
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
    cost[i][j] = cost for pairing left node i with right node j
    Lmate[i] = index of right node that left node i pairs with
    Rmate[j] = index of left node that right node j pairs with
/// The values in cost[i][j] may be positive or negative. To perform // maximization, simply negate the cost[][] matrix.
#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>
using namespace std;
typedef vector < double > VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
  int n = int(cost.size());
  // construct dual feasible solution
  VD u(n);
  VD v(n);
  for (int i = 0; i < n; i++) {
    u[i] = cost[i][0];
    for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
  // construct primal solution satisfying complementary slackness
  Lmate = VI(n, -1);
  Rmate = VI(n, -1);
  int mated = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
   if (Rmate[j] != -1) continue;</pre>
      if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {</pre>
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break:
  VD dist(n);
  VI dad(n);
  VI seen(n);
  // repeat until primal solution is feasible
  while (mated < n) {
    // find an unmatched left node
    int s = 0:
    while (Lmate[s] != -1) s++;
    // initialize Diikstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
for (int k = 0; k < n; k++)</pre>
      dist[k] = cost[s][k] - u[s] - v[k];
```

```
int j = 0;
  while (true) {
    // find closest
    for (int k = 0; k < n; k++) {
      if (seen[k]) continue;
      if (j == -1 || dist[k] < dist[j]) j = k;</pre>
    seen[j] = 1;
    // termination condition
    if (Rmate[j] == -1) break;
    // relax neighbors
    const int i = Rmate[j];
    for (int k = 0; k < n; k++) {
      if (seen[k]) continue;
      const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
      if (dist[k] > new_dist) {
        dist[k] = new_dist;
        dad[k] = j;
  // update dual variables
  for (int k = 0; k < n; k++) {
   if (k == j || !seen[k]) continue;
const int i = Rmate[k];
    v[k] += dist[k] - dist[j];
    u[i] -= dist[k] - dist[j];
  u[s] += dist[j];
  // augment along path
  while (dad[j] \ge 0) {
    const int d = dad[j];
    Rmate[j] = Rmate[d];
    Lmate[Rmate[j]] = j;
    j = d;
 Rmate[j] = s;
Lmate[s] = j;
  mated++;
double value = 0;
for (int i = 0; i < n; i++)
  value += cost[i][Lmate[i]];
return value:
```

1.6 Max bipartite matchine

```
// This code performs maximum bipartite matching.
// Running time: O(|E| |V|) -- often much faster in practice
     INPUT: w[i][j] = edge between row node i and column node j
    OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
             mc[j] = assignment for column node j, -1 if unassigned
             function returns number of matches made
#include <vector>
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
  for (int j = 0; j < w[i].size(); j++) {</pre>
    if (w[i][j] && !seen[j]) {
      seen[j] = true;
      if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {</pre>
       mr[i] = j;
mc[j] = i;
        return true:
  return false;
```

1.7 Global min-cut

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
      0(|V|^3)
// INPUT:
       - graph, constructed using AddEdge()
      - (min cut value, nodes in half of min cut)
#include <cmath>
#include <vector>
#include <iostream>
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
const int INF = 10000000000:
pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;
  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {</pre>
      prev = last;
      last = -1;
      for (int j = 1; j < N; j++)
        if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
      if (i == phase-1) {
        for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j]; for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
        used[last] = true;
        cut.push back(last):
        if (best_weight == -1 || w[last] < best_weight) {</pre>
          best cut = cut;
          best_weight = w[last];
      } else {
        for (int j = 0; j < N; j++)
          w[j] += weights[last][j];
        added[last] = true;
  return make_pair(best_weight, best_cut);
// The following code solves UVA problem #10989: Bomb, Divide and Conquer
int main() {
  int N;
  for (int i = 0; i < N; i++) {
    int n, m;
    cin >> n >> m;
    VVI weights(n, VI(n));
    for (int j = 0; j < m; j++) {
      int a, b, c;
      weights[a-1][b-1] = weights[b-1][a-1] = c;
    pair<int, VI> res = GetMinCut(weights);
    cout << "Case #" << i+1 << ": " << res.first << endl;
```

1.8 Graph cut inference

// END CUT

```
// Special-purpose \{0,1\} combinatorial optimization solver for
// problems of the following by a reduction to graph cuts:
           minimize
                              sum_i psi_i(x[i])
// x[1]...x[n] in {0,1} + sum_{i < j} phi_{ij}(x[i], x[j])
    psi_i : {0, 1} --> R
phi_{ij} : {0, 1} x {0, 1} --> R
// such that
    phi_{ij}(0,0) + phi_{ij}(1,1) \le phi_{ij}(0,1) + phi_{ij}(1,0) (*)
// This can also be used to solve maximization problems where the
// direction of the inequality in (\star) is reversed.
// INPUT: phi -- a matrix such that phi[i][j][u][v] = phi_{ij}(u, v)
           psi -- a matrix such that psi[i][u] = psi_i(u)
            x -- a vector where the optimal solution will be stored
// OUTPUT: value of the optimal solution
// To use this code, create a GraphCutInference object, and call the
// DoInference() method. To perform maximization instead of minimization,
// ensure that #define MAXIMIZATION is enabled.
#include <vector>
#include <iostream>
using namespace std;
typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;
typedef vector<VVVI> VVVVI;
const int INF = 1000000000;
 // comment out following line for minimization
#define MAXIMIZATION
struct GraphCutInference {
  int N;
  VVI cap, flow;
  VI reached;
  int Augment(int s, int t, int a) {
    reached[s] = 1;
    if (s == t) return a;
    if (int k = 0; k < N; k++) {
   if (reached[k]) continue;
   if (int a = min(a, cap[s][k] - flow[s][k])) {
      if (int b = Augment(k, t, aa)) {</pre>
           flow[s][k] += b;
flow[k][s] -= b;
           return b;
    return 0;
  int GetMaxFlow(int s, int t) {
    N = cap.size();
    flow = VVI(N, VI(N));
    reached = VI(N);
    int totflow = 0;
    while (int amt = Augment(s, t, INF)) {
       totflow += amt;
       fill(reached.begin(), reached.end(), 0);
    return totflow;
  int DoInference(const VVVVI &phi, const VVI &psi, VI &x) {
    int M = phi.size();
    cap = VVI(M+2, VI(M+2));
VI b(M);
    int c = 0;
    for (int i = 0; i < M; i++) {</pre>
      b[i] += psi[i][1] - psi[i][0];
```

```
c += psi[i][0];
      for (int j = 0; j < i; j++)
        b[i] += phi[i][j][1][1] - phi[i][j][0][1];
      for (int j = i+1; j < M; j++) {
        cap[i][j] = phi[i][j][0][1] + phi[i][j][1][0] - phi[i][j][0][0] - phi[i][j][1][1];
        b[i] += phi[i][j][1][0] - phi[i][j][0][0];
        c += phi[i][j][0][0];
#ifdef MAXIMIZATION
    for (int i = 0; i < M; i++) {
  for (int j = i+1; j < M; j++)</pre>
        cap[i][j] *= -1;
      b[i] \star = -1;
    c *= -1;
#endif
    for (int i = 0; i < M; i++) {</pre>
      if (b[i] >= 0) {
        cap[M][i] = b[i];
      } else {
        cap[i][M+1] = -b[i];
        c += b[i];
    int score = GetMaxFlow(M, M+1);
    fill(reached.begin(), reached.end(), 0);
    Augment (M, M+1, INF);
    x = VI(M);
    for (int i = 0; i < M; i++) x[i] = reached[i] ? 0 : 1;</pre>
    score += c;
#ifdef MAXIMIZATION
    score *= -1;
#endif
    return score;
};
int main() {
  // solver for "Cat vs. Dog" from NWERC 2008
  int numcases;
  cin >> numcases;
  for (int caseno = 0; caseno < numcases; caseno++) {
    int c, d, v;
    cin >> c >> d >> v;
    VVVVI phi(c+d, VVVI(c+d, VVI(2, VI(2))));
    VVI psi(c+d, VI(2));
for (int i = 0; i < v; i++) {</pre>
      char p, q;
      int u, v;
      cin >> p >> u >> q >> v;
      u--: v--:
        phi[u][c+v][0][0]++;
         phi[c+v][u][0][0]++;
      } else {
        phi[v][c+u][1][1]++;
         phi[c+u][v][1][1]++;
    GraphCutInference graph;
    VT x:
    cout << graph DoInference(phi, psi, x) << endl;</pre>
  return 0;
```

2 Geometry

2.1 Convex hull

```
// Compute the 2D convex hull of a set of points using the monotone chain // algorithm. Eliminate redundant points from the hull if REMOVE_REDUNDANT is // #defined.
```

```
// Running time: O(n log n)
     INPUT: a vector of input points, unordered.
     OUTPUT: a vector of points in the convex hull, counterclockwise, starting
               with bottommost/leftmost point
#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>
 // BEGIN CUT
#include <map>
// END CUT
using namespace std;
#define REMOVE_REDUNDANT
typedef double T;
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) < make_pair(rhs.y,rhs.x); }</pre>
  bool operator==(const PT &rhs) const { return make_pair(y,x) == make_pair(rhs.y,rhs.x); }
};
T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }
#ifdef REMOVE REDUNDANT
bool between (const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
#endif
void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  vectors:v up, dn,
for (int i = 0; i < pts.size(); i++) {
   while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
   while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();</pre>
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  pts = dn:
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);
#ifdef REMOVE REDUNDANT
  if (pts.size() <= 2) return;</pre>
  dn.clear();
  dn.push_back(pts[0]);
dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {</pre>
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
    dn.push back(pts[i]);
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  pts = dn;
#endif
// BEGIN CUT
// The following code solves SPOJ problem #26: Build the Fence (BSHEEP)
int main() {
  int t;
  scanf("%d", &t);
  for (int caseno = 0; caseno < t; caseno++) {</pre>
    int n;
    scanf("%d", &n);
    vector<PT> v(n);
    for (int i = 0; i < n; i++) scanf("%lf%lf", &v[i].x, &v[i].y);</pre>
    map<PT,int> index;
    for (int i = n-1; i >= 0; i--) index[v[i]] = i+1;
ConvexHull(h);
    double len = 0;
    for (int i = 0; i < h.size(); i++) {</pre>
      double dx = h[i].x - h[(i+1) h.size()].x;
      double dy = h[i].y - h[(i+1)%h.size()].y;
      len += sqrt (dx*dx+dy*dy);
    if (caseno > 0) printf("\n");
```

```
printf("%.2f\n", len);
for (int i = 0; i < h.size(); i++) {
   if (i > 0) printf(" ");
     printf("%d", index[h[i]]);
   }
printf("\n");
}
// END CUT
```

2.2 Miscellaneous geometry

```
// C++ routines for computational geometry.
#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>
using namespace std;
double INF = 1e100;
double EPS = 1e-12;
struct PT (
  double x, y;
  PT (double x, double y) : x(x), y(y) {}
  PT (const PT &p) : x(p.x), y(p.y)
  PT operator + (const PT &p) const { return PT(x+p.x, y+p.y);
  PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); ]
  PT operator * (double c)
                                const { return PT(x*c, y*c );
  PT operator / (double c)
                                const { return PT(x/c, y/c ); }
double dot(PT p, PT q)
                            { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)
                            { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    return os << "(" << p.x << "," << p.y << ")";
// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90 (PT p)
                        { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a) *dot(c-a, b-a) /dot(b-a, b-a);
// project point c onto line segment through a and b
PT ProjectPointSegment (PT a, PT b, PT c) {
  double r = dot(b-a, b-a);
  if (fabs(r) < EPS) return a;</pre>
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;</pre>
  if (r > 1) return b;
  return a + (b-a) *r;
// compute distance from c to segment between a and b
double DistancePointSegment (PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane (double x, double y, double z,
                           double a, double b, double c, double d)
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
&& fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
```

```
// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||</pre>
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
     return false;
    return true:
 if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true:
// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
 b=b-a; d=c-d; c=c-a;
  assert (dot (b, b) > EPS && dot (d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
 b = (a+b)/2;
  c = (a+c)/2;
 return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// \ {\it integer \ arithmetic \ by \ taking \ care \ of \ the \ division \ appropriately}
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon (const vector <PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++) {</pre>
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
     p[j].y \le q.y \&\& q.y < p[i].y) \&\&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
  return c:
// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)</pre>
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)</pre>
     return true;
    return false;
// compute intersection of line through points a and b with
// circle centered at c with radius r >
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
 b = b-a;
  a = a-c;
 double A = dot(b, b);
double B = dot(a, b);
double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;</pre>
  ret.push back(c+a+b*(-B+sgrt(D+EPS))/A);
  if (D > EPS)
   ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
if (d > r+R || d+min(r, R) < max(r, R)) return ret;</pre>
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret:
```

```
// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
  counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {</pre>
   int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  return area / 2.0;
double ComputeArea(const vector<PT> &p) {
  return fabs (ComputeSignedArea(p));
PT ComputeCentroid(const vector<PT> &p) {
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++) {
  int j = (i+1) % p.size();</pre>
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  return c / scale:
// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {</pre>
    for (int k = i+1; k < p.size(); k++) {</pre>
     int j = (i+1) % p.size();
int l = (k+1) % p.size();
     if (i == 1 \mid \mid j == k) continue;
     if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
  return true;
int main() {
  // expected: (-5,2)
  cerr << RotateCCW90(PT(2,5)) << endl;</pre>
  // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;</pre>
  // expected: (-5.2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;</pre>
  // expected: (5.2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;</pre>
  // expected: (5,2) (7.5,3) (2.5,1)
  << ProjectPointSegment (PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;
  // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;</pre>
  cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;
  // expected: 0 0 1
  << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;
  // expected: (1,2)
  cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;</pre>
  // expected: (1.1)
  cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;</pre>
  vector<PT> v:
  v.push_back(PT(0,0));
  v.push_back(PT(5,0));
  v.push_back(PT(5,5));
  v.push back(PT(0.5));
```

```
// expected: 1 1 1 0 0
cerr << PointInPolygon(v, PT(2,2)) << " "</pre>
       << PointInPolygon(v, PT(2,0)) << " "
       << PointInPolygon(v, PT(0,2)) << " "
       << PointInPolygon(v, PT(5,2)) << " "
       << PointInPolygon(v, PT(2,5)) << endl;
// expected: 0 1 1 1 1
cerr << PointOnPolygon(v, PT(2,2)) << " "</pre>
      << PointOnPolygon(v, PT(2,0)) << " "
       << PointOnPolygon(v, PT(0,2)) << " "
       << PointOnPolygon(v, PT(5,2)) << " "
       << PointOnPolygon(v, PT(2,5)) << endl;
// expected: (1,6)
                  (5,4) (4,5)
                 blank line
                  (4,5) (5,4)
                  blank line
                 (4,5) (5,4)
vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;</pre>
canc i = 0, i < u.size(); i+r) ceii < u[i] << " "; cerr << endl;
u = CircleCircleIntersection (PT(1,1), PT(8,8), 5, 5);
for (int i = 0; i < u.size(); i+r) cerr << u[i] << " "; cerr << endl;</pre>
u = CircleCircleIntersection(PT(1,1), PT(4.5, 4.5), 10, sqrt(2.0)/2.0);

for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);

for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
// area should be 5.0
// centroid should be (1.1666666, 1.166666)
PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
vector<PT> p(pa, pa+4);
PT c = ComputeCentroid(p);
cerr << "Area: " << ComputeArea(p) << endl;
cerr << "Centroid: " << c << endl;
return 0;
```

3 Numerical algorithms

3.1 Number theory (modular, Chinese remainder, linear Diophantine)

```
// This is a collection of useful code for solving problems that
// involve modular linear equations. Note that all of the
// algorithms described here work on nonnegative integers.
#include <iostream>
#include <vector>
#include <algorithm>
using namespace std;
typedef vector<int> VI;
typedef pair<int, int> PII;
// return a % b (positive value)
int mod(int a, int b)
        return ((a%b) + b) % b;
// computes gcd(a,b)
int gcd(int a, int b) {
        while (b) { int t = a%b; a = b; b = t; }
// computes lcm(a,b)
int lcm(int a, int b)
        return a / gcd(a, b) *b;
// (a^b) mod m via successive squaring
int powermod(int a, int b, int m)
        int ret = 1;
        while (b)
```

```
if (b & 1) ret = mod(ret*a, m);
                 a = mod(a*a, m);
// returns g = gcd(a, b); finds x, y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
        int xx = y = 0;
         int yy = x = 1;
         while (b) {
                 int q = a / b;
int t = b; b = a%b; a = t;
t = xx; xx = x - q*xx; x = t;
                 t = yy; yy = y - q*yy; y = t;
         return a;
// finds all solutions to ax = b \pmod{n}
VI modular_linear_equation_solver(int a, int b, int n) {
         VI ret;
         int g = extended_euclid(a, n, x, y);
         if (!(b%g)) {
                 x = mod(x*(b / g), n);
for (int i = 0; i < g; i++)
                          ret.push_back(mod(x + i*(n / g), n));
         return ret;
 // computes b such that ab = 1 \pmod{n}, returns -1 on failure
int mod_inverse(int a, int n) {
        int x, y;
         int g = extended_euclid(a, n, x, y);
         if (q > 1) return -1;
         return mod(x, n);
// Chinese remainder theorem (special case): find z such that // z % m1 = r1, z % m2 = r2. Here, z is unique modulo M = lcm(m1, m2).
// Return (z, M). On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
         int s, t;
         int g = extended_euclid(m1, m2, s, t);
         if (r1%g != r2%g) return make_pair(0, -1);
         return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i. Note that the solution is
// unique modulo M = lcm_i (m[i]). Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
         PII ret = make_pair(r[0], m[0]);
         for (int i = 1; i < m.size(); i++) {
                 ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
                 if (ret.second == -1) break;
         return ret;
// computes x and y such that ax + by = c
// computes x and y
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
         if (!a && !b)
                 if (c) return false;
                 x = 0; y = 0;
                 return true;
                 if (c % b) return false;
                 return true;
         if (!b)
                 if (c % a) return false;
                  x = c / a; y = 0;
                 return true;
         int g = gcd(a, b);
         if (c % g) return false;
         x = c / g * mod_inverse(a / g, b / g);
         y = (c - a * x) / b;
         return true;
```

```
int main() {
         // expected: 2
         cout << gcd(14, 30) << endl;
          // expected: 2 -2 1
         int x, y;
         int g = extended_euclid(14, 30, x, y);
cout << g << " " << x << " " << y << endl;
          // expected: 95 451
         VI sols = modular_linear_equation_solver(14, 30, 100);
for (int i = 0; i < sols.size(); i++) cout << sols[i] << " ";</pre>
         cout << endl:
         cout << mod_inverse(8, 9) << endl;</pre>
          // expected: 23 105
                        11 12
         PII ret = chinese_remainder_theorem(VI({ 3, 5, 7 }), VI({ 2, 3, 2 })); cout << ret.first << " " << ret.second << endl;
         ret = chinese_remainder_theorem(VI({ 4, 6 }), VI({ 3, 5 }));
         cout << ret.first << " " << ret.second << endl;</pre>
          // expected: 5 -15
         if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" << endl;</pre>
         cout << x << " " << y << endl;
         return 0:
```

3.2 Systems of linear equations, matrix inverse, determinant

```
// Gauss-Jordan elimination with full pivoting.
// Uses:
    (1) solving systems of linear equations (AX=B)
     (2) inverting matrices (AX=I)
     (3) computing determinants of square matrices
// Running time: O(n^3)
// INPUT: a[][] = an nxn matrix
              b[][] = an nxm matrix
// OUTPUT: X
              X = an nxm matrix (stored in b[][]) A^{-1} = an nxn matrix (stored in a[][])
              returns determinant of a[][]
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
const double EPS = 1e-10;
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
T GaussJordan (VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;
  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;

for (int j = 0; j < n; j++) if (!ipiv[j])
      for (int k = 0; k < n; k++) if (!ipiv[k])</pre>
        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl; exit(0); }</pre>
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
irow[i] = pj;
icol[i] = pk;
    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
```

```
for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
     c = a[p][pk];
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);</pre>
  return det;
int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = \{ \{1,2,3,4\}, \{1,0,1,0\}, \{5,3,2,4\}, \{6,1,4,6\} \};
  double B[n][m] = \{ \{1,2\}, \{4,3\}, \{5,6\}, \{8,7\} \};
  VVT a(n), b(n);
  for (int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
  double det = GaussJordan(a, b);
  // expected: 60
  cout << "Determinant: " << det << endl;</pre>
  // expected: -0.233333 0.166667 0.133333 0.0666667
                0.166667 0.166667 0.333333 -0.333333
                0.233333 0.833333 -0.133333 -0.0666667
                0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;
  for (int i = 0; i < n; i++) {
  for (int j = 0; j < n; j++)
    cout << a[i][j] << ' ';</pre>
    cout << endl;
  // expected: 1.63333 1.3
                -0.166667 0.5
                2.36667 1.7
  cout << "Solution: " << endl;
  for (int i = 0; i < n; i++) {
   for (int j = 0; j < m; j++)
  cout << b[i][j] << ' ';</pre>
    cout << endl;
```

3.3 Reduced row echelon form, matrix rank

```
// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
// Running time: O(n^3)
// INPUT: a[][] = an nxm matrix
// OUTPUT: rref[][] = an nxm matrix (stored in a[][])
            returns rank of a[][]
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
const double EPSILON = 1e-10;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
int rref(VVT &a) {
 int n = a.size():
  int m = a[0].size();
  int r = 0;
  for (int c = 0; c < m && r < n; c++) {
   int j = r;
```

```
for (int i = r + 1; i < n; i++)
      if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
    if (fabs(a[j][c]) < EPSILON) continue;</pre>
    swap(a[j], a[r]);
    T s = 1.0 / a[r][c];
    for (int j = 0; j < m; j++) a[r][j] *= s;
for (int i = 0; i < n; i++) if (i != r) {</pre>
       T t = a[i][c];
       for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];</pre>
    r++;
  return r:
int main() {
  const int n = 5, m = 4;
  double A[n][m] = {
    {16, 2, 3, 13},
    { 5, 11, 10, 8},
    { 9, 7, 6, 12},
    { 4, 14, 15, 1},
    {13, 21, 21, 13}};
  VVT a(n);
for (int i = 0; i < n; i++)</pre>
    a[i] = VT(A[i], A[i] + m);
 int rank = rref(a);
  // expected: 3
  cout << "Rank: " << rank << endl;
                 0 0 1 -3
                 0 0 0 3.10862e-15
                 0 0 0 2.22045e-15
  cout << "rref: " << endl;
 for (int i = 0; i < 5; i++) {
  for (int j = 0; j < 4; j++)
   cout << a[i][j] << ' ';
    cout << endl;
```

3.4 Fast Fourier transform

```
#include <cassert>
#include <cstdio>
#include <cmath>
struct cpx
  cpx(){}
  cpx(double aa):a(aa),b(0){}
  cpx(double aa, double bb):a(aa),b(bb){}
  double a;
  double b;
  double modsq(void) const
    return a * a + b * b;
  cpx bar(void) const
    return cpx(a, -b);
};
cpx operator + (cpx a, cpx b)
  return cpx(a.a + b.a. a.b + b.b);
cpx operator *(cpx a, cpx b)
  return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
cpx operator / (cpx a, cpx b)
  cpx r = a * b.bar();
 return cpx(r.a / b.modsq(), r.b / b.modsq());
cpx EXP (double theta)
 return cpx(cos(theta), sin(theta));
```

```
const double two_pi = 4 * acos(0);
           input array
// out:
          output array
// step:
          {SET TO 1} (used internally)
// size: length of the input/output {MUST BE A POWER OF 2}
          either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size - 1} in[j] * exp(dir * 2pi * i * j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
  if(size < 1) return;</pre>
  if(size == 1)
    out[0] = in[0];
  FFT(in, out, step * 2, size / 2, dir);
  FFT(in + step, out + size / 2, step * 2, size / 2, dir);
  for (int i = 0; i < size / 2; i++)
    cpx even = out[i];
    cpx odd = out[i + size / 2];
   out[i] = even + EXP(dir * two_pi * i / size) * odd;
    out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) / size) * odd;
// Usage:
// f[0...N-1] and g[0..N-1] are numbers
// Want to compute the convolution h, defined by
// h(n) = sum of f[k]g[n-k] (k = 0, ..., N-1).

// Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, define G and H.
// The convolution theorem says H[n] = F[n]G[n] (element-wise product).
// To compute h[] in O(N log N) time, do the following:
    1. Compute F and G (pass dir = 1 as the argument).
// 2. Get H by element-wise multiplying F and G.
    3. Get h by taking the inverse FFT (use dir = -1 as the argument)
       and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.
int main (void)
  printf("If rows come in identical pairs, then everything works.\n");
  cpx \ a[8] = \{0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0\};
  cpx b[8] = \{1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2\};
  cpx B[8];
  FFT(a, A, 1, 8, 1);
  FFT(b, B, 1, 8, 1);
  for(int i = 0 : i < 8 : i++)
   printf("%7.21f%7.21f", A[i].a, A[i].b);
  printf("\n");
  for (int i = 0; i < 8; i++)
    for (int j = 0; j < 8; j++)
      Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
    printf("%7.21f%7.21f", Ai.a, Ai.b);
  printf("\n");
  cox AB[8];
  for (int i = 0; i < 8; i++)
   AB[i] = A[i] * B[i];
  cpx aconvb[8];
  FFT (AB, aconvb, 1, 8, -1);
  for (int i = 0; i < 8; i++)
    aconvb[i] = aconvb[i] / 8;
  for (int i = 0; i < 8; i++)
    printf("%7.21f%7.21f", aconvb[i].a, aconvb[i].b);
  printf("\n");
  for(int i = 0; i < 8; i++)
    cpx aconvbi(0.0);
   for (int j = 0; j < 8; j++)
      aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
    printf("%7.21f%7.21f", aconvbi.a, aconvbi.b);
  printf("\n");
```

return 0;

3.5 Simplex algorithm

```
// Two-phase simplex algorithm for solving linear programs of the form
        maximize
        subject to Ax <= b
                        x >= 0
// INPUT: A -- an m x n matrix
           b -- an m-dimensional vector
            c -- an n-dimensional vector
            x -- a vector where the optimal solution will be stored
// OUTPUT: value of the optimal solution (infinity if unbounded
             above, nan if infeasible)
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).
#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include inits>
using namespace std;
typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const DOUBLE EPS = 1e-9:
struct LPSolver (
  int m, n;
  VI B, N;
  VVD D:
  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
     for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
     N[n] = -1; D[m + 1][n] = 1;
  void Pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
for (int i = 0; i < m + 2; i++) if (i != r)
    for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;</pre>
    D[r][s] = inv;
swap(B[r], N[s]);
  bool Simplex(int phase) {
     int x = phase == 1 ? m + 1 : m;
     while (true) {
       int s = -1;
       int s = -1,
for (int j = 0; j <= n; j++) {
   if (phase == 2 && N[j] == -1) continue;
   if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;</pre>
       if (D[x][s] > -EPS) return true;
       int r = -1:
       for (int i = 0; i < m; i++) {
         if (D[i][s] < EPS) continue;</pre>
         if (r == -1 \mid \mid D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] \mid \mid
             (D[i][n+1] / D[i][s]) == (D[r][n+1] / D[r][s]) && B[i] < B[r]) r = i;
       if (r == -1) return false;
       Pivot(r, s);
  DOUBLE Solve(VD &x) {
    int r = 0;
     for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
     if (D[r][n + 1] < -EPS) {
       Pivot(r, n);
       if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<DOUBLE>::infinity();
```

```
for (int i = 0; i < m; i++) if (B[i] == -1) {
        for (int j = 0; j <= n; j++)
          if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
int main() {
  const int n = 3;
  DOUBLE _A[m][n] =
    { 6, -1, 0 },
    \{-1, -5, 0\},
    \{-1, -5, -1\}
 DOUBLE _b[m] = { 10, -4, 5, -5 };

DOUBLE _c[n] = { 1, -1, 0 };
  VVD A(m);
  VD b(\underline{b}, \underline{b} + m);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);</pre>
  LPSolver solver (A, b, c);
  DOUBLE value = solver.Solve(x);
  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1</pre>
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];</pre>
  cerr << endl;
  return 0;
```

4 Graph algorithms

4.1 Floyd's algorithm (C++)

```
#include <bits/stdc++.h>
using namespace std;
typedef double T:
typedef vector<T> VT;
typedef vector<VT> VVT;
typedef vector<int> VI;
typedef vector<VI> VVI;
// This function runs the Floyd-Warshall algorithm for all-pairs
// shortest paths. Also handles negative edge weights. Returns true
// if a negative weight cycle is found.
// Running time: O(|V|^3)
     INPUT: w[i][j] = weight of edge from i to j
     OUTPUT: w[i][j] = shortest path from i to j
             prev[i][j] = node before j on the best path starting at i
bool FloydWarshall (VVT &w, VVI &prev) {
    int n = w.size();
    prev = VVI (n, VI(n, -1));
    for (int k = 0; k < n; k++) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
    if (w[i][j] > w[i][k] + w[k][j]) {
                    w[i][j] = w[i][k] + w[k][j];
                    prev[i][j] = k;
    // check for negative weight cycles
```

```
for(int i=0;i<n;i++)
   if (w[i][i] < 0) return false;
return true;</pre>
```

4.2 Fast Dijkstra's algorithm

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
// Running time: O(|E| log |V|)
#include <queue>
#include <cstdio>
using namespace std;
const int INF = 20000000000:
typedef pair<int, int> PII;
int main() {
        scanf("%d%d%d", &N, &s, &t);
         vector<vector<PII> > edges(N);
        for (int i = 0; i < N; i++) {
                 int M;
                 scanf("%d", &M);
                 for (int j = 0; j < M; j++) {}
                         int vertex, dist;
scanf("%d%d", &vertex, &dist);
                         edges[i].push_back(make_pair(dist, vertex)); // note order of arguments here
        // use priority queue in which top element has the "smallest" priority
        priority_queue<PII, vector<PII>, greater<PII> > Q;
        vector<int> dist(N, INF), dad(N, -1);
        Q.push(make_pair(0, s));
        dist[s] = 0;
        while (!Q.empty()) {
                 PII p = Q.top();
                 Q.pop();
                 int here = p.second;
                 if (here == t) break;
                 if (dist[here] != p.first) continue;
                 for (vector<PII>::iterator it = edges[here].begin(); it != edges[here].end(); it++) {
    if (dist[here] + it->first < dist[it->second]) {
                                  dist[it->second] = dist[here] + it->first;
                                  dad[it->second] = here;
                                  Q.push(make_pair(dist[it->second], it->second));
        printf("%d\n", dist[t]);
        if (dist[t] < INF)</pre>
                 for (int i = t; i != -1; i = dad[i])
                         printf("%d%c", i, (i == s ? '\n' : ' '));
        return 0:
Sample input:
2 1 2 3 1
3 1 4 3 3 4 1
2 0 1 2 3
2 1 5 2 1
Expected:
4 2 3 0
```

4.3 Dijkstra's algorithm

```
#include <bits/stdc++.h>
using namespace std;

// (u)int64, float
template<typename D>
```

```
struct Dijkstra {
    static constexpr D Inf = 11 << 60;
    struct Edge { size_t to; D len; };
        vector<Edge> outs;
        D dist = Inf;
#ifdef DIJKSTRA_RECORD_ROUTE
        size_t prev = -1;
#endif
    size_t N;
    vector<Vertex> vs;
    Dijkstra(size_t n) : N(n) {
        vs.resize(n);
    void add(size_t from, size_t to, D len) {
        assert (from < N);
        assert (to < N);
        assert(len >= 0);
        vs[from].outs.push_back({ to, len });
    void add_u(size_t a, size_t b, D len) {
        add(a, b, len);
        add(b, a, len);
    D solve(size_t from, size_t to) {
        vs[from].dist = 0;
#ifdef DIJKSTRA_RECORD_ROUTE
        vs[from].prev = from;
#endif
        auto comp = [&](size_t x, size_t y) {
    return vs[x].dist < vs[y].dist || (vs[x].dist == vs[y].dist && x < y);</pre>
        set<size_t, decltype(comp)> q { comp };
for (size_t i = 0; i < N; ++i) {</pre>
            q.insert(i);
        while (!q.empty()) {
            size_t i;
                 auto it = q.begin();
                 q.erase(it);
             if (i == to) {
                 goto RETURN;
             for (Edge const& e : vs[i].outs) {
                 if (vs[e.to].dist > vs[i].dist + e.len) {
                     if (q.find(e.to) != q.end()) {
                          q.erase(e.to);
                          vs[e.to].dist = vs[i].dist + e.len;
#ifdef DIJKSTRA_RECORD_ROUTE
                          vs[e.to].prev = i;
#endif
                         q.insert(e.to);
RETURN:
        return vs[to].dist;
};
```

4.4 Minimum spanning trees using Prim

```
#include <bits/stdc++.h>
using namespace std;
// (u)int64, float
template<typename D>
struct Prim {
    static constexpr D Inf = 11 << 60;</pre>
```

```
struct Edge { size_t to; D len; };
    struct Vertex {
        vector<Edge> outs;
        D dist = Inf;
        size_t prev = -1;
    size_t N;
    vector<Vertex> vs;
    // n nodes
    Prim(size_t n) : N(n) {
        vs.resize(n);
    void add(size_t from, size_t to, D len) {
        assert (from < N);
        assert (to < N);
        assert (len >= 0);
        vs[from].outs.push_back({ to, len });
    void add_u(size_t a, size_t b, D len) {
        add(a, b, len);
        add(b, a, len);
    D solve(size_t from) {
        vs[from].dist = 0;
        vs[from].prev = from;
        auto comp = [&](size_t x, size_t y) {
            return vs[x].dist < vs[y].dist || (vs[x].dist == vs[y].dist && x < y);</pre>
        set<size_t, decltype(comp)> q { comp };
        for (size_t i = 0; i < N; ++i) {</pre>
             q.insert(i);
        while (!q.empty()) {
            size_t i;
                 auto it = q.begin();
                          = *it;
                 q.erase(it);
             if (vs[i].dist == Inf) {
                 vs[i].dist = 0;
            for (Edge const& e : vs[i].outs) {
   if (vs[e.to].dist > e.len) {
                     if (q.find(e.to) != q.end()) {
                         q.erase(e.to);
                         vs[e.to].dist = e.len;
                         vs[e.to].prev = i;
                         q.insert(e.to);
        D result = 0;
        for (size_t i = 0; i < N; ++i)
    result += vs[i].dist;</pre>
        return result;
};
```

4.5 Minimum spanning trees using Kruskal

```
#include <algorithm>
#include <cassert>
#include <cassert>
#include <cstdint>
#include <set>
#include <unordered_map>
#include <unordered_map>
#include <vector>

using namespace std;

// (u)int64, float
template <typename D>
struct Kruskal {
    static constexpr D Inf = 11 << 60;</pre>
```

```
struct Edge {
    size_t n1, n2;
    D len;
    bool operator<(Edge const& o) const noexcept {</pre>
        return len < o.len || (len == o.len && (n1 < o.n1 || (n1 == o.n1 && (n2 < o.n2))));
typedef unordered_map<size_t, vector<Edge>> Forest;
vector<Edge> edges;
Kruskal(size_t n) : N(n) {}
void add(size_t n1, size_t n2, D len) {
    assert (n1 < N);
    assert (n2 < N);
    assert(len >= 0);
    if (n1 > n2) swap(n1, n2);
    edges.push_back({ n1, n2, len });
void solve(Forest& forest) {
    sort(edges.begin(), edges.end());
    vector<size_t> _ufs(N);
    for (size_t i = 0; i < N; i++)</pre>
        _ufs[i] = i;
    auto ufs_p = [&](size_t i) -> size_t* {
         while (_ufs[p] != p)
         p = _ufs[p];
_ufs[i] = p;
        return &_ufs[i];
    };
    for (Edge const& e : edges) {
        auto* ufs_n1p = ufs_p(e.n1);
auto* ufs_n2p = ufs_p(e.n2);
if (*ufs_n1p != *ufs_n2p) {
            if (*ufs_nlp > *ufs_n2p)
                 swap(*ufs_n1p, *ufs_n2p);
             auto it = forest.find(*ufs_n2p);
             if (it != forest.end()) {
                 forest[*ufs_nlp].insert(forest[*ufs_nlp].end(), it->second.begin(), it->second.end
                 forest.erase(it);
             forest[*ufs_n1p].push_back(e);
             *ufs n2p = *ufs n1p;
    return;
```

```
#include < memory.h >
struct edge{int e, nxt;};
int V. E:
edge e[MAXE], er[MAXE];
int sp[MAXV], spr[MAXV];
int group_cnt, group_num[MAXV];
bool v[MAXV];
int stk[MAXV];
void fill_forward(int x)
  int i:
  v[x]=true;
  for(i=sp[x];i;i=e[i].nxt) if(!v[e[i].e]) fill_forward(e[i].e);
  stk[++stk[0]]=x;
void fill_backward(int x)
  int i;
  v[x]=false;
  group_num[x]=group_cnt;
```

Strongly connected components

```
for(i=spr[x];i;i=er[i].nxt) if(v[er[i].e]) fill_backward(er[i].e);
void add_edge(int v1, int v2) //add edge v1->v2
  e [++E].e=v2; e [E].nxt=sp [v1]; sp [v1]=E;
 er[ E].e=v1; er[E].nxt=spr[v2]; spr[v2]=E;
void SCC()
  int i;
  stk[0]=0;
  memset(v, false, sizeof(v));
  \label{formula} \textbf{for}(\texttt{i=1}; \texttt{i<=V}; \texttt{i++}) \ \ \textbf{if}(\texttt{!v[i]}) \ \ \texttt{fill\_forward(i)};
  group cnt=0:
  for(i=stk[0];i>=1;i--) if(v[stk[i]]){group_cnt++; fill_backward(stk[i]);}
```

4.7 Eulerian path

```
struct Edge;
typedef list<Edge>::iterator iter;
struct Edge
        int next vertex;
        iter reverse_edge;
        Edge (int next vertex)
                 :next_vertex(next_vertex)
};
const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices];
                                          // adjacency list
vector<int> path:
void find_path(int v)
        while(adj[v].size() > 0)
                 int vn = adj[v].front().next_vertex;
                 adj[vn].erase(adj[v].front().reverse_edge);
                 adj[v].pop_front();
        path.push_back(v);
void add_edge(int a, int b)
        adj[a].push_front(Edge(b));
        iter ita = adj[a].begin();
adj[b].push front(Edge(a));
        iter itb = adj[b].begin();
        ita->reverse_edge = itb;
        itb->reverse_edge = ita;
```

Data structures

5.1 Suffix array

```
// Suffix array construction in O(L log^2 L) time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
// INPUT: string s
// OUTPUT: array suffix[] such that suffix[i] = index (from 0 to L-1)
            of substring s[i...L-1] in the list of sorted suffixes.
            That is, if we take the inverse of the permutation suffix[],
            we get the actual suffix array.
#include <vector>
#include <iostream>
#include <string>
using namespace std;
```

```
struct SuffixArray {
  const int L;
  string s;
  vector<vector<int> > P;
  vector<pair<int,int>,int> > M;
  SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
      P.push_back(vector<int>(L, 0));
      for (int i = 0; i < L; i++)
        M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
      sort(M.begin(), M.end());
for (int i = 0; i < L; i++)</pre>
        P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
  vector<int> GetSuffixArray() { return P.back(); }
  // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0:
    if (i == j) return L - i;
for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {</pre>
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
         \frac{1}{1} += 1 << k;
        len += 1 << k;
    return len;
// BEGIN CUT
// The following code solves UVA problem 11512: GATTACA.
#define TESTING
#ifdef TESTING
int main() {
 int T;
cin >> T;
  for (int caseno = 0; caseno < T; caseno++) {</pre>
    string s;
    cin >> s;
    SuffixArray array(s);
    vector<int> v = array.GetSuffixArray();
int bestlen = -1, bestpos = -1, bestcount = 0;
    for (int i = 0; i < s.length(); i++) {</pre>
      int len = 0, count = 0;
      for (int j = i+1, j < s.length(), j++) {
        int 1 = array.LongestCommonPrefix(i, j);
        if (1 >= len) {
          if (1 > len) count = 2: else count++:
          len = 1:
      if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen) > s.substr(i, len)) {
        bestlen = len;
        bestcount = count;
        bestpos = i;
    if (bestlen == 0) {
      cout << "No repetitions found!" << endl;</pre>
    } else {
      cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;</pre>
#else
// END CUT
int main() {
  // bobocel is the O'th suffix
      obocel is the 5'th suffix
       bocel is the 1'st suffix
        ocel is the 6'th suffix
         cel is the 2'nd suffix
         el is the 3'rd suffix
l is the 4'th suffix
  SuffixArray suffix("bobocel");
  vector<int> v = suffix.GetSuffixArrav();
  // Expected output: 0 5 1 6 2 3 4
  for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
  cout << endl:
  cout << suffix.LongestCommonPrefix(0, 2) << endl;</pre>
```

```
// BEGIN CUT
#endif
// END CUT
```

5.2 Binary Indexed Tree

```
#include <iostream>
using namespace std;
#define LOGSZ 17
int tree[(1<<LOGSZ)+1];</pre>
int N = (1 << LOGSZ);
// add v to value at x
void set(int x, int v) {
  while (x \le N)
   tree[x] += v;
   x += (x & -x);
// get cumulative sum up to and including x
int get(int x) {
 int res = 0;
  while(x) {
   res += tree[x];
   x -= (x & -x);
 return res:
// get largest value with cumulative sum less than or equal to x:
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
  int idx = 0, mask = N;
  while (mask && idx < N) {
    int t = idx + mask;
    if(x >= tree[t]) {
     idx = t;
     x -= tree[t];
   mask >>= 1:
  return idx:
```

5.3 Union-find set

```
#include <bits/stdc++.h>
using namespace std;
struct UnionFind {
  vector<int> C;
  UnionFind(int n) : C(n) { for (int i = 0; i < n; i++) C[i] = i; }
  int find(int x) { return (C[x] == x) ? x : C[x] = find(C[x]); }
  void merge(int x, int y) { C[find(x)] = find(y); }
};</pre>
```

5.4 KD-tree

```
using namespace std;
// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();
// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};
bool operator==(const point &a, const point &b)
    return a.x == b.x && a.y == b.y;
// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
    return a.x < b.x;
// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
    return a.y < b.y;
// squared distance between points
ntype pdist2(const point &a, const point &b)
    ntype dx = a.x-b.x, dy = a.y-b.y;
    return dx*dx + dy*dy;
// bounding box for a set of points
struct bbox
    ntype x0, x1, y0, y1;
    bbox(): x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}
    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {
            x0 = min(x0, v[i].x); x1 = max(x1, v[i].x);
            y0 = min(y0, v[i].y); y1 = max(y1, v[i].y);
    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
                                return pdist2(point(x0, y0), p);
            if (p, v < v0)
            else if (p.y > y1) return pdist2(point(x0, y1), p);
                                return pdist2(point(x0, p.y), p);
            else
        else if (p.x > x1) {
                                return pdist2(point(x1, y0), p);
            if (p.y < y0)
            else if (p.y > y1) return pdist2(point(x1, y1), p);
                                return pdist2(point(x1, p.y), p);
        else
            if (p.y < y0)
                                return pdist2(point(p.x, y0), p);
            else if (p.y > y1) return pdist2(point(p.x, y1), p);
            else
                                return 0;
// stores a single node of the kd-tree, either internal or leaf
struct kdnode
    bool leaf;
                    // true if this is a leaf node (has one point)
    point pt;
                    // the single point of this is a leaf
                    // bounding box for set of points in children
    bbox bound;
    kdnode *first, *second; // two children of this kd-node
    kdnode() : leaf(false), first(0), second(0) {}
    ~kdnode() { if (first) delete first; if (second) delete second; }
    // intersect a point with this node (returns squared distance)
   ntype intersect(const point &p) {
       return bound.distance(p);
    // recursively builds a kd-tree from a given cloud of points
    void construct(vector<point> &vp)
        // compute bounding box for points at this node
```

```
bound.compute(vp);
         // if we're down to one point, then we're a leaf node
        if (vp.size() == 1) {
            leaf = true;
            pt = vp[0];
            // split on x if the bbox is wider than high (not best heuristic...)
            if (bound.x1-bound.x0 >= bound.y1-bound.y0)
                sort(vp.begin(), vp.end(), on_x);
            // otherwise split on y-coordinate
            else
                sort(vp.begin(), vp.end(), on_y);
            // divide by taking half the array for each child
            // (not best performance if many duplicates in the middle)
            int half = vp.size()/2;
            vector<point> vl(vp.begin(), vp.begin()+half);
            vector<point> vr(vp.begin()+half, vp.end());
            first = new kdnode(); first->construct(v1);
            second = new kdnode(); second->construct(vr);
1:
// simple kd-tree class to hold the tree and handle queries
struct kdtree
    kdnode *root:
    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnode();
        root->construct(v);
    "kdtree() { delete root; }
    // recursive search method returns squared distance to nearest point
    ntype search(kdnode *node, const point &p)
        if (node->leaf) {
            // commented special case tells a point not to find itself
             if (p == node->pt) return sentry;
              else
                return pdist2(p, node->pt);
        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);
        // choose the side with the closest bounding box to search first
         // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {
            ntype best = search(node->first, p);
            if (bsecond < best)</pre>
               best = min(best, search(node->second, p));
            return best:
            ntype best = search(node->second, p);
            if (bfirst < best)</pre>
                best = min(best, search(node->first, p));
            return best;
    // squared distance to the nearest
    ntype nearest (const point &p) {
        return search (root, p):
};
// some basic test code here
int main()
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand()%100000, rand()%100000));
    kdtree tree(vp):
    // query some points
    for (int i = 0; i < 10; ++i) {
        point q(rand()%100000, rand()%100000);
        cout << "Closest squared distance to (" << q.x << ", " << q.y << ")"
             << " is " << tree.nearest(q) << endl;
```

5.5 Splay tree

```
#include <cstdio>
#include <algorithm>
using namespace std:
const int N_MAX = 130010;
const int oo = 0x3f3f3f3f3f;
struct Node
  Node *ch[2], *pre;
  int val, size;
  bool isTurned;
} nodePool[N_MAX], *null, *root;
Node *allocNode(int val)
  static int freePos = 0;
  Node *x = &nodePool[freePos ++];
  x->val = val, x->isTurned = false;
  x->ch[0] = x->ch[1] = x->pre = null;
  x->size = 1;
  return x;
inline void update(Node *x)
  x->size = x->ch[0]->size + x->ch[1]->size + 1;
inline void makeTurned(Node *x)
  if(x == null)
    return:
  swap(x->ch[0], x->ch[1]);
x->isTurned ^= 1;
inline void pushDown(Node *x)
  if(x->isTurned)
    makeTurned(x->ch[0]);
    makeTurned(x->ch[1]);
x->isTurned ^= 1;
inline void rotate(Node *x, int c)
  Node *v = x -> pre;
  x->pre = y->pre;
  if(y->pre != null)
    y->pre->ch[y == y->pre->ch[1]] = x;
  y - ch[!c] = x - ch[c];
  if (x->ch[c] != null)
    x->ch[c]->pre = y;
  x->ch[c] = y, y->pre = x;
  update(y);
  if(y == root)
    root = x:
void splay(Node *x, Node *p)
  while (x->pre != p)
    if(x->pre->pre == p)
      rotate(x, x == x->pre->ch[0]);
      Node *y = x->pre, *z = y->pre;
      if(y == z->ch[0])
        if(x == y->ch[0])
          rotate(y, 1), rotate(x, 1);
          rotate(x, 0), rotate(x, 1);
      else
```

```
if(x == y->ch[1])
          rotate(y, 0), rotate(x, 0);
          rotate(x, 1), rotate(x, 0);
  update(x);
void select(int k, Node *fa)
  Node *now = root;
  while(1)
    pushDown (now);
    int tmp = now->ch[0]->size + 1;
    if(tmp == k)
     break;
    else if (tmp < k)
      now = now -> ch[1], k -= tmp;
    else
      now = now->ch[0];
  splay(now, fa);
Node *makeTree(Node *p, int 1, int r)
 if(1 > r)
   return null;
  int \ mid = (1 + r) / 2;
  Node *x = allocNode(mid);
  x->pre = p;
  x->ch[0] = makeTree(x, 1, mid - 1);
x->ch[1] = makeTree(x, mid + 1, r);
  update(x);
  return x;
int main()
 int n, m;
null = allocNode(0);
  null->size = 0;
  root = allocNode(0);
  root->ch[1] = allocNode(oo);
  root->ch[1]->pre = root;
  update (root);
  scanf("%d%d", &n, &m);
  root->ch[1]->ch[0] = makeTree(root->ch[1], 1, n);
  splay(root->ch[1]->ch[0], null);
  while (m --)
    int a. b:
    scanf("%d%d", &a, &b);
    a ++, b ++;
    select(a - 1, null);
    select(b + 1, root);
    makeTurned(root->ch[1]->ch[0]);
  for(int i = 1; i \le n; i ++)
   select(i + 1, null);
printf("%d ", root->val);
```

5.6 Lowest common ancestor

```
if (n >= 1<< 8) { n >>= 8; p += 8; }
    if (n >= 1 << 4) { n >>= 4; p += 4; }
if (n >= 1 << 2) { n >>= 2; p += 2; }
    if (n >= 1<< 1) {
void DFS(int i, int 1)
    L[i] = 1;
    for(int j = 0; j < children[i].size(); j++)</pre>
        DFS(children[i][j], 1+1);
int LCA(int p, int q)
     // ensure node p is at least as deep as node q
    if(L[p] < L[q])
        swap(p, q);
     // "binary search" for the ancestor of node p situated on the same level as q
    for(int i = log_num_nodes; i >= 0; i--)
   if(L[p] - (1<<i) >= L[q])
             p = A[p][i];
    if(p == q)
        return p;
     // "binary search" for the LCA
    for(int i = log_num_nodes; i >= 0; i--)
         if(A[p][i] != -1 && A[p][i] != A[q][i])
             p = A[p][i];
             q = A[q][i];
    return A[p][0];
int main(int argc,char* argv[])
     // read num nodes, the total number of nodes
    log_num_nodes=1b(num_nodes);
    for(int i = 0; i < num nodes; i++)</pre>
        // read p, the parent of node i or -1 if node i is the root
        A[i][0] = p;
        if(p != -1)
             children[p].push_back(i);
        else
             root = i:
     // precompute A using dynamic programming
    for(int j = 1; j <= log_num_nodes; j++)
for(int i = 0; i < num_nodes; i++)</pre>
             if(A[i][j-1] != -1)
                 A[i][j] = A[A[i][j-1]][j-1];
                 A[i][j] = -1;
     // precompute L
    DFS(root, 0);
    return 0:
```

6 Miscellaneous

6.1 Longest increasing subsequence

```
// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
// INPUT: a vector of integers
// OUTPUT: a vector containing the longest increasing subsequence
#include <iostream>
#include <vector>
#include <vector>
```

```
#include <algorithm>
using namespace std;
typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;
#define STRICTLY INCREASING
VI LongestIncreasingSubsequence(VI v) {
 VPII best;
 VI dad(v.size(), -1);
for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG</pre>
    PII item = make_pair(v[i], 0);
    VPII::iterator it = lower_bound(best.begin(), best.end(), item);
    item.second = i;
#else
    PII item = make_pair(v[i], i);
    VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
    if (it == best.end()) {
      dad[i] = (best.size() == 0 ? -1 : best.back().second);
      best.push_back(item);
    } else {
     dad[i] = it == best.begin() ? -1 : prev(it)->second;
      *it = item:
  for (int i = best.back().second; i >= 0; i = dad[i])
    ret.push_back(v[i]);
  reverse(ret.begin(), ret.end());
  return ret;
```

6.2 Dates

```
// Routines for performing computations on dates. In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.
#include <iostream>
#include <string>
using namespace std;
string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};
// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y) {
  return
    1461 * (v + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y) {
  int x, n, i, j;
  x = id + 68569;
  n = 4 * x / 146097:
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x = 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = \frac{1}{1} + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
// converts integer (Julian day number) to day of week
string intToDay (int jd) {
  return dayOfWeek[jd % 7];
int main (int argc, char **argv) {
  int jd = dateToInt (3, 24, 2004);
  int m, d, y;
intToDate (jd, m, d, y);
  string day = intToDay (jd);
```

```
// expected output:

// 2453089

// 3/24/2004

// Wed

cout << jd << endl

<< m << "/" << d << "/" << y << endl

<< day << endl;
```

6.3 Prime numbers

```
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool IsPrimeSlow (LL x)
  if(x<=1) return false;</pre>
  if(x<=3) return true;</pre>
  if (!(x%2) || !(x%3)) return false;
 LL s=(LL) (sqrt((double)(x))+EPS);
for(LL i=5;i<=s;i+=6)
   if (!(x%i) || !(x%(i+2))) return false;
  return true;
  Primes less than 1000:
                             59
                 103
                       107
                             109
                                  113
                                                    137
                                                         139
                                                               149
                 167
                       173
                             179
                                  181
                                        191
                                              193
                                                    197
                                                         199
           229
                 233
                       239
                             241
                                  251
                                              263
                                                    269
                                                         271
                                  317
397
                                        331
                                              337
           293
                 307
                       311
                             313
                                                    347
                                                         349
                 379
                             389
                                        401
                                              409
           373
                       383
                                                    419
                                                         421
                                                               431
                                                                     4.3.3
                 449
                       457
                                  463
557
                                        467
563
                                              479
                                                    487
     439
           443
                             461
                                                         491
                                                               499
                            547
                                                         577
     509
                       541
                                              569
                                                   571
                                                               587
           521
                 523
     599
           601
                 607
                       613
                             617
                                  619
                                        631
                                              641
                                                    643
                                                         647
                                                               653
                                         709
                                              719
                 761
                       769
                                  787
                                         797
                                              809
                                                    811
                                                         821
                 853
                       857
                            859
                                  863
                                        877
                                              881
                                                   883
                                                         887
                 937
                       941
                            947
                                  953
                                        967
                                              971
                                                    977
     The largest prime smaller than 10 is 7.
     The largest prime smaller than 100 is 97.
     The largest prime smaller than 1000 is 997.
     The largest prime smaller than 10000 is 9973. The largest prime smaller than 100000 is 99991.
     The largest prime smaller than 1000000 is 999983. The largest prime smaller than 10000000 is 9999991.
     The largest prime smaller than 100000000 is 99999989.
     The largest prime smaller than 1000000000 is 999999937.
     The largest prime smaller than 10000000000 is 9999999967.
     The largest prime smaller than 10000000000 is 99999999977.
     The largest prime smaller than 1000000000000 is 999999999971.
      The largest prime smaller than 1000000000000 is 99999999999973.
      The largest prime smaller than 100000000000000 is 99999999999937.
      The largest prime smaller than 1000000000000000 is 999999999999997
```

6.4 C++ input/output

```
#include <bits/stdc++.h>
using namespace std;
int main()
{
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);</pre>
```

```
// Output a '+' before positive values
cout.setf(ios::showpos);
cout < 100 << " " << -100 << endl;
cout.unsetf(ios::showpos);

// Output numerical values in hexadecimal
cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;</pre>
```

6.5 Knuth-Morris-Pratt

```
Finds all occurrences of the pattern string p within the
text string t. Running time is O(n + m), where n and m
are the lengths of p and t, respecitvely.
#include <iostream>
#include <string>
#include <vector>
using namespace std;
typedef vector<int> VI:
void buildPi(string& p, VI& pi)
  pi = VI(p.length());
  int k = -2;
  for(int i = 0; i < p.length(); i++) {</pre>
    while (k \ge -1 \&\& p[k+1] != p[i])
      k = (k == -1) ? -2 : pi[k];
    pi[i] = ++k;
int KMP(string& t, string& p)
  buildPi(p, pi);
  int k = -1;
  for(int i = 0; i < t.length(); i++) {</pre>
    while (k >= -1 &  p[k+1] != t[i])
      k = (k == -1) ? -2 : pi[k];
    if(k == p.length() - 1) {
      // p matches t[i-m+1, ..., i]
      cout << "matched at index " << i-k << ": ";
      cout << t.substr(i-k, p.length()) << endl;</pre>
      k = (k == -1) ? -2 : pi[k];
  return 0:
int main()
  string a = "AABAACAADAABAABA", b = "AABA";
  KMP(a, b); // expected matches at: 0, 9, 12
```

6.6 Latitude/longitude

```
/*
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
*/
#include <iostream>
#include <cmath>
using namespace std;
struct 11
{
    double r, lat, lon;
};
struct rect
{
    double x, y, z;
}
```

```
11 convert(rect& P)
{
    11 Q;
    Q.r = sqrt(P.x*P.x*P.y*P.y*P.z*P.z);
    Q.lat = 180/M.PI*asin(P.z/Q.r);
    Q.lon = 180/M.PI*acos(P.x/sqrt(P.x*P.x*P.y*P.y));

    return Q;
}

rect convert(ll& Q)
{
    rect P;
    P.x = Q.r*cos(Q.lon*M.PI/180)*cos(Q.lat*M.PI/180);
    P.y = Q.r*sin(Q.lon*M.PI/180)*cos(Q.lat*M.PI/180);
    P.z = Q.r*sin(Q.lat*M.PI/180);
    return P;
}

int main()
{
    rect A;
    ll B;
    A.x = -1.0; A.y = 2.0; A.z = -3.0;
    B = convert(A);
    cout << B.r << " " << B.lat << " " << B.lon << endl;
    A = convert(B);
    cout << A.x << " " << A.y << " " << A.z << endl;
}</pre>
```

6.7 Vim settings

```
set enc=utf-8
set fenc=utf-8
scriptencoding utf-8
set fencs=utf-8,ucs-bom,gb18030,gbk,gb2312,cp936
syntax on
filetype plugin on
filetype indent on
set mouse=a
set nocompatible
set tabstop=4
set shiftwidth=4
```

```
set expandtab
set smarttab
set autoindent
set textwidth=1000
set showmatch
set ruler
set hlsearch
set incsearch
set ignorecase
set smartcase
set number
set relativenumber
set fdm=marker
set scrolloff=10
set showcmd
set backspace=indent,eol,start
set formatoptions=q,r
set nowrap
set foldmethod=indent
set foldlevelstart=10
set wildmenu
set omnifunc=syntaxcomplete#Complete
"the trail config failed in generate latex "remember to add in the site" set list listchars=tab:>-,trail:
set t_Co=256
set background=dark
nnoremap ; :
inoremap <silent> jj <ESC>
nnoremap <silent> <HOME> ^
inoremap <silent> <HOME> <ESC>^i
vnoremap <silent> <HOME> 0w
nnoremap <silent> <F9> :set nowrap<CR>
nnoremap <silent> <F10> :set wrap<CR>
```