Chapter 2

有穷自动机

1. Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{0, 1\}$.

   (a) All strings containing at most three 0s.

   (b) All strings containing the substring 000.

   (c) All strings not containing the substring 000.

   (d) All strings in which every run of 0s has length at least 3.

   (e) All strings in which no substring 000 appears before a 1.

(f) The language with only one string 000.

(g) Every string except 000.

(h) All strings w such that in *every prefix* of w, the number of 0s and 1s differ by at most 1.

(i) All strings w such that in every prefix of w, the number of 0s and 1s differ by at most 2.

(j) All strings in which the substring 000 appears an even number of times. (For example, 0001000 and 0000 are in this language, but 00000 is not.)

2. Design an NFA for $L = \{a^i b^j \mid i, j \geq 0 \text{ and } i + j \text{ is even.}\}$

3. [Execise 2.2.2] We defined $\hat{\delta}$ by breaking the input string into any string followed by a single symbol (in the inductive part, Equation 2.1). However, we informally think of $\hat{\delta}$ as describing what happens along a path with a certain string of labels, and if so, then it should not matter how we break the input string in the definition of $\hat{\delta}$. Show that in fact, $\hat{\delta}(q,xy) = \hat{\delta}(\hat{\delta}(q,x),y)$ for any state q and strings x and y. Hint: Perform an induction on $|y|$.

4. [Execise 2.2.3] Show that for any state q, string x, and input symbol a, $\hat{\delta}(q,ax) = \hat{\delta}(\delta(q,a),x)$. Hint: Use Exercise 2.2.2.

5. [Execise 2.2.4] Give DFA's accepting the following languages over the alphabet $\{0,1\}$.

   a) The set of all strings ending in 00.

   b) The set of all strings with three consecutive 0's (not necessarily at the end).

   c) The set of strings with 011 as a substring.

6. [Execise 2.2.5] Design DFA.

   a) The set of all strings such that each block of five consecutive symbols contains at least two 0's.

   b) The set of all strings whose tenth symbol from the right end is a 1.

   c) The set of strings that either begin or end (or both) with 01.

   d) The set of strings such that the number of 0's is divisible by 5, and the number of 1's is divisible by 3.

7. [Execise 2.2.6] Give DFA's accepting the following languages over the alphabet $\{0, 1\}$:

   a) The set of all strings beginning with a 1 that, when interpreted as binary integer, is a multiple of 5. for example, strings 101(5), 1010(10), and 1111(15) are in the language; 0, 100(4) and 111(7) are not.

b) The set of all strings that, when interpreted *in reverse* as a binary integer, is divisible by 5. Examples of string in the language are 0, 10011(25), 1001100(25), and 0101(10).

8. [Execise 2.2.7] Let A be a DFA and q a particular state of A, such that $\delta(q,a) = q$ for all input symbols a. Show by induction on the length of the input that for all input strings w, $\hat{\delta}(q,w) = q$.

9. [Execise 2.2.8] Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q,a) = q$.

a) Show by induction on n that for all $n \geq 0$, $\hat{\delta}(q,a^n) = q$, where a^n is the string consisting of n a's.

b) Show that either $\{a\}^* \subseteq L(A)$ or $\{a\}^* \cap \mathbf{L}(A) = \varnothing$.

10. [Execise 2.2.9] Let $A = (Q,\Sigma,\delta,q_0,\{q_f\})$ be a DFA, and suppose that for all a in Σ we have $\delta(q_0,a) = \delta(q_f,a)$.

(a) Show that for all $w \neq \varepsilon$ we have $\hat{\delta}(q_0,w) = \hat{\delta}(q_f,w)$.

(b) Show that if x is a nonempty string in $L(A)$, then for all $k > 0$, x^k (i.e. x written k times) is also in $L(A)$.

11. [Execise 2.2.10] Consider the DFA with the following transition table:

    Informally describe the language accepted by this DFA, and prove by induction on the length of an input string that your description is correct. *Hint*: When setting up the inductive hypothesis, it is wise to make a statement about what inputs get you to each state, not just what inputs get you to the accepting state.

	0	1
→ A	A	B
*B	B	A

12. [Execise 2.3.1] Convert to a DFA the following NFA:

	0	1
$\rightarrow p$	$\{p,q\}$	$\{p\}$
q	$\{r\}$	$\{r\}$
r	$\{s\}$	$\varnothing$
$*s$	$\{s\}$	$\{s\}$

13. [Execise 2.3.2] Convert to a DFA the following NFA:

	0	1
$\rightarrow p$	$\{q,s\}$	$\{q\}$
$*q$	$\{r\}$	$\{q,r\}$
r	$\{s\}$	$\{p\}$
$*s$	$\varnothing$	$\{p\}$

14. [Execise 2.3.3] Convert the following NFA to a DFA and informally describe the language it accepts.

	0	1
$\rightarrow p$	$\{p,q\}$	$\{p\}$
q	$\{r,s\}$	$\{t\}$
r	$\{p,r\}$	$\{t\}$
$*s$	$\varnothing$	$\varnothing$
$*t$	$\varnothing$	$\varnothing$

15. [Execise 2.3.4] Give NFA, try to take advantage of nondeterminism as much as possible.

    (a) The set of strings over alphabet $\{0,1,\cdots,9\}$ such that the final digit has appear before.

    (b) The set of strings over alphabet $\{0,1,\ldots,9\}$ such that the final digit has *not* appeared before.

    (c) The set of strings of 0's and 1's such that there are two 0's separated by a number of positions that is a mutiple of 4. (Note that 0 is an allowable multiple of 4.)

chunyu@hit.edu.cn
http://iilab.net/chunyu