1 ALGEBRA

1. The function $f(x) = x|x|$ is
   (a) continuous and differentiable at $x = 0$.
   (b) continuous but not differentiable at $x = 0$.
   (c) differentiable but not continuous at $x = 0$.
   (d) neither differentiable nor continuous at $x = 0$.
2. The objective function $Z = ax + by$ of an LPP has maximum value 42 at $(4, 6)$ and minimum value 19 at $(3, 2)$. Which of the following is true?
   (a) $a = 9, b = 1$
   (b) $a = 5, b = 2$
   (c) $a = 3, b = 5$
   (d) $a = 5, b = 3$
3. The corner points of the feasible region of a linear programming problem are $(0, 4)$, $(8, 0)$, and $(\frac{20}{3}, \frac{4}{3})$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z - minimum value of Z) is equal to
   (a) 40
   (b) 96
   (c) 120
   (d) 136
4. Solve the following linear programming problem graphically:
   Maximize : $Z = x + 2y$
   subject to constraints:
$$\begin{aligned} x + 2y &\geq 100, \\ 2x - y &\leq 0, \\ 2x + y &\leq 200, \\ x \geq 0, y &\geq 0. \end{aligned}$$
5. A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of a for which $f(a) = \sqrt{7}$.
6. The use of electric vehicles will curb air pollution in the long run.

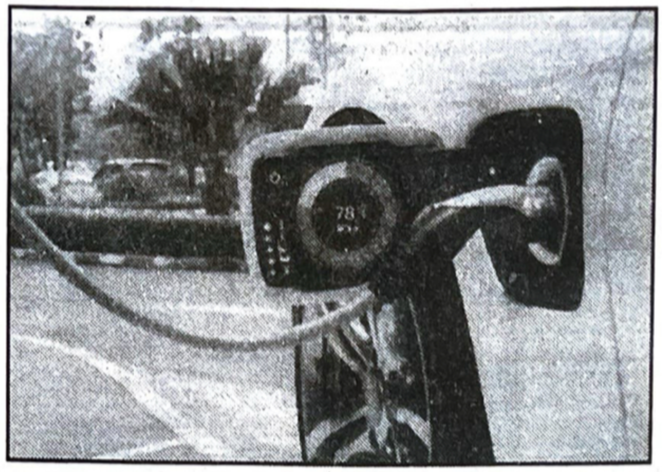


Figure 1: electric charging point

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V:

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2 \tag{1}$$

where t represents the time and t=1, 2, 3....corresponds to year 2001, 2002, 2003, ...respectively.
Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vechicles in the year 2000 ? Justify.

(ii) Prove that the function $V(t)$ is an increasing function.

2 DIFFERENTIATION

1. If $\frac{d}{dx}f(x) = 2x + \frac{3}{x}$ and $f(1) = 1$, then $f(x)$ is

   (a) $x^2 + 3\log|x| + 1$

   (b) $x^2 + 3\log|x|$

   (c) $2 - \frac{3}{x^2}$

   (d) $x^2 + 3\log|x| - 4$

2. Degree of the differential equation $\sin x + \cos(\frac{dy}{dx}) = y^2$ is

   (a) 2

(b) 1
(c) not defined
(d) 0

3. The integrating factor of the differential equation

$$(1-y^2)\frac{dx}{dy}+yx=ay,(-1<y<1)is \tag{2}$$

(a) $\frac{1}{y^2-1}$
(b) $\frac{1}{\sqrt{y^2-1}}$
(c) $\frac{1}{1-y^2}$
(d) $\frac{1}{\sqrt{1-y^2}}$

4. Anti-derivative of $\frac{\tan x-1}{\tan x+1}$ with respect to x is:

(a) $\sec^2(\frac{\pi}{4}-x)+c$
(b) $-\sec^2(\frac{\pi}{4}-x)+c$
(c) $\log\left|\sec(\frac{\pi}{4}-x)\right|+c$
(d) $-\log\left|\sec(\frac{\pi}{4}-x)\right|+c$

5. If $\tan\left(\frac{x+y}{x-y}\right)=k$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{-y}{x}$
(b) $\frac{y}{x}$
(c) $\sec^2(\frac{y}{x})$
(d) $-\sec^2(\frac{y}{x})$

6. If $y=\sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2=0$.

7. If $f(x)=\begin{cases} ax+b; & 0<x\leq 1 \\ 2x^2-x; & 1<x<2 \end{cases}$ is a differentiable function in $(0,2)$, then find the values of a and b.

8. Find the general solution of the differential equation:

$$(xy-x^2)\,dy=y^2\,dx. \tag{3}$$

9. Find the general solution of the differential equation:

$$(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}. \tag{4}$$

3 GEOMETRY

1. Equation of line passing through origin and making 30°, 60° and 90° with x, y, z axes respectively is

   (a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$

   (b) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$

   (c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$

   (d) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

2. If the equation of a line is $x = ay + b$, $z = cy + d$, then find the direction ratios of the line and a point on the line.

3. If the circumference of a circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius.

4. Find the equations of the diagonals of the parallelogram $PQRS$ whose vertices are $P(4, 2, -6)$, $Q(5, -3, 1)$, $R(12, 4, 5)$ and $S(11, 9, -2)$. Use these equations to find the point of intersection of diagonals.

5. A line l passes through point $(-1, 3, -2)$ and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.

6. Engine dispalcement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore

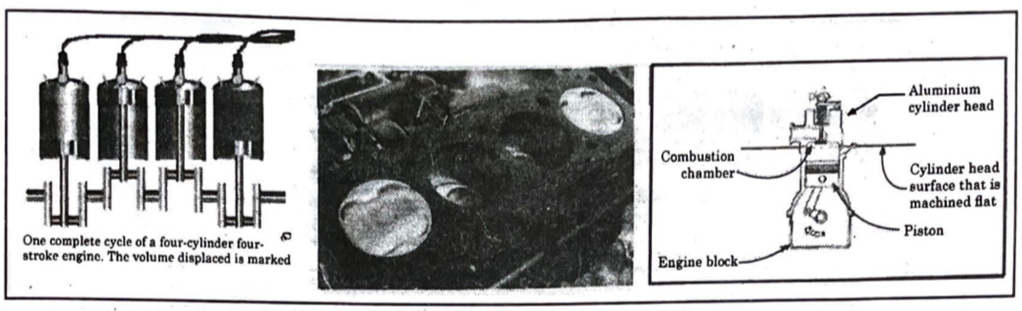


Figure 2: Cylinder bore

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi cm^2$.
Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

(ii) Find $\frac{dV}{dr}$.

(iii) Find the radius of the cylinder when its volume is maximum.

(iv) For maximum volume, $h > r$. State true or false and justify.

4 INTEGRATION

1. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})} dx$
2. Evaluate $\int_{-1}^{1} |x^4 - x|\, dx$.
3. Find $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\, dx$.
4. Find $\int e^x (\frac{1-\sin x}{1-\cos x})\, dx$
5. Using Integration, find the area of the triangle whose vertices are $(-1, 1)$, $(0, 5)$ and $(3, 2)$.

5 MATRIX

1. If (a, b), (c, d), and (e, f) are the vertices of $\triangle ABC$ and Δ denotes the area of $\triangle ABC$, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to

   (a) $2\Delta^2$

   (b) $4\Delta^2$

   (c) 2Δ

   (d) 4Δ

2. If A is a 2×3 matrix such that AB and AB' both are defined, then order of the matrix B is

   (a) 2×2

   (b) 2×1

   (c) 3×2

   (d) 3×3

3. If $\begin{pmatrix} 2 & 0 \\ 5 & 4 \end{pmatrix}$ = P + Q, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to

   (a) $\begin{pmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{pmatrix}$

   (b) $\begin{pmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & \frac{5}{2} \\ \frac{-5}{2} & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & \frac{-5}{2} \\ \frac{5}{2} & 4 \end{pmatrix}$

4. If $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{pmatrix}$ is a non-singular matrix and $a \in A$, then the set A is

   (a) $\mathbb{R}$

   (b) $\{0\}$

   (c) $\{4\}$

   (d) $\mathbb{R} - \{4\}$

5. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is

   (a) 1

   (b) -1

   (c) 2

   (d) 0

6. If $A = \begin{pmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{pmatrix}$, then find AB and use it to solve the following system of equations:

$$\begin{aligned} x - 2y &= 3 \\ 2x - y - z &= 2 \\ -2y + z &= 3 \end{aligned}$$

7. If $f(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$

6 PROBABILITY

1. If A and B are two events such that $P(A/B) = 2 \times P(B/A)$ and $P(A) + P(B) = \frac{2}{3}$, then $P(B)$ is equal to

   (a) $\frac{2}{9}$

   (b) $\frac{7}{9}$

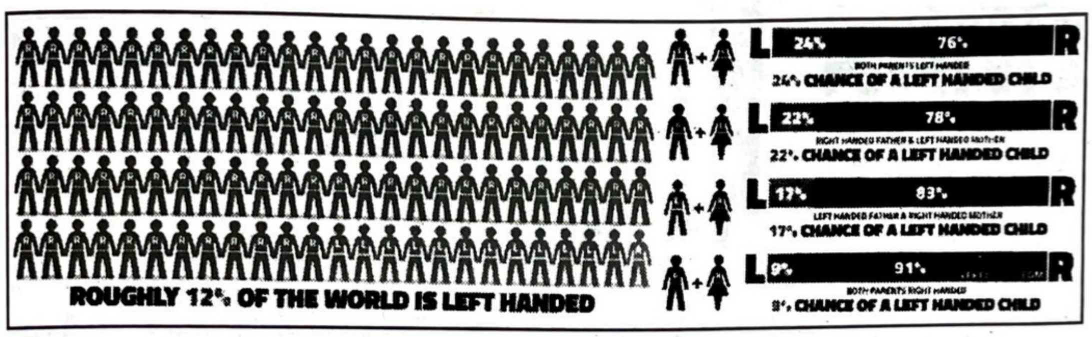
   (c) $\frac{4}{9}$

   (d) $\frac{5}{9}$

2. Two balls are drawn at random one by one with replacement from an urn containing equal numbers of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

3. A and B throw a die alternately till one of them gets a 6 and wins the game. Find their respective probabilities of winning, if A starts the game first.

4. Recent studies suggest that roughly 12% of the world population is left handed



Depending upon the parents, the chances of having a left handed child are as folws;

(a) When both father and mother are left handed:
Chances of left handed child is 24%.

(b) When father is right handed and mother is left handed:
Chances of left handrd child is 22%.

(c) When father is left handed and mother is right handed:
Chances of left handed child is 17%.

(d) When both faher and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed
Based on the above information, answer the following questions:

(i) Find $P(L/C)$

(ii) Find $P(\overline{L}/A)$

(iii) Find $P(A/L)$

(iv) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

7 TRIGONOMETRY

1. Evaluate $\sin^{-1}(\sin \frac{3\pi}{4}) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$.

2. Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.

3. **Assertion (A) :** If a line makes angles α, β, γ with the positive direction of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.
   Reason (R) : The sum of squares of the direction cosines of a line is 1.

4. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2.
   Reason (R) : Range of the principal value branch of $\cos^{-1} x$ is $[\frac{-\pi}{2}, \frac{\pi}{2}]$.

8 VECTORS

1. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are $(2, 1, -1)$ and $(4, 4, -7)$, is
   (a) $2\hat{i} + 3\hat{j} - 6\hat{k}$
   (b) $-2\hat{i} - 3\hat{j} + 6\hat{k}$
   (c) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$
   (d) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

2. If $|\overrightarrow{a} \times \overrightarrow{b}| = \sqrt{3}$ and $\overrightarrow{a} \cdot \overrightarrow{b} = -3$, then angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is
   (a) $\frac{2\pi}{3}$
   (b) $\frac{\pi}{6}$
   (c) $\frac{\pi}{3}$
   (d) $\frac{5\pi}{6}$

3. If in $\triangle ABC$, $\overrightarrow{BA} = 2\overrightarrow{a}$ and $\overrightarrow{BC} = 3\overrightarrow{b}$, then $\overrightarrow{AC}$ is
   (a) $2\overrightarrow{a} + 3\overrightarrow{b}$
   (b) $2\overrightarrow{a} - 3\overrightarrow{b}$
   (c) $3\overrightarrow{b} - 2\overrightarrow{a}$
   (d) $-2\overrightarrow{a} - 3\overrightarrow{b}$

4. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ are three non-zero unequal vectors such that $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{a} \cdot \overrightarrow{c}$, then find the angle between $\overrightarrow{a}$ and $\overrightarrow{b} - \overrightarrow{c}$.