1 Supply-Use equilibrium

This section provides the equations defining the supply use - equilibrium for the domestic and imported products and at the aggregate level. It also derives the GDP according to various definitions. All equations are therefore accounting identity.

Each identity is expressed in value and in volume. The value equation defines therefore a price index. By convention, the variable Z is always expressed in volume. PZ is its price. Therefore the quantity expressed in value is $Z^{VAL} = PZ * Z$. In most case, values are expresed as $PZ * Z$. When there is a risk that a variable in volume is equal to zero in simulation, we shall defined value as Z^{VAL} to avoid a division by zero issue. A typical exemple would be the value-added of sector s: $VA_s^{VAL} = PY_s\, Y_s - PCI_s\, CI_s$.

1.1 Use side

1.1.1 Domestic and foreign equilibrium for commodities c (value & volume):

Market price for the domestically produced commodity c

$$\begin{aligned} PQD_c\, QD_c = {} & PMGSD_c\, MGSD_c + PCID_c\, CID_c + PCHD_c\, CHD_c \\ & + PGD_c\, GD_c + PID_c\, ID_c + PXD_c\, XD_c + PDSD_c\, DSD_c \end{aligned} \tag{1.1}$$

Quantity of domestically produced commodity c expressed at market price

$$QD_c = MGSD_c + CID_c + CHD_c + GD_c + ID_c + XD_c + DSD_c \tag{1.2}$$

Market price for imported commodity c

$$\begin{aligned} PQM_c\, QM_c = {} & PMGSM_c\, MGSM_c + PCIM_c\, CIM_c + PCHM_c\, CHM_c \\ & + PGM_c\, GM_c + PIM_c\, IM_c + PXM_c\, XM_c + PDSM_c\, DSM_c \end{aligned} \tag{1.3}$$

Quantity of imported commodity c expressed at market price

$$QM_c = MGSM_c + CIM_c + CHM_c + GM_c + IM_c + XM_c + DSM_c \tag{1.4}$$

1.1.2 Margins supplied (value & volume):

The margins supplied by commodity m corresponds to the margins supplied by this commodity. By definition, the margins supplied is the sum of the margins paid (or used) on the commodities c.

The margins paid on domestic and imported products ($MGPD_{m,c}$ and $MGPM_{m,c}$) are defined with behavorial equations in the producer block. They follow the domestic and imported production of commodity c (YQ_c and M_c) more or less proportionnally depending on the possibility of substitutions between margins. The margins paid are then aggregated to define the margins supplied, MGS_m. The latter is then disagrated between the domestic and imported margins supplied ($MGSD_c$ and $MGSM_c$). See specification in the producer block.

Market price of the margins supplied by commodity m

$$PMGS_m \; MGS_m = \sum_c PMGP_{m,c} \; MGP_{m,c} \tag{1.5}$$

Margins supplied by commodity m, expressed at market price

$$MGS_m = \sum_c MGP_{m,c} \tag{1.6}$$

The margins supplied correspond to the sum of the margins paid to commodity m over all the commodities c

Market price of the margins supplied by commodity m (for verification)

$$PMGS_m^{bis} \; MGS_m^{bis} = PMGSD_m \; MGSD_m + PMGSM_m \; MGSM_m \tag{1.7}$$

Same variable as $PMGS_m$ above to check the accounting consistency.

Margins supplied by commodity m, expressed at market price (for verification)

$$MGS_m^{bis} = MGSD_m + MGSM_m \tag{1.8}$$

Same variable as MGS_m above to check the accounting consistency.

1.1.3 Aggregation of imports and domestic production for commodity c per use, expressed at market price (value & volume)

This subsection aggregates imports and domestic production for commodity c for various uses. It does not do it for Exports, Households and Government final consumption (X, CH and G) because these aggregates are already defined in behaviour equations (see Trade international, Consumer and Governement blocks). Expressed in value, this subsection also defines the prices indexes at market price for commodity c per use as a weighted average of imported and domestic production per uses: *i.e.* for Q (production of commodities), CI (intermediary consumption), I (private investment) and DS (change in inventories).

Market price of the production of commodity c

$$PQ_c\, Q_c = PQD_c\, QD_c + PQM_c\, QM_c \tag{1.9}$$

Production of commodity c, expressed at market price

$$Q_c = QD_c + QM_c \tag{1.10}$$

Market price of the intermediate consumption of commodity c

$$PCI_c\, CI_c = PCID_c\, CID_c + PCIM_c\, CIM_c \tag{1.11}$$

Intermediate consumption of commodity c, expressed at market price

$$CI_c = CID_c + CIM_c \tag{1.12}$$

Market price of the investment in commodity c

$$PI_c\, I_c = PID_c\, ID_c + PIM_c\, IM_c \tag{1.13}$$

Investment in commodity c, expressed at market price

$$I_c = ID_c + IM_c \tag{1.14}$$

Market price of the change in inventories of commodity c

$$PDS_c\, DS_c = PDSD_c\, DSD_c + PDSM_c\, DSM_c \tag{1.15}$$

Change in inventories of commodity c, expressed at market price

$$DS_c = DSD_c + DSM_c \tag{1.16}$$

1.1.4 Agregation on sectors: production of commodity c per use for domestic and imported products, expressed at market price (value & volume)

This subsection provides the aggregates for commodity c for various uses, for domestic and imported products. They are calculated through the aggregation of the corresponding sectoral data on the sector index.

Market price for the domestically produced commodity c used as intermediary consumption

$$PCID_c \, CID_c = \sum_s PCID_{c,s} \, CID_{c,s} \tag{1.17}$$

Quantity of domestically produced commodity c used as intermediary consumption, expressed at market price

$$CID_c = \sum_s CID_{c,s} \tag{1.18}$$

Market price for imported commodity c used as intermediary consumption

$$PCIM_c \, CIM_c = \sum_s PCIM_{c,s} \, CIM_{c,s} \tag{1.19}$$

Quantity of imported commodity c used as intermediary consumption, expressed at market price

$$CIM_c = \sum_s CIM_{c,s} \tag{1.20}$$

Market price for domestically produced commodity c used as investment

$$PID_c \, ID_c = \sum_s PID_{c,s} \, ID_{c,s} \tag{1.21}$$

Quantity of imported commodity c used as investment, expressed at market price

$$ID_c = \sum_s ID_{c,s} \tag{1.22}$$

Market price for imported commodity c used as investment

$$PIM_c \, IM_c = \sum_s PIM_{c,s} \, IM_{c,s} \tag{1.23}$$

Quantity of imported commodity c used as investment, expressed at market price

$$IM_c = \sum_s IM_{c,s} \tag{1.24}$$

1.1.5 Agregation on commodities: imported, domestic and aggregate intermediate consumption and investment of sector s, expressed at market price (value & volume)

This subsection provides the intermediate consumption and investment of sector s (imported, domestically produced and aggregated). They are calculated through the aggregation of the corresponding sectoral data on the commodity index.

Market price of domestically produced intermediate consumption of sector s

$$PCID_s\, CID_s = \sum_c PCID_{c,s}\, CID_{c,s} \tag{1.25}$$

Domestically produced intermediate consumption of sector s, expressed at market price

$$CID_s = \sum_c CID_{c,s} \tag{1.26}$$

Market price of imported intermediate consumption of sector s

$$PCIM_s\, CIM_s = \sum_c PCIM_{c,s}\, CIM_{c,s} \tag{1.27}$$

Imported intermediate consumption of sector s, expressed at market price

$$CIM_s = \sum_c CIM_{c,s} \tag{1.28}$$

Market price of intermediate consumption of sector s

$$PCI_s\, CI_s = PCID_s\, CID_s + PCIM_s\, CIM_s \tag{1.29}$$

Intermediate consumption of sector s, expressed at market price

$$CI_s = CID_s + CIM_s \tag{1.30}$$

Market price of intermediate consumption of sector s (for verification)

$$PCI^{bis}.CI^{bis} = \sum_{s} PCI_s \; CI_s \tag{1.31}$$

Intermediate consumption of sector s, expressed at market price (for verification)

$$CI^{bis} = \sum_{s} CI_s \tag{1.32}$$

Market price of domestically produced investment of sector s

$$PID_s \; ID_s = \sum_{c} PID_{c,s} \; ID_{c,s} \tag{1.33}$$

Domestically produced investment of sector s, expressed at market price

$$ID_s = \sum_{c} ID_{c,s} \tag{1.34}$$

Market price of imported investment of sector s

$$PIM_s \; IM_s = \sum_{c} PIM_{c,s} \; IM_{c,s} \tag{1.35}$$

Imported investment of sector s, expressed at market price

$$IM_s = \sum_{c} IM_{c,s} \tag{1.36}$$

Market price of investment of sector s

$$PI_s \; I_s = PID_s \; ID_s + PIM_s \; IM_s \tag{1.37}$$

Investment of sector s, expressed at market price

$$I_s = ID_s + IM_s \tag{1.38}$$

Market price of investment of sector s (for verification)

$$PI^{bis}.I^{bis} = \sum_{s} PI_s \; I_s \tag{1.39}$$

Investment of sector s, expressed at market price (for verification)

$$I^{bis} = \sum_{s} I_s \tag{1.40}$$

1.1.6 Aggregation on commodities: imports and domestic aggregate production per use, expressed at market price (value & volume)

This subsection provides the aggregate production for various uses, for domestic and imported products. They are calculated through the aggregation of commodity c production per use on the commodity index.

Aggregate market price for domestically produced commodities

$$PQD.QD = \sum_c PQD_c \, QD_c \tag{1.41}$$

Aggregate domestically produced commodities, expressed at market price

$$QD = \sum_c QD_c \tag{1.42}$$

Aggregate market price for imported commodities

$$PQM.QM = \sum_c PQM_c \, QM_c \tag{1.43}$$

Aggregate imported commodities, expressed at market price

$$QM = \sum_c QM_c \tag{1.44}$$

Aggregate market price for the margins supplied on domestically produced commodities

$$PMGSD.MGSD = \sum_c PMGSD_c \, MGSD_c \tag{1.45}$$

Aggregate margins supplied on domestically produced commodities, expressed at market price

$$MGSD = \sum_c MGSD_c \tag{1.46}$$

Aggregate market price for the margins supplied on imported commodities

$$PMGSM.MGSM = \sum_c PMGSM_c \, MGSM_c \tag{1.47}$$

Aggregate margins supplied on imported commodities, expressed at market price

$$MGSM = \sum_{c} MGSM_c \tag{1.48}$$

Aggregate market price for domestically produced intermediate consumption

$$PCID.CID = \sum_{c} PCID_c \, CID_c \tag{1.49}$$

Aggregate domestically produced intermediate consumption, expressed at market price

$$CID = \sum_{c} CID_c \tag{1.50}$$

Aggregate market price for imported intermediate consumption

$$PCIM.CIM = \sum_{c} PCIM_c \, CIM_c \tag{1.51}$$

Aggregate imported intermediate consumption, expressed at market price

$$CIM = \sum_{c} CIM_c \tag{1.52}$$

Aggregate market price for domestically produced households final consumption

$$PCHD.CHD = \sum_{c} PCHD_c \, CHD_c \tag{1.53}$$

Aggregate domestically produced final consumption, expressed at market price

$$CHD = \sum_{c} CHD_c \tag{1.54}$$

Aggregate market price for imported households final consumption

$$PCHM.CHM = \sum_{c} PCHM_c \, CHM_c \tag{1.55}$$

Aggregate imported households final consumption, expressed at market price

$$CHM = \sum_{c} CHM_c \tag{1.56}$$

Aggregate market price for domestically produced Government final consumption

$$PGD.GD = \sum_c PGD_c \, GD_c \tag{1.57}$$

Aggregate domestically produced Government final consumption, expressed at market price

$$GD = \sum_c GD_c \tag{1.58}$$

Aggregate market price for imported Government final consumption

$$PGM.GM = \sum_c PGM_c \, GM_c \tag{1.59}$$

Aggregate imported Government final consumption, expressed at market price

$$GM = \sum_c GM_c \tag{1.60}$$

Aggregate market price for domestically produced investment

$$PID.ID = \sum_c PID_c \, ID_c \tag{1.61}$$

Aggregate domestically produced investment, expressed at market price

$$ID = \sum_c ID_c \tag{1.62}$$

Aggregate market price for imported investment

$$PIM.IM = \sum_c PIM_c \, IM_c \tag{1.63}$$

Aggregate imported investment, expressed at market price

$$IM = \sum_c IM_c \tag{1.64}$$

Aggregate market price for domestically produced exports

$$PXD.XD = \sum_c PXD_c \, XD_c \tag{1.65}$$

Aggregate domestically produced exports, expressed at market price

$$XD = \sum_c XD_c \tag{1.66}$$

Aggregate market price for imported exports (re-exports)

$$PXM.XM = \sum_c PXM_c \, XM_c \tag{1.67}$$

Aggregate imported exports (re-exports), expressed at market price

$$XM = \sum_c XM_c \tag{1.68}$$

Aggregate market price for domestically produced change in inventories

$$PDSD.DSD = \sum_c PDSD_c \, DSD_c \tag{1.69}$$

Aggregate domestically produced change in inventories, expressed at market price

$$DSD = \sum_c DSD_c \tag{1.70}$$

Aggregate market price for imported change in inventories

$$PDSM.DSM = \sum_c PDSM_c \, DSM_c \tag{1.71}$$

Aggregate imported change in inventories, expressed at market price

$$DSM = \sum_c DSM_c \tag{1.72}$$

1.1.7 Aggregation of domestic and imported production per use, expressed at market price (value & volume)

This subsection provides the aggregate production for various uses by summing the corresponding domestic and imported aggregates.

Aggregate market price for production

$$PQ.Q = PQD.QD + PQM.QM \tag{1.73}$$

Aggregate production, expressed at market price

$$Q = QD + QM \tag{1.74}$$

Aggregate market price for supplied margins

$$PMGS.MGS = PMGSD.MGSD + PMGSM.MGSM \tag{1.75}$$

Aggregate supplied margins

$$MGS = MGSD + MGSM \tag{1.76}$$

Aggregate market price for intermediate consumption

$$PCI.CI = PCID.CID + PCIM.CIM \tag{1.77}$$

Aggregate intermediate consumption, expressed at market price

$$CI = CID + CIM \tag{1.78}$$

Aggregate market price for household final (consumer price index)

$$PCH.CH = PCHD.CHD + PCHM.CHM \tag{1.79}$$

Aggregate household final consumption, expressed at market price

$$CH = CHD + CHM \tag{1.80}$$

Aggregate market price for Government final consumption

$$PG.G = PGD.GD + PGM.GM \tag{1.81}$$

Aggregate Government final consumption, expressed at market price

$$G = GD + GM \tag{1.82}$$

Aggregate market price for investment

$$PI.I = PID.ID + PIM.IM \tag{1.83}$$

Aggregate investment, expressed at market price

$$I = ID + IM \tag{1.84}$$

Aggregate market price for exports

$$PX.X = PXD.XD + PXM.XM \tag{1.85}$$

Aggregate exports, expressed at market price

$$X = XD + XM \tag{1.86}$$

Aggregate market price for change in inventories

$$PDS.DS = PDSD.DSD + PDSM.DSM \tag{1.87}$$

Aggregate change in inventories, expressed at market price

$$DS = DSD + DSM \tag{1.88}$$

1.2 Supply side

1.2.1 Domestic and foreign equilibrium for commodities c (value & volume):

Production of commodity c, expressed at basic price

$$YQ_c\ PYQ_c + NTAXCD_c^{VAL} + PMGPD_c\ MGPD_c = PQD_c\ QD_c \quad (1.89)$$

Basic price of the production of commodity c (for verification)

$$PYQ_c^{bis}\ YQ_c + NTAXCD_c^{VAL} + PMGPD_c\ MGPD_c = PQD_c\ QD_c \quad (1.90)$$

This price is already defined as a weighted average of the production price of the sectors producing commodity c in the price block: $PYQ.YQ = \sum_c PYQ_c\ YQ_c$. To verify the accounting consistancy, we define it here under an alias name.

Production of commodity c, expressed at basic price (for verification)

$$YQ_c^{bis} + NTAXCD_c + MGPD_c = QD_c \quad (1.91)$$

Same variable as YQ_c above to check the accounting consistency.

Imports of commodity c, expressed at basic price

$$M_c\ PM_c + NTAXCM_c^{VAL} + PMGPM_c\ MGPM_c = PQM_c\ QM_c \quad (1.92)$$

Basic price of imports of commodity c (for verification)

$$PM_c^{bis}\ M_c + NTAXCM_c^{VAL} + PMGPM_c\ MGPM_c = PQM_c\ QM_c \quad (1.93)$$

This price is already defined in the price block as $PM_c = EXR.PWD_c$. To verify the accounting consistancy, we define it here under an alias name.

Imports of commodity c, expressed at basic price (for verification)

$$M_c^{bis} + NTAXCM_c + MGPM_c = QM_c \quad (1.94)$$

Same variable as M_c above to check the accounting consistency.

1.2.2 Margins paid (value & volume)

Price of the margins paid on domestically produced commodity c

$$PMGPD_c \; MGPD_c = \sum_m PMGPD_{m,c} \; MGPD_{m,c} \tag{1.95}$$

Margins paid on the domestically produced commodity c

$$MGPD_c = \sum_m MGPD_{m,c} \tag{1.96}$$

Price of the margins paid on imported commodity c

$$PMGPM_c \; MGPM_c = \sum_m PMGPM_{m,c} \; MGPM_{m,c} \tag{1.97}$$

Margins paid on imported commodity c

$$MGPM_c = \sum_m MGPM_{m,c} \tag{1.98}$$

Price of the margins paid to commodity m on commodity c

$$PMGP_{m,c} \; MGP_{m,c} = PMGPD_{m,c} \; MGPD_{m,c} + PMGPM_{m,c} \; MGPM_{m,c} \tag{1.99}$$

Margins paid to commodity m on commodity c

$$MGP_{m,c} = MGPD_{m,c} + MGPM_{m,c} \tag{1.100}$$

1.2.3 Aggregation on commodities: supply side aggregates (value & volume)

Aggregate price of the margins paid on domestically produced commodity

$$PMGPD.MGPD = \sum_c PMGPD_c \; MGPD_c \tag{1.101}$$

Margins paid on domestically produced commodities

$$MGPD = \sum_c MGPD_c \tag{1.102}$$

Aggregate price of the margins paid on imported commodities

$$PMGPM.MGPM = \sum_c PMGPM_c \, MGPM_c \tag{1.103}$$

Margins paid on imported commodities

$$MGPM = \sum_c MGPM_c \tag{1.104}$$

Aggregate basic price of domestic production

$$PYQ.YQ = \sum_c PYQ_c \, YQ_c \tag{1.105}$$

Domestic production, expressed at basic price

$$YQ = \sum_c YQ_c \tag{1.106}$$

Aggregate basic price of imports

$$PM.M = \sum_c PM_c \, M_c \tag{1.107}$$

Imports, expressed at basic price

$$M = \sum_c M_c \tag{1.108}$$

1.2.4 Supply indicators of sector s (value & volume):

Production of sector s, expressed at basic price

$$Y_s = \sum_c Y_{c,s} \tag{1.109}$$

The production price of sector s is defined in the producer block as a behavour equation. It can not therefore be defined here as an index.

Value-added of sector s expressed in value

$$VA_s^{VAL} = PY_s \, Y_s - PCI_s \, CI_s \tag{1.110}$$

Value-added of sector s

$$VA_s = Y_s - CI_s \tag{1.111}$$

Gross operating surplus of sector s expressed in value

$$GOS_s^{VAL} = VA_s^{VAL} - PWAGES_s\, WAGES_s - PRSC_s\, RSC_s - NTAXS_s^{VAL} \tag{1.112}$$

The standard definition of the Gross Operating Surplus (GOS) generally include tax on profits. For simplicity, we assume that $NTAXS_s$ includes all net taxes on capital (i.e. tax on production and profits). In our definition, the tax on profit is therefore excluded from the GOS. This should be taken into account if one wants to use the GOS as a basis for the tax on profits.

Gross operating surplus of sector s

$$GOS_s = VA_s - WAGES_s - RSC_s - NTAXS_s \tag{1.113}$$

Net operating surplus of sector s expressed in value

$$NOS_s^{VAL} = GOS_s^{VAL} - PK_{s,t-1}\, \delta_s\, F_{K,s,t-1} \tag{1.114}$$

Net operating surplus of sector s

$$NOS_s = GOS_s - PK_{s,t_0-1}\, \delta_s\, F_{K,s,t-1} \tag{1.115}$$

1.2.5 Aggregation on sectors: supply indicators of all sectors (value & volume)

Basic price of aggregate production

$$PY.Y = \sum_s PY_s\, Y_s \tag{1.116}$$

Aggregate production, expressed at basic price

$$Y = \sum_s Y_s \tag{1.117}$$

Value-added price

$$PVA.VA = \sum_s VA_s^{VAL} \tag{1.118}$$

Aggregate value-added

$$VA = \sum_s VA_s \tag{1.119}$$

Gross wage index paid by sectors

$$PWAGES.WAGES = \sum_s PWAGES_s \, WAGES_s \tag{1.120}$$

The gross wage includes employees (but not employers)' social contribution

Aggregate gross wages paid by sectors

$$WAGES = \sum_s WAGES_s \tag{1.121}$$

Price of the aggregate gross operating surplus

$$PGOS.GOS = \sum_s GOS_s^{VAL} \tag{1.122}$$

Aggregate gross operating surplus

$$GOS = \sum_s GOS_s \tag{1.123}$$

Price of the aggregate net operating surplus

$$PNOS.NOS = \sum_s NOS_s^{VAL} \tag{1.124}$$

Aggregate net operating surplus

$$NOS = \sum_s NOS_s \tag{1.125}$$

1.3 Gross Domestic Product (GDP)

In this subsection, GDP is calculated according to different approaches. All approaches lead to same result.

1.3.1 Expenditure approach

Price of GDP (expenditure definition)

$$PGDP.GDP = PCH.CH + PG.G + PI.I + PX.X + PDS.DS - PM.M \tag{1.126}$$

According to the expenditure approach, GDP is calculated as the sum of the different components in the final uses of goods and services.

GDP (expenditure definition)

$$GDP = CH + G + I + X + DS - M \tag{1.127}$$

Price of GDP of commodity c (expenditure definition)

$$\begin{aligned} PGDP_c\, GDP_c = {} & PCH_c\, CH_c + PG_c\, G_c + PI_c\, I_c \\ & + PX_c\, X_c + PDS_c\, DS_c - PM_c\, M_c \end{aligned} \tag{1.128}$$

GDP of commodity c (expenditure definition)

$$GDP_c = CH_c + G_c + I_c + X_c + DS_c - M_c \tag{1.129}$$

Price of GDP (expenditure definition, for verification)

$$PGDP^{bis}.GDP^{bis} = \sum_c PGDP_c\, GDP_c \tag{1.130}$$

GDP (expenditure definition, for verification)

$$GDP^{bis} = \sum_c GDP_c \tag{1.131}$$

1.3.2 Production approach

Price of GDP (production definition)

$$PGDP^{ter}.GDP^{ter} = PVA.VA + PNTAXC.NTAXC \tag{1.132}$$

According to the production approach, GDP is calculated as the sum of the value added plus the total net taxes on commodities.

GDP (production definition)

$$GDP^{ter} = VA + NTAXC \tag{1.133}$$

1.3.3 Income approach

Price of GDP (income definition)

$$PGDP4.GDP4 = PGOS.GOS + PWAGES.WAGES + PRSC.RSC + NTAXS^{VAL} + PNTAXC.NTAXC \tag{1.134}$$

According to the income approach, GDP is calculated as the sum of all the economic incomes (from labor and capital) corrected by the social and taxes transfers.

GDP (income definition)

$$GDP4 = GOS + WAGES + RSC + NTAXS + NTAXC \tag{1.135}$$

2 Other equations

2.1 Adjustment equations and anticipation

Mark-up in the sector s

$$\mu_s = \alpha_s^{\mu}\ \mu_s^n + (1 - \alpha_s^{\mu})\ \mu_{s,t-1} \tag{2.1}$$

Expected inflation.

$$\Delta\left(\log P^e\right) = \alpha^{Pe,P1}.\Delta\left(\log P_{t-1}\right) + \left(1 - \alpha^{Pe,P1}\right).\Delta\left(\log P_{t-1}^e\right) \tag{2.2}$$

This equation defines the expected inflation and not the expected price. P^e does not necessary converge to P. If the wage equation is a WS curve, even in the very long term it may not converge.

Expected production

$$\Delta\left(\log Y_s^e\right) = \alpha_s^{Ye,Y}\ \Delta\left(\log Y_s\right) + \left(1 - \alpha_s^{Ye,Y}\right)\ \Delta\left(\log Y_{s,t-1}^e\right) \tag{2.3}$$

Quantity of Labor, Energy and Material inputs in sector s

$$\log F_{f,s} = \alpha_{f,s}^{0,F}\ \log F_{f,s}^n + \left(1 - \alpha_{f,s}^{0,F}\right)\ \left(\log F_{f,s,t-1} + \Delta\left(\log F_{f,s}^e\right)\right) \tag{2.4}$$

Expected quantity of Labor, Energy and Material inputs in sector s

$$\Delta\left(\log F^{e}_{f,s}\right) = \alpha^{1,F}_{f,s}\ \Delta\left(\log F^{e}_{f,s,t-1}\right) + \alpha^{2,F}_{f,s}\ \Delta\left(\log F_{f,s,t-1}\right) + \alpha^{3,F}_{f,s}\ \Delta\left(\log F^{n}_{f,s}\right) \tag{2.5}$$

Capital stock of sector s

$$F_{K,s} = (1-\delta_s)\ F_{K,s,t-1} + IA_s \tag{2.6}$$

Investment in sector s

$$\begin{aligned}\Delta\left(\log IA_s\right) = {} & \alpha^{IA,Ye}_{s}\ \Delta\left(\log Y^{e}_{s}\right) + \alpha^{IA,IA1}_{s}\ \Delta\left(\log IA_{s,t-1}\right) \\ & + \alpha^{IA,SUBST}_{s}\ \Delta\left(SUBST^{F}_{K,s}\right) \\ & + \alpha^{IA,Kn}_{s}\ \left(\log F^{n}_{K,s,t-1} - \log F_{K,s,t-1}\right)\end{aligned} \tag{2.7}$$

Households final consumption of commodity c

$$\log CH_c = \alpha^{0,CH}_{c}\ \log CH^{n}_{c} + \left(1-\alpha^{0,CH}_{c}\right)\ \left(\log CH_{c,t-1} + \Delta\left(\log CH^{e}_{c}\right)\right) \tag{2.8}$$

Expected households final consumption of commodity c

$$\begin{aligned}\Delta\left(\log CH^{e}_{c}\right) = {} & \alpha^{1,CH}_{c}\ \Delta\left(\log CH^{e}_{c,t-1}\right) + \alpha^{2,CH}_{c}\ \Delta\left(\log CH_{c,t-1}\right) \\ & + \alpha^{3,CH}_{c}\ \Delta\left(\log CH^{n}_{c}\right)\end{aligned} \tag{2.9}$$

Production price of sector s

$$\log PY_s = \alpha^{0,PY}_{s}\ \log PY^{n}_{s} + \left(1-\alpha^{0,PY}_{s}\right)\ \left(\log PY_{s,t-1} + \Delta\left(\log PY^{e}_{s}\right)\right) \tag{2.10}$$

Expected production priceof sector s

$$\begin{aligned}\Delta\left(\log PY^{e}_{s}\right) = {} & \alpha^{1,PY}_{s}\ \Delta\left(\log PY^{e}_{s,t-1}\right) + \alpha^{2,PY}_{s}\ \Delta\left(\log PY_{s,t-1}\right) \\ & + \alpha^{3,PY}_{s}\ \Delta\left(\log PY^{n}_{s}\right)\end{aligned} \tag{2.11}$$

Wages of the sector s

$$\Delta\left(\log W_s\right) = \alpha^{W,Wn}_{s}\ \Delta\left(\log W^{n}_{s}\right) + \alpha^{W,W1}_{s}\ \Delta\left(\log W_{s,t-1}\right) - \alpha^{W,W1Wn1}_{s}\ \log\frac{W_{s,t-1}}{W^{n}_{s,t-1}} \tag{2.12}$$

Labor participation ratio

$$PARTR = \alpha^{0,PARTR}.PARTR^{n} + \left(1 - \alpha^{0,PARTR}\right).PARTR_{t-1} \quad (2.13)$$

Interest rate

$$R = \alpha^{0,R}.R^{n} + \left(1 - \alpha^{0,R}\right).R_{t-1} \quad (2.14)$$

Households property income in value

$$\begin{aligned}\log PROP^{INC,H,VAL} = {} & \alpha^{0,PROP,INC,H,VAL}.\log PROP^{INC,H,VAL,n} \\ & + \left(1 - \alpha^{0,PROP,INC,H,VAL}\right).\Big(\log PROP_{t-1}^{INC,H,VAL} \\ & + \Delta\left(\log PROP^{INC,H,VAL,e}\right)\Big)\end{aligned} \quad (2.15)$$

Expected Households property income in value

$$\begin{aligned}\Delta\left(\log PROP^{INC,H,VAL,e}\right) = {} & \alpha^{1,PROP,INC,H,VAL}.\Delta\left(\log PROP_{t-1}^{INC,H,VAL,e}\right) \\ & + \alpha^{2,PROP,INC,H,VAL}.\Delta\left(\log PROP_{t-1}^{INC,H,VAL}\right) \\ & + \alpha^{3,PROP,INC,H,VAL}.\Delta\left(\log PROP^{INC,H,VAL,n}\right)\end{aligned} \quad (2.16)$$

Government property incomes in value

$$\begin{aligned}\log PROP^{INC,G,VAL} = {} & \alpha^{0,PROP,INC,G,VAL}.\log PROP^{INC,G,VAL,n} \\ & + \left(1 - \alpha^{0,PROP,INC,G,VAL}\right).\Big(\log PROP_{t-1}^{INC,G,VAL} \\ & + \Delta\left(\log PROP^{INC,G,VAL,e}\right)\Big)\end{aligned} \quad (2.17)$$

Expected Government property incomes in value

$$\begin{aligned}\Delta\left(\log PROP^{INC,G,VAL,e}\right) = {} & \alpha^{1,PROP,INC,G,VAL}.\Delta\left(\log PROP_{t-1}^{INC,G,VAL,e}\right) \\ & + \alpha^{2,PROP,INC,G,VAL}.\Delta\left(\log PROP_{t-1}^{INC,G,VAL}\right) \\ & + \alpha^{3,PROP,INC,G,VAL}.\Delta\left(\log PROP^{INC,G,VAL,n}\right)\end{aligned} \quad (2.18)$$

2.2 Substitutions

Substitution effect of the production factor f in the sector s

$$SUBST^{F}_{f,s} = \alpha^{6,F}_{f,s}\ SUBST^{F,n}_{f,s} + \left(1 - \alpha^{6,F}_{f,s}\right)\ SUBST^{F}_{f,s,t-1} \tag{2.19}$$

Substitution effect of the domestic margin paid m for the commodity c

$$SUBST^{MGPD}_{m,c} = \alpha^{6,MGPD}_{m,c}\ SUBST^{MGPD,n}_{m,c} + \left(1 - \alpha^{6,MGPD}_{m,c}\right)\ SUBST^{MGPD}_{m,c,t-1} \tag{2.20}$$

Substitution effect on the imported margin paid m for the commodity c

$$SUBST^{MGPM}_{m,c} = \alpha^{6,MGPM}_{m,c}\ SUBST^{MGPM,n}_{m,c} + \left(1 - \alpha^{6,MGPM}_{m,c}\right)\ SUBST^{MGPM}_{m,c,t-1} \tag{2.21}$$

Substitution effect on the energy intermediate consumption ce in the sector s

$$SUBST^{CI}_{ce,s} = \alpha^{6,CI}_{ce,s}\ SUBST^{CI,n}_{ce,s} + \left(1 - \alpha^{6,CI}_{ce,s}\right)\ SUBST^{CI}_{ce,s,t-1} \tag{2.22}$$

Substitution effect on the transportation intermediate consumption ce in the sector s

$$SUBST^{CI}_{ct,s} = \alpha^{6,CI}_{ct,s}\ SUBST^{CI,n}_{ct,s} + \left(1 - \alpha^{6,CI}_{ct,s}\right)\ SUBST^{CI}_{ct,s,t-1} \tag{2.23}$$

Substitution effect on the imported margin supplied for the commodity m

$$SUBST^{MGSM}_{m} = \alpha^{6,MGSM}_{m}\ SUBST^{MGSM,n}_{m} + \left(1 - \alpha^{6,MGSM}_{m}\right)\ SUBST^{MGSM}_{m,t-1} \tag{2.24}$$

Substitution effect on the imported households final consumption for the commodity c

$$SUBST^{CHM}_{c} = \alpha^{6,CHM}_{c}\ SUBST^{CHM,n}_{c} + \left(1 - \alpha^{6,CHM}_{c}\right)\ SUBST^{CHM}_{c,t-1} \tag{2.25}$$

Substitution effect on the imported government final consumption for the commodity c

$$SUBST^{GM}_{c} = \alpha^{6,GM}_{c}\ SUBST^{GM,n}_{c} + \left(1 - \alpha^{6,GM}_{c}\right)\ SUBST^{GM}_{c,t-1} \tag{2.26}$$

Substitution effect on the government final consumption for the imported commodity c

$$SUBST_c^{XM} = \alpha_c^{6,XM}\ SUBST_c^{XM,n} + \left(1 - \alpha_c^{6,XM}\right)\ SUBST_{c,t-1}^{XM} \quad (2.27)$$

Substitution effect on the intermediate consumption for the imported commodity c in the sector s

$$SUBST_{c,s}^{CIM} = \alpha_{c,s}^{6,CIM}\ SUBST_{c,s}^{CIM,n} + \left(1 - \alpha_{c,s}^{6,CIM}\right)\ SUBST_{c,s,t-1}^{CIM} \quad (2.28)$$

Substitution effect on the investment for the imported commodity c in the sector s

$$SUBST_{c,s}^{IM} = \alpha_{c,s}^{6,IM}\ SUBST_{c,s}^{IM,n} + \left(1 - \alpha_{c,s}^{6,IM}\right)\ SUBST_{c,s,t-1}^{IM} \quad (2.29)$$

Substitution effect on the exports of the commodity c

$$SUBST_c^{X} = \alpha_c^{6,X}\ SUBST_c^{X,n} + \left(1 - \alpha_c^{6,X}\right)\ SUBST_{c,t-1}^{X} \quad (2.30)$$

3 Exogenous variables

3.1. $share_s^{free}$ – **Percentage of freely allocated permits to sector** s

For each sector, the quantity of free permits is defined as a share of emissions that are eligible to the emission trading scheme.

3.2. $alpha^{exo}$ – **Alpha variable (used as a test for exogenous variable documentation)**

Test exogenous variable

4 Glossary

$alpha^{exo}$	Alpha variable (used as a test for exogenous variable documentation)	3.2,	24
CH	Aggregate household final consumption, expressed at market price	1.80,	11
CH_c	Households final consumption of commodity c	2.8,	20
CH_c^e	Expected households final consumption of commodity c	2.9,	20
CHD	Aggregate domestically produced final consumption, expressed at market price	1.54,	8
CHM	Aggregate imported households final consumption, expressed at market price	1.56,	8
CI	Aggregate intermediate consumption, expressed at market price	1.78,	11
CI_c	Intermediate consumption of commodity c, expressed at market price	1.12,	3
CI_s	Intermediate consumption of sector s, expressed at market price	1.30,	5
CI^{bis}	Intermediate consumption of sector s, expressed at market price (for verification)	1.32,	6
CID	Aggregate domestically produced intermediate consumption, expressed at market price	1.50,	8
CID_c	Quantity of domestically produced commodity c used as intermediary consumption, expressed at market price	1.18,	4
CID_s	Domestically produced intermediate consumption of sector s, expressed at market price	1.26,	5
CIM	Aggregate imported intermediate consumption, expressed at market price	1.52,	8
CIM_c	Quantity of imported commodity c used as intermediary consumption, expressed at market price	1.20,	4

CIM_s	Imported intermediate consumption of sector s, expressed at market price	1.28, 5
DS	Aggregate change in inventories, expressed at market price	1.88, 12
DS_c	Change in inventories of commodity c, expressed at market price	1.16, 3
DSD	Aggregate domestically produced change in inventories, expressed at market price	1.70, 10
DSM	Aggregate imported change in inventories, expressed at market price	1.72, 10
$F_{f,s}$	Quantity of Labor, Energy and Material inputs in sector s	2.4, 19
$F_{K,s}$	Capital stock of sector s	2.6, 20
$F^e_{f,s}$	Expected quantity of Labor, Energy and Material inputs in sector s	2.5, 20
G	Aggregate Government final consumption, expressed at market price	1.82, 11
GD	Aggregate domestically produced Government final consumption, expressed at market price	1.58, 9
GDP	GDP (expenditure definition)	1.127, 18
GDP_c	GDP of commodity c (expenditure definition)	1.129, 18
$GDP4$	GDP (income definition)	1.135, 19
GDP^{bis}	GDP (expenditure definition, for verification)	1.131, 18
GDP^{ter}	GDP (production definition)	1.133, 18
GM	Aggregate imported Government final consumption, expressed at market price	1.60, 9
GOS	Aggregate gross operating surplus	1.123, 17
GOS_s	Gross operating surplus of sector s	1.113, 16
GOS_s^{VAL}	Gross operating surplus of sector s expressed in value	1.112, 16
I	Aggregate investment, expressed at market price	1.84, 12

I_c	Investment in commodity c, expressed at market price	1.14, 3
I_s	Investment of sector s, expressed at market price	1.38, 6
IA_s	Investment in sector s	2.7, 20
I^{bis}	Investment of sector s, expressed at market price (for verification)	1.40, 6
ID	Aggregate domestically produced investment, expressed at market price	1.62, 9
ID_c	Quantity of imported commodity c used as investment, expressed at market price	1.22, 4
ID_s	Domestically produced investment of sector s, expressed at market price	1.34, 6
IM	Aggregate imported investment, expressed at market price	1.64, 9
IM_c	Quantity of imported commodity c used as investment, expressed at market price	1.24, 5
IM_s	Imported investment of sector s, expressed at market price	1.36, 6
M	Imports, expressed at basic price	1.108, 15
M_c	Imports of commodity c, expressed at basic price	1.92, 13
μ_s	Mark-up in the sector s	2.1, 19
M_c^{bis}	Imports of commodity c, expressed at basic price (for verification)	1.94, 13
$MGP_{m,c}$	Margins paid to commodity m on commodity c	1.100, 14
$MGPD$	Margins paid on domestically produced commodities	1.102, 14
$MGPD_c$	Margins paid on the domestically produced commodity c	1.96, 14
$MGPM$	Margins paid on imported commodities	1.104, 15
$MGPM_c$	Margins paid on imported commodity c	1.98, 14
MGS	Aggregate supplied margins	1.76, 11

MGS_m	Margins supplied by commodity m, expressed at market price	1.6, 2
MGS_m^{bis}	Margins supplied by commodity m, expressed at market price (for verification)	1.8, 2
$MGSD$	Aggregate margins supplied on domestically produced commodities, expressed at market price	1.46, 7
$MGSM$	Aggregate margins supplied on imported commodities, expressed at market price	1.48, 8
NOS	Aggregate net operating surplus	1.125, 17
NOS_s	Net operating surplus of sector s	1.115, 16
NOS_s^{VAL}	Net operating surplus of sector s expressed in value	1.114, 16
P^e	Expected inflation.	2.2, 19
$PARTR$	Labor participation ratio	2.13, 21
PCH	Aggregate market price for household final (consumer price index)	1.79, 11
$PCHD$	Aggregate market price for domestically produced households final consumption	1.53, 8
$PCHM$	Aggregate market price for imported households final consumption	1.55, 8
PCI	Aggregate market price for intermediate consumption	1.77, 11
PCI_c	Market price of the intermediate consumption of commodity c	1.11, 3
PCI_s	Market price of intermediate consumption of sector s	1.29, 5
PCI^{bis}	Market price of intermediate consumption of sector s (for verification)	1.31, 6
$PCID$	Aggregate market price for domestically produced intermediate consumption	1.49, 8
$PCID_c$	Market price for the domestically produced commodity c used as intermediary consumption	1.17, 4

$PCID_s$	Market price of domestically produced intermediate consumption of sector s	1.25, 5
$PCIM$	Aggregate market price for imported intermediate consumption	1.51, 8
$PCIM_c$	Market price for imported commodity c used as intermediary consumption	1.19, 4
$PCIM_s$	Market price of imported intermediate consumption of sector s	1.27, 5
PDS	Aggregate market price for change in inventories	1.87, 12
PDS_c	Market price of the change in inventories of commodity c	1.15, 3
$PDSD$	Aggregate market price for domestically produced change in inventories	1.69, 10
$PDSM$	Aggregate market price for imported change in inventories	1.71, 10
PG	Aggregate market price for Government final consumption	1.81, 11
PGD	Aggregate market price for domestically produced Government final consumption	1.57, 9
$PGDP$	Price of GDP (expenditure definition)	1.126, 18
$PGDP_c$	Price of GDP of commodity c (expenditure definition)	1.128, 18
$PGDP4$	Price of GDP (income definition)	1.134, 19
$PGDP^{bis}$	Price of GDP (expenditure definition, for verification)	1.130, 18
$PGDP^{ter}$	Price of GDP (production definition)	1.132, 18
PGM	Aggregate market price for imported Government final consumption	1.59, 9
$PGOS$	Price of the aggregate gross operating surplus	1.122, 17
PI	Aggregate market price for investment	1.83, 11
PI_c	Market price of the investment in commodity c	1.13, 3
PI_s	Market price of investment of sector s	1.37, 6

PI^{bis}	Market price of investment of sector s (for verification)	1.39, 6
PID	Aggregate market price for domestically produced investment	1.61, 9
PID_c	Market price for domestically produced commodity c used as investment	1.21, 4
PID_s	Market price of domestically produced investment of sector s	1.33, 6
PIM	Aggregate market price for imported investment	1.63, 9
PIM_c	Market price for imported commodity c used as investment	1.23, 4
PIM_s	Market price of imported investment of sector s	1.35, 6
PM	Aggregate basic price of imports	1.107, 15
PM_c^{bis}	Basic price of imports of commodity c (for verification)	1.93, 13
$PMGP_{m,c}$	Price of the margins paid to commodity m on commodity c	1.99, 14
$PMGPD$	Aggregate price of the margins paid on domestically produced commodity	1.101, 14
$PMGPD_c$	Price of the margins paid on domestically produced commodity c	1.95, 14
$PMGPM$	Aggregate price of the margins paid on imported commodities	1.103, 15
$PMGPM_c$	Price of the margins paid on imported commodity c	1.97, 14
$PMGS$	Aggregate market price for supplied margins	1.75, 11
$PMGS_m$	Market price of the margins supplied by commodity m	1.5, 2
$PMGS_m^{bis}$	Market price of the margins supplied by commodity m (for verification)	1.7, 2
$PMGSD$	Aggregate market price for the margins supplied on domestically produced commodities	1.45, 7

$PMGSM$	Aggregate market price for the margins supplied on imported commodities	1.47, 7
$PNOS$	Price of the aggregate net operating surplus	1.124, 17
PQ	Aggregate market price for production	1.73, 11
PQ_c	Market price of the production of commodity c	1.9, 3
PQD	Aggregate market price for domestically produced commodities	1.41, 7
PQD_c	Market price for the domestically produced commodity c	1.1, 1
PQM	Aggregate market price for imported commodities	1.43, 7
PQM_c	Market price for imported commodity c	1.3, 1
$PROP^{INC,G,VAL}$	Government property incomes in value	2.17, 21
$PROP^{INC,G,VAL,e}$	Expected Government property incomes in value	2.18, 21
$PROP^{INC,H,VAL}$	Households property income in value	2.15, 21
$PROP^{INC,H,VAL,e}$	Expected Households property income in value	2.16, 21
PVA	Value-added price	1.118, 16
$PWAGES$	Gross wage index paid by sectors	1.120, 17
PX	Aggregate market price for exports	1.85, 12
PXD	Aggregate market price for domestically produced exports	1.65, 9
PXM	Aggregate market price for imported exports (re-exports)	1.67, 10
PY	Basic price of aggregate production	1.116, 16
PY_s	Production price of sector s	2.10, 20
PY_s^e	Expected production priceof sector s	2.11, 20
PYQ	Aggregate basic price of domestic production	1.105, 15
PYQ_c^{bis}	Basic price of the production of commodity c (for verification)	1.90, 13
Q	Aggregate production, expressed at market price	1.74, 11

Q_c	Production of commodity c, expressed at market price	1.10,	3
QD	Aggregate domestically produced commodities, expressed at market price	1.42,	7
QD_c	Quantity of domestically produced commodity c expressed at market price	1.2,	1
QM	Aggregate imported commodities, expressed at market price	1.44,	7
QM_c	Quantity of imported commodity c expressed at market price	1.4,	1
R	Interest rate	2.14,	21
$share_s^{free}$	Percentage of freely allocated permits to sector s	3.1,	24
$SUBST_c^{CHM}$	Substitution effect on the imported households final consumption for the commodity c	2.25,	22
$SUBST_{ce,s}^{CI}$	Substitution effect on the energy intermediate consumption ce in the sector s	2.22,	22
$SUBST_{ct,s}^{CI}$	Substitution effect on the transportation intermediate consumption ce in the sector s	2.23,	22
$SUBST_{c,s}^{CIM}$	Substitution effect on the intermediate consumption for the imported commodity c in the sector s	2.28,	23
$SUBST_{f,s}^{F}$	Substitution effect of the production factor f in the sector s	2.19,	22
$SUBST_c^{GM}$	Substitution effect on the imported government final consumption for the commodity c	2.26,	22
$SUBST_{c,s}^{IM}$	Substitution effect on the investment for the imported commodity c in the sector s	2.29,	23
$SUBST_{m,c}^{MGPD}$	Substitution effect of the domestic margin paid m for the commodity c	2.20,	22
$SUBST_{m,c}^{MGPM}$	Substitution effect on the imported margin paid m for the commodity c	2.21,	22
$SUBST_m^{MGSM}$	Substitution effect on the imported margin supplied for the commodity m	2.24,	22

$SUBST_c^X$	Substitution effect on the exports of the commodity c	2.30, 23
$SUBST_c^{XM}$	Substitution effect on the government final consumption for the imported commodity c	2.27, 23
VA	Aggregate value-added	1.119, 17
VA_s	Value-added of sector s	1.111, 16
VA_s^{VAL}	Value-added of sector s expressed in value	1.110, 15
W_s	Wages of the sector s	2.12, 20
$WAGES$	Aggregate gross wages paid by sectors	1.121, 17
X	Aggregate exports, expressed at market price	1.86, 12
XD	Aggregate domestically produced exports, expressed at market price	1.66, 10
XM	Aggregate imported exports (re-exports), expressed at market price	1.68, 10
Y	Aggregate production, expressed at basic price	1.117, 16
Y_s	Production of sector s, expressed at basic price	1.109, 15
Y_s^e	Expected production	2.3, 19
YQ	Domestic production, expressed at basic price	1.106, 15
YQ_c	Production of commodity c, expressed at basic price	1.89, 13
YQ_c^{bis}	Production of commodity c, expressed at basic price (for verification)	1.91, 13