ThreeME Version 3
Multi-sector Macroeconomic Model for the Evaluation of Environmental and Energy policy

A full description

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Abstract

ThreeME is a country model especially designed to evaluate the medium and long term impact of environmental and energy policies at the macroeconomic and sector levels. To do so ThreeME combines two important features. Firstly, it has the main characteristics of neo-Keynesian models by assuming a slow adjustment of effective quantities and prices to their notional level, a Taylor rule and a Philips curve. Compared to standard multi-sector CGE models, this has the advantage to allow for the existence of under-optimum equilibria such as the presence of involuntary unemployment. Secondly, ThreeME is a hybrid model in the sense that it combines the top-down approach of general equilibrium macroeconomic models with elements of bottom-up energy models developed by engineers. As in bottom-up models, the amount of energy consumed is related to its use, that is the number of buildings or cars, and the energy class to which they belong. This hypothesis is more realistic compared to the assumption made in the majority of top-down models where energy consumption is usually directly related to income through a nested structure of utility function. This document provides a full and detailed description of the third version of the ThreeME model.

Key words: neo-Keynesian modeling, hybrid modeling, macroeconomic CGE modeling, energy and environmental policy modeling

JEL code: C68, E12, E17, E27, E37, E47, D57, D58

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Chapter 1

General Introduction

Top-down Computable General Equilibrium (CGE) models are often used to evaluate a wide range of economic problems. They have the advantage to combine tractability with a high level of detail, being able to distinguish different sectors, commodities, countries or type of consumers [1]. Their ability to account for an important number of economic sectors is part of their success especially for the analysis of the economic impact of environmental and energy policies. Several CGE models have been designed to study this topic: e.g. GREEN (Burniaux et al., 1992), GEMINI-E3 (Bernard and Vielle, 2008), GEM-E3 (Capros et al., 2013), IMACLIM (Ghersi et al., 2009). With the development of detailed multi-regional Input-Output (IO) databases with environmental extensions [2], recent CGE models include an increasingly number of sectors: e.g. ENV-Linkages (Chateau et al., 2014), GTAP (Corong et al., 2017), EXIOMOD (Bulavskaya et al., 2016) or FIDELIO (Rocchi et al., 2019).

But standard CGE models have two important drawbacks. First, they rely on very restrictive assumptions relative to the functioning of the economy especially in the short and medium run. Standard CGE models are supply models that rely on the Walras hypothesis of perfect flexibility prices and quantities that ensures the full and optimal use of production factors and thus rule out permanent or transitory under-optimum equilibrium such as the presence of involuntary unemployment. They neglect important effects that are supported by empirical evidences such as demand side Keynesian multipliers. In this setting, additional public investment and expenditures have always a negative impact on the economy because of their eviction effect on private expenditures.

Neo-Keynesian CGE models try to give a more realistic representation of the actual functioning of the economy by taking explicitly into account the slow adjustments of prices and quantities to their optimum. This allows for transitory or permanent under-optimum equilibrium. In theses models, the dynamic adjustment of prices and quantities is generally estimated econometrically. It seems that this effort is made to

[1]For a survey on CGE models see Shoven and Whalley (1984, 1992); Böhringer and Löschel (2006); Ballard et al. (2009); Hosoe et al. (2010); Burfisher (2017).

[2]World IO databases include the GTAP database [`www.gtap.agecon.purdue.edu`, (Peters et al., 2011)], WIOD [`www.wiod.org`, Timmer et al. (2012); Dietzenbacher et al. (2013)] or EXIOBASE (`www.exiobase.eu`; Tukker et al. (2009); Wood et al. (2015)

the detriment of the disaggregation level which is often low. This is typically the case for macroeconomic models for the French economy used in forecasting departments: e.g. MESANGE of the French ministry of Economy has three sectors (Allard-Prigent et al., 2002; Klein and Simon, 2010), BDF-FR of the French central Bank has two sectors (Lemoine et al., 2019); its predecessor MASCOTTE (Baghli et al., 2004) or E-Mod of the OFCE (Chauvin et al., 2002) have only one. Moreover, neo-Keynesian macroeconomic models generally do not distinguish between the different types of energy or of transport which are particularly important for the assessment of environmental and energy policy [3]. They are thus likely to neglect the effect of activity transfers in terms of growth and employment from high to low intensive energy sectors.

A second limit particularly important for the analysis of the economic impact of environmental and energy policies is that CGE models provide an insufficient representation of endogenous energy efficiency phenomena. For instance, households consumption behaviors are generally represented through a nested structure of utility function which leads to an overly simplified relation between the level of consumption of each goods and the income of households. The model does not include saturation level regarding the consumption of certain commodities. The link between consumption and income is often log-linear, that is linear in relative terms: a 1% increase in the real revenue leads to a 1% increase in the consumption of each good. To account for non-linearities, it is usual to introduce a Linear Expenditure System (LES) utility function. A LES specification assumes that a share of the base year consumption is independent from the income level and can not be reduced. Therefore the relation between income and consumption is not linear anymore. This specification allows for distinguishing between necessity and luxurious consumption goods.

Although the LES utility function improves the realism of the modeling of consumption behavior, it still relies on the theoretical representation that each good provides a direct utility to households. This is not a realistic assumption for certain goods such as energy. As formalized theoretically by Lancaster (Lancaster, 1966b) and applied in certain hybrid models (Laitner and Hanson, 2006), households do not consume energy for their direct utility but rather for the service they provide when combined with a capital goods such as a car or a house. There is no point buying gasoline if one does not have a car. A more realistic theoretical representation is therefore to assume that energy is an input used in combination with different types of capital in a households production function. This representation accounts for the fact that in the real world certain services are not always externally purchased by households but rather directly produced by them. This is typically the case for transports. Households can directly purchase a transportation service produced by an activity such as public transport. Alternatively, they can invest in a capital by purchasing a car and buy the necessary amount of gasoline that fulfills their needs.

Compared to the assumption made in standard top-down representation, there is hardly any direct utility from energy alone. This is of course different for other goods, although many commodities have similar properties to energy: for instance,

[3] NEMESIS (Brécard et al., 2006; Zagame et al., 2013) or E3ME (Econometrics, 2014) are amongst the exceptions. They are econometric model based on a neo-Keynesian closure with a high sectoral detail.

a big share of water consumption is related to the use of appliances; the number of appliances generally depends on the number of houses; the amount construction material used is closely related to the size of houses, etc. Relating the consumption of these goods directly to utility (and consequently to the revenue), as assumed in the standard top-down representation, may therefore lead to unrealistic results, where the consumption expressed in physical units exceed its saturation level. Indeed, it is unlikely that a household will ever decide to buy 6 cars, 10 washing machines or to heat its house at 35°C even if it becomes richer in the future. Because of their monetary representation, one cannot exclude that standard top-down CGE models produce such unrealistic development in the long run due to the general increase of the standard of living. Only a physical (along with a monetary) representation allows for the inclusion of realistic floor and ceiling in the consumption of certain goods.

ThreeME (Multi-sector Macroeconomic Model for the Evaluation of Environmental and Energy policy) is a model developed by the ADEME (French Environment and Energy Management Agency), OFCE (French economic observatory) and NEO (Netherlands Economic Observatory). Originally developed for the French economy, it has been applied to other regional contexts: Mexico, Indonesia, the Netherlands, Tunisia, the Occitania region. Its main purpose is to evaluate the impact of environmental and energy policy measures on the economy at the macroeconomic and sectoral levels. It also has the ambition to overcome the two limitations of standard top-down CGE models pointed above by introducing neo-Keynesian features such as inertia of prices and quantities and bottom-up features in the modeling of consumption behaviors.

Having the general structure of neo-Keynesian macroeconomic models, ThreeME is more realistic than a standard CGE model for describing the actual dynamic of the economy at least in the short and medium run. In the long run, the model is neo-classical in the sense of Solow (Solow, 1956) since it converges toward a steady states where all variables grow at a constant rate. Compatible with a high level of disaggregation [4], it allows for the neo-Keynesian short/medium term macroeconomic modeling approach to catch-up with the most advanced CGE models in terms of sectoral analysis. In addition, its hybrid structure regarding the specification of the behavior of households overcomes the restrictions imposed by the nested utility approach assumed in standard CGE models.

The present documentation provides a full and detailed description of the third version of ThreeME. Compared to the second version of the model described in Callonnec et al. (2013, 2016) [5], the version 3 of ThreeME makes the following main improvements:

• The organization of the code and the syntax of the model have been improved and rationalized. We have attempted to generalize as much as possible the specification of the equations of the model in order to facilitate the inclusion of extensions and the application to other countries or regional contexts. To do so we have improved the modularity of the model allowing more easily for switching on and off certain blocks or features of the model. Moreover we specify the model such that a more

[4] For instance, the French version of ThreeME is disaggregated in 37 sectors with an explicit distinction between 13 types of energy sources and five types of transports.

[5] The description of the first version of ThreeME can be found in Reynès et al. (2011).

detailed feature can be added without changing the specification of the core model: the complexity is embedded within a simpler version of the model and can be switch off if not useful for a given study. This also allows for reducing the minimum data requirement to run a basic version of ThreeME while allowing for the inclusion of additional details if those are relevant.

• The approach used to compile the model database has been improved. We have developed a R program for the loading and for the calibration of the model input data. This program traces all the transformation steps implemented from the raw data source to the final data used as input in ThreeME. The objective is to ensure a complete tractability of the calibration assumptions and to facilitate modifications such as changes in the sectoral disaggregation or the integration of more recent data (see Chapter 3, p. 43).

• In order to make this version open source under best conditions, the code has been thoroughly documented. The present documentation provides all the equations of the model. To avoid omissions, reporting mistakes and to facilitate future updates, we have developed a R program that is able to convert the syntax used by the solver into a Latex code that we have compiled here into a pdf file.

The next chapter (Chapter 2) describes the main features of ThreeME by presenting first a non-technical overview and then the most important modules. Chapter 3 (p. 43) presents the calibration approach, the input data used by ThreeME and the way the code is implemented and the model resolved. In order to study the long term and dynamic properties of the model, Chapter 4 (p. 51) presents the simulation results of standard shocks (increase of public expenditures, of VAT, of the fossil fuel prices, of the world demand, decrease of the income tax, of the employers' social contributions, of the carbon tax). Finally, the complete list of equations is presented in Chapter 5 (p. 79). The last section of this Chapter 5 provides a table that defines the names of all the variables and parameters and rank them by alphabetical order.

Chapter 2

Main features of ThreeME

The current chapter describes the main features of ThreeME and is organised as follows. Section 2.1 (p. 6) presents a non technical overview of the model by summarizing its main characteristics. The following sections describe in more detail the key modules/blocks and equations of the model. Section 2.2 (p. 13) describes the specification of adjustment mechanisms used in ThreeME and provides their theoretical foundation. Section 2.3 (p. 16) presents the production function and structure used in the model. It provides the demand for production factors of ThreeME that is derived from a flexible production function (section 2.3.1, p. 17). The constraints on the level of the elasticities of substitution that imposes a nested CES production structure are provided in Section 2.3.2 (p. 18). Investment and capital accumulation are described in Section 2.3.3 (p. 19). Section 2.4 (p. 20) describes the price structure and how firms in each sector determine their production price. Section 2.5 (p. 21) presents the way the wage setting is modeled. Section 2.7 (p. 22) presents the three different variants of the model that are used to describe the consumption behaviour of households. Variant 1 assumes a LES utility function where substitution possibilities are uniform across commodities (see Section 2.7.1, p. 24). Variant 2 assumes a nested utility function where specific substitution mechanisms are assumed for housing, transport and energy (see Section 2.7.2, p. 24). Variant 3 presents an hybrid (top-down and bottom-up) approach where the consumption of energy related to housing and transport is modeled according to a bottom-up approach (see Section 2.7.3, p. 26). Section 2.8 (p. 34) presents the modeling of imports and exports while the behavior of the Government including the Central Bank (CB) is defined in Section 2.9 (p. 35). Section 2.10 (p. 36) and Section 2.11 (p. 36) describe respectively the way greenhouse gases (GHG) emissions and the physical energy balance are modeled.

•Explanatory note on equations, variables and indexes:

The main equations of ThreeME that are provided is this chapter are repeated from Chapter 5 under the same reference number. In the pdf file, this reference number has a red hyperlink to the equation defined in Chapter 5 which allows the reader for easily accessing to the block where the equation is defined.

All equation numbers in the form of "5.X.X" are equations of the model and are therefore listed in Chapter 5. Equations with no number or with another prefix numbers are not equations directly present in the model but serve the purpose of derivation.

The complete list of equations is presented in Chapter 5 (p.79). The last section of Chapter 5 includes a glossary that defines the names of every variable and parameter and rank them by alphabetical order (section 5.20, p.168). The glossary provides also the number of the equation defining a given variable and the page where the equation is presented.

Finally, the different sets for each index used in the equations are provided in section 5.1 (p.79).

2.1 Overview of the model

ThreeME is a CGE (Computable General Equilibrium) model [1]. It therefore takes into account the interaction and feedbacks between supply and demand as schematized in Figure 2.1. Demand (consumption, investment, exports) defines supply (domestic production and imports). Supply defines in return demand through the incomes generated by the production factors (labor, capital, energy, material, land, etc.).

The notion of "general equilibrium" relates to a state where supply is equal to demand in all markets. In the literature, there are two main approaches to ensure this state. In Walrasian models, the equilibrium force is the price system. Perfect flexibility of prices and quantities (production factors, consumption, etc.) ensures the instantaneous equilibrium between supply and demand. When an exogenous shock decreases the supply of a commodity, its price tends to go up, thereby stimulating additional supply and depressing demand, until supply and demand are equal again. Arrow and Debreu (1954) demonstrate the conditions under which such an equilibrium exists [2]. This equilibrium mechanism does not only operate on the product markets. Depending on the closures retained (see e.g. Shoven and Whalley, 1992), it may also apply on the production factors markets (labour, capital), on the saving market (savings equal investments) and on the foreign exchange markets (imports equal exports). Walrasian type of CGE models are static: after a shock a new equilibrium (system of prices and quantities) is found within the period of simulation [3].

The second approach is the closure retained in neo-Keynesian models. In these models, prices do not clear the markets and market "imperfections" (e.g. involuntary unemployment) are taken into account. In coherence with empirical evidence, they assume that prices and quantities are rigid in the short run and that they adjust slowly over time toward their optimal level. The general equilibrium is achieved by assuming that demand determines supply. In the short and medium run, there can be situations of disequilibrium between notional (optimal) supply and the actual supply and of

[1] For a comparison of different types of CGE models see Dixon and Jorgenson (2012).

[2] They demonstrate that the Walrasian equilibrium is a Nash equilibrium (Nash, 1950, 1953) if agents are perfectly rational, if they do not commit anticipation errors, if the production functions do not show increasing returns to scale, and if the utility functions satisfy the standard properties of continuity, non-saturation and strict convexity of their isoquants. Additional more technical properties are also required (see e.g. Hahn, 1982).

[3] Some CGE models introduce a recursive dynamic where the past savings define next year capital stocks.

underutilization of the production capacity (in particular involuntary unemployment). Compared to the Walrasian CGE models, neo-Keynesian CGE models are dynamic and therefore better suited to analyze medium term phenomena and the transition to the long run.

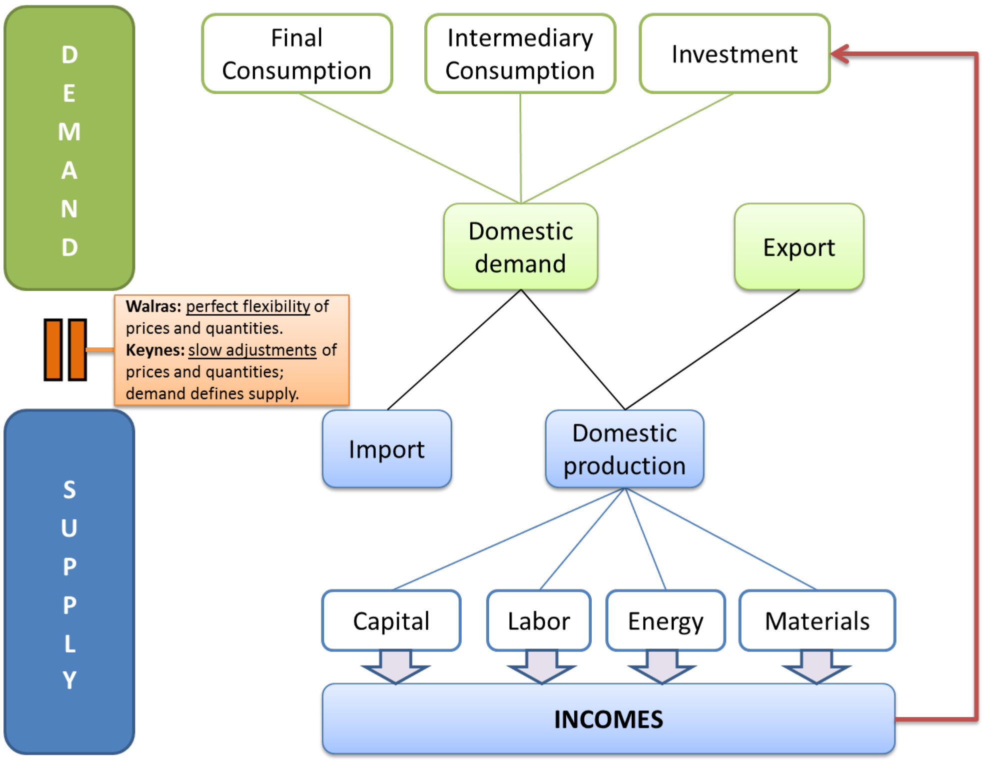


Figure 2.1: Architecture of a CGE model

2.1.1 A neo-Keynesian CGE model

ThreeME is a CGE model with neo-Keynesian features. This implies four main differences compared to a standard Walrasian CGE model.

First, the **hypothesis of slow adjustments of prices and quantities** is an important difference of neo-Keynesian models compared to Walrasien CGE models which assume perfect flexibility. In a neo-Keynesian framework, firms do not adjust instantaneously their price or their inputs (labor, capital, intermediary consumption) to the new optimum level after a shock in the economy. The same holds for the consumption of households, the wage setting or the adjustment of the interest rate. These slow adjustments result from rational decisions in the presence of various adjustment constrains related to physical or temporal boundaries, uncertainties, adjustment costs and rational choices. Building the optimal production capacity generally takes time because it takes time to hire and train the competent personnel or to build a new factory. In period of low demand, reducing immediately production capacity may not be optimal in the long run since it may lead to a loss of competences that will be costly to rebuild when the activity is high again.

The changes in consumption patterns are also the result of a slow process. In order to switch from fossil energy to renewable energy after an increase in the oil price, the firm will have to first change its production process and the consumer will have to first buy an electric car or to change his heating system. The fact that the consumption of households follows income with lags is known as an old empirical observation (see e.g. Tinbergen, 1942). Several consumption theories have been proposed to explain the slow adjustment of consumption to income observed empirically. Keynesian models emphasise the role of consumption habits or precautionary saving in response to uncertainty regarding future incomes (see e.g. Duesenberry, 1948; Modigliani, 1949; Brown, 1952). Neoclassical models emphasise inter-temporal decisions with the permanent income hypothesis proposed by Friedman (1957) or the life-cycle hypothesis proposed by Modigliani (1966).

Many studies have shown empirically that prices adjust slowly (see e.g. Carlton, 1986; Cecchetti, 1985; Kashyap, 1995). The theoretical literature on price setting puts forward the "menu costs" related to changing the selling price that prevent from adjusting it instantaneously: costs of printing menus, re-tagging items, updating computer systems but also strategic consideration such as the risk of losing clients after the change in price. This has led to various staggered price models where prices adjust slowly to their optimum level (see e.g. Taylor, 1979, 1980; Calvo, 1983). Other authors have argued that the changes in price are all the more costly that they are large because large increases are less accepted by consumers than small increases. In order to take this into account Rotemberg (1982b,a) proposes to use quadratic adjustment cost models. The firm defines the optimal price as a trade-off between the cost of adjusting and the cost of not been adjusted. These models were originally developed to model the slow adjustment of investment (Eisner and Strotz, 1963; Gould, 1968; Lucas, 1967; Schramm, 1970; Treadway, 1971). Using such a quadratic adjustment costs model, Section 2.2 (p. 13) derives the general specification used in ThreeME to model the various adjustment processes.

Second, **prices, wages and the interest rate do not clear supply and demand** on the goods, labor and saving markets even in the long run:

•Assuming imperfect (oligopolistic) competition, the optimal production price is defined as a mark-up over marginal production costs. Because of the price rigidity described above, production adjusts to the demand in the short run. However, the mark-up (and therefore the price) is sensitive to the tensions between production capacities (supply) and the demand. A positive (resp. negative) shock leads to an increase (resp. decrease) in the capacity utilization ratio, and therefore in the mark-up and in the price.

•The labor market is also characterized by imperfect competition that prevents wages from equalizing the supply and the demand for labor. Following the theory of the equilibrium unemployment rate or *NAIRU* (Non Accelerating Inflation Rate of Unemployment) initiated by Phelps (1967, 1968) or Friedman (1968), permanent disequilibrium between supply and demand and therefore involuntary unemployment is a possible outcome. We assume a general specification of the wage equation that includes the augmented Phillips curve (Phillips, 1958) and Wage Setting (WS) curve *à la* Layard *et al.* proposed in the 1980's (Layard et al., 2005).

•Walrasian CGE models generally assume that the interest rate adjust to balance savings and investments. This view has been criticized by Keynes and (neo-)Keynesian economists in particular because the equality between savings and investments is an accounting identity that is always satisfy (Lambsdorff, 2011). Moreover it provides an unrealistic representation of the functioning modern economies where the Central Bank has empirically a strong influence on the interest rates. In a modern economy, the credit supply is not only defined by the accumulated savings. Retail banks can supply more credit than their available deposit by borrowing from the CB. This is the modern form of money creation. As lender of last resort, the CB can influence the level of the interest rate by adjusting its interest rates on the money it lends to or borrows from the retail banks. Using the neo-Keynesian framework, ThreeME assumes that the interest rate is set by CB according to a Taylor rule (Taylor, 1993).

Third, and as corollary, **investment is not constrained by savings**. Whereas most Walrasian CGE models are static models, some of them introduce a recursive dynamic (see e.g. Dixon and Rimmer, 2002; Diao et al., 2012; Dixon et al., 2013) where the savings of the previous period define investment, capital and *in fine* production. The perfect flexibility of prices guarantees the Say's law (Say, 1836): a product always finds its demand. Supply is always at full production capacity. A policy increasing public spending or public investment has a negative impact on the economy: it has an eviction effect on private consumption and investment because it is not possible to increase production. This hypothesis is contradicted by stylized empirical facts: in reality most companies choose not to produce at their full production capacity. The capacity utilization ratio fluctuates historically around 80 percent [4]. This suggest that the optimal stock of capital is higher than the level that is limiting for production. It is therefore possible to increase the level of production without having to increase the stock of capital immediately as assumed in Walrasian CGE models. A neo-Keynesian CGE model as ThreeME do not assume full utilisation of available production factors whether this comes from spare production capacities or from unemployment. By including slow adjustment on capital and labor, the link imposed by the production function between the levels of labor and capital and the level of production is more a long term optimal relation than a strict constraint. This gives room for a positive multiplier effect of an increase in public spending. The eviction effect is limited and spread over time. It comes from the inflation pressure generated by a higher utilization of the available production factors.

Fourth, ThreeME includes **forward-looking** features:

•We assume that the anticipations are a mix of adaptive ("backward-looking") and rational ("forward-looking") expectations. Mixing forward and backward-looking ex-

[4] See for instance the historical data for various US industries: `https://fredblog.stlouisfed.org/2019/01/capacity-utilization/` or `http://myf.red/g/lI8W`

pectations has become a standard hypothesis in the neo-Keynesian literature since the failure of the purely forward-looking models to explain the persistence of inflation (Fuhrer and Moore, 1995a,b; Fuhrer, 1997).

•In order to account for inter-temporal maximization behaviour of certain consumers, the desired saving rate may depend positively on the real interest rate as predicted by the permanent income hypothesis (Friedman, 1957) or the life cycle model (Modigliani, 1966) and positively on the Government's debt ratio as predicted by the Ricardian equivalence (Ricardo, 1888; Buchanan, 1976; Barro, 1974, 1979).

Neo-Keynesian vs. New Keyesian DSGE models

With the above features, ThreeME is a neo-Keynesian model. It is not a so-called "new Keynesian" model as defined in the Dynamic Stochastic General Equilibrium (DSGE) literature (see e.g. Clarida et al., 1999; Galí, 2008; Goodfriend and King, 1997; Goodfriend, 2007; Woodford, 2011). In particular, ThreeME does not assume a functioning of the economy with perfect information (about the future) and with perfectly rational and forward-looking agents. Although forward-looking features can be included, the role of adaptive expectation, adjustment constraints and not clearing prices is predominant in the model. DSGE models allows for elegant theoretical statements about the inter-temporal optimum of an economy with perfect information and perfectly rational individuals. But this is at the cost of strong and unrealistic assumptions compared to the actual functioning of the economy and the actual rationality of individuals. They have been widely criticised in the recent years for their lack of practical applications (see e.g. Mankiw, 2006; Solow, 2010; Romer, 2015; Stiglitz, 2018). Moreover these assumptions leads to models that are more of classical than Keynesian influence. In this respect the terminology of "new Keynesian" used to qualify DSGE models looks improper.

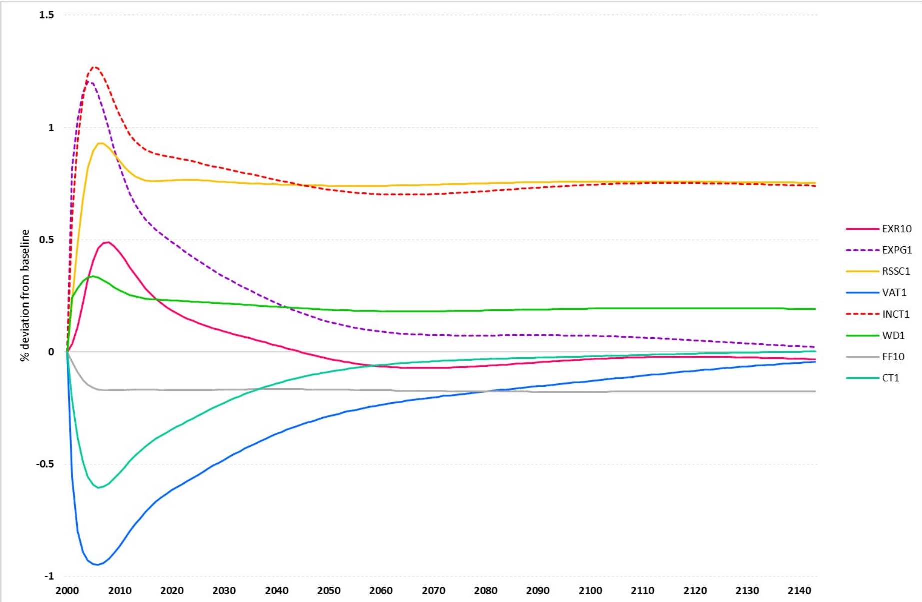
2.1.2 Long term properties

In the long run, ThreeME converge toward a steady states where all variables grow at a constant rate. In the long run, the model is neo-classical in the sense of Solow (Solow, 1956). The long term is driven by the supply constrain. All adjustment processes are achieved: there is no error of anticipation and the effective quantities coincide with the optimal ones. The prices are fully adjusted and all markets are in equilibrium. The unemployment reaches its structural level. The economy thus converges toward a stable equilibrium growth path à la Solow (1956) where all real variables grow at the same rate defined as the sum of the growth rate of the technical progress and of the population. Therefore per capita real variables grow at the same rate as the technical progress. All prices grow at the rate of inflation which is defined by the exogenous rate of inflation in the rest of the world. The endogenous dynamic of the model is determined by capital accumulation, the specification of the anticipation and of the adjustment dynamic.

Chapter 4 (p. 51) describes and discusses the long term properties of the model.

Figure 2.2 below provides an overview illustration of these long term dynamics through the GDP reactions to different shocks (for more details see Figure 4.1 and the corresponding section 4.1).

Figure 2.2: Evolution of the GDP depending on various shocks



2.1.3 Main interactions

The overall structure of the model is schematized in Figure 2.3.

In the short term, ThreeME has the main characteristics of a standard neo-Keynesian macroeconomic AS-AD model in an open economy. An important one is that demand determines supply. The demand is composed of (intermediate and final) consumption, investment and export whereas the supply comes from imports and the domestic production. As a feed-back with eventually some lags, the supply affects the demand through several mechanisms. The level of production determines the quantity of inputs used by the firms and thus the quantity of their intermediate consumptions and investment which are two components of the demand. It determines the level of employment as well and consequently the households final consumption. Another effect of employment on demand goes through the wage setting via the unemployment rate which is also determined by the active population. The active population is mainly determined by exogenous factors such as demography but also by endogenous factors: because of discouraged worker effects, the unemployment rate may affect the labor participation rate and thus the active population.

The unemployment rate is an important determinant of the wages dynamic which is defined by a Phillips or a WS curve. The inflationary property of the model is determined by the feedbacks between wages, production cost and prices. Prices

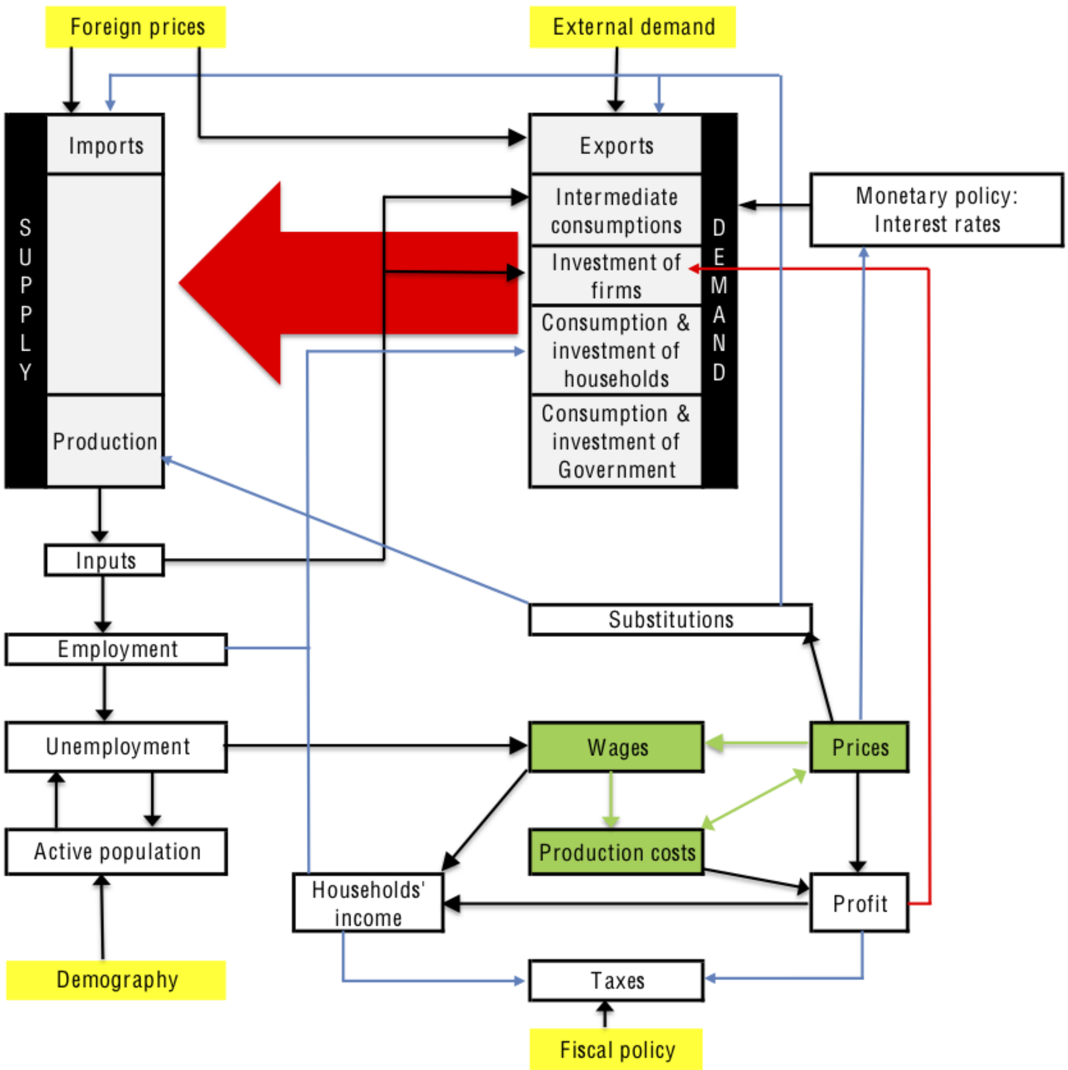


Figure 2.3: Overall structure of ThreeME

are assumed to adjust slowly to their optimum level that corresponds to a mark-up over marginal costs. In the short term, the mark-up accounts for the tensions between production capacities (supply) and demand, which is a standard market property. Consequently, wages, which affect production costs, affect directly prices. Prices have in return an impact on wages because they are indexed on (consumer price) inflation. Production costs are also directly affected by prices via the cost of intermediate consumptions and of investment.

This dynamic between wages, costs and prices affects the demand through several canals. Wages affect the household consumption because they are an important part of their income. Prices and costs affect profits and therefore households' consumption thought dividends and investment thought the sectors' debts level.

Another canal is the monetary policy which is defined by a Taylor rule. The Central Bank determines the interest rate level based on inflation and unemployment. This has an effect on the demand via the negative effect of the real interest rate on consumption and investment. Thus, the interest rate is not determined by the equilibrium between investment and saving as assumed in Walrasian CGE models (see e.g. Shoven and Whalley, 1992).

The dynamic of prices is the driver of the substitution mechanisms of the model. The evolution of relative prices between imported and domestic goods defines the import share in the supply of each commodity. The evolution of relative prices between types of goods and services defines the structure of consumption of the economy. Importantly for the analysis of environmental and energy policies, the prices dynamics defines the substitutions between production factors (in particular between capital and energy) and the share of each energy and transport into (intermediate and final) consumptions.

2.2 Adjustment mechanisms

Unlike Walrasian models that assume that equality between supply and demand is achieved through a perfect flexibility of prices and quantities, ThreeME represents more realistically the functioning of the economy by taking into account explicitly the slow adjustment of prices and quantities (factors of production, consumption). In this neo-Keynesian framework, permanent or transitory underemployment equilibria are possible and supply is determined by demand. ThreeME assumes that the actual levels of prices and quantities gradually adjust to their notional level. The notional level corresponds to the optimal (desired or target) level that the economic agent in question (the producer for prices and the demand for production factors, the household for consumption, the Central bank for the interest rate, etc.) would choose in the absence of adjustment constraints. These constraints mainly come from adjustment costs, physical or temporal boundaries and uncertainties.

Formally, we assume that the adjustment process and expectations for prices and quantities are represented by the following equations:

$$\log X_t = \alpha^{0,X}\ \log X_t^n + \left(1 - \alpha^{0,X}\right)\ \left(\log X_{t-1} + \Delta \left(\log X_t^e\right)\right) \tag{2.2.1}$$

$$\Delta\left(\log X_t^e\right) = \alpha^{1,X}\ \Delta\left(\log X_{t-1}^e\right) + \alpha^{2,X}\ \Delta\left(\log X_{t-1}\right) + \alpha^{3,X}\ \Delta\left(\log X_t^n\right) + \alpha^{4,X}\ \Delta\left(\log X_{t+1}\right) \tag{2.2.2}$$

Where X_t is the effective value of a given variable (e.g. the production price, labor, capital, etc.), X_t^n is its notional level, X_t^e its expected (anticipated) value at period t and $\alpha^{i,X}$ are the adjustments parameters, with:

$$\alpha^{1,X} + \alpha^{2,X} + \alpha^{3,X} + \alpha^{4,X} = 1 \tag{2.2.3}$$

Equation 2.2.1 assumes a geometric adjustment process. Taking into account the anticipations guaranties that the effective variables converge to their notional levels in the long run. Equation 2.2.2 assumes that the anticipations are a mix of adaptive ("backward-looking") and rational ("forward-looking") expectations. The backward-looking component corresponds to the lagged variables. The forward-looking component includes the variable in $t+1$ and its notional level. Indeed, if the notional price increases it is rational to expect that the effective price will also increase in the future. Mixing forward and backward-looking expectations has become a standard hypothesis in the neo-Keynesian literature since the failure of the forward-looking neo-Keynesian Phillips curve to explain the persistence of inflation (Fuhrer and Moore, 1995a,b; Fuhrer, 1997).

One can see that equations 2.2.1 and 2.2.2 can be reformulated into an Error Correction Model (ECM) used in the econometric estimations to take into account the non-stationary propriety of some variables:

$$\Delta\left(\log X_t\right) = \lambda_1\ \Delta\left(\log X_{t-1}\right) + \lambda_2\ \Delta\left(\log X_t^n\right) - \lambda_3\ \log\left(\frac{X_{t-1}}{X_{t-1}^n}\right) \tag{2.2.4}$$

Because of this equivalence, ECM econometric estimations from the literature can be used to calibrate the parameters of the adjustment processes.

For this, the following constraints must hold:

$$\left\{\begin{array}{l} \alpha^{0,X} = \lambda_3 \\ \alpha^{1,X} = 0 \\ \alpha^{2,X} = \frac{\lambda_1}{(1-\lambda_3)} \\ \alpha^{3,X} = \frac{(\lambda_2-\lambda_3)}{(1-\lambda_3)} \\ \alpha^{4,X} = 0 \end{array}\right. \tag{2.2.5}$$

We also assume that the substitution effects $SUBST_t^X$ adjust slowly to the notional substitution effects $SUBST_t^{X,n}$:

$$SUBST_t^X = \alpha^{6,X}\ SUBST_t^{X,n} + \left(1-\alpha^{6,X}\right)\ SUBST_{t-1}^X \tag{2.2.6}$$

The three equations above allow a rich set of adjustment as they integrate different types of rigidity (on prices and quantities, on expectations and on substitution mechanisms). For illustrative purposes, we present the full specification of the demand for Labor L . The notional demand of Labor L_s^n is derived by minimizing production costs. It depends positively on the level of the output Y_s, negatively on the productivity $PROG_{L,s}$ and on an element gathering all the substitution phenomena with the other production factors $SUBST_s^L$:

$$\Delta\left(\log L_s^n\right) = \Delta\left(\log Y_s\right) - \Delta\left(\log PROG_{L,s}\right) + \Delta\left(SUBST_s^L\right) \tag{2.2.7}$$

$$\begin{aligned}\Delta\left(SUBST_s^{L,n}\right) = &- ES_{L,K,s}\ \varphi_{K,s,t-1}\ \Delta\left(\log\frac{C_{L,s}}{PROG_{L,s}} - \log\frac{C_{K,s}}{PROG_{K,s}}\right)\\ &- ES_{L,E,s}\ \varphi_{E,s,t-1}\ \Delta\left(\log\frac{C_{L,s}}{PROG_{L,s}} - \log\frac{C_{E,s}}{PROG_{E,s}}\right)\\ &- ES_{L,M,s}\ \varphi_{M,s,t-1}\ \Delta\left(\log\frac{C_{L,s}}{PROG_{L,s}} - \log\frac{C_{M,s}}{PROG_{M,s}}\right)\end{aligned} \quad (2.2.8)$$

Where $ES_{L,K,s}$, $ES_{L,E,s}$,$ES_{L,M,s}$ are the elasticity of substitution between labor and the other production factors respectively capital, energy, material (i.e. non-energy intermediate consumption). φ_K, φ_E, φ_M are respectively the capital, energy and materials shares in the production costs. C_K, C_L, C_E, C_M are respectively the unitary costs of production of capital, labor, energy and material. The next section provides more information on the derivation of factors demands. Finally, the adjustment mechanisms being defined according to the equations 2.2.1, 2.2.2 and 2.2.6, the three following relationships are used:

$$\log L_s = \alpha_s^{0,L}\ \log L_s^n + \left(1-\alpha_s^{0,L}\right)\ \left(\log L_{s,t-1} + \Delta\left(\log L_s^e\right)\right) \quad (2.2.9)$$

$$\Delta\left(\log L_s^e\right) = \alpha_s^{1,L}\ \Delta\left(\log L_{s,t-1}^e\right) + \alpha_s^{2,L}\ \Delta\left(\log L_{s,t-1}\right) + \alpha_s^{3,L}\ \Delta\left(\log L_s^n\right) \quad (2.2.10)$$

$$SUBST_s^L = \alpha_s^{6,L}\ SUBST_s^{L,n} + \left(1-\alpha_s^{6,L}\right)\ SUBST_{s,t-1}^L \quad (2.2.11)$$

Minimizing an adjustment cost function

The specification of the price and quantity equations assume that firms gradually adjust their effective price and quantity to the optimal level. Adjustment models provide a microeconomic foundation for the specification of the adjustment process we have selected by deriving the adjustment process from an optimal behavior. Initially developed by Eisner and Strotz (1963) and Gould (1968), these models were extended by several authors including Lucas (1967), Schramm (1970) and Treadway (1971). We derive below the static case which deliver the same adjustment equation derived from the inter-temporal model proposed by Rotemberg (1982b,a).

Adjustment models generally assume a continuous relation between the objective function Γ of the agent (i.e. the profit for a firm, the utility for the consumer, the Government or the Central Bank) and the variable X controlled by the agent (i.e. price, input demand, consumption, interest rate, tax rate, etc): $\Gamma_t\left(X_t\right)$. The notional (i.e. optimal or desired) profit (or utility) is then written $\Gamma_t\left(X_t^n\right)$ where n denotes the level desired by firms. At the neighborhood of the optimum, the second-order approximation of the difference between the effective and desired profits is written:

$$\Gamma_t\left(X_t\right)-\Gamma_t\left(X_t^n\right)=\Gamma_t^{'}\left(X_t^n\right)\ \left(X_t-X_t^n\right)+\Gamma_t^{''}\left(X_t^n\right)\ \left(X_t-X_t^n\right)^2$$

The profit being maximum for X_t^n, $\Gamma_t^{'}\left(X_t^n\right)=0$ and $\Gamma_t^{''}\left(X_t^n\right)<0$. As a first approximation, the adjustment cost, i.e. the loss of profit suffered by a company that is not in the optimum, is therefore:

$$C_D=\Gamma_t\left(X_t^n\right)-\Gamma_t\left(X_t\right)=C_D\ \left(X_t-X_t^n\right)^2$$

Where:

$$C_D=-\Gamma_t^{''}\left(X_t^n\right)$$

Suppose that the adjustment cost is proportional to the square of the speed of adjustment:

$$C_A=c_A\ \left(X_t-X_{t-1}\right)^2$$

Where:

$$c_A>0$$

Minimizing the total cost function ($C_t=C_D+C_A$) is equivalent to solving:

$$C_t^{'}\left(X_t\right)=2\ c_D\ \left(X_t-X_t^n\right)+2\ c_A\ \left(X_t-X_{t-1}\right)=0$$

The condition of the second order $\left(C_t^{''}\left(X_t\right)>0\right)$ being always verified, the optimal adjustment which minimizes the total cost has the following dynamic process:

$$X_t=\alpha\ X_t^n+(1-\alpha)\ X_{t-1}$$

With:

$$\alpha=\frac{c_D}{\left(c_D+c_A\right)}$$

With this simple model, the average adjustment time is:

$$\frac{\alpha}{(1-\alpha)}=\frac{c_D}{c_A}.$$

The adjustment is slower when the adjustment cost c_A is high compared to the cost of non being adjusted c_D.

2.3 Production function and demand for production factors

The production structure is decomposed into three levels (see Figure 2.4). The first one assumes a production function with 4 inputs (or production factors), often referred as KLEM (capital, labor, energy and materials). The first level has a fifth element: the transport and commercial margins. Stricto sensu, they cannot be considered as production factors since they intervene after the production process. Thus they are not substitutable with the production factors. But they are closely related to the level of production since once a good has been processed, it has to be transported and commercialized. At the second level, capital, energy, material and margins aggregates are further decomposed by type of commodities (e.g. energy sources). At the third level, the demand for each factor or margin is either imported or produced domestically. The demands for production factors are derived from the minimization of the firms production costs. We assume a production function with constant returns-to-scale more general than the CES (Constant Elasticity of Substitution) insofar as substitution elasticities may differ between different inputs pair (Reynès, 2019).

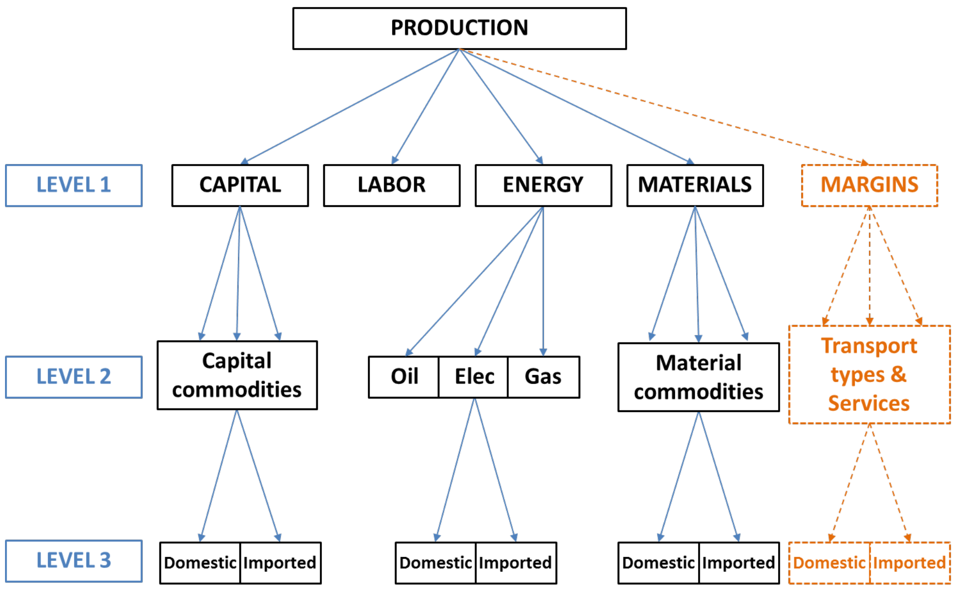


Figure 2.4: Production structure

2.3.1 Demand for production factors

The production costs minimization program leads to the following equations for the notional factors demand:

$$\Delta\left(\log F^{n}_{f,s}\right) = \Delta\left(\log Y_{s}\right) - \Delta\left(\log PROG_{f,s}\right) + \Delta\left(SUBST^{F}_{f,s}\right) \tag{5.4.8}$$

$$\Delta\left(SUBST^{F,n}_{f,s}\right) = \sum_{ff} -ES_{f,ff,s}\,\varphi_{ff,s,t-1}\,\Delta\left(\log\frac{C_{f,s}}{PROG_{f,s}} - \log\frac{C_{ff,s}}{PROG_{ff,s}}\right) \tag{5.4.9}$$

with

$$\varphi_{f,s} = \frac{C_{f,s}\,F^{n}_{f,s}}{\sum_{ff} C_{ff,s}\,F^{n}_{ff,s}} \tag{5.4.10}$$

and

$$f, ff = \{K, L, E, M\}$$

Where $F^n_{f,s}$ is the notional demand of input f ($KLEM$), $ES_{f,ff,s}$ the elasticity of substitution between the pairs of inputs f and ff, $PROG_{f,s}$ the technical progress related to input f, $C_{f,s}$ the cost/price of input f and Y_s the level of production of the sector under consideration. According to national accounts data, ThreeME assumes that each commodity may be produced by more than one sector. For instance, electricity can be produced by several sectors such as nuclear or wind power sectors. The production of each sector is defined by the following equations:

$$Y_{c,s} = \varphi^{Y}_{c,s}\, YQ_c \tag{5.4.7}$$

$$Y_s = \sum_c Y_{c,s} \tag{5.2.109}$$

Where YQ_c is the aggregated domestic production of commodity c. It is determined by the demand (intermediate and final consumption, investment, public spending, exports and stock variation). $PhiY_{c,s}$ is then the share of commodity c produced by the sector s (with $\sum_s phiY_{c,s} = 1$) and Y_s is the aggregated production of sector s.

2.3.2 Nested CES production function

In general equilibrium modeling and econometric studies, it is common to have an additional input: material (that is non energy intermediary consumption). This leads to a production technology with 4 inputs: capital (K), labor (L), energy (E) and material (M): also known as $KLEM$.Figure 2.5 shows a commonly used nested structure with 4 inputs. At the first level, material (M) can be substituted with the aggregate capital/energy/labor (KEL) with an ES of $\sigma^{NEST^{MAT,KEL}}$. At the second level, the aggregate capital/energy (KE) is a substitute to labor (L) with an ES of $\sigma^{NEST^{KE,L}}$. At the third level, capital (K) can be substituted to energy (E) with an ES of $\sigma^{NEST^{K,E}}$.

$$ES_{K,E,s} = \frac{\eta_s^{NEST^{K,E}}}{(1 - \varphi_{MAT,s} - \varphi_{L,s}) - \frac{\eta_s^{NEST^{MAT,KEL}}}{(1-\varphi_{MAT,s}) - \eta_s^{NEST^{KE,L}} \frac{\frac{\varphi_{L,s}}{1-\varphi_{MAT,s}}}{1-\varphi_{MAT,s}-\varphi_{L,s}}}} \tag{5.15.7}$$

$$ES_{K,L,s} = \frac{\eta_s^{NEST^{KE,L}} - \eta_s^{NEST^{MAT,KEL}} \varphi_{MAT,s}}{1 - \varphi_{MAT,s}} \tag{5.15.9}$$

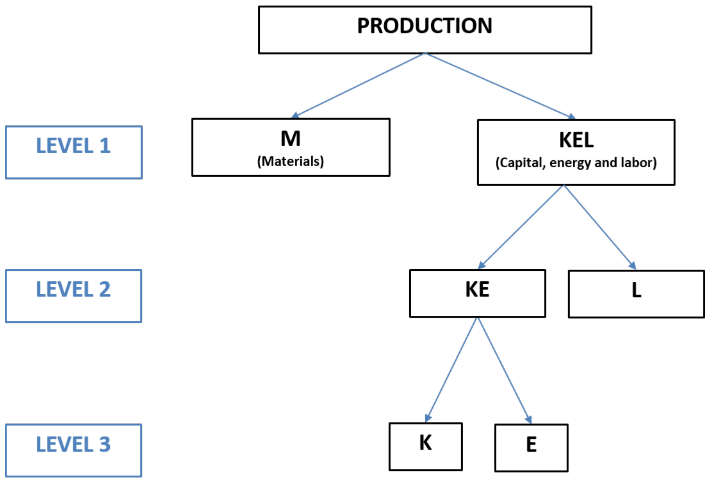


Figure 2.5: Nested CES production function

2.3.3 Investment and capital

Investment in ThreeME depends on the anticipated production, on its past dynamic, on substitution phenomena and on a correction mechanism, which guaranties that companies reach their level of long-term notional capital stock. The stock of capital is deducted from the investment according to the standard capital accumulation equation:

$$\begin{aligned}\Delta\left(\log IA_s\right) = {} & \alpha_s^{IA,Ye}\ \Delta\left(\log Y_s^e\right) + \alpha_s^{IA,IA1}\ \Delta\left(\log IA_{s,t-1}\right) \\ & + \alpha_s^{IA,SUBST}\ \Delta\left(SUBST_{K,s}^F\right) + \alpha_s^{IA,Kn}\ \left(\log F_{K,s,t-1}^n - \log F_{K,s,t-1}\right)\end{aligned} \tag{5.11.7}$$

$$F_{K,s} = \left(1-\delta_s\right)\ F_{K,s,t-1} + IA_s \tag{5.11.6}$$

Where IA is the investment, Y_s^e the anticipated production, $F_{K,s}$ and $F_{K,s}^n$ the actual and notional stocks of capital, $SUBST_{K,s}^F$ a variable gathering substitution phenomena between capital and the other inputs, and δ_s the depreciation rate of capital. Moreover, we impose the constraint $\alpha_s^{IA,IA1} + \alpha_s^{IA,Ye} = 1$ in order to guaranty the existence of the stationary equilibrium path.

This specification is a compromise between the short-term dynamics empirically observed and the consistency of the model in the long run. Like the E-MOD or MESANGE econometric models (Chauvin et al., 2002; Klein and Simon, 2010; Dufernez et al., 2017) or DSGE model (Cardani et al., 2019; Campagne and Poissonnier, 2018; Smets and Wouters, 2003), it is common to estimate an investment equation rather than capital stock equation for several reasons. Firstly, time series capital stock data are often unreliable. Secondly, this approach better represents the short-term dynamics of investment. In particular, it avoids capital destruction phenomena (negative investment) that are in practice unusual, since companies generally prefer to wait for the technical depreciation of their installed capital. Unlike E-MOD or

MESANGE, we assume in addition that investment depends on the difference between the actual and notional capital stock. This element ensures that the effective capital stock converges over time towards its notional level. In the long-term, the model is then consistent with the production function theory that establishes a relationship between the levels of production and capital stock (and not with the flow).

2.4 Price setting

The production price for each sector is set by applying a mark-up over the unit cost of production (which includes labor, capital, energy and other intermediate consumption costs) :

$$PY_s^n = CU_s^n \ (1 + \mu_s) \tag{5.3.1}$$

Where PY^n is the notional price, CU_s^n the unitary cost of production and μ_s is the effective mark-up.

$$\mu_s = \alpha_s^{\mu} \ \mu_s^n + (1 - \alpha_s^{\mu}) \ \mu_{s,t-1} \tag{5.11.1}$$

Notional mark-up of the sector$s$ (specification 1)

$$\Delta \left(\log \left(1 + \mu_s^n\right)\right) = \rho^{\mu,Y} . \Delta \left(\log CUR_s\right) \tag{5.3.2}$$

$$CUR_s = \frac{Y_s}{YCAP_s} \tag{5.3.5}$$

$$\begin{aligned} \Delta \left(\log YCAP_s\right) = & \sum_f \varphi_{f,s,t-1} \ \Delta \left(\log \left(F_{f,s} \ PROG_{f,s}\right)\right) \\ & + \alpha_s^{YCAP,Y} \ \left(\log Y_{s,t-1} - \log \left(YCAP_{s,t-1} \ CUR_{s,t_0}\right)\right) \end{aligned} \tag{5.3.4}$$

Notional mark-up of the sector s (specification 2)

$$\Delta \left(\log \left(1 + \mu_s^{n2}\right)\right) = \rho^{\mu,Y} . \left(\Delta \left(\log Y_s\right) - \Delta \left(\log Y_{s,t-1}\right)\right) \tag{5.3.3}$$

$$(1 + \mu_c) = \frac{PYQ_c \ YQ_c}{\sum_s CU_s \ Y_{c,s}} \tag{5.3.6}$$

Where Y_s is the level of production, μ_s^n and μ_s^{n2} are respectively the default notional specification and alternative notional specification mark-up. CUR_s is the capacity utilisation ratio and $YCAP_s$ is the production capacity that is defined by the production function and the effective quantities of inputs.

The default notional mark-up is a positive function of the capacity utilization ratio. One can notice that this specification is strictly equivalent to:

$$\Delta \left(\log 1 + \mu_s^n\right) = \rho^{\mu,Y} . \left(\Delta \left(\log Y_s\right) - \Delta \left(\log YCAP_s\right)\right) \tag{2.4.1}$$

Which is quite close to the alternative specification where the production capacity $YCAP_s$ is replaced by the past production $Y_{s,t-1}$.

The equation of notional price is a behavioral equation: by assuming that the addressed demand to a firm is a negative function of its price, one can easily demonstrate that the optimal price corresponds to a mark-up over the marginal cost of production. The mark-up equation reflects the fact that the returns-to-scale are decreasing in the short-term. Therefore, a non-expected increase in production results into a higher marginal cost of production and therefore into a higher notional price.

The other prices are calculated according to their accounting definition and are therefore (directly or indirectly) a function of the producer price. The price of the domestically produced commodity c is a weighted average of the production prices of activities (indexed by s) producing that commodity. For example, the price of electricity is a weighted average of the production prices of the sectors producing electricity. The price paid by the final user (consumer, government, sector, rest of the world) integrates in addition the commercial and transportation margins, and the taxes net from subsidies. Combined with the price of imports, we get the average price for each commodity paid by each end user.

2.5 Wage setting

Several studies have shown that the theoretical arguments and empirical estimates difficultly allow choosing between the two specifications. However, this difference of specification has important implications on the definition of the equilibrium unemployment rate or $NAIRU$ (Non Accelerating Inflation Rate of Unemployment) and thus on the inflationary dynamic and the long-term properties of a macroeconomic model (L'horty and Thibault, 1998; Le Bihan and Sterdyniak, 1998; Blanchard and Katz, 1999; Chagny et al., 2002; Reynès, 2006, 2010). See also the simulations in chapter 3. In ThreeME, we choose a general specification that includes the Phillips and Wage Setting (WS) curves. It assumes that the notional nominal wage (W_s^n) positively depends on the anticipated consumption price (P^e) and on the labor productivity ($PROG_s^L$), and negatively on the unemployment rate (UnR):

$$\begin{aligned} \Delta\left(\log W_s^n\right) = {} & \rho_s^{W,Cons} + \rho_s^{W,P}\ \Delta\left(\log P\right) + \rho_s^{W,Pe}\ \Delta\left(\log P^e\right) \\ & + \rho_s^{W,PROG}\ \Delta\left(\log PROG_s^L\right) - \rho_s^{W,U}\ \left(UnR - DNAIRU\right) \\ & - \rho_s^{W,DU}\ \Delta\left(UnR\right) + \rho_s^{W,L}\ \Delta\left(\log F_{L,s} - \log F_L\right) \end{aligned} \tag{5.3.7}$$

This relation can alternatively be identical, either to the Phillips curve, or to the WS curve depending on the value of the selected parameters (Heyer et al., 2007; Reynès, 2010). In the case of a Phillips curve, the level of the unemployment rate influences the growth rate of wages. The variation of the unemployment rate may also influence the growth rate of wages because of hysteresis phenomena (Blanchard and Summers, 1986). With a Phillips curve, the $NAIRU$ is:

$$NAIRU = \frac{\rho_s^{W,Cons} + (1 - \rho_s^{W,P} - \rho_s^{W,Pe}) * \Delta (\log P) + (1 - \rho_s^{W,PROG}) * \Delta \left(\log PROG_s^L\right)}{\rho_s^{W,U}} \tag{2.5.1}$$

The $NAIRU$ is predetermined in the long run when wages are fully indexed on price and productivity. When the model is calibrated at the steady state, the $NAIRU$ is simply equal to the base year unemployment rate, which may not be realistic. To have a long run "Time-Varying" $NAIRU$, we can assume that the constant is a function of the unemployment rate as sometime estimated in the literature about the "Time-Varying" $NAIRU$ (Gordon, 1997; Gordon and Stock, 1998; Heyer et al., 2007; Gordon, 2013; Watson, 2014) :

$$d(\rho_s^{W,Cons}) = \rho_s^{Cons,U} * \rho_s^{W,U} * d(UnR) \tag{2.5.2}$$

Heyer and Timbeau (2002) estimate that the value of $\rho_s^{Cons,U}$ is between 0.8 and 0.9 in the case of France.

In the case of a WS curve, the level of the unemployment rate does not influences the growth rate of wages ($\rho_s^{W,U} = 0$). Only its variation does. Therefore the link between wages and the unemployment rate is generally reformulated in level. For the model to have a consistent steady-state in the long-run, the WS curve must also impose the constraints of the wage equation introduced by Layard et al. (2005) in the 1980's, a unit indexation of wages on prices and productivity and no temporal trend:

$$\rho_s^{W,P} + \rho_s^{W,Pe} = \rho_s^{W,PROG} = 1, \rho_s^{W,Cons} = 0 \tag{2.5.3}$$

2.6 Interest rate

In the neo-Keynesian framework, it is standard to assume that the interest rate is set by the Central Bank (CB) according to a Taylor rule (Taylor, 1993). The real interest rate is a positive function of inflation and a negative function of unemployment rate (here used as a proxi of the output gap):

$$\Delta (R^n) = \rho^{Rn,Cons} + \rho^{Rn,P}.\Delta \left(\frac{\Delta (P)}{P_{t-1}}\right) - \rho^{Rn,UnR}.\Delta (UnR) \tag{5.3.13}$$

This function can be derived from the minimization of a quadratic loss function of the CB (see e.g. Friedman and Woodford, 2010). The loss function increases when inflation and unemployment deviate from the target of the CB.

2.7 Households' consumption

We assume that the notional (optimal) aggregate households consumption corresponds to a share of their current income:

$$CH^{n,VAL} = DISPINC^{AT,VAL}.(1 - MPS^n) \tag{5.5.6}$$

Where $CH^{n,VAL}$ is the aggregate notional households final consumption expressed in value, $DISPINC^{AT,VAL}$ is the households disposable income and MPS^{n} their notional marginal propensity to save.

Most theoretical models on consumption behaviours suggest that the optimal marginal propensity to save is endogenous (see e.g. Romer, 2016). (Neo-)Keynesian theories put forward the importance of savings for precautionary motive suggesting that the desired saving rate depends positively on the unemployment rate: when the probability to loose your job increases, households tend to increase their savings as often confirmed by empirical evidence (see e.g. Carroll et al., 1992, 2003) [5]. (Neo-)classical theories puts forward the inter-temporal maximization behaviour of the consumer: the optimal save rate depends positively on the real interest rate as predicted by the permanent income hypothesis (Friedman, 1957) or by the life cycle model (Modigliani, 1966) and positively on the Government's debt ratio as predicted by the Ricardian equivalence (Ricardo, 1888; Buchanan, 1976; Barro, 1974, 1979). These considerations leads to the following general specification of the notional marginal propensity to save:

$$\Delta\left(MPS^{n}\right)=\rho^{MPS,R}.\Delta\left(R-\frac{\Delta\left(P\right)}{P_{t-1}}\right)+\rho^{MPS,UnR}.\Delta\left(UnR\right) \qquad (5.5.7)$$

The so-called new Keynesian models or DSGE models generally assume an Euler consumption equation where the current consumption depends positively on the future consumption and negatively on the expected future real interest rate (see e.g. Clarida et al., 1999; Galí, 2008; Goodfriend and King, 1997; Goodfriend, 2007; Woodford, 2011). This specification corresponds to the optimal consumption derived from the neoclassical consumption model introduced by (Fisher, 1930). Contrary to our specification, in these models, consumption is not related to the current income, but the inter-temporal budget constraint because the consumer is assumed to choose its present and future consumption that maximize its lifetime satisfaction. We choose not to adopt such a specification in ThreeME for several reasons:

•Such an inter-temporal optimum is based on the very strong assumption that households do not face borrowing constraints and that they have a perfect control on the way they can allocate their consumption over time. This view is not realistic in particular for low incomes.

•Empirical evidences from behavioral economics contradict the assumption that consumer are perfectly rational forward-looking individuals (see e.g. Angeletos et al., 2001; Rabin, 1998; Camerer et al., 2005). Consumption appears more sensitive to movements in income than the permanent income hypothesis predicts. Borrowing behaviour of people using credit cards with very high interest rates is not inter-temporally rational. Studies analysing enrolment in pension systems provide the most compelling evidence. It appears that the default design proposed has a big impact on how individuals participate retirement plans. This is inconsistent with an in inter-temporally optimizing consumer.

[5]Notice that the actual saving rate may still be negatively correlated to the unemployment rate because of the slow adjustment of consumption to its notional level. The increase in unemployment leads to a decrease in income. Because of consumption habits, the decrease in consumption is generally lower than the decrease in income. This leads to a decrease of the actual saving rate.

•In addition to be unrealistic to represent consumer behavior, the neo-classical assumption is technically more difficult to solve especially for for a multi-sector model like ThreeME. Therefore our specification has the advantage of being more general since it combines neo-Keynesian and neo-classical consumption theory while been analytically more tractable.

As shown in the next sections, ThreeME include three model variants for the allocation of the aggregate consumption between the different commodities.

2.7.1 Variant 1: LES utility function

In the standard version of the model, consumption decisions between commodities are modeled through a Linear Expenditure System (LES) utility function generalized to the case of a non-unitary elasticity of substitution between the commodities (Brown and Heien, 1972). A LES specification assumes that a share of the base year consumption (NCH_c) is incompressible and therefore the relation between income and consumption is not linear. This specification allows for the distinction between the consumption of necessity and luxurious goods:

$$(CH_c^n - NCH_c)\ PCH_c = \varphi_c^{MCH}\ \left(CH^{n,VAL} - PNCH.NCH\right) \tag{5.5.8}$$

Where CH_c^n corresponds to the volume of notional households' final consumption in commodity c and PCH_c to its price. NCH_c is the incompressible volume of consumption in commodity c, $CH^{n,VAL}$ is the aggregate notional households final consumption expressed in value, $DISPINC^{AT,VAL}$ is the households disposable income and MPS^n their notional marginal propensity to save.

In the case of no incompressible expenditures ($NCH_c = 0$), households aim at allocating a share φ_c^{MCH} of their total consumption (in value), $DISPINC^{AT,VAL}$ $(1-MPS^n)$, to commodity c. This share is constant if the elasticity of substitution between the commodities is equal to one (Cobb-Douglas assumption). In this case (Cobb-Douglas utility function without incompressible consumption), commodity c consumption stay exactly proportional to income. In the case of a CES function where the elasticity of substitution is σ^{LESCES}, the marginal propensity to spend varies depending on the relative prices according to the following specification:

$$\Delta\left(\log\varphi_c^{MCH}\right) = \left(1-\eta^{LESCES}\right).\Delta\left(\log\frac{PCH_c}{PCH^{CES}}\right) \tag{5.5.12}$$

$$PCH^{CES} = \left(\sum_c \varphi_{c,t_0}^{MCH}\ PCH_c^{\left(1-\eta^{LESCES}\right)}\right)^{\left(\frac{1}{\left(1-\eta^{LESCES}\right)}\right)} \tag{5.5.14}$$

2.7.2 Variant 2: Nested utility function

In the basic version of the consumer block presented in Section 2.7.1, the consumption of every commodity is modeled using generalized LES utility function where the elasticity of substitution between every commodity is constant. An increase in the price of fossil energy increases the consumption of the other commodities uniformly whereas one expects that substitutions will mainly affect the other energy sources and households investment in more energy-efficient equipment. Figure 2.6 presents the nested utility function structure used in a second version of the model in order to better take into account energy related substitutions for households.

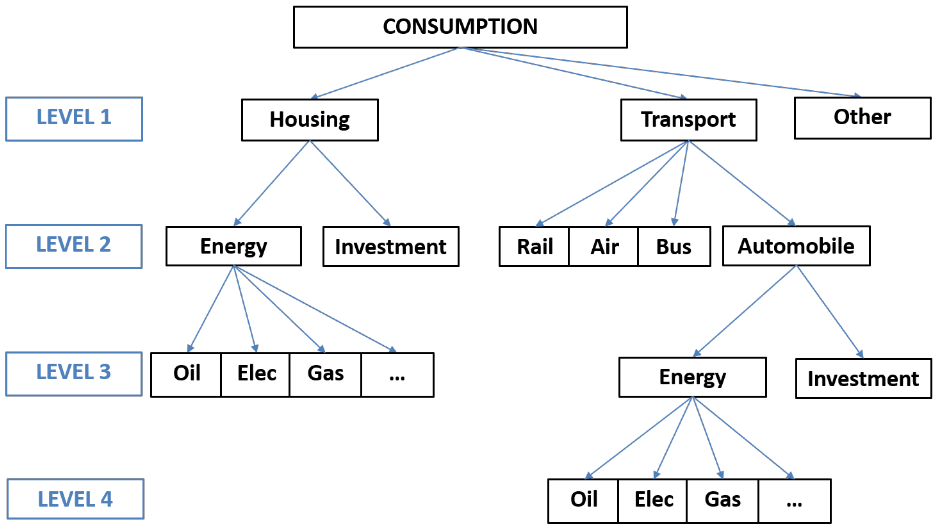


Figure 2.6: Nested utility structure for households

The first level separates the consumption related to housing and transport from the rest. We assume that the value shares of the three types of expenditures (housing, transport and other) are constant which amount to assuming an Elasticity of Substitution (ES) of one (Cobb-Douglas hypothesis).

In the other levels of the nest a CES utility function is assumed where the ES may be defined between 0 and infinity. At the second level of the nest, housing expenditures are disaggregated between energy expenditures and investment expenditures. Transport expenditures are disaggregated between transport types. The substitution possibilities between the other commodities is still modeled with a LES function as presented in Section 2.7.1

At the third level of the nest, energy housing related expenditures are disaggregated between energy types. Automobile expenditures are disaggregated between energy expenditure and investment expenditure. At the fourth level of the nest, energy transport related expenditure are disaggregated between energy types.

2.7.3 Variant 3: Hybrid approach

The standard representation of the consumer maximization behavior used in most top-down CGE models assumes that energy sources provide utility on their one. Therefore their consumption is more or less proportional to their revenue because of the hypothesis of nested utility function.

But in reality energy has no use in itself. As theoretically formalized by Lancaster (Lancaster, 1966a,b) and applied in some hybrid models (Laitner and Hanson, 2006), households buy energy to fulfill certain services such as housing (heating and the functioning of equipments) or transport. Therefore the quantity of energy consumed for heating purposes is more related to the size of the house than directly to the revenue of households. Of course, rich households generally have bigger houses and therefore their energy consumption will generally be higher. At the same time, one can expect that the energy consumption per square meters will be lower for poor households since they are generally more careful in trying to limit their energy bill. Indeed, micro data suggest that the poor households tend to lower heating temperature in their houses. The energy consumption per square meter tends to increase with the income decile but within limit since energy is more a necessity good than a luxury good. And hardly no one wants a heating temperature in their house of 35 C even very rich people.

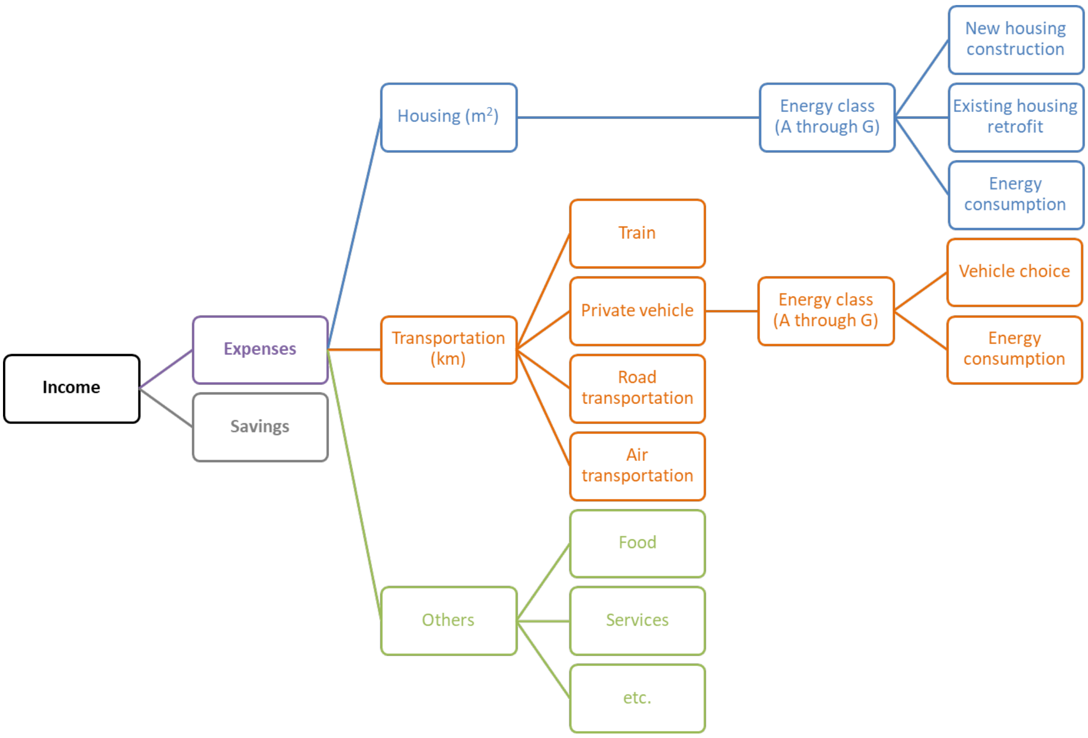


Figure 2.7: Household’s structure of expenditures

One way to model this is to assume that the production of certain services such housing and mobility is directly produced by households rather than purchase externally. Therefore we specify explicitly a household's production function for the services of housing and mobility (Figure 2.7). The block has two main components: housing, transport. We assume further that the expenses related to this production function are priority.

For the other expenditures, we use a standard LES which allows to model in a simple way the distinction between necessity/luxury goods (See section 2.7.1) or a nested utility function (See section 2.7.2). In this section, we only present the key equations of our hybrid modeling of households block. The complete block is presented in chapter 5 whereas all the notations used are defined in the glossary of terms used.

2.7.3.1 Building stock dynamic

We differentiate housing according to their energy efficiency by class, $ecl = \{1, ..., ecl\}$. If $ecl > ecl2$, the class ecl has a better energy performance than the class $ecl2$: its energy consumption per m^2 is lower. We assume that housing stock by class ecl expressed in m^2 is defined by the following dynamics:

$$\begin{aligned} \Delta\left(BUIL_{ecl}\right) = NewBUIL_{ecl} + \sum_{ecl2} REHAB_{ecl2,ecl} + \sum_{ecl2} DEP^{BUIL}_{ecl2,ecl} \\ - \left(\sum_{ecl2} REHAB_{ecl,ecl2}\right) - \left(\sum_{bcl} DEP^{BUIL}_{ecl,bcl}\right) \end{aligned} \tag{5.16.12}$$

Where, $BUIL_{ecl}$ is the building stock of the energy efficiency class ecl, $NewBUIL_{ecl}$ is the new building constructed according to class ecl, $REHAB_{ecl2,ecl}$ is the buildings rehabilitated from class $ecl2$ to ecl (with $ecl2 < ecl$ and $REHAB_{ecl,ecl} = 0$) and $DEP^{BUIL}_{ecl2,ecl}$ is the depreciation (or downgrading) building from class $ecl2$ to class ecl (with $ecl2 > ecl$). Equation 5.16.12 assumes that at each period the stock of buildings of class ecl:

- Increase by the new buildings constructed acoording to class ecl: $NewBUIL_{ecl}$
- Increase by the amount of rehabilitated buildings from a lower class $ecl2$ to class ecl: $\sum_{ecl2} REHAB_{ecl2,ecl}$
- Increase by the downgraded buildings from a higher classe to class ecl: $\sum_{ecl2} DEP^{BUIL}_{ecl2,ecl}$
- Decrease by the amount of rehabilitated building from ecl to a higher class $ecl2$: $\sum_{ecl2} REHAB_{ecl,ecl2}$
- Decrease by the downgraded buildings from class ecl to lower class: $\sum_{bcl} DEP^{BUIL}_{ecl,bcl}$, where $classbcl$ refers to destroyed building

We assume for simplicity that the number of buildings are related to the size of the population:

$$\Delta\left(\log BUIL\right)=\Delta\left(\log POP\right)+\Delta\left(\log M2perCapita\right) \tag{5.16.9}$$

Equation 5.16.12 and 5.16.9 are dynamically consistant since they imply that:

$$\sum_{ecl} BUIL_{ecl} = BUIL \tag{2.7.1}$$

Provided this is verified and correctly calibrated for the initial period.

To provide a better intuition, the stock dynamic is charted in Figure 2.8. Blue arrows represent the depreciation mechanism. As time goes along, high energy classes loses efficiency and gets downgraded until they gets eventually destructed (pool $BUIL_{bcl}$). As in the model, this chart presents the general case where the downgrading is possible to any lower class. In reality, this process is generally gradual and buildings of high classes will go successively to lower classes instead of been directly destructed. Orange arrows represent the rehabilitation mechanism: by investing in renovation, households have the possibility to increase the energy efficiency of their house. Here too various transitions are possible, e.g. from class D to C, then from C to B or directly from class D to A. Naturally, the strongest the rehabilitation, the higher the cost. Finally, black arrows represent the (re)-construction process. There are new buildings because the total housing park increases ($\Delta BUIL$) and because destroyed buildings ($BUIL_{bcl}$) are reconstructed. Here as well, although new buildings are possible in any category, in practice, new construction follows high energy efficiency standards.

At each period, a proportion of the buildings of category ecl is rehabilitated:

$$\tau_{ecl}^{REHAB} = \frac{\sum_{ecl2} REHAB_{ecl,ecl2}}{BUIL_{ecl}} \tag{2.7.2}$$

This proportion may not be constant over time. For instance, it may increase as the energy price increases because this gives an incentive toward more energy efficiency renovation. This can be modeled by assuming that τ_{ecl}^{REHAB} is endogeous and depends on the user cost of the building. Variation in that τ_{ecl}^{REHAB} may also be exogenous due to the imposition of stricter energy efficiency requirements.

These considerations lead to the following specification:

$$\tau_{ecl}^{REHAB,N} = \tau_{ecl}^{REHAB,MAX} + \frac{\tau_{ecl}^{REHAB,MIN} - \tau_{ecl}^{REHAB,MAX}}{1+e^{\tau_{ecl}-\sigma_{ecl}\,Payback_{ecl}^{REHAB}}} \tag{5.16.38}$$

$$Payback_{ecl}^{REHAB} = \frac{UC_{ecl}^{K,REHAB}\,BUIL_{ecl}^{D} - UC_{ecl}^{K}\,BUIL_{ecl}^{D}}{UC_{ecl}^{E} - UC_{ecl}^{E,REHAB}} \tag{5.16.41}$$

As explained previously, the rehabilitation of a building of a given class ecl can be done to different higher classes. It would be logical to assume that the choice between the higher classes is endogenous and depends on the relative cost of each option of renovation. However, because of the lack of data, it is difficult to model and calibrate this arbitrage. Moreover, this choice may be strongly determined by technical renovation standards with a small influence of relative prices. Therefore, we assume that

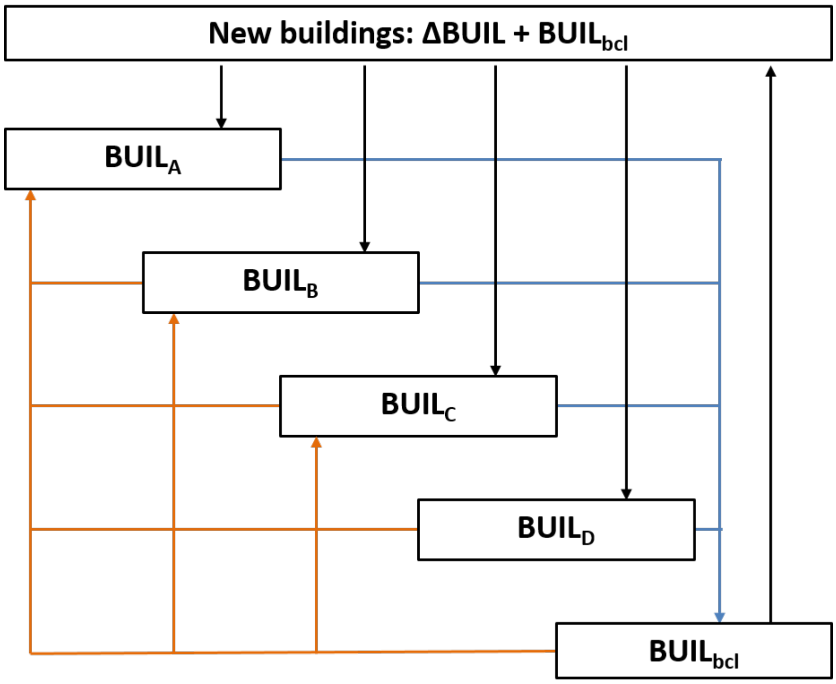


Figure 2.8: Building structure

this choice is exogenous, that is the share of class *ecl* buildings rehabilitated to class $ecl2$ ($\varphi^{REHAB}_{ecl,ecl2}$) is exogenous:

$$REHAB_{ecl,ecl2} = \varphi^{REHAB}_{ecl,ecl2}\ \tau^{REHAB}_{ecl}\ BUIL_{ecl,t-1} \tag{5.16.16}$$

In Equation 5.16.38, we assume that the proportion of the buildings of category *ecl* to be rehabilitated depends on the user cost of buildings. We assume that the latter corresponds to the annual cost of the investment ($UC^{REHAB}_{h,k}$) which consists of two components: (1) the annual cost of the investment itself including eventual interests ($UC^{K\_REHAB}_{h,k}$), (2) annual energy cost ($UC^{E\_REHAB}_{h,k}$). This leads to the following relation:

$$UC^{REHAB}_{ecl} = UC^{K,REHAB}_{ecl} + UC^{E,REHAB}_{ecl} \tag{5.16.47}$$

$$UC^{E,REHAB}_{ecl} = \sum_{ecl2} \varphi^{REHAB}_{ecl,ecl2}\ UC^{E}_{ecl2} \tag{5.16.48}$$

Regarding the decision of rehabilitating the house to a higher class, the above user cost is compared to the user cost of a building remaining in class *ecl*:

$$UC_{ecl} = UC^{K}_{ecl} + UC^{E}_{ecl} \tag{5.16.53}$$

The annual investment and energy costs are defined by the following equations:

$$UC^{K,REHAB}_{ecl} = \sum_{ecl2} \varphi^{REHAB}_{ecl,ecl2} \, UC^{K,REHAB}_{ecl,ecl2} \tag{5.16.49}$$

$$UC^{K,REHAB}_{ecl,ecl2} = \left(1 - R^{SUB}_{ecl,ecl2}\right) \left(\frac{PREHAB_{ecl,ecl2}}{REHAB^{D}_{ecl2} \left(R^{CASH,REHAB}_{ecl} + \frac{R^{LOAN,REHAB}_{ecl} \, R^{I,REHAB}_{ecl,t-1} \, LD^{REHAB}_{ecl}}{1-\left(1+R^{I,REHAB}_{ecl,t-1}\right)^{\left(-LD^{REHAB}_{ecl}\right)}} \right)} \right) \tag{5.16.50}$$

$$R^{LOAN,REHAB}_{ecl} = 1 - R^{CASH,REHAB}_{ecl} \tag{5.16.51}$$

$$UC^{K}_{ecl} = \left(\frac{PREHAB_{ecl,ecl}}{BUIL^{D}_{ecl}} \right) \left(R^{CASH}_{ecl} + \frac{R^{LOAN}_{ecl} \, R^{I,BUIL}_{ecl,t-1} \, LD_{ecl}}{1 - \left(1 + R^{I,BUIL}_{ecl,t-1}\right)^{(-LD_{ecl})}} \right) \tag{5.16.54}$$

$$R^{LOAN}_{ecl} = 1 - R^{CASH}_{ecl} \tag{5.16.55}$$

$$UC^{E}_{ecl} = PENER^{m2}_{ecl} \left(\frac{\left(\left(1 + g^{PENER^{m2,e}}_{ecl}\right)^{\left(BUIL^{D}_{ecl}\right)} \right) - 1}{g^{PENER^{m2,e}}_{ecl} \, BUIL^{D}_{ecl}} \right) \tag{5.16.56}$$

$$PENER^{m2}_{ecl} = PENER^{BUIL}_{ecl} \, \frac{ENER^{BUIL}_{ecl}}{BUIL_{ecl}} \tag{5.16.57}$$

$$g^{PENER^{m2,e}}_{ecl} = \alpha^{g_1^{PENER,m2,e}} g^{PENER^{m2}}_{ecl,t-1} + \left(1 - \alpha^{g_1^{PENER,m2,e}}\right) g^{PENER^{m2,e}}_{ecl,t-1} \tag{5.16.58}$$

Where R^{CASH}_{ecl} is the share of investment that is paid cash, R^{LOAN}_{ecl} the share of investment that is paid with a loan, R^{I}_{ecl} the interest rate, LD_{ecl} the duration of the loan and $PENER^{m2}_{ecl}$ the energy price per square meter paid in class *ecl* buildings.

Expenditures related to housing for buildings *ecl* at a given period therefore includes the expenses related to debt (interest and reimbursement) the investment paid in cash and the cost of energy:

Housing expenditures in class ecl buildings (in value):

$$
\begin{aligned}
EXP_{ecl}^{HOUSING,Val} = \Big(& DEBT_{ecl,t-1}^{REHAB,Val} \left(R_{ecl,t-1}^{I,REHAB} + R_{ecl,t-1}^{RMBS,REHAB} \right) \\
& + R_{ecl}^{CASH,REHAB} \; PREHAB_{ecl} \; REHAB_{ecl} \\
& + DEBT_{ecl,t-1}^{NewB,Val} \left(R_{ecl,t-1}^{I,NewBUIL} + R_{ecl,t-1}^{RMBS,NewBUIL} \right) \\
& + R_{ecl}^{CASH,NewBUIL} \; PNewBUIL_{ecl} \; NewBUIL_{ecl} \\
& + PENER_{ecl}^{BUIL} \; ENER_{ecl}^{BUIL} \Big)
\end{aligned} \tag{5.16.61}
$$

Debt related to the rehabilitation of a class ecl building (in value):

$$
\begin{aligned}
DEBT_{ecl}^{REHAB,Val} = & \left(1 - R_{ecl}^{RMBS,REHAB}\right) \; DEBT_{ecl,t-1}^{REHAB,Val} \\
& + R_{ecl}^{LOAN,REHAB} \; PREHAB_{ecl} \; REHAB_{ecl}
\end{aligned} \tag{5.16.59}
$$

Debt related to the purchase of a new building of a class ecl (in value):

$$
\begin{aligned}
DEBT_{ecl}^{NewB,Val} = & \left(1 - R_{ecl}^{RMBS,NewBUIL}\right) \; DEBT_{ecl,t-1}^{NewB,Val} \\
& + R_{ecl}^{LOAN,NewBUIL} \; PNewBUIL_{ecl} \; NewBUIL_{ecl}
\end{aligned} \tag{5.16.60}
$$

The investment price for rehabilitation and new building is indexed on the consumer price for commodity construction of building ($CCON$):

$$\varDelta\left(\log PREHAB_{ecl,ecl2}\right) = \varDelta\left(\log PCH_{CCON}\right) \tag{5.16.4}$$

$$\varDelta\left(\log PNewBUIL_{ecl}\right) = \varDelta\left(\log PCH_{CCON}\right) \tag{5.16.3}$$

2.7.3.2 Automobile and Transport

The arbitrage between transport modes is defined in 5 level. Level 1 determine the evolution of air, long distance and short distance transport. In level 2, the consumer may substitute between long distance transport types (automobile and train) and short distance transport types (automobile and bus). Level 3 defines the arbitrage between automobile class whereas level 4 defines the arbitrage between electric and thermic automobile. In level 5, substitution between types of thermic automobiles may be introduced.

•Level 1: Air, long distance and short distance transport

We assume that transport needs in level 1 are mainly driven by an income effect and a price effect. Therefore the number of traveler-km for each type of transport increase with income and decrease with transport prices:

$$\begin{aligned}\Delta\left(\log km^{traveler}_{cair}\right) &= \theta^{kmtravcair^{DISPINC}}.\Delta\left(\log\frac{DISPINC^{AT,VAL}}{PCH}\right)\\ &\quad - \theta^{kmtravcair^{Pkmtrav}}.\Delta\left(\log\frac{P^{km,traveler}_{cair}}{PCH}\right)\end{aligned} \tag{5.17.6}$$

$$\begin{aligned}\Delta\left(\log km^{traveler,LD}\right) &= \theta^{kmtravLD^{DISPINC}}.\Delta\left(\log\frac{DISPINC^{AT,VAL}}{PCH}\right)\\ &\quad - \theta^{kmtravLD^{Pkmtrav}}.\Delta\left(\log\frac{P^{km,traveler,LD}}{PCH}\right)\end{aligned} \tag{5.17.7}$$

$$\begin{aligned}\Delta\left(\log km^{traveler,SD}\right) &= \theta^{kmtravSD^{DISPINC}}.\Delta\left(\log\frac{DISPINC^{AT,VAL}}{PCH}\right)\\ &\quad - \theta^{kmtravSD^{Pkmtrav}}.\Delta\left(\log\frac{P^{km,traveler,SD}}{PCH}\right)\end{aligned} \tag{5.17.9}$$

•Level 2: Substitution between long distance (automobile and train) and short distance transport types (automobile and bus)

$$\begin{aligned}\Delta\left(\log km^{trav,auto,LD}\right) &= \Delta\left(\log km^{traveler,LD}\right) + \theta^{kmtrav^{autoLD,crai}}.\Big(1\\ &\quad - \varphi^{km^{trav,auto,LD}}_{t-1}\Big)\ \Delta\Big(\log P^{km,trav,auto,LD}\\ &\quad - \log P^{km,traveler}_{crai}\Big)\end{aligned} \tag{5.17.12}$$

$$\begin{aligned}\Delta\left(\log km^{traveler}_{crai}\right) &= \Delta\left(\log km^{traveler,LD}\right)\\ &\quad - \theta^{kmtrav^{autoLD,crai}}.\varphi^{km^{trav,auto,LD}}_{t-1}\ \Delta\Big(\log P^{km,traveler}_{crai}\\ &\quad - \log P^{km,trav,auto,LD}\Big)\end{aligned} \tag{5.17.13}$$

$$\begin{aligned}\Delta\left(\log km^{trav,auto,SD}\right) &= \Delta\left(\log km^{traveler,SD}\right) - \theta^{kmtrav^{autoSD,croa}}.\Big(1\\ &\quad - \varphi^{km^{trav,auto,SD}}_{t-1}\Big)\ \Delta\left(\log P^{km,trav,auto,SD}\right.\\ &\quad \left.- \log P^{km,traveler}_{croa}\right)\end{aligned} \tag{5.17.14}$$

$$\Delta\left(\log km_{croa}^{traveler}\right) = \Delta\left(\log km^{traveler,SD}\right) - \theta^{kmtrav^{autoSD,croa}} . \varphi_{t-1}^{km^{trav,auto,SD}} \Delta\left(\log P_{croa}^{km,traveler} - \log P^{km,trav,auto,SD}\right) \tag{5.17.16}$$

•From level 2 to level 3

$$\Delta\left(\log km^{AUTO,LD}\right) = \Delta\left(\log km^{trav,auto,LD}\right) - \Delta\left(\log travperauto^{LD}\right) \tag{5.17.17}$$

$$\Delta\left(\log km^{AUTO,SD}\right) = \Delta\left(\log km^{trav,auto,SD}\right) - \Delta\left(\log travperauto^{SD}\right) \tag{5.17.18}$$

$$km^{AUTO} = km^{AUTO,LD} + km^{AUTO,SD} \tag{5.17.19}$$

$$\Delta\left(\log AUTO\right) = \Delta\left(\log km^{AUTO}\right) - \Delta\left(\log kmPerAuto\right) \tag{5.17.20}$$

$$NewAUTO = \Delta\left(AUTO\right) + AUTO_{DES} \tag{5.17.22}$$

•Level 3: Arbitrage between automobile class

In level 3, we define the arbitrage between automobile class.

$$NewAUTO_{ecl} = \varphi_{ecl}^{NewAUTO}\ NewAUTO \tag{5.17.23}$$

$$\Delta\left(U_{ecl}^{AUTO}\right) = -\theta^{U^{AUTO,UC,K}} . \Delta\left(UC_{ecl}^{K,AUTO}\right) - \theta^{U^{AUTO,UC,E}} . \Delta\left(UC_{ecl}^{E,AUTO}\right) \tag{5.17.24}$$

$$SUM^{exp,U,AUTO} = \sum_{ecl} e^{U_{ecl}^{AUTO}} \tag{5.17.25}$$

$$\varphi_{ecl}^{NewAUTO^{n}} = \frac{e^{U_{ecl}^{AUTO}}}{SUM^{exp,U,AUTO}} \tag{5.17.26}$$

•Level 4: Arbitrage between electric and thermic automobile per classes

$$NewAUTO_{ecl,cele} = \varphi_{ecl,cele}^{NewAUTO}\ NewAUTO_{ecl} \tag{5.17.28}$$

$$\frac{\Delta\left(\varphi_{ecl,cele}^{NewAUTO^{n}}\right)}{\left(1 - \varphi_{ecl,cele,t-1}^{NewAUTO^{n}}\right)} = innovation_{ecl}^{exo} + innovation_{ecl} + imitation_{ecl}\ \varphi_{ecl,cele,t-1}^{NewAUTO^{n}} \tag{5.17.30}$$

In order to account for future expected innovation in the electric vehicles, We adopt a mechanism of innovation as defined by the Bass diffusion model (Bass, 1969). This model represent the dynamic of innovation as a function of the utility for consumer to switch vehicles type (between thermic and electric propulsion) associated to the fuels price.

$$\begin{aligned}&\Delta\left(innovation_{ecl}\right)\\&=\eta^{BASS}_{ecl}\ \Delta\left(\frac{\left(2.UC^{AUTO}_{ecl,cele}\right)^{\left(-\nu^{diffusion}_{ecl}\right)}}{\left(\left(2.UC^{AUTO}_{ecl,cele}\right)^{\left(-\nu^{diffusion}_{ecl}\right)}\right)+\left(\left(UC^{AUTO}_{ecl,th}\right)^{\left(-\nu^{diffusion}_{ecl}\right)}\right)}\right)\end{aligned} \tag{5.17.31}$$

$$NewAUTO_{ecl,th}=NewAUTO_{ecl}-NewAUTO_{ecl,cele} \tag{5.17.32}$$

2.8 Foreign trade

Exports are determined by the external demand addressed to domestic products and the ratio between the export and world prices:

$$\Delta\left(\log X_c\right)=\Delta\left(\log WD_c\right)+\Delta\left(SUBST^{X}_{c}\right) \tag{5.7.25}$$

$$\Delta\left(SUBST^{X,n}_{c}\right)=-\eta^{X}_{c}\ \Delta\left(\log PX_c-\log\left(EXR.PWD_c\right)\right) \tag{5.7.26}$$

Where WD_c is the world demand, PWD_c its price. PX_c is the export price that depends on the production costs and which reflects the price-competitiveness of the domestic products. EXR is the exchange rate; σ^{X}_{c} is the price-elasticity (assumed constant).

We assume imperfect substitution between domestic and imported goods (Armington, 1969). The demand for domestic and imported products is :

$$AD_c=\left(1-\varphi^{AM}_{c}\right)\ A_c \tag{2.8.1}$$

$$\varphi^{AM}_{c}=\frac{1}{\left(1+\frac{AD_c}{AM_{c,t0}}\ \exp SUBST^{AM}_{c}\right)} \tag{2.8.2}$$

$$SUBST^{AM}_{c}=\alpha^{6,AM}_{c}\ SUBST^{AM,n}_{c}+\left(1-\alpha^{6,AM}_{c}\right)\ SUBST^{AM}_{c,t-1} \tag{2.8.3}$$

$$\Delta\left(SUBST^{AM,n}_{c}\right)=-\sigma^{AM}_{c}\ \Delta\left(\log PAD_c-\log PAM_c\right) \tag{2.8.4}$$

$$AM_c=\varphi^{AM}_{c}\ A_c \tag{2.8.5}$$

Where A_c represents the demand for each type of use (intermediary consumption, investment, consumption, public spending, exports, etc.). AM_c and AD_c are the imports and the domestic products demanded for each type of use A, PAM_c and PAD_c are their respective prices. φ_c^{AM} is the import share of commodity c for each type of use A . The elasticity of substitution σ_c^{AM} by type of use A of a given commodity c can potentially be different, which allows a high degree of flexibility.

2.9 Government

According to the French national accounts, public administrations refer to the central and regional government services and social security administration. In ThreeME, we have aggregated these three components in order to focus on transfers between public administrations, household and sectors. These transfers are accounted for in the government's resources $INC^{G,VAL}$ and expenditures $SPEND^{G,VAL}$:

$$\begin{aligned} INC^{G,VAL} = {} & PNTAXC.NTAXC + NTAXS^{VAL} \\ & + INC^{SOC,TAX,VAL} + PRSC.RSC + PROP^{INC,G,VAL} \end{aligned} \tag{5.6.20}$$

With:

- The aggregate net taxes on commodity c expressed in value: $PNTAXC.NTAXC$
- The aggregate net taxes on production of sectors expressed in value: $NTAXS^VAL$
- The income and social taxes expressed in value: $INS^{SOC,TAX,VAL}$
- The aggregate employers' social security contribution paid by sector: $PRSC.RSC$
- The property income of the Government expressed in value: $PROP^{INC,G,VAL}$

$$SPEND^{G,VAL} = PG.G + SOC^{BENF,VAL} + DEBT_{t-1}^{G,VAL} \left(\varphi_{t-1}^{RD^G} + r_{t-1}^{DEBT,G}\right) \tag{5.6.22}$$

with:

- The government final consumption: $PG.G$
- The Social benefits expressed in value: $SOC^{BENF,VAL}$
- The interest paid by the government on its debt plus the share of debt reimbursed every year: $DEBT_{t-1}^{G,VAL} \left(\varphi_{t-1}^{RD^G} + r_{t-1}^{DEBT,G}\right)$

2.10 Greenhouse gases emissions

In France, the anthropogenic CO_2 emissions represent about 70% of the total gross greenhouse gases (GHG). They come from the burning of fossil fuels and decarbonation process. The modeling of the demand for fossil energy in ThreeME is detailed by economic agent and by sector for each energy source (coal, oil and gas). This allows for a precise estimation of the variation in the national CO_2 emissions. The calculation of emissions level consists in multiplying the fossil energy demand by the corresponding emission coefficients. These coefficients are specific for each economic actor, each sector and each energy sources depending on their carbon intensity.

In other words, the CO_2 emissions due to the combustion of fossil energy by sectors and households are proportional to the quantity of fossil fuel energy consumed. They are therefore calculated according to the following equations:

Emissions of the greenhouse gas ghg related to the intermediary consumption of commodity c by sector s

$$\Delta\left(\log EMS^{CI}_{ghg,c,s}\right) = \Delta\left(\log\left(CI_{c,s}\ IEMS^{CI}_{ghg,c,s}\right)\right) \tag{5.9.1}$$

where $IEMS^{CI}_{ghg,c,s}$ is the corresponding emission intensity calibrated to 1 in the base year. It may change over time because of the increase of the share of biofuels.

Emissions of the greenhouse gas ghg related to the household consumption c

$$\Delta\left(\log EMS^{CH}_{ghg,c}\right) = \Delta\left(\log\left(CH_{c}\ IEMS^{CH}_{ghg,c}\right)\right) \tag{5.9.4}$$

Emissions of the greenhouse gas ghg related to the final production of sector s.(which mainly corresponds to the emissions from agriculture).

$$\Delta\left(\log EMS^{Y}_{ghg,s}\right) = \Delta\left(\log\left(Y_{s}\ IEMS^{Y}_{ghg,s}\right)\right) \tag{5.9.3}$$

CO_2 emissions from decarbonation during the production process for the non mineral metallic products, as the glass or ceramic sectors, is assumed proportional to the quantity of intermediate raw material used in the production process:

$$\Delta\left(\log EMS^{MAT}_{ghg,s}\right) = \Delta\left(\log\left(F_{MAT,s}\ IEMS^{MAT}_{ghg,s}\right)\right) \tag{5.9.2}$$

2.11 Energy balance

Incorporating an energy block following the standards used for the construction of an energy balance in a macroeconomic model is tricky because certain definitions and concepts used in energy balance differ from the ones used in national account data. Eurostat publishes a database on Physical Energy Flow Accounts (PEFA) that aims to be consistent with the national accounts [6]. PEFA are based on energy balance data but several corrections are implemented in order to improve the consistency with the monetary flows national accounts data. One important correction is related to definition of the economy. Like the national accounts, PEFA follow the resident principle: they records all energy flows associated with activities of resident units regardless where these activities actually take place geographically. On the contrary, energy balance statistics are based on the territory principle: they records the energy flows that occurred in a given country regardless the economic residence of the agent (consumer, energy and non-energy sectors). While the production of energy products is recorded widely in the same way in both systems, the use of energy for transport is recorded differently. For instance, foreign trucks' diesel consumption is excluded of the national consumption in PEFA and considered as an export. On the contrary it is considered as national consumption in the energy balance statistics.

For the consistency, the energy block of ThreeME should be based on PEFA instead of the energy balance statistics. But the use of PEFA has also drawbacks. Some data are incomplete and the corrections made to the energy balance statistics are not straightforward. PEFA are available only for European countries which limits the generality of the module when used for other countries. Moreover, energy scenarios are generally developed by energy models based on energy balance statistics. ThreeME simulation should be consistent with these scenarios. For transparency reason, we have chosen to calibrate our energy block on energy balance statistics. PEFA are used to improve the sectoral detail of the energy balance.

2.11.1 Energy balance versus economic equilibrium

In order to facilitate the link with the rest of the model and the national accounts definitions, the energy module of ThreeME is based on a reformulation of the energy balance into meaningful economic concepts. The energy balance published by Eurostat uses the following definition for energy supply [7]:

- Total Primary Energy Supply (TPES) = Primary production + Imports - Exports + Change in stock (energy balance definition)

In order to simplify the notations, *Primary production* refers to the primary *indigenous* production including the *recovered and recycled products. Exports* include the *international maritime bunkers* and *international aviation* which are actually not considered in ThreeME.

This concept of primary supply is different from the concept of supply used in national accounts. Here an increase of exports reduces the energy supply of the country. In the national accounts, exports are considered as a use (or a demand) and their increase increases the supply of the country. This is consistent with an economic definition of supply since exports increases the activity of the producing country instead of reducing it.

[6] See Physical Energy Flow Accounts (PEFA), Manual 2014, Draft, Eurostat, https://ec.europa.eu/eurostat/documents/1798247/6191537/PEFA-Manual-2014-v20140515.pdf

[7] See https://www.iea.org/statistics/resources/balancedefinitions/

Stock changes according to energy balance definition reflect the **difference between opening** stock levels at the first day of the year and **closing** levels on the last day of the year of stocks on national territory held by producers, importers, energy transformation industries and large consumers. A stock build is shown as a **negative** number, and a stock draw as a positive number. For this reason, the change in stock appears with a positive sign on the supply side. The national accounts provides a more intuitive definition of stock changes. It is defined as the **difference between closing** stock level on the last day of the year and the **opening** level at the first day of the year. A stock build is shown as a **positive** number, and a stock draw as a negative number. Therefore change in stock according to the national accounts definition has a positive sign but on the demand side. Assuming there are expressed in the same unit, the correspondence between the two definitions is:

- Change in stock (national accounts definition) = - Change in stock (energy balance definition)

In the energy balance, supply is equal to its use according to the following equation:

- TPES = Net energy transformation + Energy sector own use + Distribution losses + Final consumption

Where: Net energy transformation = Energy transformation input - Energy transformation output

The *energy transformation input* is the amount of energy used to produce another energy commodity. It is reported as an intermediary consumption in the national account (e.g. coal used to produce electricity). The *energy transformation output* is the amount of energy transformed from another energy commodity. It is reported as production in the national account (e.g. electricity produced from coal). The *net energy transformation* is negative for secondary energy like electricity or gasoline because the output of these commodities is always higher than its use as an input to produce other energies. On the contrary the net energy transformation is positive for primary energies like natural gas, coal or crude oil since they are not an output of the energy transformation process. At the aggregate level, net energy transformation measures the amounts of energy lost during the transformation process of energy commodities.

The concept of use of the energy balance (right-hand-side of the above equation) strongly differs from the economic definition. The *energy transformation output* decreases the use in the energy balance whereas it is actually a supply concept: it appears with a positive sign on the supply side in the national account. The distinction between primary and secondary products is irrelevant economically speaking. Both are the production economic sectors and therefore increase the supply.

Energy sector own use is the energy consumed by energy sectors themselves for heating, pumping, traction, and lighting purposes. The energy used for transformation purposes is therefore excluded. For instance, the fuel oil used for electricity generation or transformed by refineries is reported in *energy transformation input.* But the fuel oil used by the same refineries for their own heat needs is reported in *energy sector own use.* It is reported as an intermediary consumption in the national account.

Energy sector own use includes also the auto-consumption of energy sector: e.g. own electricity consumed by the electricity producers. These auto-consumptions are made without any financial transactions: they only reduce the (net) production sold to the network. In the national accounts, the Input-Output (IO) tables also record auto-consumption but only if a financial transaction has occurred. A typical example is when an power plant sells electricity to a distribution company. As the production and distribution of electricity are part of the same sector in the national accounts, these financial transactions are reported as auto-consumption in the IO tables. Because of this difference in definition, it is incorrect to relate the physical auto-consumption of electricity sectors (defined in the energy balance) to the monetary auto-consumption (defined in the national accounts).

The *distribution losses* are related to electricity and heat. For electricity, they include the losses in energy distribution, transmission and transport.

Final consumption is the last component of the use in the energy balance and can be decomposed as follows:

- Final consumption = Final energy consumption + Final non-energy consumption + Statistical differences between energy supply and energy use

The *statistical differences between energy supply and energy use* come from the fact the data for supply and consumption come from different sources.

Both the national accounts and the energy balance use the terminology *final consumption* but the definitions differ. In the energy balance, it includes the consumption of households and of economic sectors. In the national accounts, final consumption concerns only the consumption of households and of the government but not of economic sectors. The energy consumption of economic sectors is defined as *intermediary consumption*. Moreover, the national accounts do not make a distinction between the intermediary consumption of energy and non-energy sector whereas the energy balance does: in *energy sector own use* for energy sectors and in *final consumption* for non-energy sectors.

To sum up, *Energy intermediary consumptions* as defined in the national accounts correspond to three different concepts in the energy balance:

1. *Energy transformation input*
2. *Energy sector own use* **excluding** the auto-consumptions.
3. *Final consumption* of non-energy sectors (*final* in the sense of the energy balance).

In ThreeME, the substitutions mechanism between energy and the other inputs (in particular with capital), between energy sources, between imported and domestic products or between exported and foreign products are defined in the economics module of the models (producer, consumer, international trade). The variables of the energy balance are therefore defined by the economic modules. In order to facilitate the link between the energy balance in physical units to the national accounts equilibrium defined in ThreeME, we have to rewrite the energy balance consistently. For this, we first need to make a distinction between the *gross supply* and the *net* *supply* of an energy commodity. The *gross supply* includes the domestic and imported primary and secondary production (as defined in the energy balance):

- Gross supply = Primary production + Energy transformation output (secondary production) + Imports

The *gross supply* is not relevant economically for the end user because it includes distribution losses and auto-consumption. What is relevant for the end user is what she actually purchases, i.e. the domestic and imported supply net from distribution losses and auto-consumption:

- Net supply = Domestic net production + Imported net production = Gross supply - Distribution losses - Auto-consumption

With this definition, we can rewrite the energy balance in an economically consistent way where the net supply of an energy commodity should be equal to its end use:

- Net supply = Energy transformation input + Energy sector own use (excluding auto-consumption) + Final consumption + Exports + Change in stock (national account definition)

2.11.2 Main energy balance equations in ThreeME

The energy demand and supply equations are defined accordingly to the PEFA energy balance definition described above. The energy demand is segmented by type of end use between intermediate consumption (CI_{toe}), households final consumption (CH_{toe}) and exports (X_{toe})

$$\Delta\left(\log CI^{toe}_{ce,s}\right) = \Delta\left(\log CI_{ce,s}\right) \tag{5.10.2}$$

The net energy supply equals energy use and is consistent with the national account concept of production, that is the physical quantity actually bought by the end user.

$$Y^{toe}_{ce} + M^{toe}_{ce} = CI^{toe}_{ce} + CH^{toe}_{ce} + X^{toe}_{ce} \tag{5.10.5}$$

and each sector s produces a share of energy ce where $\varphi^{Y^{toe}}_{ce,s}$ is given by the energy balance PEFA accounts.

$$Y^{toe}_{ce,s} = \varphi^{Y^{toe}}_{ce,s}\ Y^{toe}_{ce} \tag{5.10.7}$$

Finally, this equation ensures that if the production price of energy ce is common amongst producers, the national account definition share is equal to the energy balance definition

$$\varphi^{Y}_{ce,s} = \frac{PY^{toe}_{ce,s,t_0}\ \varphi^{Y^{toe}}_{ce,s}}{\sum_{ss}\left(PY^{toe}_{ce,ss,t_0}\ \varphi^{Y^{toe}}_{ce,ss}\right)} \tag{5.10.9}$$

In order to account for the distribution losses and the auto-consumption which are not encompassed in the equation 5.10.5, we introduce another layer in the national accounting energy production definition which add these components. The gross production of energy ce by sector s expressed in physical units is:

$$YG^{toe}_{ce,s} = Y^{toe}_{ce,s} + DLY^{toe}_{ce,s} + AC^{toe}_{ce,s} \tag{5.10.10}$$

For simplicity, we assume that the distribution losses are strictly proportional to the end use of this commodity, such that:

$$\Delta\left(\log DL^{toe}_{ce}\right) = \Delta\left(\log\left(CI^{toe}_{ce} + CH^{toe}_{ce} + X^{toe}_{ce}\right)\right) \tag{5.10.12}$$

The distribution losses are then disaggregated among producers of energy ce proportionally to their share into the total domestic and foreign net production

$$DLY^{toe}_{ce,s} = DL^{toe}_{ce}\ \frac{Y^{toe}_{ce,s}}{(Y^{toe}_{ce} + M^{toe}_{ce})} \tag{5.10.13}$$

The auto-consumption mainly concerns the electricity consumption by electricity producers, which are differentiated by sector s in the equation, allowing for taking into account their heterogeneity in production technology

$$\Delta\left(\log AC^{toe}_{ce,s}\right) = \Delta\left(\log Y^{toe}_{ce,s}\right) \tag{5.10.15}$$

Chapter 3

Calibration

The calibration of ThreeME is based on a number of databases. A Computable General Equilibrium (CGE) model is typically calibrated on a Supply Use Table (SUT) as this contains the economic transactions between agents including governments, sectors and households. The ThreeME SUT is then combined with other national accounts data to close the household and government accounts. Energy and emissions data in physical units are added to extend the SUT to an environmentally extended SUT (EE-SUT). Finally, additional data is used to increase the number of energy related products and sectors.

The ThreeME database has the following characteristics as summarized in Table 3.1. These characteristics refer to the maximum level of detail in ThreeME. The actual level of detail as applied in case studies can differ from the level of detail as described in Table 3.1. For each simulation of ThreeME, the database is aggregated to meet the needs of the research question at hand.

The data calibration is done using a flexible tool developed in the R programming language. This tool has several advantages. First, all assumptions are fully traceable. It replicates the ThreeME database using only the raw data files in their original format as downloaded from their data source. Thus, all data processing assumptions can be found in the R code. Second, the tool has a flexible setup with built-in options for adjusting the data source, year, country, sectors and products. As a result the database can be customized for each ThreeME simulation in a relatively easy manner. The core of the calibration tool consists of eight main steps as depicted in Figure 3.1.

3.1 Step 1: Read in supply use tables and other data

The first step is to read in all raw data files. Currenlty, ThreeME has two database versions. The reason for this is that two different supply use databases are used, each with their own advantages and disadvantages. Database version 1 is built using WIOD SUTs (Timmer et al., 2015) whereas database version 2 is built using Eurostat SUTs. The advantage of WIOD data is its wider country coverage. It covers next

Table 3.1: ThreeME data characteristics

	ThreeME database v1	**ThreeME database v2**
Calibration year	2010	2015
Regions	EU-27 MS, AUS, BRA, CAN, CHN, IDN, IND, JPN, KOR, MEX, RUS, TUR	EU-28 MS
Currency	EUR	EUR
Sectors	86 sectors (35 NACE Rev 1.1 sectors + 22 energy sectors + 2 transport sectors + 27 other sectors)	125 sectors (65 NACE Rev 2 sectors + 56 energy sectors + 4 transport sectors)
Products	139 products (59 CPA2002 products + 41 energy products + 2 transport products + 37 other products)	89 products (65 CPA2008 products + 20 energy products + 4 transport products)
Energy products	45 energy products	20 energy products
GHG Emissions	CO2, CH4, N2O, SF6, HFC and PFC	CO2, CH4, N2O, NF3, SF6, HFC and PFC
Data sources	WIOD 2013 release, EXIOBASE v3, IPCC emission factors, IEA heat values and OECD national accounts	Eurostat supply use tables, OECD national accounts, IEA energy balances, electricity balances, Physical Energy Flow Accounts, IEA energy prices, IPCC emissions factors, IEA heat values and Structural Business Statistics

to the EU MS also a number of non European countries[1]. On the other hand, the main advantage of Eurostat data is the availability of more recent years. Also the continuation of the database in the future is guaranteed, as Eurostat data is the official national data, making it possible to update the ThreeME database to more recent years in the future. Depending on the purpose of the simulation of ThreeME, the calibration is done using a specific database version.

3.2 Step 2: Physical energy SUTs

In ThreeME, the energy in physical units is treated as a layer rather than an extension. In other words, the full supply and use table is available for energy in physical units by energy product and sector. To create this energy layer, several energy databases

[1] Also the WIOD data is trade linked, meaning that the different country SUTs are combined into one multi-regional SUT (MR-SUT). One of the advantages of a MR-SUT allows for footprint calculation, although it is not directly relevant for ThreeME model.

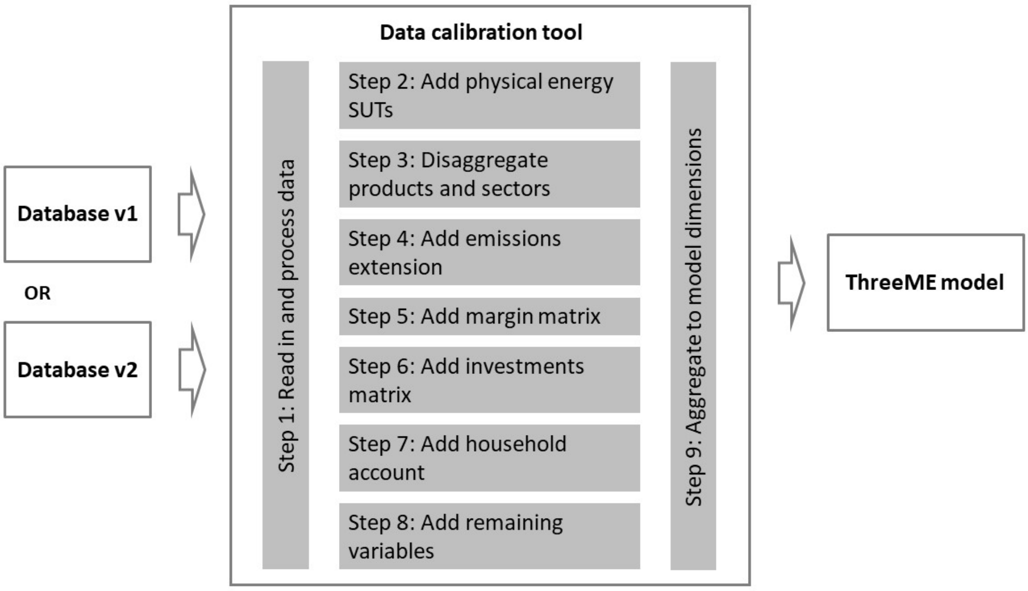


Figure 3.1: Schematic overview data calibration tool

are used, which are summarized in Table 3.2. Note that this energy layer is only available for the ThreeME database version 2.

The main data source is the IEA Energy Balances (European Union, 2017). The energy balances contain energy data expressed in TOE at 63 SIEC[2] energy product level. The data is provided for different energy flows, from energy supply and transformation to final consumption. Each flow corresponds to one of the economic flows in the ThreeME model. The correspondence between the physical energy flows and economic flows is shown in Table 3.3. For instance the intermediate consumption of energy in TOE in ThreeME, is built up using data from the transformation input, final non-energy consumption and final energy consumption from the IEA Energy Balances. In the raw data the supply and imports of energy are given in gross units. ThreeME uses both the net and gross supply of energy. The difference between the two consist of auto consumption and distribution losses.

In the energy balances, the final consumption of energy is given for 24 different economic sectors. However, the Eurostat SUT economic data is provided at 65 sector level. In order to match these two sectoral classifications, Physical Energy Flow Accounts (PEFA) are used. PEFA provides physical energy flow data in a national accounts framework. It provides the energy use in Joules by 20 SIEC energy products and 65 NACE sectors. The IEA Energy Balances and PEFA are crossed to obtain a table of energy use in TOE by 20 energy products and 65 economic sectors.

The data for the different electricity sectors need an additional processing step.

[2]Standard international energy product classification

Table 3.2: data sources energy block

Data source	**Energy products**	**Energy sectors**	**Eurostat ID**
IEA Energy Balances	63 SIEC products	24 sectors	nrg_bal_c and nrg_bal_peh
Physical Energy Flow Accounts	20 SIEC products	65 NACE Rev.2 sectors	env_ac_pefasu
Eurostat Electricity and heat production	11 SIEC products		nrg_ind_peh

Table 3.3: correspondence between the physical and economic energy flows

Economic energy flow in ThreeME	**Physical energy flow in energy balance**
Supply, imports and exports	Total energy supply Transformation output
Auto consumption	Energy sector
Distribution losses	Distribution losses
Intermediate consumption	Transformation input Final non-energy consumption Final energy consumption

The IEA Energy Balances contain detailed data on the gross electricity production. The energy inputs used to generate the electricity are specified at the 63 SIEC product level. However, in the national accounts framework, the electricity data needs to be provided in net production rather than gross production. Therefore, the Eurostat electricity and heat production data is used to convert the IEA electricity data from gross into net production.

The energy data includes a number of corrections:

– Nuclear heat. In the IEA Energy Balances nuclear heat is considered a primary energy source supplied domestically (IEA, 2020). In practice however, nuclear heat is created in the reactors and the primary energy source is in fact the nuclear fuel which is often imported. Therefore, in the ThreeME database the supply of nuclear heat by the mining sector is set to zero.

– Fuel consumption by households. In the IEA energy balances transport fuel consumed by private vehicles is included in the Final consumption of Road transport sector. National accounts however classify this consumption as final consumption of households. In the ThreeME database we follow the logic of national accounts. In order to separate private fuel consumption from the public and commercial road transport we rely on respective shares from PEFA data.

3.3 Step 3: Disaggregation of energy relevant sectors

The third step is to disaggregate energy relevant sectors in the monetary SUTs. Both WIOD and Eurostat SUTs have insufficient number of energy commodities and sectors for the application purposes of ThreeME. To disaggregate these sectors, additional detailed energy data is used. The WIOD SUTs in the ThreeME database version 1 use EXIOBASE v3 (Stadler et al., 2018) for disaggregation, while the Eurostat SUTs in database version 2 use the physical energy SUTs, created in Step 2, for further disaggregation.

EXIOBASE SUTs contain 163 sectors and 200 products, with a focus on resource related sectors and commodities. Disaggregation shares are derived from the EXIOBASE data, which are used to expand the number of products and sectors in the WIOD SUTs. This is only done for the energy relevant sectors, such as electricity, waste and transport. The number of sectors is increased from 35 to 86 and the number of products is increased from 59 to 139 products.

The disaggregation of the ThreeME database version 2 is different from the disaggregation in version 1. The starting point are the physical energy SUTs created in the previous step. First, the physical energy SUTs are converted to monetary energy SUTs using IEA energy prices. This produes an estimated monetary SUT with 20 energy relevant products and 65 sectors. Missing values in the energy prices data are replaced manually with values from literature to the extent possible. For the remaining missing values average prices are used from other energy products for which there is data available.

The 20 energy products are added to the existing list of CPA products in the monetary SUTs, resulting in 85 products. For the energy sectors the approach is based on the physical supply layer. Each combination of the 20 products and 65 sectors, for which there is a non zero supply of energy, is used to create a new energy sector in the monetary SUT. Note that there is only supply of energy in the sectors B, C19, C20, C24, D and O. This results in 56 new energy sectors. For instance, there are three new sectors related to the production of natural gas. The first one relates to the natural gas produced in the NACE sector B, the second to sector C19 and the third to sector D. The three sectors are named accordingly: 'B - natural gas', 'C19 - natural gas' and 'D - natural gas'. These 56 new energy sectors are appended to the existing 65 NACE sectors in the monetary SUT, resulting in 121 sectors.

In order to keep the new SUTs balanced, the values of the new energy related products and sectors are deducted from the original aggregate energy related NACE products and sectors from Eurostat. The remaining values of the aggregate energy related sectors are assumed to be the non-energy component of an energy product. In case the remaining values turn out negative, the non-energy component is assumed to be zero, and the monetary flows of energy related sectors and products are proportionally re-scaled to add up to the Eurostat SUT aggregates. Thus, the following aggregate energy related NACE products now only refer to the non-energy component:

- B - Mining and quarrying

– C19 - Manufacture of coke and refined petroleum products

– C20 - Manufacture of chemicals and chemical products

– C24 - Manufacture of basic metals

– D - Electricity, gas, steam and air conditioning supply

– O - Public administration and defence; compulsory social security

Next to the energy sectors, also the transport sectors are disaggregated. Data from Eurostat Structural Business Statistics is used. This increases the number of products from 85 to 89 and the number of sectors from 121 to 125.

Finally, the resulting monetary SUTs are made consistent with the energy SUTs. There are cases where there is energy consumption or supply in a specific product-sector combination in the energy SUT but there is no economic transaction in the monetary SUT. In these instances, the energy SUT data is imposed on the monetary SUT. The new monetary SUT is then rebalanced using a RAS procedure (McDougall, 1999). The rebalancing is required in the supply and the intermediate consumption blocks.

3.4 Step 4: Emissions

The fourth step is to add emissions as an extension to the SUTs. ThreeME needs the GHG emissions data in tonnes both per emitting sector and per energy commodity. In official databases emissions are often only provided per emitting sector and not per energy commodity. The emissions data in ThreeME is therefore estimated.

In database version 1 the starting point is the physical use table from EXIOBASE which provides intermediate and final use per sector and energy commodity in tonnes of energy. The energy use in tonnes is converted to Joules using IEA (2005) heat values and then converted to emissions in tonnes using IPCC (2006) emission factors. This results in a matrix of GHG emissions in tonnes per energy commodity over the rows and per emitting sector over the columns. The emissions matrix are then scaled to the EXIOBASE emissions data. This is done at the sectoral level for different types of emissions, namely combustion, material and production related emissions.

Emissions data in database version 2 is estimated in a different fashion. The first difference is that the physical use table is taken from Step 2, rather than from EXIOBASE. The physical use table is then converted from energy to emissions in the same way as in version 1. The second difference is that the emissions matrix is scaled to Eurostat sectoral emissions instead of EXIOBASE emissions. However, EXIOBASE data is used to obtain the shares between combustion, material and production related emissions.

3.5 Step 5: Margins

Trade and transport products are partly used as a margin charged over the use of products. In SUTs margins are provided as *supplied* margins per margin type and separately also as *paid* margins per product over which the margin has to be paid. However, ThreeME needs the margins per margin type and per product at the same time. Therefore, a matrix of margins is estimated using a RAS procedure. The column totals of this matrix equal the total *supply* of margins per margin type and the row totals equal the total *paid* margins used per product.

3.6 Step 6: Investment matrix

The investment matrix is reconciled simply using the total investments per product combined with intermediate consumption shares per product and sector. Thus the investment matrix per product and sector is calculated by applying proportionate shares of the intermediate consumption on the investments per product.

3.7 Step 7: Household accounts

The step adds the household account present in the fourth quadrant of the SAM. These include the household income from property income, social benefits, employer social contributions and transfers. OECD data is used in this step. Note that in ThreeME wages and salaries include income from self-employed persons.

3.8 Step 8: Other variables

ThreeME has three product taxes, namely VAT, subsidies and other taxes. One fixed subsidy rate is assumed for all products and this rate is derived from the OECD national accounts database. The VAT rates are product specific and are taken from the European Commission (2018). Finally, there are 'other taxes' which are estimated as a residual, as the three product taxes should sum up to the total product taxes from the supply table.

IMF (2018) data is used for the government debt. Finally, gross operating surplus is recalculated as a residual, to ensure balance between use and supply. In this way, any data discrepancies caused by for instance adjusting 'wages and salaries' to include self-employed persons are compensated for in the gross operating surplus.

3.9 Step 9: Sector and commodity disaggregation

In the last step, the database is aggregated to the required level of detail, in particular for the number of sectors and commodities. This aggregation step removes details that are not relevant for a specific simulation. Keeping only the details needed, leads to more robust results, less computation time and clearer interpretation of results. The default list of aggregated sectors is shown in Table 3.4, with the energy sectors at the end of the list. Finally, the aggregated database is exported to both the ThreeME model as well as to Excel for manual inspection.

Table 3.4: Default list of aggregated sectors in ThreeME

Sectors	**Commodities**
Agriculture and fishing	Products of agriculture, Hunting, and Fishing Biomass and other energy
Forestry	Products of Forestry
Manufacture of food products and beverages	Food products and beverages
Manufacture of motor vehicles, trailers and semi-trailers	Motor vehicles, trailers and semi-trailers
Manufacture of glass, ceramic and cement products	Glass and ceramic products
Manufacture of paper and paperboard	Paper and paperboard
Manufacture of chemicals	Chemicals
Manufacture of plastics products	Plastics products
Metallurgy	Metals
Manufacture of other goods for industry	Other goods for industry
Manufacture of other goods for consumers	Other goods for consumers
Construction of buildings and civil engineering	Buildings and Civil engineering
Rail transport	Rail transport
Road transport	Road transport
Water transport	Water transport
Air transport	Air transport
Business services	Business services
Public services	Public services
Mining	Non-energy mining Coal Crude oil Natural gas
Production of refined oil Production of biofuel Distribution of natural gas Production and distribution of biogas	Transport fuels Heating fuel Natural gas
Transmission and distribution of electricity Nuclear electricity generation Oil-fired electricity generation Gas-fired electricity generation Coal-fired electricity generation Wind electricity generation Solar electricity generation Hydroelectric electricity generation Cogeneration (CHP) electricity generation and other heat supply Electricity - Others	Electricity Heat

Chapter 4

Long term and dynamic properties

ThreeME is a neo-keynesian model since it accounts for potential viscosities in the adjustment dynamics of prices and quantities. Nevertheless, in the long run, the model has neoclassical properties since it converges towards a steady state equilibrium *à la* Solow (1956). In such a case, every variable of the model (GDP, output, consumption, capital, etc) grows at a constant rate. The long term real growth rate is equal to the sum of the population growth rate and the labour productivity rate. All prices grow at the pace of the inflation target.

For the model to converge towards such a long-term steady state, we have to make several assumptions related to the evolution of exogenous variables: the population, the technical progress, the external demand and foreign prices have to grow at a constant rate; the rates and ratios (related to labour participation, tax, interest, capital depreciation, etc) should be constant. Furthermore, some homogeneity conditions should hold in order to guarantee that the specification of each equation is in line with a steady state. Hence, the production function must be homogeneous of degree 1 (constant return-to-scale) for the production to grow in the long run as the sum of the population growth rate and the labour productivity. Additionally, two equations defining the long term behaviour of the same variable should be consistent with each other. For instance, the real wage is defined by both the price equation and the wage equation. For the inflation rate to be stable, the real wage should grow in the long term as the labour productivity, which in turn implies that the unemployment rate should be constant in the long run. In the case of a Wage Setting (WS) curve, it means that the indexation of wages on prices and labour productivity should be unitary (see Chapter 1 for more detail).

Homogeneity constraints are necessary for the long term steady state to exist but they are not sufficient for the model dynamics to converge towards this steady state. In that respect, we also need to rely on dynamic constraints. It entails that, following a shock, a corrective mechanism should make the model converge towards a new equilibrium. In the ThreeME model, international trade contributes to this overall system stabilisation. For example, a positive demand shock will generate a drop of unemployment and an increase in inflation. The degradation of competitiveness progressively counterbalances the positive demand effect, hence making the model converge towards a new equilibrium. For this adjustment to exist, the Armington elasticity of substitution between domestic and foreign products should satisfy the Marshall-Lerner conditions: the absolute sum of the export and import demand elasticities should be greater than unity (Marshall, 1930; Lerner, 1944, 1952).

In this section, we analyze the dynamics and long term properties of the ThreeME model. In that respect, we compute alternative scenarios designed "naively" on purpose. More precisely, we do not take into account some indirect effects, often difficult to quantify, that would make the interpretation of the results less straightforward. For instance, we do not assume that the increase in the oil price generates inflation also in the rest of the world. We therefore overestimate the decrease in competitiveness of the French economy compared to the rest of the world. But this simplification allows to shed a better light on the model's cogs and wheels as well as on its underlying hypotheses.

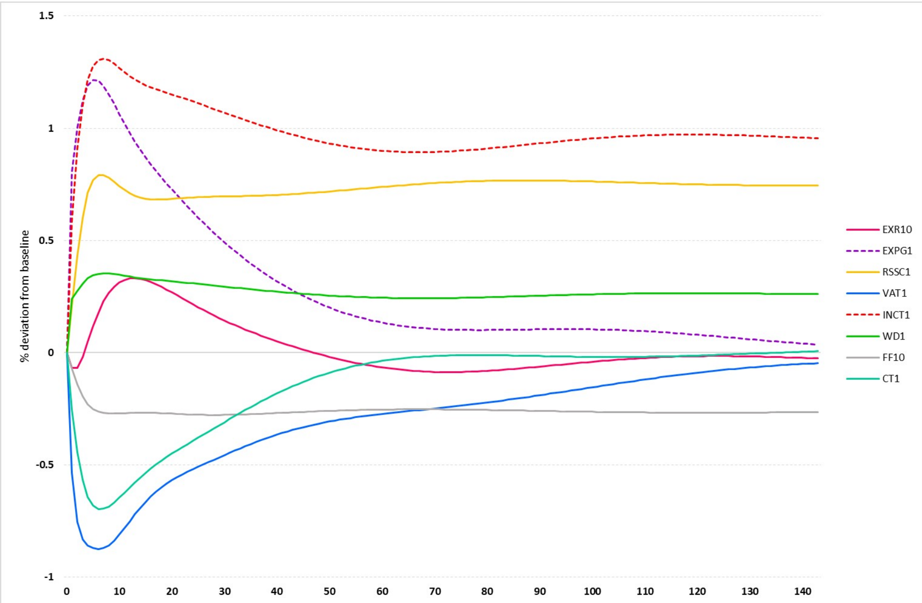
In a first section, we detail and interpret the results of such naive scenarios in the framework of the French economy. We then investigate in the second section the impact of the specification of the wage equation on the long term of the model.Finally, we study in the last section how the long term properties of the model are derived.

4.1 Simulations of standard shocks

Based on the French calibration, we simulate eight alternative scenarios, also denominated as "shocks". All shocks consists in a modification of the steady state at baseyear and are calibrated in terms of *ex ante* GDP. Among them, we can distinguish between public policy shocks that stem from the government's decision to implement new economic measures and external shocks which can be taken as the consequences of global phenomena impacting the entire French economy all of a sudden. The public policy shocks we consider are the five following: *(i)* an increase of public expenditure of one point of *ex ante* GDP [EXPG1]; *(ii)* a decrease of the household income tax of one point of ex ante GDP [INCT1]; *(iii)* a decrease of Social Employer Contributions of one point of ex ante GDP [RSSC1]; *(iv)* an increase of the value-added tax of one point of ex ante GDP [VAT1]; *(v)* an increase of the carbon tax of one point of ex ante GDP [CT1]. On the other hand, the external shocks at stake are: *(i)* an increase of fossil fuel prices of 10 percent [FF10]; *(ii)* a permanent increase of the world demand of 1 percent [WD1]; *(iii)* a permanent devaluation of the euro of 10 percent [EXR10].

A preliminary analysis of the GDP's evolution under these scenarios show that the model is in line with the previously detailed long-term constraints. Indeed, figure 4.1 shows that the GDP converges at some point to its long-term equilibrium for all scenarios. That being said, the same model can predict different outputs and paths for the same shock depending on the modelling choices that were made. To that extent, it is worth mentioning that we do not intend to provide, in the following section, a sensitivity analysis of the chosen parameters or specification hypotheses. We rather try to outline a standardized analysis of various shocks under the same assumptions and figure out what are the mechanisms at stake bringing the economy back to a steady state.

Figure 4.1: Evolution of the GDP depending on various shocks



Considering these parameters and assumptions as given, we can disentangle several types of outputs. More precisely, it is important to analyze both the short-term and the long-term effects. For instance, two different shocks can have the same impact on the short run, be this effect positive or negative. But then, it is also important to consider whether this short-run effect will gradually vanish leading the economy back to its previous steady state or whether the short-run effect entails a permanent modification of the steady state. On figure 4.1 we can see that the shock of household income tax [INCT1] and the shock of public expenditure [EXPG1] have more or less the same impact in the first years following the shock but the latter does not modify the long-term steady state while the former leads to a permanently higher steady-state GDP. More generally, among the eight alternative scenarios we simulate, five have a positive impact on GDP in the short run and four modify the steady-state GDP permanently.

4.1.1 Increase of public expenditure

First, we simulate a permanent increase of public expenditure by 1 (percentage) point of *ex ante* GDP implemented in year 1. More precisely, this increase is split into an increase of government expenditure on the one hand and an increase of social benefits on the other. The increase of social benefits accounts for 46% of the total increase. This increase is a pure demand shock. Results are shown in Table 4.1.

In the short run, the increase in public expenditure entails a rapid and strong increase in household consumption. The accrued demand has a positive impact on production. Hence, the demand shock goes hand in hand with an increase of employment, which in turn magnifies the augmentation of household consumption. The decrease of the unemployment rate puts an upward pressure on wages leading to higher labor and production costs. Quite logically, this increase in production cost triggers an increase in consumption prices. In a first moment, this does not counterbalance the increase in wages : household disposable income increases as well which reinforces the initial positive demand effect.

On the other hand, the increase in production costs and in the added value price index have a negative impact on exports. Indeed, five years after the introduction of the shock exports have declined by 0.27 point. Concomitantly, since higher wages and a lower unemployment rate sustain household consumption, imports increase. This is reinforced by the substitutions of domestic production to imports due to the greater competitiveness of foreign producers. This means part of the increase in demand is evicted towards foreign goods. In the end, the combination of these two opposite effects leads to a degradation of the trade balance. All in all, we observe a quite important positive effect on GDP with an increase of 0.81 point in the first year. This multiplier would be higher than one if the shock was exclusively about an increase of government expenditure. Because of the slow adjustment of consumption to income, the effect of increasing social benefits is delayed in time.

The various multiplicative mechanisms described above make the deviation from baseline bigger and bigger until it reaches a peak five years after the initial shock (1.22 point). Indeed, at some point, the inflationary pressures becomes high enough to generate negative impacts on the economic activity that counterbalance the initial positive impulse. The unemployment reaches its lowest point. It entails a decrease in job creations, hence a degradation of aggregate consumption and the beginning of a generalized slowdown. On the long run, the aggregate output converges towards the same steady state as before. The increase in public expenditure does not structurally modify the economy.

4.1.2 Decrease of the household income tax

This alternative scenario consists in a permanent decrease of the revenues generated by the income tax by 1 point of *ex ante* GDP. It is tantamount to a positive demand shock since it is about an immediate augmentation of the disposable income of households. Table 4.2 sums up the main results.

The decrease of the income tax unsurprisingly leads to a greater disposable income for households in the short-run. This increase is channeled both by consumption and savings. More precisely, household domestic consumption increases by 1.18 point and imports increase by 0.62 point in the first year. The saving rate also increases by 0.49 as compared to the baseline scenario. On the other hand, the newly accrued demand fuels production and unemployment decreases. To that extent, not only does the initial positive demand plays at the intensive margin but it also plays at the extensive margin and triggers a virtuous circle through the quantitative expansion of waged employment. As a result, over the years following the shock, household consumption keeps increasing, reaching a 2.77 point deviation peak ten years after the initial shock.

That being said, the decrease of unemployment goes hand in hand with an increase of wages leading to higher prices, which in the end scale down the positive demand impact. It also contributes to inflationary pressures and to a degradation of the trade balance since exports become less competitive. Nonetheless, those negative impacts are not sufficiently important for them to counterbalance the positive effects detailed earlier, hence putting the economy on a higher long-term steady-state GDP as before. The reduction of the household income tax has a permanent impact on the economy. This is tightly linked to the wage equation specification. We used the Wage Setting (WS) estimated in the macro-econometric model MESANGE model (Allard-Prigent et al., 2002; Klein and Simon, 2010). It assumes that wages are fully indexed on the income tax rate and the employer social contribution tax rate. A reduction in the income tax rate leads therefore to a permanent reduction of the equilibrium rate of unemployment and therefore to a positive multiplier in the long run. This contrasts with the transitory multiplier of the public expenditure shock that does not impact the equilibrium rate of unemployment (see previous section). It is worth underlining that with a Phillips curve the long term impact would be null (see the second section of this chapter).

Table 4.1: Increase of public expenditure by 1 point of ex ante GDP

	1 year	2 years	3 years	5 years	10 years	LT
GDP in volume	0.81	1.00	1.12	1.22	1.06	0.03
Household consumption	0.59	1.00	1.27	1.55	1.51	0.02
Investment	0.39	0.67	0.85	1.04	1.10	0.00
Exports	-0.02	-0.06	-0.12	-0.27	-0.63	-0.03
Imports	0.54	0.82	1.01	1.21	1.26	0.03
Household disposable income	0.87	1.11	1.29	1.49	1.47	0.02
Saving rate	0.25	0.10	0.02	-0.05	-0.04	0.00
Household consumption price index	0.10	0.25	0.40	0.71	1.22	0.04
Production price index	0.12	0.30	0.49	0.86	1.46	0.05
Added value price index	0.15	0.35	0.58	1.02	1.72	0.06
Intermediate consumption price index	0.10	0.24	0.40	0.70	1.20	0.04
Export price index	0.10	0.25	0.42	0.76	1.29	0.05
Import price index	0.00	0.00	0.00	0.00	0.00	0.00
Gross nominal wage	0.09	0.24	0.42	0.83	1.57	0.06
Real cost of labor	-0.06	-0.12	-0.16	-0.19	-0.15	0.00
Wage employment (in thousands)	119.68	215.86	282.73	343.48	295.32	10.70
Unemployment rate	-0.36	-0.61	-0.77	-0.89	-0.73	-0.02
Trade balance (in points of GDP)	-0.10	-0.13	-0.15	-0.14	-0.10	0.00
Public budget balance (in points of GDP)	-0.55	-0.30	-0.12	0.08	0.12	-0.02

Table 4.2: Decrease of the household income tax by 1 point of ex ante GDP

	1 year	2 years	3 years	5 years	10 years	LT
GDP in volume	0.58	0.91	1.10	1.28	1.27	0.95
Household consumption	1.18	1.84	2.22	2.61	2.77	2.37
Investment	0.30	0.61	0.85	1.13	1.34	0.60
Exports	-0.01	-0.05	-0.10	-0.24	-0.62	-0.70
Imports	0.62	1.04	1.30	1.59	1.78	1.48
Household disposable income	1.75	1.99	2.21	2.51	2.71	2.37
Saving rate	0.49	0.14	-0.01	-0.09	-0.05	0.00
Household consumption price index	0.07	0.19	0.33	0.63	1.23	1.12
Production price index	0.08	0.23	0.40	0.77	1.48	1.33
Added value price index	0.10	0.27	0.48	0.92	1.75	1.57
Intermediate consumption price index	0.07	0.18	0.33	0.63	1.21	1.09
Export price index	0.07	0.20	0.36	0.69	1.31	1.17
Import price index	0.00	0.00	0.00	0.00	0.00	0.00
Gross nominal wage	0.06	0.17	0.33	0.72	1.56	1.46
Real cost of labor	-0.04	-0.10	-0.14	-0.19	-0.18	-0.10
Wage employment (in thousands)	76.83	166.24	241.15	327.28	328.10	336.56
Unemployment rate	-0.23	-0.48	-0.67	-0.86	-0.81	-0.56
Trade balance (in points of GDP)	-0.13	-0.20	-0.23	-0.25	-0.22	-0.21
Public budget balance (in points of GDP)	-0.72	-0.48	-0.30	-0.09	-0.03	-0.33

4.1.3 Decrease of employers' social contributions

We simulate a decrease of the employer social security contributions by an overall of 1 point of *ex ante* GDP, that is to say a positive supply shock. This equates to a decrease of about 7% of the contribution rate for all sectors of activities. Results are shown in Table 4.3. This shock implies a decrease of the labor cost, which in turn impacts several other aggregates. First, the decrease of the labor cost leads to a smaller production cost, hence greater real disposable incomes for households which goes hand in hand with more consumption.

At the same time, we observe a decrease of the unemployment rate : one year after the shock, the unemployment rate has decreased by 0.09 point and by 0.58 point after five years. There is no doubt it is partly due to the increase in household consumption. Nonetheless, it might also be explained to some extent by the substituability between production factors since after the decrease of the employer social security rate, labor becomes less expensive as compared to other production factors whose prices have not changed. Hence, it is more demanded than before and makes unemployment decrease.

The decrease in labor and production costs also contribute to improving firm competitiveness : exports increase by 0.08 point the first year following the shock and by 0.53 point after 5 years. On the contrary, even if it may appear less intuitive, imports also increase. This is due to the fact that the positive income effect of the domestic demand increase counterbalances the negative substitution effect which entails that foreign goods become more expensive than domestic goods than before. In the end, imports increase. Despite the improvement of the competitiveness, the trade balance in value decreases: France exports more but at a lower price. On the contrary the trade balance in volume increases (result not shown).

4.1.4 Increase of the VAT

This shock consists in an increase by 1 point of *ex ante* GDP of the VAT for all commodities. Since these additional revenues are not redistributed, this scenario is recessive. It affects the economy negatively before it converges again towards it previous steady-state GDP. These results are detailed in Table 4.4.

A VAT shock immediately depreciates the purchasing power of households since it increases the consumption prices, which increase by 1.82 point in the first year. In reaction to this increase, households decrease their consumption levels of domestic and foreign goods. Due to some inertia in their consumption patterns, savings as well are temporally reduced due to the slow adjustment of consumption to real income in the short run.

Furthermore, the contracted demand impacts the supply side: employment falls down which in turn negatively weigh on consumption levels. This low activity decreases gross nominal wages in the first year despite their indexation on the consumer price. The improvement of trade balance does not entail positive feedbacks since it relies only on a huge contraction of imports due to a weak domestic demand that counterbalances the contraction of exports. Hence, the initial shock progressively expands to the entire economy and the global activity reaches its lowest point five years later. The initial shock is then almost entirely transmitted to the economy with a negative deviation of GDP of 0.87 point from the baseline scenario.

On the long run, gross nominal wages grow since they are positively indexed on the evolution of consumption prices. This leads households to consume more, relaxing the constraint on the supply side. Employment stop decreasing, spurring the economic recovery and hence opening the way to a slow convergence towards the previous equilibrium of economic activity.

4.1.5 Increase of the carbon tax

In this scenario, we assume that France unilaterally implements a carbon tax of 1 point of *ex ante* GDP, without taking into account any form of border adjustment. This type of policy aims at reducing carbon dioxide emissions through the indirect distortion of fossil energy prices. The additional revenues generated by the carbon tax are not redistributed in any way. They entirely go to the reduction of the government's deficit. To that extent, it amounts to a negative demand shock since it generates an immediate increase of energy consumption prices and therefore a reduction of the purchasing power of households. It is also a supply shock to the extent that intermediate consumption become more expensive, which directly affects the production process, left alone feedbacks from the demand side. We show the main results in Table 4.5.

Since we assume the introduction of the carbon tax only concerns France, we can observe a loss of competitiveness eventually leading to a decrease in French exports. Indeed, the introduction of the carbon tax leads to higher production costs under the assumption that firms' mark-ups remain stable. Firms whose production chain is highly intensive in energy have no choice but to rise their selling price, which in turn impacts households.

Besides, the carbon tax depreciates households' purchasing power since they are final consumer of energy too. The increase in energy prices leads to both a decrease in energy consumption and a decrease of their real disposable income. All in all, the increase in the household consumption price index reaches more than 1% in the third year, which has a negative impact on household consumption. Households decrease their consumption of both domestic and foreign goods.

In the end, the positive impact on the trade balance is positive because of (a) the contraction of imports related to the decrease in household disposable income and (b) the increase in the export price that compensate the decrease in exports. Nonetheless, the global impact on the economic activity is negative and gets worse and worse over the first five years - from -0.25 point of GDP in the first year to -0.68 point five years after the initial shock. In particular, the recessive impact of the non-redistributed tax is amplified by the decrease in investments and the overall degradation of the economic activity leads to job losses.

To sum up, in the short-run, the introduction of a carbon tax has quite the same macroeconomic impacts than an increase in fossil fuel prices except from the fact that it also contributes to reducing the public deficit. This reduction decreases over time from 0.61 point of GDP in the first year following the shock to 0.16 point five years later. This decrease accounts for two phenomena: the negative impact of the carbon tax on the economic activity on the one hand and the reduction of the tax base related to the emissions reduction on the other. The next chapter on the double dividend focus in more details on the impact of a carbon tax on the economy under various assumptions.

Table 4.3: Decrease of Social Employer Contributions by 1 point of ex ante GDP

	1 year	2 years	3 years	5 years	10 years	LT
GDP in volume	0.21	0.43	0.60	0.77	0.74	0.75
Household consumption	0.36	0.73	0.98	1.15	0.89	0.83
Investment	0.07	0.23	0.38	0.54	0.38	-0.11
Exports	0.08	0.19	0.31	0.53	0.91	1.26
Imports	0.15	0.32	0.44	0.50	0.25	0.07
Household disposable income	0.54	0.85	1.02	1.09	0.86	0.83
Saving rate	0.15	0.11	0.03	-0.06	-0.03	0.00
Household consumption price index	-0.45	-0.77	-0.99	-1.29	-1.74	-2.07
Production price index	-0.52	-0.89	-1.15	-1.48	-2.00	-2.40
Added value price index	-0.61	-1.05	-1.35	-1.73	-2.33	-2.80
Intermediate consumption price index	-0.43	-0.74	-0.95	-1.23	-1.67	-1.99
Export price index	-0.42	-0.74	-0.98	-1.29	-1.73	-2.07
Import price index	0.00	0.00	0.00	0.00	0.00	0.00
Gross nominal wage	0.02	-0.05	-0.20	-0.54	-1.21	-1.70
Real cost of labor	-0.87	-0.51	-0.35	-0.30	-0.37	-0.38
Wage employment (in thousands)	31.09	81.60	135.65	217.76	261.09	375.36
Unemployment rate	-0.09	-0.24	-0.38	-0.58	-0.65	-0.62
Trade balance (in points of GDP)	-0.13	-0.23	-0.29	-0.34	-0.31	-0.27
Public budget balance (in points of GDP)	-0.65	-0.43	-0.30	-0.22	-0.36	-0.32

Table 4.4: Increase of the VAT by 1 point of ex ante GDP

	1 year	2 years	3 years	5 years	10 years	LT
GDP in volume	-0.54	-0.75	-0.83	-0.87	-0.81	-0.04
Household consumption	-1.00	-1.35	-1.44	-1.41	-1.05	-0.05
Investment	-0.50	-0.85	-1.02	-1.07	-0.79	0.06
Exports	-0.04	-0.08	-0.13	-0.26	-0.70	-0.09
Imports	-0.58	-0.84	-0.91	-0.84	-0.47	0.00
Household disposable income	-1.46	-1.40	-1.35	-1.31	-1.02	-0.05
Saving rate	-0.42	-0.04	0.09	0.09	0.03	0.00
Household consumption price index	1.82	1.89	2.00	2.31	2.95	0.23
Production price index	0.20	0.31	0.46	0.86	1.76	0.16
Added value price index	0.24	0.38	0.54	1.00	2.05	0.19
Intermediate consumption price index	0.15	0.25	0.37	0.71	1.46	0.14
Export price index	0.20	0.30	0.41	0.73	1.52	0.14
Import price index	0.00	0.00	0.00	0.00	0.00	0.00
Gross nominal wage	-0.04	0.39	0.83	1.48	2.44	0.21
Real cost of labor	-0.29	0.02	0.29	0.47	0.38	0.02
Wage employment (in thousands)	-56.63	-119.81	-169.57	-228.00	-264.64	-21.88
Unemployment rate	0.17	0.34	0.47	0.60	0.66	0.04
Trade balance (in points of GDP)	0.20	0.28	0.32	0.35	0.35	0.02
Public budget balance (in points of GDP)	0.46	0.33	0.23	0.10	0.16	0.02

Table 4.5: Increase of the carbon tax by 1 point of ex ante GDP

	1 year	2 years	3 years	5 years	10 years	LT
GDP in volume	-0.25	-0.44	-0.57	-0.68	-0.65	0.01
Household consumption	-0.47	-0.76	-0.90	-0.93	-0.62	0.00
Investment	-0.34	-0.70	-0.98	-1.31	-1.37	0.04
Exports	-0.09	-0.21	-0.33	-0.54	-0.82	0.00
Imports	-0.41	-0.70	-0.86	-0.92	-0.66	0.00
Household disposable income	-0.69	-0.84	-0.89	-0.86	-0.60	0.00
Saving rate	-0.20	-0.07	0.01	0.06	0.02	0.00
Household consumption price index	0.75	0.98	1.15	1.38	1.62	0.00
Production price index	0.42	0.71	0.91	1.20	1.53	0.00
Added value price index	-0.28	0.08	0.34	0.72	1.23	0.00
Intermediate consumption price index	1.12	1.34	1.48	1.67	1.83	0.00
Export price index	0.49	0.82	1.04	1.29	1.52	0.00
Import price index	0.00	0.00	0.00	0.00	0.00	0.00
Gross nominal wage	-0.01	0.17	0.40	0.83	1.40	-0.01
Real cost of labor	0.27	0.09	0.06	0.11	0.17	0.00
Wage employment (in thousands)	-13.68	-40.37	-70.76	-120.84	-160.14	-3.19
Unemployment rate	0.04	0.12	0.20	0.32	0.40	0.01
Trade balance (in points of GDP)	0.21	0.34	0.40	0.43	0.36	0.00
Public budget balance (in points of GDP)	0.61	0.41	0.29	0.16	0.14	0.01

4.1.6 Increase of fossil fuel prices

We simulate a 10% permanent increase of fossil fuel prices. By fossil fuels, we mean every fossil sources of energy that is to say oil, gas and coal. This shock is an exogenous inflationary shock. It impacts both the supply side and the demand side. The overall foreign prices and foreign demand framework is assumed to be left unchanged except from this modification of fossil fuel prices. Results are detailed in Table 4.6.

Since France has no domestic production of fossil fuels and systematically imports what it needs for its consumptions, a shock on fossil fuel prices is tantamount to an increase of import prices in the very short run : in the first year, the import price index increases by a bit more than 1%. Import prices have then several channels of diffusion.

A first impact is about domestic production costs. More specifically, more expensive imports entail that production becomes costlier. This production costs increase induces, in the medium run, an increase in household consumption prices on the one hand and in export prices on the other since firms try to partly transfer their increase in production costs to selling prices. This means that the household purchasing power is depreciated and that exports become less competitive. Recall that we naively assume that the price of the other commodities in rest of the world is not affected by this shock. This tends to overestimate the loss in competitiveness faced by France.

Another impact is that the household disposable income is further reduced due to the direct impact of more expensive imports on their purchasing power. The decrease in demand exerts a downward pressure on domestic production, hence opening the way to a vicious recessive circle: employment decreases, intensifying the decrease in household consumption hence hardening firms production conditions. As employment decreases, the real cost of labor begins to decrease too which assures that the economy slowly reaches a new steady state.

4.1.7 Permanent increase of the world demand to France

Table 4.7 shows the results stemming from the simulation of a 1% permanent increase of world demand addressed to France. This shock is tantamount to a pure demand shock. To that extent, we expect it generate impacts that are similar to the scenario where public expenditures increases. A major difference though is that it does not increase the public deficit.

The increase of the world demand consists in an immediate increase of exports. The first year following the shock, exports increase by 0.99% as compared to the baseline scenario. This is very close to the 1% increase in the exogenous part of exports assumed in this scenario because the export price index remains almost unchanged with a slight increase of 0.05%. Since exports increase with hardly any price eviction, the trade balance appreciates. All these changes contribute to the 0.24 point increase of the economic activity in the first year. This economic improvement generates a virtuous circle with more jobs created and more investment to sustain the growth trend. Five years after the shock, the unemployment rate has decreased by 0.26 point and investment has increased by 0.41%.

On the medium run, the decrease of the unemployment rate goes hand in hand with higher and higher wages. This increase in labor costs leads to higher production prices, which impacts in turn household consumption prices. Ten years after the initial shock, production prices have increased by 0.47% as compared to the baseline scenario. The deviation of household consumption prices from the baseline scenario is about 0.36%. This inevitably slows down the economic boom since it puts a downward pressure on household consumption. Nonetheless, the shock has a small but permanent effect on the steady state. On the long run, the GDP is still 0.26% higher in the alternative scenario than in the baseline.

Table 4.6: Increase of fossil fuel prices by 10%

	1 year	2 years	3 years	5 years	10 years	LT
GDP in volume	-0.07	-0.14	-0.19	-0.25	-0.27	-0.26
Household consumption	-0.15	-0.28	-0.35	-0.39	-0.28	-0.21
Investment	-0.10	-0.23	-0.36	-0.53	-0.63	-0.40
Exports	0.02	0.01	-0.03	-0.11	-0.29	-0.49
Imports	-0.13	-0.25	-0.33	-0.40	-0.32	-0.19
Household disposable income	-0.23	-0.32	-0.36	-0.36	-0.27	-0.21
Saving rate	-0.07	-0.04	0.00	0.02	0.01	0.00
Household consumption price index	0.24	0.38	0.49	0.66	0.91	1.13
Production price index	0.17	0.32	0.43	0.61	0.91	1.17
Added value price index	-0.13	0.01	0.14	0.34	0.69	1.01
Intermediate consumption price index	0.48	0.62	0.73	0.89	1.13	1.34
Export price index	0.21	0.38	0.51	0.69	0.94	1.16
Import price index	1.04	1.03	1.03	1.03	1.03	1.03
Gross nominal wage	0.00	0.06	0.15	0.36	0.76	1.04
Real cost of labor	0.13	0.04	0.02	0.03	0.07	0.03
Wage employment (in thousands)	-3.99	-12.12	-22.01	-40.09	-63.01	-94.10
Unemployment rate	0.01	0.04	0.06	0.11	0.16	0.16
Trade balance (in points of GDP)	-0.19	-0.11	-0.07	-0.02	-0.02	-0.04
Public budget balance (in points of GDP)	-0.04	-0.08	-0.11	-0.13	-0.11	-0.08

Table 4.7: Permanent increase of the world demand to France by 1%

	1 year	**2 years**	**3 years**	**5 years**	**10 years**	**LT**
GDP in volume	0.24	0.28	0.31	0.34	0.35	0.26
Household consumption	0.07	0.14	0.21	0.30	0.36	0.25
Investment	0.18	0.28	0.34	0.41	0.48	0.24
Exports	0.99	0.98	0.96	0.91	0.79	0.78
Imports	0.26	0.36	0.41	0.48	0.55	0.46
Household disposable income	0.10	0.17	0.23	0.30	0.35	0.25
Saving rate	0.03	0.03	0.01	0.00	-0.01	0.00
Household consumption price index	0.03	0.08	0.12	0.22	0.39	0.35
Production price index	0.04	0.09	0.15	0.26	0.47	0.42
Added value price index	0.05	0.11	0.18	0.31	0.55	0.49
Intermediate consumption price index	0.03	0.08	0.12	0.21	0.38	0.34
Export price index	0.05	0.10	0.15	0.25	0.42	0.37
Import price index	0.00	0.00	0.00	0.00	0.00	0.00
Gross nominal wage	0.03	0.07	0.13	0.25	0.49	0.46
Real cost of labor	-0.02	-0.04	-0.06	-0.06	-0.06	-0.03
Wage employment (in thousands)	35.80	63.59	82.59	101.37	99.16	106.46
Unemployment rate	-0.11	-0.18	-0.23	-0.26	-0.25	-0.18
Trade balance (in points of GDP)	0.18	0.17	0.17	0.16	0.17	0.17
Public budget balance (in points of GDP)	0.09	0.16	0.20	0.24	0.26	0.17

Table 4.8: Permanent devaluation of the exchange rate by 5%

	1 year	2 years	3 years	5 years	10 years	LT
GDP in volume	-0.07	-0.07	-0.02	0.12	0.31	-0.03
Household consumption	-0.62	-0.88	-0.91	-0.65	0.08	-0.04
Investment	-0.13	-0.20	-0.21	-0.11	0.29	0.02
Exports	0.74	1.16	1.37	1.42	0.81	-0.06
Imports	-0.55	-0.81	-0.87	-0.71	-0.07	-0.01
Household disposable income	-0.91	-0.93	-0.83	-0.54	0.09	-0.04
Saving rate	-0.26	-0.04	0.07	0.10	0.01	0.00
Household consumption price index	1.19	1.56	1.92	2.64	4.18	5.09
Production price index	0.53	0.97	1.39	2.22	4.00	5.08
Added value price index	-0.26	0.26	0.75	1.71	3.81	5.07
Intermediate consumption price index	1.30	1.68	2.03	2.72	4.20	5.09
Export price index	0.77	1.35	1.82	2.63	4.18	5.10
Import price index	5.00	5.00	5.00	5.00	5.00	5.00
Gross nominal wage	0.01	0.35	0.80	1.75	3.84	5.08
Real cost of labor	0.26	0.09	0.05	0.04	0.03	0.01
Wage employment (in thousands)	8.58	22.64	40.48	76.09	99.47	-13.06
Unemployment rate	-0.03	-0.07	-0.11	-0.21	-0.25	0.02
Trade balance (in points of GDP)	-0.80	-0.47	-0.28	-0.09	0.01	0.01
Public budget balance (in points of GDP)	-0.25	-0.26	-0.22	-0.07	0.25	0.01

4.2 Wage Setting and long term properties

Several studies have shown that the theoretical arguments and empirical estimates difficultly allow choosing between the two specifications. However, this difference of specification has important implications on the definition of the equilibrium unemployment rate or $NAIRU$ (Non Accelerating Inflation Rate of Unemployment) and thus on the inflationary dynamic and the long-term properties of a macroeconomic model (L'horty and Thibault, 1998; Le Bihan and Sterdyniak, 1998; Blanchard and Katz, 1999; Chagny et al., 2002; Reynès, 2006, 2010). See also the simulations in chapter 3.

For the purpose of comparison, the previous section retain the specification of the Wage Setting (WS) estimated in the macro-econometric model MESANGE model (Allard-Prigent et al., 2002; Klein and Simon, 2010). Following the work initiated by Layard et al. (2005) in the 1980's, it assumes that the wage negotiation process leads to an indexation of wages on the tax wedge which corresponds to the share of direct and indirect taxes into the total labour cost (see e.g. Cahuc et al., 2014, Chap. 12 on Income redistribution). A reduction in taxes leads therefore to a permanent reduction of the equilibrium rate of unemployment and therefore a positive multiplier in the long run. This contrasts with the transitory multiplier of the public expenditure shock that does not impact the equilibrium rate of unemployment (see previous section). The impact the tax wedge on wage setting has been criticised by Le Bihan and Sterdyniak (1998); Chagny et al. (2002). In particular the uniformity of the impact is contradicted empirically Tyrväinen (1995); Chagny et al. (2002). For instance, there is no reason that unions gets increases in wages when the employers' contributions increases. On the contrary, reductions are expected as employers seek to pass on the increase in contribution rates to wages to limit the increase in costs.

[HERE: Graph with employers' contributions shock with WS and Phillips curve: GDP, Unemployment rate, consumer price]

The magnitude and persistence of the long-term shock will depend on the assumptions taken into account in the specification of the wage equation. With a Wage-Setting curve (WS), the equilibrium unemployment rate increases permanently following an increase in the price of fossil fuels, whereas this increase is only transitory with a Phillips curve. Also we see in that the increase in the unemployment rate at 35 is 0.28 point with a WS curve and 0 point with a Phillips curve.

With a WS curve, the rise in unemployment is permanent while the unemployment rate returns to its initial level with a Phillips curve after about 30 years. The rise in unemployment manages to stabilize the rise in prices with a WS curve while it reverses the inflationary dynamic in the case of a Phillips curve, so that the consumer price falls in the long term. Rising unemployment has a stronger impact on inflation with a Phillips curve, the rate of wage growth depends negatively on the level of the unemployment rate and not only on a variation as in the case of a WS curve.

[HERE: Graph with oil price shock with WS and Phillips curve: GDP, Unemployment rate, consumer price]

[HERE: Graph with public expenditure shock with Phillips curve with constant and Time-Varying NAIRU: GDP, Unemployment rate, consumer price]

Figure 4.2: Evolution of GDP (in volume) following a decrease of Social Employer Contribution by 1 point of ex ante GDP

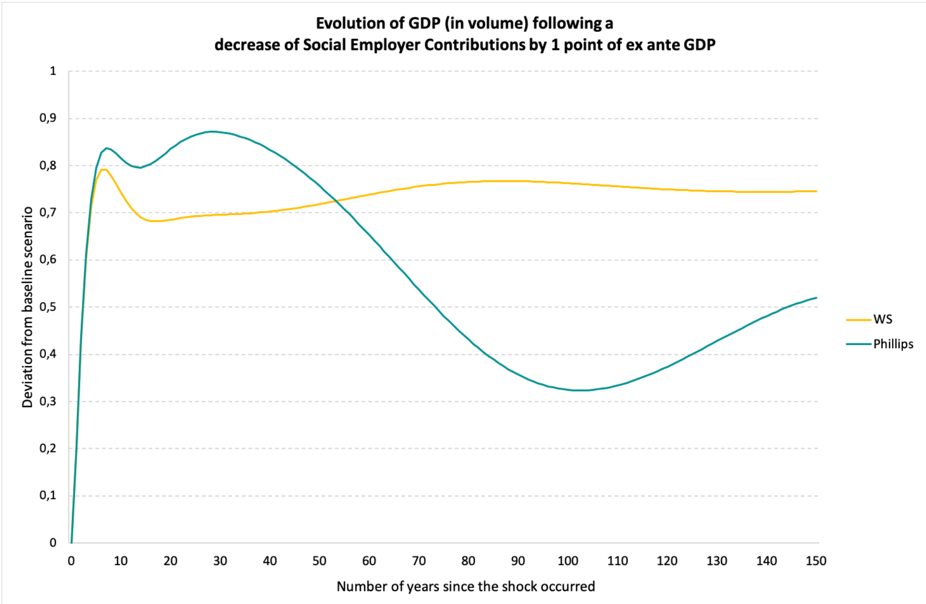


Figure 4.3: Evolution of unemployment rate following a decrease of Social Employer Contribution by 1 point of ex ante GDP

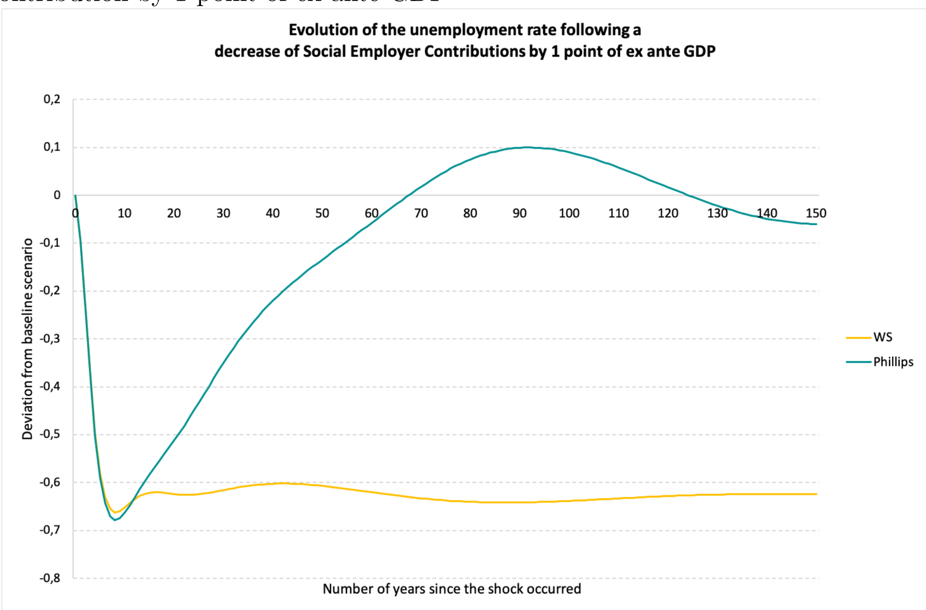


Figure 4.4: Evolution of consumer price index following a decrease of Social Employer Contribution by 1 point of ex ante GDP

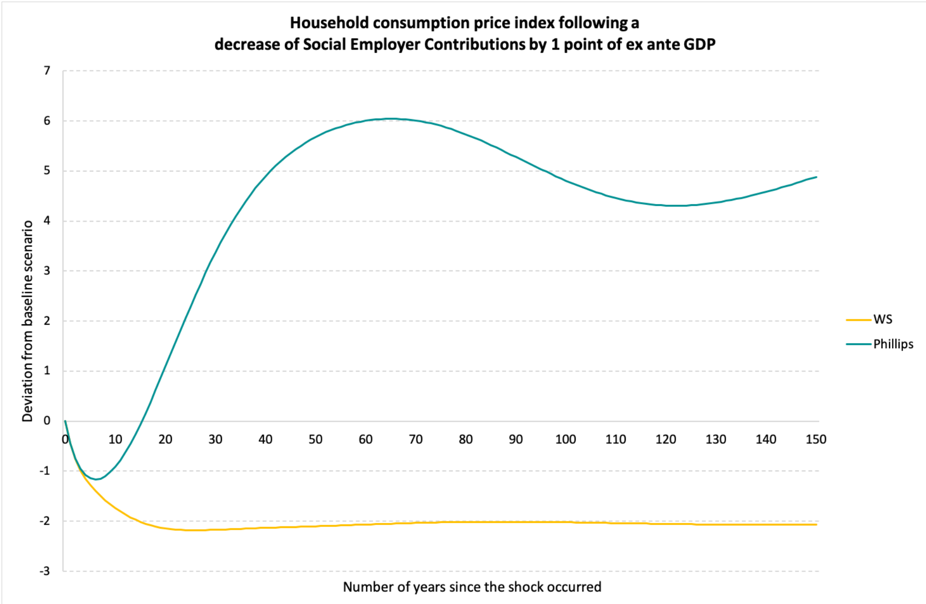


Figure 4.5: Evolution of GDP (in volume) following a 10% increase in fossil fuel prices

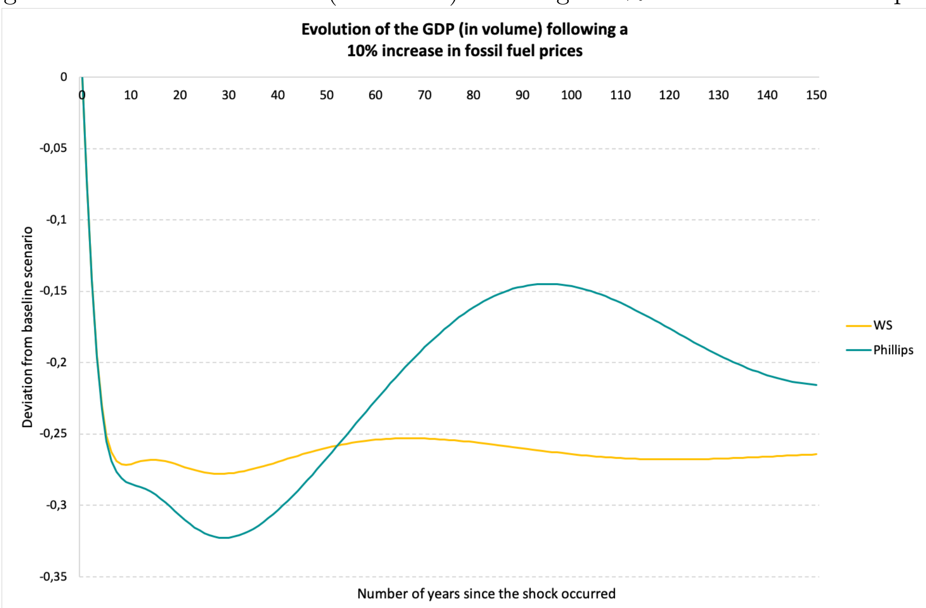


Figure 4.6: Evolution of unemployment rate following a 10% increase in fossil fuel prices

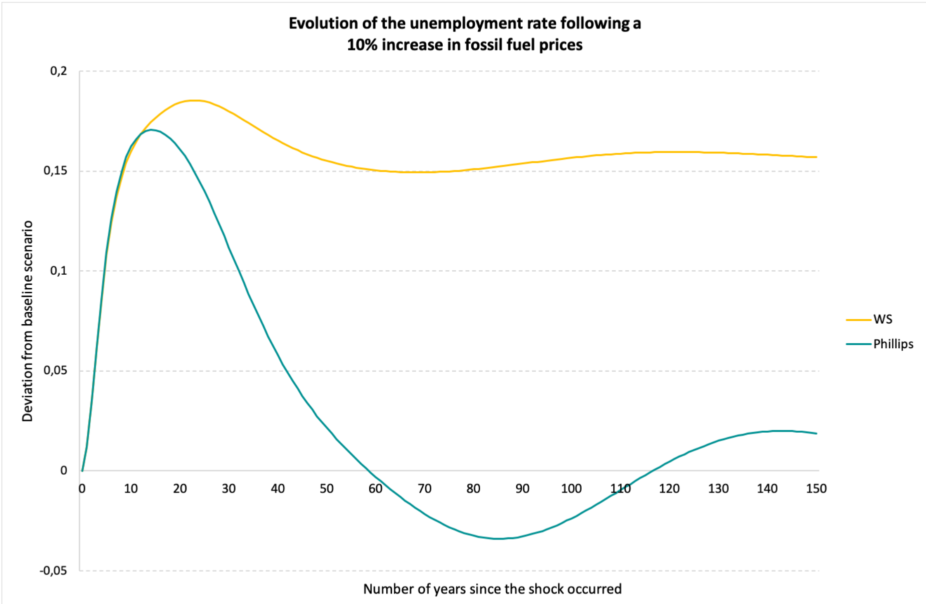


Figure 4.7: Evolution of consumer price index following a 10% increase in fossil fuel prices

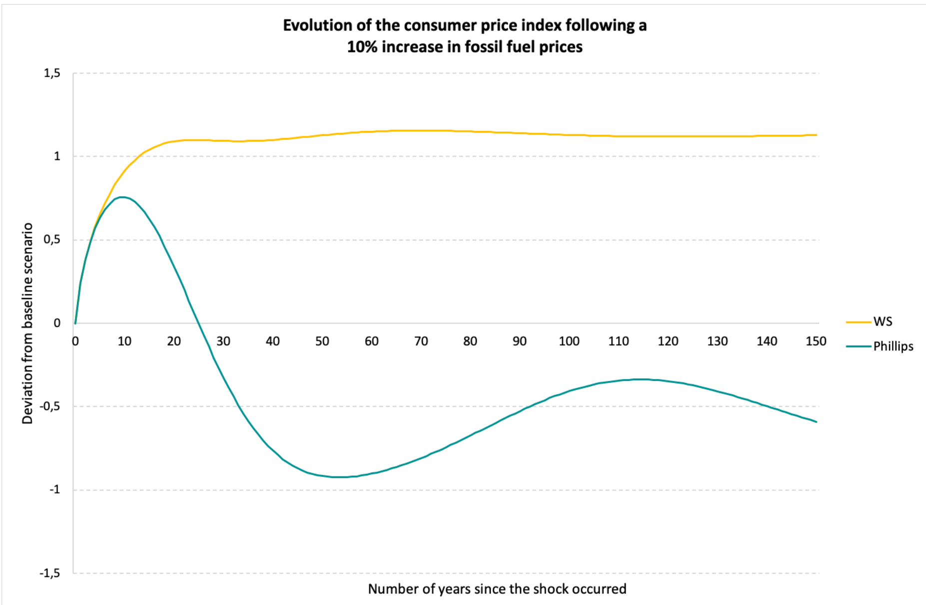


Figure 4.8: Evolution of GDP (in volume) following a 1 GDP point increase in public expenditure

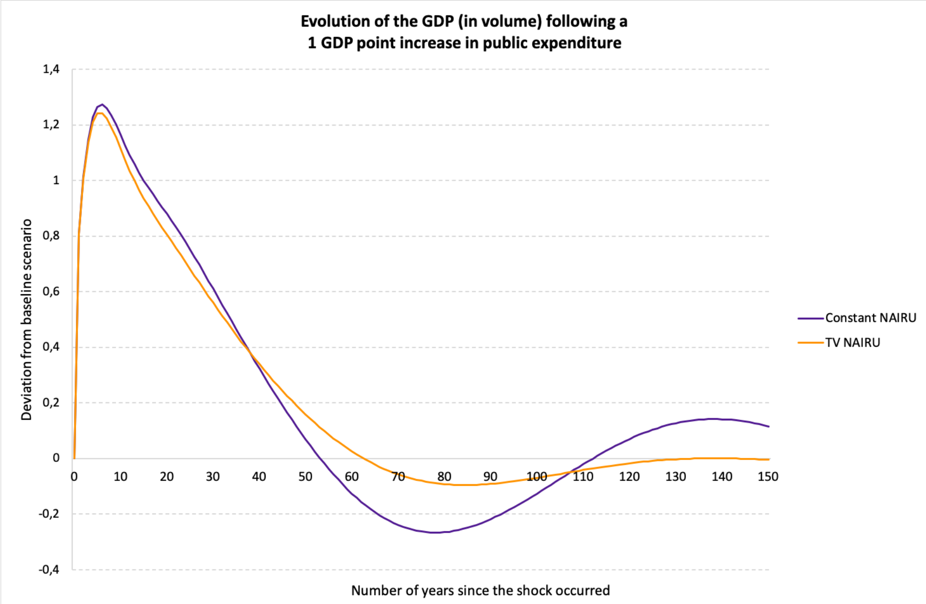


Figure 4.9: Evolution of unemployment rate following a 1 GDP point increase in public expenditure

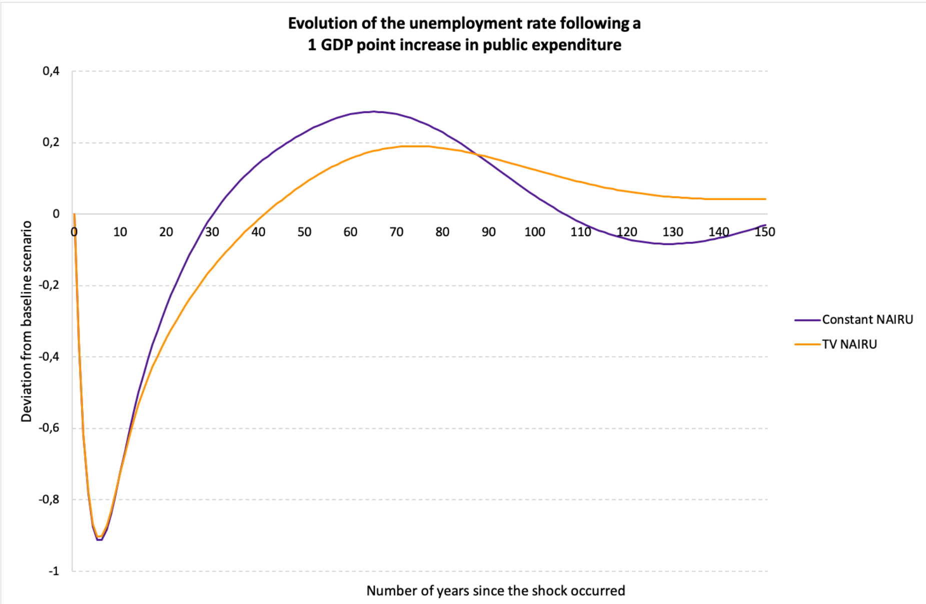
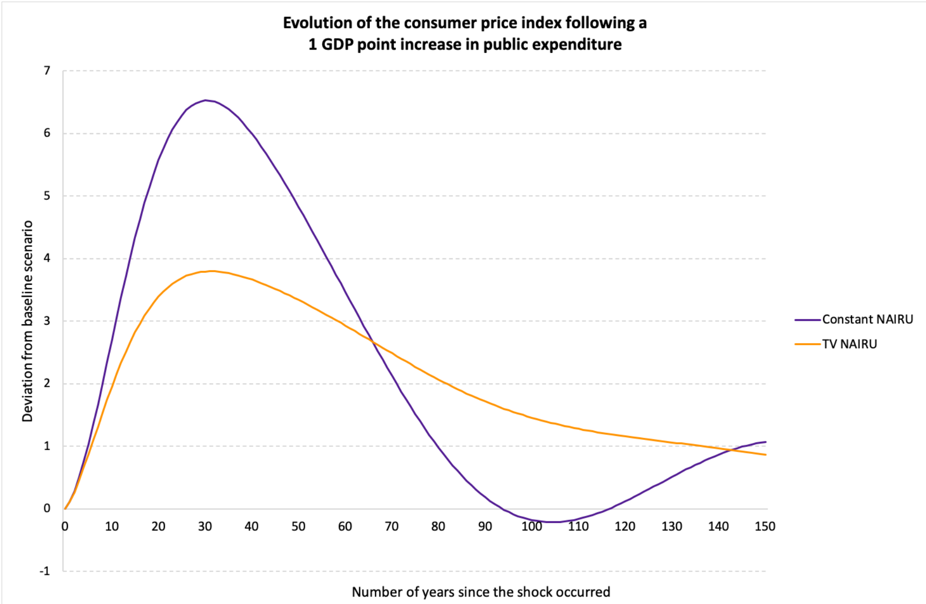


Figure 4.10: Evolution of consumer price index following a 1 GDP point increase in public expenditure



4.3 Derivation of the long term properties of the model

The long term steady state of the model is generally defined as a state where all variables grow at a constant rate (Deleau et al., 1980, 1981; Malgrange, 1983; Deleau et al., 1990; Loufir et al., 1994). This state is coherent with the representation of a stable economy able to maintain a given configuration forever. This implies that rates such as the unemployment or labor participation ratios, tax rates are constant in the long run. This is coherent with the fact that these ratios lie by definition between 0 and 100% and thus cannot be affected by a trend forever.

Most shares should also be constant. For instance, the shares of investment or of consumption into GDP should be constant. Otherwise the effect of one of these two determinants of the GDP vanishes over time. The same argument holds for the share of one sector in the total in terms of labor or production: we expect an economy where all sectors remain in the long run, which implies that some economic mechanisms ensures stable share for each sectors.

Some exceptions are possible. As empirically observed, it seems realistic that the share of labor into the GDP decreases over time because of the technical progress. But the share of the efficient labor, that is including the technical progress, remains constant. Because of the globalization of the economy, the ratio between export and production may also increase permanently in the long run. But in the long run this effect is expected to be compensated by the increase in the ratio between import and production so that the share of the external balance into production still remains constant.

In the long run, all relative prices are expected to be constant. This implies that all prices grow at the same rate. This ensures that the economy is not affected by substitution mechanisms in the long run: firms do not want to change the share of each production factors into production and consumers are satisfied with share of each good into their aggregate consumption. It implies also that each agent is satisfied with their share of the global revenue: firms do not want to change the growth rate of their price whereas employees do not want to change the growth rate of their wage.

Assuming that ν, τ, μ, π andω are the growth rates of the population, of the technical progress, of the real economy (i.e. of the GDP), of prices (i.e. inflation), and of wages, the long run value of these rates cannot be chosen independently. First, the growth rate of the real economy should be equal to the sum of the growth rate of the population and of the technical progress:$\mu = \nu + \tau$. This condition is a direct consequence of the hypothesis of constant return to scales (homogeneous of degree 1) of the production function. In the long run, relative price are constant and the labor demand implies that production grows at the sum of the growth rates of labor and technical progress$\Delta y_{jt} = \Delta l_{jt} + \Delta p_{jt}^{rog}$. In addition, the stability of unemployment rate implies that labor grows at the same rate as the population. In the long run, the price equation implies that the growth rate of wages should be equal to the sum of inflation and of the growth rate of the technical progress. This holds only if some economic mechanisms imply that the unemployment rate converge to the NAIRU. The latter depends on the parameter of the Phillips curve:

$$U_\infty = \left(\rho_1 - (1-\rho_2)\pi - (1-\rho_3)\tau/\rho_5\right) \tag{4.3.1}$$

In the model several stabilizing equation guaranty that the economy return to stationary path after a shock. Inflationary shocks degrade the external position of France by decreasing export and increasing imports. In addition, the Taylor rule combined with the negative impact of the real interest rate on the demand prevents inflationary shock to lead to an explosive inflation dynamic. The negative impact of the real interest rate on the activity has several possible canals:

- Consumption: in coherence with a life-cycle model and the possibility of an intertemporal allocation of their resource, households may increase their savings when the real interest rate increases and thus reduces their consumptions. They may also have Ricardian behavior in the long run by internalizing the government and firms' budget constraints. They may thus adjust their consumption in such way that the ratio between their savings and the national debt is constant in order to insure the sustainability of the debt.
- Investment: firms may choose their investment level that is coherent with the stability of their debt into the value-added.
- Tax and public spending: the government is expected to choose the tax rate and public spending levels that are coherent with a stable debt into the GDP.

The consistency of a dynamic model with a stationary equilibrium requires long term constraints which depend on the type of mathematical equation. We briefly detailed the main cases that are encountered in ThreeME and how the model can be calibrated in order to be at the stationary state from the first period of the simulation onward. It should be noted that this assumption is made in order to verify the coherence of the data calibration and of the dynamic properties of the model.

Some of the constraints needed to guarantee a permanent stationary state are quite restrictive. For instance, it implies a strict relation between stock and flow values (e.g. capital and investment, see below) which may not be satisfied at the base year. Also, the growth rates of population, and of technical progress should always be constant, which is not verify empirically. For this reason, we simulate also realistic baseline scenarios where the model is fully calibrated on empirical data and on realistic projections for the exogenous variables.

4.3.1 Additive equations

In the model, many relations enter in an additive form:

$$Y_t = \sum_{i=1}^{I} X_{it} \tag{4.3.2}$$

These are in general definitions such as the GDP decomposition or income, etc. In case of an additive equation 4.3.2, the variable Y grows at the rate μ from the first period onward if all its components X grow also at that rate:

$$Y_t = (\sum_{i=1}^{I} X_{i,0}).(1+\mu)^t = Y_0(1+\mu)^t \tag{4.3.3}$$

Moreover in that case all ratios between variables ($\frac{X_i}{Y}$ and $\frac{Xi}{Xj}$) are constant over time. In the case of the GDP equation this seems a realistic long run property. Otherwise the share of each component in the GDP is not stable over time and the long run growth rate of the GDP corresponds to the component X highest growth rate. Indeed, if the X-variables do not grow at the same rate, the growth rate of $Y(\mu)$ converges to the highest X-variable growth rate. And the share of the X-variable with a lower growth rate tends toward zero. This mathematical property may imply unrealistic constraint on the model if one wants to be at the steady states at the first period of the simulation. This is particularly true if one wishes to calibrate the model on real data. We can give 2 examples: for instance, it is unrealistic to assume that a negative inventory change will decrease indefinitely because the level of inventories becomes at some point negative. One possibility is to amend the calibration in order to impose a zero-inventory change at the base years.

In the real world, most countries' imports and exports do not grow at the rate of the GDP but at a higher rate because of the trade globalization. In fact Equation 4.3.2 allows that several X-variables grow at a different rate than Y in the long run as long their sum grows at the same rate as Y. Consequently, imports and exports may grow faster that the GDP forever as long as their effect cancel out, that is as long as the foreign trade balance grows at the rate of the GDP. If the long run foreign trade balance is zero, imports and exports grow at the same rate. If not, they grow at the same rate asymptotically, the smallest (in absolute value) growing faster. This implies mechanism that imposes import and export to grow consistently.

The most common way is to assume that the exchange rate adjusts in order to reach the external balance objective.

4.3.2 Unit elasticity logarithm equations

Many relations in the model impose a unit-elasticity specified in logarithm form:

$$\ln(Y_t) = \ln(X_t) + \alpha \tag{4.3.4}$$

This specification is used for all production factor demand since we systematically assume a constant return-to-scale technology. If the coefficient α is calibrated in the initial period as a simple inversion of equation 4.3.4 and constant over time, this specification implies that Y always grows at the same rate as X.

In the production factors demand, α depends on the relative prices and thus may vary over time in case of shock or if they are not in equilibrium in the initial period. In that case, the growth rate of Y and X differs over time but they tend to converge toward each other provided that mechanisms in the price equation guaranty the long run stability of relative prices.

4.3.3 Accumulation equations

The model contains several accumulation equations: capital stock dynamic, public and private debt, household savings. All can be represented with the following equation:

$$Y_t = Y_{t-1}(1 + \beta) + X_t \tag{4.3.5}$$

In the case of capital accumulation, β is the depreciation rate and is negative. In the case of debt or saving equation,β is the interest rate and is thus positive. At the steady states, X should grow at the same rate as Y. Consequently, being at the stationary states from the first period onward implies that X cannot be calibrated on real data. At the stationary state, X is calibrated as an inversion of Equation4.3.5:

$$Y = \left(\frac{\mu - \beta}{1 + \mu}\right)^{-1} X \tag{4.3.6}$$

Chapter 5

Equations of ThreeME

This chapter provides all the equations of the model. The names of all the variables and parameters are defined in the last section of this chapter (section 5.20, p. 168) and ranked in alphabetical order. This last section provides also the number of the equation defining a given variable and the page where this equation is located.

5.1 Sets and indexes

This section provides the different sets for each index used in the equation description.

t is the time operator that may be omitted when no confusion arises, *e.g.* $Z = Z_t$. Variables in first difference are referred as: $\Delta(Z_t) = Z_t - Z_{t-1}$. Therefore the logarithm difference of a variable is approximately its growth rate: $\Delta(\log Z_t) \approx Z_t/Z_{t-1} - 1$. All parameters written in Greek letter are positive. n as an exponent refers to the notional value of a given variable that is the optimal value desired by the maximization agent: *e.g.* Z^n is the notional value of variable Z. Because of adjustment constraint, effective values adjust slowly to their notional value.

Production factors f

$$f \in \{K, L, E, M\} \tag{5.1.1}$$

Greenhouse Gases ghg

$$ghg \in \{CO2, CH4, N2O, SF6, HFC, PFC\} \tag{5.1.2}$$

Car energy labels ecl

$$ecl \in \{CA, CB, CC, CD, CE, CF, CG\} \tag{5.1.3}$$

Building energy labels bcl

$$bcl \in \{CA, CB, CC, CD, CE, CF, CG, DES\} \tag{5.1.4}$$

Sectors (or activities) s

$$s \in S \tag{5.1.5}$$

Commodities c, cc

$$c, cc \in C \tag{5.1.6}$$

Margins m, mm

$$m, mm \in CMARG \subset C \tag{5.1.7}$$

Commodities used as material input cm

$$cm \in CMAT \subset C \tag{5.1.8}$$

commodities used as transportation input ct

$$ct \in CTRSP \subset C \tag{5.1.9}$$

commodities used as energy input ce

$$ce \in CE \subset C \tag{5.1.10}$$

5.2 Supply-Use equilibrium

This section provides the equations defining the supply use - equilibrium for the domestic and imported products and at the aggregate level. It also derives the GDP according to various definitions. All equations are therefore accounting identity.

Each identity is expressed in value and in volume. The value equation defines therefore a price index. By convention, the variable Z is always expressed in volume. PZ is its price. Therefore the quantity expressed in value is $Z^{VAL} = PZ * Z$. In most case, values are expressed as $PZ * Z$. When there is a risk that a variable in volume is equal to zero in simulation, we shall defined value as Z^{VAL} to avoid a division by zero issue. A typical example would be the value-added of sector s: $VA_s^{VAL} = PY_s\, Y_s - PCI_s\, CI_s$.

5.2.1 Use side

5.2.1.1 Domestic and foreign equilibrium for commodities c (value & volume):

Market price for the domestically produced commodity c

$$\begin{aligned} PQD_c\, QD_c = {} & PMGSD_c\, MGSD_c + PCID_c\, CID_c + PCHD_c\, CHD_c \\ & + PGD_c\, GD_c + PID_c\, ID_c + PXD_c\, XD_c + PDSD_c\, DSD_c \end{aligned} \tag{5.2.1}$$

Quantity of domestically produced commodity c **expressed at market price**

$$QD_c = MGSD_c + CID_c + CHD_c + GD_c + ID_c + XD_c + DSD_c \tag{5.2.2}$$

Market price for imported commodity c

$$\begin{aligned} PQM_c\, QM_c = {} & PMGSM_c\, MGSM_c + PCIM_c\, CIM_c + PCHM_c\, CHM_c \\ & + PGM_c\, GM_c + PIM_c\, IM_c + PXM_c\, XM_c + PDSM_c\, DSM_c \end{aligned} \tag{5.2.3}$$

Quantity of imported commodity c expressed at market price

$$QM_c = MGSM_c + CIM_c + CHM_c + GM_c + IM_c + XM_c + DSM_c \tag{5.2.4}$$

5.2.1.2 Margins supplied (value & volume):

The margins supplied by commodity m corresponds to the margins supplied by this commodity. By definition, the margins supplied is the sum of the margins paid (or used) on the commodities c.

The margins paid on domestic and imported products ($MGPD_{m,c}$ and $MGPM_{m,c}$) are defined with behavioral equations in the producer block. They follow the domestic and imported production of commodity c (YQ_c and M_c) more or less proportionally depending on the possibility of substitutions between margins. The margins paid are then aggregated to define the margins supplied, MGS_m. The latter is then disaggregated between the domestic and imported margins supplied ($MGSD_c$ and $MGSM_c$). See specification in the producer block.

Market price of the margins supplied by commodity m

$$PMGS_m\, MGS_m = \sum_c PMGP_{m,c}\, MGP_{m,c} \tag{5.2.5}$$

Margins supplied by commodity m, expressed at market price

$$MGS_m = \sum_c MGP_{m,c} \tag{5.2.6}$$

The margins supplied correspond to the sum of the margins paid to commodity m over all the commodities c

Market price of the margins supplied by commodity m (for verification)

$$PMGS_m^{bis}\, MGS_m^{bis} = PMGSD_m\, MGSD_m + PMGSM_m\, MGSM_m \tag{5.2.7}$$

Same variable as $PMGS_m$ above to check the accounting consistency.

Margins supplied by commodity m, expressed at market price (for verification)

$$MGS_m^{bis} = MGSD_m + MGSM_m \tag{5.2.8}$$

Same variable as MGS_m above to check the accounting consistency.

5.2.1.3 Aggregation of imports and domestic production for commodity c per use, expressed at market price (value & volume)

This subsection aggregates imports and domestic production for commodity c for various uses. It does not do it for Exports, Households and Government final consumption (X, CH and G) because these aggregates are already defined in behaviour equations (see Trade international, Consumer and Government blocks). Expressed in value, this subsection also defines the prices indexes at market price for commodity c per use as a weighted average of imported and domestic production per uses: *i.e.* for Q (production of commodities), CI (intermediary consumption), I (private investment) and DS (change in inventories).

Market price of the production of commodity c

$$PQ_c\, Q_c = PQD_c\, QD_c + PQM_c\, QM_c \tag{5.2.9}$$

Production of commodity c, expressed at market price

$$Q_c = QD_c + QM_c \tag{5.2.10}$$

Market price of the intermediate consumption of commodity c

$$PCI_c\, CI_c = PCID_c\, CID_c + PCIM_c\, CIM_c \tag{5.2.11}$$

Intermediate consumption of commodity c, expressed at market price

$$CI_c = CID_c + CIM_c \tag{5.2.12}$$

Market price of the investment in commodity c

$$PI_c\, I_c = PID_c\, ID_c + PIM_c\, IM_c \tag{5.2.13}$$

Investment in commodity c, expressed at market price

$$I_c = ID_c + IM_c \tag{5.2.14}$$

Market price of the change in inventories of commodity c

$$PDS_c\, DS_c = PDSD_c\, DSD_c + PDSM_c\, DSM_c \tag{5.2.15}$$

Change in inventories of commodity c, expressed at market price

$$DS_c = DSD_c + DSM_c \tag{5.2.16}$$

5.2.1.4 Aggregation on sectors: production of commodity c per use for domestic and imported products, expressed at market price (value & volume)

This subsection provides the aggregates for commodity c for various uses, for domestic and imported products. They are calculated through the aggregation of the corresponding sectorial data on the sector index.

Market price for the domestically produced commodity c used as intermediary consumption

$$PCID_c \, CID_c = \sum_s PCID_{c,s} \, CID_{c,s} \tag{5.2.17}$$

Quantity of domestically produced commodity c used as intermediary consumption, expressed at market price

$$CID_c = \sum_s CID_{c,s} \tag{5.2.18}$$

Market price for imported commodity c used as intermediary consumption

$$PCIM_c \, CIM_c = \sum_s PCIM_{c,s} \, CIM_{c,s} \tag{5.2.19}$$

Quantity of imported commodity c used as intermediary consumption, expressed at market price

$$CIM_c = \sum_s CIM_{c,s} \tag{5.2.20}$$

Market price for domestically produced commodity c used as investment

$$PID_c \, ID_c = \sum_s PID_{c,s} \, ID_{c,s} \tag{5.2.21}$$

Quantity of imported commodity c used as investment, expressed at market price

$$ID_c = \sum_s ID_{c,s} \tag{5.2.22}$$

Market price for imported commodity c used as investment

$$PIM_c \, IM_c = \sum_s PIM_{c,s} \, IM_{c,s} \tag{5.2.23}$$

Quantity of imported commodity c used as investment, expressed at market price

$$IM_c = \sum_s IM_{c,s} \tag{5.2.24}$$

5.2.1.5 Aggregation on commodities: imported, domestic and aggregate intermediate consumption and investment of sector s, expressed at market price (value & volume)

This subsection provides the intermediate consumption and investment of sector s (imported, domestically produced and aggregated). They are calculated through the aggregation of the corresponding sectorial data on the commodity index.

Market price of domestically produced intermediate consumption of sector s

$$PCID_s\, CID_s = \sum_c PCID_{c,s}\, CID_{c,s} \tag{5.2.25}$$

Domestically produced intermediate consumption of sector s, expressed at market price

$$CID_s = \sum_c CID_{c,s} \tag{5.2.26}$$

Market price of imported intermediate consumption of sector s

$$PCIM_s\, CIM_s = \sum_c PCIM_{c,s}\, CIM_{c,s} \tag{5.2.27}$$

Imported intermediate consumption of sector s, expressed at market price

$$CIM_s = \sum_c CIM_{c,s} \tag{5.2.28}$$

Market price of intermediate consumption of sector s

$$PCI_s\, CI_s = PCID_s\, CID_s + PCIM_s\, CIM_s \tag{5.2.29}$$

Intermediate consumption of sector s, expressed at market price

$$CI_s = CID_s + CIM_s \tag{5.2.30}$$

Market price of intermediate consumption of sector s (for verification)

$$PCI^{bis}.CI^{bis} = \sum_s PCI_s\, CI_s \tag{5.2.31}$$

Intermediate consumption of sector s, expressed at market price (for verification)

$$CI^{bis} = \sum_s CI_s \tag{5.2.32}$$

Market price of domestically produced investment of sector s

$$PID_s\, ID_s = \sum_c PID_{c,s}\, ID_{c,s} \tag{5.2.33}$$

Domestically produced investment of sector s, expressed at market price

$$ID_s = \sum_c ID_{c,s} \tag{5.2.34}$$

Market price of imported investment of sector s

$$PIM_s\, IM_s = \sum_c PIM_{c,s}\, IM_{c,s} \tag{5.2.35}$$

Imported investment of sector s, expressed at market price

$$IM_s = \sum_c IM_{c,s} \tag{5.2.36}$$

Market price of investment of sector s

$$PI_s\, I_s = PID_s\, ID_s + PIM_s\, IM_s \tag{5.2.37}$$

Investment of sector s, expressed at market price

$$I_s = ID_s + IM_s \tag{5.2.38}$$

Market price of investment of sector s (for verification)

$$PI^{bis}.I^{bis} = \sum_s PI_s\, I_s \tag{5.2.39}$$

Investment of sector s, expressed at market price (for verification)

$$I^{bis} = \sum_s I_s \tag{5.2.40}$$

5.2.1.6 Aggregation on commodities: imports and domestic aggregate production per use, expressed at market price (value & volume)

This subsection provides the aggregate production for various uses, for domestic and imported products. They are calculated through the aggregation of commodity c production per use on the commodity index.

Aggregate market price for domestically produced commodities

$$PQD.QD = \sum_c PQD_c\, QD_c \tag{5.2.41}$$

Aggregate domestically produced commodities, expressed at market price

$$QD = \sum_c QD_c \tag{5.2.42}$$

Aggregate market price for imported commodities

$$PQM.QM = \sum_c PQM_c\, QM_c \tag{5.2.43}$$

Aggregate imported commodities, expressed at market price

$$QM = \sum_c QM_c \tag{5.2.44}$$

Aggregate market price for the margins supplied on domestically produced commodities

$$PMGSD.MGSD = \sum_c PMGSD_c\, MGSD_c \tag{5.2.45}$$

Aggregate margins supplied on domestically produced commodities, expressed at market price

$$MGSD = \sum_{c} MGSD_c \tag{5.2.46}$$

Aggregate market price for the margins supplied on imported commodities

$$PMGSM.MGSM = \sum_{c} PMGSM_c \, MGSM_c \tag{5.2.47}$$

Aggregate margins supplied on imported commodities, expressed at market price

$$MGSM = \sum_{c} MGSM_c \tag{5.2.48}$$

Aggregate market price for domestically produced intermediate consumption

$$PCID.CID = \sum_{c} PCID_c \, CID_c \tag{5.2.49}$$

Aggregate domestically produced intermediate consumption, expressed at market price

$$CID = \sum_{c} CID_c \tag{5.2.50}$$

Aggregate market price for imported intermediate consumption

$$PCIM.CIM = \sum_{c} PCIM_c \, CIM_c \tag{5.2.51}$$

Aggregate imported intermediate consumption, expressed at market price

$$CIM = \sum_{c} CIM_c \tag{5.2.52}$$

Aggregate market price for domestically produced households final consumption

$$PCHD.CHD = \sum_{c} PCHD_c \, CHD_c \tag{5.2.53}$$

Aggregate domestically produced final consumption, expressed at market price

$$CHD = \sum_{c} CHD_c \tag{5.2.54}$$

Aggregate market price for imported households final consumption

$$PCHM.CHM = \sum_{c} PCHM_c \, CHM_c \tag{5.2.55}$$

Aggregate imported households final consumption, expressed at market price

$$CHM = \sum_c CHM_c \tag{5.2.56}$$

Aggregate market price for domestically produced Government final consumption

$$PGD.GD = \sum_c PGD_c \, GD_c \tag{5.2.57}$$

Aggregate domestically produced Government final consumption, expressed at market price

$$GD = \sum_c GD_c \tag{5.2.58}$$

Aggregate market price for imported Government final consumption

$$PGM.GM = \sum_c PGM_c \, GM_c \tag{5.2.59}$$

Aggregate imported Government final consumption, expressed at market price

$$GM = \sum_c GM_c \tag{5.2.60}$$

Aggregate market price for domestically produced investment

$$PID.ID = \sum_c PID_c \, ID_c \tag{5.2.61}$$

Aggregate domestically produced investment, expressed at market price

$$ID = \sum_c ID_c \tag{5.2.62}$$

Aggregate market price for imported investment

$$PIM.IM = \sum_c PIM_c \, IM_c \tag{5.2.63}$$

Aggregate imported investment, expressed at market price

$$IM = \sum_c IM_c \tag{5.2.64}$$

Aggregate market price for domestically produced exports

$$PXD.XD = \sum_c PXD_c \, XD_c \tag{5.2.65}$$

Aggregate domestically produced exports, expressed at market price

$$XD = \sum_c XD_c \tag{5.2.66}$$

Aggregate market price for imported exports (re-exports)

$$PXM.XM = \sum_c PXM_c \, XM_c \tag{5.2.67}$$

Aggregate imported exports (re-exports), expressed at market price

$$XM = \sum_c XM_c \tag{5.2.68}$$

Aggregate market price for domestically produced change in inventories

$$PDSD.DSD = \sum_c PDSD_c \, DSD_c \tag{5.2.69}$$

Aggregate domestically produced change in inventories, expressed at market price

$$DSD = \sum_c DSD_c \tag{5.2.70}$$

Aggregate market price for imported change in inventories

$$PDSM.DSM = \sum_c PDSM_c \, DSM_c \tag{5.2.71}$$

Aggregate imported change in inventories, expressed at market price

$$DSM = \sum_c DSM_c \tag{5.2.72}$$

5.2.1.7 Aggregation of domestic and imported production per use, expressed at market price (value & volume)

This subsection provides the aggregate production for various uses by summing the corresponding domestic and imported aggregates.

Aggregate market price for production

$$PQ.Q = PQD.QD + PQM.QM \tag{5.2.73}$$

Aggregate production, expressed at market price

$$Q = QD + QM \tag{5.2.74}$$

Aggregate market price for supplied margins

$$PMGS.MGS = PMGSD.MGSD + PMGSM.MGSM \tag{5.2.75}$$

Aggregate supplied margins

$$MGS = MGSD + MGSM \tag{5.2.76}$$

Aggregate market price for intermediate consumption

$$PCI.CI = PCID.CID + PCIM.CIM \tag{5.2.77}$$

Aggregate intermediate consumption, expressed at market price

$$CI = CID + CIM \tag{5.2.78}$$

Aggregate market price for household final (consumer price index)

$$PCH.CH = PCHD.CHD + PCHM.CHM \tag{5.2.79}$$

Aggregate household final consumption, expressed at market price

$$CH = CHD + CHM \tag{5.2.80}$$

Aggregate market price for Government final consumption

$$PG.G = PGD.GD + PGM.GM \tag{5.2.81}$$

Aggregate Government final consumption, expressed at market price

$$G = GD + GM \tag{5.2.82}$$

Aggregate market price for investment

$$PI.I = PID.ID + PIM.IM \tag{5.2.83}$$

Aggregate investment, expressed at market price

$$I = ID + IM \tag{5.2.84}$$

Aggregate market price for exports

$$PX.X = PXD.XD + PXM.XM \tag{5.2.85}$$

Aggregate exports, expressed at market price

$$X = XD + XM \tag{5.2.86}$$

Aggregate market price for change in inventories

$$PDS.DS = PDSD.DSD + PDSM.DSM \tag{5.2.87}$$

Aggregate change in inventories, expressed at market price

$$DS = DSD + DSM \tag{5.2.88}$$

5.2.2 Supply side

5.2.2.1 Domestic and foreign equilibrium for commodities c (value & volume):

Production of commodity c, expressed at basic price

$$YQ_c \, PYQ_c + NTAXCD_c^{VAL} + PMGPD_c \, MGPD_c = PQD_c \, QD_c \quad (5.2.89)$$

Basic price of the production of commodity c (for verification)

$$PYQ_c^{bis} \, YQ_c + NTAXCD_c^{VAL} + PMGPD_c \, MGPD_c = PQD_c \, QD_c \quad (5.2.90)$$

This price is already defined as a weighted average of the production price of the sectors producing commodity c in the price block: $PYQ.YQ = \sum_c PYQ_c \, YQ_c$. To verify the accounting consistency, we define it here under an alias name.

Production of commodity c, expressed at basic price (for verification)

$$YQ_c^{bis} + NTAXCD_c + MGPD_c = QD_c \quad (5.2.91)$$

Same variable as YQ_c above to check the accounting consistency.

Imports of commodity c, expressed at basic price

$$M_c \, PM_c + NTAXCM_c^{VAL} + PMGPM_c \, MGPM_c = PQM_c \, QM_c \quad (5.2.92)$$

Basic price of imports of commodity c (for verification)

$$PM_c^{bis} \, M_c + NTAXCM_c^{VAL} + PMGPM_c \, MGPM_c = PQM_c \, QM_c \quad (5.2.93)$$

This price is already defined in the price block as $PM_c = EXR.PWD_c$. To verify the accounting consistency, we define it here under an alias name.

Imports of commodity c, expressed at basic price (for verification)

$$M_c^{bis} + NTAXCM_c + MGPM_c = QM_c \quad (5.2.94)$$

Same variable as M_c above to check the accounting consistency.

5.2.2.2 Margins paid (value & volume)

Price of the margins paid on domestically produced commodity c

$$PMGPD_c \, MGPD_c = \sum_m PMGPD_{m,c} \, MGPD_{m,c} \quad (5.2.95)$$

Margins paid on the domestically produced commodity c

$$MGPD_c = \sum_m MGPD_{m,c} \quad (5.2.96)$$

Price of the margins paid on imported commodity c

$$PMGPM_c\, MGPM_c = \sum_m PMGPM_{m,c}\, MGPM_{m,c} \tag{5.2.97}$$

Margins paid on imported commodity c

$$MGPM_c = \sum_m MGPM_{m,c} \tag{5.2.98}$$

Price of the margins paid to commodity m on commodity c

$$PMGP_{m,c}\, MGP_{m,c} = PMGPD_{m,c}\, MGPD_{m,c} + PMGPM_{m,c}\, MGPM_{m,c} \tag{5.2.99}$$

Margins paid to commodity m on commodity c

$$MGP_{m,c} = MGPD_{m,c} + MGPM_{m,c} \tag{5.2.100}$$

5.2.2.3 Aggregation on commodities: supply side aggregates (value & volume)

Aggregate price of the margins paid on domestically produced commodity

$$PMGPD.MGPD = \sum_c PMGPD_c\, MGPD_c \tag{5.2.101}$$

Margins paid on domestically produced commodities

$$MGPD = \sum_c MGPD_c \tag{5.2.102}$$

Aggregate price of the margins paid on imported commodities

$$PMGPM.MGPM = \sum_c PMGPM_c\, MGPM_c \tag{5.2.103}$$

Margins paid on imported commodities

$$MGPM = \sum_c MGPM_c \tag{5.2.104}$$

Aggregate basic price of domestic production

$$PYQ.YQ = \sum_c PYQ_c\, YQ_c \tag{5.2.105}$$

Domestic production, expressed at basic price

$$YQ = \sum_c YQ_c \tag{5.2.106}$$

Aggregate basic price of imports

$$PM.M = \sum_c PM_c\, M_c \tag{5.2.107}$$

Imports, expressed at basic price

$$M = \sum_c M_c \tag{5.2.108}$$

5.2.2.4 Supply indicators of sector s (value & volume):

Production of sector s, expressed at basic price

$$Y_s = \sum_c Y_{c,s} \tag{5.2.109}$$

The production price of sector s is defined in the producer block as a behavior equation. It can not therefore be defined here as an index.

Value-added of sector s expressed in value

$$VA_s^{VAL} = PY_s\, Y_s - PCI_s\, CI_s \tag{5.2.110}$$

Value-added of sector s

$$VA_s = Y_s - CI_s \tag{5.2.111}$$

Gross operating surplus of sector s expressed in value

$$GOS_s^{VAL} = VA_s^{VAL} - PWAGES_s\, WAGES_s - PRSC_s\, RSC_s - NTAXS_s^{VAL} \tag{5.2.112}$$

The standard definition of the Gross Operating Surplus (GOS) generally include tax on profits. For simplicity, we assume that $NTAXS_s$ includes all net taxes on capital (i.e. tax on production and profits). In our definition, the tax on profit is therefore excluded from the GOS. This should be taken into account if one wants to use the GOS as a basis for the tax on profits.

Gross operating surplus of sector s

$$GOS_s = VA_s - WAGES_s - RSC_s - NTAXS_s \tag{5.2.113}$$

Net operating surplus of sector s expressed in value

$$NOS_s^{VAL} = GOS_s^{VAL} - PK_{s,t-1}\, \delta_s\, F_{K,s,t-1} \tag{5.2.114}$$

Net operating surplus of sector s

$$NOS_s = GOS_s - PK_{s,t_0-1}\, \delta_s\, F_{K,s,t-1} \tag{5.2.115}$$

5.2.2.5 Aggregation on sectors: supply indicators of all sectors (value & volume)

Basic price of aggregate production

$$PY.Y = \sum_s PY_s\, Y_s \tag{5.2.116}$$

Aggregate production, expressed at basic price

$$Y = \sum_s Y_s \tag{5.2.117}$$

Value-added price

$$PVA.VA = \sum_s VA_s^{VAL} \tag{5.2.118}$$

Aggregate value-added

$$VA = \sum_s VA_s \tag{5.2.119}$$

Gross wage index paid by sectors

$$PWAGES.WAGES = \sum_s PWAGES_s\, WAGES_s \tag{5.2.120}$$

The gross wage includes employees (but not employers)' social contribution

Aggregate gross wages paid by sectors

$$WAGES = \sum_s WAGES_s \tag{5.2.121}$$

Price of the aggregate gross operating surplus

$$PGOS.GOS = \sum_s GOS_s^{VAL} \tag{5.2.122}$$

Aggregate gross operating surplus

$$GOS = \sum_s GOS_s \tag{5.2.123}$$

Price of the aggregate net operating surplus

$$PNOS.NOS = \sum_s NOS_s^{VAL} \tag{5.2.124}$$

Aggregate net operating surplus

$$NOS = \sum_s NOS_s \tag{5.2.125}$$

5.2.3 Gross Domestic Product (GDP)

In this subsection, GDP is calculated according to different approaches. All approaches lead to same result.

5.2.3.1 Expenditure approach

Price of GDP (expenditure definition)

$$PGDP.GDP = PCH.CH + PG.G + PI.I + PX.X + PDS.DS - PM.M \tag{5.2.126}$$

According to the expenditure approach, GDP is calculated as the sum of the different components in the final uses of goods and services.

GDP (expenditure definition)

$$GDP = CH + G + I + X + DS - M \tag{5.2.127}$$

Price of GDP of commodity c (expenditure definition)

$$PGDP_c\, GDP_c = PCH_c\, CH_c + PG_c\, G_c + PI_c\, I_c + PX_c\, X_c + PDS_c\, DS_c - PM_c\, M_c \tag{5.2.128}$$

GDP of commodity c (expenditure definition)

$$GDP_c = CH_c + G_c + I_c + X_c + DS_c - M_c \tag{5.2.129}$$

Price of GDP (expenditure definition, for verification)

$$PGDP^{bis}.GDP^{bis} = \sum_c PGDP_c\, GDP_c \tag{5.2.130}$$

GDP (expenditure definition, for verification)

$$GDP^{bis} = \sum_c GDP_c \tag{5.2.131}$$

5.2.3.2 Production approach

Price of GDP (production definition)

$$PGDP^{ter}.GDP^{ter} = PVA.VA + PNTAXC.NTAXC \tag{5.2.132}$$

According to the production approach, GDP is calculated as the sum of the value added plus the total net taxes on commodities.

GDP (production definition)

$$GDP^{ter} = VA + NTAXC \tag{5.2.133}$$

5.2.3.3 Income approach

Price of GDP (income definition)

$$\begin{aligned} PGDP4.GDP4 = {} & PGOS.GOS + PWAGES.WAGES + PRSC.RSC \\ & + NTAXS^{VAL} + PNTAXC.NTAXC \end{aligned} \tag{5.2.134}$$

According to the income approach, GDP is calculated as the sum of all the economic incomes (from labor and capital) corrected by the social and taxes transfers.

GDP (income definition)

$$GDP4 = GOS + WAGES + RSC + NTAXS + NTAXC \tag{5.2.135}$$

5.3 Prices

This section provides the equations defining the prices, wages and the interest rates.

5.3.1 Behavioral equations

5.3.1.1 Price setting

Notional production price of sector s

$$PY_s^n = CU_s^n \; (1+\mu_s) \tag{5.3.1}$$

Notional mark-up of the sector s (specification 1)

$$\Delta \left(\log \left(1+\mu_s^n\right)\right) = \rho^{\mu,Y}.\Delta \left(\log CUR_s\right) \tag{5.3.2}$$

The notional mark-up is a positive function of the capacity utilization ratio. One can notice that this specification is strictly equivalent to: $\Delta \left(\log 1+\mu_s^n\right) = \rho^{\mu,Y}.\left(\Delta \left(\log Y_s\right) - \Delta \left(\log YCAP_s\right)\right)$ which is quite close to the specification 2 below where the production capacity $YCAP_s$ is replaced by the past production $Y_{s,t-1}$.

Notional mark-up of the sector s (specification 2)

$$\Delta \left(\log \left(1+\mu_s^{n2}\right)\right) = \rho^{\mu,Y}.\left(\Delta \left(\log Y_s\right) - \Delta \left(\log Y_{s,t-1}\right)\right) \tag{5.3.3}$$

This specification can be used instead of the default specification 1 above.

Production capacity of the sector s

$$\begin{aligned}\Delta \left(\log YCAP_s\right) = &\sum_f \varphi_{f,s,t-1} \; \Delta \left(\log \left(F_{f,s} \; PROG_{f,s}\right)\right) \\ &+ \alpha_s^{YCAP,Y} \; \left(\log Y_{s,t-1} - \log \left(YCAP_{s,t-1} \; CUR_{s,t_0}\right)\right)\end{aligned} \tag{5.3.4}$$

The production capacity is defined by the production function and the effective quantities of inputs.

Capacity utilization ratio of the sector s

$$CUR_s = \frac{Y_s}{YCAP_s} \tag{5.3.5}$$

Average mark-up on commodity c

$$(1+\mu_c) = \frac{PYQ_c \; YQ_c}{\sum_s CU_s \; Y_{c,s}} \tag{5.3.6}$$

5.3.1.2 Wage setting

Notional wage in sector s

$$\begin{aligned}\Delta\left(\log W_s^n\right) = {} & \rho_s^{W,Cons} + \rho_s^{W,P}\ \Delta\left(\log P\right) + \rho_s^{W,Pe}\ \Delta\left(\log P^e\right) \\ & + \rho_s^{W,PROG}\ \Delta\left(\log PROG_s^L\right) - \rho_s^{W,U}\ \left(UnR - DNAIRU\right) \\ & - \rho_s^{W,DU}\ \Delta\left(UnR\right) + \rho_s^{W,L}\ \Delta\left(\log F_{L,s} - \log F_L\right)\end{aligned} \tag{5.3.7}$$

This general specification combines various wage equations found in the literature: the Phillips curve and the WS curve. The WS curve á la Layard et al. (2005) requires the following constraints:

$$\rho_s^{W,P} + \rho_s^{W,Pe} = \rho_s^{W,PROG} = 1, \rho_s^{W,U} = \rho_s^{W,Cons} = 0$$

In the case of a Phillips curve, the $NAIRU$ is predetermined in the long run when wages are fully indexed on price and productivity. To have a long run "time-varying" $NAIRU$ under these hypotheses, we have to assume that the constant is a function of the unemployment rate (see Heyer et al. 2007):

$$\Delta\left(\rho_s^{W,Cons}\right) = \rho_s^{Cons,U}\ \rho_s^{W,U}\ \Delta\left(UnR\right) \tag{5.3.8}$$

Average wage

$$W.F_L = \left(\sum_s W_s\ F_{L,s}\right) \tag{5.3.9}$$

Consumer Price Index

$$P = PCH \tag{5.3.10}$$

Gross wages paid by sector s including employees (but not employers)' social contribution

$$WAGES_s\ PWAGES_s = W_s\ F_{L,s} \tag{5.3.11}$$

To derive the volume, we assume that the price of wages is the consumer price.

Price index for gross wages in sector s

$$PWAGES_s = P \tag{5.3.12}$$

5.3.1.3 Interest rate setting

Notional key interest rate of the Central Bank (Taylor rule)

$$\Delta\left(R^n\right) = \rho^{Rn,Cons} + \rho^{Rn,P}.\Delta\left(\frac{\Delta\left(P\right)}{P_{t-1}}\right) - \rho^{Rn,UnR}.\Delta\left(UnR\right) \tag{5.3.13}$$

Interest rate paid on capital by sector s

$$\Delta\left(R_s\right) = \Delta\left(R\right) \tag{5.3.14}$$

We assume a constant premium on the key interest rate of the Central Bank.

Interest rate paid by the Government on its debt

$$\Delta\left(r^{DEBT,G}\right) = \Delta\left(r\right) \tag{5.3.15}$$

5.3.2 Costs

Notional unit cost of production in sector s

$$CU^n_s\ Y_s = \sum_f C_{f,s}\ F^n_{f,s} + NTAXS^{VAL}_s \tag{5.3.16}$$

The notional price is based on the notional unit cost of production instead of the effective one. This leads to a more stable dynamic and gives a better representation of expectations.

Unit cost of production in sector s

$$CU_s\ Y_s = \sum_f C_{f,s}\ F_{f,s} + NTAXS^{VAL}_s \tag{5.3.17}$$

Labor cost in sector s

$$C_{L,s}\ F_{L,s} = W_s\ F_{L,s} + RSC_s\ PRSC_s \tag{5.3.18}$$

Capital cost in sector s

$$C_{K,s} = PK_s\ (\delta_s + r_s) \tag{5.3.19}$$

It is preferable to calculate the user cost of capital based on the price of capital rather than on the price of investment. Indeed the price of the average capital installed is lower than the one of investment because of inflation. Using the price of investment tends to overestimate the cost of capital because it assumes that the debt contracted to finance past investments is indexed on inflation which is not the case in reality. Moreover using the price of investment often leads to unstable dynamics because it overestimates the impact of inflationary shocks on the cost of capital.

Price of capital in sector s

$$PK_s\ F_{K,s} = (1-\delta_s)\ PK_{s,t-1}\ F_{K,s,t-1} + PI_s\ I_s \tag{5.3.20}$$

The price of capital is calibrated by rewriting the above equation in the long run. Its baseyear value is always smaller than 1 because it is calibrated as follows:

$$PK_s = \frac{PI_s * (\delta_s + g^{REAL}) * (1+g^{PRICES})}{(Rdep_s - 1 + (1+g^{REAL}) * (1+GR^{PRICES}))}$$

Energy costs in sector s

$$C_{E,s} = PE_s \tag{5.3.21}$$

In first approximation the cost of energy correspond to the energy price. However if the producer is forward-looking, she will integrate the anticipation of price increase in its definition of the user cost of energy and consider the average cost (eventually discounted) over the life time of the equipment. In this case of a discounta rate of zero, the specification becomes:

Energy costs in sector s (forward-looking specification)

$$C^{bis}_{E,s} = PE_s \, \frac{\left(\frac{PE^e_s}{PE^e_{s,t-1}}\right)^{\left(\frac{1}{\delta_s}\right)-1}}{\left(\frac{PE^e_s}{PE^e_{s,t-1}}-1\right)\,\left(\frac{1}{\delta_s}\right)} \tag{5.3.22}$$

Expected energy price inflation

$$\Delta\left(\log PE^e_s\right) = \alpha^{PEe,PE1}_s \, \Delta\left(\log PE_{s,t-1}\right) + \left(1-\alpha^{PEe,PE1}_s\right) \, \Delta\left(\log PE^e_{s,t-1}\right) \tag{5.3.23}$$

Price of energy in sector s

$$PE_s \, F_{E,s} = \sum_{ce} PCI_{ce,s} \, CI_{ce,s} \tag{5.3.24}$$

Materials costs in sector s

$$C_{MAT,s} = PMAT_s \tag{5.3.25}$$

Price of materials in sector s

$$PMAT_s \, F_{MAT,s} = \sum_{cm} PCI_{cm,s} \, CI_{cm,s} \tag{5.3.26}$$

Aggregate cost of capital

$$C_K \, F_K = \sum_{s} C_{K,s} \, F_{K,s} \tag{5.3.27}$$

Aggregate cost of labor

$$C_L \, F_L = \sum_{s} C_{L,s} \, F_{L,s} \tag{5.3.28}$$

Aggregate cost of energy

$$C_E \, F_E = \sum_{s} C_{E,s} \, F_{E,s} \tag{5.3.29}$$

Aggregate cost of materials

$$C_{MAT} \, F_{MAT} = \sum_{s} C_{MAT,s} \, F_{MAT,s} \tag{5.3.30}$$

5.3.3 From basic to market prices for various uses

In the basic version of the model presented in this sub-section, we do not differentiate the market prices for various uses. For instance, the market price of household's final consumption is the same as the ones of intermediary consumption and exports. This hypotheses is often made in CGE models because it is easier to calibrate. But it is not very realistic for two reasons. First, the import content of household consumption is generally higher than the one of exports. Second, household consumption, intermediary consumption and exports are not subject to the same taxes. For instance, there is generally no VAT on exports and intermediary consumption. These hypotheses are changed in the tax block.

Price of domestically produced commodity c, expressed at basic price

$$PYQ_c\, YQ_c = \sum_s PY_s\, Y_{c,s} \tag{5.3.31}$$

Price of imported commodity c, expressed at basic price

$$PM_c = EXR.PWD_c \tag{5.3.32}$$

Price of domestically produced commodity c, expressed at market price

$$PYQS_c\, YQS_c = PYQ_c\, YQ_c + PMGPD_c\, MGPD_c + NTAXCD_c^{VAL} \tag{5.3.33}$$

YQS_c is the volume of the production expressed at market price. It should not be seen as a composite of several “goods“: production at base price, margins and taxes. Its does not increase when the volume of the margins and taxes increase. The price does instead. This is equivalent to assuming that YQS_c is always proportional to and YQ_c since the volume of margins and taxes depends on the latter. Writing it following the specification composite of several goods, $YQS_c = YQ_c + MGPD_c + NTAXCD_c$, would lead to inaccurate results since a decrease in the quantity of margins used per unit of production would not lead to a decrease of the selling price.

Production of commodity c, expressed at market price

$$YQS_c = YQ_c + MGPD_c + NTAXCD_c \tag{5.3.34}$$

Price of imported commodity c, expressed at market price

$$PMS_c\, MS_c = PM_c\, M_c + NTAXCM_c^{VAL} + PMGPM_c\, MGPM_c \tag{5.3.35}$$

Imports of commodity c, expressed at market price

$$MS_c = M_c + NTAXCM_c + MGPM_c \tag{5.3.36}$$

Market price of the margins paid to commodity m on domestically produced commodity c

$$PMGPD_{m,c}\, MGS_m = PMGSD_m\, MGSD_m + PMGSM_m\, MGSM_m \tag{5.3.37}$$

We assume that the margins paid on domestic and imported commodities can be produced by domestic and foreign (using the import share of the margin supplied). The price of the margins paid to commodity m is assumed common to all commodity c.

Market price of the margins paid to commodity m on imported commodity c

$$PMGPM_{m,c} = PMGPD_{m,c} \tag{5.3.38}$$

This price is the same as the one paid on domestic commodity because of the assumption given in the previous equation.

Market price of margins supplied by domestically produced commodity c

$$PMGSD_c = PYQS_c \tag{5.3.39}$$

Market price of margins supplied by imported commodity c

$$PMGSM_c = PMS_c \tag{5.3.40}$$

Market price of domestically produced intermediate consumption c **purchased by sector** s

$$PCID_{c,s} = PYQS_c \tag{5.3.41}$$

Market price of imported intermediate consumption c **purchased by sector** s

$$PCIM_{c,s} = PMS_c \tag{5.3.42}$$

Market price of domestically produced households final consumption c

$$PCHD_c = PYQS_c \tag{5.3.43}$$

Market price of imported households final consumption c

$$PCHM_c = PMS_c \tag{5.3.44}$$

Market price of domestically produced Government final consumption c

$$PGD_c = PYQS_c \tag{5.3.45}$$

Market price of imported Government final consumption c

$$PGM_c = PMS_c \tag{5.3.46}$$

Market price of domestically produced investment c **purchased by sector** s

$$PID_{c,s} = PYQS_c \tag{5.3.47}$$

Market price of imported investment c **purchased by sector** s

$$PIM_{c,s} = PMS_c \tag{5.3.48}$$

Market price of domestically produced exports c

$$PXD_c = PYQS_c \tag{5.3.49}$$

Market price of imported exports (re-exports) c

$$PXM_c = PMS_c \tag{5.3.50}$$

Market price of domestically produced change in inventories c

$$PDSD_c = PYQS_c \tag{5.3.51}$$

Market price of imported change in inventories c

$$PDSM_c = PMS_c \tag{5.3.52}$$

5.3.4 Average market prices for various uses (aggregation of domestic and imported uses)

Market price of intermediate consumption c purchased by sector s

$$PCI_{c,s}\,CI_{c,s} = PCID_{c,s}\,CID_{c,s} + PCIM_{c,s}\,CIM_{c,s} \tag{5.3.53}$$

Market price of households final consumption c

$$PCH_c\,CH_c = PCHD_c\,CHD_c + PCHM_c\,CHM_c \tag{5.3.54}$$

Market price of Government final consumption c

$$PG_c\,G_c = PGD_c\,GD_c + PGM_c\,GM_c \tag{5.3.55}$$

Market price of exports c

$$PX_c\,X_c = PXD_c\,XD_c + PXM_c\,XM_c \tag{5.3.56}$$

5.4 Producer

This section provides the equations defining the producer behaviour.

5.4.1 Margins

Margins paid to commodity m on the domestic commodity c

$$\Delta\left(\log MGPD_{m,c}\right) = \Delta\left(\log YQ_c\right) + \Delta\left(SUBST^{MGPD}_{m,c}\right) \tag{5.4.1}$$

The demand for margins m paid on commodity c depends on the demand for the commodity c and on the substitution between margins type.

Notional substitution between margin m and the other margin types mm paid on domestic commodity c

$$SUBST^{MGPD,n}_{m,c} = \sum_{mm} -\eta^{MGPD}_{m,mm,c}\,\varphi^{MGPD}_{mm,c,t-1}\,\Delta\left(\log PMGPD_{m,c} - \log PMGPD_{mm,c}\right) \tag{5.4.2}$$

Share of margin type m into the total margins paid on domestic commodity c

$$\varphi^{MGPD}_{m,c} = \frac{PMGPD_{m,c}\,MGPD_{m,c}}{\sum_{mm} PMGPD_{mm,c}\,MGPD_{mm,c}} \tag{5.4.3}$$

Margins paid to commodity m on the imported commodity c

$$\Delta\left(\log MGPM_{m,c}\right) = \Delta\left(\log M_c\right) + \Delta\left(SUBST^{MGPM}_{m,c}\right) \tag{5.4.4}$$

Notional substitution between margin m and the other margin types mm paid on imported commodity c

$$SUBST^{MGPM,n}_{m,c} = \sum_{mm} -\eta^{MGPM}_{m,mm,c}\,\varphi^{MGPM}_{mm,c,t-1}\,\Delta\left(\log PMGPM_{m,c} - \log PMGPM_{mm,c}\right) \tag{5.4.5}$$

Share of margin type m into the total margins paid on imported commodity c

$$\varphi_{m,c}^{MGPM} = \frac{PMGPM_{m,c}\ MGPM_{m,c}}{\sum_{mm} PMGPM_{mm,c}\ MGPM_{mm,c}} \tag{5.4.6}$$

5.4.2 Production factors

Production of commodity c by sector s

$$Y_{c,s} = \varphi_{c,s}^{Y}\ YQ_c \tag{5.4.7}$$

We assume that each sector s may produce more than one commodity c. Therefore the production Y of commodity c by sector s depends on the parameter $\varphi_{c,s}^{Y}$ which represents the share of sector s in the total production of commodity c.

Demand for production factor f by sector s

$$\Delta\left(\log F_{f,s}^{n}\right) = \Delta\left(\log Y_s\right) - \Delta\left(\log PROG_{f,s}\right) + \Delta\left(SUBST_{f,s}^{F}\right) \tag{5.4.8}$$

Notional substitution between input f and the other inputs ff

$$\Delta\left(SUBST_{f,s}^{F,n}\right) = \sum_{ff} -ES_{f,ff,s}\ \varphi_{ff,s,t-1}\ \Delta\left(\log\frac{C_{f,s}}{PROG_{f,s}} - \log\frac{C_{ff,s}}{PROG_{ff,s}}\right) \tag{5.4.9}$$

Cost share of input f for sector s

$$\varphi_{f,s} = \frac{C_{f,s}\ F_{f,s}^{n}}{\sum_{ff} C_{ff,s}\ F_{ff,s}^{n}} \tag{5.4.10}$$

Aggregate production factors f

$$F_f = \sum_{s} F_{f,s} \tag{5.4.11}$$

Investment in commodity c by sector s

$$\Delta\left(\log I_{c,s}\right) = \Delta\left(\log IA_s\right) \tag{5.4.12}$$

For a given sector, we assume that the investment structure is fixed over time. In other words, the investment good is a composite of several commodities in fixed proportion.

Energy consumption ce of sector s

$$\Delta\left(\log CI_{ce,s}\right) = \Delta\left(\log F_{E,s}\right) + \Delta\left(SUBST_{ce,s}^{CI}\right) \tag{5.4.13}$$

Notional substitution between energy commodity ce and the other energy commodities cee in the sector s

$$\Delta\left(SUBST_{ce,s}^{CI,n}\right) = \sum_{cee} -\eta_{ce,cee,s}^{NRJ}\ \varphi_{E,cee,s,t-1}\ \Delta\left(\log PCI_{ce,s} - \log PCI_{cee,s}\right) \tag{5.4.14}$$

Share of energy ce into the total energy use of sector s

$$\varphi_{E,ce,s} = \frac{PCI_{ce,s}\ CI_{ce,s}}{\sum_{cee} PCI_{cee,s}\ CI_{cee,s}} \tag{5.4.15}$$

Material consumption cmo of sector s

$$\Delta\left(\log CI_{cmo,s}\right) = \Delta\left(\log F_{MAT,s}\right) \tag{5.4.16}$$

We assume that intermediary consumption that are not transport or energy commodities are not substitutables (Leontief technology).

Transport demand of sector s

$$\Delta\left(\log TRSP_{s}\right) = \Delta\left(\log F_{MAT,s}\right) \tag{5.4.17}$$

Demand for transport commodity ct by sector s

$$\Delta\left(\log CI_{ct,s}\right) = \Delta\left(\log TRSP_{s}\right) + \Delta\left(SUBST^{CI}_{ct,s}\right) \tag{5.4.18}$$

Notional substitution between the transport ct and the other transports mt in the sector s

$$\Delta\left(SUBST^{CI,n}_{ct,s}\right) = \sum_{ctt} -\eta^{TRSP}_{ct,ctt,s}\ \varphi^{TRSP}_{ctt,s,t-1}\ \Delta\left(\log PCI_{ct,s} - \log PCI_{ctt,s}\right) \tag{5.4.19}$$

Share for transport ct into the total transport use of sector s

$$\varphi^{TRSP}_{ct,s} = \frac{PCI_{ct,s}\ CI_{ct,s}}{\sum_{ctt} PCI_{ctt,s}\ CI_{ctt,s}} \tag{5.4.20}$$

Technical progress of the production factor f in sector s

$$PROG_{f,s} = PROG_{f,s,t-1}\ \left(1 + g^{PROG}_{f,s}\right) \tag{5.4.21}$$

Energy efficiency gains in sector s

$$g^{PROG_{E,s}} = g^{PROG_{E,s}}_{t_0} + \rho^{PROG_{E,PE}}\ \left(\log PE_{s} - \log P\right) \tag{5.4.22}$$

This specification states that the productivity gain of the energy input in sector s depends on an exogenous trend and an endogenous price-induced component. Energy efficiency gains increase when the real energy price increases. This effect is assumed as irreversible: a decrease in the real energy price does not lead to a decrease in energy efficiency gains.

5.5 Consumer

This section provides the equations defining the households' income structure as well as their consumption and savings behaviour.

5.5.1 Households' income

Disposable income before tax expressed in value

$$\begin{aligned} DISPINC^{BT,VAL} = {} & PWAGES.WAGES + PROP^{INC,H,VAL} \\ & + SOC^{BENF,VAL} + TRSF^{HH,VAL} \end{aligned} \tag{5.5.1}$$

The disposable income before tax is used as base for the income tax.

Disposable income after tax expressed in value

$$DISPINC^{AT,VAL} = DISPINC^{BT,VAL} - INC^{SOC,TAX,VAL} \tag{5.5.2}$$

The definition of the disposable income after tax corresponds to the definition of "gross disposable income" defined in the annual account by institutional sector of Eurostat (b.6.g).

Income and social taxes expressed in value

$$INC^{SOC,TAX,VAL} = RINC^{SOC,TAX}.DISPINC^{BT,VAL} - T2VAL^{CH}.REDIS^{CT,H} \tag{5.5.3}$$

Property incomes expressed in value

$$PROP^{INC,H,VAL,n} = \varphi^{PROP^{INC,H}}.PNOS.NOS \tag{5.5.4}$$

Social benefits expressed in value

$$SOC^{BENF,VAL} = RR^{POP}.W_{t_0}\ PROG^{L}.P.POP + RR^{Un}.W.Un + DSOC^{BENF,VAL} \tag{5.5.5}$$

5.5.2 Households' expenditures

Aggregate notional households final consumption expressed in value

$$CH^{n,VAL} = DISPINC^{AT,VAL}.\left(1 - MPS^{n}\right) \tag{5.5.6}$$

Notional marginal propensity to save

$$\Delta\left(MPS^{n}\right) = \rho^{MPS,R}.\Delta\left(R - \frac{\Delta\left(P\right)}{P_{t-1}}\right) + \rho^{MPS,UnR}.\Delta\left(UnR\right) \tag{5.5.7}$$

Households' final consumption c

$$\left(CH_c^{n} - NCH_c\right)\ PCH_c = \varphi_c^{MCH}\ \left(CH^{n,VAL} - PNCH.NCH\right) \tag{5.5.8}$$

We assume a Linear Expenditure System (LES) utility function to model consumption decisions between the commodity types. A LES specification assumes that a share of the base year consumption (NCH_c) is incompressible and therefore the relation between income and consumption is not linear. This specification allows for the distinction between the consumption of necessity and luxurious goods.

Price of necessary households consumption c

$$PNCH_c = PCH_c \tag{5.5.9}$$

Price of aggregate necessary households consumption

$$PNCH.NCH = \sum_c PNCH_c \, NCH_c \tag{5.5.10}$$

Aggregate necessary households final consumption

$$NCH = \sum_c NCH_c \tag{5.5.11}$$

Share of commodity c into the total marginal household consumption

$$\Delta\left(\log \varphi_c^{MCH}\right) = \left(1 - \eta^{LESCES}\right).\Delta\left(\log \frac{PCH_c}{PCH^{CES}}\right) \tag{5.5.12}$$

The household marginal propensity to spend in commodity c, φ_c^{MCH}, is generally constant in a LES setting assuming implicitly an elasticity of substitution of one between commodities. We assume here a more general case where the elasticity of substitution can vary from zero to infinity.

Share of commodity c into the total household consumption

$$\varphi_c^{CH} = \frac{CH_c}{CH} \tag{5.5.13}$$

Notice that if $NCH_c = 0$, $\varphi_c^{CH} = \varphi_c^{MCH}$.

CES consumption price index

$$PCH^{CES} = \left(\sum_c \varphi_{c,t_0}^{MCH} \, PCH_c^{\left(1-\eta^{LESCES}\right)}\right)^{\left(\frac{1}{\left(1-\eta^{LESCES}\right)}\right)} \tag{5.5.14}$$

Households savings expressed in value

$$SAV^{H,VAL} = DISPINC^{AT,VAL} - PCH.CH \tag{5.5.15}$$

Households savings rate

$$RSAV^{H,VAL} = \frac{SAV^{H,VAL}}{DISPINC^{AT,VAL}} \tag{5.5.16}$$

Households savings stock

$$Stock^{SAV,H,VAL} = Stock_{t-1}^{SAV,H,VAL} + SAV^{H,VAL} \tag{5.5.17}$$

5.6 Government

5.6.1 Incomes

5.6.1.1 Taxes and social contributions

We assume that the volume of the tax varies only when the volume of the tax bases (e.g. production, consumption) varies. Hence an increase in the tax rate does not increase the volume of the tax but increases its price. This is consistent with the specification of the production price: increasing the tax rate on production increases the production price but not production.

Net taxes on domestically produced commodity c expressed in value

$$NTAXCD_c^{VAL} = RNTAXCD_c\ PYQ_c\ YQ_c \tag{5.6.1}$$

Net taxes on domestically produced commodity c

$$NTAXCD_c = RNTAXCD_{c,t_0}\ YQ_c \tag{5.6.2}$$

Net taxes on imported commodity c expressed in value

$$NTAXCM_c^{VAL} = RNTAXCM_c\ PM_c\ M_c \tag{5.6.3}$$

Net taxes on imported commodity c

$$NTAXCM_c = RNTAXCM_{c,t_0}\ M_c \tag{5.6.4}$$

Net taxes on commodity c expressed in value

$$NTAXC_c^{VAL} = NTAXCD_c^{VAL} + NTAXCM_c^{VAL} \tag{5.6.5}$$

Net taxes on commodity c

$$NTAXC_c = NTAXCD_c + NTAXCM_c \tag{5.6.6}$$

Net taxes on the production of sector s expressed in value

$$\begin{aligned} NTAXS_s^{VAL} = {} & RNTAXS_s\ PY_s\ Y_s + T2VAL_s^{MAT} \\ & + T2VAL_s^{Y} - T2VAL^{SEC} . \frac{F_{L,s}}{F_L\ REDIS^{CT,LS}} \end{aligned} \tag{5.6.7}$$

Net taxes on the production of sector s

$$NTAXS_s = RNTAXS_{s,t_0}\ Y_s + T2_s^{MAT} + T2_s^{Y} \tag{5.6.8}$$

Employers' social security contribution paid by sector s

$$RSC_s\ PRSC_s = W_s\ F_{L,s}\ RRSC_s - T2VAL^{SEC} . \frac{F_{L,s}}{F_L\ REDIS^{CT,RRSC}} \tag{5.6.9}$$

Price of the employers' social security contribution paid by sector s

$$PRSC_s = P \tag{5.6.10}$$

5.6.1.2 Aggregate taxes and social contributions

Aggregate net taxes on commodity c expressed in value

$$PNTAXC.NTAXC = \sum_c NTAXC_c^{VAL} \tag{5.6.11}$$

Aggregate net taxes on commodity c

$$NTAXC = \sum_c NTAXC_c \tag{5.6.12}$$

Aggregate net taxes on the production of sectors expressed in value

$$NTAXS^{VAL} = \sum_s NTAXS_s^{VAL} \tag{5.6.13}$$

Aggregate net taxes on the production of sectors

$$NTAXS = \sum_s NTAXS_s \tag{5.6.14}$$

Price of the aggregate employers' social security contribution paid by sector s

$$PRSC.RSC = \sum_s PRSC_s\ RSC_s \tag{5.6.15}$$

Aggregate employers' social security contribution paid by sectors

$$RSC = \sum_s RSC_s \tag{5.6.16}$$

Average employers' social security contribution rate

$$RRSC = \frac{PRSC.RSC}{W.F_L} \tag{5.6.17}$$

$$RRSC_s^{ef} = RSC_s\ \frac{PRSC_s}{(W_s\ F_{L,s})} \tag{5.6.18}$$

5.6.1.3 Other and aggregate incomes

Notional property incomes of the Government expressed in value

$$PROP^{INC,G,VAL,n} = \varphi^{PROP^{INC,G}}.PNOS.NOS \tag{5.6.19}$$

Aggregate incomes of the Government expressed in value

$$\begin{aligned} INC^{G,VAL} = &\ PNTAXC.NTAXC + NTAXS^{VAL} \\ &+ INC^{SOC,TAX,VAL} + PRSC.RSC + PROP^{INC,G,VAL} \end{aligned} \tag{5.6.20}$$

5.6.2 spending

Government final consumption of commodity c

$$\Delta\left(\log G_c\right) = \Delta\left(\log EXPG\right) \tag{5.6.21}$$

Aggregate spending of the Government expressed in value

$$SPEND^{G,VAL} = PG.G + SOC^{BENF,VAL} + DEBT^{G,VAL}_{t-1}\,\left(\varphi^{RD^G}_{t-1} + r^{DEBT,G}_{t-1}\right) \tag{5.6.22}$$

5.6.3 Deficit and debt

Savings of the Government expressed in value

$$SAV^{G,VAL} = INC^{G,VAL} - SPEND^{G,VAL} \tag{5.6.23}$$

It corresponds to the net lending/borrowing, which is the published deficit/savings of the Government.

Primary balance of the Government expressed in value (deficit)

$$Bal^{G,Prim,VAL} = SAV^{G,VAL} + DEBT^{G,VAL}_{t-1}\,\left(\varphi^{RD^G}_{t-1} + r^{DEBT,G}_{t-1}\right) \tag{5.6.24}$$

It corresponds to the savings excluding the reimbursement and the interest on the debt.

Primary balance of the Government expressed in value (deficit) (for verification)

$$Bal^{G,Prim,VAL,bis} = INC^{G,VAL} - \left(PG.G + SOC^{BENF,VAL}\right) \tag{5.6.25}$$

Total balance of the Government expressed in value (deficit)

$$Bal^{G,Tot,VAL} = Bal^{G,Prim,VAL} - DEBT^{G,VAL}_{t-1}\,r^{DEBT,G}_{t-1} \tag{5.6.26}$$

Government's debt expressed in value

$$DEBT^{G,VAL} = DEBT^{G,VAL}_{t-1}\,\left(1 - \varphi^{RD^G}_{t-1}\right) - SAV^{G,VAL} \tag{5.6.27}$$

It corresponds to the previous year debt minus the reimbursement of the debt and the government savings.

Government's savings rate expressed in value (in percent of GDP)

$$RSAV^{G,VAL} = \frac{SAV^{G,VAL}}{(PGDP.GDP)} \tag{5.6.28}$$

Primary balance of the Government expressed in value (in percent of GDP)

$$RBal^{G,Prim,VAL} = \frac{Bal^{G,Prim,VAL}}{(PGDP.GDP)} \tag{5.6.29}$$

Total balance of the Government expressed in value (in percent of GDP)

$$RBal^{G,Tot,VAL} = \frac{Bal^{G,Tot,VAL}}{(PGDP.GDP)} \tag{5.6.30}$$

Ratio of the Government's debt expressed in value (in percent of GDP)

$$RDEBT^{G,VAL} = \frac{DEBT^{G,VAL}}{(PGDP.GDP)} \tag{5.6.31}$$

5.7 International Trade

This section provides the equations defining the allocation between domestic and imported goods per use.The differentiation per use allows for distinguishing import share per use and therefore a more realistic representation of the economy than model that assume a common import share. Indeed, the import share of export is generally smaller than for consumption.

5.7.1 Domestic demand

Received margins on domestically produced commodity m

$$MGSD_m = \left(1 - \varphi_m^{MGSM}\right)\ MGS_m \tag{5.7.1}$$

Private final consumption of domestically produced commodity c

$$CHD_c = \left(1 - \varphi_c^{CHM}\right)\ CH_c \tag{5.7.2}$$

Public final consumption of domestically produced commodity c

$$GD_c = \left(1 - \varphi_c^{GM}\right)\ G_c \tag{5.7.3}$$

Margins supplied from imported commodity m

$$MGSM_m = \varphi_m^{MGSM}\ MGS_m \tag{5.7.4}$$

Private final consumption of imported commodity c

$$CHM_c = \varphi_c^{CHM}\ CH_c \tag{5.7.5}$$

Public final consumption of imported commodity c

$$GM_c = \varphi_c^{GM}\ G_c \tag{5.7.6}$$

Import share of commodity c on supplied margins

$$\varphi_m^{MGSM} = \frac{1}{\left(1 + \frac{MGSD_m}{MGSM_{m,t_0}}\ e^{SUBST_m^{MGSM}}\right)} \tag{5.7.7}$$

Import share of commodity c for household final consumption

$$\varphi_c^{CHM} = \frac{1}{\left(1 + \frac{CHD_c}{CHM_{c,t_0}}\ e^{SUBST_c^{CHM}}\right)} \tag{5.7.8}$$

Import share φ_c of commodity c on the government final consumption

$$\varphi_c^{GM} = \frac{1}{\left(1 + \frac{GD_c}{GM_{c,t_0}}\ e^{SUBST_c^{GM}}\right)} \tag{5.7.9}$$

Notional substitution effect induced by a change in the relative price between imported and domestically produced commodity c for margins supplied

$$\Delta\left(SUBST_c^{MGSM,n}\right) = -\eta_c^{MGSM}\ \Delta\left(\log PMGSD_c - \log PMGSM_c\right) \tag{5.7.10}$$

Notional substitution effect induced by a change in the relative price between imported and domestically produced commodity c for households final consumption

$$\Delta\left(SUBST_c^{CHM,n}\right) = -\eta_c^{CHM}\ \Delta\left(\log PCHD_c - \log PCHM_c\right) \tag{5.7.11}$$

Notional substitution effect induced by a change in the relative price between imported and domestically produced commodity c for government final consumption

$$\Delta\left(SUBST_c^{GM,n}\right) = -\eta_c^{GM}\ \Delta\left(\log PGD_c - \log PGM_c\right) \tag{5.7.12}$$

Intermediary consumption from sector s in domestically produced commodity c

$$CID_{c,s} = \left(1 - \varphi_{c,s}^{CIM}\right)\ CI_{c,s} \tag{5.7.13}$$

Investment from sector s in domestically produced commodity c

$$ID_{c,s} = \left(1 - \varphi_{c,s}^{IM}\right)\ I_{c,s} \tag{5.7.14}$$

Intermediary consumption from sector s in imported commodity c

$$CIM_{c,s} = \varphi_{c,s}^{CIM}\ CI_{c,s} \tag{5.7.15}$$

Investment from sector s in imported commodity c

$$IM_{c,s} = \varphi_{c,s}^{IM}\ I_{c,s} \tag{5.7.16}$$

Import share of intermediary consumption from sector s in domestically produced commodity c

$$\varphi_{c,s}^{CIM} = \frac{1}{\left(1 + \frac{CID_{c,s}}{CIM_{c,s,t_0}}\ e^{SUBST_{c,s}^{CIM}}\right)} \tag{5.7.17}$$

Import share of intermediary consumption from sector s in imported commodity c

$$\varphi_{c,s}^{IM} = \frac{1}{\left(1 + \frac{ID_{c,s}}{IM_{c,s,t_0}}\ e^{SUBST_{c,s}^{IM}}\right)} \tag{5.7.18}$$

Notional substitution effect induced by a change in the relative price between imported and domestic intermediary consumption in commodity c from the sector s

$$\Delta\left(SUBST_{c,s}^{CIM,n}\right) = -\eta_{c,s}^{CIM}\ \Delta\left(\log PCID_{c,s} - \log PCIM_{c,s}\right) \tag{5.7.19}$$

Notional substitution effect induced by a change in the relative price between imported and domestic investment in commodity c from the sector s

$$\Delta\left(SUBST_{c,s}^{IM,n}\right) = -\eta_{c,s}^{IM}\ \Delta\left(\log PID_{c,s} - \log PIM_{c,s}\right) \tag{5.7.20}$$

5.7.2 Exports

Exports of domestically produced commodity c

$$XD_c = \left(1 - \varphi_c^{XM}\right)\ X_c \tag{5.7.21}$$

Exports of imported commodity c

$$XM_c = \varphi_c^{XM}\ X_c \tag{5.7.22}$$

Import share of commodity c exports

$$\varphi_c^{XM} = \frac{1}{\left(1 + \frac{XD_c}{XM_{c,t_0}}\ e^{SUBST_c^{XM}}\right)} \tag{5.7.23}$$

Notional substitution effect induced by a change in the relative price between imported and domestic products c for exports

$$\Delta\left(SUBST_c^{XM,n}\right) = -\eta_c^{XM}\ \Delta\left(\log PXD_c - \log PXM_c\right) \tag{5.7.24}$$

Foreign demand for exports of commodity c

$$\Delta\left(\log X_c\right) = \Delta\left(\log WD_c\right) + \Delta\left(SUBST_c^{X}\right) \tag{5.7.25}$$

Notional substitution effect induced by a change in the relative price between export prices and (converted in domestic currency) international prices for the commodity c

$$\Delta\left(SUBST_c^{X,n}\right) = -\eta_c^{X}\ \Delta\left(\log PX_c - \log\left(EXR.PWD_c\right)\right) \tag{5.7.26}$$

Balance of trade of commodity c

$$Bal_c^{Trade,VAL} = PX_c\ X_c - PM_c\ M_c \tag{5.7.27}$$

Aggregate balance of trade

$$Bal^{Trade,VAL} = \sum_c Bal_c^{Trade,VAL} \tag{5.7.28}$$

Balance of trade (in percent of GDP)

$$RBal^{Trade,VAL} = \frac{Bal^{Trade,VAL}}{(PGDP.GDP)} \tag{5.7.29}$$

5.8 Demography

This section provides the equations defining the occupation of the working-age population

Working-age population

$$\Delta\left(\log WAPop\right) = \Delta\left(\log POP\right) \tag{5.8.1}$$

The working age population linearly grows with the total population.

Labor force

$$LF = PARTR.WAPop \tag{5.8.2}$$

The Labor force depends on a participation rate of the working-age population.

Labor force participation ratio

$$\Delta\left(PARTR^{n}\right) = \Delta\left(PARTR^{trend}\right) - \rho^{PART,UnR}.\Delta\left(UnR\right) \tag{5.8.3}$$

Because of discouraged worker effect, the participation ratio depends negatively on the unemployment rate.

Employment (ILO definition)

$$\Delta\left(\log empl\right) = \Delta\left(\log F_{L}\right) \tag{5.8.4}$$

In general, labor according to the national account differs from the employment according to the ILO definition. One reason is that labor is expressed in FTE (Full Time Equivalent). To calculate the unemployment rate, one needs to use the employment according to the International Labor Organization (ILO) definition. We assume that the average work duration is constant over time, implying stability of the employment to labor ratio.

Unemployment

$$Un = LF - Empl \tag{5.8.5}$$

Unemployment is determined as the difference between the total active population with the one which is employed.

Unemployment rate

$$UnR = \frac{Un}{LF} \tag{5.8.6}$$

The Unemployment rate is defined as the ratio between the total unemployment and the active population.

5.9 Greenhouse gases emissions

This section provides the equations defining the path of GreenHouse Gases (GHG) emissions. All emission types are expressed in CO_2-equivalent (GWP factors are taken from the fifth IPCC report) to facilitate aggregation. For the same emission type (e.g. CO_2), several equation are defined depending on the emission source: intermediary consumption, household consumption or production.

5.9.1 Behavioural equations

Emissions of the greenhouse gas ghg **related to the intermediary consumption of commodity** c **by sector** s

$$\Delta\left(\log EMS^{CI}_{ghg,c,s}\right) = \Delta\left(\log\left(CI_{c,s}\, IEMS^{CI}_{ghg,c,s}\right)\right) \tag{5.9.1}$$

In practice only a few intermediaries generate emissions (e.g. coal, gas, petrol). $IEMS^{CI}_{ghg,c,s}$ is the corresponding emission intensity calibrated to 1 in the baseyear. It may change over time because of the increase in the share of biofuels.

Emissions of the greenhouse gas ghg **related to the materials consumption of sector** s

$$\Delta\left(\log EMS^{MAT}_{ghg,s}\right) = \Delta\left(\log\left(F_{MAT,s}\, IEMS^{MAT}_{ghg,s}\right)\right) \tag{5.9.2}$$

This mainly corresponds to the CO_2 emissions from decarbonation.

Emissions of the greenhouse gas ghg **related to the final production of sector** s

$$\Delta\left(\log EMS^{Y}_{ghg,s}\right) = \Delta\left(\log\left(Y_s\, IEMS^{Y}_{ghg,s}\right)\right) \tag{5.9.3}$$

This mainly corresponds to the emissions from agriculture.

Emissions of the greenhouse gas ghg **related to the household consumption** c

$$\Delta\left(\log EMS^{CH}_{ghg,c}\right) = \Delta\left(\log\left(CH_c\, IEMS^{CH}_{ghg,c}\right)\right) \tag{5.9.4}$$

5.9.2 Aggregation equations

Emissions of the greenhouse gas ghg **related to the intermediary consumption of commodity** c

$$EMS^{CI}_{ghg,c} = \sum_s EMS^{CI}_{ghg,c,s} \tag{5.9.5}$$

Emissions of the greenhouse gas ghg **related to the intermediary consumption by sector** s

$$EMS^{CI}_{ghg,s} = \sum_c EMS^{CI}_{ghg,c,s} \tag{5.9.6}$$

Emissions of the greenhouse gas ghg related to the intermediary consumption

$$EMS^{CI}_{ghg} = \sum_{s} EMS^{CI}_{ghg,s} \tag{5.9.7}$$

Emissions of the greenhouse gas ghg related to the intermediary consumption (for verification)

$$EMS^{CI,bis}_{ghg} = \sum_{c} EMS^{CI}_{ghg,c} \tag{5.9.8}$$

Emissions of the greenhouse gas ghg related to the total material consumption

$$EMS^{MAT}_{ghg} = \sum_{s} EMS^{MAT}_{ghg,s} \tag{5.9.9}$$

Emissions of the greenhouse gas ghg related to the final production

$$EMS^{Y}_{ghg} = \sum_{s} EMS^{Y}_{ghg,s} \tag{5.9.10}$$

Emissions of the greenhouse gas ghg related to the household final consumption

$$EMS^{CH}_{ghg} = \sum_{c} EMS^{CH}_{ghg,c} \tag{5.9.11}$$

Aggregate emissions of the greenhouse gas ghg

$$EMS_{ghg} = EMS^{CI}_{ghg} + EMS^{MAT}_{ghg} + EMS^{Y}_{ghg} + EMS^{CH}_{ghg} \tag{5.9.12}$$

Aggregate emissions related to the intermediary consumption

$$EMS^{CI} = \sum_{ghg} EMS^{CI}_{ghg} \tag{5.9.13}$$

Aggregate emissions related to the material consumption

$$EMS^{MAT} = \sum_{ghg} EMS^{MAT}_{ghg} \tag{5.9.14}$$

Aggregate emissions related to the final production

$$EMS^{Y} = \sum_{ghg} EMS^{Y}_{ghg} \tag{5.9.15}$$

Aggregate emissions related to the households final consumption

$$EMS^{CH} = \sum_{ghg} EMS^{CH}_{ghg} \tag{5.9.16}$$

Aggregate emissions

$$EMS = EMS^{CI} + EMS^{MAT} + EMS^{Y} + EMS^{CH} \tag{5.9.17}$$

Aggregate emissions by type of gas ghg

$$EMS^{bis} = \sum_{ghg} EMS_{ghg} \tag{5.9.18}$$

5.10 Energy balance

This section provides the equations defining the energy balance in physical units. The unit used is the tonne of oil equivalent (toe). The International Energy Agency (IEA) defines the following correspondence: 1 toe = 11.63 megawatt-hours (MWh) = 41.868 gigajoules (GJ). Chapter 1 of the documention discusses in detail the correspondance and differences between the energy balance and the national account definitions and equilibrium. It also provides the hypotheses used to derived the simplified energy balance presented below.

5.10.1 End uses

End use for energy ce of sectors, expressed in toe

$$CI_{ce}^{toe} = \sum_{s} CI_{ce,s}^{toe} \tag{5.10.1}$$

End use for energy ce of sector s, expressed in toe

$$\Delta \left(\log CI_{ce,s}^{toe}\right) = \Delta \left(\log CI_{ce,s}\right) \tag{5.10.2}$$

This variable regroups the following energy balance variables: energy transformation input, energy sector own use excluding the auto-consumption and “final consumption“ of non-energy sectors.

End use for energy ce of households, expressed in toe

$$\Delta \left(\log CH_{ce}^{toe}\right) = \Delta \left(\log CH_{ce}\right) \tag{5.10.3}$$

External end use for energy ce, expressed in toe

$$\Delta \left(\log X_{ce}^{toe}\right) = \Delta \left(\log X_{ce}\right) \tag{5.10.4}$$

5.10.2 Net supply (domestic and imported)

The total net supply is equal to the total end use. Net supply is the supply consistent with the national account concept of production, that is the physical quantity actually bought by the end user. Compared to the gross supply, it has been corrected from the distribution losses and the auto-consumption of energy producers that do not have any corresponding financial transaction.

Net production of energy ce, expressed in toe

$$Y_{ce}^{toe} + M_{ce}^{toe} = CI_{ce}^{toe} + CH_{ce}^{toe} + X_{ce}^{toe} \tag{5.10.5}$$

Net production regroups the primary (indigenous) production (including Recovered & recycled products) and the secondary production (energy transformation output) of the energy balance. Depending on the energy source, this variable is (mainly) a primary or a secondary production: e.g., primary for crude oil and natural gaz;

secondary, for electricity.

Imported net production of energy ce, expressed in toe

$$\Delta\left(\log M^{toe}_{ce}\right) = \Delta\left(\log M_{ce}\right) \tag{5.10.6}$$

Net production of energy ce by sector s, expressed in toe

$$Y^{toe}_{ce,s} = \varphi^{Y^{toe}}_{ce,s}\; Y^{toe}_{ce} \tag{5.10.7}$$

Net production of energy ce by sector s, expressed in toe (for verification)

$$Y^{toe,bis}_{ce} = \sum_{s} Y^{toe}_{ce,s} \tag{5.10.8}$$

Share of energy ce produced by sector s (national account definition)

$$\varphi^{Y}_{ce,s} = \frac{PY^{toe}_{ce,s,t_0}\; \varphi^{Y^{toe}}_{ce,s}}{\sum_{ss}\left(PY^{toe}_{ce,ss,t_0}\; \varphi^{Y^{toe}}_{ce,ss}\right)} \tag{5.10.9}$$

If the (net) production price of energy ce is common amongst producers, the national account definition share, $\varphi^{Y}_{c,s}$, is equal to the energy balance definition share $\varphi^{Y^{toe}}_{c,s}$.

5.10.3 Gross supply (domestic and imported)

In the energy balance, supply corresponds to a gross production since it includes physical quantities that are not bought by the end user (distribution losses and the auto-consumption of energy producers). They are included here in order to derive the same level of supply as published in the energy balance.

Gross production of energy ce by sector s, expressed in toe

$$YG^{toe}_{ce,s} = Y^{toe}_{ce,s} + DLY^{toe}_{ce,s} + AC^{toe}_{ce,s} \tag{5.10.10}$$

Likewise the net production, the gross production regroups the primary (indigenous) production (including Recovered & recycled products) and the secondary production (energy transformation output) of the energy balance.

Gross imported energy ce, expressed in toe

$$MG^{toe}_{ce} = M^{toe}_{ce} + DLM^{toe}_{ce} \tag{5.10.11}$$

Distribution losses of energy ce, expressed in toe

$$\Delta\left(\log DL^{toe}_{ce}\right) = \Delta\left(\log\left(CI^{toe}_{ce} + CH^{toe}_{ce} + X^{toe}_{ce}\right)\right) \tag{5.10.12}$$

For simplicity, we assume that the distribution losses related of energy ce are proportional to the end use of this commodity.

Distribution losses of energy ce produced by sector s, expressed in toe

$$DLY_{ce,s}^{toe} = DL_{ce}^{toe} \frac{Y_{ce,s}^{toe}}{(Y_{ce}^{toe} + M_{ce}^{toe})} \tag{5.10.13}$$

Distribution losses are disaggregated between producers of energy ce proportionally to their share into the total domestic and foreign net production.

Distribution losses of imported energy ce, expressed in toe

$$DLM_{ce}^{toe} = DL_{ce}^{toe} \frac{M_{ce}^{toe}}{(Y_{ce}^{toe} + M_{ce}^{toe})} \tag{5.10.14}$$

Auto-consumption of energy ce by producer s, expressed in toe

$$\Delta\left(\log AC_{ce,s}^{toe}\right) = \Delta\left(\log Y_{ce,s}^{toe}\right) \tag{5.10.15}$$

These are mainly the auto-consumption of electricity by electricity producers. This equation allows for taking into account the fact that the loss rate differs per technology.

Gross production of energy ce, expressed in toe

$$YG_{ce}^{toe} = \sum_{s} YG_{ce,s}^{toe} \tag{5.10.16}$$

Gross production of energy ce by sector s, expressed in toe (for verification)

$$YG_{ce}^{toe,bis} = Y_{ce}^{toe} + DLY_{ce}^{toe} + AC_{ce}^{toe} \tag{5.10.17}$$

Distribution losses of energy ce of domestic producers, expressed in toe

$$DLY_{ce}^{toe} = \sum_{s} DLY_{ce,s}^{toe} \tag{5.10.18}$$

Auto-consumption of energy ce, expressed in toe

$$AC_{ce}^{toe} = \sum_{s} AC_{ce,s}^{toe} \tag{5.10.19}$$

5.10.4 Indicators

These indicators by definition sum up to one and can also be interpreted as shares of the gross energy production YG_{ce}^{toe}.

Net production rate for energy ce

$$RY_{ce}^{toe} = \frac{Y_{ce}^{toe}}{YG_{ce}^{toe}} \tag{5.10.20}$$

Distribution losses rate of energy ce

$$RDLY_{ce}^{toe} = \frac{DLY_{ce}^{toe}}{YG_{ce}^{toe}} \tag{5.10.21}$$

Auto-consumption rate of energy ce

$$RAC_{ce}^{toe} = \frac{AC_{ce}^{toe}}{YG_{ce}^{toe}} \tag{5.10.22}$$

5.11 Other equations

5.11.1 Adjustment equations and anticipation

Mark-up in the sector s

$$\mu_s = \alpha_s^{\mu}\ \mu_s^n + (1-\alpha_s^{\mu})\ \mu_{s,t-1} \tag{5.11.1}$$

Expected inflation

$$\Delta\left(\log P^e\right) = \alpha^{Pe,P1}.\Delta\left(\log P_{t-1}\right) + \left(1-\alpha^{Pe,P1}\right).\Delta\left(\log P_{t-1}^e\right) \tag{5.11.2}$$

This equation defines the expected inflation and not the expected price. P^e does not necessary converge to P. If the wage equation is a WS curve, even in the very long term it may not converge.

Expected production

$$\Delta\left(\log Y_s^e\right) = \alpha_s^{Ye,Y}\ \Delta\left(\log Y_s\right) + \left(1-\alpha_s^{Ye,Y}\right)\ \Delta\left(\log Y_{s,t-1}^e\right) \tag{5.11.3}$$

Quantity of Labor, Energy and Material inputs in sector s

$$\log F_{f,s} = \alpha_{f,s}^{0,F}\ \log F_{f,s}^n + \left(1-\alpha_{f,s}^{0,F}\right)\ \left(\log F_{f,s,t-1} + \Delta\left(\log F_{f,s}^e\right)\right) \tag{5.11.4}$$

Expected quantity of Labor, Energy and Material inputs in sector s

$$\Delta\left(\log F_{f,s}^e\right) = \alpha_{f,s}^{1,F}\ \Delta\left(\log F_{f,s,t-1}^e\right) + \alpha_{f,s}^{2,F}\ \Delta\left(\log F_{f,s,t-1}\right) + \alpha_{f,s}^{3,F}\ \Delta\left(\log F_{f,s}^n\right) \tag{5.11.5}$$

Capital stock of sector s

$$F_{K,s} = (1-\delta_s)\ F_{K,s,t-1} + IA_s \tag{5.11.6}$$

Investment in sector s

$$\begin{aligned}\Delta\left(\log IA_s\right) = {} & \alpha_s^{IA,Ye}\ \Delta\left(\log Y_s^e\right) + \alpha_s^{IA,IA1}\ \Delta\left(\log IA_{s,t-1}\right) \\ & + \alpha_s^{IA,SUBST}\ \Delta\left(SUBST_{K,s}^F\right) + \alpha_s^{IA,Kn}\ \left(\log F_{K,s,t-1}^n - \log F_{K,s,t-1}\right)\end{aligned} \tag{5.11.7}$$

Households final consumption of commodity c

$$\log CH_c = \alpha_c^{0,CH}\ \log CH_c^n + \left(1-\alpha_c^{0,CH}\right)\ \left(\log CH_{c,t-1} + \Delta\left(\log CH_c^e\right)\right) \tag{5.11.8}$$

Expected households final consumption of commodity c

$$\Delta\left(\log CH_c^e\right) = \alpha_c^{1,CH}\ \Delta\left(\log CH_{c,t-1}^e\right) + \alpha_c^{2,CH}\ \Delta\left(\log CH_{c,t-1}\right) + \alpha_c^{3,CH}\ \Delta\left(\log CH_c^n\right) \tag{5.11.9}$$

Production price of sector s

$$\log PY_s = \alpha_s^{0,PY}\ \log PY_s^n + \left(1-\alpha_s^{0,PY}\right)\ \left(\log PY_{s,t-1} + \Delta\left(\log PY_s^e\right)\right) \tag{5.11.10}$$

Expected production price of sector s

$$\Delta\left(\log PY_s^e\right) = \alpha_s^{1,PY}\ \Delta\left(\log PY_{s,t-1}^e\right) + \alpha_s^{2,PY}\ \Delta\left(\log PY_{s,t-1}\right) + \alpha_s^{3,PY}\ \Delta\left(\log PY_s^n\right) \tag{5.11.11}$$

Wages of the sector s

$$\Delta\left(\log W_s\right) = \alpha_s^{W,Wn}\ \Delta\left(\log W_s^n\right) + \alpha_s^{W,W1}\ \Delta\left(\log W_{s,t-1}\right) - \alpha_s^{W,W1Wn1}\ \log\frac{W_{s,t-1}}{W_{s,t-1}^n} \tag{5.11.12}$$

Labor participation ratio

$$PARTR = \alpha^{0,PARTR}.PARTR^n + \left(1-\alpha^{0,PARTR}\right).PARTR_{t-1} \tag{5.11.13}$$

Interest rate

$$R = \alpha^{0,R}.R^n + \left(1-\alpha^{0,R}\right).R_{t-1} \tag{5.11.14}$$

Households property income in value

$$\begin{aligned}\log PROP^{INC,H,VAL} = {} & \alpha^{0,PROP,INC,H,VAL}.\log PROP^{INC,H,VAL,n}\\ & + \left(1-\alpha^{0,PROP,INC,H,VAL}\right).\left(\log PROP_{t-1}^{INC,H,VAL}\right.\\ & \left. + \Delta\left(\log PROP^{INC,H,VAL,e}\right)\right)\end{aligned} \tag{5.11.15}$$

Expected Households property income in value

$$\begin{aligned}\Delta\left(\log PROP^{INC,H,VAL,e}\right) = {} & \alpha^{1,PROP,INC,H,VAL}.\Delta\left(\log PROP_{t-1}^{INC,H,VAL,e}\right)\\ & + \alpha^{2,PROP,INC,H,VAL}.\Delta\left(\log PROP_{t-1}^{INC,H,VAL}\right)\\ & + \alpha^{3,PROP,INC,H,VAL}.\Delta\left(\log PROP^{INC,H,VAL,n}\right)\end{aligned} \tag{5.11.16}$$

Government property incomes in value

$$\begin{aligned}\log PROP^{INC,G,VAL} = {} & \alpha^{0,PROP,INC,G,VAL}.\log PROP^{INC,G,VAL,n}\\ & + \left(1-\alpha^{0,PROP,INC,G,VAL}\right).\left(\log PROP_{t-1}^{INC,G,VAL}\right.\\ & \left. + \Delta\left(\log PROP^{INC,G,VAL,e}\right)\right)\end{aligned} \tag{5.11.17}$$

Expected Government property incomes in value

$$\begin{aligned}\Delta\left(\log PROP^{INC,G,VAL,e}\right) = {} & \alpha^{1,PROP,INC,G,VAL}.\Delta\left(\log PROP_{t-1}^{INC,G,VAL,e}\right)\\ & + \alpha^{2,PROP,INC,G,VAL}.\Delta\left(\log PROP_{t-1}^{INC,G,VAL}\right)\\ & + \alpha^{3,PROP,INC,G,VAL}.\Delta\left(\log PROP^{INC,G,VAL,n}\right)\end{aligned} \tag{5.11.18}$$

5.11.2 Substitutions

Substitution effect of the production factor f in the sector s

$$SUBST^{F}_{f,s} = \alpha^{6,F}_{f,s}\ SUBST^{F,n}_{f,s} + \left(1 - \alpha^{6,F}_{f,s}\right)\ SUBST^{F}_{f,s,t-1} \tag{5.11.19}$$

Substitution effect of the domestic margin paid m for the commodity c

$$SUBST^{MGPD}_{m,c} = \alpha^{6,MGPD}_{m,c}\ SUBST^{MGPD,n}_{m,c} + \left(1 - \alpha^{6,MGPD}_{m,c}\right)\ SUBST^{MGPD}_{m,c,t-1} \tag{5.11.20}$$

Substitution effect on the imported margin paid m for the commodity c

$$SUBST^{MGPM}_{m,c} = \alpha^{6,MGPM}_{m,c}\ SUBST^{MGPM,n}_{m,c} + \left(1 - \alpha^{6,MGPM}_{m,c}\right)\ SUBST^{MGPM}_{m,c,t-1} \tag{5.11.21}$$

Substitution effect on the energy intermediate consumption ce in the sector s

$$SUBST^{CI}_{ce,s} = \alpha^{6,CI}_{ce,s}\ SUBST^{CI,n}_{ce,s} + \left(1 - \alpha^{6,CI}_{ce,s}\right)\ SUBST^{CI}_{ce,s,t-1} \tag{5.11.22}$$

Substitution effect on the transportation intermediate consumption ce in the sector s

$$SUBST^{CI}_{ct,s} = \alpha^{6,CI}_{ct,s}\ SUBST^{CI,n}_{ct,s} + \left(1 - \alpha^{6,CI}_{ct,s}\right)\ SUBST^{CI}_{ct,s,t-1} \tag{5.11.23}$$

Substitution effect on the imported margin supplied for the commodity m

$$SUBST^{MGSM}_{m} = \alpha^{6,MGSM}_{m}\ SUBST^{MGSM,n}_{m} + \left(1 - \alpha^{6,MGSM}_{m}\right)\ SUBST^{MGSM}_{m,t-1} \tag{5.11.24}$$

Substitution effect on the imported households final consumption for the commodity c

$$SUBST^{CHM}_{c} = \alpha^{6,CHM}_{c}\ SUBST^{CHM,n}_{c} + \left(1 - \alpha^{6,CHM}_{c}\right)\ SUBST^{CHM}_{c,t-1} \tag{5.11.25}$$

Substitution effect on the imported government final consumption for the commodity c

$$SUBST^{GM}_{c} = \alpha^{6,GM}_{c}\ SUBST^{GM,n}_{c} + \left(1 - \alpha^{6,GM}_{c}\right)\ SUBST^{GM}_{c,t-1} \tag{5.11.26}$$

Substitution effect on the exports for the imported commodity c

$$SUBST^{XM}_{c} = \alpha^{6,XM}_{c}\ SUBST^{XM,n}_{c} + \left(1 - \alpha^{6,XM}_{c}\right)\ SUBST^{XM}_{c,t-1} \tag{5.11.27}$$

Substitution effect on the intermediate consumption for the imported commodity c in the sector s

$$SUBST^{CIM}_{c,s} = \alpha^{6,CIM}_{c,s}\ SUBST^{CIM,n}_{c,s} + \left(1 - \alpha^{6,CIM}_{c,s}\right)\ SUBST^{CIM}_{c,s,t-1} \tag{5.11.28}$$

Substitution effect on the investment for the imported commodity c in the sector s

$$SUBST^{IM}_{c,s} = \alpha^{6,IM}_{c,s}\ SUBST^{IM,n}_{c,s} + \left(1 - \alpha^{6,IM}_{c,s}\right)\ SUBST^{IM}_{c,s,t-1} \tag{5.11.29}$$

Substitution effect on the exports of the commodity c

$$SUBST^{X}_{c} = \alpha^{6,X}_{c}\ SUBST^{X,n}_{c} + \left(1 - \alpha^{6,X}_{c}\right)\ SUBST^{X}_{c,t-1} \tag{5.11.30}$$

5.12 Carbon tax

5.12.1 Incomes generated by intermediary consumption emissions

Carbon tax in value on intermediary consumption emissions depending on ghg emission types, commodity c in sector s

$$T2VAL^{CI}_{ghg,c,s} = R2^{CI}_{ghg,c,s}\ EMS^{CI}_{ghg,c,s} \tag{5.12.1}$$

Carbon tax in volume on intermediary consumption emissions depending on ghg emission types, commodity c in sector s

$$T2^{CI}_{ghg,c,s} = R2^{CI}_{ghg,c,s,t_0}\ EMS^{CI}_{ghg,c,s} \tag{5.12.2}$$

Carbon tax in value on intermediary consumption emissions depending on ghg emission types and commodity c

$$T2VAL^{CI}_{ghg,c} = \sum_s T2VAL^{CI}_{ghg,c,s} \tag{5.12.3}$$

Carbon tax in value on intermediary consumption emissions depending on ghg emission types and domestic commodity c

$$T2VAL^{CID}_{ghg,c} = \sum_s T2VAL^{CI}_{ghg,c,s}\ \frac{CID_{c,s}}{(CI_{c,s} + eps)} \tag{5.12.4}$$

Carbon tax in volume on intermediary consumption emissions depending on ghg emission types and domestic commodity c

$$T2^{CID}_{ghg,c} = \sum_s T2^{CI}_{ghg,c,s}\ \frac{CID_{c,s}}{(CI_{c,s} + eps)} \tag{5.12.5}$$

Carbon tax in value on intermediary consumption emissions depending on ghg emission types and imported commodity c

$$T2VAL^{CIM}_{ghg,c} = \sum_s T2VAL^{CI}_{ghg,c,s}\ \frac{CIM_{c,s}}{(CI_{c,s} + eps)} \tag{5.12.6}$$

Carbon tax in volume on intermediary consumption emissions depending on ghg emission types and imported commodity c

$$T2^{CIM}_{ghg,c} = \sum_s T2^{CI}_{ghg,c,s}\ \frac{CIM_{c,s}}{(CI_{c,s} + eps)} \tag{5.12.7}$$

Carbon tax in value on intermediary consumption emissions depending on ghg emission types and sector s

$$T2VAL^{CI}_{ghg,s} = \sum_c T2VAL^{CI}_{ghg,c,s} \tag{5.12.8}$$

Carbon tax in value on intermediary consumption emissions depending on commodity c and sector s

$$T2VAL^{CI}_{c,s} = \sum_{ghg} T2VAL^{CI}_{ghg,c,s} \tag{5.12.9}$$

Carbon tax in value on intermediary consumption emissions depending on ghg emission types

$$T2VAL^{CI}_{ghg} = \sum_{c} T2VAL^{CI}_{ghg,c} \tag{5.12.10}$$

Carbon tax in value on intermediary consumption emissions depending on commodity c

$$T2VAL^{CI}_{c} = \sum_{ghg} T2VAL^{CI}_{ghg,c} \tag{5.12.11}$$

Carbon tax in value on intermediary consumption emissions depending on domestic commodity c

$$T2VAL^{CID}_{c} = \sum_{ghg} T2VAL^{CID}_{ghg,c} \tag{5.12.12}$$

Carbon tax in volume on intermediary consumption emissions depending on domestic commodity c

$$T2^{CID}_{c} = \sum_{ghg} T2^{CID}_{ghg,c} \tag{5.12.13}$$

Carbon tax in value on intermediary consumption emissions depending on imported commodity c

$$T2VAL^{CIM}_{c} = \sum_{ghg} T2VAL^{CIM}_{ghg,c} \tag{5.12.14}$$

Carbon tax in volume on intermediary consumption emissions depending on imported commodity c

$$T2^{CIM}_{c} = \sum_{ghg} T2^{CIM}_{ghg,c} \tag{5.12.15}$$

Carbon tax in value on intermediary consumption emissions depending on sector s

$$T2VAL^{CI}_{s} = \sum_{ghg} T2VAL^{CI}_{ghg,s} \tag{5.12.16}$$

Carbon tax in value on intermediary consumption emissions

$$T2VAL^{CI} = \sum_{s} T2VAL^{CI}_{s} \tag{5.12.17}$$

5.12.2 Incomes generated by material emissions

Carbon tax in value on material emissions depending on ghg emission types, commodity c in sector s

$$T2VAL^{MAT}_{ghg,s} = R2^{MAT}_{ghg,s}\ EMS^{MAT}_{ghg,s} \tag{5.12.18}$$

Carbon tax in volume on material emissions depending on ghg emission types, commodity c in sector s

$$T2^{MAT}_{ghg,s} = R2^{MAT}_{ghg,s,t_0}\ EMS^{MAT}_{ghg,s} \tag{5.12.19}$$

Carbon tax in value on material emissions depending on ghg emission types

$$T2VAL^{MAT}_{ghg} = \sum_{s} T2VAL^{MAT}_{ghg,s} \tag{5.12.20}$$

Carbon tax in volume on material emissions depending on ghg emission types

$$T2^{MAT}_{ghg} = \sum_{s} T2^{MAT}_{ghg,s} \tag{5.12.21}$$

Carbon tax in value on material emissions depending on commodity c

$$T2VAL^{MAT}_{s} = \sum_{ghg} T2VAL^{MAT}_{ghg,s} \tag{5.12.22}$$

Carbon tax in volume on material emissions depending on commodity c

$$T2^{MAT}_{s} = \sum_{ghg} T2^{MAT}_{ghg,s} \tag{5.12.23}$$

Carbon tax in value on material emissions

$$T2VAL^{MAT} = \sum_{s} T2VAL^{MAT}_{s} \tag{5.12.24}$$

5.12.3 Incomes generated by production emissions

Carbon tax in value on production emissions depending on ghg emission types, commodity c in sector s

$$T2VAL^{Y}_{ghg,s} = R2^{Y}_{ghg,s}\ EMS^{Y}_{ghg,s} \tag{5.12.25}$$

Carbon tax in volume on production emissions depending on ghg emission types, commodity c in sector s

$$T2^{Y}_{ghg,s} = R2^{Y}_{ghg,s,t_0}\ EMS^{Y}_{ghg,s} \tag{5.12.26}$$

Carbon tax in value on production emissions depending on ghg emission types

$$T2VAL^{Y}_{ghg} = \sum_{s} T2VAL^{Y}_{ghg,s} \tag{5.12.27}$$

Carbon tax in volume on production emissions depending on ghg emission types

$$T2^{Y}_{ghg} = \sum_{s} T2^{Y}_{ghg,s} \tag{5.12.28}$$

Carbon tax in value on production emissions depending on sector s

$$T2VAL^{Y}_{s} = \sum_{ghg} T2VAL^{Y}_{ghg,s} \tag{5.12.29}$$

Carbon tax in volume on production emissions depending on sector s

$$T2^{Y}_{s} = \sum_{ghg} T2^{Y}_{ghg,s} \tag{5.12.30}$$

Carbon tax in value on production emissions

$$T2VAL^{Y} = \sum_{s} T2VAL^{Y}_{s} \tag{5.12.31}$$

5.12.3.1 Incomes generated by production emissions, intermediary consumption emissions and material emissions

Total carbon tax in value collected depending on sector s

$$T2VAL^{SEC}_{s} = T2VAL^{Y}_{s} + T2VAL^{CI}_{s} + T2VAL^{MAT}_{s} \tag{5.12.32}$$

Total carbon tax in value collected over firms

$$T2VAL^{SEC} = \sum_{s} T2VAL^{SEC}_{s} \tag{5.12.33}$$

5.12.3.2 Incomes generated by households' emissions

Carbon tax in value on households' consumption depending on ghg emission types and commodity c

$$T2VAL^{CH}_{ghg,c} = R2^{CH}_{ghg,c}\, EMS^{CH}_{ghg,c} \tag{5.12.34}$$

Carbon tax in volume on households' consumption depending on ghg emission types and commodity c

$$T2^{CH}_{ghg,c} = R2^{CH}_{ghg,c,t_0}\, EMS^{CH}_{ghg,c} \tag{5.12.35}$$

Carbon tax in value on households' consumption depending on ghg emission types

$$T2VAL^{CH}_{ghg} = \sum_{c} T2VAL^{CH}_{ghg,c} \tag{5.12.36}$$

Carbon tax in value on households' consumption depending on commodity c

$$T2VAL^{CH}_{c} = \sum_{ghg} T2VAL^{CH}_{ghg,c} \tag{5.12.37}$$

Carbon tax in volume on households' consumption depending on commodity c

$$T2_c^{CH} = \sum_{ghg} T2_{ghg,c}^{CH} \tag{5.12.38}$$

Carbon tax in value on households' consumption depending on ghg emission types and commodity c

$$T2VAL^{CH} = \sum_{c} T2VAL_c^{CH} \tag{5.12.39}$$

5.13 Taxes on commodities & Prices per uses

There are two versions regarding the specification of prices and taxes on commodities: (1) the basic version where taxes are not differentiated between uses (e.g. intermediary, final consumption, export, etc.) and type (e.g. VAT, subsidies, etc.); (2) the advanced version where this differentiation is made.

This file provides the equations defining with more detail taxes and prices. In the basic version, taxes on commodities are not differentiated between uses. A constant average tax rate on each commodity is assumed. This has the advantage to require less data for the calibration. But this assumption is often unrealistic since taxes on commodities are generally differentiated per uses. For instance, VAT applies on final consumption but not on intermediary consumption or exports. Subsidies are generally higher for domestic than imported products. When the equations of this section are activated in ThreeME, the heterogeneity regarding the tax rates is taken into account. Additional data are required to disaggregate taxes. The average tax rate is not constant anymore and becomes an endogenous variable that depends on the evolution of structure of taxes and subsidies. To preserve the accountancy consistency of the model, prices have also to be amended compared to the basic version.

5.13.1 Taxes on commodities

5.13.1.1 Average tax rates on commodities

Net taxes on domestically produced commodity c expressed in value

$$\begin{aligned} NTAXCD_c^{VAL} = {} & PVATD_c \, VATD_c + POTHCTD_c \, OTHCTD_c \\ & + PSUBCD_c \, SUBCD_c + T2VAL_c^{CH} \, \frac{CHD_c}{(CH_c + eps) + T2VAL_c^{CID}} \end{aligned} \tag{5.13.1}$$

Net taxes on domestically produced commodity c

$$NTAXCD_c = VATD_c + OTHCTD_c + SUBCD_c + T2_c^{CH} \, \frac{CHD_c}{(CH_c + eps) + T2_c^{CID}} \tag{5.13.2}$$

This is not defined anymore by simply assuming a constant rate over the production value: $NTAXCD_c^{VAL} = RNTAXCD_c * PYQ_c * YQ_c$. Different taxes are now defined depending on their basis (see below).

Average tax rate on domestically produced commodity c

$$RNTAXCD_c = \frac{NTAXCD_c^{VAL}}{(PYQ_c\, YQ_c)} \tag{5.13.3}$$

This rate become now endogenous by inverting the equation of $NTAXCD_c^{VAL}$. This specification ensures the accounting equality between the various GDP definition.

Net taxes on imported commodity c expressed in value

$$\begin{aligned} NTAXCM_c^{VAL} &= PVATM_c\, VATM_c + POTHCTM_c\, OTHCTM_c \\ &\quad + PSUBCM_c\, SUBCM_c + T2VAL_c^{CH}\, \frac{CHM_c}{CH_c + T2VAL_c^{CIM}} \end{aligned} \tag{5.13.4}$$

Net taxes on imported commodity c

$$NTAXCM_c = VATM_c + OTHCTM_c + SUBCM_c + T2_c^{CH}\, \frac{CHM_c}{CH_c + T2_c^{CIM}} \tag{5.13.5}$$

Average tax rate on imported commodity c

$$RNTAXCM_c = \frac{NTAXCM_c^{VAL}}{(PM_c\, M_c)} \tag{5.13.6}$$

5.13.1.2 Value-added tax (value & volume)

Price of value-added tax on domestically produced commodity c

$$PVATD_c\, VATD_c = RVATD_c\, PYQSBVAT_c\, \frac{CHD_c}{(1 + RVATD_{c,t_0})} \tag{5.13.7}$$

Value-added tax on domestically produced commodity c

$$VATD_c = RVATD_{c,t_0}\, \frac{CHD_c}{(1 + RVATD_{c,t_0})} \tag{5.13.8}$$

Price of value-added tax on imported commodity c

$$PVATM_c\, VATM_c = RVATM_c\, PMSBVAT_c\, \frac{CHM_c}{(1 + RVATM_{c,t_0})} \tag{5.13.9}$$

Value-added tax on imported commodity c

$$VATM_c = RVATM_{c,t_0}\, \frac{CHM_c}{(1 + RVATM_{c,t_0})} \tag{5.13.10}$$

5.13.1.3 Other taxes on commodities (value & volume)

Price of other taxes on domestically produced commodity c

$$POTHCTD_c \, OTHCTD_c = ROTHCTD_c \, PYQ_c \, YQ_c \tag{5.13.11}$$

Other taxes on domestically produced commodity c

$$OTHCTD_c = ROTHCTD_{c,t_0} \, YQ_c \tag{5.13.12}$$

Price other taxes on imported commodity c

$$POTHCTM_c \, OTHCTM_c = ROTHCTM_c \, PM_c \, M_c \tag{5.13.13}$$

Other taxes on imported commodity c

$$OTHCTM_c = ROTHCTM_{c,t_0} \, M_c \tag{5.13.14}$$

5.13.1.4 Subsidies on commodities (value & volume)

$$PSUBCD_c \, SUBCD_c = RSUBCD_c \, YQ_c \tag{5.13.15}$$

Subsidies on domestically produced commodity c

$$SUBCD_c = RSUBCD_{c,t_0} \, YQ_c \tag{5.13.16}$$

We assume that subsidies are proportional to on the volume of production which is often the cases (in particular for agriculture). Consequently the price of the subvention grows like the subvention rate. For simplicity, we assume that at the steady state, the subvention rate grows at the rate of inflation.

Price of subsidies on imported commodity c

$$PSUBCM_c \, SUBCM_c = RSUBCM_c \, M_c \tag{5.13.17}$$

Subsidies on imported commodity c

$$SUBCM_c = RSUBCM_{c,t_0} \, M_c \tag{5.13.18}$$

5.13.2 Prices of commodities

Prices are amended to be consistent with the above specification of taxes.

Selling price before VAT for domestically produced commodity c

$$\begin{aligned} PYQSBVAT_c \, YQSBVAT_c = {} & PYQ_c \, YQ_c + POTHCTD_c \, OTHCTD_c \\ & + PSUBCD_c \, SUBCD_c + PMGPD_c \, MGPD_c \end{aligned} \tag{5.13.19}$$

Production of commodity c expressed at market price before VAT

$$YQSBVAT_c = YQ_c + OTHCTD_c + SUBCD_c + MGPD_c \tag{5.13.20}$$

$YQSBVAT_c$ is the volume of the production expressed at market price before VAT. It should **not** be seen as a composite of several g̈oods:̈ production at basic price, margins and taxes. Indeed, it does not increase when the volume of the commercial and transport margins increase. The price does instead. Consequently, $YQSBVAT_c$ is always proportional to YQ_c. Notice the consistency with the specification of the volume of a tax or a subvention: the tax rate does not increase the volume of the tax but increases its price. The volume of the tax increases only when the volume of the tax base (e.g. consumption, production) increases.

Selling price before VAT for imported commodity c

$$\begin{aligned} PMSBVAT_c\, MSBVAT_c = PM_c\, M_c + POTHCTM_c\, OTHCTM_c \\ + PSUBCM_c\, SUBCM_c + PMGPM_c\, MGPM_c \end{aligned} \tag{5.13.21}$$

Imports of commodity c **expressed at market price before VAT**

$$MSBVAT_c = M_c + OTHCTM_c + SUBCM_c + MGPM_c \tag{5.13.22}$$

Market price of margins supplied by domestically produced commodity c

$$PMGSD_c = PYQSBVAT_c \tag{5.13.23}$$

Market price of margins supplied by imported commodity c

$$PMGSM_c = PMSBVAT_c \tag{5.13.24}$$

Market price of domestically produced intermediate consumption c **purchased by sector** s

$$PCID_{c,s} = PYQSBVAT_c + \frac{T2VAL^{CI}_{c,s}}{CI_{c,s}} \tag{5.13.25}$$

Market price of imported intermediate consumption c **purchased by sector** s

$$PCIM_{c,s} = PMSBVAT_c + \frac{T2VAL^{CI}_{c,s}}{CI_{c,s}} \tag{5.13.26}$$

Market price of domestically produced households final consumption c

$$PCHD_c = PYQSBVAT_c\, \frac{1 + RVATD_c}{1 + RVATD_{c,t_0}} \tag{5.13.27}$$

Market price of imported households final consumption c

$$PCHM_c = PMSBVAT_c\, \frac{1 + RVATM_c}{1 + RVATM_{c,t_0}} \tag{5.13.28}$$

Market price of domestically produced Government final consumption c

$$PGD_c = PYQSBVAT_c \tag{5.13.29}$$

Market price of imported Government final consumption c

$$PGM_c = PMSBVAT_c \tag{5.13.30}$$

Market price of domestically produced investment c purchased by sector s

$$PID_{c,s} = PYQSBVAT_c \tag{5.13.31}$$

Market price of imported investment c purchased by sector s

$$PIM_{c,s} = PMSBVAT_c \tag{5.13.32}$$

Market price of domestically produced exports c

$$PXD_c = PYQSBVAT_c \tag{5.13.33}$$

Market price of imported exports (re-exports) c

$$PXM_c = PMSBVAT_c \tag{5.13.34}$$

Market price of domestically produced change in inventories c

$$PDSD_c = PYQSBVAT_c \tag{5.13.35}$$

Market price of imported change in inventories c

$$PDSM_c = PMSBVAT_c \tag{5.13.36}$$

5.13.2.1 Aggregation of taxes

Aggregate market price for value-added tax paid on domestically produced comodities

$$PVATD.VATD = \sum_c PVATD_c\, VATD_c \tag{5.13.37}$$

Aggregate value-added tax paid on domestically produced comodities

$$VATD = \sum_c VATD_c \tag{5.13.38}$$

Aggregate market price for other product tax paid on domestically produced comodities

$$POTHCTD.OTHCTD = \sum_c POTHCTD_c\, OTHCTD_c \tag{5.13.39}$$

Aggregate other product tax paid on domestically produced comodities

$$OTHCTD = \sum_c OTHCTD_c \tag{5.13.40}$$

Aggregate market price for subsidies on domestically produced commodities

$$PSUBCD.SUBCD = \sum_c PSUBCD_c\, SUBCD_c \tag{5.13.41}$$

Aggregate subsidies on domestically produced commodities

$$SUBCD = \sum_c SUBCD_c \tag{5.13.42}$$

Aggregate market price for value-added tax paid on imported comodities

$$PVATM.VATM = \sum_c PVATM_c\, VATM_c \tag{5.13.43}$$

Aggregate value-added tax paid on imported comodities

$$VATM = \sum_c VATM_c \tag{5.13.44}$$

Aggregate market price for Other product Tax on imported commododities

$$POTHCTM.OTHCTM = \sum_c POTHCTM_c\, OTHCTM_c \tag{5.13.45}$$

Aggregate Other product Tax on imported commododities

$$OTHCTM = \sum_c OTHCTM_c \tag{5.13.46}$$

Aggregate market price for subsidies on imported commododities

$$PSUBCM.SUBCM = \sum_c PSUBCM_c\, SUBCM_c \tag{5.13.47}$$

Aggregate subsidies on imported commodities

$$SUBCM = \sum_c SUBCM_c \tag{5.13.48}$$

Aggregate market price for Value-added Tax on total households final consumption

$$PVAT.VAT = PVATD.VATD + PVATM.VATM \tag{5.13.49}$$

Aggregate Value-added Tax on total households final consumption

$$VAT = VATD + VATM \tag{5.13.50}$$

Aggregate market price for other product Tax

$$POTHCT.OTHCT = POTHCTD.OTHCTD + POTHCTM.OTHCTM \tag{5.13.51}$$

Aggregate Other product Tax

$$OTHCT = OTHCTD + OTHCTM \tag{5.13.52}$$

Aggregate market price for subsidies on commodities

$$PSUBC.SUBC = PSUBCD.SUBCD + PSUBCM.SUBCM \tag{5.13.53}$$

Aggregate subsidies on commodities

$$SUBC = SUBCD + SUBCM \tag{5.13.54}$$

5.13.2.2 Average tax rates

Average VAT rate on domestically produced commodity

$$RVATD = \frac{PVATD.VATD}{PCHD.CHD - PVATD.VATD} \tag{5.13.55}$$

Average rate of other taxes on domestically produced commodity

$$ROTHCTD = \frac{POTHCTD.OTHCTD}{PYQ.YQ} \tag{5.13.56}$$

Average rate of subsidies on domestically produced commodity

$$RSUBCD = \frac{(PSUBCD.SUBCD)}{YQ} \tag{5.13.57}$$

Average VAT rate on imported commodity

$$RVATM = \frac{PVATM.VATM}{PCHM.CHM - PVATM.VATM} \tag{5.13.58}$$

Average rate of other taxes on imported commodity

$$ROTHCTM = \frac{POTHCTM.OTHCTM}{PM.M} \tag{5.13.59}$$

Average rate of subsidies on imported commodity

$$RSUBCM = \frac{(PSUBCM.SUBCM)}{M} \tag{5.13.60}$$

Average VAT rate

$$RVAT = \frac{PVAT.VAT}{PCH.CH - PVAT.VAT} \tag{5.13.61}$$

Average rate of other taxes on commodity

$$ROTHCT = \frac{POTHCT.OTHCT}{PYQ.YQ + PM.M} \tag{5.13.62}$$

Average rate of subsidies on commodity

$$RSUBC = \frac{PSUBC.SUBC}{YQ + M} \tag{5.13.63}$$

5.14 Nested utility function for the consumer

This section provides the equations defining the consumers nested utility function. In the basic version of the consumer block (see Section Consumer), the consumption of every commodity is modeled using generalized LES utility function where the elasticity of substitution between every commodity is constant. An increase in the price of fossil energy will increase the consumption of the other commodities uniformly whereas one expects that substitutions will mainly affect the other energy sources and households investment in more energy-efficient equipment. This block amends the basic version by introducing a nested utility function.

The first level separates the consumption related to housing (CH^{HOUS}) and transport (CH^{TRSP}) from the rest. We assume that the value shares of the three types of expenditures (housing, transport and other) are constant which amount to assuming an Elasticity of Substitution (ES) of one (Cobb-Douglas hypothesis).

At the second level of the nest, housing expenditures (CH^{HOUS}) are disaggregated between energy expenditures ($CH^{HOUSENER}$) and investment expenditures ($CH^{HOUSINV}$). Transport expenditures (CH^{TRSP}) are disaggregated between transport types.

At the third level of the nest, energy housing related expenditures ($CH^{HOUSENER}$) are disaggregated between energy types ($CH^{HOUSENER}_{ce}$). Automobile expenditures (CH^{TRSP}_{auto}) are disaggregated between energy expenditure ($CH^{TRSPENER}$) and investment expenditure ($CH^{TRSPINV}$).

At the fourth level of the nest, energy transport related expenditure ($CH^{TRSPENER}$) are disaggregated between energy types ($CH^{HOUSENER}_{ce}$). Except for the first level were the ES is one, in the other levels of the nest, the ES can be chosen freely.

5.14.1 Link with the consumer block of ThreeME

When this block is activated, the expenditures related to construction, transport, vehicles and energy commodities are not defined anymore by the basic version of the consumer block (see Section Consumer). To do so the necessary (minimum) households' final consumption for this commodity is endogenized and calibrated at their total at the base year: $NCH_c = CH_c$ for $c \in \{ccon, ct, cveh, ce\}$. In other words, we assume that these expenditures have the priority over the other ones.

Necessary (minimum) households' final consumption for construction commodity *ccon*

$$NCH_{ccon} = \frac{CH^{HOUSINV,VAL}}{PNCH_{ccon}} \tag{5.14.1}$$

Necessary (minimum) households' final consumption for transport commodities *ct*

$$NCH_{ct} = \frac{CH^{TRSP,VAL}_{ct}}{PCH_{ct}} \tag{5.14.2}$$

Necessary (minimum) households' final consumption for vehicles commodity *cveh*

$$NCH_{cveh} = \frac{CH^{TRSPINV,VAL}}{PNCH_{cveh}} \tag{5.14.3}$$

Necessary (minimum) households' final consumption for energy commodities ce

$$NCH_{ce} = CH_{ce}^{HOUS} + CH_{ce}^{TRSP} \tag{5.14.4}$$

5.14.1.1 Level 1: Disaggregation of aggregate expenditures

Households' final consumption related to housing in value

$$\Delta\left(\log CH^{HOUS,VAL}\right) = \Delta\left(\log CH^{n,VAL}\right) \tag{5.14.5}$$

Households' final consumption related to transport in value

$$\Delta\left(\log CH^{TRSP,VAL}\right) = \Delta\left(\log CH^{n,VAL}\right) \tag{5.14.6}$$

5.14.2 Disaggregation of housing expenditures

5.14.2.1 Level 2: Disaggregation between investment and energy expenditures related to housing

Households' final consumption related to housing investment in value

$$CH^{HOUSINV,VAL} = \varphi^{MCH^{HOUSINV}}.CH^{HOUS,VAL} \tag{5.14.7}$$

Households' final consumption of energy related to housing in value

$$CH^{HOUSENER,VAL} = CH^{HOUS,VAL} - CH^{HOUSINV,VAL} \tag{5.14.8}$$

Share of housing investment expenditures into the total housing expenditures

$$\varphi^{MCH^{HOUSINV}} = \frac{1}{\left(1 + \frac{CH^{HOUSENER,VAL}}{CH^{HOUSINV,VAL},t_0}.e^{SUBST^{HOUSINV}}\right)} \tag{5.14.9}$$

Substitution effect induced by a change in the relative price between investment and energy housing expenditures

$$\Delta\left(SUBST^{HOUSINV}\right) = \left(1 - \eta^{HOUS^{INV,ENER}}\right).\Delta\left(\log\frac{PCH^{HOUSENER}}{PCH^{HOUSINV}}\right) \tag{5.14.10}$$

Price of the aggregate energy expenditure related to housing

$$PCH^{HOUSENER} = PCH^{HOUSENER,CES} \tag{5.14.11}$$

Price of investment expenditure related to housing

$$PCH^{HOUSINV} = PCH_{ccon} \tag{5.14.12}$$

5.14.2.2 Level 3: Disaggregation of energy expenditures related to housing

Households' final consumption of energy commodity ce related to housing

$$CH_{ce}^{HOUS}\ PCH_{ce}^{HOUS} = \varphi_{ce}^{MCH^{HOUS}}\ CH^{HOUSENER,VAL} \tag{5.14.13}$$

Share of energy consumption ce into the total energy consumption related to housing

$$\Delta\left(\log \varphi_{ce}^{MCH^{HOUS}}\right) = \left(1 - \eta^{HOUS^{ENER}}\right).\Delta\left(\log \frac{PCH_{ce}^{HOUS}}{PCH^{HOUSENER,CES}}\right) \tag{5.14.14}$$

Price of the aggregate energy expenditures related to housing

$$PCH^{HOUSENER,CES} = \left(\sum_{ce} \varphi_{ce,t_0}^{MCH,HOUS}\ PCH_{ce}^{HOUS\left(1-\sigma^{HOUS,ENER}\right)}\right)^{\frac{1}{1-\sigma^{HOUS,ENER}}} \tag{5.14.15}$$

Price of energy consumption ce related to housing

$$PCH_{ce}^{HOUS} = PCH_{ce} \tag{5.14.16}$$

5.14.3 Disaggregation of transport expenditures

5.14.3.1 Level 2: Disaggregation between transport expenditures

Households' final consumption of transport $chtrsp$ in value

$$CH_{chtrsp}^{TRSP,VAL} = \varphi_{chtrsp}^{MCH^{TRSP}}\ CH^{TRSP,VAL} \tag{5.14.17}$$

Share of transport consumption $chtrsp$ into the total transport expenditures

$$\Delta\left(\log \varphi_{chtrsp}^{MCH^{TRSP}}\right) = \left(1 - \eta^{CHTRSP}\right).\Delta\left(\log \frac{PCH_{chtrsp}^{TRSP}}{PCH^{TRSP,CES}}\right) \tag{5.14.18}$$

Price of the aggregate transport expenditures

$$PCH^{TRSP,CES} = \left(\sum_{chtrsp} \varphi_{chtrsp,t_0}^{MCH,TRSP}\ PCH_{chtrsp}^{TRSP\left(1-\sigma^{CHTRSP}\right)}\right)^{\left(\frac{1}{\left(1-\sigma^{CHTRSP}\right)}\right)} \tag{5.14.19}$$

Price of transport consumption of commodity ct

$$PCH_{ct}^{TRSP} = PCH_{ct} \tag{5.14.20}$$

Price of the aggregate automobile expenditures

$$PCH_{auto}^{TRSP} = \left(\frac{CH_{t_0}^{TRSPINV,VAL}}{CH_{auto,t_0}^{TRSP,VAL}} \; PCH^{TRSPINV\left(1-\sigma^{TRSP,INV,ENER}\right)} + \frac{CH_{t_0}^{TRSPENER,VAL}}{CH_{auto,t_0}^{TRSP,VAL}} \; PCH^{TRSPENER\left(1-\sigma^{TRSP,INV,ENER}\right)} \right)^{\frac{1}{1-\sigma^{TRSP,INV,ENER}}} \tag{5.14.21}$$

5.14.3.2 Level 3: Disaggregation between investment and energy related to automobile transport expenditures

Households' final consumption related to automobile transport in value

$$CH^{TRSPINV,VAL} = \varphi^{MCH^{TRSPINV}}.CH_{auto}^{TRSP,VAL} \tag{5.14.22}$$

Households' final consumption of energy related to automobile transport in value

$$CH^{TRSPENER,VAL} = CH_{auto}^{TRSP,VAL} - CH^{TRSPINV,VAL} \tag{5.14.23}$$

Share of automobile transport investment expenditures into the total automobile transport expenditures

$$\varphi^{MCH^{TRSPINV}} = \frac{1}{\left(1 + \frac{CH^{TRSPENER,VAL}}{CH^{TRSPINV,VAL},t_0}.e^{SUBST^{TRSPINV}}\right)} \tag{5.14.24}$$

Share of automobile transport investment expenditures into the total automobile transport expenditures

$$\Delta\left(SUBST^{TRSPINV}\right) = \left(1 - \eta^{TRSP^{INV,ENER}}\right).\Delta\left(\log \frac{PCH^{TRSPENER}}{PCH^{TRSPINV}}\right) \tag{5.14.25}$$

Price of the aggregate energy expenditure related to automobile transport

$$PCH^{TRSPENER} = PCH^{TRSPENER,CES} \tag{5.14.26}$$

Price of investment expenditure related to automobile transport

$$PCH^{TRSPINV} = PCH_{cveh} \tag{5.14.27}$$

5.14.3.3 Level 4: Disaggregation between energy expenditures related to automobile transport

Households' final consumption of energy commodity ce related to automobile transport

$$CH_{ce}^{TRSP} \; PCH_{ce}^{TRSP} = \varphi_{ce}^{MCH^{TRSP}} \; CH^{TRSPENER,VAL} \tag{5.14.28}$$

Share of energy consumption ce into the total energy consumption related to automobile transport

$$\Delta\left(\log\varphi_{ce}^{MCH^{TRSP}}\right) = \left(1-\eta^{TRSP^{ENER}}\right).\Delta\left(\log\frac{PCH_{ce}^{TRSP}}{PCH^{TRSPENER,CES}}\right) \tag{5.14.29}$$

Price of the aggregate energy expenditures related to automobile transport

$$PCH^{TRSPENER,CES} = \left(\sum_{ce}\varphi_{ce,t_0}^{MCH,TRSP}\ PCH_{ce}^{TRSP\left(1-\sigma^{TRSP,ENER}\right)}\right)^{\frac{1}{1-\sigma^{TRSP,ENER}}} \tag{5.14.30}$$

Price of energy consumption ce related to automobile transport

$$PCH_{ce}^{TRSP} = PCH_{ce} \tag{5.14.31}$$

5.15 Nested CES Production function

This file provides the equations defining a nested CES production function. The nested structure is a follows: K (capital), L (labor), E (energy), M (material).

The specification of the Elasticity of Substitution (ES) used between two inputs is the one presented in Reyns (2018, Appendix B), From the ES in the different levels of the nest, it is possible to determine the actual elasticity between two inputs. Since the ES between two inputs is symmetric, we have:

ES between material and capital

$$ES_{MAT,K,s} = \eta_s^{NEST^{MAT,KEL}} \tag{5.15.1}$$

ES between capital and material

$$ES_{K,MAT,s} = ES_{MAT,K,s} \tag{5.15.2}$$

ES between material and energy

$$ES_{MAT,E,s} = \eta_s^{NEST^{MAT,KEL}} \tag{5.15.3}$$

ES between energy and material

$$ES_{E,MAT,s} = ES_{MAT,E,s} \tag{5.15.4}$$

ES between material and labor

$$ES_{MAT,L,s} = \eta_s^{NEST^{MAT,KEL}} \tag{5.15.5}$$

ES between labor and material

$$ES_{L,MAT,s} = ES_{MAT,L,s} \tag{5.15.6}$$

ES between capital and energy

$$ES_{K,E,s} = \frac{\eta_s^{NEST^{K,E}}}{(1 - \varphi_{MAT,s} - \varphi_{L,s}) - \frac{\eta_s^{NEST^{MAT,KEL}}}{(1-\varphi_{MAT,s}) - \eta_s^{NEST^{KE,L}} \frac{\frac{\varphi_{L,s}}{1-\varphi_{MAT,s}}}{1-\varphi_{MAT,s}-\varphi_{L,s}}}} \tag{5.15.7}$$

ES between energy and capital

$$ES_{E,K,s} = ES_{K,E,s} \tag{5.15.8}$$

ES between capital and labor

$$ES_{K,L,s} = \frac{\eta_s^{NEST^{KE,L}} - \eta_s^{NEST^{MAT,KEL}} \quad \varphi_{MAT,s}}{1 - \varphi_{MAT,s}} \tag{5.15.9}$$

ES between labor and capital

$$ES_{L,K,s} = ES_{K,L,s} \tag{5.15.10}$$

ES between energy and labor

$$ES_{E,L,s} = ES_{K,L,s} \tag{5.15.11}$$

ES between labor and energy

$$ES_{L,E,s} = ES_{E,L,s} \tag{5.15.12}$$

5.16 Housing

5.16.1 Link with the rest of the model

Necessary (minimum) households' final consumption for construction commodity $ccon$

$$NCH_{ccon} = PNewBUIL_{t_0} \; NewBUIL + PREHAB_{t_0} \; REHAB \tag{5.16.1}$$

Necessary (minimum) households' final consumption for energy commodities ceb **related to buildings**

$$NCH_{ceb}^{BUIL} = PENER_{ceb,t_0}^{BUIL} \; ENER_{ceb}^{BUIL} \tag{5.16.2}$$

Price of class ecl **new building per square meter**

$$\Delta \left(\log PNewBUIL_{ecl}\right) = \Delta \left(\log PCH_{CCON}\right) \tag{5.16.3}$$

Price of investment for rehabilitating a class ecl **building to class** $ecl2$ **per square meter**

$$\Delta \left(\log PREHAB_{ecl,ecl2}\right) = \Delta \left(\log PCH_{CCON}\right) \tag{5.16.4}$$

Price of investment for rehabilitating a class *ecl* building in the same class *ecl* per square meter

$$\Delta\left(\log PREHAB_{ecl,ecl}\right) = \Delta\left(\log PCH_{CCON}\right) \tag{5.16.5}$$

Interest rate paid for an investment in a new class *ecl* building

$$\Delta\left(R_{ecl}^{I,NewBUIL}\right) = \Delta\left(R\right) \tag{5.16.6}$$

Interest rate paid for an investment in the rehabilitation of a class *ecl* building

$$\Delta\left(R_{ecl}^{I,REHAB}\right) = \Delta\left(R\right) \tag{5.16.7}$$

Interest rate paid for a (maintenance) investment of a class *ecl* building

$$\Delta\left(R_{ecl}^{I,BUIL}\right) = \Delta\left(R\right) \tag{5.16.8}$$

This corresponds to the interest rate paid to maintain the building in its class whereas the previous interest rate is the one paid to rehabilitate the building to a higher class.

5.16.2 Building stock dynamic (in m2)

Total building stock

$$\Delta\left(\log BUIL\right) = \Delta\left(\log POP\right) + \Delta\left(\log M2perCapita\right) \tag{5.16.9}$$

It follows proportionally the size of the population (POP) and the number of square meter per inhabitant ($M2perCapita$).

Number of square meter per person

$$\Delta\left(\log M2perCapita\right) = \rho.\Delta\left(\log \frac{DISPINC^{AT,VAL}}{\frac{PCH}{POP}}\right) \tag{5.16.10}$$

Total building stock (for verification)

$$BUIL^{bis} = \sum_{ecl} BUIL_{ecl} \tag{5.16.11}$$

Stock of building of the energy efficiency class *ecl*

$$\begin{aligned}\Delta\left(BUIL_{ecl}\right) = NewBUIL_{ecl} + \sum_{ecl2} REHAB_{ecl2,ecl} + \sum_{ecl2} DEP_{ecl2,ecl}^{BUIL} \\ - \left(\sum_{ecl2} REHAB_{ecl,ecl2}\right) - \left(\sum_{bcl} DEP_{ecl,bcl}^{BUIL}\right)\end{aligned} \tag{5.16.12}$$

The variation of stocks of building per class depends on several variables:
- It increases by the amount of the new buildings constructed in class *ecl*: $NewBUIL_{ecl}$
- It increases by the amount of rehabilitated buildings from a lower class to class *ecl*: $\sum REHAB_{ecl2,ecl}$
- It increases by the downgraded buildings from a higher class to class *ecl*: $\sum DEP^{BUIL}_{ecl2,ecl}$
- It decreases by the amount of rehabilitated buildings from class ecl to a higher class: $\sum REHAB_{ecl2,ecl}$
- It decreases by the downgraded buildings from class *ecl* to lower class: $\sum DEP^{BUIL}_{ecl,bcl}$

New buildings constructed according to class *ecl*

$$NewBUIL_{ecl} = \varphi^{NewBUIL}_{ecl} \ (\Delta (BUIL) + BUIL_{DES}) \tag{5.16.13}$$

Buildings depreciating from class *ecl* **to class** *bcl*

$$DEP^{BUIL}_{ecl,bcl} = \delta^{BUIL}_{ecl,bcl} \ BUIL_{ecl,t-1} \tag{5.16.14}$$

Destroyed buildings

$$BUIL_{DES} = \sum_{ecl} DEP^{BUIL}_{ecl,DES} \tag{5.16.15}$$

Buildings rehabilitated from class *ecl* **to class** *ecl*2

$$REHAB_{ecl,ecl2} = \varphi^{REHAB}_{ecl,ecl2} \ \tau^{REHAB}_{ecl} \ BUIL_{ecl,t-1} \tag{5.16.16}$$

Total new buildings

$$NEWBUIL = \sum_{ecl} NEWBUIL_{ecl} \tag{5.16.17}$$

Average price of new buildings

$$PNEWBUIL.NEWBUIL = \sum_{ecl} PNEWBUIL_{ecl} \ NEWBUIL_{ecl} \tag{5.16.18}$$

Class *ecl* **rehabilitated buildings**

$$REHAB_{ecl} = \sum_{ecl2} REHAB_{ecl,ecl2} \tag{5.16.19}$$

Price of class *ecl* **rehabilitated buildings**

$$PREHAB_{ecl} \ REHAB_{ecl} = \sum_{ecl2} \left(1 - R^{SUB}_{ecl,ecl2}\right) \ PREHAB_{ecl,ecl2} \ REHAB_{ecl,ecl2} \tag{5.16.20}$$

Total of buildings rehabilitated

$$REHAB = \sum_{ecl} REHAB_{ecl} \tag{5.16.21}$$

Price buildings rehabilitated

$$PREHAB.REHAB = \sum_{ecl} PREHAB_{ecl} \ REHAB_{ecl} \tag{5.16.22}$$

5.16.3 Energy consumption of buildings

Consumption of energy *ceb* by class *ecl* buildings expressed in toe

$$ENER^{BUIL}_{ecl,ceb} = ENERperM2_{ecl,ceb}\ BUIL_{ecl} \tag{5.16.23}$$

Consumption of energy *ceb* per square meter in class *ecl* buildings

$$\begin{aligned}\Delta\left(\log ENERperM2_{ecl,ceb}\right) = -\zeta_{ecl,ceb}\ \Delta(\log PCH_{ceb} \\ - \log PCH)\ \left(\Delta\left(\log PCH_{ceb} - \log PCH\right) \\ > 0\right) + \Delta\left(SUBST^{NRJperM2}_{ecl,ceb}\right)\end{aligned} \tag{5.16.24}$$

Substitution between energy commodity *ceb* and the other energy commodities *cebb* consumed in class *ecl* buildings

$$\Delta\left(SUBST^{NRJperM2}_{ecl,ceb}\right) = \sum -\eta^{BUILNRJ}_{ecl,ceb,cebb}\ \varphi_{ecl,cebb,t-1}\ \Delta\left(\log PCH_{ceb} - \log PCH_{cebb}\right) \tag{5.16.25}$$

Share of energy expenditures by *ecl* class building

$$\varphi_{ecl,ceb} = \frac{PCH_{ceb}\ CH^{ENER,BUIL}_{ecl,ceb}}{\sum_{cebb} PCH_{cebb}\ CH^{ENER,BUIL}_{ecl,cebb}} \tag{5.16.26}$$

Consumption of energy *ceb* by class *ecl* buildings (in millions of euros)

$$\Delta\left(\log CH^{ENER,BUIL}_{ecl,ceb}\right) = \Delta\left(\log ENER^{BUIL}_{ecl,ceb}\right) \tag{5.16.27}$$

Price of energy *ceb* in class *ecl* buildings (euros per kWh)

$$PENER^{BUIL}_{ecl,ceb}\ ENER^{BUIL}_{ecl,ceb} = PCH_{ceb}\ CH^{ENER,BUIL}_{ecl,ceb} \tag{5.16.28}$$

Consumer price of energy in class *ecl* buildings (index)

$$PCH^{ENER,BUIL}_{ecl}\ CH^{ENER,BUIL}_{ecl} = \sum_{ceb} PCH_{ceb}\ CH^{ENER,BUIL}_{ecl,ceb} \tag{5.16.29}$$

Consumption of energy in class *ecl* buildings (in millions of euros)

$$CH^{ENER,BUIL}_{ecl} = \sum_{ceb} CH^{ENER,BUIL}_{ecl,ceb} \tag{5.16.30}$$

Consumption of energy *ceb* (in millions of euros)

$$CH^{ENER,BUIL}_{ceb} = \sum_{ecl} CH^{ENER,BUIL}_{ecl,ceb} \tag{5.16.31}$$

Energy consumption in class *ecl* buildings (in toe)

$$ENER^{BUIL}_{ecl} = \sum_{ceb} ENER^{BUIL}_{ecl,ceb} \tag{5.16.32}$$

Energy consumption in class *ecl* buildings (in millions of euros)

$$PENER_{ecl}^{BUIL}\ ENER_{ecl}^{BUIL} = \sum_{ceb} PENER_{ecl,ceb}^{BUIL}\ ENER_{ecl,ceb}^{BUIL} \tag{5.16.33}$$

Total energy consumption of buildings (in volume)

$$CH^{ENER,BUIL} = \sum_{ecl} CH_{ecl}^{ENER,BUIL} \tag{5.16.34}$$

Total energy consumption of buildings (in value)

$$PCH^{ENER,BUIL}.CH^{ENER,BUIL} = \sum_{ecl} PCH_{ecl}^{ENER,BUIL}\ CH_{ecl}^{ENER,BUIL} \tag{5.16.35}$$

Total energy consumption in class *ceb* buildings (in volume)

$$ENER_{ceb}^{BUIL} = \sum_{ecl} ENER_{ecl,ceb}^{BUIL} \tag{5.16.36}$$

Total energy consumption in class *ceb* buildings (in value)

$$PENER_{ceb}^{BUIL}\ ENER_{ceb}^{BUIL} = \sum_{ecl} PENER_{ecl,ceb}^{BUIL}\ ENER_{ecl,ceb}^{BUIL} \tag{5.16.37}$$

5.16.4 Arbitrage in buildings investment decisions

5.16.4.1 Level 1: Building rehabilitation decisions

Notional rehabilitation rate of a class *ecl* building

$$\tau_{ecl}^{REHAB,N} = \tau_{ecl}^{REHAB,MAX} + \frac{\tau_{ecl}^{REHAB,MIN} - \tau_{ecl}^{REHAB,MAX}}{1 + e^{\tau_{ecl} - \sigma_{ecl}\ Payback_{ecl}^{REHAB}}} \tag{5.16.38}$$

It corresponds to the proportion of class *ecl* building rehabilitated to a higher class. In order to avoid discontinuity, the proportion of rehabilitated building is defined according to a logistic function. A logistic function is defined by 2 regimes, Yi and Yf, the switching speed between the 2 regimes, σ, and point of inflection between the 2 regimes, $\frac{\tau}{\sigma}$.
$Y = (1 - \Phi(X))Yi + \Phi(X))Yf$ with $\Phi(X) = (1 - \mathrm{e}^{\tau - \sigma.X})^{-1}$ which is equivalent to: $Y = Yi + \frac{Yf - Yi}{1 - \mathrm{e}^{\tau - \sigma.X}}$.The point of inflection corresponds to the case where $\Phi(X) = 1/2$ that is where $X = \tau/\sigma$.
This replaces the previous specification:
- $\tau_{ecl}^{REHAB_N} = (\Delta(\tau_{ecl}^{REHAB\_trend}) - \nu_{ecl}^{REHAB}.\Delta(\log(Payback_{ecl}^{REHAB})))$ with a max and min value.
The previous specification had several drawbacks: - Discontinuity in the first derivative at the max and minimum value
- Constant elasticity $\tau_{ecl}^{REHAB_N}$ and $Payback_{ecl}^{REHAB}$ whatever the level of $\tau_{ecl}^{REHAB_N}$.
- Infeasible solution if $Payback_{ecl}^{REHAB} < 0$

First derivative of the notional rehabilitation rate of a class ecl building $\tau_{ecl}^{REHAB_N}$ with respect to the log of payback time $Payback_{ecl}^{REHAB}$

$$\nu_{ecl}^{REHAB} = \frac{\left(\tau_{ecl}^{REHAB,MAX} - \tau_{ecl}^{REHAB,MIN}\right) \sigma_{ecl}\ Payback_{ecl}^{REHAB}\ e^{au_{ecl} - \sigma_{ecl}\ Payback_{ecl}^{REHAB}}}{\left(1 + e^{\tau_{ecl} - \sigma_{ecl}\ Payback_{ecl}^{REHAB}}\right)^2} \tag{5.16.39}$$

Rehabilitation rate of a class ecl building

$$\tau_{ecl}^{REHAB} = \alpha^{0,\tau,REHAB}.\tau_{ecl}^{REHAB,N} + \left(1 - \alpha^{0,\tau,REHAB}\right).\tau_{ecl,t-1}^{REHAB} \tag{5.16.40}$$

Payback time of rehabilitating a class ecl building

$$Payback_{ecl}^{REHAB} = \frac{UC_{ecl}^{K,REHAB}\ BUIL_{ecl}^{D} - UC_{ecl}^{K}\ BUIL_{ecl}^{D}}{UC_{ecl}^{E} - UC_{ecl}^{E,REHAB}} \tag{5.16.41}$$

5.16.4.2 Level 2: Transition between building classes

The transition matrix from one building class to another is endogenous and is defined according to a discrete choice model. When the gains from rehabilitating a building toward a specific class increases, the share of buildings rehabilitated to this class increases.

Gain from the rehabilitation from a ecl to a $ecl2$ class building

$$GAIN_{ecl,ecl2}^{REHAB} = \left(UC_{ecl}^{E} - UC_{ecl2}^{E}\right) - UC_{ecl,ecl2}^{K,REHAB} \tag{5.16.42}$$

Utility from rehabilitating a ecl to a $ecl2$ class building

$$\Delta\left(U_{ecl,ecl2}^{REHAB}\right) = \rho_{ecl,ecl2}^{U,GAIN}\ \frac{\Delta\left(GAIN_{ecl,ecl2}^{REHAB}\right)}{GAIN_{ecl,ecl2,t-1}^{REHAB}} \tag{5.16.43}$$

This utility is a function of the gain from the rehabilitation. For convenience it is calibrated as equal to $\log\left(\varphi_{ecl,ecl2}^{REHAB}\right)$ at the base year.

Sum of the exponential utility from rehabilitating a ecl to a $ecl2$ class building

$$SUM_{ecl}^{exp,U,REHAB} = \sum_{ecl2} e^{U_{ecl,ecl2}^{REHAB}} \tag{5.16.44}$$

Notional share of class ecl buildings rehabilitated to class $ecl2$

$$\Delta\left(\varphi_{ecl,ecl2}^{REHAB^n}\right) = \Delta\left(\frac{e^{U_{ecl,ecl2}^{REHAB}}}{SUM_{ecl}^{exp,U,REHAB}}\right) \tag{5.16.45}$$

Share of class ecl buildings rehabilitated to class $ecl2$

$$\varphi_{ecl,ecl2}^{REHAB} = \alpha^{phi,REHAB}.\varphi_{ecl,ecl2}^{REHAB^n} + \left(1 - \alpha^{phi,REHAB}\right).\varphi_{ecl,ecl2,t-1}^{REHAB} \quad (5.16.46)$$

Notice that $\alpha^{\varphi^{REHAB}}$ is common to every class transition. This ensures that $\sum_{ecl2}\varphi_{ecl,ecl2}^{REHAB} = 1$.

5.16.4.3 User costs

User cost of a ecl class building after rehabilitation to a more energy efficient class

$$UC_{ecl}^{REHAB} = UC_{ecl}^{K,REHAB} + UC_{ecl}^{E,REHAB} \quad (5.16.47)$$

User energy cost of a ecl class building after rehabilitation to a more energy efficient class

$$UC_{ecl}^{E,REHAB} = \sum_{ecl2}\varphi_{ecl,ecl2}^{REHAB}\, UC_{ecl2}^{E} \quad (5.16.48)$$

User capital cost of a ecl class building after rehabilitation to a more energy efficient class

$$UC_{ecl}^{K,REHAB} = \sum_{ecl2}\varphi_{ecl,ecl2}^{REHAB}\, UC_{ecl,ecl2}^{K,REHAB} \quad (5.16.49)$$

User capital cost of a ecl class building after rehabilitation to a $ecl2$ class

$$UC_{ecl,ecl2}^{K,REHAB} = \left(1 - R_{ecl,ecl2}^{SUB}\right)\left(\frac{PREHAB_{ecl,ecl2}}{REHAB_{ecl2}^{D}\left(R_{ecl}^{CASH,REHAB} + \frac{R_{ecl}^{LOAN,REHAB}\; R_{ecl,t-1}^{I,REHAB}\; LD_{ecl}^{REHAB}}{1-\left(1+R_{ecl,t-1}^{I,REHAB}\right)^{\left(-LD_{ecl}^{REHAB}\right)}}\right)}\right) \quad (5.16.50)$$

Share of the class ecl building rehabilitation investment paid through a loan

$$R_{ecl}^{LOAN,REHAB} = 1 - R_{ecl}^{CASH,REHAB} \quad (5.16.51)$$

Share of the new class ecl building investment paid through a loan

$$R_{ecl}^{LOAN,NewBUIL} = 1 - R_{ecl}^{CASH,NewBUIL} \quad (5.16.52)$$

User cost of a ecl class building

$$UC_{ecl} = UC_{ecl}^{K} + UC_{ecl}^{E} \quad (5.16.53)$$

User cost of capital of a class ecl building

$$UC_{ecl}^{K} = \left(\frac{PREHAB_{ecl,ecl}}{BUIL_{ecl}^{D}}\right)\left(R_{ecl}^{CASH} + \frac{R_{ecl}^{LOAN}\; R_{ecl,t-1}^{I,BUIL}\; LD_{ecl}}{1-\left(1+R_{ecl,t-1}^{I,BUIL}\right)^{(-LD_{ecl})}}\right) \quad (5.16.54)$$

Notice that it depends on $PREHAB_{ecl,ecl}$, the price of rehabilitating (maintaining) a building of ecl in the same class.

Share of the class ecl building investment paid through a loan

$$R_{ecl}^{LOAN} = 1 - R_{ecl}^{CASH} \tag{5.16.55}$$

User energy cost of a class ecl building

$$UC_{ecl}^{E} = PENER_{ecl}^{m2} \left(\frac{\left(\left(1 + g_{ecl}^{PENER^{m2,e}}\right)^{\left(BUIL_{ecl}^{D}\right)}\right) - 1}{g_{ecl}^{PENER^{m2,e}} \; BUIL_{ecl}^{D}} \right) \tag{5.16.56}$$

Energy price per square meter paid in class ecl buildings

$$PENER_{ecl}^{m2} = PENER_{ecl}^{BUIL} \; \frac{ENER_{ecl}^{BUIL}}{BUIL_{ecl}} \tag{5.16.57}$$

Growth rate of the energy price per square meter paid in class ecl buildings

$$g_{ecl}^{PENER^{m2,e}} = \alpha^{g_1^{PENER,m2,e}} \; g_{ecl,t-1}^{PENER^{m2}} + \left(1 - \alpha^{g_1^{PENER,m2,e}}\right) \; g_{ecl,t-1}^{PENER^{m2,e}} \tag{5.16.58}$$

5.16.4.4 Debts and expenditures related to housing

Debt related to the rehabilitation of a class ecl building (in value)

$$\begin{aligned} DEBT_{ecl}^{REHAB,Val} = & \left(1 - R_{ecl}^{RMBS,REHAB}\right) \; DEBT_{ecl,t-1}^{REHAB,Val} \\ & + R_{ecl}^{LOAN,REHAB} \; PREHAB_{ecl} \; REHAB_{ecl} \end{aligned} \tag{5.16.59}$$

Debt related to the purchase of a new building of a class ecl (in value)

$$\begin{aligned} DEBT_{ecl}^{NewB,Val} = & \left(1 - R_{ecl}^{RMBS,NewBUIL}\right) \; DEBT_{ecl,t-1}^{NewB,Val} \\ & + R_{ecl}^{LOAN,NewBUIL} \; PNewBUIL_{ecl} \; NewBUIL_{ecl} \end{aligned} \tag{5.16.60}$$

Housing expenditures in class ecl buildings (in value)

$$\begin{aligned} EXP_{ecl}^{HOUSING,Val} = & \Big(DEBT_{ecl,t-1}^{REHAB,Val} \; \left(R_{ecl,t-1}^{I,REHAB} + R_{ecl,t-1}^{RMBS,REHAB}\right) \\ & + R_{ecl}^{CASH,REHAB} \; PREHAB_{ecl} \; REHAB_{ecl} \\ & + DEBT_{ecl,t-1}^{NewB,Val} \; \left(R_{ecl,t-1}^{I,NewBUIL} + R_{ecl,t-1}^{RMBS,NewBUIL}\right) \\ & + R_{ecl}^{CASH,NewBUIL} \; PNewBUIL_{ecl} \; NewBUIL_{ecl} \\ & + PENER_{ecl}^{BUIL} \; ENER_{ecl}^{BUIL}\Big) \end{aligned} \tag{5.16.61}$$

Housing expenditures (in value)

$$EXP^{HOUSING,Val} = \sum_{ecl} EXP_{ecl}^{Housing,Val} \tag{5.16.62}$$

Expenditures in rehabilitation of buildings (in value)

$$EXP^{REHAB,VAL} = PREHAB.REHAB \tag{5.16.63}$$

Expenditures in construction of new buildings (in value)

$$EXP^{NEWBUIL,VAL} = PNEWBUIL.NEWBUIL \tag{5.16.64}$$

5.17 Transport

This section provides the equations defining the hybrid block on transport.

5.17.1 Link with the rest of the model

Necessary (minimum) households' final consumption for energy commodity cea consumed by automobiles

$$\Delta\left(\log NCH_{cea}^{AUTO}\right) = \Delta\left(\log CH_{cea}^{AUTO}\right) \tag{5.17.1}$$

Necessary (minimum) households' final consumption for public transport commodities cth

$$\Delta\left(\log NCH_{cth}\right) = \Delta\left(\log km_{cth}^{traveler}\right) \tag{5.17.2}$$

Necessary (minimum) households' final consumption for vehicles commodity $cveh$

$$NCH_{cveh} = PnewAUTO_{t_0}\ NewAUTO + UC^{M,AUTO,t_0}.AUTO \tag{5.17.3}$$

Price of public transport commodities cth (train, road and air)

$$\Delta\left(\log P_{cth}^{km,traveler}\right) = \Delta\left(\log PCH_{cth}\right) \tag{5.17.4}$$

Interest rate paid for an investment in a class ecl automobile fueled with energy cea

$$\Delta\left(R_{ecl,cea}^{I,AUTO}\right) = \Delta\left(R\right) \tag{5.17.5}$$

5.17.2 Arbitrage between transport modes

The arbitrage between transport modes is defined in 5 level. Level 1 determine the evolution of air, long and short distance transport. In level 2, the consumer may substitute between long distance transport types (automobile and train) and short distance transport types (automobile and bus). Level 3 defines the arbitrage between automobile class whereas level 4 defines the arbitrage between electric and thermic automobile. In level 5, substitution between types of thermic automobiles may be introduced.

5.17.2.1 Level 1: Determination of air, long and short distance transport

Kilometer-travelers for air transport

$$\begin{aligned}\Delta\left(\log km^{traveler}_{cair}\right) = \theta^{kmtravcair^{DISPINC}}.\Delta\left(\log\frac{DISPINC^{AT,VAL}}{PCH}\right)\\ - \theta^{kmtravcair^{Pkmtrav}}.\Delta\left(\log\frac{P^{km,traveler}_{cair}}{PCH}\right)\end{aligned} \tag{5.17.6}$$

Kilometer-travelers for long distance transport (by automobile and train)

$$\begin{aligned}\Delta\left(\log km^{traveler,LD}\right) = \theta^{kmtravLD^{DISPINC}}.\Delta\left(\log\frac{DISPINC^{AT,VAL}}{PCH}\right)\\ - \theta^{kmtravLD^{Pkmtrav}}.\Delta\left(\log\frac{P^{km,traveler,LD}}{PCH}\right)\end{aligned} \tag{5.17.7}$$

Price of long distance Kilometer-traveler (automobile and train)

$$\begin{aligned}&P^{km,traveler,LD}.km^{traveler,LD}\\ &= P^{km,traveler}_{crai}\ km^{traveler}_{crai} + P^{km,trav,auto,LD}.km^{trav,auto,LD}\end{aligned} \tag{5.17.8}$$

Kilometer-travelers for short distance transport (by automobile and bus)

$$\begin{aligned}\Delta\left(\log km^{traveler,SD}\right) = \theta^{kmtravSD^{DISPINC}}.\Delta\left(\log\frac{DISPINC^{AT,VAL}}{PCH}\right)\\ - \theta^{kmtravSD^{Pkmtrav}}.\Delta\left(\log\frac{P^{km,traveler,SD}}{PCH}\right)\end{aligned} \tag{5.17.9}$$

Price of short distance Kilometer-traveler (by automobile and bus)

$$\begin{aligned}&P^{km,traveler,SD}.km^{traveler,SD}\\ &= P^{km,traveler}_{croa}\ km^{traveler}_{croa} + P^{km,trav,auto,SD}.km^{trav,auto,SD}\end{aligned} \tag{5.17.10}$$

5.17.2.2 Level 2: Arbitrage between long distance transport (automobile and train)

Share of Kilometer-travelers by automobile into the long distance Kilometer-travelers

$$\varphi^{km^{trav,auto,LD}} = \frac{P^{km,trav,auto,LD}.km^{trav,auto,LD}}{P^{km,traveler,LD}.km^{traveler,LD}} \tag{5.17.11}$$

Kilometer-travelers for long distance by automobile

$$\begin{aligned}\Delta\left(\log km^{trav,auto,LD}\right) = \Delta\left(\log km^{traveler,LD}\right) + \theta^{kmtrav^{autoLD,crai}}.\Big(1\\ - \varphi^{km^{trav,auto,LD}}_{t-1}\Big)\ \Delta\Big(\log P^{km,trav,auto,LD}\\ - \log P^{km,traveler}_{crai}\Big)\end{aligned} \tag{5.17.12}$$

Kilometer-travelers for transport by train

$$\Delta\left(\log km_{crai}^{traveler}\right) = \Delta\left(\log km^{traveler,LD}\right) - \theta^{kmtrav^{autoLD,crai}} . \varphi_{t-1}^{km^{trav,auto,LD}} \, \Delta\left(\log P_{crai}^{km,traveler} - \log P^{km,trav,auto,LD}\right) \quad (5.17.13)$$

5.17.2.3 Level 2: Arbitrage between short distance transport (automobile and bus)

Kilometer-travelers for short distance by automobile

$$\Delta\left(\log km^{trav,auto,SD}\right) = \Delta\left(\log km^{traveler,SD}\right) - \theta^{kmtrav^{autoSD,croa}} . \left(1 - \varphi_{t-1}^{km^{trav,auto,SD}}\right) \Delta\left(\log P^{km,trav,auto,SD} - \log P_{croa}^{km,traveler}\right) \quad (5.17.14)$$

Share of Kilometer-travelers by automobile into the short distance Kilometer-travelers

$$\varphi^{km^{trav,auto,SD}} = \frac{P^{km,trav,auto,SD} . km^{trav,auto,SD}}{P^{km,traveler,SD} . km^{traveler,SD}} \quad (5.17.15)$$

Kilometer-travelers for transport by road (bus)

$$\Delta\left(\log km_{croa}^{traveler}\right) = \Delta\left(\log km^{traveler,SD}\right) - \theta^{kmtrav^{autoSD,croa}} . \varphi_{t-1}^{km^{trav,auto,SD}} \, \Delta\left(\log P_{croa}^{km,traveler} - \log P^{km,trav,auto,SD}\right) \quad (5.17.16)$$

5.17.3 Transport by automobile

Kilometers for long distance by automobile

$$\Delta\left(\log km^{AUTO,LD}\right) = \Delta\left(\log km^{trav,auto,LD}\right) - \Delta\left(\log travperauto^{LD}\right) \quad (5.17.17)$$

Kilometers for short distance by automobile

$$\Delta\left(\log km^{AUTO,SD}\right) = \Delta\left(\log km^{trav,auto,SD}\right) - \Delta\left(\log travperauto^{SD}\right) \quad (5.17.18)$$

Total kilometers by automobile

$$km^{AUTO} = km^{AUTO,LD} + km^{AUTO,SD} \quad (5.17.19)$$

Automobiles stock

$$\Delta\left(\log AUTO\right) = \Delta\left(\log km^{AUTO}\right) - \Delta\left(\log kmPerAuto\right) \quad (5.17.20)$$

Automobiles stock (for verification)

$$AUTO^{bis} = \sum_{ecl} AUTO_{ecl} \quad (5.17.21)$$

New automobiles

$$NewAUTO = \Delta\left(AUTO\right) + AUTO_{DES} \quad (5.17.22)$$

5.17.3.1 Level 3: Arbitrage between automobile price classes

New automobiles of class *ecl*

$$NewAUTO_{ecl} = \varphi_{ecl}^{NewAUTO}\ NewAUTO \tag{5.17.23}$$

Utility of a automobile of class *ecl*

$$\Delta\left(U_{ecl}^{AUTO}\right) = -\theta^{U^{AUTO,UC,K}}.\Delta\left(UC_{ecl}^{K,AUTO}\right) - \theta^{U^{AUTO,UC,E}}.\Delta\left(UC_{ecl}^{E,AUTO}\right) \tag{5.17.24}$$

The utility of an automobile is a negative function of its capital and energy user costs. For convenience it is calibrated as equal to $\log \varphi_{ecl}^{NewAUTO}$ at the base year. The coefficients of the utility function are derived from the study of Durrmeyer and Samano (2017) [1].

Sum of the exponential of the automobile utilities per class

$$SUM^{exp,U,AUTO} = \sum_{ecl} e^{U_{ecl}^{AUTO}} \tag{5.17.25}$$

Notional share of class *ecl* automobile

$$\varphi_{ecl}^{NewAUTO^n} = \frac{e^{U_{ecl}^{AUTO}}}{SUM^{exp,U,AUTO}} \tag{5.17.26}$$

Share of class *ecl* automobiles

$$\varphi_{ecl}^{NewAUTO} = \alpha^{phi,NewAUTO}.\varphi_{ecl}^{NewAUTO^n} + \left(1 - \alpha^{phi,NewAUTO}\right).\varphi_{ecl,t-1}^{NewAUTO} \tag{5.17.27}$$

Notice that $\alpha^{phi,NewAUTO}$ is common to every class transition. This ensures that $\sum_{ecl2} \varphi_{ecl}^{NewAUTO} = 1$.

5.17.3.2 Level 4: Arbitrage between electric and thermic automobile per classes

New electric automobiles of class *ecl*

$$NewAUTO_{ecl,cele} = \varphi_{ecl,cele}^{NewAUTO}\ NewAUTO_{ecl} \tag{5.17.28}$$

Every class of electric car has no emission. But their energy consumption vary per km.

Share of class *ecl* electric automobile

$$\varphi_{ecl,cele}^{NewAuto} = \varphi_{ecl,cele}^{NewAuto^n} \tag{5.17.29}$$

[1] Durrmeyer, I., & Samano, M. (2017). To rebate or not to rebate: Fuel economy standards versus feebates. The Economic Journal, 128(616), 3076-3116

Notional share of class *ecl* electric automobile

$$\frac{\Delta\left(\varphi_{ecl,cele}^{NewAUTO^n}\right)}{\left(1-\varphi_{ecl,cele,t-1}^{NewAUTO^n}\right)} = innovation_{ecl}^{exo} + innovation_{ecl} + imitation_{ecl}\ \varphi_{ecl,cele,t-1}^{NewAUTO^n} \tag{5.17.30}$$

The adoption of electric automobile is modeled according to Bass Diffusion Model (Bass, 1969) [2]. The parameters of the model are calibrated using the study of Taszka et al. (2017) [3].

Bass innovation parameter for class *ecl* electric automobile

$$\begin{aligned}&\Delta\left(innovation_{ecl}\right)\\ &= \eta_{ecl}^{BASS}\ \Delta\left(\frac{\left(2.UC_{ecl,cele}^{AUTO}\right)^{\left(-\nu_{ecl}^{diffusion}\right)}}{\left(\left(2.UC_{ecl,cele}^{AUTO}\right)^{\left(-\nu_{ecl}^{diffusion}\right)}\right)+\left(\left(UC_{ecl,th}^{AUTO}\right)^{\left(-\nu_{ecl}^{diffusion}\right)}\right)}\right)\end{aligned} \tag{5.17.31}$$

New thermic automobiles of class *ecl*

$$NewAUTO_{ecl,th} = NewAUTO_{ecl} - NewAUTO_{ecl,cele} \tag{5.17.32}$$

New thermic automobiles of class *ecl* (for verification)

$$NewAUTO_{ecl,th}^{bis} = NewAUTO_{ecl,cfut} + NewAUTO_{ecl,cgas} \tag{5.17.33}$$

5.17.3.3 Level 5: Arbitrage between types of thermic automobiles

New thermic automobiles fueled with oil of class *ecl*

$$NewAUTO_{ecl,cfut} = \varphi_{ecl,cfut}^{NewAUTO}\ NewAUTO_{ecl,th} \tag{5.17.34}$$

New thermic automobiles fueled with gas of class *ecl*

$$NewAUTO_{ecl,cgas} = NewAUTO_{ecl,th} - NewAUTO_{ecl,cfut} \tag{5.17.35}$$

Share of New thermic automobiles fueled with oil of class *ecl*

$$\Delta\left(\varphi_{ecl,cfut}^{NewAUTO}\right) = 0 \tag{5.17.36}$$

For simplicity, the share of the different types of thermic automobiles is assumed constant. Arbitrage between types of thermic automobiles can be added by modifying the above equation.

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[2] Bass, F. M. (1969). A new product growth for model consumer durables. Management science, 15(5), 215-227

[3] Taszka, S., Domergue, S., Poret, M., & Monnoyer-Smith, L. (2017). Analyse coûts bénéfices des véhicules électriques, Les voitures. Cost-benefit analysis of electrical vehicles,Cars. Théma. Commissariat général au développement durable

5.17.3.4 Level 5: Stock of automobiles

Stock of automobiles of class *ecl* **fueled with energy** *cea*

$$\Delta\left(AUTO_{ecl,cea}\right) = NewAUTO_{ecl,cea} - AUTO_{ecl,cea,DES} \tag{5.17.37}$$

Class *ecl* **fueled with energy** *cea* **automobiles destroyed**

$$AUTO_{ecl,cea,DES} = \delta^{AUTO}_{ecl,cea,DES}\ AUTO_{ecl,cea,t-1} \tag{5.17.38}$$

Automobiles fueled with energy *cea*

$$AUTO_{cea} = \sum_{ecl} AUTO_{ecl,cea} \tag{5.17.39}$$

Stock of class *ecl* **automobiles**

$$AUTO_{ecl} = AUTO_{ecl,cele} + AUTO_{ecl,th} \tag{5.17.40}$$

Thermic automobiles

$$AUTO_{th} = \sum_{ecl} AUTO_{ecl,th} \tag{5.17.41}$$

Stock of class *ecl* **thermic automobiles**

$$AUTO_{ecl,th} = AUTO_{ecl,cfut} + AUTO_{ecl,cgas} \tag{5.17.42}$$

Automobiles fueled with energy *cea* **destroyed**

$$AUTO_{cea,DES} = \sum_{ecl} AUTO_{ecl,cea,DES} \tag{5.17.43}$$

Automobiles destroyed

$$AUTO_{DES} = \sum_{ecl} AUTO_{ecl,DES} \tag{5.17.44}$$

Class *ecl* **automobiles destroyed**

$$AUTO_{ecl,DES} = AUTO_{ecl,cele,DES} + AUTO_{ecl,th,DES} \tag{5.17.45}$$

Class *ecl* **thermic automobiles destroyed**

$$AUTO_{ecl,th,DES} = AUTO_{ecl,cfut,DES} + AUTO_{ecl,cgas,DES} \tag{5.17.46}$$

Thermic automobiles destroyed

$$AUTO_{th,DES} = \sum_{ecl} AUTO_{ecl,th,DES} \tag{5.17.47}$$

5.17.3.5 User cost of automobile

User energy cost of a class *ecl* automobile fueled with energy *cea*

$$UC^{E,AUTO}_{ecl,cea} = PE^{AUTO}_{ecl,cea} \frac{\left(\left(1+g^{PE^{AUTO,e}}_{ecl,cea}\right)^{AUTO^{D}_{ecl,cea}}\right)-1}{g^{PE^{AUTO,e}}_{ecl,cea}\ AUTO^{D}_{ecl,cea}} \tag{5.17.48}$$

Energy price of a class *ecl* automobile fueled with energy *cea*

$$PE^{AUTO}_{ecl,cea}\ AUTO_{ecl,cea} = PCH_{cea}\ CH^{AUTO}_{cea}\ \frac{CH^{AUTO,toe}_{ecl,cea}}{CH^{AUTO,toe}_{cea}} \tag{5.17.49}$$

Growth rate of the energy price of an automobile of class *ecl* fueled with energy *cea*

$$g^{PE^{AUTO}}_{ecl,cea} = \alpha^{g_1^{PE^{AUTO,e}}}\ g^{PE^{AUTO}_{ecl,cea,t-1}} + \left(1-\alpha^{g_1^{PE^{AUTO,e}}}\right)\ g^{PE^{AUTO,e}_{ecl,cea,t-1}} \tag{5.17.50}$$

User capital cost of a class *ecl* automobile fueled with energy *cea*

$$\begin{aligned} UC^{K,AUTO}_{ecl,cea} = &\left(\left(1-R^{SUB,AUTO}_{ecl,cea}\right)\ \frac{PNewAUTO_{ecl,cea}}{AUTO^{D}_{ecl,cea}}\right)\left(R^{CASH,AUTO}_{ecl,cea}\right. \\ &\left. + \frac{R^{LOAN,AUTO}_{ecl,cea}\ R^{I,AUTO}_{ecl,cea,t-1}\ LD^{AUTO}_{ecl,cea}}{1-\left(1+R^{I,AUTO}_{ecl,cea,t-1}\right)^{\left(-LD^{AUTO}_{ecl,cea}\right)}}\right) \end{aligned} \tag{5.17.51}$$

Price of a new thermic automobiles fueled with oil of class *ecl*

$$\Delta\left(\log PNewAUTO_{ecl,cfut}\right) = \Delta\left(\log PCH_{cveh}\right) \tag{5.17.52}$$

Price of a new thermic automobiles fueled with gas of class *ecl*

$$\Delta\left(\log PNewAUTO_{ecl,cgas}\right) = \Delta\left(\log PCH_{cveh}\right) \tag{5.17.53}$$

Price of a new electric automobiles of class *ecl*

$$PNewAUTO_{ecl,cele} = PNewAUTO_{ecl,cfut} + Pbattery + overcost^{elec} \tag{5.17.54}$$

Price of a new automobiles of class *ecl*

$$\begin{aligned} PNewAUTO_{ecl}\ NewAUTO_{ecl} = &\ PNewAUTO_{ecl,cele}\ NewAUTO_{ecl,cele} \\ &+ PNewAUTO_{ecl,th}\ NewAUTO_{ecl,th} \end{aligned} \tag{5.17.55}$$

Price of a new thermic automobiles of class *ecl*

$$\begin{aligned} PNewAUTO_{ecl,th}\ NewAUTO_{ecl,th} = &\ PNewAUTO_{ecl,cfut}\ NewAUTO_{ecl,cfut} \\ &+ PNewAUTO_{ecl,cgas}\ NewAUTO_{ecl,cgas} \end{aligned} \tag{5.17.56}$$

Price of a new automobiles

$$PNewAUTO.NewAUTO = \sum_{ecl} PNewAUTO_{ecl}\ NewAUTO_{ecl} \quad (5.17.57)$$

User maintenance cost of a class ecl automobile fueled with energy cea

$$UC^{M,AUTO}_{ecl,cea} = PCH_{cveh}\ \left(\frac{MCperkm_{ecl,cea}}{100}\right)\ \left(\frac{kmPerAuto}{1000}\right) \quad (5.17.58)$$

User cost of an automobile

$$UC^{AUTO}.AUTO = \sum_{ecl} UC^{AUTO}_{ecl}\ AUTO_{ecl} \quad (5.17.59)$$

User cost of an automobile (for verification)

$$UC^{AUTO,bis} = UC^{K,AUTO} + UC^{E,AUTO} + UC^{M,AUTO} \quad (5.17.60)$$

User capital cost of an automobile

$$UC^{K,AUTO}.AUTO = \sum_{ecl} UC^{K,AUTO}_{ecl}\ AUTO_{ecl} \quad (5.17.61)$$

User energy cost of an automobile

$$UC^{E,AUTO}.AUTO = \sum_{ecl} UC^{E,AUTO}_{ecl}\ AUTO_{ecl} \quad (5.17.62)$$

User maintenance cost of an automobile

$$UC^{M,AUTO}.AUTO = \sum_{ecl} UC^{M,AUTO}_{ecl}\ AUTO_{ecl} \quad (5.17.63)$$

User cost of an automobile of class ecl

$$UC^{AUTO}_{ecl} = UC^{K,AUTO}_{ecl} + UC^{E,AUTO}_{ecl} + UC^{M,AUTO}_{ecl} \quad (5.17.64)$$

User cost of a class ecl automobile fueled with energy cea

$$UC^{AUTO}_{ecl,cea} = UC^{K,AUTO}_{ecl,cea} + UC^{E,AUTO}_{ecl,cea} + UC^{M,AUTO}_{ecl,cea} \quad (5.17.65)$$

User cost of a thermic automobile of class ecl

$$UC^{AUTO}_{ecl,th} = UC^{K,AUTO}_{ecl,th} + UC^{E,AUTO}_{ecl,th} + UC^{M,AUTO}_{ecl,th} \quad (5.17.66)$$

User capital cost of an automobile of class ecl

$$UC^{K,AUTO}_{ecl}\ AUTO_{ecl} = UC^{K,AUTO}_{ecl,cele}\ AUTO_{ecl,cele} + UC^{K,AUTO}_{ecl,th}\ AUTO_{ecl,th} \quad (5.17.67)$$

User capital cost of a thermic automobile of class ecl

$$UC^{K,AUTO}_{ecl,th}\ AUTO_{ecl,th} = UC^{K,AUTO}_{ecl,cfut}\ AUTO_{ecl,cfut} + UC^{K,AUTO}_{ecl,cgas}\ AUTO_{ecl,cgas} \quad (5.17.68)$$

User energy cost of an automobile of class ecl

$$UC^{E,AUTO}_{ecl}\ AUTO_{ecl} = UC^{E,AUTO}_{ecl,cele}\ AUTO_{ecl,cele} + UC^{E,AUTO}_{ecl,th}\ AUTO_{ecl,th} \tag{5.17.69}$$

User energy cost of a thermic automobile of class ecl

$$UC^{E,AUTO}_{ecl,th}\ AUTO_{ecl,th} = UC^{E,AUTO}_{ecl,cfut}\ AUTO_{ecl,cfut} + UC^{E,AUTO}_{ecl,cgas}\ AUTO_{ecl,cgas} \tag{5.17.70}$$

User maintenance cost of an automobile of class ecl

$$UC^{M,AUTO}_{ecl}\ AUTO_{ecl} = UC^{M,AUTO}_{ecl,cele}\ AUTO_{ecl,cele} + UC^{M,AUTO}_{ecl,th}\ AUTO_{ecl,th} \tag{5.17.71}$$

User maintenance cost of a thermic automobile of class ecl

$$UC^{M,AUTO}_{ecl,th}\ AUTO_{ecl,th} = UC^{M,AUTO}_{ecl,cfut}\ AUTO_{ecl,cfut} + UC^{M,AUTO}_{ecl,cgas}\ AUTO_{ecl,cgas} \tag{5.17.72}$$

5.17.3.6 Households' energy consumption related to transport

Kilometers by a class ecl automobile fueled with energy cea

$$km^{AUTO}_{ecl,cea} = km^{AUTO}.\frac{AUTO_{ecl,cea}}{AUTO} \tag{5.17.73}$$

Kilometers by class ecl automobile

$$km^{AUTO}_{ecl} = \sum_{cea} km^{AUTO}_{ecl,cea} \tag{5.17.74}$$

Total kilometers by automobile

$$km^{AUTO,bis} = \sum_{ecl} km^{AUTO}_{ecl} \tag{5.17.75}$$

Energy consumption of a class ecl automobile fueled with energy cea expressed in tonne of oil equivalent

$$CH^{AUTO,toe}_{ecl,cea} = km^{AUTO}_{ecl,cea}\ toePerKm_{ecl,cea} \tag{5.17.76}$$

Energy consumption of automobiles fueled with energy cea expressed in tonne of oil equivalent

$$CH^{AUTO,toe}_{cea} = \sum_{ecl} CH^{AUTO,toe}_{ecl,cea} \tag{5.17.77}$$

Households' final consumption of electricity (expressed in monetary unit)

$$\Delta\left(\log CH^{AUTO}_{cea}\right) = \Delta\left(\log CH^{AUTO,toe}_{cea}\right) \tag{5.17.78}$$

5.17.3.7 Debts and expenditures related to automobile

Debt related to the purchase of a class *ecl* automobile fueled with energy *cea*

$$DEBT^{AUTO,VAL}_{ecl,cea} = DEBT^{AUTO,VAL}_{ecl,cea,t-1}\left(1 - R^{RMBS,AUTO}_{ecl,cea,t-1}\right) + R^{LOAN,AUTO}_{ecl,cea}\, PNewAUTO_{ecl,cea}\, NewAUTO_{ecl,cea}\left(1 - R^{SUB,AUTO}_{ecl,cea}\right) \tag{5.17.79}$$

Expenditures related to the use of a class *ecl* automobile fueled with energy *cea* (in value)

$$EXP^{AUTO,VAL}_{ecl,cea} = DEBT^{AUTO,VAL}_{ecl,cea,t-1}\left(R^{I,AUTO}_{ecl,cea,t-1} + R^{RMBS,AUTO}_{ecl,cea,t-1}\right) + R^{CASH,AUTO}_{ecl,cea}\, PNewAUTO_{ecl,cea}\, NewAUTO_{ecl,cea}\left(1 - R^{SUB,AUTO}_{ecl,cea}\right) + \left(\frac{\left(PCH_{cea}\, CH^{AUTO}_{cea}\, CH^{AUTO,toe}_{ecl,cea}\right)}{CH^{AUTO,toe}_{cea}}\right) + UC^{M,AUTO}_{ecl,cea}\, AUTO_{ecl,cea} \tag{5.17.80}$$

Expenditures related to the use of a class *ecl* automobile (in value)

$$EXP^{AUTO,VAL}_{ecl} = \sum_{cea} EXP^{AUTO,VAL}_{ecl,cea} \tag{5.17.81}$$

Expenditures related to the use of an automobile fueled with energy *cea* (in value)

$$EXP^{AUTO,VAL}_{cea} = \sum_{ecl} EXP^{AUTO,VAL}_{ecl,cea} \tag{5.17.82}$$

Expenditures related to the use of an automobile (in value)

$$EXP^{AUTO,VAL} = \sum_{ecl} EXP^{AUTO,VAL}_{ecl} \tag{5.17.83}$$

Expenditures related to the use of an automobile (for verification)

$$EXP^{AUTO,VAL,bis} = \sum_{cea} EXP^{AUTO,VAL}_{cea} \tag{5.17.84}$$

Price of kilometer-travelers for short distance by automobile

$$P^{km,AUTO}.km^{AUTO} = EXP^{AUTO,VAL}.1000 \tag{5.17.85}$$

Price of kilometer-travelers for short distance by automobile

$$P^{km,trav,auto,SD}.km^{trav,auto,SD} = P^{km,AUTO}.\frac{km^{AUTO,SD}}{1000} \tag{5.17.86}$$

Price of kilometer-travelers for long distance by automobile

$$P^{km,trav,auto,LD}.km^{trav,auto,LD} = P^{km,AUTO}.\frac{km^{AUTO,LD}}{1000} \tag{5.17.87}$$

Price of kilometer-travelers for automobile transportation

$$\begin{aligned} P^{km,trav,auto}.km^{trav,auto} = {} & P^{km,trav,auto,LD}.km^{trav,auto,LD} \\ & + P^{km,trav,auto,SD}.km^{trav,auto,SD} \end{aligned} \tag{5.17.88}$$

Total kilometer-travelers by automobile

$$km^{trav,auto} = km^{trav,auto,LD} + km^{trav,auto,SD} \tag{5.17.89}$$

5.18 Integration of an Emission Trading Scheme

Nominal price of emissions permits

$$P^{ETS,nominal} = \left(\frac{P^{ETS}}{1000000}\right).PGDP \tag{5.18.1}$$

Quantity of freely allocated permits to sector s For each sector, the quantity of free permits is defined as a share of emissions that are eligible to the emission trading scheme.

$$Q_s^{ETS,free} = share_s^{free}\ share_s^{ETS}\ EMS_s \tag{5.18.2}$$

Quantity of emissions permits required for sector s relative to its $ce2$ energy consumption

$$Q_{ce2,s}^{ETS} = \left(share_s^{ETS}\ EMS_{ce2,s}\right) - Q_s^{ETS,frenergybalance.mdlee}\ \frac{EMS_{ce2,s}}{EMS_s} \tag{5.18.3}$$

Nominal value of emissions permits bought by sector s due to $ce2$ consumption

$$ETS_{ce2,s}^{VAL} = \left(P^{ETS,nominal}.Q_{ce2,s}^{ETS}\right) \tag{5.18.4}$$

Nominal value of emissions permits required for sector s

$$ETS_s^{VAL} = \sum_{ce2} ETS_{ce2,s}^{VAL} \tag{5.18.5}$$

Total nominal value of emissions permits on the trading market

$$ETS^{VAL,TOT} = \sum_{s} ETS_s^{VAL} \tag{5.18.6}$$

5.18.1 Scenario 1 : price signal

We overwrite the equation of the price block in order to add the ETS cost (energy $ce2$ used in activity s). We only consider energy goods $ce2$ - and not ce - since intermediary consumption of electricity does not induce dioxyde emissions.

Price of $ce2$ energy consumption in sector s

$$PE_{ce2,s}\ E_{ce2,s} = PED_{ce2}\ ED_{ce2,s} + PEM_{ce2}\ EM_{ce2,s} + ETS^{VAL,SEC}_{ce2,s} \tag{5.18.7}$$

5.18.2 Scenario 2 : implicit production subvention

We overwrite the production cost equation of the price block in order to consider the freely allocated permits as an implicit subvention on production.

Nominal production prices of covered sectors

$$\begin{aligned} PY^{n}_{s}\ Y_{s} = \big(&CK_{s}\ K_{s} + CL_{s}\ L_{s}\ PROG^{L}_{s} + PE_{s}\ E_{s} + PMAT_{s}\ MAT_{s} + PIY_{s}\ IY_{s} \\ &+ PSY_{s}\ SY_{s} + PIS_{s}\ IS_{s} - P^{ETS,nominal}.Q^{ETS,free}_{s}\big)\ (1 + TMD_{s}) \end{aligned} \tag{5.18.8}$$

5.18.3 Government budget

We overwrite the equation in order to add the aggregated value of bought permits to the revenue of the government.
Government revenues

$$\begin{aligned} INC^{G,VAL} = &PNTAXC.NTAXC + NTAXS^{VAL} + INC^{SOC,TAX,VAL} \\ &+ PRSC.RSC + PROP^{INC,G,VAL} + ETS^{VAL,TOT} \end{aligned} \tag{5.18.9}$$

Employers' social security contribution paid by sector s

$$RSC_{s}\ PRSC_{s} = W_{s}\ F_{L,s}\ RRSC_{s} - ETS^{VAL}_{s} \tag{5.18.10}$$

Necessary (minimum) households' final consumption for construction commodity $ccon$

$$NCH_{ccon} = PNewBUIL_{t_0}\ NewBUIL + PREHAB_{t_0}\ REHAB + CH^{OTH}_{CCON} \tag{5.18.11}$$

5.19 Exogenous variables

5.19.1 From Prices section

5.19.1. $DNAIRU$ – **Change in the long term NAIRU**

5.19.2. $\rho^{Rn,Cons}$ – **Constant of the notional interest rate equation (Taylor rule).**

The constant is equal to zero in the baseline scenario. It can be used to simulate monetary shocks.

5.19.3. $\rho^{Rn,P}$ – **Elasticity of the notional interest rate to inflation**

5.19.4. $\rho^{Rn,UnR}$ – **Elasticity of the notional interest rate to the unemployment rate**

5.19.5. $\rho_s^{W,DU}$ – **Elasticity of the notional wage inflation to the variation of the unemployment rate for sector** s

5.19.6. $\rho_s^{W,L}$ – **Elasticity of the notional wage inflation to the growth rate of share of labor in sector** s

5.19.7. $\rho_s^{W,P}$ – **Elasticity of the notional wage inflation to prices inflation for sector** s

5.19.8. $\rho_s^{W,Pe}$ – **Elasticity of the notional wage inflation to expected price inflation for sector** s

5.19.9. $\rho_s^{W,PROG}$ – **Elasticity of the notional wage inflation to labor productivity growth for sector** s

5.19.10. $\rho_s^{W,U}$ – **Elasticity of the notional wage inflation to the unemployment rate for sector** s

5.19.11. $\rho_s^{W,X}$ – **Constant in the wage equation for sector** s

5.19.2 From Producer section

5.19.12. $\varphi_{f,s}$ – **Share of the production input** f **by sector** s

5.19.3 From Consumer section

5.19.13. η^{lesces} – **Elasticity of the linear elasticity system**

5.19.14. gr^{nomi} – **Nominal growth rate of the economy**

5.19.15. $Pwages$ – **Nominal price index**

5.19.16. $prog_s^L$ – **Labor productivity in the sector** s

5.19.17. $RINC^{SOC,TAX}$ – **Rate of social taxes to the disposable income**

5.19.18. $\rho^{MPS,R}$ – **Elasticity of the marginal propensity to share to the growth rate of the interest rate**

5.19.19. $\rho^{MPS,UnR}$ – **Elasticity of the marginal propensity to share to the growth rate of unemployment**

5.19.20. $riskprem$ – **Risk premium**

5.19.21. RR^{POP} – **Population growth rate**

5.19.22. RR^{Un} – **unemployment growth rate**

5.19.23. $TRSF^{HH,VAL}$ – **Transfers to households**

5.19.4 From Government section

5.19.24. $EXPG^{trend}$ – **Public expenditures trend from the government**

5.19.25. $\varphi^{PROP^{INC,G}}$ – **Propriety income of the government expressed in value**

5.19.26. φ^{RD^G} – **Share of debt reimbursed every year**

5.19.27. $RNTAXS_s$ – **tax rate on the production of sector** s

5.19.28. $RRSC_s$ – **Employers' social security contribution rate of the sector** s

5.19.5 From International trade section

5.19.29. η_c^{CHM} – **Elasticity of substitution between imported and domestically produced commodity c for households final consumption**

5.19.30. $\eta_{c,s}^{CIM}$ – **Elasticity of substitution between imported and domestic intermediary consumption in commodity c from the sector s**

5.19.31. η_c^{GM} – **Elasticity of substitution between imported and domestically produced commodity c for government final consumption**

5.19.32. $\eta_{c,s}^{IM}$ – **Elasticity of substitution between imported and domestic investment in commodity c from the sector s**

5.19.33. η_c^{MGSM} – **Elasticity of substitution between imported and domestically produced commodity c for margins supplied**

5.19.34. η_c^{XM} – **Elasticity of substitution between domestic and imported commodity c for exports**

5.19.35. η_c^{X} – **Elasticity of substitution between french exports and rest of the world exports**

5.19.36. EXR – **Exchange rate of the euro with the rest of world's currency**

5.19.6 From Demography section

5.19.37. $PARTR^{TREND}$ – **Participation rate to the labor force trend**

5.19.38. POP – **Total population**

5.19.39. $\rho^{PATR,UnR}$ – **Elasticity of unemployed population with the labor force participation**

5.19.7 From GHG Emissions section

5.19.40. $IEMS_{ghg,c}^{CH}$ – **Emissions intensity of the greenhouse gas ghg related to the household final consumption of commodity c**

5.19.41. $IEMS_{ghg,c,s}^{CI}$ – **Emissions intensity of the greenhouse gas ghg related to the intermediary consumption of commodity c by sector s**

5.19.42. $IEMS^{MAT}_{ghg,s}$ – **Emissions intensity of the greenhouse gas ghg related to the material consumption of sector s**

5.19.43. $IEMS^{Y}_{ghg,s}$ – **Emissions intensity of the greenhouse gas ghg related to the final production of sector s**

5.19.8 From Energy balance section

5.19.44. $\varphi^{Y^{toe}}_{ce,s}$ – **Share of energy ce produced by sector s (energy balance definition)**

5.19.45. $PY^{toe}_{ce,s}$ – **Net production price of energy ce supplied by sector s**

5.19.9 From Adjustment section

5.19.46. $\alpha^{Pe,P1}$ – **Adjustment parameter on the formation of inflation expectation**

5.19.47. α^{μ}_{s} – **Adjustment parameter between the notional and the previous period value of the mark-up in the sector s**

5.19.48. $\alpha^{W,Wn}_{s}$ – **Adjustment parameter on the notional value of the wages in the sector s**

5.19.49. $\alpha^{W,W1}_{s}$ – **Adjustment parameter on the previous period value of the wages in the sector s**

5.19.50. $\alpha^{W,W1Wn1}_{s}$ – **Adjustment parameter on the ratio between the previous period value of the wages and the previous period notional value in the sector s**

5.19.51. $\alpha^{Ye,Y}_{s}$ – **Adjustment parameter on the formation of production of the sector s expectation**

5.19.52. $\alpha^{0}_{f,s}$ – **Adjustment parameter between the notional and the expected value of the variable Ff,s,t**

5.19.53. $\alpha^{0,CH}_{c}$ – **Adjustment parameter between the notional and the expected value of household consumption of commodity c**

5.19.54. $\alpha_s^{0,IA}$ – **Adjustment parameter between the notional and the expected value of investment in the sector** s

5.19.55. $\alpha^{0,PARTR}$ – **Adjustment parameter on the notional value of the labor participation ratio**

5.19.56. $\alpha^{0,PROP,INC,H,VAL}$ – **Adjustment parameter on the notional value of the Households property income**

5.19.57. $\alpha^{0,R}$ – **Adjustment parameter on the notional value of the interest rate**

5.19.58. $\alpha_s^{0,PY}$ – **Adjustment parameter on the notional value of production price in the sector** s

5.19.59. $\alpha_{f,s}^{1}$ – **Adjustment parameter between the expected and the real value of a variable** $F_{f,s,t-1}^{e}$

5.19.60. $\alpha_c^{1,CH}$ – **Adjustment parameter on the previous period expected value of household consumption of commodity** c

5.19.61. $\alpha_s^{1,IA}$ – **Adjustment parameter on the previous period expected value of investment in the sector** s

5.19.62. $\alpha^{1,PROP,INC,G,VAL}$ – **Adjustment parameter on the expected value of the government property income**

5.19.63. $\alpha_s^{1,PY}$ – **Adjustment parameter on the previous period expected value of production price in the sector** s

5.19.64. $\alpha_{f,s}^{2}$ – **Adjustment parameter between the previous period real value and the current value of a variable** $F_{f,s,t-1}$

5.19.65. $\alpha_c^{2,CH}$ – **Adjustment parameter on the previous period real value of household consumption of commodity** c

5.19.66. $\alpha_s^{2,IA}$ – **Adjustment parameter on the previous period real value of investment in the sector** s

5.19.67. $\alpha_s^{2,PY}$ – **Adjustment parameter on the previous period value of production price in the sector** s

5.19.68. $\alpha^{3}_{f,s}$ – **Adjustment parameter between the notional and the real value of the variable** $F^{n}_{f,s,t-1}$

5.19.69. $\alpha^{3,CH}_{c}$ – **Adjustment parameter on the notional value of household consumption of commodity** c

5.19.70. $\alpha^{3,IA}_{s}$ – **Adjustment parameter on the notional value of investment in the sector** s

5.19.71. $\alpha^{3,PY}_{s}$ – **Adjustment parameter on the notional value of production price in the sector** s

5.19.72. $\alpha^{6}_{f,s}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the production factor** f **in the sector** s

5.19.73. $\alpha^{6,CI}_{ce,s}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the energy intermediate consumption** ce **in the sector** s

5.19.74. $\alpha^{6,CI}_{ct,s}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the transportation intermediate consumption** ce **in the sector** s

5.19.75. $\alpha^{6,CHM}_{c}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the imported households final consumption for the commodity** c

5.19.76. $\alpha^{6,CIM}_{c,s}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the intermediate consumption for the imported commodity** c **in the sector** s

5.19.77. $\alpha^{6,GM}_{c}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the imported government final consumption for the commodity** c

5.19.78. $\alpha^{6,IM}_{c,s}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the investment for the imported commodity** c **in the sector** s

5.19.79. $\alpha^{6,MGSM}_{m}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the imported margin supplied for**

the commodity m

5.19.80. $\alpha^{6,MGPD}_{m,c}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the domestic margin paid** m **for the commodity** c

5.19.81. $\alpha^{6,MGPM}_{m,c}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the imported margin paid** m **for the commodity** c

5.19.82. $\alpha^{6,X}_{c}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the exports of the commodity** c

5.19.83. $\alpha^{6,XM}_{c}$ – **Adjustment parameter between the notional and the real value for the substitution effect on the exports for the imported commodity** c

5.19.10 From Taxes and Prices exception

5.19.84. $R2^{CH}_{ghg,c}$ – **Carbon tax rate on the greenhouse gas** ghg **associated to the household final consumption of the sector** s

5.19.85. $R2^{CI}_{ghg,c,s}$ – **Carbon tax rate on the greenhouse gas** ghg **associated to the intermediary consumption of commodity** c **by the sector** s

5.19.86. $R2^{MAT}_{ghg,s}$ – **Carbon tax rate on the greenhouse gas** ghg **associated to the material consumption of the sector** s

5.19.87. $R2^{Y}_{ghg,s}$ – **Carbon tax rate on the greenhouse gas** ghg **associated to the final production of the sector** s

5.19.11 From ETS section

5.19.88. $share^{free}_{s}$ – **Percentage of freely allocated permits to sector** s

5.19.89. $share^{ETS}_{s}$ – **Percentage of emissions covered by an ETS scheme for the sector** s

5.19.12 From Nested utility function section

5.19.90. η^{CHTRSP} – **Elasticity of substitution between transport expenditures**

5.19.91. $\eta^{HOUS^{ENER}}$ – **Elasticity of substitution between energy housing expenditures**

5.19.92. $\eta^{HOUS^{INV,ENER}}$ – **Elasticity of substitution between investment and expenditures in energy housing**

5.19.93. $\eta^{TRSP^{INV,ENER}}$ – **Elasticity of substitution between investment and expenditures in transport**

5.19.13 From Housing section

5.19.94. $\alpha^{GR,PENER,m2,e,1}$ – **Adjustment parameter between the growth rate of the energy price per square meter between its past period and its expected value**

5.19.95. $\alpha^{phi,REHAB}$ – **Adjustment parameter between the rehabilitation of a building of a class** ecl **to a class** $ecl2$

5.19.96. $\alpha^{0,\tau,REHAB}$ – **Adjustment parameter between the notional and the real value of rehabilitation rate for buildings**

5.19.97. $BUIL^{D}_{ecl}$ – **Lifetime of a building of class** ecl **(definition ?)**

5.19.98. $\delta^{BUIL}_{ecl,bcl}$ – **Depreciation rate of building stock from class** ecl **to class** bcl

5.19.99. $\zeta_{ecl,ceb}$ – **Elasticity of substitution between the energy commodity** ceb **and the surface of housing per capita in class** ecl **buldings**

5.19.100. $\eta^{BUILNRJ}_{ecl,ceb,cebb}$ – **Elasticity of substitution between the energy commodity** ceb **and the other energy commodities** $cebb$ **in class** ecl **buldings**

5.19.101. g^{NOMI} – **Growth rate of the nominal interest rate**

5.19.102. LD^{REHAB}_{ecl} – **Duration of the loan funding rehabilitation**

5.19.103. LD_{ecl} – **Duration of the loan for housing of class** ecl **investment**

5.19.104. $R_{ecl}^{CASH,REHAB}$ – **Share of investment for rehabilitation directly paid by the households**

5.19.105. $R_{ecl}^{CASH,NewBUIL}$ – **Share of investment for the construction of a building directly paid by the households**

5.19.106. $R_{ecl}^{RMBS,REHAB}$ – **Share of investment for rehabilitation directly paid by the households**

5.19.107. $R_{ecl}^{RMBS,NewBUIL}$ – **Share of the reimbursement of a loan related to the purchase of building of class** ecl

5.19.108. $R_{ecl,ecl2}^{SUB}$ – **Share of public subsidy for the rehabilitation of a building from class** ecl **to class** $ecl2$

5.19.109. $\rho_{ecl,ecl2}^{U,GAIN}$ – **Utility elasticity of the financial gain induced by the rehabilitating a building from class** ecl **to class** $ecl2$

5.19.110. σ_{ecl} – **Switching speed between the 2 regimes of the logistic function representing the notional rehabilitation rate of a class** ecl **building**

5.19.111. $\tau_{ecl}^{REHAB,MIN}$ – **Lower bound of the rehabilitation rate for buildings of class** ecl

5.19.112. $\tau_{ecl}^{REHAB,MAX}$ – **Upper bound of the rehabilitation rate for buildings of class** ecl

5.19.14 From Transport section

5.19.113. $\alpha^{phi,NewAUTO}$ – **Adjustment parameter between the notional share and the past period value of** ecl **class automobiles in the year's sales**

5.19.114. $\alpha^{GR,PE,AUTO,e,1}$ – **Adjustment parameter on the growth rate of the energy prices**

5.19.115. $AUTO_{ecl,cea}^{D}$ – **Life duration of a car of class** ecl **fueled with energy** cea

5.19.116. $\delta^{AUTO}_{ecl,cea,DES}$ – **Depreciation rate of automobile of class** ecl **into other classes** cea **and destruction** DES

5.19.117. $kmPerAuto$ – **Average distance in km driven by cars in a year**

5.19.118. $km^{traveler}_{cth}$ – **kilometer-traveler of public transport commodity** cth

5.19.119. $LD^{AUTO}_{ecl,cea}$ – **kilometer-traveler of public transport commodity** cth

5.19.120. $innovation^{exo}_{ecl}$ – **exogenous innovation trend in electric cars**

5.19.121. $MCperkm_{ecl,cea}$ – **Maintenance cost per km for the automobile of class** ecl **fueled with energy** cea

5.19.122. η^{BASS}_{ecl} – **Elasticity between the the innovation in electric cars and the bass diffusion model equation**

5.19.123. $\nu^{diffusion}_{ecl}$ – **Elasticity between the the innovation stock in electric cars and its diffusion**

5.19.124. $R^{CASH,AUTO}_{ecl,cea}$ – **Share of the price for a new car directly financed by the household**

5.19.125. $R^{LOAN,AUTO}_{ecl,cea}$ – **Share of the price of the purchase of a new car financed through a loan**

5.19.126. $R^{RMBS,AUTO}_{ecl,cea}$ – **Share of the reimbursement of a loan related to the purchase of a car**

5.19.127. $R^{SUB,AUTO}_{ecl,cea}$ – **Share of the price of the purchase of a car financed by a public subsidy**

5.19.128. $\theta^{kmtravcair^{DISPINC}}$ – **Elasticity of substitution between long-distance air transport and income**

5.19.129. $\theta^{kmtravcair^{Pkmtrav}}$ – **Price elasticity of long-distance air transport**

5.19.130. $\theta^{kmtravLD^{DISPINC}}$ – **Elasticity of substitution between long-distance (by road or train) transports and income**

5.19.131. $\theta^{kmtravLD^{Pkmtrav}}$ – **Price elasticity of short-distance (by road or**

train) of transport commodity

5.19.132. $\theta^{kmtravSD^{DISPINC}}$ – **Elasticity of substitution between long-distance (by road or train) transports and income**

5.19.133. $\theta^{kmtravSD^{Pkmtrav}}$ – **Price elasticity of short-distance (by road or train) of transport commodity**

5.19.134. $\theta^{kmtrav^{autoLD,crai}}$ – **Elasticity between road and rail transport use for long distance trips**

5.19.135. $\theta^{kmtrav^{autoSD,crai}}$ – **Elasticity between road and rail transport use for short distance trips**

5.19.136. $\theta^{kmtrav^{autoSD,croa}}$ – **Elasticity between automobile and bus use for short distance trips**

5.19.137. $\theta^{U^{AUTO,UC,E}}$ – **Elasticity between the marginal utility from automobile use and its energy cost**

5.19.138. $\theta^{U^{AUTO,UC,K}}$ – **Elasticity between the marginal utility from automobile use and its capital cost**

5.19.139. $toePerKm_{ecl,cea}$ – **Energy consumption in ton equivalent petrol per km for the automobile of class** ecl **fueled with energy** cea

5.20 List of variables and parameters

The first column provides the variable name as it appears both in the ThreeME equations and in the EViews code. The definition of the variable is given in the second column, and the third column provides the equation number and the corresponding page within this document.

$AC^{toe}_{ce,s}$	Auto-consumption of energy ce by producer s, expressed in toe	5.10.15, 117
AC^{toe}_{ce}	Auto-consumption of energy ce, expressed in toe	5.10.19, 117
$\alpha^{GR,PE,AUTO,e,1}$	Adjustment parameter on the growth rate of the energy prices	5.19.114, 165
$\alpha^{GR,PENER,m2,e,1}$	Adjustment parameter between the growth rate of the energy price per square meter between its past period and its expected value	5.19.94, 164
α^{μ}_{s}	Adjustment parameter between the notional and the previous period value of the mark-up in the sector s	5.19.47, 160
$\alpha^{Pe,P1}$	Adjustment parameter on the formation of inflation expectation	5.19.46, 160
$\alpha^{phi,NewAUTO}$	Adjustment parameter between the notional share and the past period value of ecl class automobiles in the year's sales	5.19.113, 165
$\alpha^{phi,REHAB}$	Adjustment parameter between the rehabilitation of a building of a class ecl to a class $ecl2$	5.19.95, 164
$\alpha^{W,W1}_{s}$	Adjustment parameter on the previous period value of the wages in the sector s	5.19.49, 160
$\alpha^{W,W1Wn1}_{s}$	Adjustment parameter on the ratio between the previous period value of the wages and the previous period notional value in the sector s	5.19.50, 160
$\alpha^{W,Wn}_{s}$	Adjustment parameter on the notional value of the wages in the sector s	5.19.48, 160
$\alpha^{Ye,Y}_{s}$	Adjustment parameter on the formation of production of the sector s expectation	5.19.51, 160
$\alpha^{0,CH}_{c}$	Adjustment parameter between the notional and the expected value of household consumption of commodity c	5.19.53, 160
$\alpha^{0,IA}_{s}$	Adjustment parameter between the notional and the expected value of investment in the sector s	5.19.54, 161
$\alpha^{0,PARTR}$	Adjustment parameter on the notional value of the labor participation ratio	5.19.55, 161
$\alpha^{0,PROP,INC,H,VAL}$	Adjustment parameter on the notional value of the Households property income	5.19.56, 161
$\alpha^{0,PY}_{s}$	Adjustment parameter on the notional value of production price in the sector s	5.19.58, 161

$\alpha^{0,R}$	Adjustment parameter on the notional value of the interest rate	5.19.57, 161
$\alpha^{0,\tau,REHAB}$	Adjustment parameter between the notional and the real value of rehabilitation rate for buildings	5.19.96, 164
$\alpha^{0}_{f,s}$	Adjustment parameter between the notional and the expected value of the variable Ff,s,t	5.19.52, 160
$\alpha^{1,CH}_{c}$	Adjustment parameter on the previous period expected value of household consumption of commodity c	5.19.60, 161
$\alpha^{1,IA}_{s}$	Adjustment parameter on the previous period expected value of investment in the sector s	5.19.61, 161
$\alpha^{1,PROP,INC,G,VAL}$	Adjustment parameter on the expected value of the government property income	5.19.62, 161
$\alpha^{1,PY}_{s}$	Adjustment parameter on the previous period expected value of production price in the sector s	5.19.63, 161
$\alpha^{1}_{f,s}$	Adjustment parameter between the expected and the real value of a variable $F^{e}_{f,s,t-1}$	5.19.59, 161
$\alpha^{2,CH}_{c}$	Adjustment parameter on the previous period real value of household consumption of commodity c	5.19.65, 161
$\alpha^{2,IA}_{s}$	Adjustment parameter on the previous period real value of investment in the sector s	5.19.66, 161
$\alpha^{2,PY}_{s}$	Adjustment parameter on the previous period value of production price in the sector s	5.19.67, 161
$\alpha^{2}_{f,s}$	Adjustment parameter between the previous period real value and the current value of a variable $F_{f,s,t-1}$	5.19.64, 161
$\alpha^{3,CH}_{c}$	Adjustment parameter on the notional value of household consumption of commodity c	5.19.69, 162
$\alpha^{3,IA}_{s}$	Adjustment parameter on the notional value of investment in the sector s	5.19.70, 162
$\alpha^{3,PY}_{s}$	Adjustment parameter on the notional value of production price in the sector s	5.19.71, 162
$\alpha^{3}_{f,s}$	Adjustment parameter between the notional and the real value of the variable $F^{n}_{f,s,t-1}$	5.19.68, 162
$\alpha^{6,CHM}_{c}$	Adjustment parameter between the notional and the real value for the substitution effect on the imported households final consumption for the commodity c	5.19.75, 162
$\alpha^{6,CI}_{ce,s}$	Adjustment parameter between the notional and the real value for the substitution effect on the energy intermediate consumption ce in the sector s	5.19.73, 162
$\alpha^{6,CI}_{ct,s}$	Adjustment parameter between the notional and the real value for the substitution effect on the transportation intermediate consumption ce in the sector s	5.19.74, 162

$\alpha^{6,CIM}_{c,s}$	Adjustment parameter between the notional and the real value for the substitution effect on the intermediate consumption for the imported commodity c in the sector s	5.19.76, 162
$\alpha^{6,GM}_{c}$	Adjustment parameter between the notional and the real value for the substitution effect on the imported government final consumption for the commodity c	5.19.77, 162
$\alpha^{6,IM}_{c,s}$	Adjustment parameter between the notional and the real value for the substitution effect on the investment for the imported commodity c in the sector s	5.19.78, 162
$\alpha^{6,MGPD}_{m,c}$	Adjustment parameter between the notional and the real value for the substitution effect on the domestic margin paid m for the commodity c	5.19.80, 163
$\alpha^{6,MGPM}_{m,c}$	Adjustment parameter between the notional and the real value for the substitution effect on the imported margin paid m for the commodity c	5.19.81, 163
$\alpha^{6,MGSM}_{m}$	Adjustment parameter between the notional and the real value for the substitution effect on the imported margin supplied for the commodity m	5.19.79, 162
$\alpha^{6,X}_{c}$	Adjustment parameter between the notional and the real value for the substitution effect on the exports of the commodity c	5.19.82, 163
$\alpha^{6,XM}_{c}$	Adjustment parameter between the notional and the real value for the substitution effect on the exports for the imported commodity c	5.19.83, 163
$\alpha^{6}_{f,s}$	Adjustment parameter between the notional and the real value for the substitution effect on the production factor f in the sector s	5.19.72, 162
$AUTO$	Automobiles stock	5.17.20, 147
$AUTO^{D}_{ecl,cea}$	Life duration of a car of class ecl fueled with energy cea	5.19.115, 165
$AUTO_{cea,DES}$	Automobiles fueled with energy cea destroyed	5.17.43, 150
$AUTO_{cea}$	Automobiles fueled with energy cea	5.17.39, 150
$AUTO_{DES}$	Automobiles destroyed	5.17.44, 150
$AUTO_{ecl,cea,DES}$	Class ecl fueled with energy cea automobiles destroyed	5.17.38, 150
$AUTO_{ecl,cea}$	Stock of automobiles of class ecl fueled with energy cea	5.17.37, 150
$AUTO_{ecl,DES}$	Class ecl automobiles destroyed	5.17.45, 150
$AUTO_{ecl,th,DES}$	Class ecl thermic automobiles destroyed	5.17.46, 150
$AUTO_{ecl,th}$	Stock of class ecl thermic automobiles	5.17.42, 150
$AUTO_{ecl}$	Stock of class ecl automobiles	5.17.40, 150
$AUTO_{th,DES}$	Thermic automobiles destroyed	5.17.47, 150
$AUTO_{th}$	Thermic automobiles	5.17.41, 150

$AUTO^{bis}$	Automobiles stock (for verification)	5.17.21, 147
$Bal^{G,Prim,VAL}$	Primary balance of the Government expressed in value (deficit)	5.6.24, 108
$Bal^{G,Prim,VAL,bis}$	Primary balance of the Government expressed in value (deficit) (for verification)	5.6.25, 108
$Bal^{G,Tot,VAL}$	Total balance of the Government expressed in value (deficit)	5.6.26, 108
$Bal^{Trae,VAL}$	Aggregate balance of trade	5.7.28, 111
$Bal_c^{Trae,VAL}$	Balance of trade of commodity c	5.7.27, 111
$BUIL$	Total building stock	5.16.9, 138
$BUIL_{ecl}^D$	Lifetime of a building of class *ecl* (definition ?)	5.19.97, 164
$BUIL_{DES}$	Destroyed buildings	5.16.15, 139
$BUIL_{ecl}$	Stock of building of the energy efficiency class *ecl*	5.16.12, 138
$BUIL^{bis}$	Total building stock (for verification)	5.16.11, 138
$C_{E,s}$	Energy costs in sector s	5.3.21, 97
C_E	Aggregate cost of energy	5.3.29, 98
$C_{K,s}$	Capital cost in sector s	5.3.19, 97
C_K	Aggregate cost of capital	5.3.27, 98
$C_{L,s}$	Labor cost in sector s	5.3.18, 97
C_L	Aggregate cost of labor	5.3.28, 98
$C_{MAT,s}$	Materials costs in sector s	5.3.25, 98
C_{MAT}	Aggregate cost of materials	5.3.30, 98
$C_{E,s}^{bis}$	Energy costs in sector s (forward-looking specification)	5.3.22, 98
CH	Aggregate household final consumption, expressed at market price	5.2.80, 89
$CH_{cea}^{AUTO,toe}$	Energy consumption of automobiles fueled with energy *cea* expressed in tonne of oil equivalent	5.17.77, 153
$CH_{ecl,cea}^{AUTO,toe}$	Energy consumption of a class *ecl* automobile fueled with energy *cea* expressed in tonne of oil equivalent	5.17.76, 153
CH_{cea}^{AUTO}	Households' final consumption of electricity (expressed in monetary unit)	5.17.78, 153
CH_c^e	Expected households final consumption of commodity c	5.11.9, 118
$CH^{ENER,BUIL}$	Total energy consumption of buildings (in volume)	5.16.34, 141
$CH_{ceb}^{ENER,BUIL}$	Consumption of energy *ceb* (in millions of euros)	5.16.31, 140
$CH_{ecl,ceb}^{ENER,BUIL}$	Consumption of energy *ceb* by class *ecl* buildings (in millions of euros)	5.16.27, 140
$CH_{ecl}^{ENER,BUIL}$	Consumption of energy in class *ecl* buildings (in millions of euros)	5.16.30, 140

$CH^{HOUS,VAL}$	Households' final consumption related to housing in value	5.14.5, 133
CH^{HOUS}_{ce}	Households' final consumption of energy commodity ce related to housing	5.14.13, 134
$CH^{HOUSENER,VAL}$	Households' final consumption of energy related to housing in value	5.14.8, 133
$CH^{HOUSINV,VAL}$	Households' final consumption related to housing investment in value	5.14.7, 133
$CH^{n,VAL}$	Aggregate notional households final consumption expressed in value	5.5.6, 104
CH^{n}_{c}	Households' final consumption c	5.5.8, 104
CH^{toe}_{ce}	End use for energy ce of households, expressed in toe	5.10.3, 115
$CH^{TRSP,VAL}$	Households' final consumption related to transport in value	5.14.6, 133
$CH^{TRSP,VAL}_{chtrsp}$	Households' final consumption of transport $chtrsp$ in value	5.14.17, 134
CH^{TRSP}_{ce}	Households' final consumption of energy commodity ce related to automobile transport	5.14.28, 135
$CH^{TRSPENER,VAL}$	Households' final consumption of energy related to automobile transport in value	5.14.23, 135
$CH^{TRSPINV,VAL}$	Households' final consumption related to automobile transport in value	5.14.22, 135
CH_{c}	Households final consumption of commodity c	5.11.8, 118
CHD	Aggregate domestically produced final consumption, expressed at market price	5.2.54, 86
CHD_{c}	Private final consumption of domestically produced commodity c	5.7.2, 109
CHM	Aggregate imported households final consumption, expressed at market price	5.2.56, 87
CHM_{c}	Private final consumption of imported commodity c	5.7.5, 109
CI	Aggregate intermediate consumption, expressed at market price	5.2.78, 89
$CI^{toe}_{ce,s}$	End use for energy ce of sector s, expressed in toe	5.10.2, 115
CI^{toe}_{ce}	End use for energy ce of sectors, expressed in toe	5.10.1, 115
CI_{c}	Intermediate consumption of commodity c, expressed at market price	5.2.12, 82
$CI_{ce,s}$	Energy consumption ce of sector s	5.4.13, 102
$CI_{cmo,s}$	Material consumption cmo of sector s	5.4.16, 103
$CI_{ct,s}$	Demand for transport commodity ct by sector s	5.4.18, 103
CI_{s}	Intermediate consumption of sector s, expressed at market price	5.2.30, 84

CI^{bis}	Intermediate consumption of sector s, expressed at market price (for verification)	5.2.32, 84
CID	Aggregate domestically produced intermediate consumption, expressed at market price	5.2.50, 86
$CID_{c,s}$	Intermediary consumption from sector s in domestically produced commodity c	5.7.13, 110
CID_c	Quantity of domestically produced commodity c used as intermediary consumption, expressed at market price	5.2.18, 83
CID_s	Domestically produced intermediate consumption of sector s, expressed at market price	5.2.26, 84
CIM	Aggregate imported intermediate consumption, expressed at market price	5.2.52, 86
$CIM_{c,s}$	Intermediary consumption from sector s in imported commodity c	5.7.15, 110
CIM_c	Quantity of imported commodity c used as intermediary consumption, expressed at market price	5.2.20, 83
CIM_s	Imported intermediate consumption of sector s, expressed at market price	5.2.28, 84
CU_s^n	Notional unit cost of production in sector s	5.3.16, 97
CU_s	Unit cost of production in sector s	5.3.17, 97
CUR_s	Capacity utilization ratio of the sector s	5.3.5, 95
$DEBT_{ecl,cea}^{AUTO,VAL}$	Debt related to the purchase of a class ecl automobile fueled with energy cea	5.17.79, 154
$DEBT^{G,VAL}$	Government's debt expressed in value	5.6.27, 108
$DEBT_{ecl}^{NewB,Val}$	Debt related to the purchase of a new building of a class ecl (in value)	5.16.60, 144
$DEBT_{ecl}^{REHAB,Val}$	Debt related to the rehabilitation of a class ecl building (in value)	5.16.59, 144
$DEP_{ecl,bcl}^{BUIL}$	Buildings depreciating from class ecl to class bcl	5.16.14, 139
$DISPINC^{AT,VAL}$	Disposable income after tax expressed in value	5.5.2, 104
$DISPINC^{BT,VAL}$	Disposable income before tax expressed in value	5.5.1, 104
DL_{ce}^{toe}	Distribution losses of energy ce, expressed in toe	5.10.12, 116
DLM_{ce}^{toe}	Distribution losses of imported energy ce, expressed in toe	5.10.14, 117
$DLY_{ce,s}^{toe}$	Distribution losses of energy ce produced by sector s, expressed in toe	5.10.13, 117
DLY_{ce}^{toe}	Distribution losses of energy ce of domestic producers, expressed in toe	5.10.18, 117
$DNAIRU$	Change in the long term NAIRU	5.19.1, 157
DS	Aggregate change in inventories, expressed at market price	5.2.88, 89

DS_c	Change in inventories of commodity c, expressed at market price	5.2.16, 82
DSD	Aggregate domestically produced change in inventories, expressed at market price	5.2.70, 88
DSM	Aggregate imported change in inventories, expressed at market price	5.2.72, 88
empl	Employment (ILO definition)	5.8.4, 112
EMS	Aggregate emissions	5.9.17, 114
EMS^{CH}	Aggregate emissions related to the households final consumption	5.9.16, 114
$EMS^{CH}_{ghg,c}$	Emissions of the greenhouse gas *ghg* related to the household consumption c	5.9.4, 113
EMS^{CH}_{ghg}	Emissions of the greenhouse gas *ghg* related to the household final consumption	5.9.11, 114
EMS^{CI}	Aggregate emissions related to the intermediary consumption	5.9.13, 114
$EMS^{CI}_{ghg,c,s}$	Emissions of the greenhouse gas *ghg* related to the intermediary consumption of commodity c by sector s	5.9.1, 113
$EMS^{CI}_{ghg,c}$	Emissions of the greenhouse gas *ghg* related to the intermediary consumption of commodity c	5.9.5, 113
$EMS^{CI}_{ghg,s}$	Emissions of the greenhouse gas *ghg* related to the intermediary consumption by sector s	5.9.6, 113
EMS^{CI}_{ghg}	Emissions of the greenhouse gas *ghg* related to the intermediary consumption	5.9.7, 114
$EMS^{CI,bis}_{ghg}$	Emissions of the greenhouse gas *ghg* related to the intermediary consumption (for verification)	5.9.8, 114
EMS^{MAT}	Aggregate emissions related to the material consumption	5.9.14, 114
$EMS^{MAT}_{ghg,s}$	Emissions of the greenhouse gas *ghg* related to the materials consumption of sector s	5.9.2, 113
EMS^{MAT}_{ghg}	Emissions of the greenhouse gas *ghg* related to the total material consumption	5.9.9, 114
EMS^{Y}	Aggregate emissions related to the final production	5.9.15, 114
$EMS^{Y}_{ghg,s}$	Emissions of the greenhouse gas *ghg* related to the final production of sector s	5.9.3, 113
EMS^{Y}_{ghg}	Emissions of the greenhouse gas *ghg* related to the final production	5.9.10, 114
EMS_{ghg}	Aggregate emissions of the greenhouse gas *ghg*	5.9.12, 114
EMS^{bis}	Aggregate emissions by type of gas *ghg*	5.9.18, 114
$ENER^{BUIL}_{ceb}$	Total energy consumption in class *ceb* buildings (in volume)	5.16.36, 141

$ENER^{BUIL}_{ecl,ceb}$	Consumption of energy *ceb* by class *ecl* buildings expressed in toe	5.16.23, 140
$ENER^{BUIL}_{ecl}$	Energy consumption in class *ecl* buildings (in toe)	5.16.32, 140
$ENERperM2_{ecl,ceb}$	Consumption of energy *ceb* per square meter in class *ecl* buildings	5.16.24, 140
η^{BASS}_{ecl}	Elasticity between the the innovation in electric cars and the bass diffusion model equation	5.19.122, 166
$\eta^{BUILNRJ}_{ecl,ceb,cebb}$	Elasticity of substitution between the energy commodity *ceb* and the other energy commodities *cebb* in class *ecl* buldings	5.19.100, 164
η^{CHM}_{c}	Elasticity of substitution between imported and domestically produced commodity *c* for households final consumption	5.19.29, 159
η^{CHTRSP}	Elasticity of substitution between transport expenditures	5.19.90, 164
$\eta^{CIM}_{c,s}$	Elasticity of substitution between imported and domestic intermediary consumption in commodity *c* from the sector *s*	5.19.30, 159
η^{GM}_{c}	Elasticity of substitution between imported and domestically produced commodity *c* for government final consumption	5.19.31, 159
$\eta^{HOUS^{ENER}}$	Elasticity of substitution between energy housing expenditures	5.19.91, 164
$\eta^{HOUS^{INV,ENER}}$	Elasticity of substitution between investment and expenditures in energy housing	5.19.92, 164
$\eta^{IM}_{c,s}$	Elasticity of substitution between imported and domestic investment in commodity *c* from the sector *s*	5.19.32, 159
η^{lesces}	Elasticity of the linear elasticity system	5.19.13, 158
η^{MGSM}_{c}	Elasticity of substitution between imported and domestically produced commodity *c* for margins supplied	5.19.33, 159
$\eta^{TRSP^{INV,ENER}}$	Elasticity of substitution between investment and expenditures in transport	5.19.93, 164
η^{X}_{c}	Elasticity of substitution between french exports and rest of the world exports	5.19.35, 159
η^{XM}_{c}	Elasticity of substitution between domestic and imported commodity *c* for exports	5.19.34, 159
$ES_{E,K,s}$	ES between energy and capital	5.15.8, 137
$ES_{E,L,s}$	ES between energy and labor	5.15.11, 137
$ES_{E,MAT,s}$	ES between energy and material	5.15.4, 136
$ES_{K,E,s}$	ES between capital and energy	5.15.7, 137
$ES_{K,L,s}$	ES between capital and labor	5.15.9, 137
$ES_{K,MAT,s}$	ES between capital and material	5.15.2, 136
$ES_{L,E,s}$	ES between labor and energy	5.15.12, 137

$ES_{L,K,s}$	ES between labor and capital	5.15.10, 137
$ES_{L,MAT,s}$	ES between labor and material	5.15.6, 136
$ES_{MAT,E,s}$	ES between material and energy	5.15.3, 136
$ES_{MAT,K,s}$	ES between material and capital	5.15.1, 136
$ES_{MAT,L,s}$	ES between material and labor	5.15.5, 136
$\zeta_{ecl,ceb}$	Elasticity of substitution between the energy commodity ceb and the surface of housing per capita in class ecl buldings	5.19.99, 164
$ETS^{VAL,TOT}$	Total nominal value of emissions permits on the trading market	5.18.6, 155
$ETS^{VAL}_{ce2,s}$	Nominal value of emissions permits bought by sector s due to $ce2$ consumption	5.18.4, 155
ETS^{VAL}_{s}	Nominal value of emissions permits required for sector s	5.18.5, 155
$EXP^{AUTO,VAL}$	Expenditures related to the use of an automobile (in value)	5.17.83, 154
$EXP^{AUTO,VAL}_{cea}$	Expenditures related to the use of an automobile fueled with energy cea (in value)	5.17.82, 154
$EXP^{AUTO,VAL}_{ecl,cea}$	Expenditures related to the use of a class ecl automobile fueled with energy cea (in value)	5.17.80, 154
$EXP^{AUTO,VAL}_{ecl}$	Expenditures related to the use of a class ecl automobile (in value)	5.17.81, 154
$EXP^{AUTO,VAL,bis}$	Expenditures related to the use of an automobile (for verification)	5.17.84, 154
$EXP^{HOUSING,Val}$	Housing expenditures (in value)	5.16.62, 145
$EXP^{HOUSING,Val}_{ecl}$	Housing expenditures in class ecl buildings (in value)	5.16.61, 144
$EXP^{NEWBUIL,VAL}$	Expenditures in construction of new buildings (in value)	5.16.64, 145
$EXP^{REHAB,VAL}$	Expenditures in rehabilitation of buildings (in value)	5.16.63, 145
$EXPG^{tren}$	Public expenditures trend from the government	5.19.24, 158
EXR	Exchange rate of the euro with the rest of world's currency	5.19.36, 159
$F^{e}_{f,s}$	Expected quantity of Labor, Energy and Material inputs in sector s	5.11.5, 118
$F^{n}_{f,s}$	Demand for production factor f by sector s	5.4.8, 102
$F_{f,s}$	Quantity of Labor, Energy and Material inputs in sector s	5.11.4, 118
F_{f}	Aggregate production factors f	5.4.11, 102
$F_{K,s}$	Capital stock of sector s	5.11.6, 118
G	Aggregate Government final consumption, expressed at market price	5.2.82, 89
G_{c}	Government final consumption of commodity c	5.6.21, 108

$GAIN^{REHAB}_{ecl,ecl2}$	Gain from the rehabilitation from a *ecl* to a *ecl*2 class building	5.16.42, 142
GD	Aggregate domestically produced Government final consumption, expressed at market price	5.2.58, 87
GD_c	Public final consumption of domestically produced commodity c	5.7.3, 109
GDP	GDP (expenditure definition)	5.2.127, 94
GDP_c	GDP of commodity c (expenditure definition)	5.2.129, 94
$GDP4$	GDP (income definition)	5.2.135, 94
GDP^{bis}	GDP (expenditure definition, for verification)	5.2.131, 94
GDP^{ter}	GDP (production definition)	5.2.133, 94
GM	Aggregate imported Government final consumption, expressed at market price	5.2.60, 87
GM_c	Public final consumption of imported commodity c	5.7.6, 109
GOS	Aggregate gross operating surplus	5.2.123, 93
GOS^{VAL}_s	Gross operating surplus of sector s expressed in value	5.2.112, 92
GOS_s	Gross operating surplus of sector s	5.2.113, 92
gr^{nomi}	Nominal growth rate of the economy	5.19.14, 158
g^{NOMI}	Growth rate of the nominal interest rate	5.19.101, 164
$g^{PE^{AUTO,e}}_{ecl,cea}$	Growth rate of the energy price of an automobile of class *ecl* fueled with energy *cea*	5.17.50, 151
$g^{PENER^{m2,e}}_{ecl}$	Growth rate of the energy price per square meter paid in class *ecl* buildings	5.16.58, 144
$g^{PROG}_{E,s}$	Energy efficiency gains in sector s	5.4.22, 103
I	Aggregate investment, expressed at market price	5.2.84, 89
$I_{c,s}$	Investment in commodity c by sector s	5.4.12, 102
I_c	Investment in commodity c, expressed at market price	5.2.14, 82
I_s	Investment of sector s, expressed at market price	5.2.38, 85
IA_s	Investment in sector s	5.11.7, 118
I^{bis}	Investment of sector s, expressed at market price (for verification)	5.2.40, 85
ID	Aggregate domestically produced investment, expressed at market price	5.2.62, 87
$ID_{c,s}$	Investment from sector s in domestically produced commodity c	5.7.14, 110
ID_c	Quantity of imported commodity c used as investment, expressed at market price	5.2.22, 83

ID_s	Domestically produced investment of sector s, expressed at market price	5.2.34, 84
$IEMS^{CH}_{ghg,c}$	Emissions intensity of the greenhouse gas ghg related to the household final consumption of commodity c	5.19.40, 159
$IEMS^{CI}_{ghg,c,s}$	Emissions intensity of the greenhouse gas ghg related to the intermediary consumption of commodity c by sector s	5.19.41, 159
$IEMS^{MAT}_{ghg,s}$	Emissions intensity of the greenhouse gas ghg related to the material consumption of sector s	5.19.42, 160
$IEMS^{Y}_{ghg,s}$	Emissions intensity of the greenhouse gas ghg related to the final production of sector s	5.19.43, 160
IM	Aggregate imported investment, expressed at market price	5.2.64, 87
$IM_{c,s}$	Investment from sector s in imported commodity c	5.7.16, 110
IM_c	Quantity of imported commodity c used as investment, expressed at market price	5.2.24, 83
IM_s	Imported investment of sector s, expressed at market price	5.2.36, 85
$INC^{G,VAL}$	Aggregate incomes of the Government expressed in value	5.6.20, 107
$INC^{G,VAL}$ (2)	Government revenues	5.18.9, 156
$INC^{SOC,TAX,VAL}$	Income and social taxes expressed in value	5.5.3, 104
$innovation^{exo}_{ecl}$	exogenous innovation trend in electric cars	5.19.120, 166
$innovation_{ecl}$	Bass innovation parameter for class ecl electric automobile	5.17.31, 149
km^{AUTO}	Total kilometers by automobile	5.17.19, 147
$km^{AUTO,LD}$	Kilometers for long distance by automobile	5.17.17, 147
$km^{AUTO,SD}$	Kilometers for short distance by automobile	5.17.18, 147
$km^{AUTO}_{ecl,cea}$	Kilometers by a class ecl automobile fueled with energy cea	5.17.73, 153
km^{AUTO}_{ecl}	Kilometers by class ecl automobile	5.17.74, 153
$km^{AUTO,bis}$	Total kilometers by automobile	5.17.75, 153
$km^{trav,auto}$	Total kilometer-travelers by automobile	5.17.89, 155
$km^{trav,auto,LD}$	Kilometer-travelers for long distance by automobile	5.17.12, 146
$km^{trav,auto,SD}$	Kilometer-travelers for short distance by automobile	5.17.14, 147
$km^{traveler,LD}$	Kilometer-travelers for long distance transport (by automobile and train)	5.17.7, 146
$km^{traveler,SD}$	Kilometer-travelers for short distance transport (by automobile and bus)	5.17.9, 146
$km^{traveler}_{cair}$	Kilometer-travelers for air transport	5.17.6, 146
$km^{traveler}_{crai}$	Kilometer-travelers for transport by train	5.17.13, 147
$km^{traveler}_{croa}$	Kilometer-travelers for transport by road (bus)	5.17.16, 147
$km^{traveler}_{cth}$	kilometer-traveler of public transport commodity cth	5.19.118, 166

$kmPerAuto$	Average distance in km driven by cars in a year	5.19.117, 166
$LD^{AUTO}_{ecl,cea}$		5.19.119, 166
LD^{REHAB}_{ecl}	Duration of the loan funding rehabilitation	5.19.102, 164
LD_{ecl}	Duration of the loan for housing of class ecl investment	5.19.103, 165
LF	Labor force	5.8.2, 112
M	Imports, expressed at basic price	5.2.108, 91
M^{toe}_{ce}	Imported net production of energy ce, expressed in toe	5.10.6, 116
M_c	Imports of commodity c, expressed at basic price	5.2.92, 90
$M2perCapita$	Number of square meter per person	5.16.10, 138
μ^n_s	Notional mark-up of the sector s (specification 1)	5.3.2, 95
μ^{n2}_s	Notional mark-up of the sector s (specification 2)	5.3.3, 95
μ_c	Average mark-up on commodity c	5.3.6, 95
μ_s	Mark-up in the sector s	5.11.1, 118
M^{bis}_c	Imports of commodity c, expressed at basic price (for verification)	5.2.94, 90
$MCperkm_{ecl,cea}$	Maintenance cost per km for the automobile of class ecl fueled with energy cea	5.19.121, 166
MG^{toe}_{ce}	Gross imported energy ce, expressed in toe	5.10.11, 116
$MGP_{m,c}$	Margins paid to commodity m on commodity c	5.2.100, 91
$MGPD$	Margins paid on domestically produced commodities	5.2.102, 91
$MGPD_c$	Margins paid on the domestically produced commodity c	5.2.96, 90
$MGPD_{m,c}$	Margins paid to commodity m on the domestic commodity c	5.4.1, 101
$MGPM$	Margins paid on imported commodities	5.2.104, 91
$MGPM_c$	Margins paid on imported commodity c	5.2.98, 91
$MGPM_{m,c}$	Margins paid to commodity m on the imported commodity c	5.4.4, 101
MGS	Aggregate supplied margins	5.2.76, 88
MGS_m	Margins supplied by commodity m, expressed at market price	5.2.6, 81
MGS^{bis}_m	Margins supplied by commodity m, expressed at market price (for verification)	5.2.8, 81
$MGSD$	Aggregate margins supplied on domestically produced commodities, expressed at market price	5.2.46, 86
$MGSD_m$	Received margins on domestically produced commodity m	5.7.1, 109
$MGSM$	Aggregate margins supplied on imported commodities, expressed at market price	5.2.48, 86

$MGSM_m$	Margins supplied from imported commodity m	5.7.4, 109
MPS^n	Notional marginal propensity to save	5.5.7, 104
MS_c	Imports of commodity c, expressed at market price	5.3.36, 99
$MSBVAT_c$	Imports of commodity c expressed at market price before VAT	5.13.22, 128
NCH	Aggregate necessary households final consumption	5.5.11, 105
NCH_{cea}^{AUTO}	Necessary (minimum) households' final consumption for energy commodity *cea* consumed by automobiles	5.17.1, 145
NCH_{ceb}^{BUIL}	Necessary (minimum) households' final consumption for energy commodities *ceb* related to buildings	5.16.2, 137
NCH_{ccon}	Necessary (minimum) households' final consumption for construction commodity *ccon*	5.14.1, 132
NCH_{ccon} (2)	Necessary (minimum) households' final consumption for construction commodity *ccon*	5.16.1, 137
NCH_{ccon} (3)	Necessary (minimum) households' final consumption for construction commodity *ccon*	5.18.11, 156
NCH_{ce}	Necessary (minimum) households' final consumption for energy commodities *ce*	5.14.4, 133
NCH_{ct}	Necessary (minimum) households' final consumption for transport commodities *ct*	5.14.2, 132
NCH_{cth}	Necessary (minimum) households' final consumption for public transport commodities *cth*	5.17.2, 145
NCH_{cveh}	Necessary (minimum) households' final consumption for vehicles commodity *cveh*	5.14.3, 132
NCH_{cveh} (2)	Necessary (minimum) households' final consumption for vehicles commodity *cveh*	5.17.3, 145
$NewAUTO$	New automobiles	5.17.22, 147
$NewAUTO_{ecl,th}^{,bis}$	New thermic automobiles of class *ecl* (for verification)	5.17.33, 149
$NewAUTO_{ecl,cele}$	New electric automobiles of class *ecl*	5.17.28, 148
$NewAUTO_{ecl,cfut}$	New thermic automobiles fueled with oil of class *ecl*	5.17.34, 149
$NewAUTO_{ecl,cgas}$	New thermic automobiles fueled with gas of class *ecl*	5.17.35, 149
$NewAUTO_{ecl,th}$	New thermic automobiles of class *ecl*	5.17.32, 149
$NewAUTO_{ecl}$	New automobiles of class *ecl*	5.17.23, 148
$NEWBUIL$	Total new buildings	5.16.17, 139
$NewBUIL_{ecl}$	New buildings constructed according to class *ecl*	5.16.13, 139
NOS	Aggregate net operating surplus	5.2.125, 93
NOS_s^{VAL}	Net operating surplus of sector s expressed in value	5.2.114, 92
NOS_s	Net operating surplus of sector s	5.2.115, 92

$NTAXC$	Aggregate net taxes on commodity c	5.6.12, 107
$NTAXC_c^{VAL}$	Net taxes on commodity c expressed in value	5.6.5, 106
$NTAXC_c$	Net taxes on commodity c	5.6.6, 106
$NTAXCD_c^{VAL}$	Net taxes on domestically produced commodity c expressed in value	5.6.1, 106
$NTAXCD_c^{VAL}$ (2)	Net taxes on domestically produced commodity c expressed in value	5.13.1, 125
$NTAXCD_c$	Net taxes on domestically produced commodity c	5.6.2, 106
$NTAXCD_c$ (2)	Net taxes on domestically produced commodity c	5.13.2, 125
$NTAXCM_c^{VAL}$	Net taxes on imported commodity c expressed in value	5.6.3, 106
$NTAXCM_c^{VAL}$ (2)	Net taxes on imported commodity c expressed in value	5.13.4, 126
$NTAXCM_c$	Net taxes on imported commodity c	5.6.4, 106
$NTAXCM_c$ (2)	Net taxes on imported commodity c	5.13.5, 126
$NTAXS$	Aggregate net taxes on the production of sectors	5.6.14, 107
$NTAXS^{VAL}$	Aggregate net taxes on the production of sectors expressed in value	5.6.13, 107
$NTAXS_s^{VAL}$	Net taxes on the production of sector s expressed in value	5.6.7, 106
$NTAXS_s$	Net taxes on the production of sector s	5.6.8, 106
$\nu_{ecl}^{iffusion}$	Elasticity between the the innovation stock in electric cars and its diffusion	5.19.123, 166
ν_{ecl}^{REHAB}	First derivative of the notional rehabilitation rate of a class ecl building $\tau_{ecl}^{REHAB_N}$ with respect to the log of payback time $Payback_{ecl}^{REHAB}$	5.16.39, 142
$OTHCT$	Aggregate Other product Tax	5.13.52, 130
$OTHCTD$	Aggregate other product tax paid on domestically produced comodities	5.13.40, 129
$OTHCTD_c$	Other taxes on domestically produced commodity c	5.13.12, 127
$OTHCTM$	Aggregate Other product Tax on imported commododities	5.13.46, 130
$OTHCTM_c$	Other taxes on imported commodity c	5.13.14, 127
P	Consumer Price Index	5.3.10, 96
P^e	Expected inflation	5.11.2, 118
$P^{ETS,nominal}$	Nominal price of emissions permits	5.18.1, 155
$P^{km,AUTO}$	Price of kilometer-travelers for short distance by automobile	5.17.85, 154
$P^{km,trav,auto}$	Price of kilometer-travelers for automobile transportation	5.17.88, 155
$P^{km,trav,auto,LD}$	Price of kilometer-travelers for long distance by automobile	5.17.87, 155
$P^{km,trav,auto,SD}$	Price of kilometer-travelers for short distance by automobile	5.17.86, 154

$P^{km,traveler,LD}$	Price of long distance Kilometer-traveler (automobile and train)	5.17.8, 146
$P^{km,traveler,SD}$	Price of short distance Kilometer-traveler (by automobile and bus)	5.17.10, 146
$P_{cth}^{km,traveler}$	Price of public transport commodities cth (train, road and air)	5.17.4, 145
$PARTR$	Labor participation ratio	5.11.13, 119
$PARTR^{n}$	Labor force participation ratio	5.8.3, 112
$PARTR^{TREND}$	Participation rate to the labor force trend	5.19.37, 159
$Payback_{ecl}^{REHAB}$	Payback time of rehabilitating a class ecl building	5.16.41, 142
PCH	Aggregate market price for household final (consumer price index)	5.2.79, 89
PCH^{CES}	CES consumption price index	5.5.14, 105
$PCH^{ENER,BUIL}$	Total energy consumption of buildings (in value)	5.16.35, 141
$PCH_{ecl}^{ENER,BUIL}$	Consumer price of energy in class ecl buildings (index)	5.16.29, 140
PCH_{ce}^{HOUS}	Price of energy consumption ce related to housing	5.14.16, 134
$PCH^{HOUSENER}$	Price of the aggregate energy expenditure related to housing	5.14.11, 133
$PCH^{HOUSENER,CES}$	Price of the aggregate energy expenditures related to housing	5.14.15, 134
$PCH^{HOUSINV}$	Price of investment expenditure related to housing	5.14.12, 133
$PCH^{TRSP,CES}$	Price of the aggregate transport expenditures	5.14.19, 134
PCH_{auto}^{TRSP}	Price of the aggregate automobile expenditures	5.14.21, 135
PCH_{ce}^{TRSP}	Price of energy consumption ce related to automobile transport	5.14.31, 136
PCH_{ct}^{TRSP}	Price of transport consumption of commodity ct	5.14.20, 134
$PCH^{TRSPENER}$	Price of the aggregate energy expenditure related to automobile transport	5.14.26, 135
$PCH^{TRSPENER,CES}$	Price of the aggregate energy expenditures related to automobile transport	5.14.30, 136
$PCH^{TRSPINV}$	Price of investment expenditure related to automobile transport	5.14.27, 135
PCH_{c}	Market price of households final consumption c	5.3.54, 101
$PCHD$	Aggregate market price for domestically produced households final consumption	5.2.53, 86
$PCHD_{c}$	Market price of domestically produced households final consumption c	5.3.43, 100
$PCHD_{c}$ (2)	Market price of domestically produced households final consumption c	5.13.27, 128

$PCHM$	Aggregate market price for imported households final consumption	5.2.55, 86
$PCHM_c$	Market price of imported households final consumption c	5.3.44, 100
$PCHM_c$ (2)	Market price of imported households final consumption c	5.13.28, 128
PCI	Aggregate market price for intermediate consumption	5.2.77, 89
$PCI_{c,s}$	Market price of intermediate consumption c purchased by sector s	5.3.53, 101
PCI_c	Market price of the intermediate consumption of commodity c	5.2.11, 82
PCI_s	Market price of intermediate consumption of sector s	5.2.29, 84
PCI^{bis}	Market price of intermediate consumption of sector s (for verification)	5.2.31, 84
$PCID$	Aggregate market price for domestically produced intermediate consumption	5.2.49, 86
$PCID_{c,s}$	Market price of domestically produced intermediate consumption c purchased by sector s	5.3.41, 100
$PCID_{c,s}$ (2)	Market price of domestically produced intermediate consumption c purchased by sector s	5.13.25, 128
$PCID_c$	Market price for the domestically produced commodity c used as intermediary consumption	5.2.17, 83
$PCID_s$	Market price of domestically produced intermediate consumption of sector s	5.2.25, 84
$PCIM$	Aggregate market price for imported intermediate consumption	5.2.51, 86
$PCIM_{c,s}$	Market price of imported intermediate consumption c purchased by sector s	5.3.42, 100
$PCIM_{c,s}$ (2)	Market price of imported intermediate consumption c purchased by sector s	5.13.26, 128
$PCIM_c$	Market price for imported commodity c used as intermediary consumption	5.2.19, 83
$PCIM_s$	Market price of imported intermediate consumption of sector s	5.2.27, 84
PDS	Aggregate market price for change in inventories	5.2.87, 89
PDS_c	Market price of the change in inventories of commodity c	5.2.15, 82
$PDSD$	Aggregate market price for domestically produced change in inventories	5.2.69, 88
$PDSD_c$	Market price of domestically produced change in inventories c	5.3.51, 100
$PDSD_c$ (2)	Market price of domestically produced change in inventories c	5.13.35, 129

$PDSM$	Aggregate market price for imported change in inventories	5.2.71, 88
$PDSM_c$	Market price of imported change in inventories c	5.3.52, 100
$PDSM_c$ (2)	Market price of imported change in inventories c	5.13.36, 129
$PE^{AUTO}_{ecl,cea}$	Energy price of a class ecl automobile fueled with energy cea	5.17.49, 151
PE^e_s	Expected energy price inflation	5.3.23, 98
$PE_{ce2,s}$	Price of $ce2$ energy consumption in sector s	5.18.7, 156
PE_s	Price of energy in sector s	5.3.24, 98
$PENER^{BUIL}_{ceb}$	Total energy consumption in class ceb buildings (in value)	5.16.37, 141
$PENER^{BUIL}_{ecl,ceb}$	Price of energy ceb in class ecl buildings (euros per kWh)	5.16.28, 140
$PENER^{BUIL}_{ecl}$	Energy consumption in class ecl buildings (in millions of euros)	5.16.33, 141
$PENER^{m2}_{ecl}$	Energy price per square meter paid in class ecl buildings	5.16.57, 144
PG	Aggregate market price for Government final consumption	5.2.81, 89
PG_c	Market price of Government final consumption c	5.3.55, 101
PGD	Aggregate market price for domestically produced Government final consumption	5.2.57, 87
PGD_c	Market price of domestically produced Government final consumption c	5.3.45, 100
PGD_c (2)	Market price of domestically produced Government final consumption c	5.13.29, 128
$PGDP$	Price of GDP (expenditure definition)	5.2.126, 93
$PGDP_c$	Price of GDP of commodity c (expenditure definition)	5.2.128, 94
$PGDP4$	Price of GDP (income definition)	5.2.134, 94
$PGDP^{bis}$	Price of GDP (expenditure definition, for verification)	5.2.130, 94
$PGDP^{ter}$	Price of GDP (production definition)	5.2.132, 94
PGM	Aggregate market price for imported Government final consumption	5.2.59, 87
PGM_c	Market price of imported Government final consumption c	5.3.46, 100
PGM_c (2)	Market price of imported Government final consumption c	5.13.30, 129
$PGOS$	Price of the aggregate gross operating surplus	5.2.122, 93
φ^{CH}_c	Share of commodity c into the total household consumption	5.5.13, 105
φ^{CHM}_c	Import share of commodity c for household final consumption	5.7.8, 110
$\varphi^{CIM}_{c,s}$	Import share of intermediary consumption from sector s in domestically produced commodity c	5.7.17, 110
φ^{GM}_c	Import share φ_c of commodity c on the government final consumption	5.7.9, 110

$\varphi^{IM}_{c,s}$	Import share of intermediary consumption from sector s in imported commodity c	5.7.18, 111
$\varphi^{km^{trav,auto,LD}}$	Share of Kilometer-travelers by automobile into the long distance Kilometer-travelers	5.17.11, 146
$\varphi^{km^{trav,auto,SD}}$	Share of Kilometer-travelers by automobile into the short distance Kilometer-travelers	5.17.15, 147
$\varphi^{MCH^{HOUS}}_{ce}$	Share of energy consumption ce into the total energy consumption related to housing	5.14.14, 134
$\varphi^{MCH^{HOUSINV}}$	Share of housing investment expenditures into the total housing expenditures	5.14.9, 133
$\varphi^{MCH^{TRSP}}_{ce}$	Share of energy consumption ce into the total energy consumption related to automobile transport	5.14.29, 136
$\varphi^{MCH^{TRSP}}_{chtrsp}$	Share of transport consumption $chtrsp$ into the total transport expenditures	5.14.18, 134
$\varphi^{MCH^{TRSPINV}}$	Share of automobile transport investment expenditures into the total automobile transport expenditures	5.14.24, 135
φ^{MCH}_{c}	Share of commodity c into the total marginal household consumption	5.5.12, 105
$\varphi^{MGPD}_{m,c}$	Share of margin type m into the total margins paid on domestic commodity c	5.4.3, 101
$\varphi^{MGPM}_{m,c}$	Share of margin type m into the total margins paid on imported commodity c	5.4.6, 102
φ^{MGSM}_{m}	Import share of commodity c on supplied margins	5.7.7, 109
$\varphi^{NewAUTO^n}_{ecl,cele}$	Notional share of class ecl electric automobile	5.17.30, 149
$\varphi^{NewAUTO^n}_{ecl}$	Notional share of class ecl automobile	5.17.26, 148
$\varphi^{NewAUTO}_{ecl,cele}$	Share of class ecl electric automobile	5.17.29, 148
$\varphi^{NewAUTO}_{ecl,cfut}$	Share of New thermic automobiles fueled with oil of class ecl	5.17.36, 149
$\varphi^{NewAUTO}_{ecl}$	Share of class ecl automobiles	5.17.27, 148
$\varphi^{PROP^{INC,G}}$	Propriety income of the government expressed in value	5.19.25, 158
φ^{RD^G}	Share of debt reimbursed every year	5.19.26, 158
$\varphi^{REHAB^n}_{ecl,ecl2}$	Notional share of class ecl buildings rehabilitated to class $ecl2$	5.16.45, 142
$\varphi^{REHAB}_{ecl,ecl2}$	Share of class ecl buildings rehabilitated to class $ecl2$	5.16.46, 143
$\varphi^{TRSP}_{ct,s}$	Share for transport ct into the total transport use of sector s	5.4.20, 103
φ^{XM}_{c}	Import share of commodity c exports	5.7.23, 111
$\varphi_{E,ce,s}$	Share of energy ce into the total energy use of sector s	5.4.15, 103

$\varphi_{ecl,ceb}$	Share of energy expenditures by *ecl* class building	5.16.26, 140
$\varphi_{f,s}$	Cost share of input f for sector s	5.4.10, 102
$\varphi_{f,s}$ (2)	Share of the production input f by sector s	5.19.12, 157
$\varphi^{Y^{toe}}_{ce,s}$	Share of energy *ce* produced by sector s (energy balance definition)	5.19.44, 160
$\varphi^{Y}_{ce,s}$	Share of energy *ce* produced by sector s (national account definition)	5.10.9, 116
PI	Aggregate market price for investment	5.2.83, 89
PI_c	Market price of the investment in commodity c	5.2.13, 82
PI_s	Market price of investment of sector s	5.2.37, 85
PI^{bis}	Market price of investment of sector s (for verification)	5.2.39, 85
PID	Aggregate market price for domestically produced investment	5.2.61, 87
$PID_{c,s}$	Market price of domestically produced investment c purchased by sector s	5.3.47, 100
$PID_{c,s}$ (2)	Market price of domestically produced investment c purchased by sector s	5.13.31, 129
PID_c	Market price for domestically produced commodity c used as investment	5.2.21, 83
PID_s	Market price of domestically produced investment of sector s	5.2.33, 84
PIM	Aggregate market price for imported investment	5.2.63, 87
$PIM_{c,s}$	Market price of imported investment c purchased by sector s	5.3.48, 100
$PIM_{c,s}$ (2)	Market price of imported investment c purchased by sector s	5.13.32, 129
PIM_c	Market price for imported commodity c used as investment	5.2.23, 83
PIM_s	Market price of imported investment of sector s	5.2.35, 84
PK_s	Price of capital in sector s	5.3.20, 97
PM	Aggregate basic price of imports	5.2.107, 91
PM_c	Price of imported commodity c, expressed at basic price	5.3.32, 99
$PMAT_s$	Price of materials in sector s	5.3.26, 98
PM^{bis}_c	Basic price of imports of commodity c (for verification)	5.2.93, 90
$PMGP_{m,c}$	Price of the margins paid to commodity m on commodity c	5.2.99, 91
$PMGPD$	Aggregate price of the margins paid on domestically produced commodity	5.2.101, 91
$PMGPD_c$	Price of the margins paid on domestically produced commodity c	5.2.95, 90

$PMGPD_{m,c}$	Market price of the margins paid to commodity m on domestically produced commodity c	5.3.37, 99
$PMGPM$	Aggregate price of the margins paid on imported commodities	5.2.103, 91
$PMGPM_c$	Price of the margins paid on imported commodity c	5.2.97, 91
$PMGPM_{m,c}$	Market price of the margins paid to commodity m on imported commodity c	5.3.38, 99
$PMGS$	Aggregate market price for supplied margins	5.2.75, 88
$PMGS_m$	Market price of the margins supplied by commodity m	5.2.5, 81
$PMGS_m^{bis}$	Market price of the margins supplied by commodity m (for verification)	5.2.7, 81
$PMGSD$	Aggregate market price for the margins supplied on domestically produced commodities	5.2.45, 85
$PMGSD_c$	Market price of margins supplied by domestically produced commodity c	5.3.39, 100
$PMGSD_c$ (2)	Market price of margins supplied by domestically produced commodity c	5.13.23, 128
$PMGSM$	Aggregate market price for the margins supplied on imported commodities	5.2.47, 86
$PMGSM_c$	Market price of margins supplied by imported commodity c	5.3.40, 100
$PMGSM_c$ (2)	Market price of margins supplied by imported commodity c	5.13.24, 128
PMS_c	Price of imported commodity c, expressed at market price	5.3.35, 99
$PMSBVAT_c$	Selling price before VAT for imported commodity c	5.13.21, 128
$PNCH$	Price of aggregate necessary households consumption	5.5.10, 105
$PNCH_c$	Price of necessary households consumption c	5.5.9, 105
$PNewAUTO$	Price of a new automobiles	5.17.57, 152
$PNewAUTO_{ecl,cele}$	Price of a new electric automobiles of class *ecl*	5.17.54, 151
$PNewAUTO_{ecl,cfut}$	Price of a new thermic automobiles fueled with oil of class *ecl*	5.17.52, 151
$PNewAUTO_{ecl,cgas}$	Price of a new thermic automobiles fueled with gas of class *ecl*	5.17.53, 151
$PNewAUTO_{ecl,th}$	Price of a new thermic automobiles of class *ecl*	5.17.56, 151
$PNewAUTO_{ecl}$	Price of a new automobiles of class *ecl*	5.17.55, 151
$PNEWBUIL$	Average price of new buildings	5.16.18, 139
$PNewBUIL_{ecl}$	Price of class *ecl* new building per square meter	5.16.3, 137
$PNOS$	Price of the aggregate net operating surplus	5.2.124, 93
$PNTAXC$	Aggregate net taxes on commodity c expressed in value	5.6.11, 107
POP	Total population	5.19.38, 159

$POTHCT$	Aggregate market price for other product Tax	5.13.51, 130
$POTHCTD$	Aggregate market price for other product tax paid on domestically produced comodities	5.13.39, 129
$POTHCTD_c$	Price of other taxes on domestically produced commodity c	5.13.11, 127
$POTHCTM$	Aggregate market price for Other product Tax on imported commododities	5.13.45, 130
$POTHCTM_c$	Price other taxes on imported commodity c	5.13.13, 127
PQ	Aggregate market price for production	5.2.73, 88
PQ_c	Market price of the production of commodity c	5.2.9, 82
PQD	Aggregate market price for domestically produced commodities	5.2.41, 85
PQD_c	Market price for the domestically produced commodity c	5.2.1, 80
PQM	Aggregate market price for imported commodities	5.2.43, 85
PQM_c	Market price for imported commodity c	5.2.3, 81
$PREHAB$	Price buildings rehabilitated	5.16.22, 139
$PREHAB_{ecl,ecl}$	Price of investment for rehabilitating a class ecl building in the same class ecl per square meter	5.16.5, 138
$PREHAB_{ecl,ecl2}$	Price of investment for rehabilitating a class ecl building to class $ecl2$ per square meter	5.16.4, 137
$PREHAB_{ecl}$	Price of class ecl rehabilitated buildings	5.16.20, 139
$prog^L_s$	Labor productivity in the sector s	5.19.16, 158
$PROG_{f,s}$	Technical progress of the production factor f in sector s	5.4.21, 103
$PROP^{INC,G,VAL}$	Government property incomes in value	5.11.17, 119
$PROP^{INC,G,VAL,e}$	Expected Government property incomes in value	5.11.18, 119
$PROP^{INC,G,VAL,n}$	Notional property incomes of the Government expressed in value	5.6.19, 107
$PROP^{INC,H,VAL}$	Households property income in value	5.11.15, 119
$PROP^{INC,H,VAL,e}$	Expected Households property income in value	5.11.16, 119
$PROP^{INC,H,VAL,n}$	Property incomes expressed in value	5.5.4, 104
$PRSC$	Price of the aggregate employers' social security contribution paid by sector s	5.6.15, 107
$PRSC_s$	Price of the employers' social security contribution paid by sector s	5.6.10, 106
$PSUBC$	Aggregate market price for subsidies on commodities	5.13.53, 130
$PSUBCD$	Aggregate market price for subsidies on domestically produced commodities	5.13.41, 129
$PSUBCD_c$		5.13.15, 127

$PSUBCM$	Aggregate market price for subsidies on imported commodities	5.13.47, 130
$PSUBCM_c$	Price of subsidies on imported commodity c	5.13.17, 127
PVA	Value-added price	5.2.118, 93
$PVAT$	Aggregate market price for Value-added Tax on total households final consumption	5.13.49, 130
$PVATD$	Aggregate market price for value-added tax paid on domestically produced comodities	5.13.37, 129
$PVATD_c$	Price of value-added tax on domestically produced commodity c	5.13.7, 126
$PVATM$	Aggregate market price for value-added tax paid on imported comodities	5.13.43, 130
$PVATM_c$	Price of value-added tax on imported commodity c	5.13.9, 126
$PWAGES$	Gross wage index paid by sectors	5.2.120, 93
$Pwages$	Nominal price index	5.19.15, 158
$PWAGES_s$	Price index for gross wages in sector s	5.3.12, 96
PX	Aggregate market price for exports	5.2.85, 89
PX_c	Market price of exports c	5.3.56, 101
PXD	Aggregate market price for domestically produced exports	5.2.65, 87
PXD_c	Market price of domestically produced exports c	5.3.49, 100
PXD_c (2)	Market price of domestically produced exports c	5.13.33, 129
PXM	Aggregate market price for imported exports (re-exports)	5.2.67, 88
PXM_c	Market price of imported exports (re-exports) c	5.3.50, 100
PXM_c (2)	Market price of imported exports (re-exports) c	5.13.34, 129
PY	Basic price of aggregate production	5.2.116, 92
PY_s^e	Expected production price of sector s	5.11.11, 119
PY_s^n	Notional production price of sector s	5.3.1, 95
PY_s^n (2)	Nominal production prices of covered sectors	5.18.8, 156
$PY_{ce,s}^{toe}$	Net production price of energy ce supplied by sector s	5.19.45, 160
PY_s	Production price of sector s	5.11.10, 118
PYQ	Aggregate basic price of domestic production	5.2.105, 91
PYQ_c	Price of domestically produced commodity c, expressed at basic price	5.3.31, 99
PYQ_c^{bis}	Basic price of the production of commodity c (for verification)	5.2.90, 90
$PYQS_c$	Price of domestically produced commodity c, expressed at market price	5.3.33, 99

$PYQSBVAT_c$	Selling price before VAT for domestically produced commodity c	5.13.19, 127
Q	Aggregate production, expressed at market price	5.2.74, 88
$Q_s^{ETS,free}$	Quantity of freely allocated permits to sector s	5.18.2, 155
$Q_{ce2,s}^{ETS}$	Quantity of emissions permits required for sector s relative to its $ce2$ energy consumption	5.18.3, 155
Q_c	Production of commodity c, expressed at market price	5.2.10, 82
QD	Aggregate domestically produced commodities, expressed at market price	5.2.42, 85
QD_c	Quantity of domestically produced commodity c expressed at market price	5.2.2, 80
QM	Aggregate imported commodities, expressed at market price	5.2.44, 85
QM_c	Quantity of imported commodity c expressed at market price	5.2.4, 81
R	Interest rate	5.11.14, 119
$R_{ecl,cea}^{CASH,AUTO}$	Share of the price for a new car directly financed by the household	5.19.124, 166
$R_{ecl}^{CASH,NewBUIL}$	Share of investment for the construction of a building directly paid by the households	5.19.105, 165
$R_{ecl}^{CASH,REHAB}$	Share of investment for rehabilitation directly paid by the households	5.19.104, 165
$r^{DEBT,G}$	Interest rate paid by the Government on its debt	5.3.15, 96
$R_{ecl,cea}^{I,AUTO}$	Interest rate paid for an investment in a class ecl automobile fueled with energy cea	5.17.5, 145
$R_{ecl}^{I,BUIL}$	Interest rate paid for a (maintenance) investment of a class ecl building	5.16.8, 138
$R_{ecl}^{I,NewBUIL}$	Interest rate paid for an investment in a new class ecl building	5.16.6, 138
$R_{ecl}^{I,REHAB}$	Interest rate paid for an investment in the rehabilitation of a class ecl building	5.16.7, 138
$R_{ecl,cea}^{LOAN,AUTO}$	Share of the price of the purchase of a new car financed through a loan	5.19.125, 166
$R_{ecl}^{LOAN,NewBUIL}$	Share of the new class ecl building investment paid through a loan	5.16.52, 143
$R_{ecl}^{LOAN,REHAB}$	Share of the class ecl building rehabilitation investment paid through a loan	5.16.51, 143
R_{ecl}^{LOAN}	Share of the class ecl building investment paid through a loan	5.16.55, 144
R^n	Notional key interest rate of the Central Bank (Taylor rule)	5.3.13, 96

$R^{RMBS,AUTO}_{ecl,cea}$	Share of the reimbursement of a loan related to the purchase of a car	5.19.126, 166
$R^{RMBS,NewBUIL}_{ecl}$	Share of the reimbursement of a loan related to the purchase of building of class *ecl*	5.19.107, 165
$R^{RMBS,REHAB}_{ecl}$	Share of investment for rehabilitation directly paid by the households	5.19.106, 165
$R^{SUB,AUTO}_{ecl,cea}$	Share of the price of the purchase of a car financed by a public subsidy	5.19.127, 166
$R^{SUB}_{ecl,ecl2}$	Share of public subsidy for the rehabilitation of a building from class *ecl* to class *ecl*2	5.19.108, 165
R_s	Interest rate paid on capital by sector s	5.3.14, 96
$R2^{CH}_{ghg,c}$	Carbon tax rate on the greenhouse gas *ghg* associated to the household final consumption of the sector s	5.19.84, 163
$R2^{CI}_{ghg,c,s}$	Carbon tax rate on the greenhouse gas *ghg* associated to the intermediary consumption of commodity c by the sector s	5.19.85, 163
$R2^{MAT}_{ghg,s}$	Carbon tax rate on the greenhouse gas *ghg* associated to the material consumption of the sector s	5.19.86, 163
$R2^{Y}_{ghg,s}$	Carbon tax rate on the greenhouse gas *ghg* associated to the final production of the sector s	5.19.87, 163
RAC^{toe}_{ce}	Auto-consumption rate of energy *ce*	5.10.22, 117
$RBal^{G,Prim,VAL}$	Primary balance of the Government expressed in value (in percent of GDP)	5.6.29, 108
$RBal^{G,Tot,VAL}$	Total balance of the Government expressed in value (in percent of GDP)	5.6.30, 109
$RBal^{Trae,VAL}$	Balance of trade (in percent of GDP)	5.7.29, 111
$RDEBT^{G,VAL}$	Ratio of the Government's debt expressed in value (in percent of GDP)	5.6.31, 109
$RDLY^{toe}_{ce}$	Distribution losses rate of energy *ce*	5.10.21, 117
$REHAB$	Total of buildings rehabilitated	5.16.21, 139
$REHAB_{ecl,ecl2}$	Buildings rehabilitated from class *ecl* to class *ecl*2	5.16.16, 139
$REHAB_{ecl}$	Class *ecl* rehabilitated buildings	5.16.19, 139
$Rep^{AUTO}_{ecl,cea,DES}$	Depreciation rate of automobile of class *ecl* into other classes *cea* and destruction DES	5.19.116, 166
$Rep^{BUIL}_{ecl,bcl}$	Depreciation rate of building stock from class *ecl* to class *bcl*	5.19.98, 164
$\rho^{MPS,R}$	Elasticity of the marginal propensity to share to the growth rate of the interest rate	5.19.18, 158
$\rho^{MPS,UnR}$	Elasticity of the marginal propensity to share to the growth rate of unemployment	5.19.19, 158

$\rho^{PATR,UnR}$	Elasticity of unemployed population with the labor force participation	5.19.39, 159
$\rho^{Rn,Cons}$	Constant of the notional interest rate equation (Taylor rule).	5.19.2, 157
$\rho^{Rn,P}$	Elasticity of the notional interest rate to inflation	5.19.3, 157
$\rho^{Rn,UnR}$	Elasticity of the notional interest rate to the unemployment rate	5.19.4, 157
$\rho^{U,GAIN}_{ecl,ecl2}$	Utility elasticity of the financial gain induced by the rehabilitating a building from class ecl to class $ecl2$	5.19.109, 165
$\rho^{W,Cons}_{s}$		5.3.8, 96
$\rho^{W,DU}_{s}$	Elasticity of the notional wage inflation to the variation of the unemployment rate for sector s	5.19.5, 157
$\rho^{W,L}_{s}$	Elasticity of the notional wage inflation to the growth rate of share of labor in sector s	5.19.6, 157
$\rho^{W,P}_{s}$	Elasticity of the notional wage inflation to prices inflation for sector s	5.19.7, 157
$\rho^{W,Pe}_{s}$	Elasticity of the notional wage inflation to expected price inflation for sector s	5.19.8, 157
$\rho^{W,PROG}_{s}$	Elasticity of the notional wage inflation to labor productivity growth for sector s	5.19.9, 157
$\rho^{W,U}_{s}$	Elasticity of the notional wage inflation to the unemployment rate for sector s	5.19.10, 157
$\rho^{W,X}_{s}$	Constant in the wage equation for sector s	5.19.11, 157
$RINC^{SOC,TAX}$	Rate of social taxes to the disposable income	5.19.17, 158
$riskprem$	Risk premium	5.19.20, 158
$RNTAXCD_c$	Average tax rate on domestically produced commodity c	5.13.3, 126
$RNTAXCM_c$	Average tax rate on imported commodity c	5.13.6, 126
$RNTAXS_s$	tax rate on the production of sector s	5.19.27, 158
$ROTHCT$	Average rate of other taxes on commodity	5.13.62, 131
$ROTHCTD$	Average rate of other taxes on domestically produced commodity	5.13.56, 131
$ROTHCTM$	Average rate of other taxes on imported commodity	5.13.59, 131
RR^{POP}	Population growth rate	5.19.21, 158
RR^{Un}	unemployment growth rate	5.19.22, 158
$RRSC$	Average employers' social security contribution rate	5.6.17, 107
$RRSC^{ef}_{s}$		5.6.18, 107
$RRSC_s$	Employers' social security contribution rate of the sector s	5.19.28, 158
$RSAV^{G,VAL}$	Government's savings rate expressed in value (in percent of GDP)	5.6.28, 108

$RSAV^{H,VAL}$	Households savings rate	5.5.16, 105
RSC	Aggregate employers' social security contribution paid by sectors	5.6.16, 107
RSC_s	Employers' social security contribution paid by sector s	5.6.9, 106
RSC_s (2)	Employers' social security contribution paid by sector s	5.18.10, 156
$RSUBC$	Average rate of subsidies on commodity	5.13.63, 131
$RSUBCD$	Average rate of subsidies on domestically produced commodity	5.13.57, 131
$RSUBCM$	Average rate of subsidies on imported commodity	5.13.60, 131
$RVAT$	Average VAT rate	5.13.61, 131
$RVATD$	Average VAT rate on domestically produced commodity	5.13.55, 131
$RVATM$	Average VAT rate on imported commodity	5.13.58, 131
RY_{ce}^{toe}	Net production rate for energy ce	5.10.20, 117
$SAV^{G,VAL}$	Savings of the Government expressed in value	5.6.23, 108
$SAV^{H,VAL}$	Households savings expressed in value	5.5.15, 105
$share_s^{ETS}$	Percentage of emissions covered by an ETS scheme for the sector s	5.19.89, 163
$share_s^{free}$	Percentage of freely allocated permits to sector s	5.19.88, 163
σ_{ecl}	Switching speed between the 2 regimes of the logistic function representing the notional rehabilitation rate of a class ecl building	5.19.110, 165
$SOC^{BENF,VAL}$	Social benefits expressed in value	5.5.5, 104
$SPEND^{G,VAL}$	Aggregate spending of the Government expressed in value	5.6.22, 108
$Stock^{SAV,H,VAL}$	Households savings stock	5.5.17, 105
$SUBC$	Aggregate subsidies on commodities	5.13.54, 130
$SUBCD$	Aggregate subsidies on domestically produced commodities	5.13.42, 130
$SUBCD_c$	Subsidies on domestically produced commodity c	5.13.16, 127
$SUBCM$	Aggregate subsidies on imported commodities	5.13.48, 130
$SUBCM_c$	Subsidies on imported commodity c	5.13.18, 127
$SUBST_c^{CHM,n}$	Notional substitution effect induced by a change in the relative price between imported and domestically produced commodity c for households final consumption	5.7.11, 110
$SUBST_c^{CHM}$	Substitution effect on the imported households final consumption for the commodity c	5.11.25, 120
$SUBST_{ce,s}^{CI,n}$	Notional substitution between energy commodity ce and the other energy commodities cee in the sector s	5.4.14, 102

$SUBST^{CI,n}_{ct,s}$	Notional substitution between the transport ct and the other transports mt in the sector s	5.4.19, 103
$SUBST^{CI}_{ce,s}$	Substitution effect on the energy intermediate consumption ce in the sector s	5.11.22, 120
$SUBST^{CI}_{ct,s}$	Substitution effect on the transportation intermediate consumption ce in the sector s	5.11.23, 120
$SUBST^{CIM,n}_{c,s}$	Notional substitution effect induced by a change in the relative price between imported and domestic intermediary consumption in commodity c from the sector s	5.7.19, 111
$SUBST^{CIM}_{c,s}$	Substitution effect on the intermediate consumption for the imported commodity c in the sector s	5.11.28, 120
$SUBST^{F,n}_{f,s}$	Notional substitution between input f and the other inputs ff	5.4.9, 102
$SUBST^{F}_{f,s}$	Substitution effect of the production factor f in the sector s	5.11.19, 120
$SUBST^{GM,n}_{c}$	Notional substitution effect induced by a change in the relative price between imported and domestically produced commodity c for government final consumption	5.7.12, 110
$SUBST^{GM}_{c}$	Substitution effect on the imported government final consumption for the commodity c	5.11.26, 120
$SUBST^{HOUSINV}$	Substitution effect induced by a change in the relative price between investment and energy housing expenditures	5.14.10, 133
$SUBST^{IM,n}_{c,s}$	Notional substitution effect induced by a change in the relative price between imported and domestic investment in commodity c from the sector s	5.7.20, 111
$SUBST^{IM}_{c,s}$	Substitution effect on the investment for the imported commodity c in the sector s	5.11.29, 120
$SUBST^{MGPD,n}_{m,c}$	Notional substitution between margin m and the other margin types mm paid on domestic commodity c	5.4.2, 101
$SUBST^{MGPD}_{m,c}$	Substitution effect of the domestic margin paid m for the commodity c	5.11.20, 120
$SUBST^{MGPM,n}_{m,c}$	Notional substitution between margin m and the other margin types mm paid on imported commodity c	5.4.5, 101
$SUBST^{MGPM}_{m,c}$	Substitution effect on the imported margin paid m for the commodity c	5.11.21, 120
$SUBST^{MGSM,n}_{c}$	Notional substitution effect induced by a change in the relative price between imported and domestically produced commodity c for margins supplied	5.7.10, 110
$SUBST^{MGSM}_{m}$	Substitution effect on the imported margin supplied for the commodity m	5.11.24, 120
$SUBST^{NRJperM2}_{ecl,ceb}$	Substitution between energy commodity ceb and the other energy commodities $cebb$ consumed in class ecl buildings	5.16.25, 140

$SUBST^{TRSPINV}$	Share of automobile transport investment expenditures into the total automobile transport expenditures	5.14.25, 135
$SUBST_c^{X,n}$	Notional substitution effect induced by a change in the relative price between export prices and (converted in domestic currency) international prices for the commodity c	5.7.26, 111
$SUBST_c^{X}$	Substitution effect on the exports of the commodity c	5.11.30, 120
$SUBST_c^{XM,n}$	Notional substitution effect induced by a change in the relative price between imported and domestic products c for exports	5.7.24, 111
$SUBST_c^{XM}$	Substitution effect on the exports for the imported commodity c	5.11.27, 120
$SUM^{exp,U,AUTO}$	Sum of the exponential of the automobile utilities per class	5.17.25, 148
$SUM_{ecl}^{exp,U,REHAB}$	Sum of the exponential utility from rehabilitating a ecl to a $ecl2$ class building	5.16.44, 142
$T2_c^{CH}$	Carbon tax in volume on households' consumption depending on commodity c	5.12.38, 125
$T2_{ghg,c}^{CH}$	Carbon tax in volume on households' consumption depending on ghg emission types and commodity c	5.12.35, 124
$T2_{ghg,c,s}^{CI}$	Carbon tax in volume on intermediary consumption emissions depending on ghg emission types, commodity c in sector s	5.12.2, 121
$T2_c^{CID}$	Carbon tax in volume on intermediary consumption emissions depending on domestic commodity c	5.12.13, 122
$T2_{ghg,c}^{CID}$	Carbon tax in volume on intermediary consumption emissions depending on ghg emission types and domestic commodity c	5.12.5, 121
$T2_c^{CIM}$	Carbon tax in volume on intermediary consumption emissions depending on imported commodity c	5.12.15, 122
$T2_{ghg,c}^{CIM}$	Carbon tax in volume on intermediary consumption emissions depending on ghg emission types and imported commodity c	5.12.7, 121
$T2_{ghg,s}^{MAT}$	Carbon tax in volume on material emissions depending on ghg emission types, commodity c in sector s	5.12.19, 123
$T2_{ghg}^{MAT}$	Carbon tax in volume on material emissions depending on ghg emission types	5.12.21, 123
$T2_s^{MAT}$	Carbon tax in volume on material emissions depending on commodity c	5.12.23, 123
$T2_{ghg,s}^{Y}$	Carbon tax in volume on production emissions depending on ghg emission types, commodity c in sector s	5.12.26, 123
$T2_{ghg}^{Y}$	Carbon tax in volume on production emissions depending on ghg emission types	5.12.28, 124

$T2_s^Y$	Carbon tax in volume on production emissions depending on sector s	5.12.30, 124
$T2VAL^{CH}$	Carbon tax in value on households' consumption depending on ghg emission types and commodity c	5.12.39, 125
$T2VAL_c^{CH}$	Carbon tax in value on households' consumption depending on commodity c	5.12.37, 124
$T2VAL_{ghg,c}^{CH}$	Carbon tax in value on households' consumption depending on ghg emission types and commodity c	5.12.34, 124
$T2VAL_{ghg}^{CH}$	Carbon tax in value on households' consumption depending on ghg emission types	5.12.36, 124
$T2VAL^{CI}$	Carbon tax in value on intermediary consumption emissions	5.12.17, 122
$T2VAL_{c,s}^{CI}$	Carbon tax in value on intermediary consumption emissions depending on commodity c and sector s	5.12.9, 122
$T2VAL_c^{CI}$	Carbon tax in value on intermediary consumption emissions depending on commodity c	5.12.11, 122
$T2VAL_{ghg,c,s}^{CI}$	Carbon tax in value on intermediary consumption emissions depending on ghg emission types, commodity c in sector s	5.12.1, 121
$T2VAL_{ghg,c}^{CI}$	Carbon tax in value on intermediary consumption emissions depending on ghg emission types and commodity c	5.12.3, 121
$T2VAL_{ghg,s}^{CI}$	Carbon tax in value on intermediary consumption emissions depending on ghg emission types and sector s	5.12.8, 121
$T2VAL_{ghg}^{CI}$	Carbon tax in value on intermediary consumption emissions depending on ghg emission types	5.12.10, 122
$T2VAL_s^{CI}$	Carbon tax in value on intermediary consumption emissions depending on sector s	5.12.16, 122
$T2VAL_c^{CID}$	Carbon tax in value on intermediary consumption emissions depending on domestic commodity c	5.12.12, 122
$T2VAL_{ghg,c}^{CID}$	Carbon tax in value on intermediary consumption emissions depending on ghg emission types and domestic commodity c	5.12.4, 121
$T2VAL_c^{CIM}$	Carbon tax in value on intermediary consumption emissions depending on imported commodity c	5.12.14, 122
$T2VAL_{ghg,c}^{CIM}$	Carbon tax in value on intermediary consumption emissions depending on ghg emission types and imported commodity c	5.12.6, 121
$T2VAL^{MAT}$	Carbon tax in value on material emissions	5.12.24, 123
$T2VAL_{ghg,s}^{MAT}$	Carbon tax in value on material emissions depending on ghg emission types, commodity c in sector s	5.12.18, 123
$T2VAL_{ghg}^{MAT}$	Carbon tax in value on material emissions depending on ghg emission types	5.12.20, 123

$T2VAL_s^{MAT}$	Carbon tax in value on material emissions depending on commodity c	5.12.22, 123
$T2VAL^{SEC}$	Total carbon tax in value collected over firms	5.12.33, 124
$T2VAL_s^{SEC}$	Total carbon tax in value collected depending on sector s	5.12.32, 124
$T2VAL^{Y}$	Carbon tax in value on production emissions	5.12.31, 124
$T2VAL_{ghg,s}^{Y}$	Carbon tax in value on production emissions depending on ghg emission types, commodity c in sector s	5.12.25, 123
$T2VAL_{ghg}^{Y}$	Carbon tax in value on production emissions depending on ghg emission types	5.12.27, 123
$T2VAL_s^{Y}$	Carbon tax in value on production emissions depending on sector s	5.12.29, 124
$\tau_{ecl}^{REHAB,MAX}$	Upper bound of the rehabilitation rate for buildings of class ecl	5.19.112, 165
$\tau_{ecl}^{REHAB,MIN}$	Lower bound of the rehabilitation rate for buildings of class ecl	5.19.111, 165
$\tau_{ecl}^{REHAB,N}$	Notional rehabilitation rate of a class ecl building	5.16.38, 141
τ_{ecl}^{REHAB}	Rehabilitation rate of a class ecl building	5.16.40, 142
$\theta^{kmtrav^{autoLD,crai}}$	Elasticity between road and rail transport use for long distance trips	5.19.134, 167
$\theta^{kmtrav^{autoSD,crai}}$	Elasticity between road and rail transport use for short distance trips	5.19.135, 167
$\theta^{kmtrav^{autoSD,croa}}$	Elasticity between automobile and bus use for short distance trips	5.19.136, 167
$\theta^{kmtravcair^{DISPINC}}$	Elasticity of substitution between long-distance air transport and income	5.19.128, 166
$\theta^{kmtravcair^{Pkmtrav}}$	Price elasticity of long-distance air transport	5.19.129, 166
$\theta^{kmtravLD^{DISPINC}}$	Elasticity of substitution between long-distance (by road or train) transports and income	5.19.130, 166
$\theta^{kmtravLD^{Pkmtrav}}$	Price elasticity of short-distance (by road or train) of transport commodity	5.19.131, 166
$\theta^{kmtravSD^{DISPINC}}$	Elasticity of substitution between long-distance (by road or train) transports and income	5.19.132, 167
$\theta^{kmtravSD^{Pkmtrav}}$	Price elasticity of short-distance (by road or train) of transport commodity	5.19.133, 167
$\theta^{U^{AUTO,UC,E}}$	Elasticity between the marginal utility from automobile use and its energy cost	5.19.137, 167
$\theta^{U^{AUTO,UC,K}}$	Elasticity between the marginal utility from automobile use and its capital cost	5.19.138, 167

$toePerKm_{ecl,cea}$	Energy consumption in ton equivalent petrol per km for the automobile of class *ecl* fueled with energy *cea*	5.19.139,167
$TRSF^{HH,VAL}$	Transfers to households	5.19.23, 158
$TRSP_s$	Transport demand of sector *s*	5.4.17, 103
U^{AUTO}_{ecl}	Utility of a automobile of class *ecl*	5.17.24, 148
$U^{REHAB}_{ecl,ecl2}$	Utility from rehabilitating a *ecl* to a *ecl*2 class building	5.16.43, 142
UC^{AUTO}	User cost of an automobile	5.17.59, 152
$UC^{AUTO}_{ecl,cea}$	User cost of a class *ecl* automobile fueled with energy *cea*	5.17.65, 152
$UC^{AUTO}_{ecl,th}$	User cost of a thermic automobile of class *ecl*	5.17.66, 152
UC^{AUTO}_{ecl}	User cost of an automobile of class *ecl*	5.17.64, 152
$UC^{AUTO,bis}$	User cost of an automobile (for verification)	5.17.60, 152
$UC^{E,AUTO}$	User energy cost of an automobile	5.17.62, 152
$UC^{E,AUTO}_{ecl,cea}$	User energy cost of a class *ecl* automobile fueled with energy *cea*	5.17.48, 151
$UC^{E,AUTO}_{ecl,th}$	User energy cost of a thermic automobile of class *ecl*	5.17.70, 153
$UC^{E,AUTO}_{ecl}$	User energy cost of an automobile of class *ecl*	5.17.69, 153
$UC^{E,REHAB}_{ecl}$	User energy cost of a *ecl* class building after rehabilitation to a more energy efficient class	5.16.48, 143
UC^{E}_{ecl}	User energy cost of a class *ecl* building	5.16.56, 144
$UC^{K,AUTO}$	User capital cost of an automobile	5.17.61, 152
$UC^{K,AUTO}_{ecl,cea}$	User capital cost of a class *ecl* automobile fueled with energy *cea*	5.17.51, 151
$UC^{K,AUTO}_{ecl,th}$	User capital cost of a thermic automobile of class *ecl*	5.17.68, 152
$UC^{K,AUTO}_{ecl}$	User capital cost of an automobile of class *ecl*	5.17.67, 152
$UC^{K,REHAB}_{ecl,ecl2}$	User capital cost of a *ecl* class building after rehabilitation to a *ecl*2 class	5.16.50, 143
$UC^{K,REHAB}_{ecl}$	User capital cost of a *ecl* class building after rehabilitation to a more energy efficient class	5.16.49, 143
UC^{K}_{ecl}	User cost of capital of a class *ecl* building	5.16.54, 143
$UC^{M,AUTO}$	User maintenance cost of an automobile	5.17.63, 152
$UC^{M,AUTO}_{ecl,cea}$	User maintenance cost of a class *ecl* automobile fueled with energy *cea*	5.17.58, 152
$UC^{M,AUTO}_{ecl,th}$	User maintenance cost of a thermic automobile of class *ecl*	5.17.72, 153
$UC^{M,AUTO}_{ecl}$	User maintenance cost of an automobile of class *ecl*	5.17.71, 153
UC^{REHAB}_{ecl}	User cost of a *ecl* class building after rehabilitation to a more energy efficient class	5.16.47, 143

UC_{ecl}	User cost of a *ecl* class building	5.16.53, 143
Un	Unemployment	5.8.5, 112
UnR	Unemployment rate	5.8.6, 112
VA	Aggregate value-added	5.2.119, 93
VA_s^{VAL}	Value-added of sector s expressed in value	5.2.110, 92
VA_s	Value-added of sector s	5.2.111, 92
VAT	Aggregate Value-added Tax on total households final consumption	5.13.50, 130
$VATD$	Aggregate value-added tax paid on domestically produced comodities	5.13.38, 129
$VATD_c$	Value-added tax on domestically produced commodity c	5.13.8, 126
$VATM$	Aggregate value-added tax paid on imported comodities	5.13.44, 130
$VATM_c$	Value-added tax on imported commodity c	5.13.10, 126
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W_s	Wages of the sector s	5.11.12, 119
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$WAGES_s$	Gross wages paid by sector s including employees (but not employers)' social contribution	5.3.11, 96
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X_{ce}^{toe}	External end use for energy ce, expressed in toe	5.10.4, 115
X_c	Foreign demand for exports of commodity c	5.7.25, 111
XD	Aggregate domestically produced exports, expressed at market price	5.2.66, 87
XD_c	Exports of domestically produced commodity c	5.7.21, 111
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