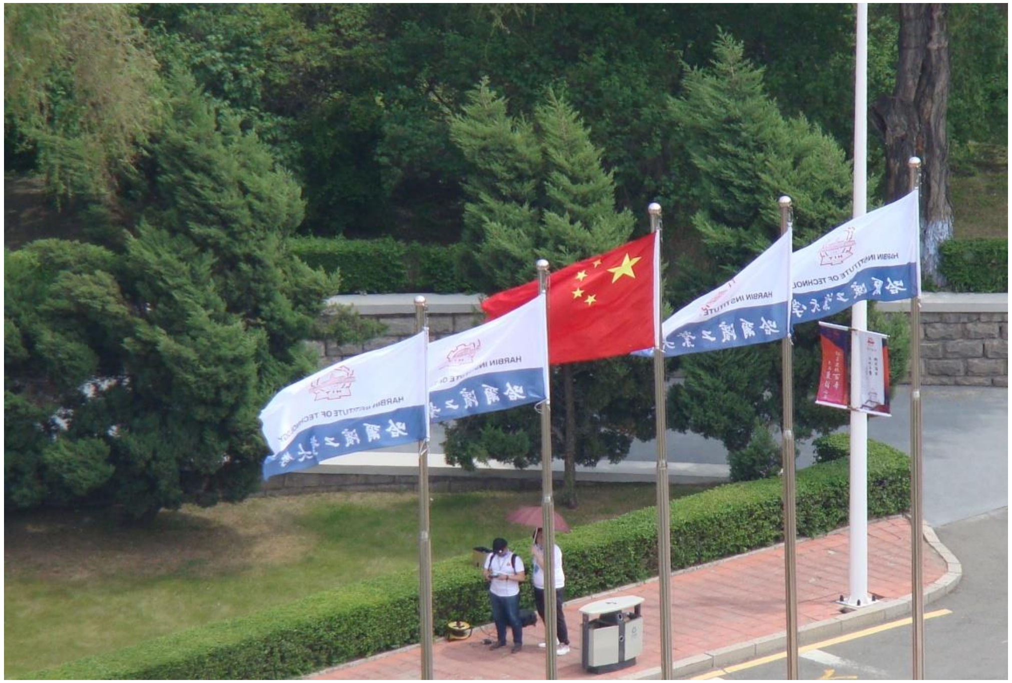
Afternoon

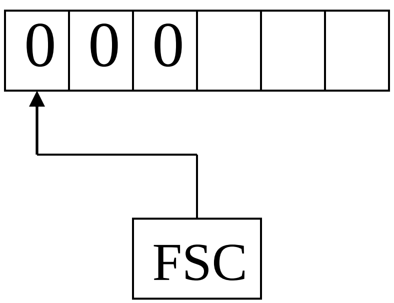
Turing Machine

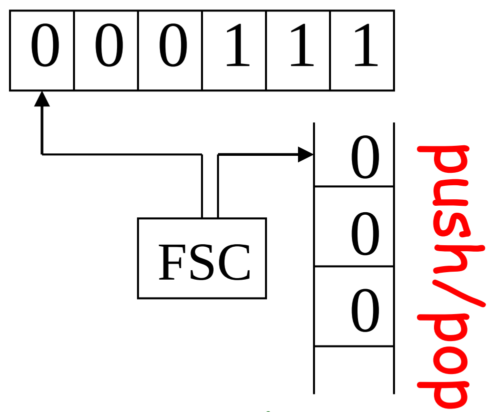
- Definition
- Construction
- Language

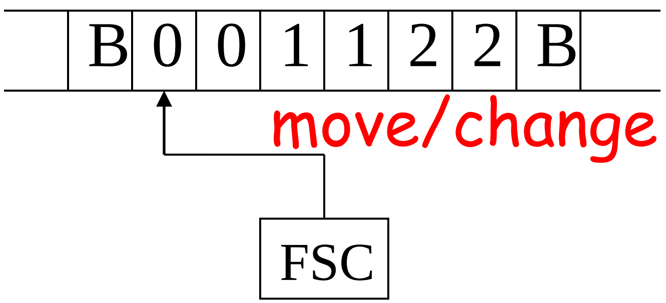


Different automata

- FA $\Rightarrow$ PDA $\Rightarrow$ TM







FA	PDA	TM
$\{0^n \mid n \geq 1\}$	$\{0^n 1^n \mid n \geq 1\}$	$\{0^n 1^n 2^n \mid n \geq 1\}$
$(Q, \Sigma, \delta, q_0, F)$	$(Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$	$(Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Difinition

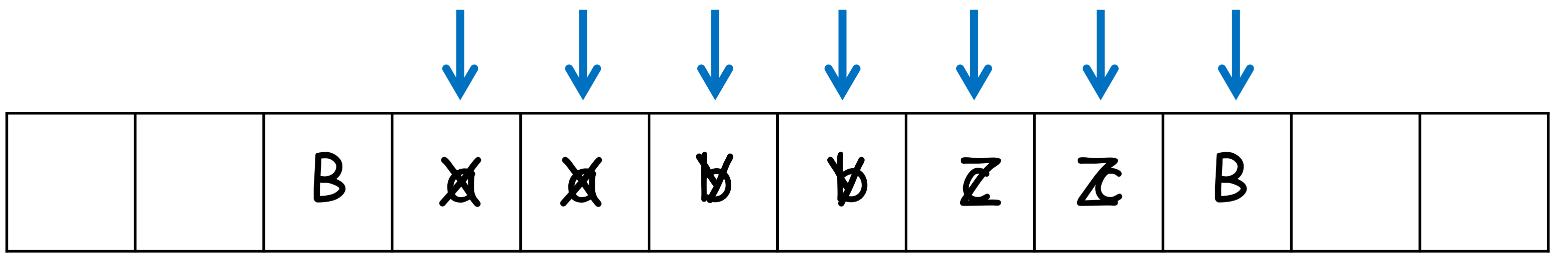
TM is a seven-tuple $P = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

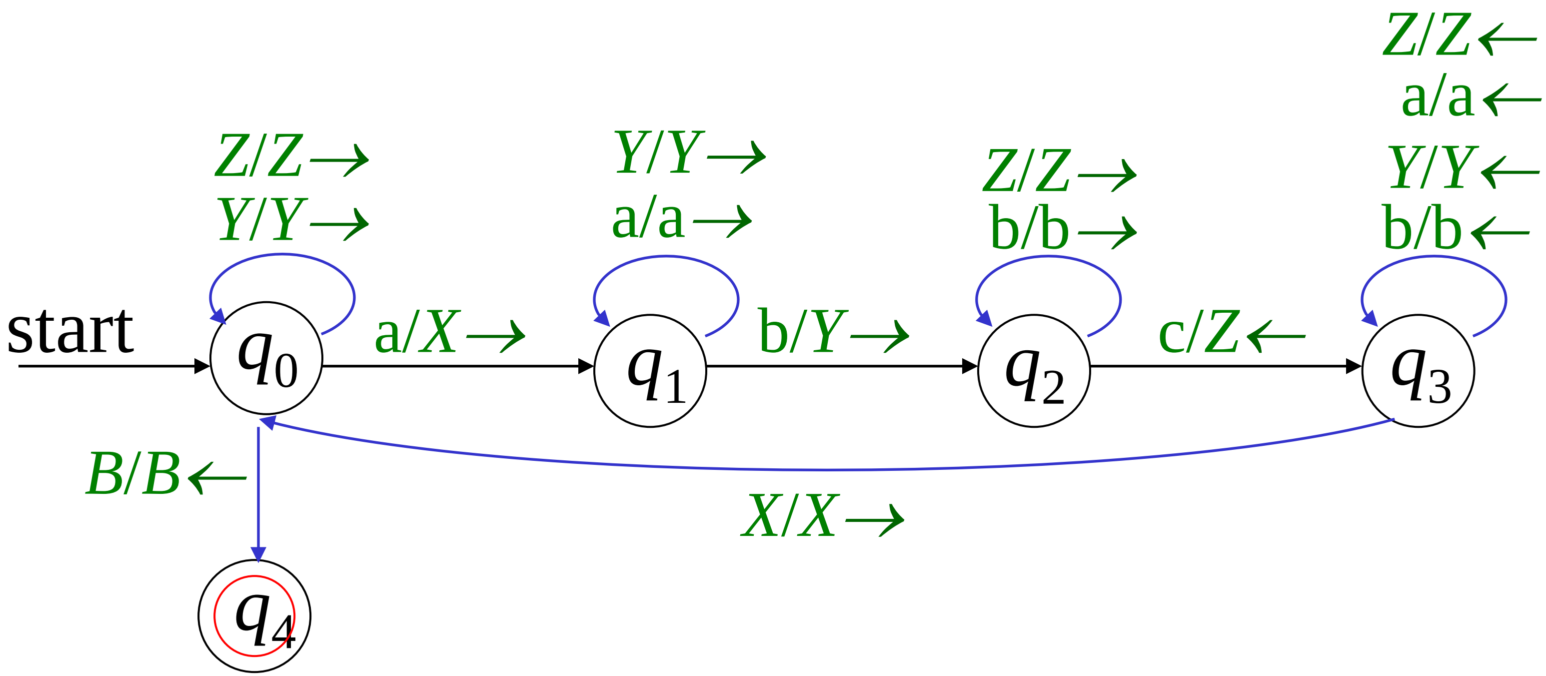
- Q is finite set of states
- Σ is finite set of input symbols
- Γ is finite set of tape symbols
- δ is transition function : $Q \times \Gamma \Rightarrow Q \times \Gamma \times \{R, L\}$

  $$\delta(q, X) = (p, Y, D)$$
- q_0 is start state
- B is blank symbol
- F is finite set of final state

Example 1 TM for

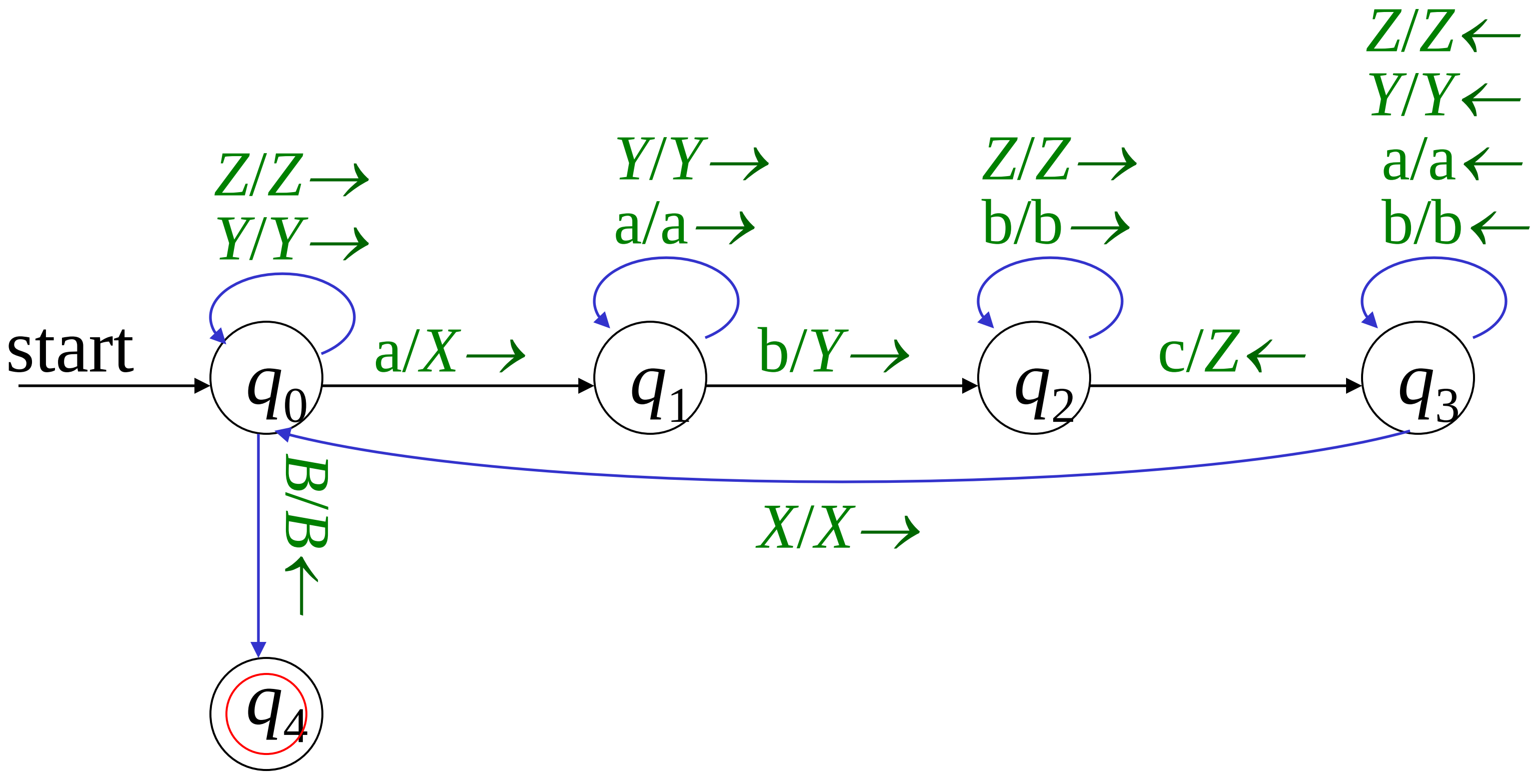
$L = \{ a^n b^n c^n \mid n \geq 0 \}$





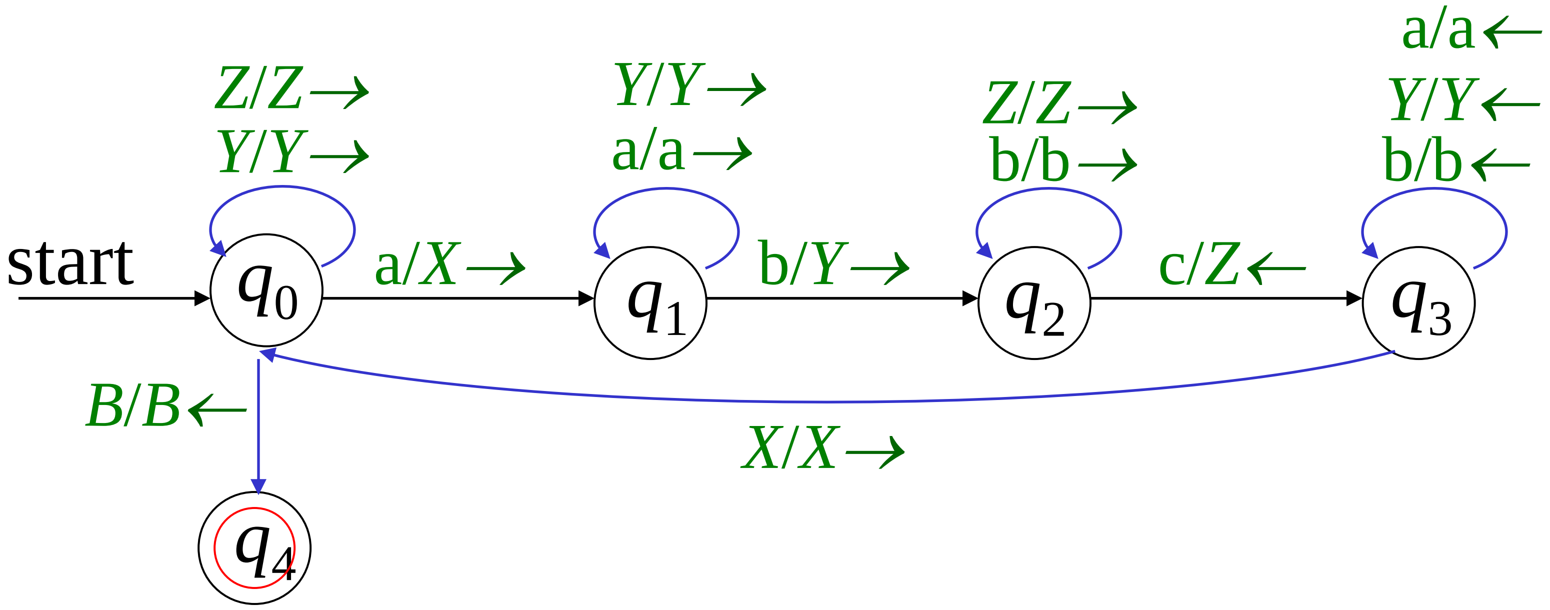
Example 1 TM for 'a^{n}b^{n}c^n '

$M=(\{q_0,q_1,q_2,q_3,q_4\},\{a,b,c\},\{a,b,c,B,X,Y,Z\},\delta, q_0,B,\{q_4\})$

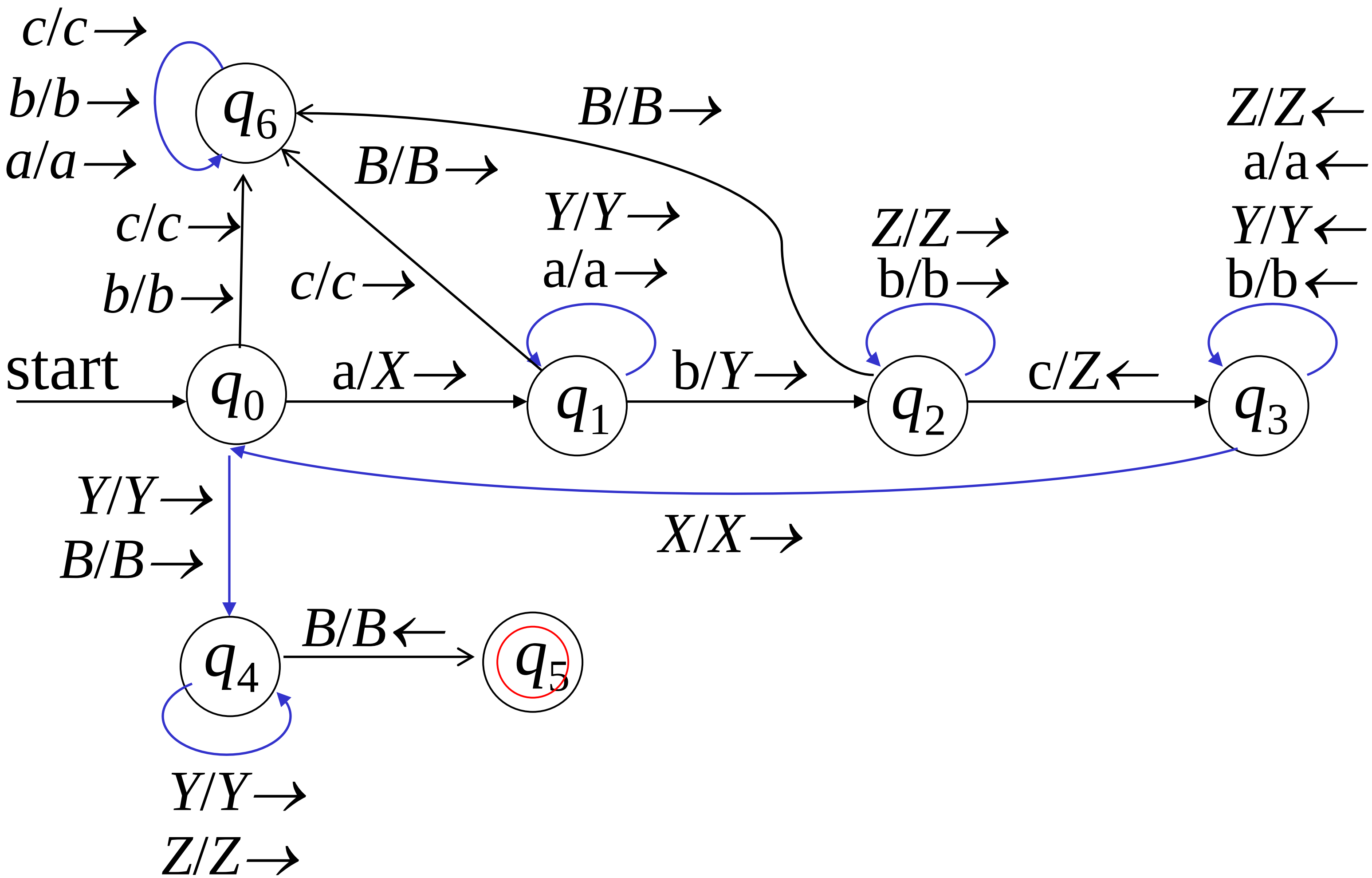


Example 1 TM for 'a^{n}b^{n}c^n'

- ε ?
- $a^2b^2c^2a^3b^3c^3$? ?
- $a^2b^3c^4$, ac, bbc ? ? ?



Example 1 TM for 'a^{n}b^{n}c^n '



Instantaneous

- how to describe the configuration of TM
  - sequence of symbols in tape
  - state of TM
  - read/wtite head of TM
  - $X_1 \ldots X_{i-1} q X_i X_{i+1} \ldots X_n$

Instantaneous

- ID of the above TM for w=aabbcc$\in\{a^n b^n c^n \mid n \geq 1\}$

$q_0aabbcc \vdash Xq_1abbcc \vdash X\,aq_1bbcc \vdash XaYq_2bcc$

$XaYbq_2cc \vdash X\,aYq_3bZc \vdash X\,aq_3YbZc \vdash Xq_3aYbZc \vdash$

$q_3XaYbZc \vdash Xq_0aYbZc \vdash XXq_1YbZc \vdash XXYq_1bZc \;\vdash$

$XXYYq_2Zc \vdash XXYYZq_2c \vdash XXYYq_3ZZ \vdash \ldots$

Language of TM

- language accepted by a TM

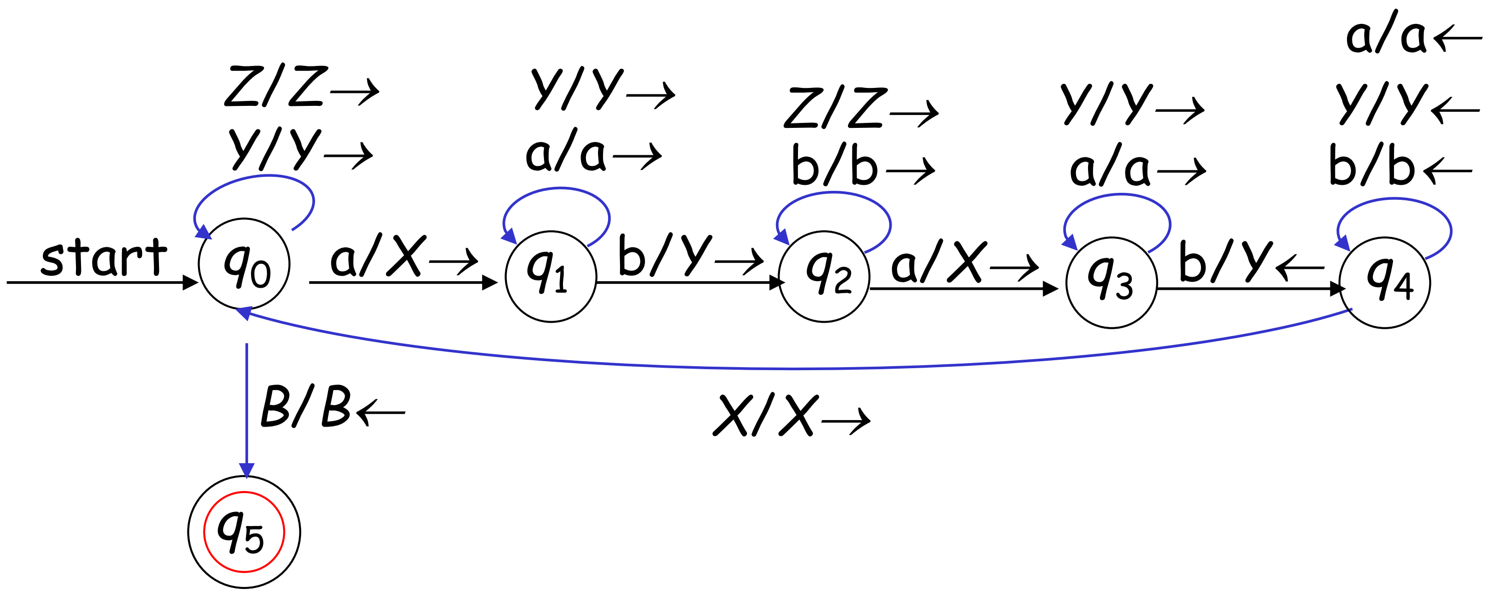
  $\{w \mid q_0 w \overset{*}{\vdash} \alpha p \beta \text{ , } p \in F \text{ , } \alpha, \beta \in \Gamma^*\}$

- The language accepted by TM is called *recursively enumerable(RE)* language.

Example 2 TM for

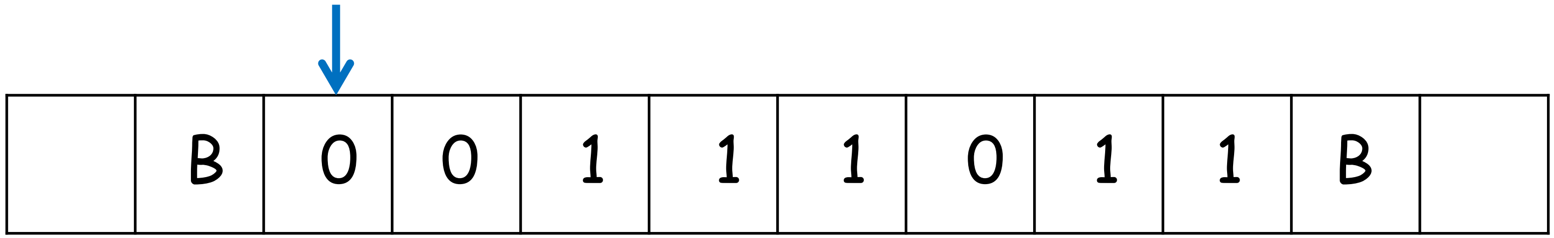
$L = \{ a^n b^n a^n b^n \mid n \geq 0 \}$

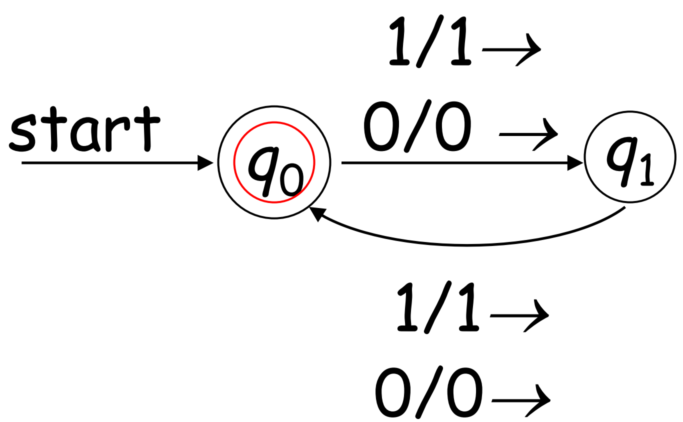
	B	a	a	b	b	a	a	b	b	B	



Example 3 TM for

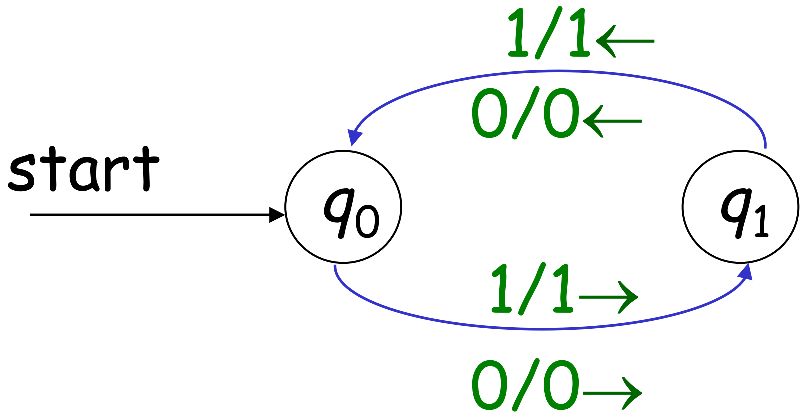
$L = \{ w \mid w \in \{0,1\}^* \text{ and } |w| \text{ is even} \}$





Halting

We say a TM halts if it enters a state q, scanning a tape symbol X, and there is no move in this situation .



Example 4

Given two positive integers x and y, design a TM to compute $x + y$.

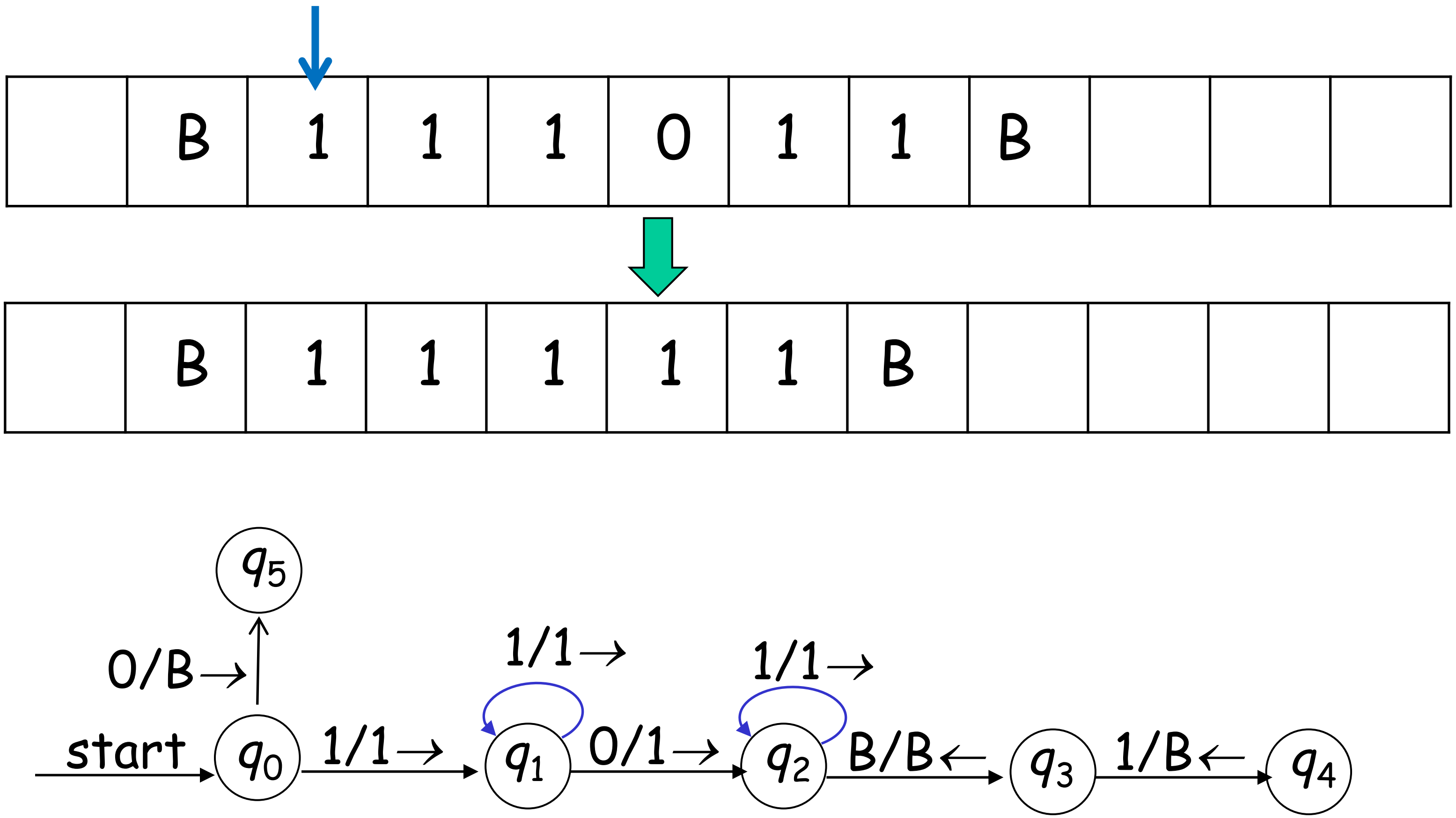
Notation for x and y

$$x \Rightarrow w(x) \in \{1\}^+ \text{ and } |w(x)| = x$$

$$x + y \Rightarrow w(x+y) \in \{1\}^+ \text{ and } |w(x+y)| = x+y$$

3 — 111,   2 — 11,   3 + 2 — 11111

Example 4



Example 5

Compute the function *nomus* $(m, n)=\max(m-n,0)$

- put 1^m01^n into tape as input
- delete a 1 from 1^m and a 1 from 1^n
- three cases :

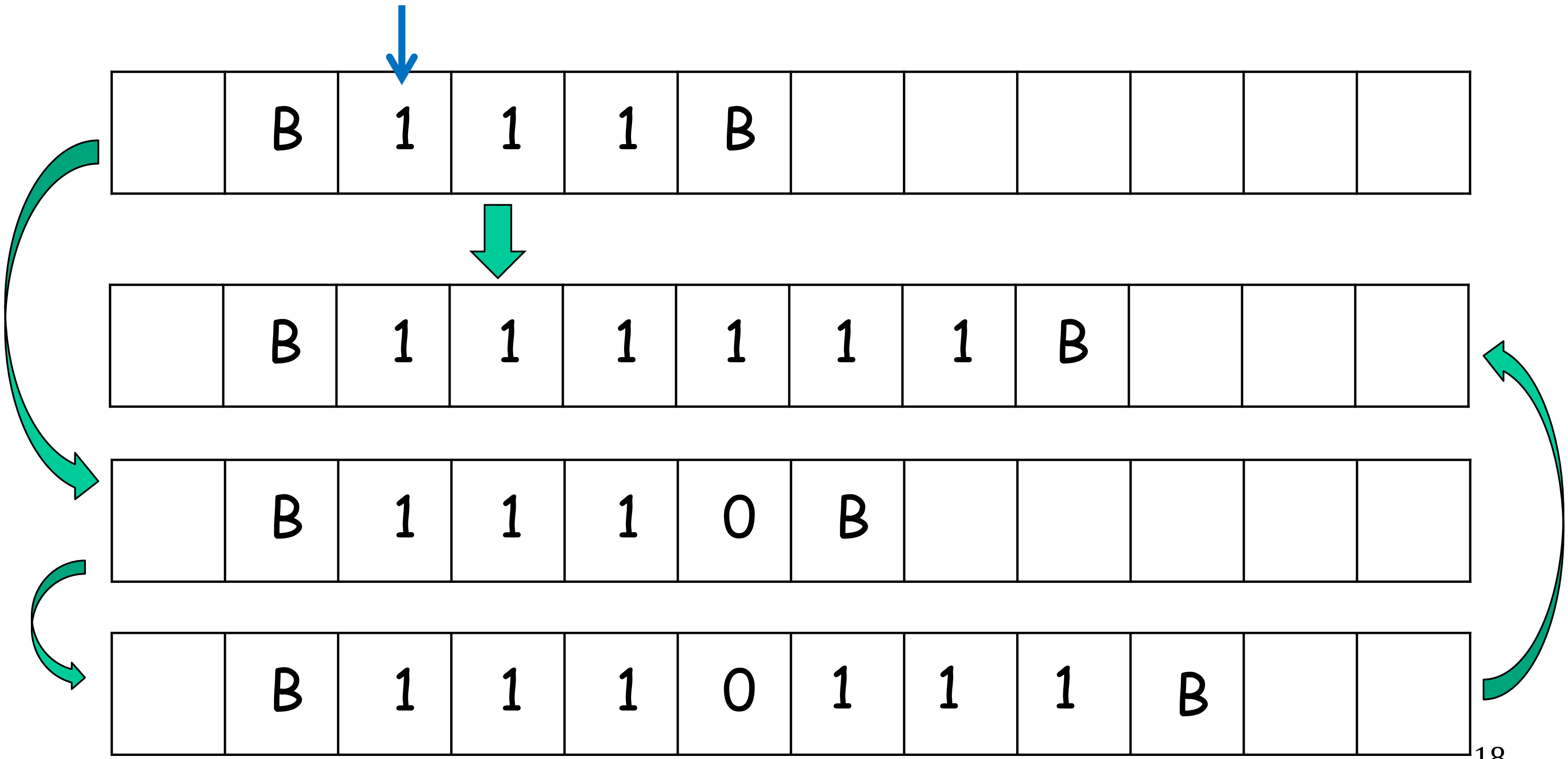
  $m>n \Leftrightarrow 1^{m-n-1}$ at the left of 1

  $m=n \Leftrightarrow$ no 1

  $m<n \Leftrightarrow 1^{n-m}$ at the right of 1

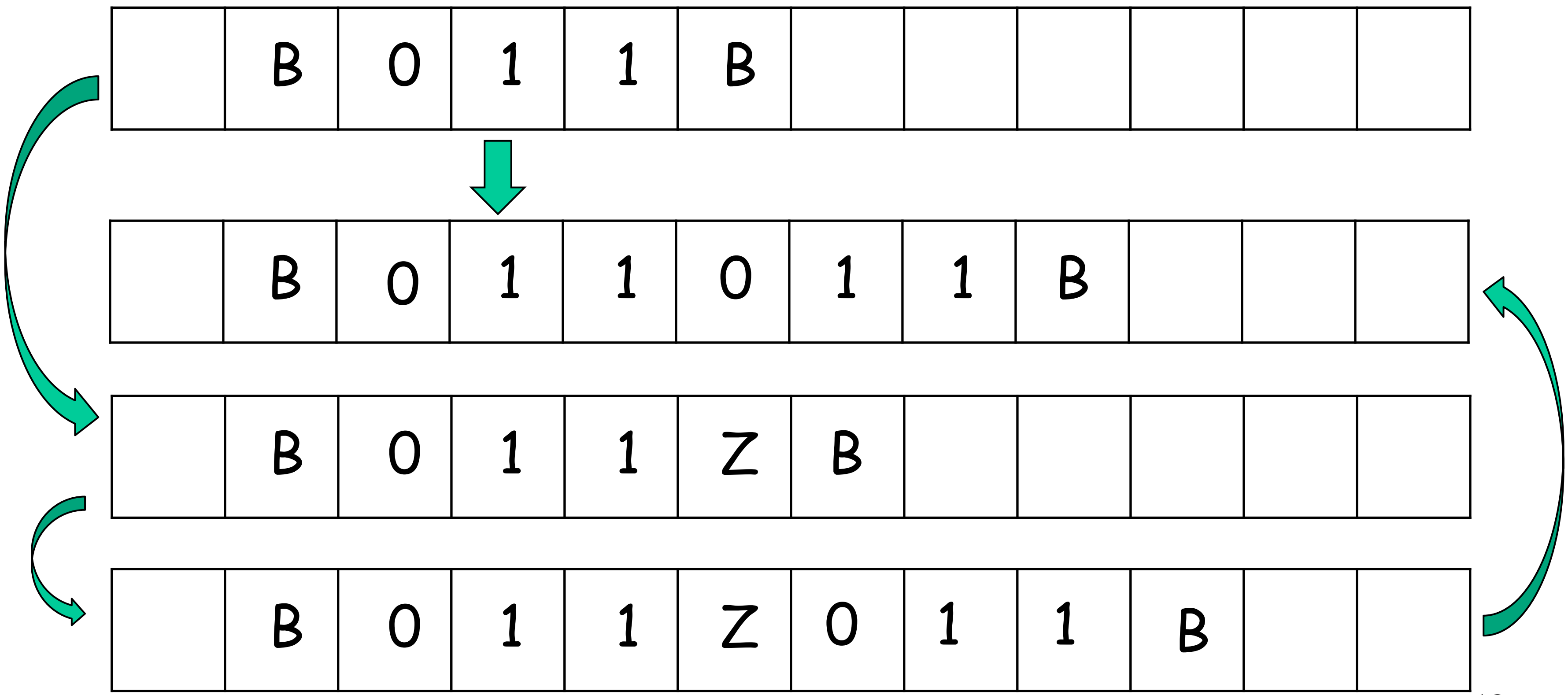
Example 6

Construct a TM to compute the function $f(w)=ww$ where $w \in \{1\}^+$.



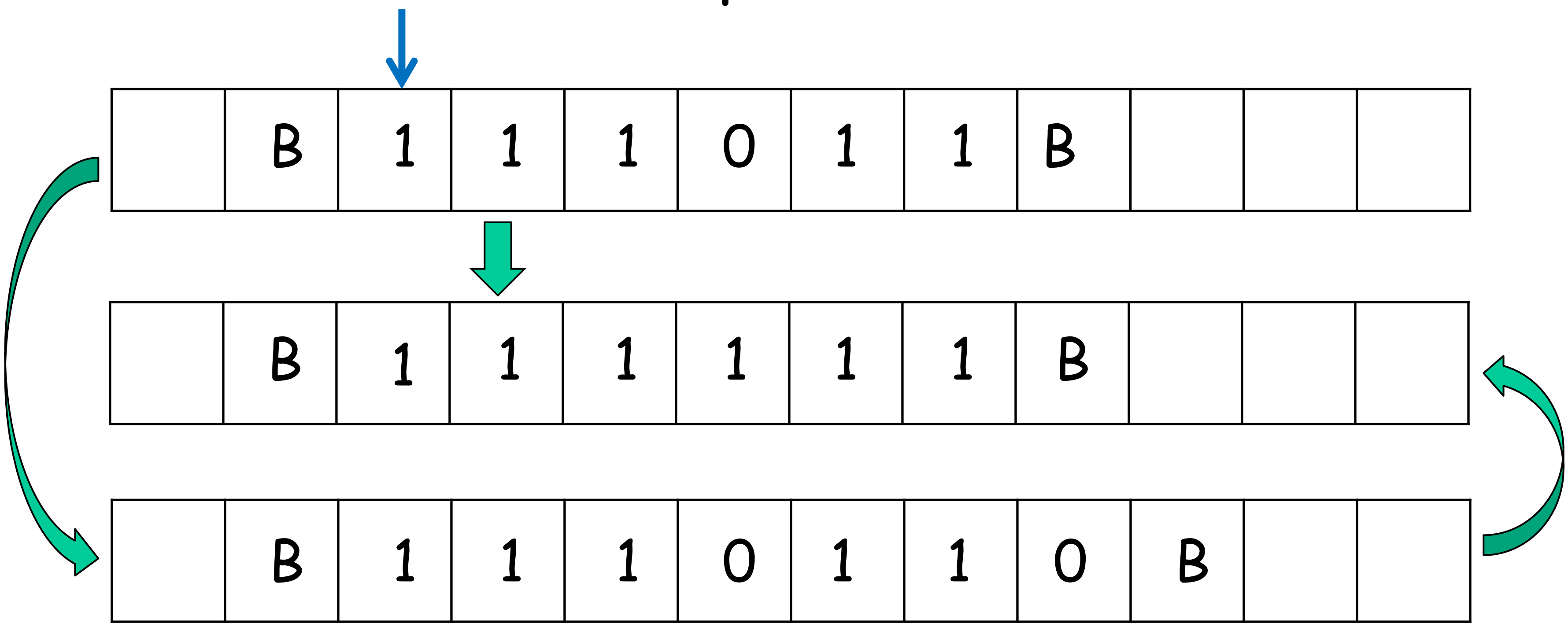
Example 7

Construct a TM to compute the function $f(w)=ww$ where $w \in \{0, 1\}^+$.



Example 8

Construct a TM to compute $m \times n$.



$3 \times 2 \Rightarrow 2 + 2 + 2$

Example 8

B	1	1	1	0	1	1	0	B			

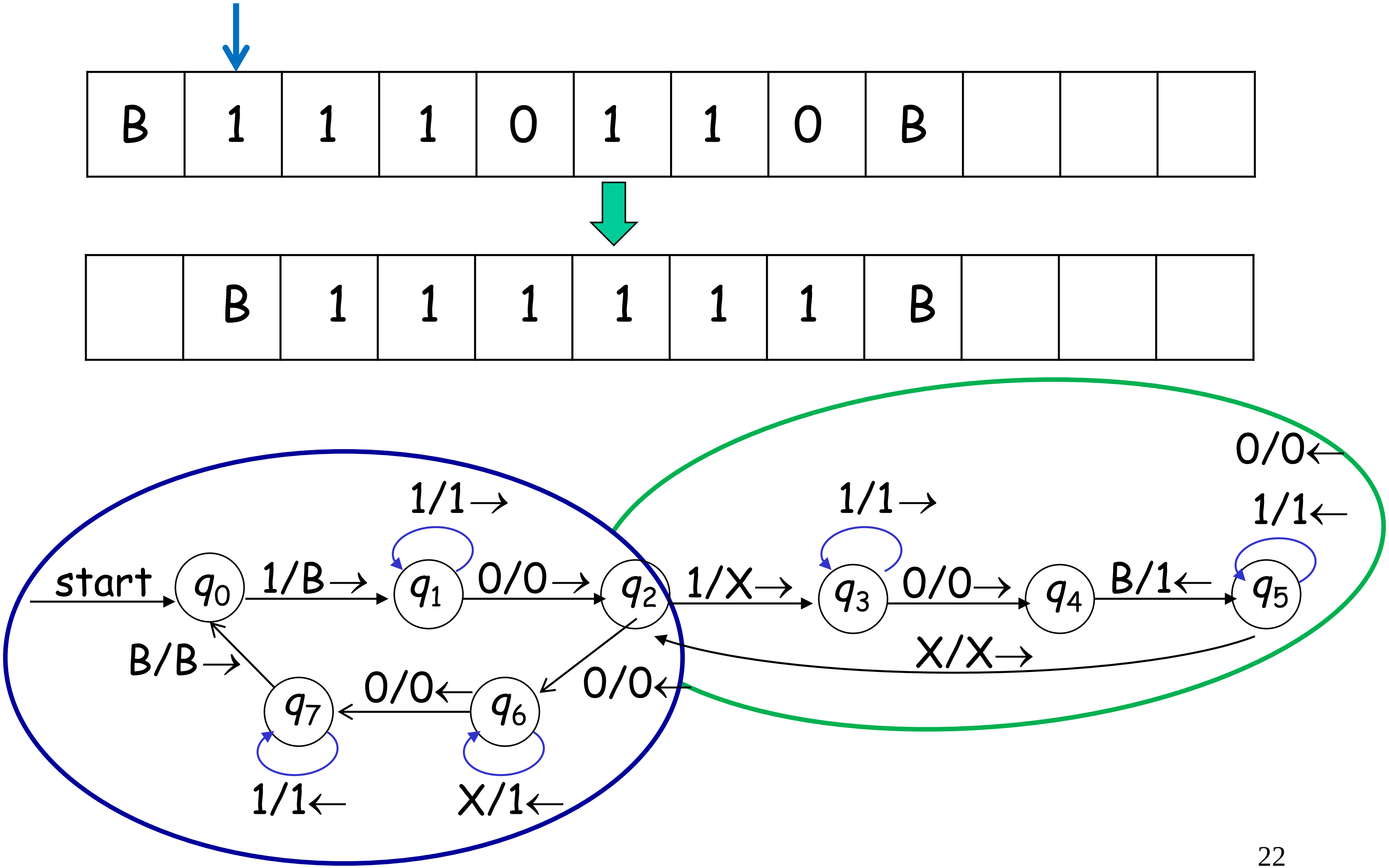
	B	1	1	0	1	1	0	1	1	B	

		B	1	0	1	1	0	1	1	1	1	B

	B	0	1	1	0	1	1	1	1	1	1	B

					B	1	1	1	1	1	1	B

Example 8

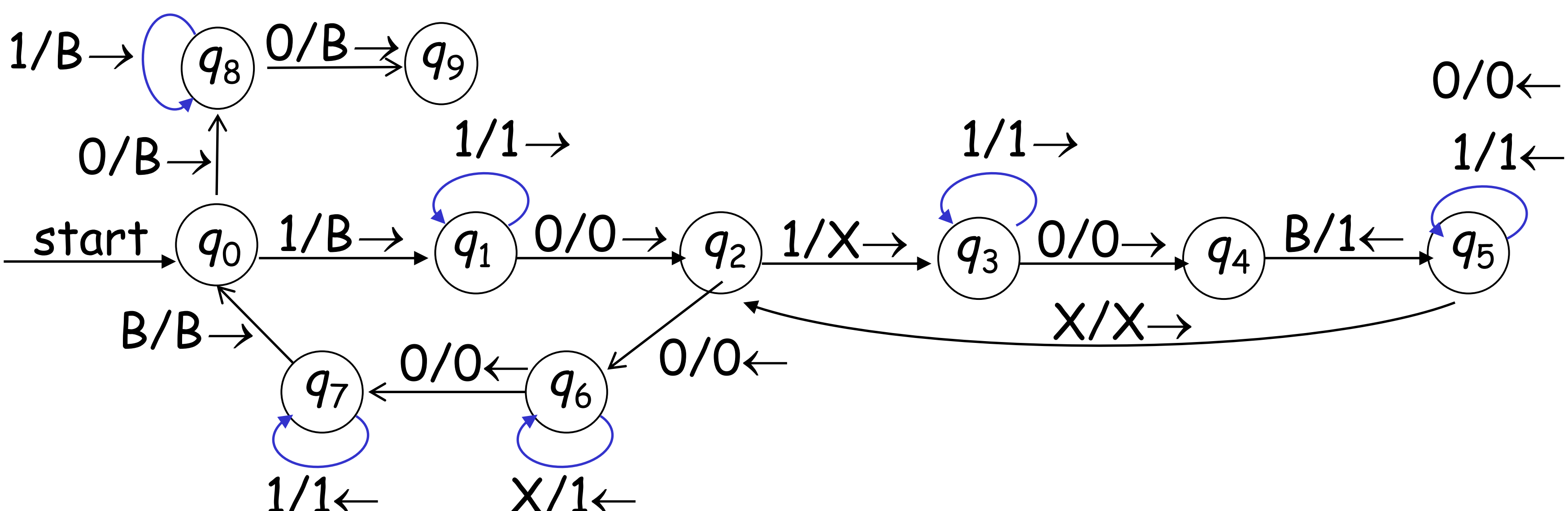


B 1 1 1 0 1 1 0 B
B 1 1 1 1 1 1 B
start
q0
q1
q2
q3
q4
q5
q6
q7
1/B→
1/1→
0/0→
1/X→
1/1→
0/0→
B/1←
0/0←
1/1←
X/X→
0/0←
B/B→
0/0←
1/1←
X/1←

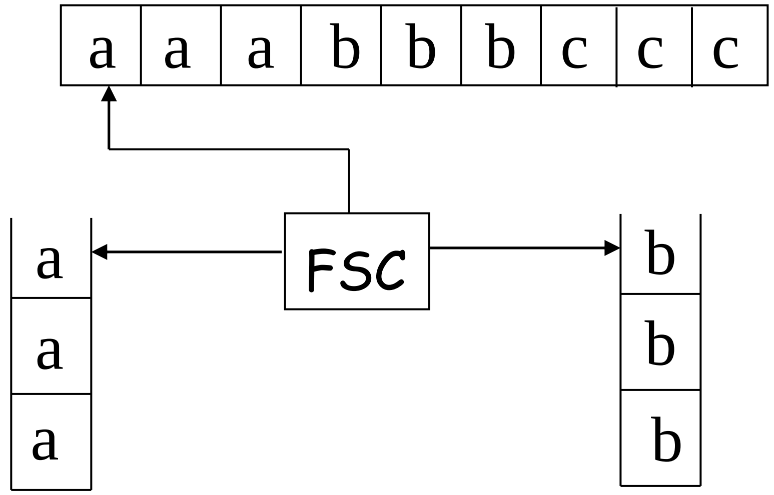
Example 8

B	0	1	1	0	1	1	1	1	1	1	B

	B	1	1	1	1	1	1	B			



Two Stack Machine

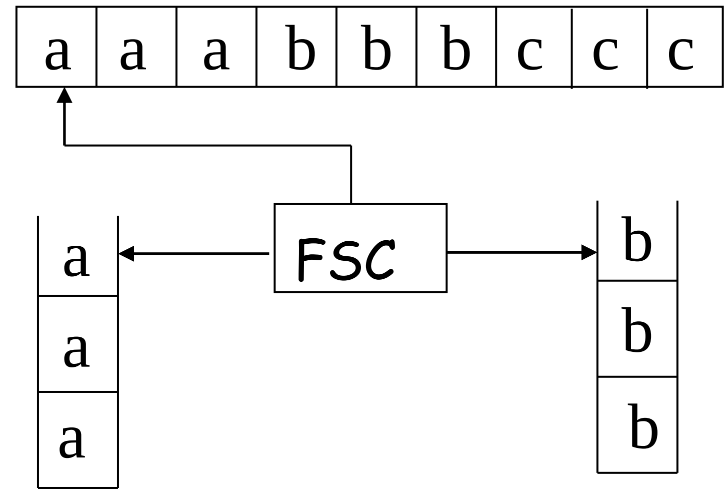


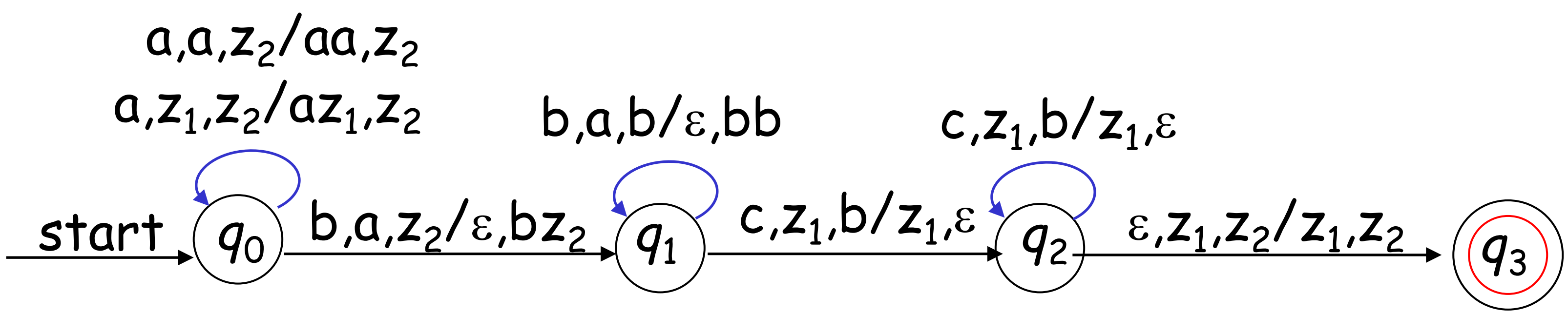
$$\delta(q, a, X, Y) = (p, \alpha, \beta)$$

Example 9

Construct a two stack machine for

$$L = \{ a^n b^n c^n \mid n \geq 0 \}$$

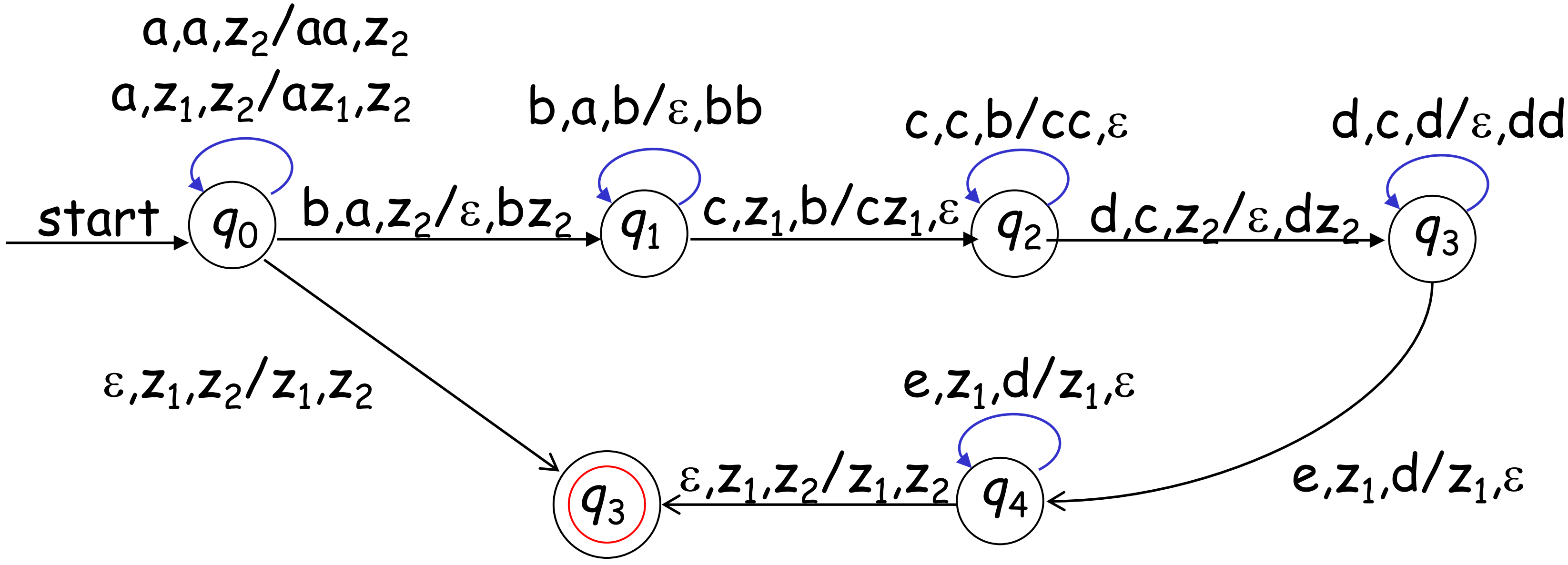




Example 10

Construct a two stack machine for

$$L = \{ a^n b^n c^n d^n e^n \mid n \geq 0 \}$$



Good good study

day day up !

Never confuse education with intelligence.
Intelligence isn’t ability to remember and repeat like they teach you in school.
Intelligence is the ability to learn from experience, solve problems, and use our knowledge to adapt to new situation.

—— Richard Feynman