Afternoon

ε-NFA and Minimization of DFA

- Definition
- Minimize DFA
- Exercises

Formal Definition

ε - NFA is a five-tuple ,

such as $M = (Q, \Sigma, \delta, q_0, F)$

Where Q is a finite set of *states* ,

Σ is a finite set of *input symbols* ,

q_0 is a *start state* ,

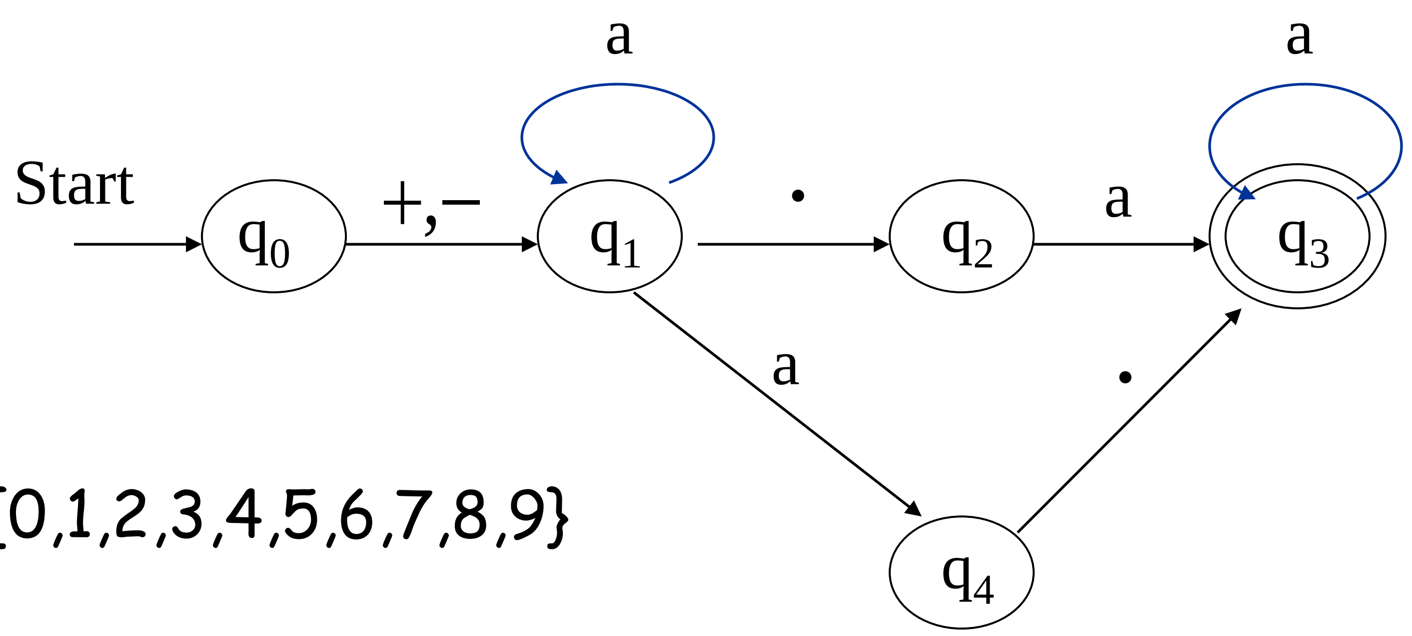
F is *a set of final state* ,

δ is *transition function* , which is a mapping

from $Q \times (\Sigma \cup \{ \varepsilon \})$ to 2^Q.

Example 1

Describe the language accepted by this NFA :



$a \in \{0,1,2,3,4,5,6,7,8,9\}$

{ +23. 01, -69. 0, +. 0, -10. , +00. 000, ... }

23. 01 / 69. 0 / . 0 / 10. ?

Example 1

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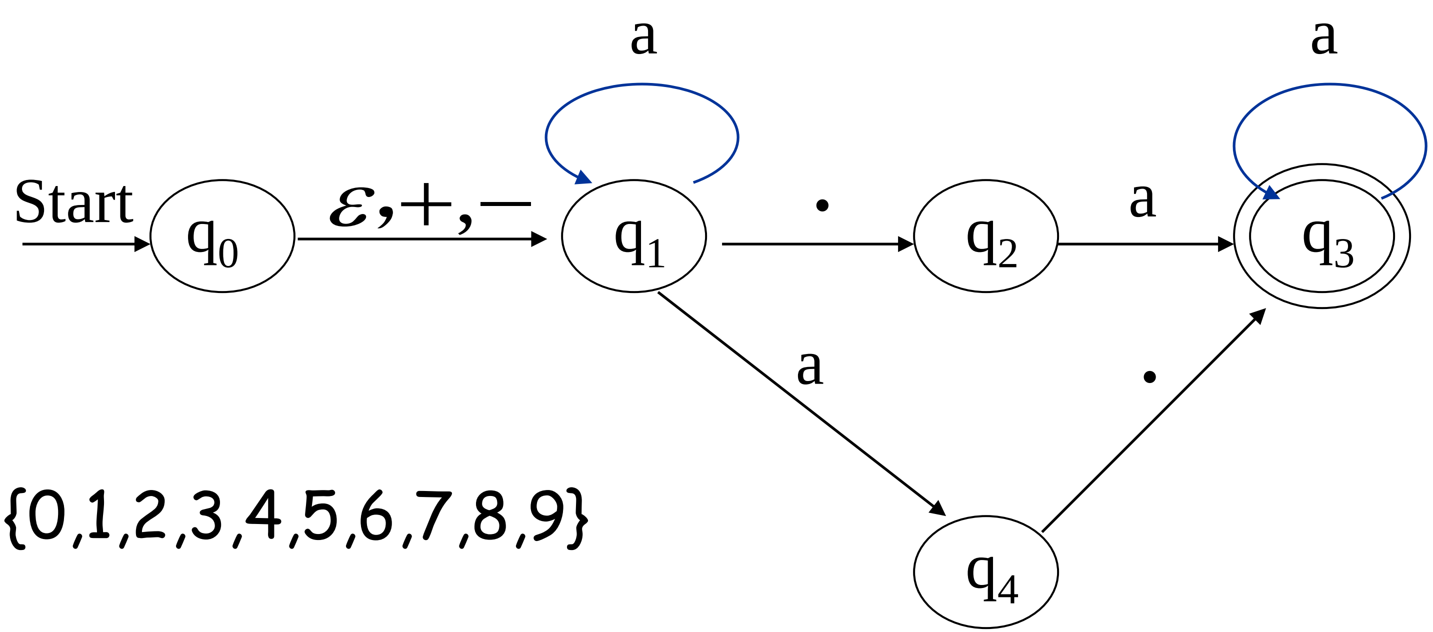

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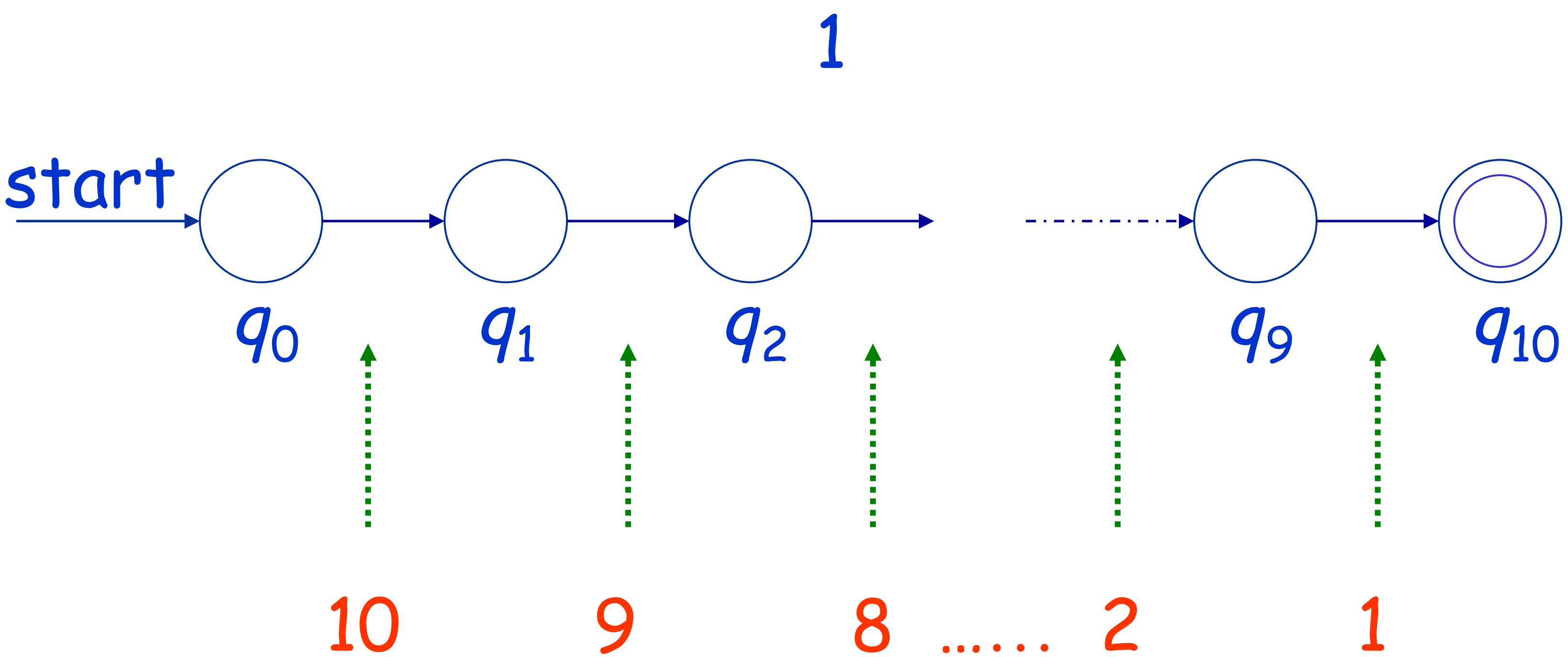
23. 01 / 69. 0 / . 0 / 10. !

Example 2 ε-NFA for

The set of strings of 0's and 1's such that at least one of the last ten positions is a 1 .



Example 2 NFA for

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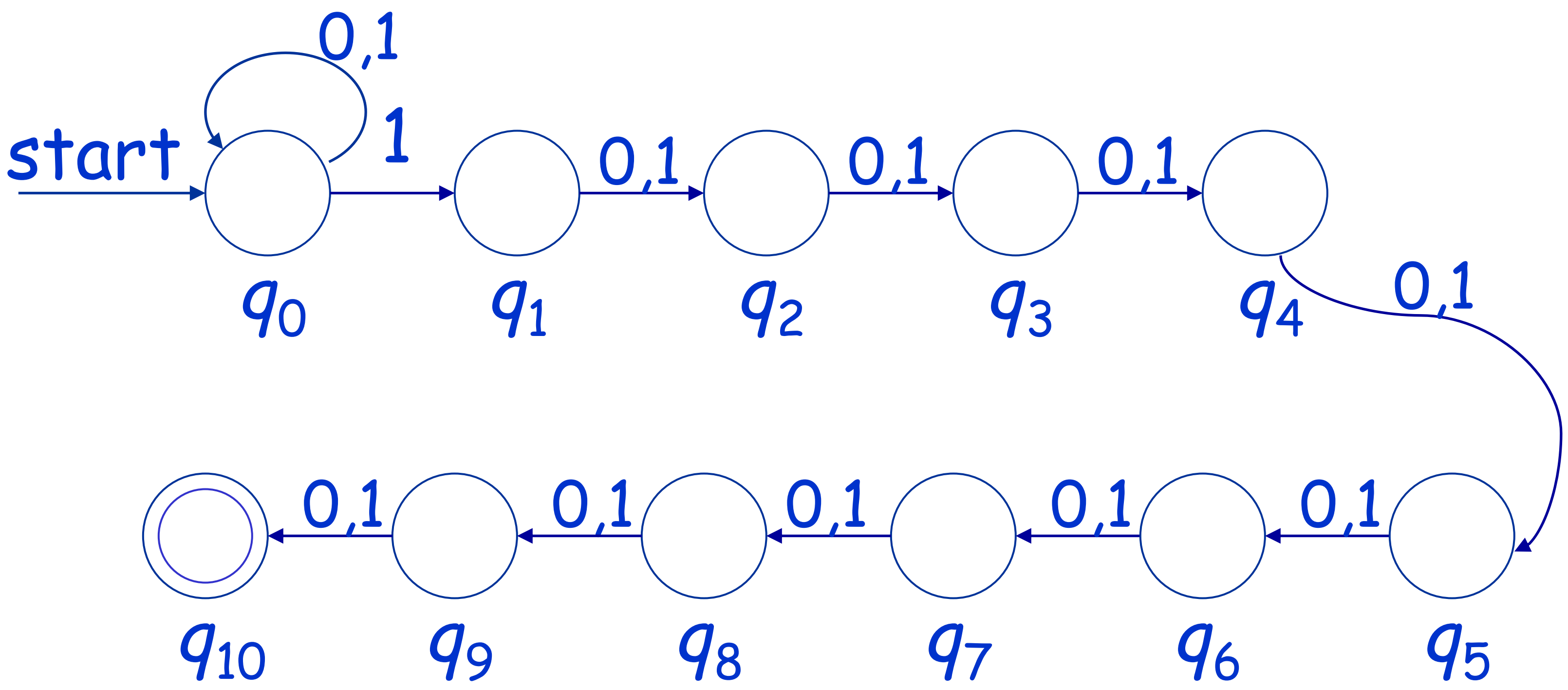
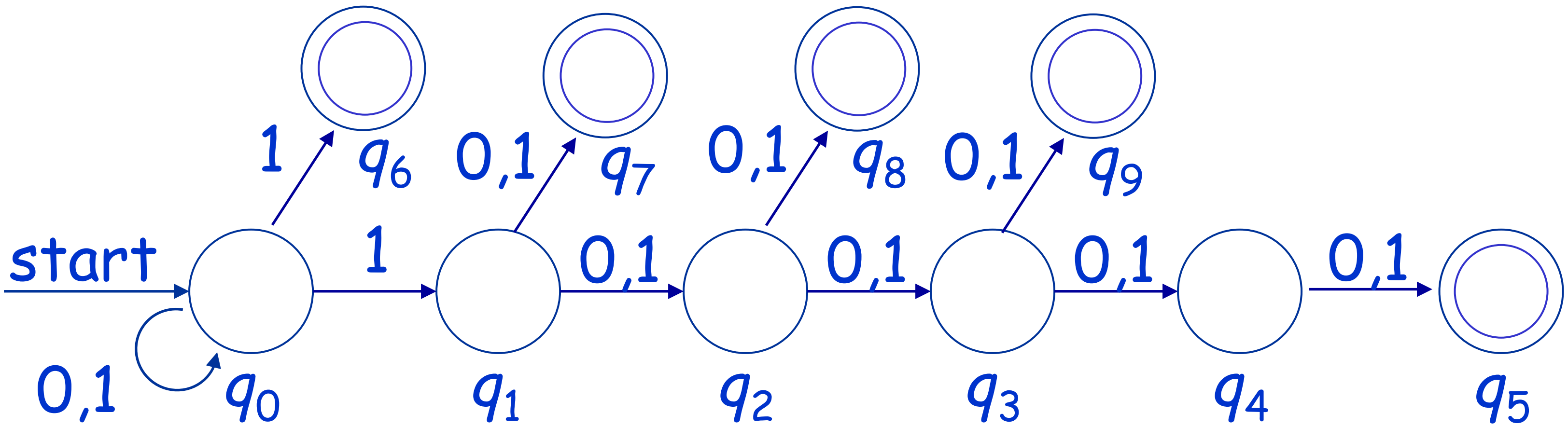
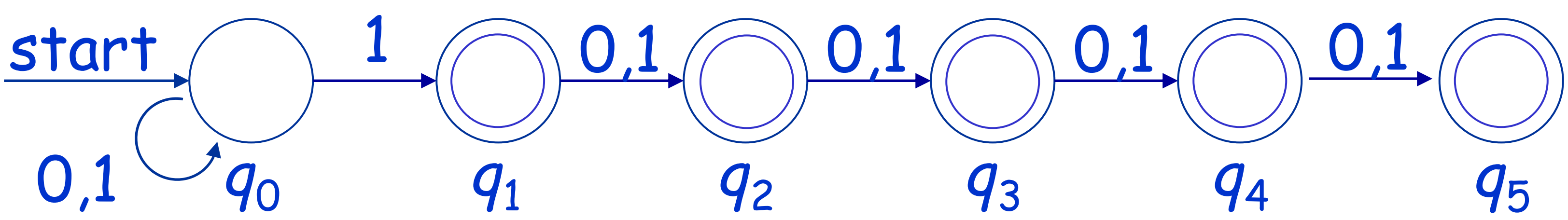


Example 2 NFA for

The set of strings of 0's and 1's such that at least one of the last five positions is a 1 .





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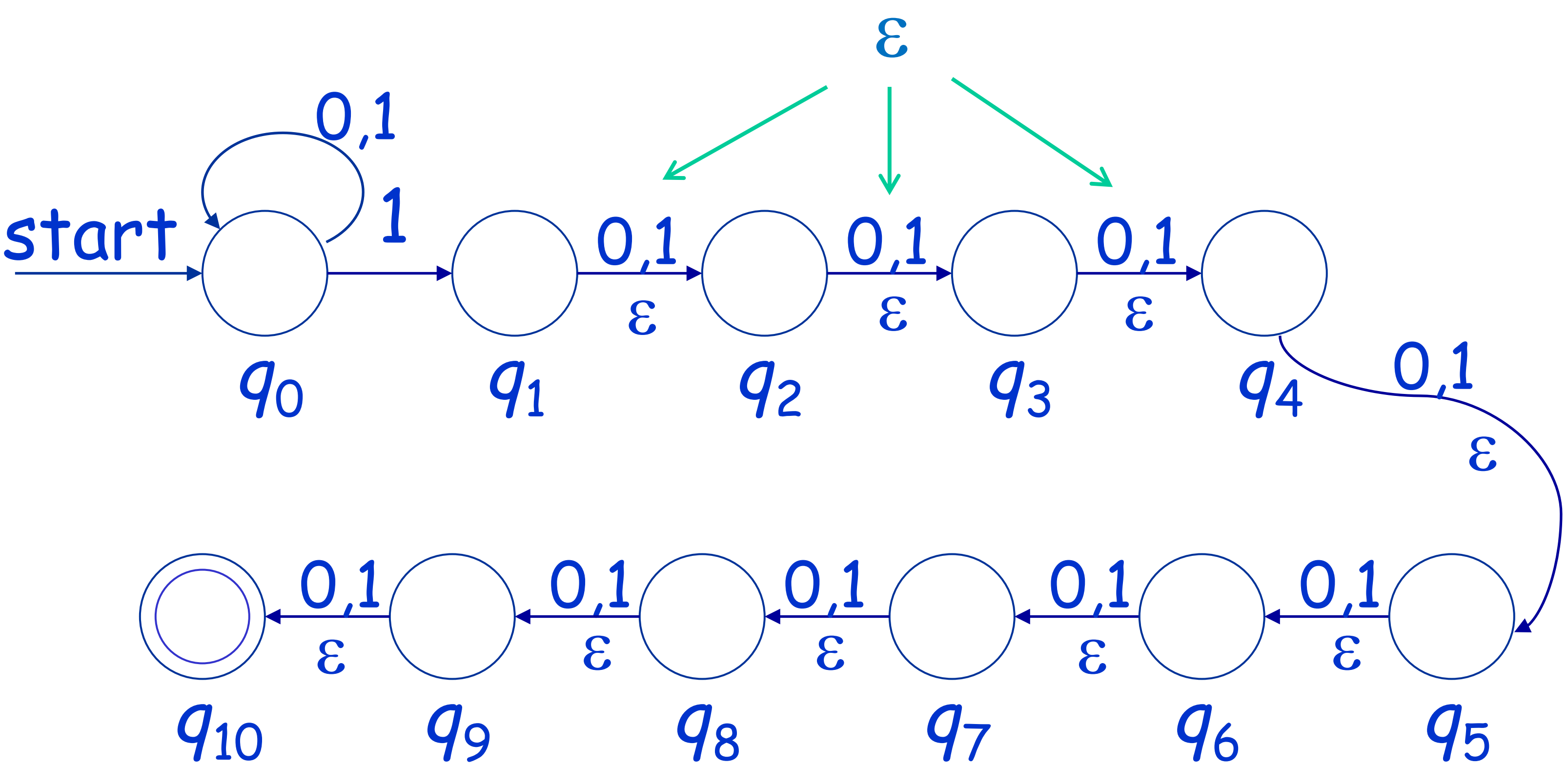
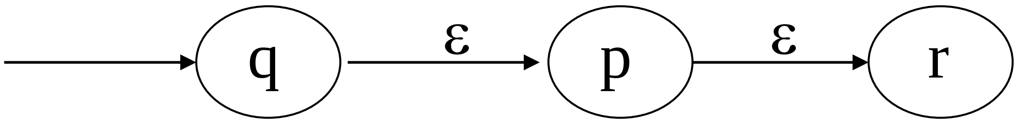


ε - closure

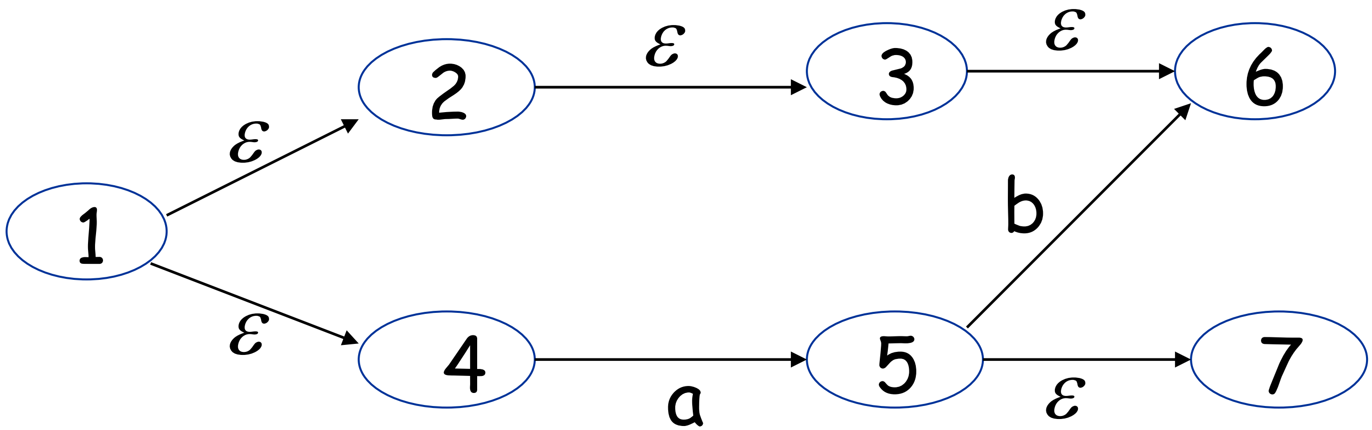
BASIS : State q is in ECLOSE(q)

INDUCTION : If state p is in ECLOSE(q),and there is a transition from state p to state r labeled ε, then r is in ECLOSE(q).



E(r) = { r } , E(p) = { p, r }, E(q) = { p, q, r }

ε - closure

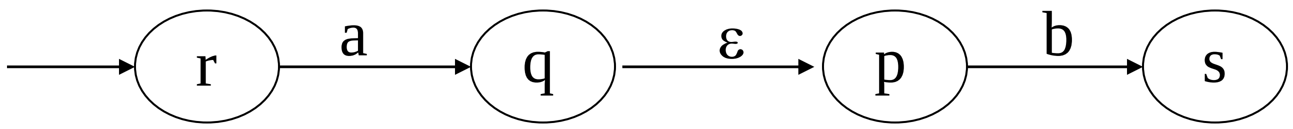


E(7) = { 7 } , E(6) = { 6 }, E(5) = { 5, 7 }

E(4) = { 4 } , E(3) = { 3, 6 },

E(2) = { 2, 3, 6 } , E(1) = { 1, 2, 4, 3, 6 }

ε - transition



$\delta(r, a) = ?$ $\delta(q, b) = ?$

To which from state r with input symbol a ?

To which from state q with input symbol b ?

Extending δ to string

BASIS : $\hat{\delta}(q, \varepsilon) = Eclose(q)$.

INDUCTION :

Surpose $w = xa$, $\hat{\delta}(q, x) = \{p_1, p_2, \ldots, p_k\}$

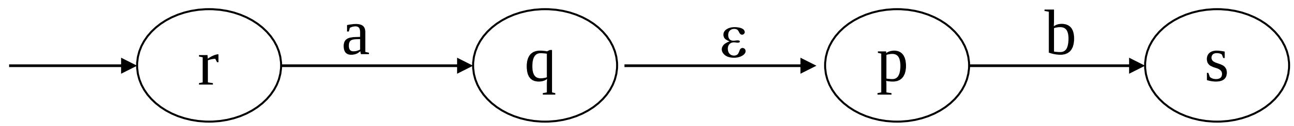
Let $\bigcup_{i=1}^{k} \delta(p_i, a) = \{r_1, r_2, \ldots, r_m\}$

NFA

Then $\hat{\delta}(q, w) = \bigcup_{i=1}^{m} Eclose(r_i)$

ε-NFA

Extending δ to string



w = ab

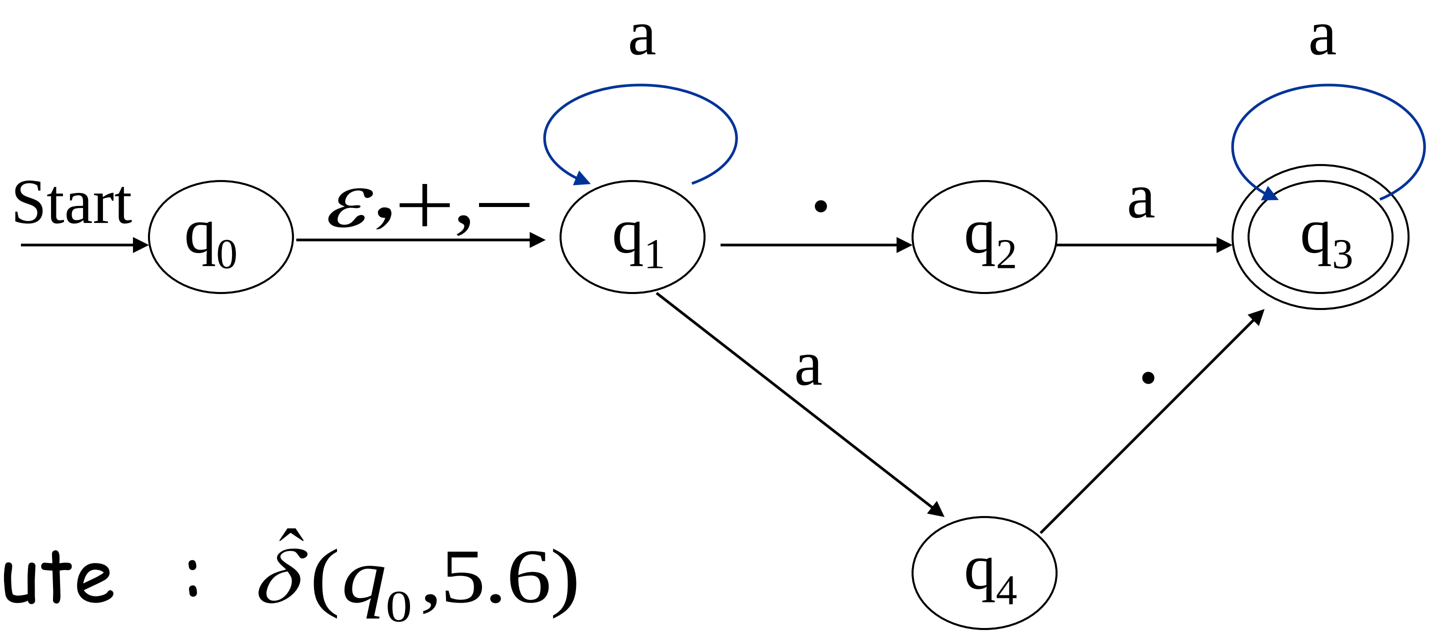
Eclose(r) = { r }, δ(r, a) = {q}

Eclose(q) = { q, p } $\Rightarrow \hat{\delta}(r, a) = \{ q, p \}$

$$\delta\left(\hat{\delta}(r, a), b\right) = \delta\left(\{ q, p \}, b\right) = \{ s \}$$

Eclose(s) = {s} $\Rightarrow \hat{\delta}(r, w) = \hat{\delta}(r, ab) = \{ s \}$

Example 3



Compute : $\hat{\delta}(q_0, 5.6)$

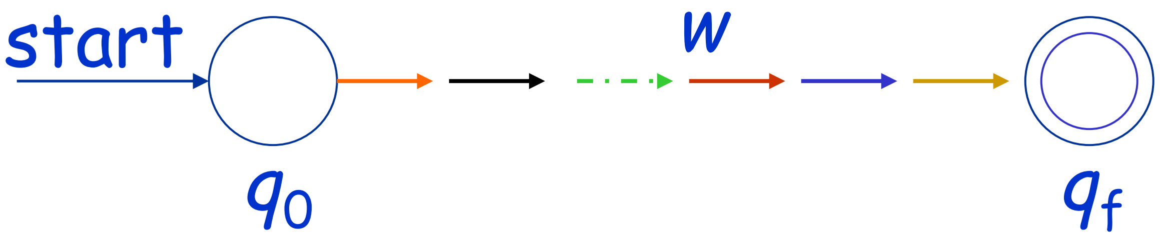
$$\hat{\delta}(q_0,\ 5.\ 6) = \left\{ \mathrm{Eclose}(q) \mid q \in \bigcup_{p \in \hat{\delta}(q_0,\ 5.)} \delta(p,\ 6) \right\}$$

Eclose(q_0) = {q_0, q_1 }, $\hat{\delta}(q_0,\ 5) = \hat{\delta}(Eclose(q_0),\ 5)$

Language of ε-NFA

Definition The *language of an ε-NFA A* is denoted $L(A)$, and defined by

$$L(A) = \{w \mid \hat{\delta}(q_0, w) \cap F \neq \phi\}$$



There is at least a path, labeled with w, from start state to final state.

Equivalence of states

- equivalent states

$$\forall w \in \Sigma^*, \hat{\delta}(p,w) \in F \Leftrightarrow \hat{\delta}(q,w) \in F$$

- notice

We never mentioned $\hat{\delta}(p,w)$ = $\hat{\delta}(q,w)$

- distinguishable states

$$\exists w \in \Sigma^*, \hat{\delta}(p,w) \in F \Leftrightarrow \neg \hat{\delta}(q,w) \in F$$

Equivalence of states

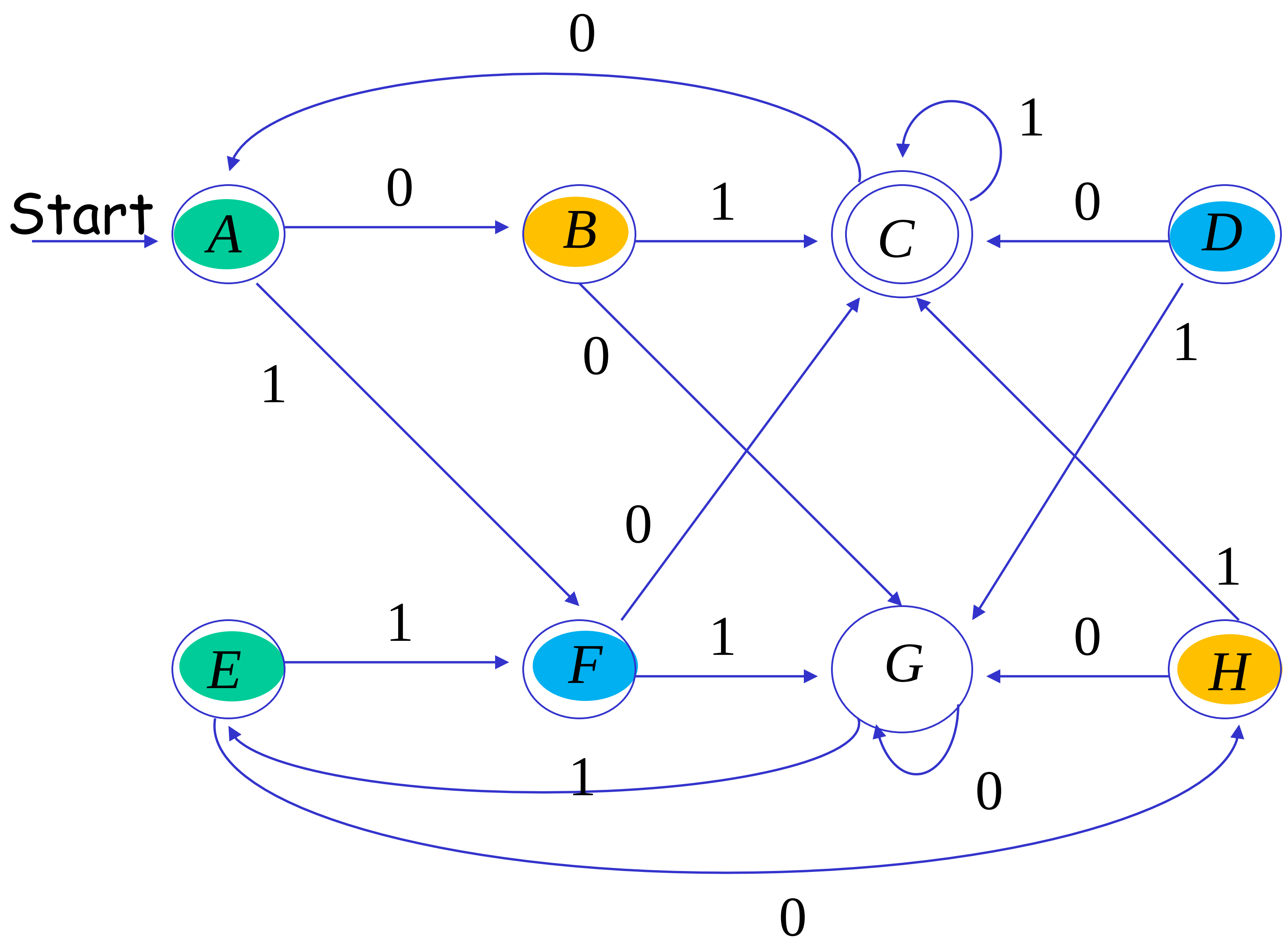


Table-filling algorithm

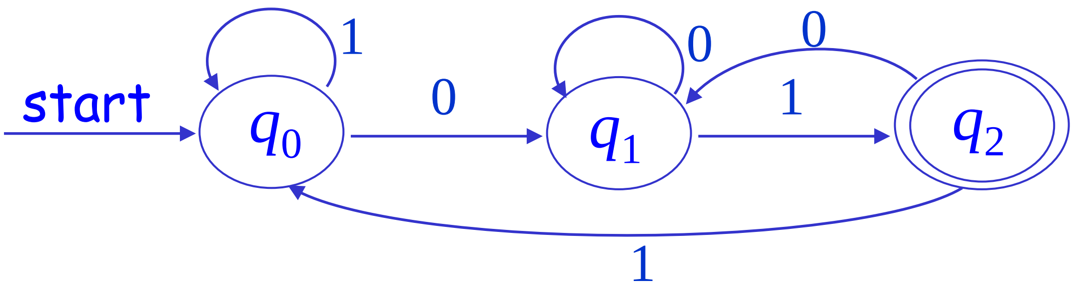
- **Basis** If p is accepting and q is not accepting, then p and q are distinguishable.
- **Induction** Let $r = \delta(p, a)$, $s = \delta(q, a)$, r and s are distinguishable. Then p and q are distinguishable.
- **Example**

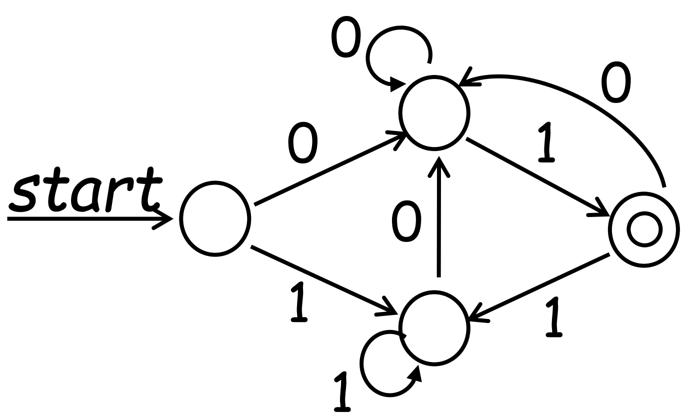
	A	B	C	D	E	F	G
B	×						
C	×	×					
D	×	×	×				
E		×	×				
F	×	×	×		×		
G	×	×	×	×	×	×	
H	×		×	×	×	×	×

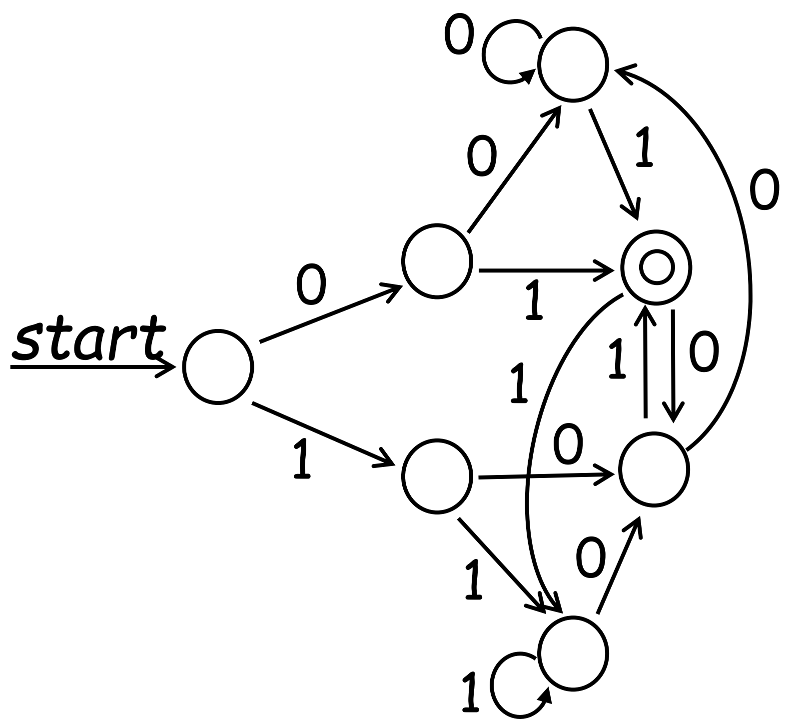
Minimization of DFA's

- What is minimization of DFA
- Algorithm for minimization
  - partition remaining states into equivalent blocks
  - take blocks as states
- Minimum-state DFA for a regular language is unique

Example 4 Minimize DFA







Exercises

Construct DFA for following languages :

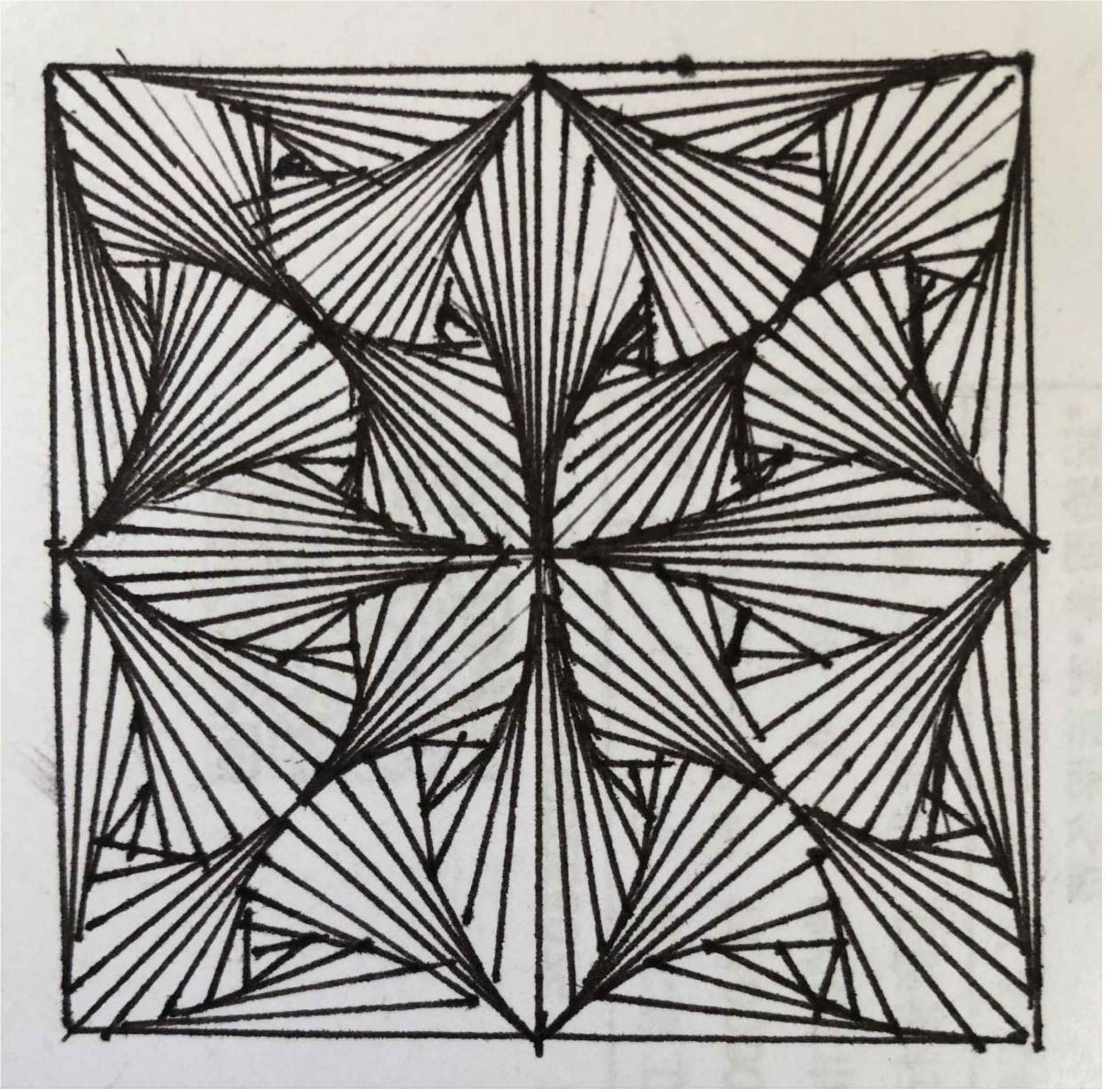
a) $\{ 0 \}^*$

b) $\{w \mid w \in \{0,1\}^*$ and begin with 0 $\}$

c) $\{w \mid w$ consists of any number of 0's followed by any number of 1's $\}$

d) $\{ \varepsilon \}$

e) ϕ



Good good study

day day up !