

Morning



手机关机或静音放包/兜里

Formal Language And Automata

- ♥ Languages
- ♥ Automata
- ♥ Computation



Language

What is a language ?

This is a sentence.

This is also a sentence.

So we have

{ sentence 1, sentence 2, sentence 3, ... }

the set of sentences \Leftrightarrow Language

Sentence/String

Sentence, String, Word

$(1 + 2) * (13 - 7)$

To stay at home and save lives.

不聚集，戴口罩，勤洗手。

0, 1, 00, 01, 10, 000, 001, 1010, 00111100

String = sequence of symbols

chosen from the alphabet Σ

Alphabet

Alphabet = set of symbols

$\{a, b, c, \dots, x, y, z\}$

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, *, /, (,)\}$

ASCII, 中文国标, $\Sigma = \{0, 1\}$

symbols \Rightarrow strings \Rightarrow language

Rules/Grammar

Rules : by which strings are composed/generated
rules for English :

$\langle \text{sentence} \rangle \rightarrow \langle \text{noun-phrase} \rangle \langle \text{predicate} \rangle$

$\langle \text{noun-phrase} \rangle \rightarrow \langle \text{article} \rangle \langle \text{noun} \rangle$

$\langle \text{predicate} \rangle \rightarrow \langle \text{verb} \rangle$

$\langle \text{article} \rangle \rightarrow a|an|the$

$\langle \text{noun} \rangle \rightarrow \text{wolf}|\text{sheep}$

$\langle \text{verb} \rangle \rightarrow \text{love}|\text{eat}$

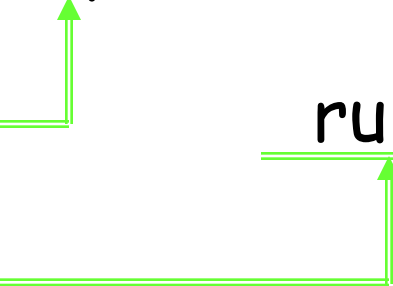
I have a dream.

Example 1

$L = \{w \mid w \text{ consists of 0's and 1's, and end with 0}\}$

alphabet : {0,1}

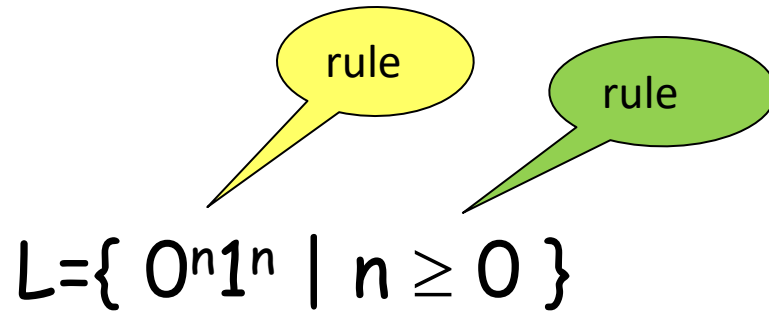
rule



$L = \{ 0, 00, 10, 000, 010, 100, 110, 0000, \dots \}$

$1111100 \in L, 1 \notin L, 0001 \notin L, 20 \notin L$

Example 2

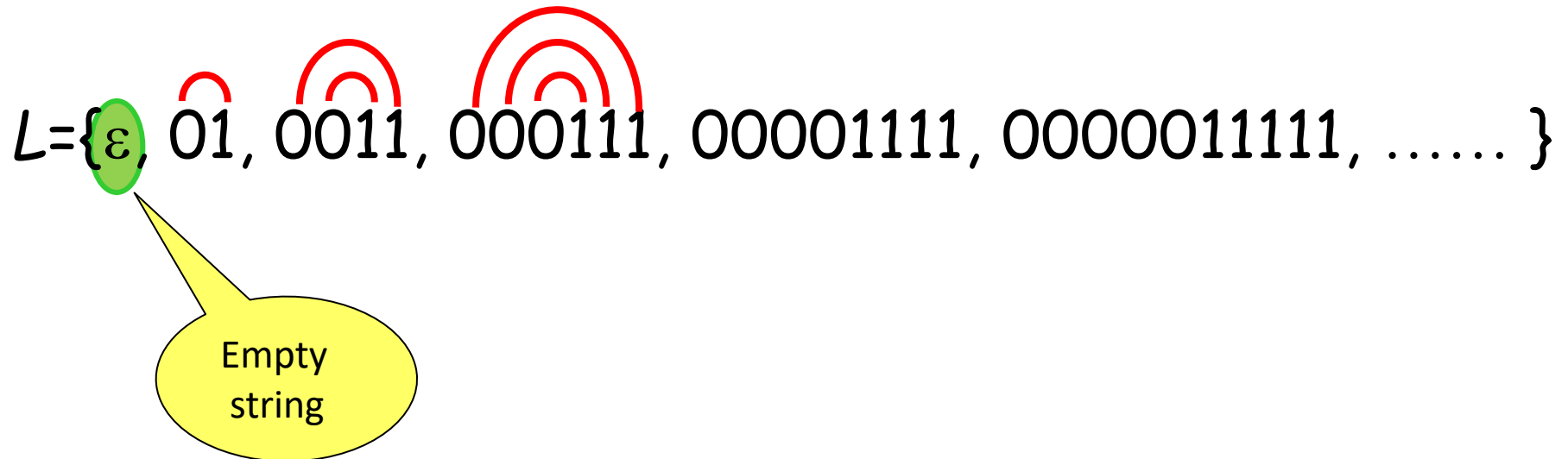


rule

rule

$$L = \{ 0^n 1^n \mid n \geq 0 \}$$

alphabet = {0,1}



Empty string

$$L = \{ \varepsilon, 01, 0011, 000111, 00001111, 0000011111, \dots \}$$

Example 3

$L = \{ w \mid w \text{ is a sentence in English} \}$

Everyone loves his/her motherland. ✓

Dog bites man. ✓

Man bites dog. ?

- ♣ Formal language focus on form of string
not semantics/meaning

Operations of String

$$w = a_1a_2\ldots a_m$$

abc

$$v = b_1b_2\ldots b_n$$

123456

◆ Concatenation

$$wv = a_1a_2\ldots a_mb_1b_2\ldots b_n$$

abc123456

$$\begin{array}{c} \text{✂} \\ vw = b_1b_2\ldots b_na_1a_2\ldots a_m \end{array}$$

123456abc

◆ Reverse

$$w^R = a_na_{n-1}\ldots a_1$$

cba

Operations on Languages

♦ Usual set operations

$$L_1 \cup L_2 = \{ w \mid w \in L_1 \text{ or } w \in L_2 \}$$

$$L_1 \cap L_2 = \{ w \mid w \in L_1 \text{ and } w \in L_2 \}$$

$$L_1 - L_2 = \{ w \mid w \in L_1 \text{ and } w \notin L_2 \}$$

♦ Reverse

$$L^R = \{ w^R \mid w \in L \}$$

♦ Concatenation

$$L_1 L_2 = \{ wv \mid w \in L_1 \text{ and } v \in L_2 \}$$

Example 4

$$L = \{ ab, abc, abcd \} \Rightarrow L^R = \{ ba, cba, dcba \}$$

$$L = \{ a^n b^n \mid n \geq 1 \} \Rightarrow L^R = \{ b^n a^n \mid n \geq 1 \}$$

$$L = \{ a^n b^n \mid n \geq 1 \}, K = \{ 0^n 1^n \mid n \geq 1 \}$$

$$LK = \{ a^n b^n 0^n 1^n \mid n \geq 1 \}$$



$$LK = \{ a^n b^n 0^m 1^m \mid n \geq 1, m \geq 1 \}$$

$$L^2 = ?$$

* / Star Operation on Languages

$$\Sigma = \{0, 1\}$$

$$\Sigma^* = \Sigma^0 \cup \Sigma \cup \Sigma^2 \cup \Sigma^3 \cup \dots \cup \Sigma^n \cup \dots$$

$$\Sigma^0 = \{\varepsilon\}, \quad \Sigma^n = \underbrace{\Sigma \Sigma \dots \Sigma}_n$$

$$\{0,1\}^* = \{\varepsilon\} \cup \{0,1\} \cup \{0,1\}^2 \cup \dots \cup \{0,1\}^n \cup \dots$$

$$= \{ \varepsilon, \underline{0}, \underline{1}, \underline{00}, \underline{01}, \underline{10}, \underline{11}, 000, 001, 010, 011, 111, \dots \}$$

Empty string / language

Denote ε as empty string

$$|\varepsilon| = 0, \quad w\varepsilon = \varepsilon w = w$$

Denote ϕ as empty language

$$\phi = \{\}, \quad \phi L = L\phi = \phi$$

Denote $\Sigma^+ = \Sigma \cup \Sigma^2 \cup \Sigma^3 \cup \dots \cup \Sigma^n \cup \dots$

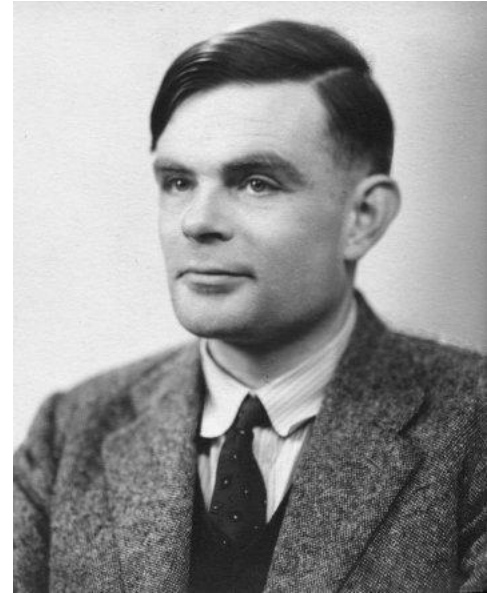
$$= \Sigma^* - \Sigma^0$$

$$= \Sigma^* - \{\varepsilon\}$$

Automata

Alan Marthison Turing

- On Computable Numbers
With an Application to
the Entscheidungs Problem
- Turing Machine



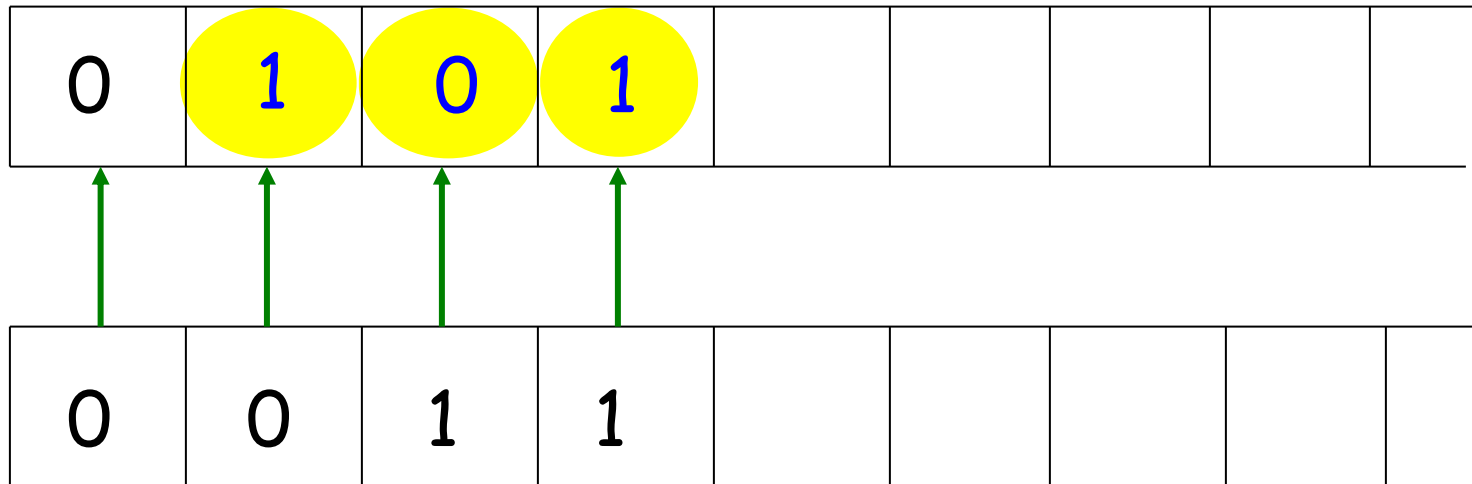
Automata

- ◆ Finite Automata
 - Deterministic Finite Automata
 - Non-deterministic Finite Automata
- ◆ Push Down Automata
- ◆ Turing Mashine

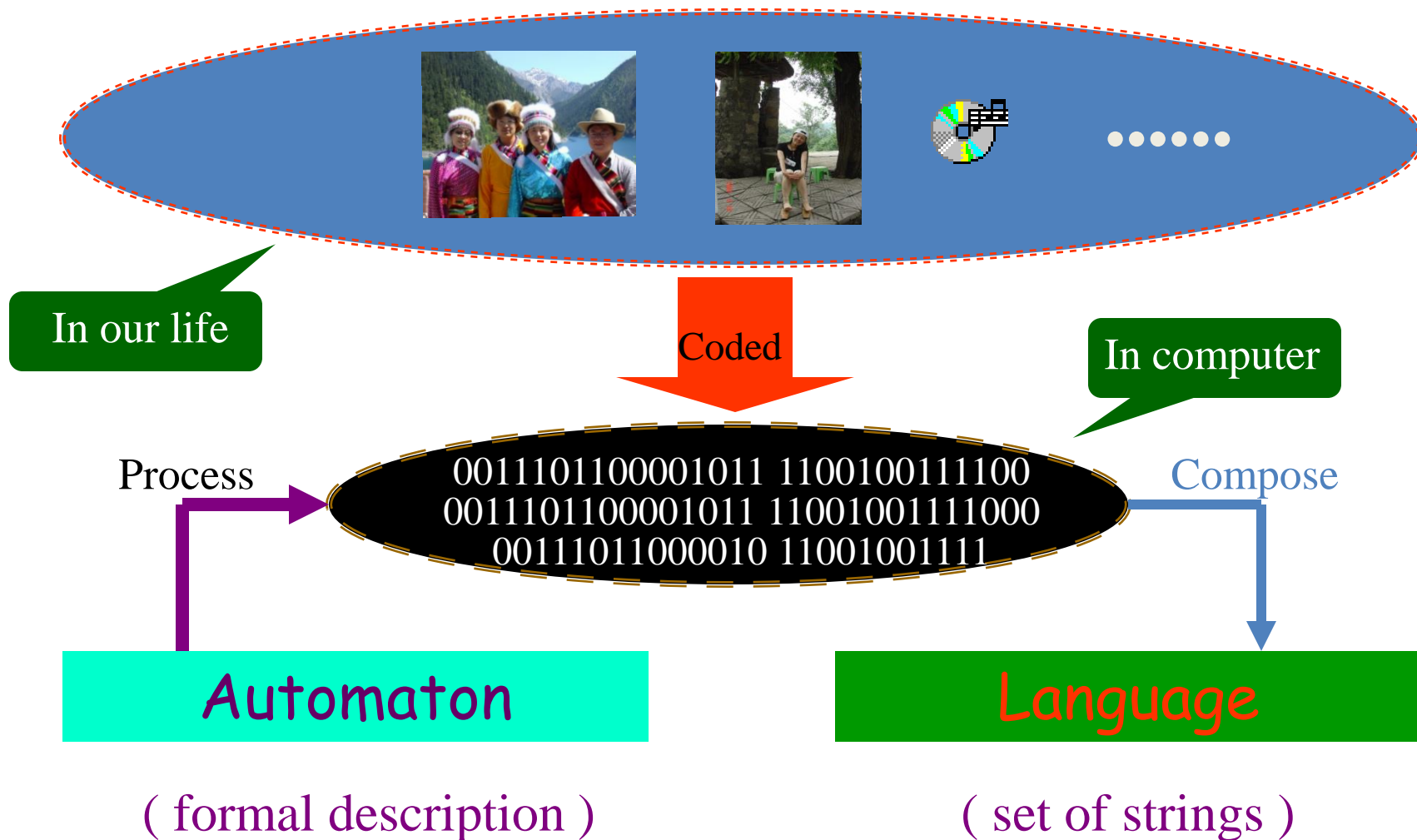
Computation

$$\begin{array}{r} 2 \\ + 3 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 0010 \\ + 0011 \\ \hline 0101 \end{array}$$



Computation



Computation

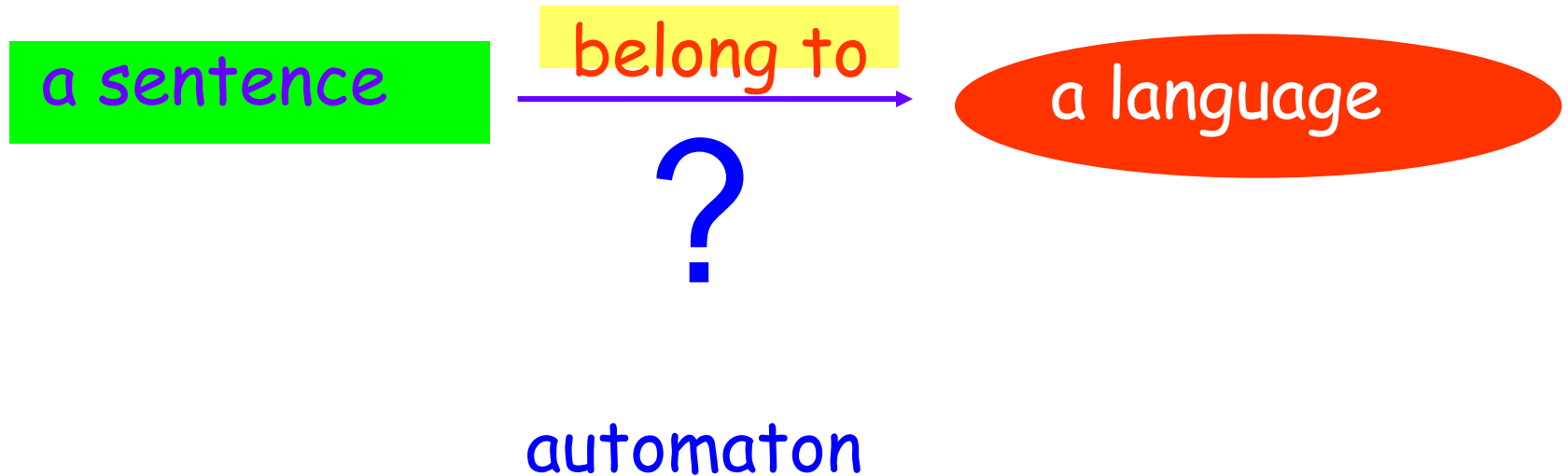
- ◆ Computable Problems
 - write a program to solve
- ◆ Intractable Problems
 - find some way to work around

Undecidable Problem

```
main ( )
{
    Int n, total, x, y, z ;
    scanf("%d", &n);
    total=3;
    while(1){
        for(x=1;x<=total-2;x++)
            for(y=1;y<=total-x-1;y++){
                z=total-x-y;
                if(exp(x,n)+exp(y,n)==exp(z,n))
                    printf("hello,world\n");
            }
        total++;
    }
}
```


$$x^n + y^n = z^n$$

Undecidable Problem



Content

Automata

Languages

Grammars

Construction

Properties

Design

Finite
Automaton

Regular
Language

Regular
Expression

Recognize

Generate

Push Down
Automaton

Context Free
Language

Context Free
Grammar

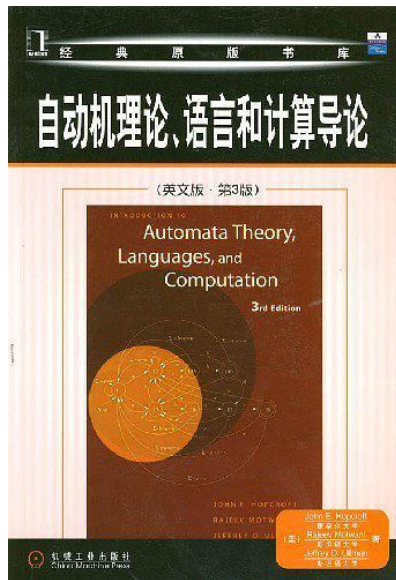
Turing
Machine

Recursively
Enumerable

(Phrase
Grammar)

Text book

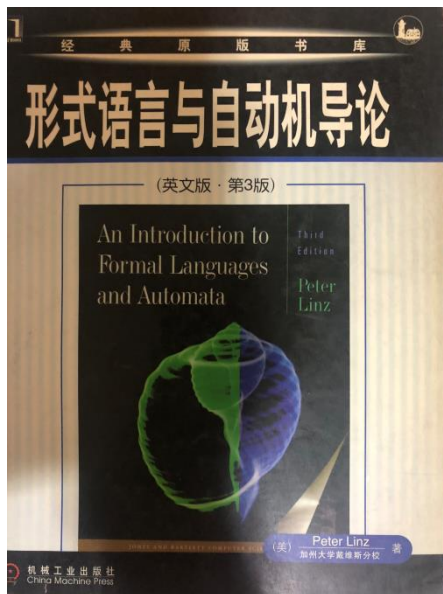
1. Introduction to Automata Theory , Languages , and Computation (Third Edition)



—— John E. Hopcroft
Rajeev Motwani
Jeffrey D. Ullman

2. An Introduction to Formal Languages and Automata (Third Edition)

—— Peter Linz



Goal

1. Understanding "theoretical" concepts
----- method of formal description
2. To realize mathematics
3. Improving reading ability in English

Homework

- ♦ All exercises showed on qq-group
- ♦ Write on A4 papers
- ♦ Discussions - 4 times

Honor and Collaboration

- ◆ Collaboration is strongly encouraged
- ◆ Solutions must be written independently
- ◆ Responsible for Understanding and explaining

Examination

- ◆ Only final exam
- ◆ Closed exam

Nothing allowed except one pen



Grading Policy

- ◆ Homework : 20% //including Class Performance
- ◆ Final exam : 80%



Information

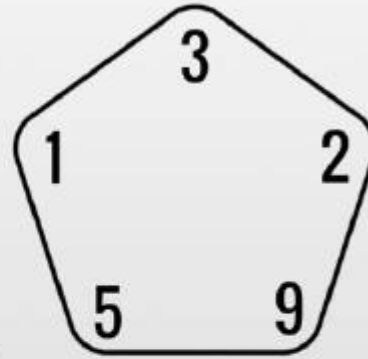
- ◆ Tutor : 孙大烈
- ◆ Office : 综合楼 520
- ◆ E-mail : sdl@hit.edu.cn
- ◆ 课程群 : 自动机/104804608 (qq)
- ◆ MOOC :



群名称:自动机
群 号:104804608

- <https://www.icourse163.org/learn/HIT-1206319802>
- ◆ 画状态图: <http://madebyevan.com/fsm/>

CAN YOU FIND THE MISSING NUMBER?



Good good study
day day up!