Stanford University ICPC Team Notebook (2015-16)

Contents

1 Combinatorial optimization

1.1 Edmonds-Karp max-flow (normal, SPFA min-cost)

```cpp
#include <bits/stdc++.h>
using namespace std;

template<typename D>
struct EdmondsKarp {
    static constexpr D Inf = 1l << 60;

    struct Edge {
        size_t from, to; D cap;
    };

    size_t N;

    vector<Edge> edges;
    vector<vector<size_t>> outs;

    EdmondsKarp(size_t n) : N(n) {
        outs.resize(n);
    }

    void add(size_t from, size_t to, D cap) {
        assert(from < N);
        assert(to < N);
        if (cap > 0) {
            outs[from].emplace_back(edges.size());
            edges.emplace_back(Edge { from, to, cap });
            outs[to].emplace_back(edges.size());
            edges.emplace_back(Edge { to, from, 0 });
        }
    }

    vector<D> amount;
    vector<size_t> route;
    vector<size_t> visiting;

    D solve(size_t from, size_t to) {
        amount.resize(N);
        route.resize(N);
        visiting.resize(N);

        D flow = 0;

        while (true) {
            fill(amount.begin(), amount.end(), 0);

            size_t head = 0, tail = 0;

            amount[from] = Inf;
            visiting[tail++] = from;

            while (head < tail && !amount[to]) {
                size_t i = visiting[head++];

                for (size_t j = 0; j < outs[i].size(); ++j) {
                    Edge &e = edges[outs[i][j]];

                    if (!amount[e.to] && e.cap) {
                        amount[e.to] = min(amount[i], e.cap);
                        route[e.to] = outs[i][j];

                        visiting[tail++] = e.to;
                    }
                }
            }

            if (!amount[to]) break;

            for (size_t i = to; i != from; i = edges[route[i]].from) {
                edges[route[i]].cap -= amount[to];
                edges[route[i] ^ 1].cap += amount[to];
            }

            flow += amount[to];
        }

        return flow;
    }
};
```

1.2 Dinic max-flow

```cpp
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
//     O(|V|^2 |E|)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source and sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain actual flow values, look at edges with capacity > 0
//       (zero capacity edges are residual edges).
```

```
#include<cstdio>
#include<vector>
#include<queue>
using namespace std;
typedef long long LL;

struct Edge {
  int u, v;
  LL cap, flow;
  Edge() {}
  Edge(int u, int v, LL cap): u(u), v(v), cap(cap), flow(0) {}
};

struct Dinic {
  int N;
  vector<Edge> E;
  vector<vector<int>> g;
  vector<int> d, pt;

  Dinic(int N): N(N), E(0), g(N), d(N), pt(N) {}

  void AddEdge(int u, int v, LL cap) {
    if (u != v) {
      E.emplace_back(u, v, cap);
      g[u].emplace_back(E.size() - 1);
      E.emplace_back(v, u, 0);
      g[v].emplace_back(E.size() - 1);
    }
  }

  bool BFS(int S, int T) {
    queue<int> q({S});
    fill(d.begin(), d.end(), N + 1);
    d[S] = 0;
    while(!q.empty()) {
      int u = q.front(); q.pop();
      if (u == T) break;
      for (int k: g[u]) {
        Edge &e = E[k];
        if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
          d[e.v] = d[e.u] + 1;
          q.emplace(e.v);
        }
      }
    }
    return d[T] != N + 1;
  }

  LL DFS(int u, int T, LL flow = -1) {
    if (u == T || flow == 0) return flow;
    for (int &i = pt[u]; i < g[u].size(); ++i) {
      Edge &e = E[g[u][i]];
      Edge &oe = E[g[u][i]^1];
      if (d[e.v] == d[e.u] + 1) {
        LL amt = e.cap - e.flow;
        if (flow != -1 && amt > flow) amt = flow;
        if (LL pushed = DFS(e.v, T, amt)) {
          e.flow += pushed;
          oe.flow -= pushed;
          return pushed;
        }
      }
    }
    return 0;
  }

  LL MaxFlow(int S, int T) {
    LL total = 0;
    while (BFS(S, T)) {
      fill(pt.begin(), pt.end(), 0);
      while (LL flow = DFS(S, T))
        total += flow;
    }
    return total;
  }
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (FASTFLOW)

int main()
{
  int N, E;
  scanf("%d%d", &N, &E);
  Dinic dinic(N);
  for(int i = 0; i < E; i++)
  {
    int u, v;
    LL cap;
    scanf("%d%d%lld", &u, &v, &cap);
    dinic.AddEdge(u - 1, v - 1, cap);
    dinic.AddEdge(v - 1, u - 1, cap);
  }
  printf("%lld\n", dinic.MaxFlow(0, N - 1));
  return 0;
}

// END CUT
```

1.3 Push-relabel max-flow

```
// Adjacency list implementation of FIFO push relabel maximum flow
// with the gap relabeling heuristic.  This implementation is
// significantly faster than straight Ford-Fulkerson.  It solves
// random problems with 10000 vertices and 1000000 edges in a few
// seconds, though it is possible to construct test cases that
// achieve the worst-case.
//
// Running time:
//     O(|V|^3)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source
//     - sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain the actual flow values, look at all edges with
//       capacity > 0 (zero capacity edges are residual edges).

#include <cmath>
#include <vector>
#include <iostream>
#include <queue>

using namespace std;

typedef long long LL;

struct Edge {
  int from, to, cap, flow, index;
  Edge(int from, int to, int cap, int flow, int index) :
    from(from), to(to), cap(cap), flow(flow), index(index) {}
};

struct PushRelabel {
  int N;
  vector<vector<Edge> > G;
  vector<LL> excess;
  vector<int> dist, active, count;
  queue<int> Q;

  PushRelabel(int N) : N(N), G(N), excess(N), dist(N), active(N), count(2*N) {}

  void AddEdge(int from, int to, int cap) {
    G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
    if (from == to) G[from].back().index++;
    G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
  }

  void Enqueue(int v) {
    if (!active[v] && excess[v] > 0) { active[v] = true; Q.push(v); }
  }

  void Push(Edge &e) {
    int amt = int(min(excess[e.from], LL(e.cap - e.flow)));
    if (dist[e.from] <= dist[e.to] || amt == 0) return;
    e.flow += amt;
    G[e.to][e.index].flow -= amt;
    excess[e.to] += amt;
    excess[e.from] -= amt;
    Enqueue(e.to);
  }

  void Gap(int k) {
    for (int v = 0; v < N; v++) {
      if (dist[v] < k) continue;
      count[dist[v]]--;
      dist[v] = max(dist[v], N+1);
      count[dist[v]]++;
      Enqueue(v);
    }
  }

  void Relabel(int v) {
    count[dist[v]]--;
    dist[v] = 2*N;
    for (int i = 0; i < G[v].size(); i++)
```

```
      if (G[v][i].cap - G[v][i].flow > 0)
        dist[v] = min(dist[v], dist[G[v][i].to] + 1);
    count[dist[v]]++;
    Enqueue(v);
  }

  void Discharge(int v) {
    for (int i = 0; excess[v] > 0 && i < G[v].size(); i++) Push(G[v][i]);
    if (excess[v] > 0) {
      if (count[dist[v]] == 1)
        Gap(dist[v]);
      else
        Relabel(v);
    }
  }

  LL GetMaxFlow(int s, int t) {
    count[0] = N-1;
    count[N] = 1;
    dist[s] = N;
    active[s] = active[t] = true;
    for (int i = 0; i < G[s].size(); i++) {
      excess[s] += G[s][i].cap;
      Push(G[s][i]);
    }

    while (!Q.empty()) {
      int v = Q.front();
      Q.pop();
      active[v] = false;
      Discharge(v);
    }

    LL totflow = 0;
    for (int i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;
    return totflow;
  }
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (FASTFLOW)

int main() {
  int n, m;
  scanf("%d%d", &n, &m);

  PushRelabel pr(n);
  for (int i = 0; i < m; i++) {
   int a, b, c;
    scanf("%d%d%d", &a, &b, &c);
    if (a == b) continue;
    pr.AddEdge(a-1, b-1, c);
    pr.AddEdge(b-1, a-1, c);
  }
  printf("%Ld\n", pr.GetMaxFlow(0, n-1));
  return 0;
}

// END CUT
```

1.4 Min-cost matching

```
///////////////////////////////////////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs.  In practice, it solves 1000x1000 problems in around 1
// second.
//
//   cost[i][j] = cost for pairing left node i with right node j
//   Lmate[i] = index of right node that left node i pairs with
//   Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative.  To perform
// maximization, simply negate the cost[][] matrix.
///////////////////////////////////////////////////////////////////////

#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>

using namespace std;

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
  int n = int(cost.size());

  // construct dual feasible solution
  VD u(n);
  VD v(n);
  for (int i = 0; i < n; i++) {
    u[i] = cost[i][0];
    for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
  }
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  // construct primal solution satisfying complementary slackness
  Lmate = VI(n, -1);
  Rmate = VI(n, -1);
  int mated = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      if (Rmate[j] != -1) continue;
      if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break;
      }
    }
  }

  VD dist(n);
  VI dad(n);
  VI seen(n);

  // repeat until primal solution is feasible
  while (mated < n) {

    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    while (true) {

      // find closest
      j = -1;
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;

      // termination condition
      if (Rmate[j] == -1) break;

      // relax neighbors
      const int i = Rmate[j];
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
      if (k == j || !seen[k]) continue;
      const int i = Rmate[k];
      v[k] += dist[k] - dist[j];
      u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
      const int d = dad[j];
      Rmate[j] = Rmate[d];
      Lmate[Rmate[j]] = j;
      j = d;
    }
```

```
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
  }

  double value = 0;
  for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

  return value;
}
```

1.5 Max bipartite matching

```
// This code performs maximum bipartite matching.
//
// Running time: O(|E| |V|) -- often much faster in practice
//
//   INPUT: w[i][j] = edge between row node i and column node j
//   OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
//           mc[j] = assignment for column node j, -1 if unassigned
//           function returns number of matches made

#include <vector>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
  for (int j = 0; j < w[i].size(); j++) {
    if (w[i][j] && !seen[j]) {
      seen[j] = true;
      if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
        mr[i] = j;
        mc[j] = i;
        return true;
      }
    }
  }
  return false;
}

int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
  mr = VI(w.size(), -1);
  mc = VI(w[0].size(), -1);

  int ct = 0;
  for (int i = 0; i < w.size(); i++) {
    VI seen(w[0].size());
    if (FindMatch(i, w, mr, mc, seen)) ct++;
  }
  return ct;
}
```

1.6 Global min-cut

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
//     O(|V|^3)
//
// INPUT:
//     - graph, constructed using AddEdge()
//
// OUTPUT:
//     - (min cut value, nodes in half of min cut)

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
      prev = last;
      last = -1;
      for (int j = 1; j < N; j++)
        if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
      if (i == phase-1) {
        for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j];
        for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
        used[last] = true;
        cut.push_back(last);
        if (best_weight == -1 || w[last] < best_weight) {
          best_cut = cut;
          best_weight = w[last];
        }
      } else {
        for (int j = 0; j < N; j++)
          w[j] += weights[last][j];
        added[last] = true;
      }
    }
  }
  return make_pair(best_weight, best_cut);
}

// BEGIN CUT
// The following code solves UVA problem #10989: Bomb, Divide and Conquer
int main() {
  int N;
  cin >> N;
  for (int i = 0; i < N; i++) {
    int n, m;
    cin >> n >> m;
    VVI weights(n, VI(n));
    for (int j = 0; j < m; j++) {
      int a, b, c;
      cin >> a >> b >> c;
      weights[a-1][b-1] = weights[b-1][a-1] = c;
    }
    pair<int, VI> res = GetMinCut(weights);
    cout << "Case #" << i+1 << ": " << res.first << endl;
  }
}
// END CUT
```

1.7 Graph cut inference

```
// Special-purpose {0,1} combinatorial optimization solver for
// problems of the following by a reduction to graph cuts:
//
//        minimize         sum_i  psi_i(x[i])
//  x[1]...x[n] in {0,1}      + sum_{i < j}  phi_{ij}(x[i], x[j])
//
// where
//      psi_i : {0, 1} --> R
//   phi_{ij} : {0, 1} x {0, 1} --> R
//
// such that
//   phi_{ij}(0,0) + phi_{ij}(1,1) <= phi_{ij}(0,1) + phi_{ij}(1,0)   (*)
//
// This can also be used to solve maximization problems where the
// direction of the inequality in (*) is reversed.
//
// INPUT: phi -- a matrix such that phi[i][j][u][v] = phi_{ij}(u, v)
//        psi -- a matrix such that psi[i][u] = psi_i(u)
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution
//
// To use this code, create a GraphCutInference object, and call the
// DoInference() method.  To perform maximization instead of minimization,
// ensure that #define MAXIMIZATION is enabled.

#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;
```

```
typedef vector<VVVI> VVVVI;

const int INF = 1000000000;

// comment out following line for minimization
#define MAXIMIZATION

struct GraphCutInference {
  int N;
  VVI cap, flow;
  VI reached;

  int Augment(int s, int t, int a) {
    reached[s] = 1;
    if (s == t) return a;
    for (int k = 0; k < N; k++) {
      if (reached[k]) continue;
      if (int aa = min(a, cap[s][k] - flow[s][k])) {
        if (int b = Augment(k, t, aa)) {
          flow[s][k] += b;
          flow[k][s] -= b;
          return b;
        }
      }
    }
    return 0;
  }

  int GetMaxFlow(int s, int t) {
    N = cap.size();
    flow = VVI(N, VI(N));
    reached = VI(N);

    int totflow = 0;
    while (int amt = Augment(s, t, INF)) {
      totflow += amt;
      fill(reached.begin(), reached.end(), 0);
    }
    return totflow;
  }

  int DoInference(const VVVVI &phi, const VVI &psi, VI &x) {
    int M = phi.size();
    cap = VVI(M+2, VI(M+2));
    VI b(M);
    int c = 0;

    for (int i = 0; i < M; i++) {
      b[i] += psi[i][1] - psi[i][0];
      c += psi[i][0];
      for (int j = 0; j < i; j++)
        b[i] += phi[i][j][1][1] - phi[i][j][0][1];
      for (int j = i+1; j < M; j++) {
        cap[i][j] = phi[i][j][0][1] + phi[i][j][1][0] - phi[i][j][0][0] - phi[i][j][1][1];
        b[i] += phi[i][j][1][0] - phi[i][j][0][0];
        c += phi[i][j][0][0];
      }
    }

#ifdef MAXIMIZATION
    for (int i = 0; i < M; i++) {
      for (int j = i+1; j < M; j++)
        cap[i][j] *= -1;
      b[i] *= -1;
    }
    c *= -1;
#endif

    for (int i = 0; i < M; i++) {
      if (b[i] >= 0) {
        cap[M][i] = b[i];
      } else {
        cap[i][M+1] = -b[i];
        c += b[i];
      }
    }

    int score = GetMaxFlow(M, M+1);
    fill(reached.begin(), reached.end(), 0);
    Augment(M, M+1, INF);
    x = VI(M);
    for (int i = 0; i < M; i++) x[i] = reached[i] ? 0 : 1;
    score += c;
#ifdef MAXIMIZATION
    score *= -1;
#endif

    return score;
  }

};

int main() {

  // solver for "Cat vs. Dog" from NWERC 2008

  int numcases;
  cin >> numcases;
  for (int caseno = 0; caseno < numcases; caseno++) {
    int c, d, v;
    cin >> c >> d >> v;

    VVVVI phi(c+d, VVVI(c+d, VVI(2, VI(2))));
    VVI psi(c+d, VI(2));
    for (int i = 0; i < v; i++) {
      char p, q;
      int u, v;
      cin >> p >> u >> q >> v;
      u--; v--;
      if (p == 'C') {
        phi[u][c+v][0][0]++;
        phi[c+v][u][0][0]++;
      } else {
        phi[v][c+u][1][1]++;
        phi[c+u][v][1][1]++;
      }
    }

    GraphCutInference graph;
    VI x;
    cout << graph.DoInference(phi, psi, x) << endl;
  }

  return 0;
}
```

2 Geometry

2.1 Convex hull

```
// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm.  Eliminate redundant points from the hull if REMOVE_REDUNDANT is
// #defined.
//
// Running time: O(n log n)
//
//   INPUT:   a vector of input points, unordered.
//   OUTPUT:  a vector of points in the convex hull, counterclockwise, starting
//            with bottommost/leftmost point

#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>
// BEGIN CUT
#include <map>
// END CUT

using namespace std;

#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) < make_pair(rhs.y,rhs.x); }
  bool operator==(const PT &rhs) const { return make_pair(y,x) == make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  for (int i = 0; i < pts.size(); i++) {
```

```
    while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
    while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  }
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
  if (pts.size() <= 2) return;
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
    dn.push_back(pts[i]);
  }
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  }
  pts = dn;
#endif
}

// BEGIN CUT
// The following code solves SPOJ problem #26: Build the Fence (BSHEEP)

int main() {
  int t;
  scanf("%d", &t);
  for (int caseno = 0; caseno < t; caseno++) {
    int n;
    scanf("%d", &n);
    vector<PT> v(n);
    for (int i = 0; i < n; i++) scanf("%lf%lf", &v[i].x, &v[i].y);
    vector<PT> h(v);
    map<PT,int> index;
    for (int i = n-1; i >= 0; i--) index[v[i]] = i+1;
    ConvexHull(h);

    double len = 0;
    for (int i = 0; i < h.size(); i++) {
      double dx = h[i].x - h[(i+1)%h.size()].x;
      double dy = h[i].y - h[(i+1)%h.size()].y;
      len += sqrt(dx*dx+dy*dy);
    }

    if (caseno > 0) printf("\n");
    printf("%.2f\n", len);
    for (int i = 0; i < h.size(); i++) {
      if (i > 0) printf(" ");
      printf("%d", index[h[i]]);
    }
    printf("\n");
  }
}

// END CUT
```

2.2 Miscellaneous geometry

```
// C++ routines for computational geometry.

#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
  PT operator + (const PT &p)  const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x*c,   y*c  ); }
  PT operator / (double c)     const { return PT(x/c,   y/c  ); }
};

double dot(PT p, PT q)     { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)   { return dot(p-q,p-q); }
double cross(PT p, PT q)   { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  return os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p)   { return PT(-p.y,p.x); }
PT RotateCW90(PT p)    { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                          double a, double b, double c, double d)
{
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
```

```
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
      return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion.  Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}

int main() {

  // expected: (-5,2)
  cerr << RotateCCW90(PT(2,5)) << endl;

  // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;

  // expected: (-5,2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

  // expected: (5,2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

  // expected: (5,2) (7.5,3) (2.5,1)
  cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

  // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

  // expected: 1 0 1
  cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 0 0 1
  cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 1 1 1 0
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;

  // expected: (1,2)
  cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;

  // expected: (1,1)
  cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

  vector<PT> v;
  v.push_back(PT(0,0));
  v.push_back(PT(5,0));
  v.push_back(PT(5,5));
  v.push_back(PT(0,5));

  // expected: 1 1 1 0 0
  cerr << PointInPolygon(v, PT(2,2)) << " "
       << PointInPolygon(v, PT(2,0)) << " "
       << PointInPolygon(v, PT(0,2)) << " "
       << PointInPolygon(v, PT(5,2)) << " "
       << PointInPolygon(v, PT(2,5)) << endl;

  // expected: 0 1 1 1 1
  cerr << PointOnPolygon(v, PT(2,2)) << " "
       << PointOnPolygon(v, PT(2,0)) << " "
       << PointOnPolygon(v, PT(0,2)) << " "
       << PointOnPolygon(v, PT(5,2)) << " "
       << PointOnPolygon(v, PT(2,5)) << endl;

  // expected: (1,6)
  //           (5,4) (4,5)
  //           blank line
  //           (4,5) (5,4)
  //           blank line
  //           (4,5) (5,4)
  vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;

  // area should be 5.0
  // centroid should be (1.1666666, 1.166666)
  PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
  vector<PT> p(pa, pa+4);
  PT c = ComputeCentroid(p);
  cerr << "Area: " << ComputeArea(p) << endl;
  cerr << "Centroid: " << c << endl;

  return 0;
}
```

3 Numerical algorithms

3.1 Number theory (modular, Chinese remainder, linear Diophantine)

```
// This is a collection of useful code for solving problems that
// involve modular linear equations.  Note that all of the
// algorithms described here work on nonnegative integers.

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;

// return a % b (positive value)
int mod(int a, int b) {
	return ((a%b) + b) % b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
	while (b) { int t = a%b; a = b; b = t; }
	return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
	return a / gcd(a, b)*b;
}

// (a^b) mod m via successive squaring
int powermod(int a, int b, int m)
{
	int ret = 1;
	while (b)
	{
		if (b & 1) ret = mod(ret*a, m);
		a = mod(a*a, m);
		b >>= 1;
	}
	return ret;
}

// returns g = gcd(a, b); finds x, y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
	int xx = y = 0;
	int yy = x = 1;
	while (b) {
		int q = a / b;
		int t = b; b = a%b; a = t;
		t = xx; xx = x - q*xx; x = t;
		t = yy; yy = y - q*yy; y = t;
	}
	return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
	int x, y;
	VI ret;
	int g = extended_euclid(a, n, x, y);
	if (!(b%g)) {
		x = mod(x*(b / g), n);
		for (int i = 0; i < g; i++)
			ret.push_back(mod(x + i*(n / g), n));
	}
	return ret;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
	int x, y;
	int g = extended_euclid(a, n, x, y);
	if (g > 1) return -1;
	return mod(x, n);
}

// Chinese remainder theorem (special case): find z such that
// z % m1 = r1, z % m2 = r2.  Here, z is unique modulo M = lcm(m1, m2).
// Return (z, M).  On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
	int s, t;
	int g = extended_euclid(m1, m2, s, t);
	if (r1%g != r2%g) return make_pair(0, -1);
	return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
}

// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i.  Note that the solution is
// unique modulo M = lcm_i (m[i]).  Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
	PII ret = make_pair(r[0], m[0]);
	for (int i = 1; i < m.size(); i++) {
		ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
		if (ret.second == -1) break;
	}
	return ret;
}

// computes x and y such that ax + by = c
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
	if (!a && !b)
	{
		if (c) return false;
		x = 0; y = 0;
		return true;
	}
	if (!a)
	{
		if (c % b) return false;
		x = 0; y = c / b;
		return true;
	}
	if (!b)
	{
		if (c % a) return false;
		x = c / a; y = 0;
		return true;
	}
	int g = gcd(a, b);
	if (c % g) return false;
	x = c / g * mod_inverse(a / g, b / g);
	y = (c - a*x) / b;
	return true;
}

int main() {
	// expected: 2
	cout << gcd(14, 30) << endl;

	// expected: 2 -2 1
	int x, y;
	int g = extended_euclid(14, 30, x, y);
	cout << g << " " << x << " " << y << endl;

	// expected: 95 451
	VI sols = modular_linear_equation_solver(14, 30, 100);
	for (int i = 0; i < sols.size(); i++) cout << sols[i] << " ";
	cout << endl;

	// expected: 8
	cout << mod_inverse(8, 9) << endl;

	// expected: 23 105
	//           11 12
	PII ret = chinese_remainder_theorem(VI({ 3, 5, 7 }), VI({ 2, 3, 2 }));
	cout << ret.first << " " << ret.second << endl;
	ret = chinese_remainder_theorem(VI({ 4, 6 }), VI({ 3, 5 }));
	cout << ret.first << " " << ret.second << endl;

	// expected: 5 -15
	if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" << endl;
	cout << x << " " << y << endl;
	return 0;
}
```

3.2 Systems of linear equations, matrix inverse, determinant

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
//   (1) solving systems of linear equations (AX=B)
```

```
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
//
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;

  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])
      for (int k = 0; k < n; k++) if (!ipiv[k])
        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl; exit(0); }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
    irow[i] = pj;
    icol[i] = pk;

    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
      c = a[p][pk];
      a[p][pk] = 0;
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
  }

  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
  }

  return det;
}

int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
  double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
  VVT a(n), b(n);
  for (int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
  }

  double det = GaussJordan(a, b);

  // expected: 60
  cout << "Determinant: " << det << endl;

  // expected: -0.233333 0.166667 0.133333 0.0666667
  //           0.166667 0.166667 0.333333 -0.333333
  //           0.233333 0.833333 -0.133333 -0.0666667
  //           0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }

  // expected: 1.63333 1.3
  //           -0.166667 0.5
  //           2.36667 1.7
  //           -1.85 -1.35
  cout << "Solution: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
      cout << b[i][j] << ' ';
    cout << endl;
  }
}
```

3.3 Reduced row echelon form, matrix rank

```
// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting.  This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxm matrix
//
// OUTPUT:   rref[][] = an nxm matrix (stored in a[][])
//           returns rank of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPSILON = 1e-10;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

int rref(VVT &a) {
  int n = a.size();
  int m = a[0].size();
  int r = 0;
  for (int c = 0; c < m && r < n; c++) {
    int j = r;
    for (int i = r + 1; i < n; i++)
      if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
    if (fabs(a[j][c]) < EPSILON) continue;
    swap(a[j], a[r]);

    T s = 1.0 / a[r][c];
    for (int j = 0; j < m; j++) a[r][j] *= s;
    for (int i = 0; i < n; i++) if (i != r) {
      T t = a[i][c];
      for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
    }
    r++;
  }
  return r;
}

int main() {
  const int n = 5, m = 4;
  double A[n][m] = {
    {16,  2,  3, 13},
    { 5, 11, 10,  8},
    { 9,  7,  6, 12},
    { 4, 14, 15,  1},
    {13, 21, 21, 13}};
  VVT a(n);
  for (int i = 0; i < n; i++)
    a[i] = VT(A[i], A[i] + m);

  int rank = rref(a);

  // expected: 3
  cout << "Rank: " << rank << endl;

  // expected: 1 0 0 1
  //           0 1 0 3
  //           0 0 1 -3
  //           0 0 0 3.10862e-15
  //           0 0 0 2.22045e-15
  cout << "rref: " << endl;
  for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 4; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }
}
```

3.4 Fast Fourier transform

```
#include <cassert>
#include <cstdio>
#include <cmath>

struct cpx
{
  cpx(){}
  cpx(double aa):a(aa),b(0){}
  cpx(double aa, double bb):a(aa),b(bb){}
  double a;
  double b;
  double modsq(void) const
  {
    return a * a + b * b;
  }
  cpx bar(void) const
  {
    return cpx(a, -b);
  }
};

cpx operator +(cpx a, cpx b)
{
  return cpx(a.a + b.a, a.b + b.b);
}

cpx operator *(cpx a, cpx b)
{
  return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}

cpx operator /(cpx a, cpx b)
{
  cpx r = a * b.bar();
  return cpx(r.a / b.modsq(), r.b / b.modsq());
}

cpx EXP(double theta)
{
  return cpx(cos(theta),sin(theta));
}

const double two_pi = 4 * acos(0);

// in:     input array
// out:    output array
// step:   {SET TO 1} (used internally)
// size:   length of the input/output {MUST BE A POWER OF 2}
// dir:    either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size - 1} in[j] * exp(dir * 2pi * i * j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{
  if(size < 1) return;
  if(size == 1)
  {
    out[0] = in[0];
    return;
  }
  FFT(in, out, step * 2, size / 2, dir);
  FFT(in + step, out + size / 2, step * 2, size / 2, dir);
  for(int i = 0 ; i < size / 2 ; i++)
  {
    cpx even = out[i];
    cpx odd = out[i + size / 2];
    out[i] = even + EXP(dir * two_pi * i / size) * odd;
    out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) / size) * odd;
  }
}

// Usage:
// f[0...N-1] and g[0..N-1] are numbers
// Want to compute the convolution h, defined by
// h[n] = sum of f[k]g[n-k] (k = 0, ..., N-1).
// Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, define G and H.
// The convolution theorem says H[n] = F[n]G[n] (element-wise product).
// To compute h[] in O(N log N) time, do the following:
//   1. Compute F and G (pass dir = 1 as the argument).
//   2. Get H by element-wise multiplying F and G.
//   3. Get h by taking the inverse FFT (use dir = -1 as the argument)
//      and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.

int main(void)
{
  printf("If rows come in identical pairs, then everything works.\n");

  cpx a[8] = {0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0};
  cpx b[8] = {1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2};
  cpx A[8];
  cpx B[8];
  FFT(a, A, 1, 8, 1);
  FFT(b, B, 1, 8, 1);

  for(int i = 0 ; i < 8 ; i++)
  {
    printf("%7.2lf%7.2lf", A[i].a, A[i].b);
  }
  printf("\n");
  for(int i = 0 ; i < 8 ; i++)
  {
    cpx Ai(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
      Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
    }
    printf("%7.2lf%7.2lf", Ai.a, Ai.b);
  }
  printf("\n");

  cpx AB[8];
  for(int i = 0 ; i < 8 ; i++)
    AB[i] = A[i] * B[i];
  cpx aconvb[8];
  FFT(AB, aconvb, 1, 8, -1);
  for(int i = 0 ; i < 8 ; i++)
    aconvb[i] = aconvb[i] / 8;
  for(int i = 0 ; i < 8 ; i++)
  {
    printf("%7.2lf%7.2lf", aconvb[i].a, aconvb[i].b);
  }
  printf("\n");
  for(int i = 0 ; i < 8 ; i++)
  {
    cpx aconvbi(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
      aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
    }
    printf("%7.2lf%7.2lf", aconvbi.a, aconvbi.b);
  }
  printf("\n");

  return 0;
}
```

3.5 Simplex algorithm

```
// Two-phase simplex algorithm for solving linear programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments.  Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;
```

```
  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] < N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
  cerr << endl;
  return 0;
}
```

4 Graph algorithms

4.1 Floyd's algorithm (C++)

```
#include <bits/stdc++.h>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

// This function runs the Floyd-Warshall algorithm for all-pairs
// shortest paths.  Also handles negative edge weights.  Returns true
// if a negative weight cycle is found.
//
// Running time: O(|V|^3)
//
//   INPUT:  w[i][j] = weight of edge from i to j
//   OUTPUT: w[i][j] = shortest path from i to j
//           prev[i][j] = node before j on the best path starting at i

bool FloydWarshall (VVT &w, VVI &prev){
    int n = w.size();
    prev = VVI (n, VI(n, -1));

    for (int k = 0; k < n; k++){
        for (int i = 0; i < n; i++){
            for (int j = 0; j < n; j++){
                if (w[i][j] > w[i][k] + w[k][j]){
                    w[i][j] = w[i][k] + w[k][j];
                    prev[i][j] = k;
                }
            }
        }
    }

    // check for negative weight cycles
    for(int i=0;i<n;i++)
        if (w[i][i] < 0) return false;
    return true;
}
```

4.2 Topological sort (C++)

```
// This function uses performs a non-recursive topological sort.
//
// Running time: O(|V|^2).  If you use adjacency lists (vector<map<int> >),
//               the running time is reduced to O(|E|).
//
//   INPUT:   w[i][j] = 1 if i should come before j, 0 otherwise
//   OUTPUT:  a permutation of 0,...,n-1 (stored in a vector)
//            which represents an ordering of the nodes which
//            is consistent with w
//
// If no ordering is possible, false is returned.

#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool TopologicalSort (const VVI &w, VI &order){
  int n = w.size();
  VI parents (n);
  queue<int> q;
  order.clear();

  for (int i = 0; i < n; i++){
```

```
    for (int j = 0; j < n; j++)
      if (w[j][i]) parents[i]++;
      if (parents[i] == 0) q.push (i);
  }

  while (q.size() > 0){
    int i = q.front();
    q.pop();
    order.push_back (i);
    for (int j = 0; j < n; j++) if (w[i][j]){
      parents[j]--;
      if (parents[j] == 0) q.push (j);
    }
  }

  return (order.size() == n);
}
```

4.3 Dijkstra's algorithm

```
#include <bits/stdc++.h>
using namespace std;

// (u)int64, float
template<typename D>
struct Dijkstra {
    static constexpr D Inf = 1l << 60;

    struct Edge { size_t to; D len; };
    struct Vertex {
        vector<Edge> outs;
        D dist = Inf;
#ifdef DIJKSTRA_RECORD_ROUTE
        size_t prev = -1;
#endif
    };

    size_t N;

    vector<Vertex> vs;

    // n nodes
    Dijkstra(size_t n) : N(n) {
        vs.resize(n);
    }

    void add(size_t from, size_t to, D len) {
        assert(from < N);
        assert(to < N);
        assert(len >= 0);
        vs[from].outs.push_back({ to, len });
    }

    void add_u(size_t a, size_t b, D len) {
        add(a, b, len);
        add(b, a, len);
    }

    D solve(size_t from, size_t to) {
        vs[from].dist = 0;
#ifdef DIJKSTRA_RECORD_ROUTE
        vs[from].prev = from;
#endif

        auto comp = [&](size_t x, size_t y) {
            return vs[x].dist < vs[y].dist || (vs[x].dist == vs[y].dist && x < y);
        };
        set<size_t, decltype(comp)> q { comp };
        for (size_t i = 0; i < N; ++i) {
            q.insert(i);
        }

        while (!q.empty()) {
            size_t i;
            {
                auto it = q.begin();
                i       = *it;
                q.erase(it);
            }

            if (i == to) {
                goto RETURN;
            }

            for (Edge const& e : vs[i].outs) {
                if (vs[e.to].dist > vs[i].dist + e.len) {
                    if (q.find(e.to) != q.end()) {
                        q.erase(e.to);
                        vs[e.to].dist = vs[i].dist + e.len;
#ifdef DIJKSTRA_RECORD_ROUTE
                        vs[e.to].prev = i;
#endif
                        q.insert(e.to);
                    }
                }
            }
        }

RETURN:
        return vs[to].dist;
    }
};
```

4.4 SPFA shortest paths

```
// Shortest Path

template <long N>
struct SPFA {
    struct Edge {
        long to, len;
    };

    vector<Edge> edges;
    vector<long> outs[N];

    void add(long from, long to, long len) {
        edges.push_back({to, len});
        outs[from].push_back(edges.size() - 1);
    }

    long dist[N];
    long route[N];
    long visiting[N];
    bool active[N];

    long solve(long from, long to) {
        memset(active, 0, sizeof(active));

        for (int i = 0; i < N; ++i) {
            dist[i] = 1l << 60;
        }

        long head = 0;
        long tail = 0;

        dist[from] = 0;
        visiting[(tail++) % N] = from;
        active[from] = true;

        while (head < tail) {
            long i = visiting[(head++) % N];
            active[i] = false;

            for (long j = 0; j < outs[i].size(); ++j) {
                Edge &e = edges[outs[i][j]];

                if (dist[e.to] > dist[i] + e.len) {
                    dist[e.to] = dist[i] + e.len;
                    route[e.to] = i;

                    if (!active[e.to]) {
                        visiting[(tail++) % N] = e.to;
                        active[e.to] = true;
                    }
                }
            }
        }

        return dist[to];
    }
};
```

4.5 Minimum spanning trees using Prim

```
#include <bits/stdc++.h>
using namespace std;

// (u)int64, float
template<typename D>
```

```
struct Prim {
    static constexpr D Inf = 1ll << 60;

    struct Edge { size_t to; D len; };
    struct Vertex {
        vector<Edge> outs;
        D dist = Inf;
        size_t prev = -1;
    };

    size_t N;

    vector<Vertex> vs;

    // n nodes
    Prim(size_t n) : N(n) {
        vs.resize(n);
    }

    void add(size_t from, size_t to, D len) {
        assert(from < N);
        assert(to < N);
        assert(len >= 0);
        vs[from].outs.push_back({ to, len });
    }

    void add_u(size_t a, size_t b, D len) {
        add(a, b, len);
        add(b, a, len);
    }

    D solve(size_t from) {
        vs[from].dist = 0;
        vs[from].prev = from;

        auto comp = [&](size_t x, size_t y) {
            return vs[x].dist < vs[y].dist || (vs[x].dist == vs[y].dist && x < y);
        };
        set<size_t, decltype(comp)> q { comp };
        for (size_t i = 0; i < N; ++i) {
            q.insert(i);
        }

        while (!q.empty()) {
            size_t i;
            {
                auto it = q.begin();
                i       = *it;
                q.erase(it);
            }

            if (vs[i].dist == Inf) {
                vs[i].dist = 0;
            }

            for (Edge const& e : vs[i].outs) {
                if (vs[e.to].dist > e.len) {
                    if (q.find(e.to) != q.end()) {
                        q.erase(e.to);
                        vs[e.to].dist = e.len;
                        vs[e.to].prev = i;
                        q.insert(e.to);
                    }
                }
            }
        }

        D result = 0;
        for (size_t i = 0; i < N; ++i)
            result += vs[i].dist;
        return result;
    }
};
```

4.6 Minimum spanning trees using Kruskal

```
#include <algorithm>
#include <cassert>
#include <cstdint>
#include <set>
#include <unordered_map>
#include <vector>

using namespace std;

// (u)int64, float
template <typename D>
struct Kruskal {
    static constexpr D Inf = 1ll << 60;

    struct Edge {
        size_t n1, n2;
        D len;

        bool operator<(Edge const& o) const noexcept {
            return len < o.len || (len == o.len && (n1 < o.n1 || (n1 == o.n1 && (n2 < o.n2))));
        }
    };

    typedef unordered_map<size_t, vector<Edge>> Forest;

    size_t N;
    vector<Edge> edges;

    // n nodes
    Kruskal(size_t n) : N(n) {}

    void add(size_t n1, size_t n2, D len) {
        assert(n1 < N);
        assert(n2 < N);
        assert(len >= 0);
        if (n1 > n2) swap(n1, n2);
        edges.push_back({ n1, n2, len });
    }

    void solve(Forest& forest) {
        sort(edges.begin(), edges.end());

        vector<size_t> _ufs(N);
        for (size_t i = 0; i < N; i++)
            _ufs[i] = i;

        auto ufs_p = [&](size_t i) -> size_t* {
            auto p = i;
            while (_ufs[p] != p)
                p = _ufs[p];
            _ufs[i] = p;
            return &_ufs[i];
        };

        for (Edge const& e : edges) {
            auto* ufs_n1p = ufs_p(e.n1);
            auto* ufs_n2p = ufs_p(e.n2);
            if (*ufs_n1p != *ufs_n2p) {
                if (*ufs_n1p > *ufs_n2p)
                    swap(*ufs_n1p, *ufs_n2p);

                auto it = forest.find(*ufs_n2p);
                if (it != forest.end()) {
                    forest[*ufs_n1p].insert(forest[*ufs_n1p].end(), it->second.begin(), it->second.end
                        ());
                    forest.erase(it);
                }

                forest[*ufs_n1p].push_back(e);

                *ufs_n2p = *ufs_n1p;
            }
        }

        return;
    }
};
```

4.7 Strongly connected components

```
// Strongly Connected Components

template <long N>
struct Tarjan {
    vector<long> outs[N];

    void add(long from, long to) {
        outs[from].push_back(to);
    }

    long id_self[N];
    long id_low[N];
    long route[N];

    void dfs(long (&scc)[N], long from, long &last, long &now, long &now_scc) {
        id_self[from] = now;
        id_low[from] = now;
        now += 1;
```

```
        route[from] = last;
        last = from;

        for (long j = 0; j < outs[from].size(); ++j) {
            long to = outs[from][j];

            if (!id_self[to]) {
                dfs(scc, to, last, now, now_scc);

                id_low[from] = min(id_low[from], id_low[to]);
            } else if (!scc[to]) {
                id_low[from] = min(id_low[from], id_self[to]);
            }
        }

        if (id_low[from] == id_self[from]) {
            while (last != from) {
                scc[last] = now_scc;
                last = route[last];
            }

            scc[last] = now_scc;
            last = route[last];
            now_scc += 1;
        }
    }

    void solve(long (&scc)[N]) {
        memset(id_self, 0, sizeof(id_self));

        long last = 0;
        long now = 1;
        long now_scc = 1;

        for (long i = 0; i < N; ++i) {
            if (!id_self[i]) {
                dfs(scc, i, last, now, now_scc);
            }
        }

        for (long i = 0; i < N; ++i) {
            scc[i] -= 1;
        }
    }
};
```

4.8 Eulerian path

```
// Eulerian Circuit

template <long N>
struct HierholzerUndirected {
    struct Edge {
        long to;
        bool chosen;
    };

    vector<Edge> edges;
    vector<long> outs[N];

    void add(long from, long to) {
        edges.push_back(Edge {to, false});
        outs[from].push_back(edges.size() - 1);
        edges.push_back(Edge {from, false});
        outs[to].push_back(edges.size() - 1);
    }

    long step[N];

    void dfs(vector<long> &path, long from) {
        for (; step[from] < outs[from].size(); ++step[from]) {
            Edge &e = edges[outs[from][step[from]]];

            if (!e.chosen) {
                e.chosen = true;
                edges[outs[from][step[from]] ^ 1].chosen = true;
                dfs(path, e.to);
            }
        }

        path.push_back(from);
    }

    void solve(vector<long> &path, long from) {
        memset(step, 0, sizeof(step));

        dfs(path, from);
    }
};

template <long N>
struct HierholzerDirected {
    struct Edge {
        long to;
        bool chosen;
    };

    vector<Edge> edges;
    vector<long> outs[N];

    void add(long from, long to) {
        edges.push_back(Edge {to, false});
        outs[from].push_back(edges.size() - 1);
    }

    long step[N];

    void dfs(vector<long> &path, long from) {
        for (; step[from] < outs[from].size(); ++step[from]) {
            Edge &e = edges[outs[from][step[from]]];

            if (!e.chosen) {
                e.chosen = true;
                dfs(path, e.to);
            }
        }

        path.push_back(from);
    }

    void solve(vector<long> &path, long from) {
        memset(step, 0, sizeof(step));

        dfs(path, from);
    }
};
```

4.9 Travelling Salesman Problem

```
// Travelling Salesman Problem

template <long N>
struct HeldKarp {
    struct Edge {
        long to, len;
    };

    vector<Edge> edges;
    vector<long> outs[N];

    void add(long from, long to, long len) {
        edges.push_back({to, len});
        outs[from].push_back(edges.size() - 1);
    }

    long dist[N][N];
    long best[1l << (N - 1)][N];

    long solve(long n) {
        for (long i = 0; i < n; ++i) {
            for (long j = 0; j < n; ++j) {
                dist[i][j] = 1l << 60;
            }

            dist[i][i] = 0;

            for (long j = 0; j < outs[i].size(); ++j) {
                Edge &e = edges[outs[i][j]];

                dist[i][e.to] = min(dist[i][e.to], e.len);
            }
        }

        for (long i = 0; i < n; ++i) {
            for (long j = 0; j < n; ++j) {
                for (long k = 0; k < n; ++k) {
                    dist[j][k] = min(dist[j][k], dist[j][i] + dist[i][k]);
                }
            }
        }

        for (long i = 0; i < (1l << (n - 1)); ++i) {
            for (long j = 0; j < n; ++j) {
```

```
                best[i][j] = 1ll << 60;
            }

            for (long j = 0; j < n - 1; ++j) {
                if (i == 1ll << j) {
                    best[i][j] = dist[n - 1][j];
                } else if (i & (1ll << j)) {
                    for (long k = 0; k < n - 1; ++k) {
                        if ((i ^ (1ll << j)) & (1ll << k)) {
                            best[i][j] = min(best[i][j], dist[j][k] + best[i ^ (1ll << j)][k]);
                        }
                    }
                }
            }
        }

        long result = 1ll << 60;
        for (long i = 0; i < n - 1; ++i) {
            result = min(result, dist[i][n - 1] + best[(1ll << (n - 1)) - 1][i]);
        }

        return result;
    }
};
```

5 Data structures

5.1 Suffix array

```
// Suffix array construction in O(L log^2 L) time.  Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
//
// INPUT:   string s
//
// OUTPUT:  array suffix[] such that suffix[i] = index (from 0 to L-1)
//          of substring s[i...L-1] in the list of sorted suffixes.
//          That is, if we take the inverse of the permutation suffix[],
//          we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
  const int L;
  string s;
  vector<vector<int> > P;
  vector<pair<pair<int,int>,int> > M;

  SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
      P.push_back(vector<int>(L, 0));
      for (int i = 0; i < L; i++)
        M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
      sort(M.begin(), M.end());
      for (int i = 0; i < L; i++)
        P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
    }
  }

  vector<int> GetSuffixArray() { return P.back(); }

  // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
        j += 1 << k;
        len += 1 << k;
      }
    }
    return len;
  }
};

// BEGIN CUT
// The following code solves UVA problem 11512: GATTACA.
#define TESTING
#ifdef TESTING
int main() {
  int T;
  cin >> T;
  for (int caseno = 0; caseno < T; caseno++) {
    string s;
    cin >> s;
    SuffixArray array(s);
    vector<int> v = array.GetSuffixArray();
    int bestlen = -1, bestpos = -1, bestcount = 0;
    for (int i = 0; i < s.length(); i++) {
      int len = 0, count = 0;
      for (int j = i+1; j < s.length(); j++) {
        int l = array.LongestCommonPrefix(i, j);
        if (l >= len) {
          if (l > len) count = 2; else count++;
          len = l;
        }
      }
      if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen) > s.substr(i, len)) {
        bestlen = len;
        bestcount = count;
        bestpos = i;
      }
    }
    if (bestlen == 0) {
      cout << "No repetitions found!" << endl;
    } else {
      cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;
    }
  }
}

#else
// END CUT
int main() {

  // bobocel is the 0'th suffix
  //  obocel is the 5'th suffix
  //   bocel is the 1'st suffix
  //    ocel is the 6'th suffix
  //     cel is the 2'nd suffix
  //      el is the 3'rd suffix
  //       l is the 4'th suffix
  SuffixArray suffix("bobocel");
  vector<int> v = suffix.GetSuffixArray();

  // Expected output: 0 5 1 6 2 3 4
  //                  2
  for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
  cout << endl;
  cout << suffix.LongestCommonPrefix(0, 2) << endl;
}
// BEGIN CUT
#endif
// END CUT
```

5.2 Binary Indexed Tree

```
#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
  while(x <= N) {
    tree[x] += v;
    x += (x & -x);
  }
}

// get cumulative sum up to and including x
int get(int x) {
  int res = 0;
  while(x) {
    res += tree[x];
    x -= (x & -x);
  }
  return res;
}

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
```

```
  int idx = 0, mask = N;
  while(mask && idx < N) {
    int t = idx + mask;
    if(x >= tree[t]) {
      idx = t;
      x -= tree[t];
    }
    mask >>= 1;
  }
  return idx;
}
```

5.3 Union-find set

```
#include <bits/stdc++.h>
using namespace std;
struct UnionFind {
    vector<int> C;
    UnionFind(int n) : C(n) { for (int i = 0; i < n; i++) C[i] = i; }
    int find(int x) { return (C[x] == x) ? x : C[x] = find(C[x]); }
    void merge(int x, int y) { C[find(x)] = find(y); }
};
```

5.4 KD-tree

```
// -----------------------------------------------------------------
// A straightforward, but probably sub-optimal KD-tree implmentation
// that's probably good enough for most things (current it's a
// 2D-tree)
//
//  - constructs from n points in O(n lg^2 n) time
//  - handles nearest-neighbor query in O(lg n) if points are well
//    distributed
//  - worst case for nearest-neighbor may be linear in pathological
//    case
//
// Sonny Chan, Stanford University, April 2009
// -----------------------------------------------------------------

#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>

using namespace std;

// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();

// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};

bool operator==(const point &a, const point &b)
{
    return a.x == b.x && a.y == b.y;
}

// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
{
    return a.x < b.x;
}

// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
{
    return a.y < b.y;
}

// squared distance between points
ntype pdist2(const point &a, const point &b)
{
    ntype dx = a.x-b.x, dy = a.y-b.y;
    return dx*dx + dy*dy;
}

// bounding box for a set of points
struct bbox
{
    ntype x0, x1, y0, y1;

    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}

    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {
            x0 = min(x0, v[i].x);   x1 = max(x1, v[i].x);
            y0 = min(y0, v[i].y);   y1 = max(y1, v[i].y);
        }
    }

    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
            if (p.y < y0)       return pdist2(point(x0, y0), p);
            else if (p.y > y1)  return pdist2(point(x0, y1), p);
            else                return pdist2(point(x0, p.y), p);
        }
        else if (p.x > x1) {
            if (p.y < y0)       return pdist2(point(x1, y0), p);
            else if (p.y > y1)  return pdist2(point(x1, y1), p);
            else                return pdist2(point(x1, p.y), p);
        }
        else {
            if (p.y < y0)       return pdist2(point(p.x, y0), p);
            else if (p.y > y1)  return pdist2(point(p.x, y1), p);
            else                return 0;
        }
    }
};

// stores a single node of the kd-tree, either internal or leaf
struct kdnode
{
    bool leaf;      // true if this is a leaf node (has one point)
    point pt;       // the single point of this is a leaf
    bbox bound;     // bounding box for set of points in children

    kdnode *first, *second; // two children of this kd-node

    kdnode() : leaf(false), first(0), second(0) {}
    ~kdnode() { if (first) delete first; if (second) delete second; }

    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p) {
        return bound.distance(p);
    }

    // recursively builds a kd-tree from a given cloud of points
    void construct(vector<point> &vp)
    {
        // compute bounding box for points at this node
        bound.compute(vp);

        // if we're down to one point, then we're a leaf node
        if (vp.size() == 1) {
            leaf = true;
            pt = vp[0];
        }
        else {
            // split on x if the bbox is wider than high (not best heuristic...)
            if (bound.x1-bound.x0 >= bound.y1-bound.y0)
                sort(vp.begin(), vp.end(), on_x);
            // otherwise split on y-coordinate
            else
                sort(vp.begin(), vp.end(), on_y);

            // divide by taking half the array for each child
            // (not best performance if many duplicates in the middle)
            int half = vp.size()/2;
            vector<point> vl(vp.begin(), vp.begin()+half);
            vector<point> vr(vp.begin()+half, vp.end());
            first = new kdnode();   first->construct(vl);
            second = new kdnode();  second->construct(vr);
        }
    }
};

// simple kd-tree class to hold the tree and handle queries
struct kdtree
{
    kdnode *root;

    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnode();
        root->construct(v);
    }
    ~kdtree() { delete root; }
```

```
    // recursive search method returns squared distance to nearest point
    ntype search(kdnode *node, const point &p)
    {
        if (node->leaf) {
            // commented special case tells a point not to find itself
//            if (p == node->pt) return sentry;
//            else
                return pdist2(p, node->pt);
        }

        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);

        // choose the side with the closest bounding box to search first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {
            ntype best = search(node->first, p);
            if (bsecond < best)
                best = min(best, search(node->second, p));
            return best;
        }
        else {
            ntype best = search(node->second, p);
            if (bfirst < best)
                best = min(best, search(node->first, p));
            return best;
        }
    }

    // squared distance to the nearest
    ntype nearest(const point &p) {
        return search(root, p);
    }
};

// --------------------------------------------------------------------------
// some basic test code here

int main()
{
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand()%100000, rand()%100000));
    }
    kdtree tree(vp);

    // query some points
    for (int i = 0; i < 10; ++i) {
        point q(rand()%100000, rand()%100000);
        cout << "Closest squared distance to (" << q.x << ", " << q.y << ")"
             << " is " << tree.nearest(q) << endl;
    }

    return 0;
}

// --------------------------------------------------------------------------
```

5.5 Splay tree

```
#include <cstdio>
#include <algorithm>
using namespace std;

const int N_MAX = 130010;
const int oo = 0x3f3f3f3f;
struct Node
{
  Node *ch[2], *pre;
  int val, size;
  bool isTurned;
} nodePool[N_MAX], *null, *root;

Node *allocNode(int val)
{
  static int freePos = 0;
  Node *x = &nodePool[freePos ++];
  x->val = val, x->isTurned = false;
  x->ch[0] = x->ch[1] = x->pre = null;
  x->size = 1;
  return x;
}

inline void update(Node *x)
{
  x->size = x->ch[0]->size + x->ch[1]->size + 1;
}

inline void makeTurned(Node *x)
{
  if(x == null)
    return;
  swap(x->ch[0], x->ch[1]);
  x->isTurned ^= 1;
}

inline void pushDown(Node *x)
{
  if(x->isTurned)
  {
    makeTurned(x->ch[0]);
    makeTurned(x->ch[1]);
    x->isTurned ^= 1;
  }
}

inline void rotate(Node *x, int c)
{
  Node *y = x->pre;
  x->pre = y->pre;
  if(y->pre != null)
    y->pre->ch[y == y->pre->ch[1]] = x;
  y->ch[!c] = x->ch[c];
  if(x->ch[c] != null)
    x->ch[c]->pre = y;
  x->ch[c] = y, y->pre = x;
  update(y);
  if(y == root)
    root = x;
}

void splay(Node *x, Node *p)
{
  while(x->pre != p)
  {
    if(x->pre->pre == p)
      rotate(x, x == x->pre->ch[0]);
    else
    {
      Node *y = x->pre, *z = y->pre;
      if(y == z->ch[0])
      {
        if(x == y->ch[0])
          rotate(y, 1), rotate(x, 1);
        else
          rotate(x, 0), rotate(x, 1);
      }
      else
      {
        if(x == y->ch[1])
          rotate(y, 0), rotate(x, 0);
        else
          rotate(x, 1), rotate(x, 0);
      }
    }
  }
  update(x);
}

void select(int k, Node *fa)
{
  Node *now = root;
  while(1)
  {
    pushDown(now);
    int tmp = now->ch[0]->size + 1;
    if(tmp == k)
      break;
    else if(tmp < k)
      now = now->ch[1], k -= tmp;
    else
      now = now->ch[0];
  }
  splay(now, fa);
}

Node *makeTree(Node *p, int l, int r)
{
  if(l > r)
    return null;
  int mid = (l + r) / 2;
  Node *x = allocNode(mid);
  x->pre = p;
  x->ch[0] = makeTree(x, l, mid - 1);
  x->ch[1] = makeTree(x, mid + 1, r);
  update(x);
  return x;
}
```

```
int main()
{
  int n, m;
  null = allocNode(0);
  null->size = 0;
  root = allocNode(0);
  root->ch[1] = allocNode(oo);
  root->ch[1]->pre = root;
  update(root);

  scanf("%d%d", &n, &m);
  root->ch[1]->ch[0] = makeTree(root->ch[1], 1, n);
  splay(root->ch[1]->ch[0], null);

  while(m --)
  {
    int a, b;
    scanf("%d%d", &a, &b);
    a ++, b ++;
    select(a - 1, null);
    select(b + 1, root);
    makeTurned(root->ch[1]->ch[0]);
  }

  for(int i = 1; i <= n; i ++)
  {
    select(i + 1, null);
    printf("%d ", root->val);
  }
}
```

5.6 Lowest common ancestor

```
const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;

vector<int> children[max_nodes];        // children[i] contains the children of node i
int A[max_nodes][log_max_nodes+1];      // A[i][j] is the 2^j-th ancestor of node i, or -1 if that
      ancestor does not exist
int L[max_nodes];                       // L[i] is the distance between node i and the root

// floor of the binary logarithm of n
int lb(unsigned int n)
{
    if(n==0)
        return -1;
    int p = 0;
    if (n >= 1<<16) { n >>= 16; p += 16; }
    if (n >= 1<< 8) { n >>=  8; p +=  8; }
    if (n >= 1<< 4) { n >>=  4; p +=  4; }
    if (n >= 1<< 2) { n >>=  2; p +=  2; }
    if (n >= 1<< 1) {           p +=  1; }
    return p;
}

void DFS(int i, int l)
{
    L[i] = l;
    for(int j = 0; j < children[i].size(); j++)
        DFS(children[i][j], l+1);
}

int LCA(int p, int q)
{
    // ensure node p is at least as deep as node q
    if(L[p] < L[q])
        swap(p, q);

    // "binary search" for the ancestor of node p situated on the same level as q
    for(int i = log_num_nodes; i >= 0; i--)
        if(L[p] - (1<<i) >= L[q])
            p = A[p][i];

    if(p == q)
        return p;

    // "binary search" for the LCA
    for(int i = log_num_nodes; i >= 0; i--)
        if(A[p][i] != -1 && A[p][i] != A[q][i])
        {
            p = A[p][i];
            q = A[q][i];
        }

    return A[p][0];
}

int main(int argc,char* argv[])
{
    // read num_nodes, the total number of nodes
    log_num_nodes=lb(num_nodes);

    for(int i = 0; i < num_nodes; i++)
    {
        int p;
        // read p, the parent of node i or -1 if node i is the root

        A[i][0] = p;
        if(p != -1)
            children[p].push_back(i);
        else
            root = i;
    }

    // precompute A using dynamic programming
    for(int j = 1; j <= log_num_nodes; j++)
        for(int i = 0; i < num_nodes; i++)
            if(A[i][j-1] != -1)
                A[i][j] = A[A[i][j-1]][j-1];
            else
                A[i][j] = -1;

    // precompute L
    DFS(root, 0);


    return 0;
}
```

6 Miscellaneous

6.1 Longest increasing subsequence

```
// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
//   INPUT: a vector of integers
//   OUTPUT: a vector containing the longest increasing subsequence

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;

#define STRICTLY_INCREASNG

VI LongestIncreasingSubsequence(VI v) {
  VPII best;
  VI dad(v.size(), -1);

  for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
    PII item = make_pair(v[i], 0);
    VPII::iterator it = lower_bound(best.begin(), best.end(), item);
    item.second = i;
#else
    PII item = make_pair(v[i], i);
    VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
    if (it == best.end()) {
      dad[i] = (best.size() == 0 ? -1 : best.back().second);
      best.push_back(item);
    } else {
      dad[i] = it == best.begin() ? -1 : prev(it)->second;
      *it = item;
    }
  }

  VI ret;
  for (int i = best.back().second; i >= 0; i = dad[i])
    ret.push_back(v[i]);
  reverse(ret.begin(), ret.end());
  return ret;
}
```

6.2 Dates

```cpp
// Routines for performing computations on dates.  In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.

#include <iostream>
#include <string>

using namespace std;

string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
  return
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
  int x, n, i, j;

  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
  return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
  int jd = dateToInt (3, 24, 2004);
  int m, d, y;
  intToDate (jd, m, d, y);
  string day = intToDay (jd);

  // expected output:
  //    2453089
  //    3/24/2004
  //    Wed
  cout << jd << endl
    << m << "/" << d << "/" << y << endl
    << day << endl;
}
```

6.3 Prime numbers

```cpp
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool IsPrimeSlow (LL x)
{
  if(x<=1) return false;
  if(x<=3) return true;
  if (!(x%2) || !(x%3)) return false;
  LL s=(LL)(sqrt((double)(x))+EPS);
  for(LL i=5;i<=s;i+=6)
  {
    if (!(x%i) || !(x%(i+2))) return false;
  }
  return true;
}
// Primes less than 1000:
//      2     3     5     7    11    13    17    19    23    29    31    37
//     41    43    47    53    59    61    67    71    73    79    83    89
//     97   101   103   107   109   113   127   131   137   139   149   151
//    157   163   167   173   179   181   191   193   197   199   211   223
//    227   229   233   239   241   251   257   263   269   271   277   281
//    283   293   307   311   313   317   331   337   347   349   353   359
//    367   373   379   383   389   397   401   409   419   421   431   433
//    439   443   449   457   461   463   467   479   487   491   499   503
//    509   521   523   541   547   557   563   569   571   577   587   593
//    599   601   607   613   617   619   631   641   643   647   653   659
//    661   673   677   683   691   701   709   719   727   733   739   743
//    751   757   761   769   773   787   797   809   811   821   823   827
//    829   839   853   857   859   863   877   881   883   887   907   911
//    919   929   937   941   947   953   967   971   977   983   991   997

// Other primes:
//    The largest prime smaller than 10 is 7.
//    The largest prime smaller than 100 is 97.
//    The largest prime smaller than 1000 is 997.
//    The largest prime smaller than 10000 is 9973.
//    The largest prime smaller than 100000 is 99991.
//    The largest prime smaller than 1000000 is 999983.
//    The largest prime smaller than 10000000 is 9999991.
//    The largest prime smaller than 100000000 is 99999989.
//    The largest prime smaller than 1000000000 is 999999937.
//    The largest prime smaller than 10000000000 is 9999999967.
//    The largest prime smaller than 100000000000 is 99999999977.
//    The largest prime smaller than 1000000000000 is 999999999989.
//    The largest prime smaller than 10000000000000 is 9999999999971.
//    The largest prime smaller than 100000000000000 is 99999999999973.
//    The largest prime smaller than 1000000000000000 is 999999999999989.
//    The largest prime smaller than 10000000000000000 is 9999999999999937.
//    The largest prime smaller than 100000000000000000 is 99999999999999997.
//    The largest prime smaller than 1000000000000000000 is 999999999999999989.
```

6.4 Miller-Rabin Primality Test

```cpp
// Randomized Primality Test (Miller-Rabin):
//   Error rate: 2^(-TRIAL)
//   Almost constant time. srand is needed

#include <stdlib.h>
#define EPS 1e-7

typedef long long LL;

LL ModularMultiplication(LL a, LL b, LL m)
{
        LL ret=0, c=a;
        while(b)
        {
                if(b&1) ret=(ret+c)%m;
                b>>=1; c=(c+c)%m;
        }
        return ret;
}
LL ModularExponentiation(LL a, LL n, LL m)
{
        LL ret=1, c=a;
        while(n)
        {
                if(n&1) ret=ModularMultiplication(ret, c, m);
                n>>=1; c=ModularMultiplication(c, c, m);
        }
        return ret;
}
bool Witness(LL a, LL n)
{
        LL u=n-1;
  int t=0;
        while(!(u&1)){u>>=1; t++;}
        LL x0=ModularExponentiation(a, u, n), x1;
        for(int i=1;i<=t;i++)
        {
                x1=ModularMultiplication(x0, x0, n);
                if(x1==1 && x0!=1 && x0!=n-1) return true;
                x0=x1;
        }
        if(x0!=1) return true;
        return false;
}
LL Random(LL n)
{
  LL ret=rand(); ret*=32768;
        ret+=rand(); ret*=32768;
        ret+=rand(); ret*=32768;
        ret+=rand();
  return ret%n;
}
bool IsPrimeFast(LL n, int TRIAL)
{
  while(TRIAL--)
  {
```

```
    LL a=Random(n-2)+1;
    if(Witness(a, n)) return false;
  }
  return true;
}
```

6.5 Fast exponentiation

```
/*
Uses powers of two to exponentiate numbers and matrices. Calculates
n^k in O(log(k)) time when n is a number. If A is an n x n matrix,
calculates A^k in O(n^3*log(k)) time.
*/

#include <iostream>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T power(T x, int k) {
  T ret = 1;

  while(k) {
    if(k & 1) ret *= x;
    k >>= 1; x *= x;
  }
  return ret;
}

VVT multiply(VVT& A, VVT& B) {
  int n = A.size(), m = A[0].size(), k = B[0].size();
  VVT C(n, VT(k, 0));

  for(int i = 0; i < n; i++)
    for(int j = 0; j < k; j++)
      for(int l = 0; l < m; l++)
        C[i][j] += A[i][l] * B[l][j];

  return C;
}

VVT power(VVT& A, int k) {
  int n = A.size();
  VVT ret(n, VT(n)), B = A;
  for(int i = 0; i < n; i++) ret[i][i]=1;

  while(k) {
    if(k & 1) ret = multiply(ret, B);
    k >>= 1; B = multiply(B, B);
  }
  return ret;
}

int main()
{
  /* Expected Output:
     2.37^48 = 9.72569e+17

     376 264 285 220 265
     550 376 529 285 484
     484 265 376 264 285
     285 220 265 156 264
     529 285 484 265 376 */
  double n = 2.37;
  int k = 48;

  cout << n << "^" << k << " = " << power(n, k) << endl;

  double At[5][5] = {
    { 0, 0, 1, 0, 0 },
    { 1, 0, 0, 1, 0 },
    { 0, 0, 0, 0, 1 },
    { 1, 0, 0, 0, 0 },
    { 0, 1, 0, 0, 0 } };

  vector <vector <double> > A(5, vector <double>(5));
  for(int i = 0; i < 5; i++)
    for(int j = 0; j < 5; j++)
      A[i][j] = At[i][j];

  vector <vector <double> > Ap = power(A, k);

  cout << endl;
  for(int i = 0; i < 5; i++) {
    for(int j = 0; j < 5; j++)
      cout << Ap[i][j] << " ";
    cout << endl;
  }
}
```

6.6 C++ input/output

```
#include <bits/stdc++.h>

using namespace std;

int main()
{
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);

    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}
```

6.7 Knuth-Morris-Pratt

```
/*
Finds all occurrences of the pattern string p within the
text string t. Running time is O(n + m), where n and m
are the lengths of p and t, respecitvely.
*/

#include <iostream>
#include <string>
#include <vector>

using namespace std;

typedef vector<int> VI;

void buildPi(string& p, VI& pi)
{
  pi = VI(p.length());
  int k = -2;
  for(int i = 0; i < p.length(); i++) {
    while(k >= -1 && p[k+1] != p[i])
      k = (k == -1) ? -2 : pi[k];
    pi[i] = ++k;
  }
}

int KMP(string& t, string& p)
{
  VI pi;
  buildPi(p, pi);
  int k = -1;
  for(int i = 0; i < t.length(); i++) {
    while(k >= -1 && p[k+1] != t[i])
      k = (k == -1) ? -2 : pi[k];
    k++;
    if(k == p.length() - 1) {
      // p matches t[i-m+1, ..., i]
      cout << "matched at index " << i-k << ": ";
      cout << t.substr(i-k, p.length()) << endl;
      k = (k == -1) ? -2 : pi[k];
    }
  }
  return 0;
}

int main()
{
```

```
  string a = "AABAACAADAABAABA", b = "AABA";
  KMP(a, b); // expected matches at: 0, 9, 12
  return 0;
}
```

6.8 Latitude/longitude

```
/*
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
*/

#include <iostream>
#include <cmath>

using namespace std;

struct ll
{
  double r, lat, lon;
};

struct rect
{
  double x, y, z;
};

ll convert(rect& P)
{
  ll Q;
  Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
  Q.lat = 180/M_PI*asin(P.z/Q.r);
  Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

  return Q;
}

rect convert(ll& Q)
{
  rect P;
  P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.z = Q.r*sin(Q.lat*M_PI/180);

  return P;
}

int main()
{
  rect A;
  ll B;

  A.x = -1.0; A.y = 2.0; A.z = -3.0;

  B = convert(A);
  cout << B.r << " " << B.lat << " " << B.lon << endl;

  A = convert(B);
  cout << A.x << " " << A.y << " " << A.z << endl;
}
```

6.9 Vim settings

```
set enc=utf-8
set fenc=utf-8
scriptencoding utf-8

set fencs=utf-8,ucs-bom,gb18030,gbk,gb2312,cp936

syntax on

filetype plugin on
filetype indent on

set mouse=a

set nocompatible

set tabstop=4

set shiftwidth=4

set expandtab

set smarttab

set autoindent

set textwidth=1000

set showmatch

set ruler

set hlsearch
set incsearch
set ignorecase
set smartcase

set number
set relativenumber

set fdm=marker

set scrolloff=10

set showcmd

set backspace=indent,eol,start

set formatoptions=q,r

set nowrap

set foldmethod=indent
set foldlevelstart=10

set wildmenu

set omnifunc=syntaxcomplete#Complete

"the trail config failed in generate latex
"remember to add in the site"
set list listchars=tab:>-,trail:

set t_Co=256
set background=dark

nnoremap ; :

inoremap <silent> jj <ESC>

nnoremap <silent> <HOME> ^
inoremap <silent> <HOME> <ESC>^i
vnoremap <silent> <HOME> 0w

nnoremap <silent> <F9> :set nowrap<CR>
nnoremap <silent> <F10> :set wrap<CR>
```