Dynamic Macroeconomics
Spring 2025

Problem Set 2

Do Thu An

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1. Part I: Modeling Households in Vietnam

$$
\begin{aligned}
\max_{\{c_t\}_{t=0}^{T-1}, \{a_{t+1}\}_{t=0}^{T-1}} U &= \sum_{t=0}^{T-1} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \\
\text{s.t. } a_{t+1} &= (1+r)\left(a_t + y_t - c_t\right) \\
y_t &= \begin{cases} G_t e^{(\rho \log y_{t-1} + \varepsilon_t)} & \text{if } t < t_r \\ \kappa y_{t_r - 1} & \text{if } t \geqslant t_r \end{cases} \\
a_t &\geqslant 0 \\
c_t &> 0 \\
a_T &= 0
\end{aligned}
$$

where $|\rho| < 1, G_t$ is age-specific average of income, $\varepsilon_t \sim \mathcal{N}\left(0, \sigma^2\right)$, and the initial wealth endowment, $a_0 \geqslant 0$, is given.

1.1. VHLSS data processing

The final dataset is constructed by merging 55 CSV files from the `VHLSS 2008 Data` to analyze household income, consumption, and wealth. The three aggregated household variables include total household earnings and assets for income and wealth, while consumption is measured as the average household expenditure.

The data processing steps were done in Python.

Data mapping

The chosen variables in the datasets are mapped together based on a cluster of household ID including `tinh`, `huyen`, `diaban`, and `hoso`.

Datasets and Variables selection

We identify the appropriate variables by reviewing the questionnaires in the `VHLSS 2008 Questionnaire` folder, which consists of eight sections with multiple subsections. The key content of the questionnaires and the selected variables are summarized in the following tables.

1.1.1. Demographic Data

The selected dataset used as demographics and data labelling and mapping is `muc123a.csv`.The selected columns are extracted from `muc123a` to form a new DataFrame `df`, which serves as the base for merging labels from other files.

In this file, we conduct the following data transformation and filtering steps:

1. **Create Household Size Column (`hsize`)**:

SECTION 1A. A LIST OF HOUSEHOLD MEMBERS - *Individual*

VAR	QUESTIONS	#OBS.	MEASUREMENT	FILE NAME
`tinh`	Province/City	38,253	Code	Muc1A
`huyen`	District/Town	38,253	Code	Muc1A
`xa`	Commune/Ward/Township	38,253	Code	Muc1A
`diaban`	Enumeration area	38,253	Code	Muc1A
`hoso`	Household code	38,253	Code	Muc1A
`matv`	Member code	38,253	Code	Muc1A
`m1ac2`	Gender (1=male; 2=female)	38,253	Dummy	Muc1A
`m1ac3`	The relationship with the head	38,253	Category	Muc1A
`m1ac5`	Age	38,253	Number	Muc1A

Table 1: Household Members Data - `muc123a.csv`

- The number of household members is computed using `groupby()` on household identifiers (`tinh`, `huyen`, `diaban`, `hoso`).
- The transform(`max`) function assigns the maximum `matv` value (representing the highest member ID in the household) to each row.

2. **Filter for Household Heads Who Are Male and Age $\geqslant 25$:**
   - The dataset is filtered to keep only rows where `m1ac3 == 1` (household head).
   - Further filtering ensures only male household heads (`m1ac3 == 1`) aged 25 or older (`m1ac5 >= 1`) remain.

1.1.2. Household income data

For the household aggregated income, we consider only one primary source: income from salary and wages (in thousands VND) earned through main and secondary jobs (if applicable) in `muc4a.csv`.

Muc4a. Earnings from salary/wages of main and secondary jobs

SECTION 4: INCOMES

PART 4A. EMPLOYMENT AND SALARIES AND WAGES

These questions concern all household members aged 6 or more.

VAR	QUESTIONS	#OBS.	MEASUREMENT	FILE NAME
`tinh`	Province/City	38,253	Code	Muc4A
`huyen`	District/Town	35,155	Code	Muc4A
`xa`	Commune/Ward/Township	35,155	Code	Muc4A
`diaban`	Enumeration area	35,155	Code	Muc4A
`hoso`	Household code	35,155	Code	Muc4A
`matv`	Member code	38,253	Code	Muc4A
`m4ac11`	Salaries/wages (cash & in-kind) from main job (1000 VND)	35,155	Continuous	Muc4A
`m4ac12f`	Other salary/income from main job (1000 VND)	35,155	Continuous	Muc4A
`m4ac21`	Salaries/wages from secondary job (1000 VND)	38,253	Continuous	Muc4A
`m4ac22f`	Other salary/income from secondary job (1000 VND)	38,253	Continuous	Muc4A
`m4ac25`	Additional salary/income (1000 VND)	38,253	Continuous	Muc4A

Table 2: Household Members Income Data - `muc4a.csv`

Since the survey collects responses at the individual level, we first calculate each individual's total income and then aggregate it by household ID to obtain the total household income:

1. **Calculate Individual-Level Income**: A new column, `indi_income`, is created by summing the selected income-related variables for each individual.

2. **Aggregate Household-Level Income**:
   - The dataset is grouped by household identifiers (`tinh`, `huyen`, `diaban`, `hoso`).
   - The total household income `HH_Income` is computed by summing `indi_income` for all household members.
   - Missing values (NaN) are filled with zero.
3. **Integrate Processed Income Data with the Main Dataset**: The processed income data `muc4a`, now containing both individual and household income, is merged into the main dataset `df` based on household and individual identifiers.

1.1.3. Household consumption data

The consumption data is categorized into various aspects of individual expenditures, including *:

- Expenditures on food and drinks (Section 5A1, 5A2) - `muc5a1.csv` & `muc5a2.csv`
- Expenditures on non-food items and other expenditures (Section 5B1) - `muc5b1.csv`
- Household annual consumption expenditure (Section 5B2) - `muc5b2.csv`
- Other spending that is considered as household expenditure (Section 5B3) - `muc5b3.csv`

(*) *In this model, we assume that household expenditure on healthcare and education do not affect the part of consumption that the model focus on.*

SECTION 5. EXPENDITURES

VAR	QUESTIONS	#OBS.	MEASUREMENT	FILE NAME
PART 5A. EXPENDITURES ON FOOD AND DRINKS				
5A1. EXPENDITURES ON FOOD AND DRINKS ON FESTIVE OCCASIONS				
`hoso`	Household code	134,652	Code	Muc5A1
`matv`	Member code	134,652	Code	Muc5A1
`m5a1c2b`	Food and drinks purchased for festive occasions (1000 VND)	134,652	Continuous	Muc5A1
`m5a1c3b`	Food and drinks self-supplied or received as gifts (1000 VND)	134,652	Continuous	Muc5A1
5A2. RECURRENT EXPENDITURES ON FOOD AND DRINKS				
`hoso`	Household code	134,652	Code	Muc5A2
`matv`	Member code	134,652	Code	Muc5A2
`m5a2c6`	Daily food and drinks purchased (1000 VND)	134,652	Continuous	Muc5A2
`m5a2c10`	Daily food and drinks self-supplied or received (1000 VND)	134,652	Continuous	Muc5A2
PART 5B. EXPENDITURES ON NON-FOOD ITEMS AND OTHER EXPENDITURES				
5B1. DAILY EXPENDITURES				
`hoso`	Household code	134,652	Code	Muc5B1
`matv`	Member code	134,652	Code	Muc5B1
`m5b1c4`	Non-food daily expenses received (1000 VND)	134,652	Continuous	Muc5B1
`m5b1c5`	Non-food daily expenses paid annually (1000 VND)	134,652	Continuous	Muc5B1
5B2. ANNUAL CONSUMPTION				
`hoso`	Household code	134,652	Code	Muc5B2
`matv`	Member code	134,652	Code	Muc5B2
`m5b2c2`	Annual consumption expenses paid (1000 VND)	134,652	Continuous	Muc5B2
`m5b2c3`	Annual consumption expenses self-supplied or received (1000 VND)	134,652	Continuous	Muc5B2
5B3. OTHER COSTS AS HOUSEHOLD EXPENDITURES				
`hoso`	Household code	134,652	Code	Muc5B3
`matv`	Member code	134,652	Code	Muc5B3
`m5b3c2`	Other annual household expenses (1000 VND)	134,652	Continuous	Muc5B3

Table 3: Household Expenditure Data - `muc5a1.csv`, `muc5a2.csv`, `muc5b1.csv`, `muc5b2.csv`, `muc5b3.csv`

For the expenditures in Section 5, the data processing and calculation by defining a Function `merge_expenditure` for Processing Expenditure Data:

- The function groups the data by household ID and sum expenses to obtain total expenditure per household.
- Remove duplicate household records to ensure each ID has a unique expenditure value.
- Merge the processed expenditure data back into the main dataset `df`.
- Sum these expenses into a new variable for each file (`HH_exp1`, `HH_exp2`, `HH_exp3`, `HH_exp4`, `HH_exp5`)

SECTION 7: HOUSING

VAR	QUESTIONS	#OBS.	MEASUREMENT	FILE NAME
`tinh`	Province/City	9,190	Code	Muc7
`huyen`	District/Town	9,190	Code	Muc7
`xa`	Commune/Ward/Township	9,190	Code	Muc7
`diaban`	Enumeration area	9,190	Code	Muc7
`hoso`	Household code	9,190	Code	Muc7
`m7c15`	Income from leasing land or housing (1000 VND)	9,190	Continuous	Muc7
`m7c32`	Annual water expense (1000 VND)	9,190	Continuous	Muc7
`m7c36`	Annual electricity expense (1000 VND)	9,190	Continuous	Muc7
`m7c39`	Annual garbage collection expense (1000 VND)	9,190	Continuous	Muc7

Table 4: Household Housing-Related Expenses - `muc7.csv`

The data in Section 7 is basically sum these expenses listed in table 4 into a new variable `HH_exp6`.

Finally, we compute total household consumption `HH_consumption` as the sum of all expenditure components `HH_exp1` to `HH_exp6`. Divide total household consumption `HH_consumption` by household size (hsize) to get `HH_consumption_avr`, which represents per capita consumption.

1.1.4. Household wealth data

The total wealth data comes from one source: fixed assets (`muc6a.csv`), in which the dataset is from section 6A.

SECTION 6. FIXED ASSETS AND DURABLE APPLIANCE

PART 6A. FIXED ASSETS

VAR	QUESTIONS	#OBS.	MEASUREMENT	FILE NAME
`hoso`	Household code	18,149	Code	Muc6A
`matv`	Member code	18,149	Code	Muc6A
`m6ac3`	Quantity of fixed assets owned	18,149	Continuous	Muc6A
`m6ac6`	Value of fixed assets at current price (1000 VND)	18,149	Continuous	Muc6A
`m6ac7`	Percentage of ownership of fixed assets	18,149	Percentage	Muc6A

Table 5: Household Wealth Data - `muc6a.csv`

We process the calculation for each wealth source as follows:

1. **Fixed assets:** If dealing with fixed assets, it considers:
   - The quantity of assets `m6ac3`.
   - The assets' current value `m6ac6`.
   - The ownership percentage (`m6ac7` converted to decimal).
2. Aggregate household wealth by summing up values per household ID to obtain `HH_Wealth`

1.2. Interpretation of Gt

(**) We use MatLab from this part.

Figure 1 shows the visualization of the exponentiated average log income by age group G_t.

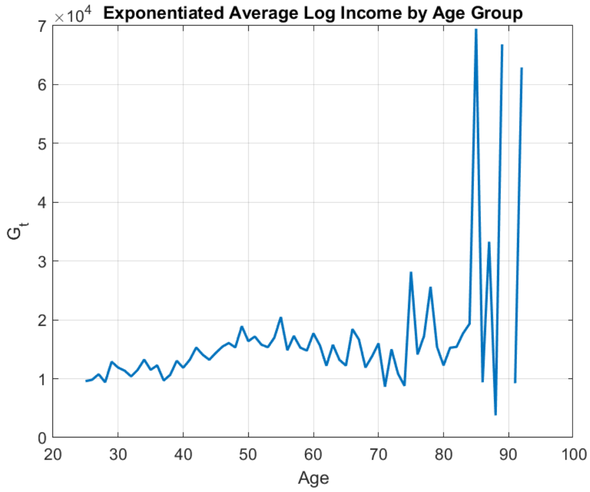


Figure 1: Age-Specific Income Index (G_t)

From figure 1, we see that G_t remains relatively stable from ages 25 (which is assumed to be the starting working age) to around 60 (retirement age). There is a slight increase over middle age, which aligns with the life-cycle hypothesis where income typically grows with experience and productivity.

However, there is high variability in the older age group. From around age 70 onwards, G_t becomes highly volatile, with significant fluctuations. This maybe due to fewer observations in older age groups, making the estimates more sensitive to outliers. Retirement effects may also contribute, where some individuals still report high income (e.g., pensions, investments) while others report very low or no income.

1.3. Baseline Stochastic Life Cycle Model

1.3.1. Gt processing

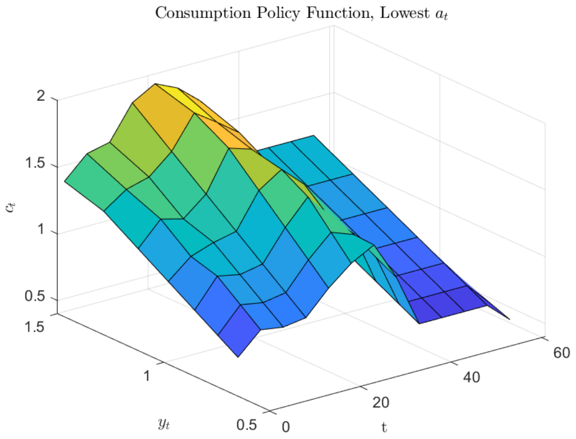
Since G_t is derived from real household-level data in the VHLSS, it captures empirically observed patterns in income across the life cycle. To integrate it into the quantitative model, we conduct the following steps:

1. Load G_t from CSV `'Gt_values.csv'` saved after the calculation above.
2. **Normalize** G_t: When computing G_t from data (e.g., average income by age from VHLSS), the raw values are in levels, often influenced by the local currency scale, inflation, or absolute income variation[1]. Thus, we normalize G_t by having all values relative to the first period, i.e., $G_1 = 1$, and subsequent values show income growth or decline over the life cycle G_t/G_1.
3. **Slice only relevant periods**: We only include values of `Gmat` for model periods 1 to 61. From figure 1, we concluded that from around age 70 onwards, G_t becomes highly volatile, with significant spikes and dips. This could be due to fewer observations in older age groups, making the estimates more sensitive to outliers. Hence, we remove the values in the old age group so that it does not affect the model's performance.

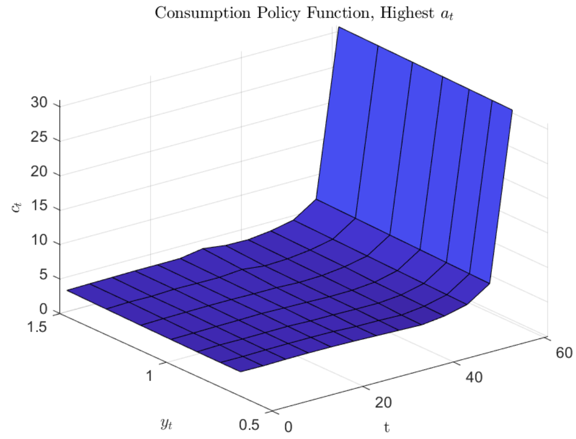
1.3.2. Policy Functions

Consumption Policy Functions:

[1]Directly plugging these values into your model can bias the scale of income and consumption, depending on the absolute magnitude. This leads to inconsistencies when you simulate households starting with zero or arbitrary initial wealth.



(a) Lowest a_t



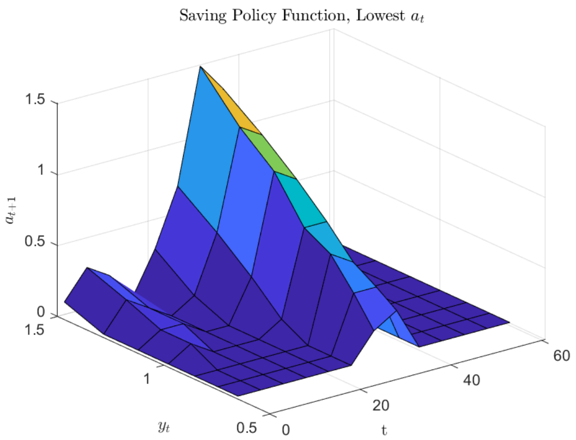
(b) Highest a_t

Figure 2: Consumption Policy Function for Lowest and Highest a_t

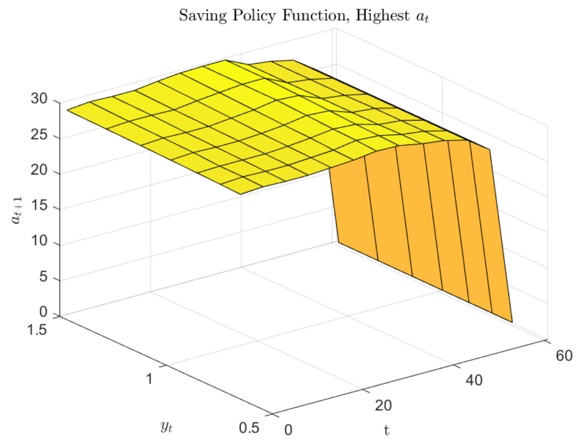
Figure 2a shows the consumption policy for households with the lowest asset holdings, in which consumption tends to be lower as the household has less wealth to draw from. The plot shows a relatively smooth increase in consumption as income increases. The plot shows a relatively smooth increase in consumption as income increases. This reflects the typical behavior of households with low wealth—they use income to smooth their consumption patterns and consume more when their income increases.

In contrast, households with higher asset holdings tend to have a more stable consumption profile (figure 2b). The consumption policy function for wealthier households shows a different behavior: they consume a higher portion of their income, especially as they age and accumulate wealth. There is a clear shift towards consuming more once a certain asset threshold is reached: These households already have significant assets, so they may reduce their savings in favor of higher consumption, especially during retirement.

Saving Policy Functions:



(a) Lowest a_t



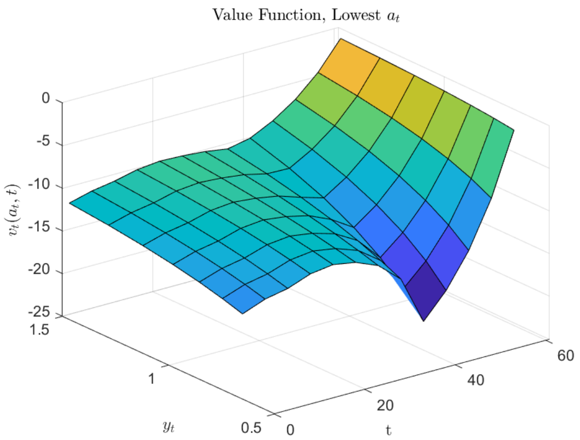
(b) Highest a_t

Figure 3: Saving Policy Function for Lowest and Highest a_t

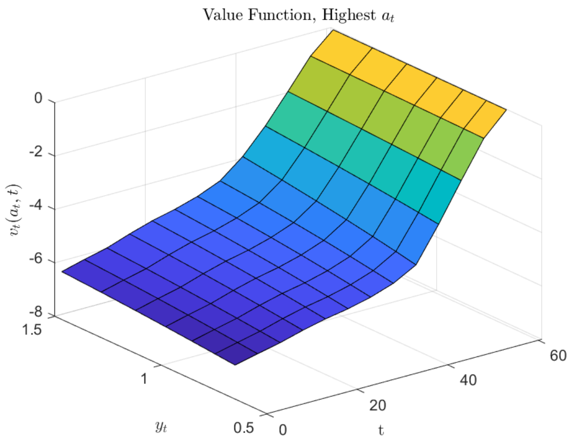
Figure 3a for the saving policy function with the lowest asset holdings shows that as income (y_t) increases, there is a relatively steep increase in savings. This suggests that households with lower initial assets are more likely to save a higher portion of their income, possibly to smooth consumption over time, especially when they have a higher income in later years. The policy function also suggests that households are more responsive to income in the earlier stages of their life cycle when their wealth is low.

For households with the highest asset holdings, the saving behavior changes (figure 3b). The saving function for high $a_t$is less sensitive to income changes, and the slope of the curve flattens. This indicates that wealthier households are less likely to save further, especially when income increases. Instead, they may focus on consumption during their retirement years (as shown in figure 2b).

Value Policy Functions:



(a) Lowest a_t



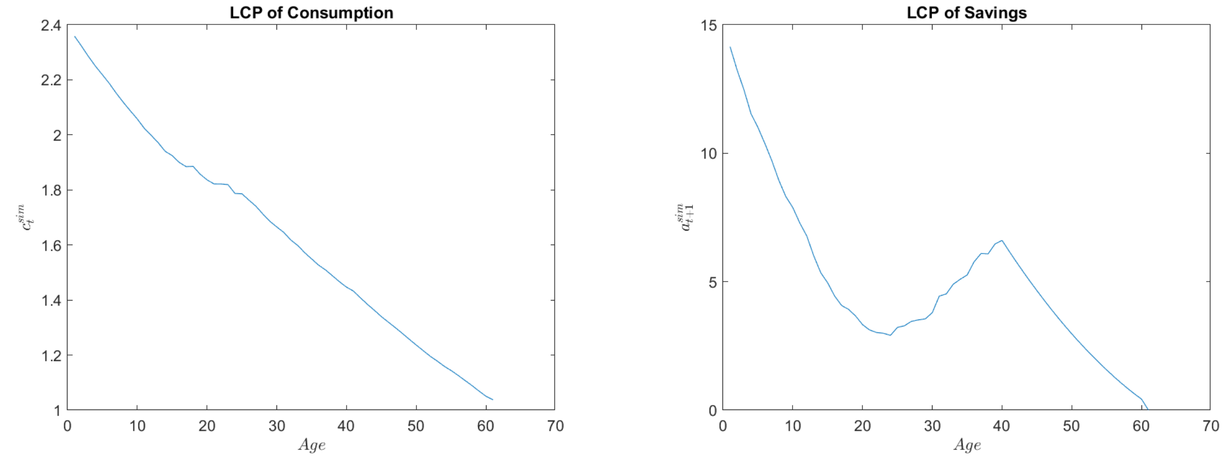
(b) Highest a_t

Figure 4: Value Function for Lowest and Highest a_t

The value function for households with the lowest asset holdings is lower, indicating that households with fewer assets are in a less favorable situation in terms of lifetime utility (figure 4a). The utility curve shows a positive relationship with wealth, but it becomes less sensitive to changes in assets as the household approaches retirement, reflecting diminishing returns to additional wealth.

The value function for wealthier households (figure 4b), the value function increases more slowly with wealth. These households have already accumulated enough assets and are less dependent on future savings or labor supply decisions. The plot shows that, for wealthier households, their utility is largely dependent on their consumption rather than the need to save.

1.3.3. Life-Cycle Profiles (Baseline: beta = 0.94, gamma = 2.00)



(a) Simulated Consumption Profile

(b) Simulated Wealth Profile

Figure 5: Life-Cycle Profiles of Simulated Consumption and Wealth

Wealth Profile (LCP of Savings): Figure 5b shows how savings (a_{t+1}) evolve over the life cycle. The plot indicates that individuals save more during their early years (ages 0 to 20), which is likely to ensure future consumption as they face lower incomes. This is followed by a period of declining wealth accumulation (ages 20 to 50), which might reflect increased consumption as they get older and have more responsibilities (such as family and housing).

After age 50, there is a significant drop in wealth, which may indicate a period of dissaving or withdrawal of accumulated wealth as the individual approaches retirement age.

Consumption Profile (LCP of Consumption): The consumption profile (c_t) shown in figure 5a declines steadily over the life cycle. Early in life, individuals consume more, as their wealth is often low, and they rely heavily on income.

As people age, their consumption steadily declines. This could be due to the increasing ability to save and accumulate wealth in the earlier years, alongside a possible decrease in income as individuals approach retirement age.

The curve is typically smooth, reflecting the consumption-smoothing behavior expected in life-cycle models, where people try to maintain a relatively stable level of consumption despite fluctuations in income.

The decrease in consumption results in the reduction in utility profile.

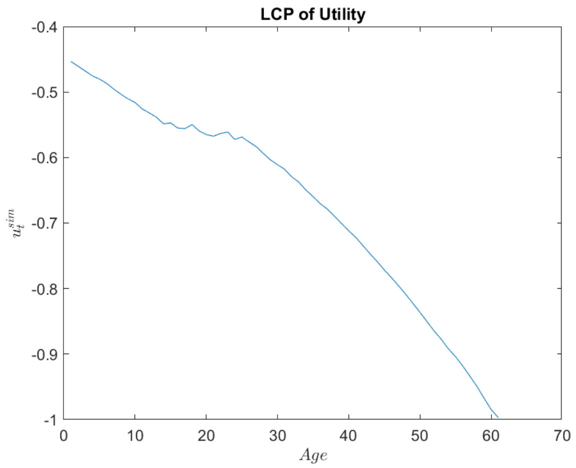


Figure 6: Life-Cycle Profile of Simulated Utility

As shown in figure 6, the decline in life cycle utility reflect the fact that individuals consume less as they age, and therefore experience diminishing satisfaction (utility) from consumption.

1.4. Simulated Life Cycle Profiles for Different Values of beta and gamma

1.4.1. Life Cycle Consumption and Wealth Profiles for Different beta values (gamma = 2.00)

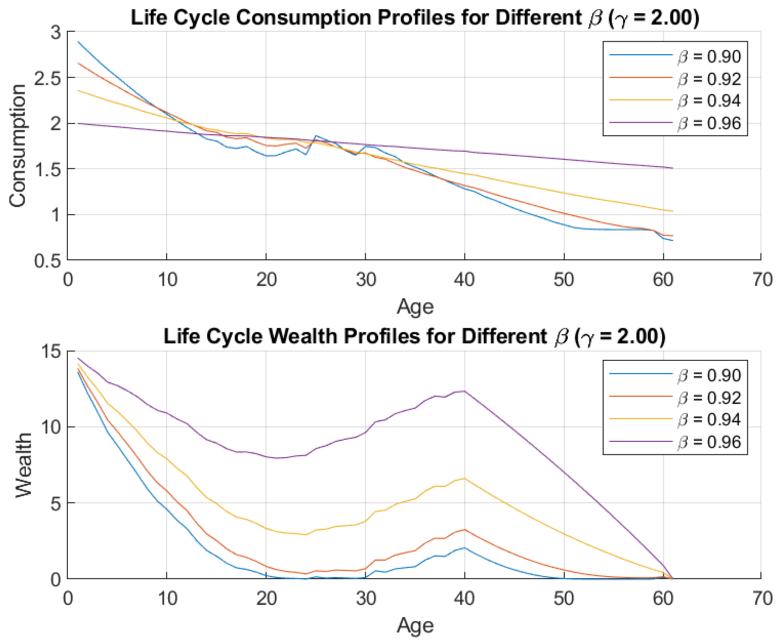


Figure 7: Life cycle profiles for consumption (*upper*) and wealth (*lower*) for different values of β.

Looking at the multiple lines in the upper plot of figure 7, households with a higher discount factor (e.g., $\beta = 0.96$) tend to smooth their consumption more evenly across their life, with a relatively steady decrease over time. Since individuals are more patient, i.e., value future utility more, these households tend to have more wealth in the later years due to their preference for saving. This associated with more savings over time (shown in the lower plot of figure 7).

While lower discount factor (e.g., $\beta = 0.90$) place less value on the future, leading to faster consumption in earlier periods. When they are less patient, households save less and consume more during their working years, resulting in lower wealth later in life.

1.4.2. Life Cycle Consumption and Wealth Profiles for Different gamma values (beta = 0.96)

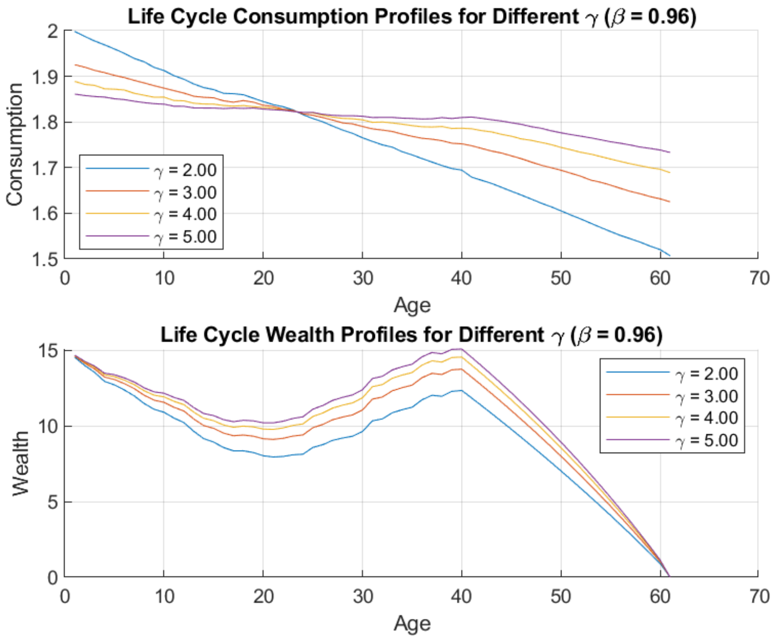


Figure 8: Life cycle profiles for consumption (*upper*) and wealth (*lower*) for different values of γ.

Figure 8 shows how consumption (upper plot) and wealth (lower plot) changes across the life cycle for households with different values of γ. As γ increases, higher values of risk aversion (e.g., $\gamma = 5.00$) lead to more cautious consumption behavior. Such high preference for saving results in higher wealth accumulation during their working years, as they prioritize stability over consumption.

Lower γ (e.g., $\gamma = 2.00$), in contrast, means that households have lower risk aversion. This leads to more willingness to decrease consumption more quickly in the future. This also associates with mpre gradual wealth accumulation as they are less concerned about the risks associated with future consumption.

1.4.3. Heatmap of Average Simulated Wealth for Different beta and gamma

Figure 9 aggregates the effects of intertemporal preferences (through β and γ) affect average households' wealth over their life cycle.

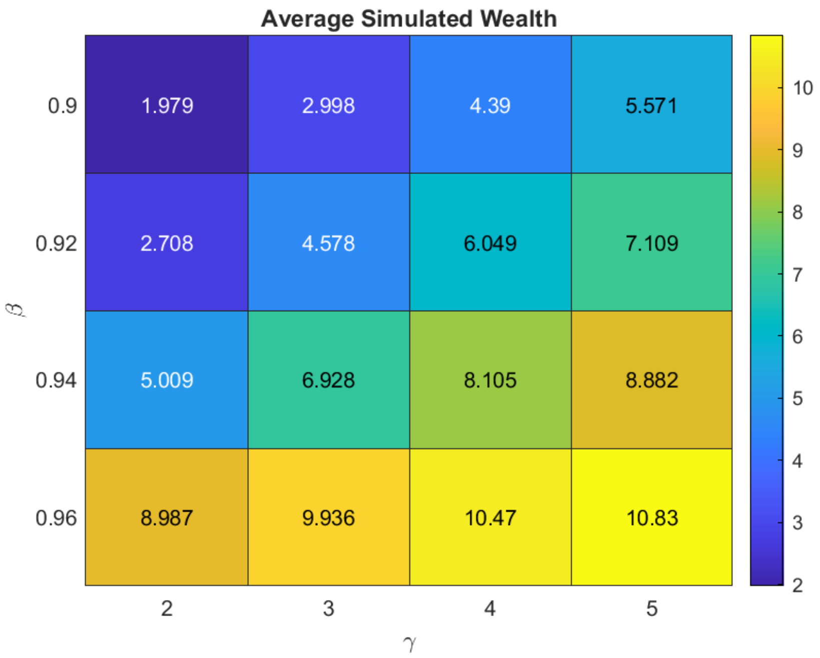


Figure 9: Heatmap of Average Simulated Wealth for Different β and γ

A general observation is that as β increases (from 0.90 to 0.96), average wealth increases for any given value of γ since they become more patient and value future savings. Increasing γ leads to more cautious saving and higher average wealth, as households with higher risk aversion tend to save more.

1.4.4. Descriptive Statistics

We present the descriptive statistic features for consumption (table 6) and wealth (table 7).

1. **Descriptive Statistics for Consumption:**

Parameters	Values	Mean	StdDev	Min	Max	Median
Beta	0.90	1.5507	0.56187	0.71608	2.8934	1.6456
	0.92	1.5719	0.5104	0.76829	2.6579	1.6273
	0.94	1.6389	0.37303	1.0378	2.3586	1.6458
	0.96	1.7548	0.1399	1.5069	1.9982	1.7574
Gamma	2.00	1.7548	0.1399	1.5069	1.9982	1.7574
	3.00	1.7825	0.084066	1.625	1.9255	1.7849
	4.00	1.7981	0.052572	1.6888	1.8888	1.7999
	5.00	1.8085	0.032166	1.7331	1.861	1.8108

Table 6: Descriptive Statistics for Consumption for Different Betas and Gammas

As β increases (from 0.90 to 0.96), the mean consumption increases from 1.5507 to 1.7548. The standard deviation decreases as β increases, moving from 0.56187 to 0.1399. This suggests that individuals with higher β values have more stable consumption patterns, as they are more focused on long-term utility and are less influenced by short-term fluctuations.

As γ increases from 2.00 to 5.00, the mean consumption gradually rises from 1.7548 to 1.8085, while the standard deviation decreases from 0.1399 to 0.032166, indicating that individuals with higher γ values (higher risk aversion) tend to consume more consistently and with less variability.

2. **Descriptive Statistics for Wealth:**

Parameters	Values	Mean	StdDev	Min	Max	Median
Beta	0.90	1.9793	3.2686	0	13.598	0.6089
	0.92	2.7082	3.3696	0	13.841	1.43
	0.94	5.0091	3.1461	0	14.149	4.4398
	0.96	8.9867	3.239	0	14.52	9.2306
Gamma	2.00	1.7548	0.1399	1.5069	1.9982	1.7574
	3.00	1.7825	0.0841	1.625	1.9255	1.7849
	4.00	1.7981	0.0526	1.6888	1.8888	1.7999
	5.00	1.8085	0.0322	1.7331	1.861	1.8108

Table 7: Descriptive Statistics for Wealth for Different Betas and Gammas

For wealth, as β increases from 0.90 to 0.96, the mean wealth increases significantly, from 1.9793 to 8.9867, indicating that individuals with higher β values accumulate more wealth over time. The standard deviation of wealth is relatively high across all β values, suggesting a wide variation in wealth accumulation, especially for higher β values. Especially, the minimum wealth remains 0 for all β values, indicating that there is the possibility of no wealth accumulation for some households.

The mean wealth increases from 1.7548 at $\gamma = 2.00$ to 1.8085 at $\gamma = 5.00$. The increase in wealth with increasing γ reflects the behavior where higher γ values tend to focus on more stable and long-term wealth accumulation rather than consuming today. Individuals with higher γ values likely save more over their life cycle to ensure they can smooth out their consumption, which increases wealth.

Overall, the model shows that there is an inverse relationship between consumption and wealth. In the early years, households focus on saving to build wealth, with high sensitivity to income. Utility increases as income and wealth grow.

In mid-life, consumption stabilizes, becoming more reliant on accumulated wealth, and the saving rate slows. The value function grows more slowly due to diminishing returns on wealth accumulation. After retirement, households' dissaving increases as they spend accumulated wealth, and consumption is mainly funded by savings and pensions, causing the value function to flatten.

1.5. Extension of the Model: Introducing Working Hours

1.5.1. Determinants of Consumption Among Vietnamese Households

Several key factors influence household consumption in Vietnam, as they do in many other East Asian economies. Among these factors, income and wealth stand out as primary determinants, aligning with classical economic theory (Keynes, 1937; Duesenberry, 1949; Friedman, 2018; Arapova, 2018). Income is the most direct and obvious determinant of consumption. Higher incomes generally lead to higher consumption levels, as households can afford more goods and services.

Additionally, there are other socioeconomic factors such as:

- **Fiscal and Monetary Policies**: Government policies, such as changes in taxation, social security, and fiscal stimulus packages, have an immediate impact on disposable income and, consequently, on consumption. For instance, Vietnam's fiscal policies aimed at raising the minimum wage or providing subsidies for rural households can stimulate consumption by increasing disposable income. Vietnam's monetary policy also plays a role in influencing interest rates, which affect both consumption and savings decisions.
- **Inflation and price levels** (e.g., food prices, housing costs) have a substantial effect on the purchasing power of households. In Vietnam, inflation, especially in food and housing sectors, affects the real income of households and can constrain their consumption ability.

- **Cultural Factors**: While not directly quantifiable for inclusion in the model, cultural preferences in Vietnam, such as a strong emphasis on saving and providing for future generations, influence consumption patterns. These cultural factors may lead to lower levels of consumption among certain segments of the population, particularly older generations, who prioritize savings over immediate consumption.

- Working hours affect income directly. If we assume a basic wage model where income is proportional to the number of hours worked.

1.5.2. Adding a Determinant to the Model

In this extension of the model, we introduce working hours as a new variable to capture the relationship between labor supply and household consumption. Traditionally, models of consumption have focused on income and wealth as primary determinants of consumption. However, working hours — representing the time household members allocate to income-generating activities — can significantly affect consumption and wealth accumulation [2]. As we extend the model to include working hours, this variable becomes crucial in Vietnam, where labor force participation rates and the type of employment (formal vs. informal) vary significantly across urban and rural areas.

Adding working hours into the model, we aim to explore how variations in labor supply impact consumption decisions. By intuition, households with longer working hours may earn higher incomes but may experience diminishing returns in terms of consumption satisfaction due to limited time for other activities. On the other hand, households with fewer working hours might enjoy more leisure time but may have lower consumption due to reduced income.

We use the working hours from `muc4a.csv` of the VHLSS dataset, which includes the following variables:

Muc4a. Earnings from salary/wages of main and secondary jobs

[2] ***Anecdotal evidence for excluded determinants***: While factors like education, family structure, and external shocks are important, they are challenging to model effectively within this framework due to data limitations. Incorporating education as a variable would require additional demographic data beyond what the current model focuses on. Similarly, cultural factors, though significant, are difficult to quantify and would complicate the model.

SECTION 4: INCOMES (*continue*)

PART 4A. EMPLOYMENT AND SALARIES AND WAGES

These questions concern all household members aged 6 or more.

VAR	QUESTIONS	#OBS.	MEASUREMENT	FILE NAME
`tinh`	Province/City	35,155	Code	Muc4A
`huyen`	District/Town	35,155	Code	Muc4A
`xa`	Commune/Ward/Township	35,155	Code	Muc4A
`diaban`	Enumeration area	35,155	Code	Muc4A
`hoso`	Household code	35,155	Code	Muc4A
`matv`	Member code	38,253	Code	Muc4A
`m4ac6`	Working months annually (main job)	35,155	Continuous	Muc4A
`m4ac7`	Average days worked per month (main job)	35,155	Continuous	Muc4A
`m4ac8`	Average hours worked per day (main job)	35,155	Continuous	Muc4A
`m4ac11`	Cash received from main job (1000 VND)	35,155	Continuous	Muc4A
`m4ac12f`	Other salary/income from main job (1000 VND)	35,155	Continuous	Muc4A
`m4ac16`	Working months annually (secondary job)	35,155	Continuous	Muc4A
`m4ac17`	Average days worked per month (secondary job)	35,155	Continuous	Muc4A
`m4ac18`	Average hours worked per day (secondary job)	35,155	Continuous	Muc4A
`m4ac21`	Cash received from secondary job (1000 VND)	35,155	Continuous	Muc4A
`m4ac22f`	Other salary/income from secondary job (1000 VND)	35,155	Continuous	Muc4A
`m4ac25`	Additional salary/income (1000 VND)	35,155	Continuous	Muc4A

Table 8: Household Members Income Data - `muc4a.csv`

Some initial observation of real data

We assume that as households work over time, they will earn some income [3]. The correlation between the working hours and consumption over time is presented through the scatterplot as follows:

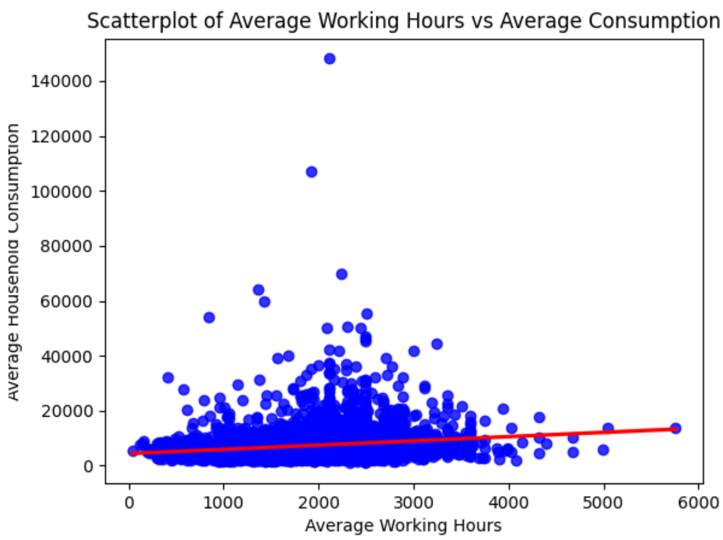


Figure 10: Scatter plot of average consumption vs average working hours

Figure 10 shows that ther is a positive relationship between average working hours and average consumption. As average working hours increase, average consumption tends to increase, albeit in a very subtle manner, as evidenced by the large scatter of points and a relatively shallow upward slope.

The OLS regression results show the relationship between average working hours (`avg_work_hours`)

[3] In the VHLSS dataset, there are households who works but do not earn any income at all. We remove all data points that do not follow the assumptions to consistent with the analysis

and average household consumption (HH_consumption_avr).

Dep. Variable:	HH_consumption_avr	**R-squared:**	0.026
Model:	OLS	**Adj. R-squared:**	0.026
Method:	Least Squares	**F-statistic:**	112.4
Date:	Fri, 18 Apr 2025	**Prob (F-statistic):**	6.14e-26
Time:	01:36:43	**Log-Likelihood:**	-42824.
No. Observations:	4230	**AIC:**	8.565e+04
Df Residuals:	4228	**BIC:**	8.567e+04
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	4413.9235	305.051	14.469	0.000	3815.862	5011.985
avg_work_hours	1.5423	0.145	10.602	0.000	1.257	1.827

Omnibus:	5358.429	**Durbin-Watson:**	1.480
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1798916.511
Skew:	6.616	**Prob(JB):**	0.00
Kurtosis:	103.157	**Cond. No.**	6.89e+03

Table 9: OLS Regression Results

We interpret the OLS regression table 9 as follows:

1. **R-squared:** The R-squared value is 0.026, which means that the model explains only 2.6% of the variation in average household consumption based on working hours. This indicates that working hours are a weak predictor of consumption in this model.

2. **Coefficient for avg_work_hours**: The coefficient is 1.5423, which means that for every additional hour worked on average, average consumption increases by 1.5423 units. While this is positive, the weak relationship (reflected in the low R-squared) suggests that working hours alone are not a strong determinant of consumption.

3. The **p-value** for avg_work_hours is 0.000, which is highly significant (well below the usual significance level of 0.05). This suggests that there is a statistically significant relationship between working hours and consumption, even though the strength of the relationship is weak (as indicated by the low R-squared).

Thus, by observing the correlation of the real data from the VHLSS, we conclude that as working hours increase, consumption increases as well. The effect of working hours, though modest, are statistically significant in predicting consumption. Next, we construct the extension in our baseline quantitative household modeling above by adding working hours.

1.5.3. Stochastic Life Cycle Model with Endogenous Labor

The representative consumer is born in period 0 and lives up to $T-1$. Her working life is from period 0 to $t_r - 1$, where she receives exogenous labor income, y_t. From period t_r onward, she is retired and receives a pension equal to some fraction, $0 < \kappa < 1$, of her labor income in the last period of her working life. She maximizes lifetime utility by choosing consumption and savings, subject to a dynamic budget constraint in each period. Her optimization problem is:

$$
\begin{aligned}
\max_{\{c_t\}_{t=1}^{T-1},\{n_t\}_{t=0}^{T-1},\{a_{t+1}\}_{t=0}^{T-1}} U = \sum_{t=1}^{T-1} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \gamma \frac{(1-n_t)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right], \\
\text{s.t. } a_{t+1} &= (1+r)(a_t + y_t - c_t), \\
y_t &= \begin{cases} G_t n_t e^{(\rho \log y_{t-1} + e_t)} & \text{if } t < t_r \\ \kappa y_{t_r-1} & \text{if } t \geqslant t_r \end{cases} \\
a_t &\geqslant 0, \\
a_0 &\geqslant 0 \text{ given} \\
a_T &= 0 \\
c_t &> 0 \\
n_t + l_t &= 1
\end{aligned} \tag{1}
$$

for all $t = 0, 1, \ldots, T-1$ and where $|\rho| < 1, G_t$ is age-specific average of income and $\varepsilon_t \sim \mathcal{N}\left(0, \sigma^2\right)$. The consumer has an initial wealth endowment, $a_0 \geqslant 0$, and the real interest rate, r, is fixed. Given an endowment of one unit of time, part of it can go to labor n_t and the rest will go to leisure l_t. Consumers derive utility from leisure but must supply labor to produce the final good.

1.5.4. Recursive Formulation of the Optimization Problem

The recursive formulation is

$$
\begin{aligned}
V_t(a_t, n_t, y_t) = \max_{c_t, 1-n_t, a_{t+1}} \frac{c_t^{1-\sigma}}{1-\sigma} + \gamma \frac{(1-n_t)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta V_{t+1}(a_{t+1}, y_{t+1}), \\
\text{s.t. } a_{t+1} &= (1+r)(a_t + y_t - c_t), \\
y_t &= \begin{cases} G_t n_t e^{(\rho \log y_{t-1} + e_t)} & \text{if } t < t_r \\ \kappa y_{t_r-1} & \text{if } t \geqslant t_r \end{cases} \\
a_t &\geqslant 0, \\
a_0 &\geqslant 0 \text{ given} \\
a_T &= 0 \\
c_t &> 0 \\
n_t + l_t &= 1
\end{aligned} \tag{2}
$$

The state space consists only of a_t while the choice space consists of c_t and a_{t+1}. Substituting the budget constraint into the utility function gives

$$
\begin{aligned}
V_t(a_t, y_t, n_t) = \max_{a_{t+1}, 1-n_{t+1}} \frac{\left[a_t + y_t - \frac{a_{t+1}}{1+r}\right]^{1-\sigma}}{1-\sigma} + \gamma \frac{(1-n_t)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta V_{t+1}(a_{t+1}, y_{t+1}), \\
\text{s.t. } a_{t+1} &= (1+r)(a_t + y_t - c_t), \\
y_t &= \begin{cases} G_t n_t e^{(\rho \log y_{t-1} + e_t)} & \text{if } t < t_r \\ \kappa y_{t_r-1} & \text{if } t \geqslant t_r \end{cases} \\
a_t &\geqslant 0, \\
a_0 &\geqslant 0 \text{ given} \\
a_T &= 0 \\
c_t &> 0 \\
n_t + l_t &= 1
\end{aligned} \tag{3}
$$

Note that the allocation of labor and leisure is a static decision. Adopting from (5.23) in Adda and Cooper (2003), the intratemporal is

$$c_t^{-\sigma}\left[G_t e^{(\rho \log y_{t-1}+e_t)}\right] = -\gamma(1-n_t)^{\frac{1}{\nu}} \tag{4}$$

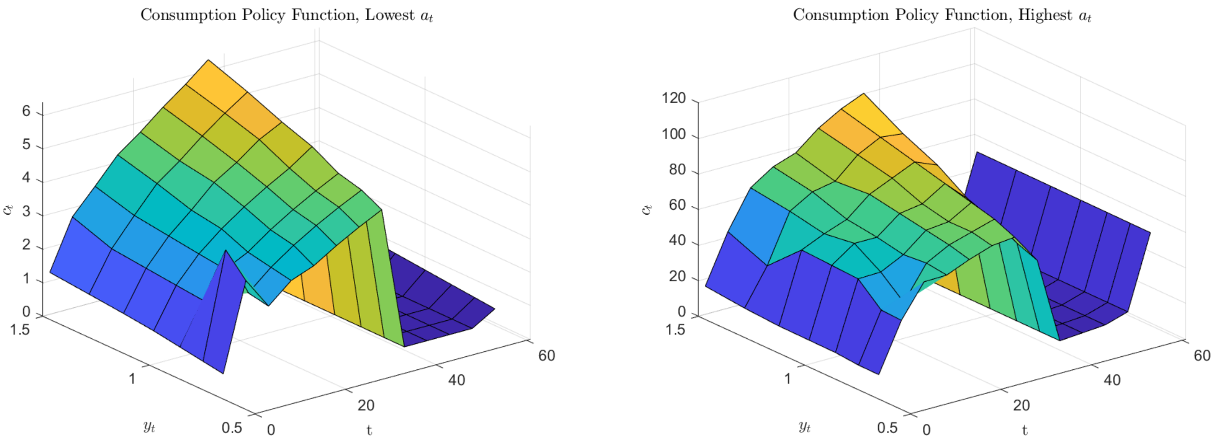
We have pre-determined grids for a_t, and we use the same grid for a_{t+1} as a_t. Given these and the equation above, we can solve n_t by using a numerical solver to minimize

$$f(n_t) = c_t^{-\sigma}\left[G_t e^{(\rho \log y_{t-1}+e_t)}\right] + \gamma(1-n_t)^{\frac{1}{\nu}} \tag{5}$$

and obtain optimal labor supply given any combination of the state variables and any potential choice of a_{t+1} associated with these. Crucially, we can do this outside of the value function iteration algorithm because the decision is static. Once we have the optimal n_t, we can treat this as a parameter for each combination of a_t, a_{t+1} and y_t.

1.5.5. Policy Functions

Consumption Policy Functions:



(a) Lowest a_t

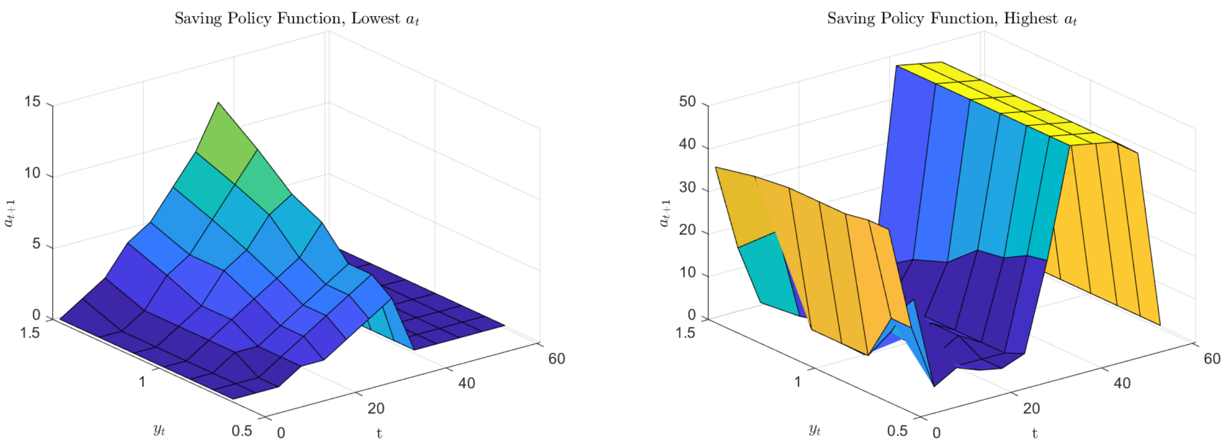
(b) Highest a_t

Figure 11: Consumption Policy Function for Lowest and Highest a_t

Figure 11a shows the consumption policy for households with the lowest asset holdings. At the lowest asset level, the consumption policy function shows a clear pattern consistent with the life-cycle model under borrowing constraints. Early in life, consumption is relatively low as households have little wealth and depend primarily on labor income. As individuals age and their income grows, consumption increases steadily, reflecting the rise in available resources and the desire to smooth consumption over the life cycle. However, as retirement approaches, the growth of consumption slows down and eventually declines after retirement. This drop is explained by the fall in income when individuals switch from labor income to a reduced pension. Limited savings accumulated during the working years mean that consumption must adjust downward to match lower post-retirement income. Throughout, the borrowing constraint prevents young households from fully smoothing consumption early in life, forcing them to consume less when income is low.

For wealthier households (figure 11b), the consumption policy function displays a markedly different trajectory. Households with substantial initial wealth consume at significantly higher levels across all ages, even in early life. Their ability to sustain high consumption reflects a relaxation of liquidity constraints: they are not limited by current labor income and can rely on their large asset holdings. Consumption rises sharply during the prime working years and peaks around the retirement age. After retirement, although labor income falls, wealthy households are able to maintain high consumption levels by drawing down their accumulated assets. This results in a much smoother and flatter consumption profile post-retirement compared to households with low assets. Overall, the consumption policy function at high asset levels illustrates the advantage of wealth in achieving better consumption smoothing throughout the life cycle.

Saving Policy Functions:



(a) Lowest a_t

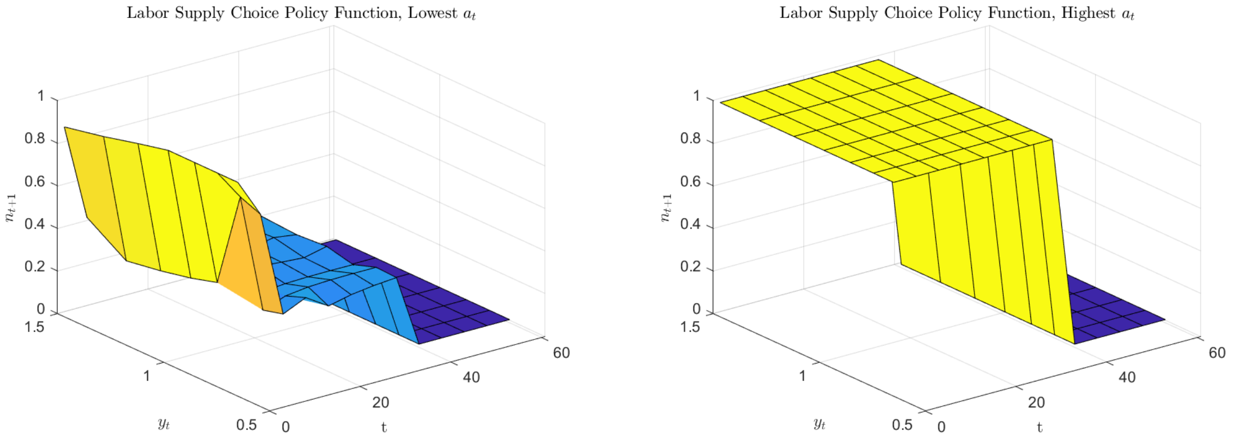
(b) Highest a_t

Figure 12: Saving Policy Function for Lowest and Highest a_t

Low-asset households save a large portion of their income, especially when their wealth is limited (figure 12a). Early in life, savings are minimal because young individuals must allocate most of their limited resources to current consumption needs. As they progress through their working years and experience income growth, these households begin to save more, building precautionary savings to prepare for future income risks and eventual retirement. However, the borrowing constraint continues to bind in many periods, limiting their ability to fully smooth consumption. Savings reach a moderate peak just before retirement, reflecting the need to compensate for the anticipated decline in income during the retirement phase. After retirement, savings rapidly deplete as individuals must rely on accumulated wealth to finance consumption in the absence of labor income, leading to a dissaving behavior where assets are gradually drawn down to zero by the end of life.

In contrast, at the highest asset level, the savings policy function exhibits a markedly different dynamic (figure 12b). Wealthy households are able to save aggressively from an early age, even while maintaining high consumption levels. They are less constrained by immediate consumption needs and are motivated by long-term wealth accumulation and intertemporal smoothing. Savings tend to rise sharply during the prime working years, peaking at or near retirement. Notably, these households do not face the same abrupt drawdown of savings after retirement; instead, they decumulate assets more gradually. Their substantial asset holdings allow for continued high consumption without the need for rapid dissaving.

Labor Supply Choice Policy Functions:



(a) Lowest a_t

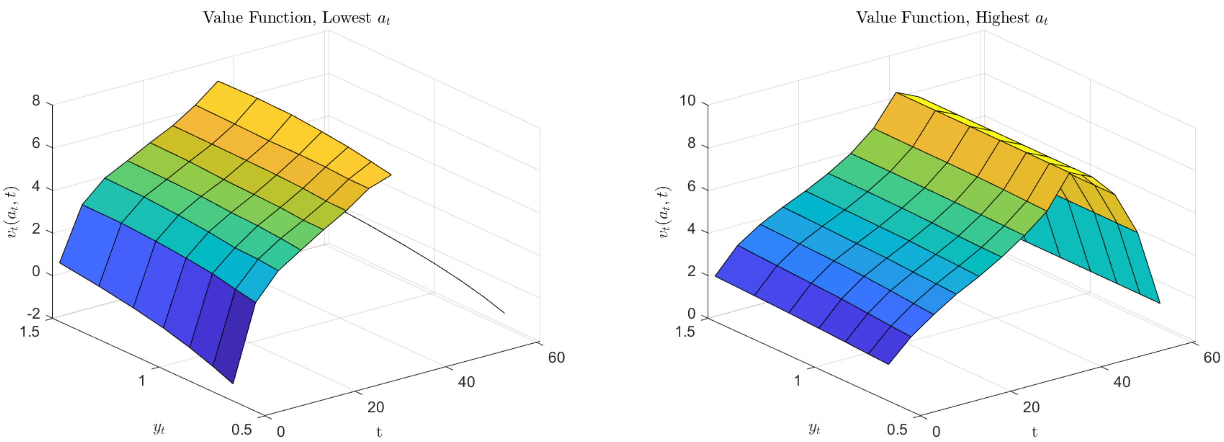
(b) Highest a_t

Figure 13: Labor Supply Choice Policy Function for Lowest and Highest a_t

The labor supply choice policy for households with the lowest asset holdings (figure 13a) shows that labor supply is high when income is low. This behavior is expected because households with few assets must rely heavily on labor income to finance consumption and savings. The need to accumulate precautionary savings and to avoid falling into borrowing constraints leads to a strong incentive to work more when young. Labor supply generally remains high throughout the working life but gradually declines as individuals approach retirement age. The decline reflects both diminishing returns to working additional hours and the gradual accumulation of enough assets to partially self-insure against income shocks. After retirement age, labor supply drops sharply to zero, as individuals leave the labor force and rely entirely on pension income and previous savings.

For wealthier households (figure 13b), the labor supply choice policy function presents a different pattern. Wealthier households, having accumulated substantial assets, can afford to supply less labor throughout their lifetime. Early in life, they still engage in some labor supply but to a much lesser extent compared to low-asset households. As their wealth grows further, these households reduce their labor supply earlier and more sharply. Well before retirement age, they can exit the labor force or significantly scale back their working hours, relying on accumulated savings and investment returns to finance consumption. After reaching retirement, labor supply fully drops to zero, but unlike low-wealth individuals, high-wealth households make this transition earlier and more smoothly, reflecting a greater degree of financial security and flexibility in their labor market participation decisions.

Value Policy Functions:



(a) Lowest a_t

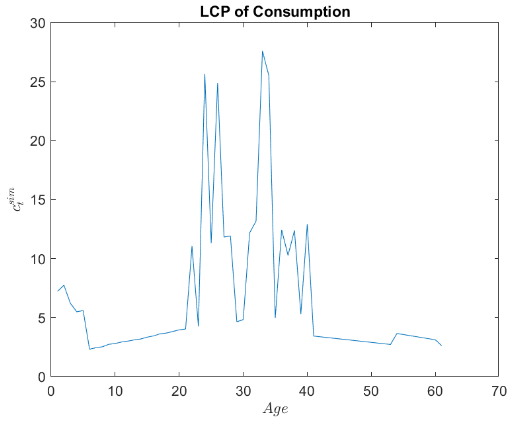
(b) Highest a_t

Figure 14: Value Function for Lowest and Highest a_t

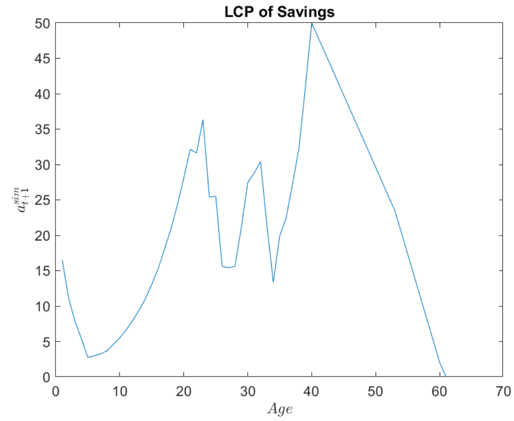
For households with the lowest asset holdings (figure 14a), the value function is relatively low, especially at younger ages. This reflects the limited resources and constrained choices faced by households with little initial wealth. Early in life, the value of being alive and making future choices is modest because the household is heavily dependent on labor income, vulnerable to income shocks, and constrained by the borrowing limit. However, as the individual progresses through the life cycle, the value function gradually rises. This increase occurs because the household builds up assets, improves its financial resilience, and gains more flexibility over consumption, savings, and labor supply. Nevertheless, the value function remains relatively sensitive to economic shocks compared to wealthier households, especially before retirement. After retirement, as labor income ceases, the value function tends to stabilize or decline slightly, reflecting reduced opportunities to adjust labor supply and the gradual depletion of assets to finance consumption.

The value function for wealthier households (figure 14b) is significantly higher across all ages. Households starting with high wealth experience greater lifetime utility because they are less constrained by borrowing limits and can better smooth consumption and leisure over time. Early in life, these households enjoy both higher consumption and more leisure, leading to a strong and steadily increasing value function. They are also more insulated against adverse income shocks, which raises the expected continuation value. The value function peaks around the retirement transition period when households have maximized their asset holdings and thus their financial security. After retirement, although there is no more labor income, the value function declines only slightly, as households draw down their substantial assets while maintaining a comfortable standard of living.

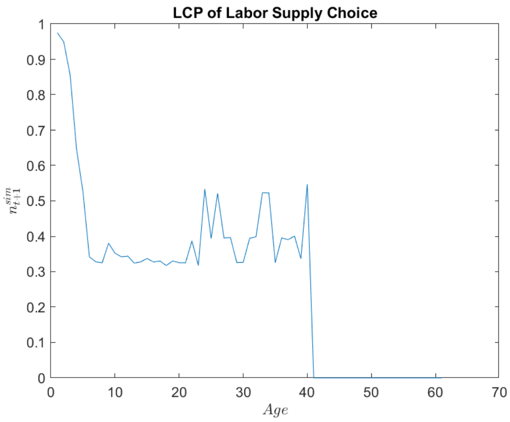
1.5.6. Baseline Life cycle profile



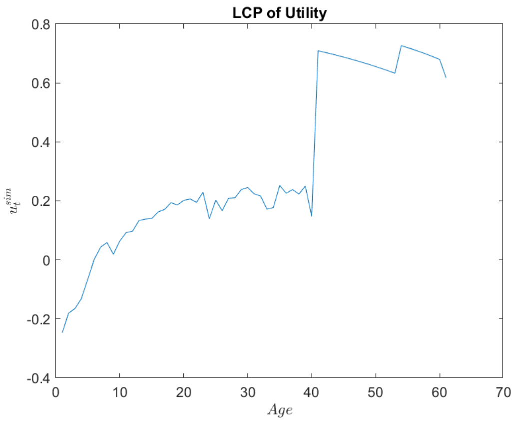
(a) LCP of Consumption



(b) LCP of Savings



(c) LCP of Labor Supply Choice



(d) LCP of Utility

Figure 15: Life Cycle Profiles for Consumption, Savings, Labor Supply, and Utility

We interpret the arbitrary life cycle functions:

Consumption Profile (LCP of Consumption): The consumption profile (c_t) shown in figure 15a shows consumption starting moderately in early years, then increasing substantially during early adulthood with notable peaks occurring in the 20s through 40s—particularly around ages 30-35 when consumption reaches its maximum levels. These peaks likely coincide with family formation, housing purchases, and career advancement periods when income is typically higher and financial obligations increase. After age 40, consumption demonstrates a marked decline, decreasing steadily as individuals approach and enter retirement years. This pattern reflects the classic life-cycle hypothesis, where consumption is smoothed relative to income but still exhibits variation based on changing needs and resources throughout different life stages. The sharp spikes in the consumption profile during working years also suggest periodic major expenditures or responses to income fluctuations, while the sustained lower consumption level during later years aligns with reduced income during retirement and potentially changing consumption needs in older age.

Wealth Profile (LCP of Savings): Figure 15c shows how savings (a_{t+1}) evolve over the life cycle. *Early in life*, individuals save very little or may even dissave slightly, as their incomes are low and immediate consumption needs are high relative to their resources. Young households prioritize current consumption over savings because they have limited access to borrowing and must finance their expenses largely from labor income. *As individuals age* and their productivity rises, income increases, allowing them to begin accumulating savings. During the prime working years, typically between early adulthood and just before retirement, savings grow significantly as individuals prepare for the eventual loss of labor income. This leads to a hump-shaped profile where savings peak just before retirement. After *retirement*, when individuals stop working and switch to a fixed pension income, the life cycle profile shows a steady dissaving phase. Households begin to run down their accumulated wealth to finance their consumption needs during retirement. This dissaving process continues until savings approach zero near the end of life, consistent with the terminal condition that assets must be exhausted at death. The profile illustrates the strong precautionary motive for saving during working years and the reliance on accumulated wealth to smooth consumption once labor income ceases.

Labor Supply Choice Profile (LCP of Labor Choice): Starting at nearly maximum participation (close to 1.0) in very early years, labor supply (figure 15c) quickly decreases to stabilize around 0.3-0.4 for most of the prime working years. This pattern reflects the typical transition from full-time commitment to the workforce toward a balanced allocation that accommodates other life priorities. Throughout the working period (approximately ages 5-40), the labor supply exhibits several modest fluctuations, potentially representing responses to changing family circumstances, education periods, or evolving career opportunities. The most striking feature occurs around age 40, where labor supply abruptly drops to zero, indicating complete retirement from the workforce. This sharp discontinuity suggests a discrete retirement decision rather than a gradual reduction in working hours. The complete absence of labor supply after this point demonstrates that in this model, individuals do not return to the labor market during retirement years, which aligns with traditional retirement patterns where pension income replaces labor earnings as the primary financial resource.

Utility Function Profile (LCP of Utility): Utility, representing the overall satisfaction or well-being an individual experiences (figure 15d). Early in life, utility is relatively low because individuals have low income and limited resources, forcing them to prioritize immediate consumption needs over savings and leisure. Consumption is constrained, and borrowing limitations prevent full smoothing of well-being across time. As individuals age and their incomes rise, utility steadily increases. Higher labor income allows for both greater consumption and savings, improving financial security and raising lifetime satisfaction. The increase in utility during the working years captures the benefits of income growth, asset accumulation, and greater flexibility in consumption and labor supply decisions. Around the retirement age, the life cycle profile often shows a noticeable jump or stabilization in utility. Although labor income falls, the transition into retirement brings an increase in leisure time, which positively contributes to utility. Individuals draw on accumulated savings to maintain consumption, smoothing out the potential drop in well-being. After retirement, utility tends to stabilize or decline slightly in old age, reflecting factors such as reduced consumption capacity, health deterioration, and the gradual depletion of savings. However, because individuals have planned their life cycle optimally, the decline in utility is typically mild

From the analyses above, we see that there is a clear inverse relationship between consumption and wealth during the life cycle. As wealth increases in the middle age, consumption grows. But, when wealth starts to decrease in retirement, consumption also declines. Labor supply is positively related to consumption in the early stages of life, as individuals work and earn income to finance their consumption. As labor supply decreases with age, wealth becomes the key factor to maintain consumption. Utility follows the wealth accumulation trajectory to some extent but is also influenced by consumption behavior. As individuals approach retirement, utility improves with consumption funded by wealth. However, as wealth depletes, utility decreases significantly, particularly as consumption starts to fall.

Particularly, individuals follow a typical life cycle pattern where they focus on earning and accumulating wealth in their early and middle years. As they approach retirement, they start consuming their savings, leading to a decline in wealth and consumption and ultimately lower utility. The transition into retirement is marked by a sharp decline in labor supply, followed by a gradual decrease in consumption and wealth until retirement funds are exhausted.

1.6. Simulated Life Cycle Profiles for Different Values of beta and gamma

1.6.1. Simulated Life Cycle Consumption and Wealth for Different Values of beta (gamma = 2.00)

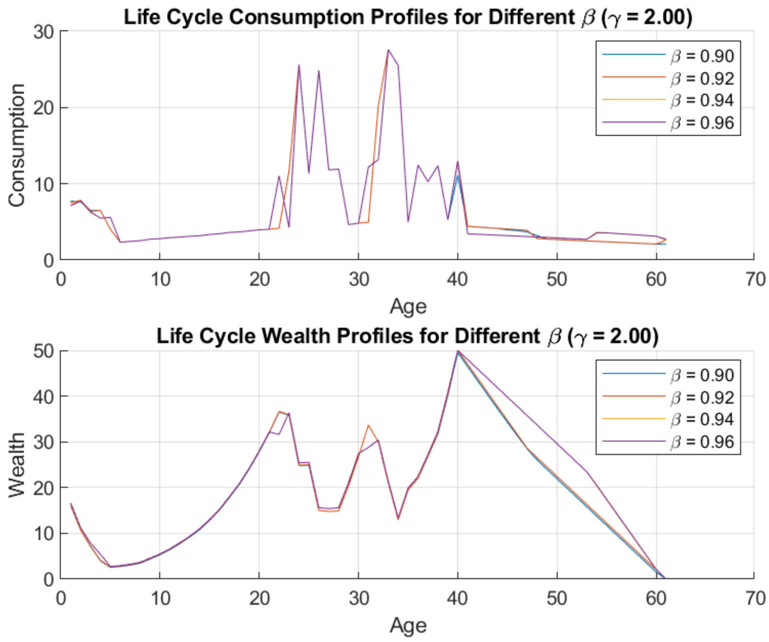


Figure 16: Life cycle profiles for consumption (*upper*) and wealth (*lower*) for different values of β.

Life Cycle Consumption Profiles for different β values: When γ is fixed at 2.00, higher β values (0.96) produce notably flatter consumption trajectories, reflecting greater patience and willingness to defer immediate gratification for future consumption. These more patient individuals maintain relatively stable consumption throughout their lives, with a gradual decline in later years. In contrast, lower β values (0.90) generate consumption paths that start significantly higher in early years but decline more steeply with age, indicating stronger preferences for present consumption over future consumption. The intermediate β values (0.92, 0.94) show consumption patterns that fall predictably between these extremes. This observation aligns with economic theory, where the discount factor represents time preference—higher β values indicate individuals who value future utility nearly as much as present utility, leading them to smooth consumption more effectively across their lifetime.

The relatively stable consumption paths of highly patient individuals (high β) reflect optimal intertemporal allocation decisions that prioritize maintaining consistent living standards throughout life, while the declining paths of impatient individuals (low β) demonstrate their preference for immediate consumption despite the future consequences of reduced spending capacity.

Life Cycle Wealth Profiles for Different β values: When γ is fixed at 2.00, individuals with higher β values (0.96) exhibit substantially greater wealth accumulation compared to their less patient counterparts. The most patient individuals ($\beta = 0.96$) accumulate peak wealth nearly seven times higher than those with the lowest patience ($\beta = 0.90$). All profiles display the characteristic hump shape predicted by life-cycle theory, with wealth rising during working years, peaking around age 40, and then declining during retirement as assets are drawn down. However, higher β values not only increase the maximum wealth level but also alter the shape of the accumulation path—more patient individuals build wealth more consistently and sustain higher asset levels throughout retirement. This reflects their greater willingness to defer consumption during working years to maintain financial resources later in life. The stark differences in wealth accumulation across β values highlight the critical role of patience in determining lifetime financial outcomes and retirement security, even when individuals face identical income patterns and risk attitudes.

In comparison to the baseline model ($\beta = 0.94$), we see that wealth tends to grow more steadily for individuals with a higher β. The wealth for those with lower β decreases more quickly, illustrating the tendency for more immediate consumption.

1.6.2. Simulated Life Cycle Consumption and Wealth for Different Values of gamma (beta = 0.96)

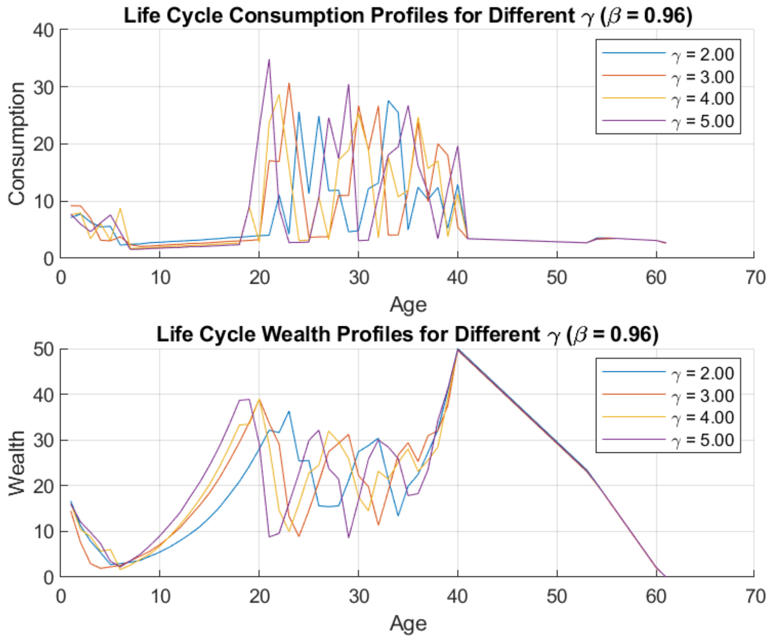


Figure 17: Life cycle profiles for consumption (*upper*) and wealth (*lower*) for different values of γ.

Life Cycle Consumption Profiles for Different γ values: With β fixed at 0.96, individuals with lower risk aversion ($\gamma = 2.00$) exhibit consumption paths that begin higher in early years but decline more steeply over time, reflecting less concern about future consumption variability. As γ increases to higher values (3.00, 4.00, and 5.00), consumption profiles become progressively flatter and more stable across the life cycle. This pattern demonstrates that more risk-averse individuals prioritize consumption smoothing to avoid potential future shortfalls. The highest risk aversion ($\gamma = 5.00$) produces the flattest consumption trajectory, with minimal decline even in later years, as these individuals are particularly motivated to maintain consistent consumption levels rather than risk significant reductions later in life. While all profiles show some consumption decline with age, the rate of decline is inversely related to risk aversion, confirming that precautionary motives strengthen as γ increases. These findings align with theoretical predictions that higher risk aversion leads to greater precautionary saving behavior, resulting in more effective consumption smoothing throughout the life cycle despite income fluctuations and retirement transitions.

The baseline model with $\gamma = 2.00$ shows more fluctuation in consumption, especially in the younger years, reflecting a moderate attitude towards risk. In contrast, higher γ values smooth consumption, as individuals grow increasingly more focused on ensuring future consumption, even at the expense of present consumption.

Life Cycle Consumption Profiles for Different γ values: With β fixed at 0.96, increasing risk aversion leads to greater precautionary savings and higher wealth accumulation. Individuals with higher γ values (4.00, 5.00) build larger wealth buffers that peak around age 40, with the most risk-averse reaching approximately 15 units of wealth compared to about 12 units for those with $\gamma = 2.00$. This pattern demonstrates the precautionary saving motive—more risk-averse individuals accumulate additional assets as insurance against potential future income shocks or consumption needs. Interestingly, while peak wealth levels differ significantly across γ values, the overall shape of the wealth profiles remains similar, with all showing accumulation during working years followed by decumulation in retirement. Additionally, wealth levels converge as individuals approach the end of life, indicating that regardless of risk preferences, assets are eventually depleted to fund retirement consumption.

The baseline model ($\gamma = 2.00$) demonstrates a balance between consumption and savings, where wealth accumulation happens progressively but with some consumption smoothing in mid-life. With higher γ values, wealth accumulation happens earlier and more aggressively, with wealth remaining high through the retirement years. This contrasts with the baseline model, where wealth decreases more rapidly towards retirement as individuals are less focused on saving for future uncertainties.

1.6.3. Heatmap of Average Simulated Wealth for Different beta and gamma

Figure 18 aggregates the effects of intertemporal preferences (through β and γ) affect average households' wealth over their life cycle.

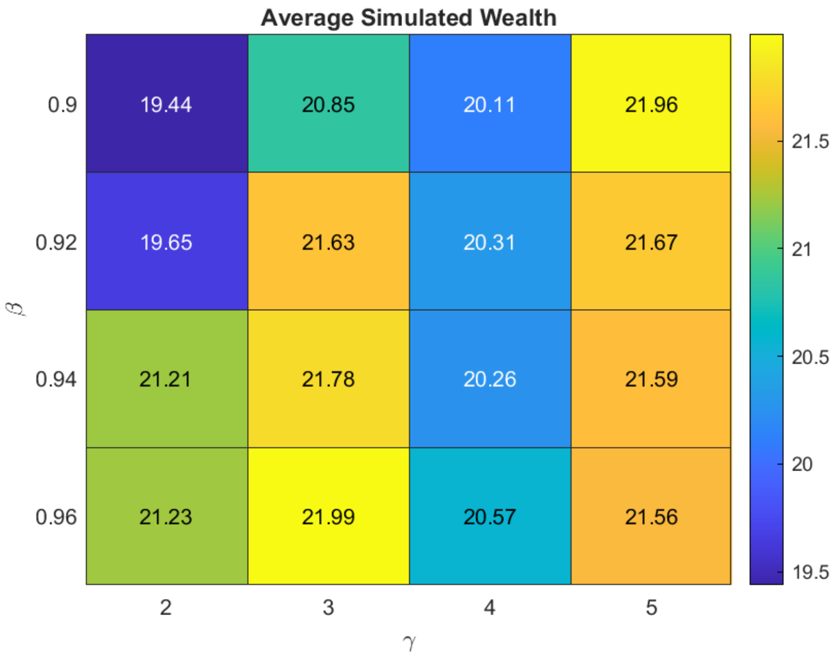


Figure 18: Heatmap of Average Simulated Wealth for Different β and γ

As β increases, the average wealth in the simulated economy increases for all levels of γ. This reflects that higher patience (higher β) leads to higher savings and wealth accumulation in the long run.

For γ, increasing values (i.e., greater risk aversion) generally correlate with higher average wealth. This suggests that individuals with higher risk aversion are more likely to save and accumulate wealth over time to ensure future stability.

The heat map clearly shows that the highest average wealth occurs when β is 0.96 and γ is 5.00. Implication from the progressive behavior is that individuals with a combination of high β (low time preference) and high γ (high risk aversion) are able to accumulate the most wealth. This indicates that a more patient and risk-averse individual will tend to save more, which aligns with conventional economic theory about savings and wealth accumulation.

1.7. Descriptive Statistics

We present the descriptive statistic features for consumption (table 10) and wealth (table 11).

1. **Descriptive Statistics for Consumption:**

Parameters	Values	Mean	StdDev	Min	Max	Median
Beta	0.90	6.4257	6.3094	2.0854	27.6	3.9578
	0.92	6.47	6.3357	2.1003	27.6	3.9826
	0.94	6.5065	6.1238	2.3258	27.598	3.5665
	0.96	6.5069	6.1231	2.3258	27.598	3.5566
Gamma	2.00	21.23	13.082	0	50	20.918
	3.00	21.992	10.545	0	47.245	21.837
	4.00	20.57	10.951	0	45.306	22.143
	5.00	21.557	9.7962	0	46.531	19.592

Table 10: Descriptive Statistics for Consumption for Different Betas and Gammas

As shown in Table 10, consumption patterns are relatively stable across varying discount factors β. A marginal increase in both mean and median consumption is observed as β increases from 0.90 to 0.96. This is consistent with theoretical expectations—households with higher patience (larger β) value future consumption more, leading to slightly smoother consumption paths. Nonetheless, the standard deviation remains notably high, indicating substantial dispersion in individual consumption levels, likely driven by income heterogeneity and life-cycle shocks.

Across different γ values, which capture risk aversion, we observe that mean consumption remains within a similar range, but the standard deviation decreases slightly as γ increases from 2.00 to 5.00. This suggests that higher risk aversion reduces extreme consumption behavior, possibly due to more conservative saving and labor supply responses. Interestingly, the median remains fairly stable, but the gap between mean and median narrows slightly at higher γ, implying a reduced right tail in consumption distribution.

These statistics indicate that while β predominantly affects the level of consumption, γ influences the dispersion and skewness. The comparison of consumption across different gamma values ($\gamma = 2.00$ to $\gamma = 5.00$) shows a similar trend where higher γ values, which reflect greater risk aversion, tend to increase both the mean and standard deviation of consumption, with more pronounced peaks and troughs in the consumption pattern. This implies that individuals with higher risk aversion consume more erratically, potentially in response to their perceived future needs.

2. **Descriptive Statistics for Wealth:**

Parameters	Values	Mean	StdDev	Min	Max	Median
Beta	0.90	19.438	12.568	0	49.49	17.857
	0.92	19.649	12.607	0	50	18.367
	0.94	21.214	13.093	0	50	20.918
	0.96	21.23	13.082	0	50	20.918
Gamma	2.00	6.5069	6.1231	2.3258	27.598	3.5566
	3.00	7.4258	7.2159	1.7947	26.375	3.1197
	4.00	8.1949	7.2614	1.6639	25.447	3.3465
	5.00	9.0939	8.8466	1.4545	35.827	4.483

Table 11: Descriptive Statistics for Wealth for Different Betas and Gammas

Table 11 shows a more pronounced effect of both β and γ on wealth accumulation. As β increases, households accumulate more wealth on average and the median wealth also rises, which is expected as more patient agents are inclined to defer consumption in favor of saving. At $\beta = 0.96$, households show the highest levels of mean and median wealth, with standard deviation peaking as well—indicating a wider spread in saving behavior among the population.

With increasing γ, we also observe higher mean and median wealth levels, suggesting that more risk-averse households opt to save more to hedge against future uncertainty. Notably, the maximum wealth and standard deviation also rise with γ, pointing to increased heterogeneity in the distribution—some households accumulate significantly more than others. This aligns with theoretical predictions where high risk aversion leads to precautionary saving behavior.

However, while higher β increases wealth across the board, higher γ introduces a broader dispersion. This indicates that while patience promotes uniform wealth accumulation, risk aversion interacts more subtly with income shocks and labor supply dynamics, amplifying inequality in asset holdings.

1.8. Sensitivity analysis

In this section, we examine how sensitive the model's outcomes are to changes in individual utility parameters, specifically those related to consumption smoothing and labor supply decisions. The key parameters we focus on are:

- γ: The coefficient of relative risk aversion, which determines the curvature of the utility function. Higher values of γ imply stronger aversion to fluctuations in consumption and a greater desire for consumption smoothing over time.
- ν: The Frisch elasticity of labor supply, which captures the responsiveness of labor effort to changes in wages or productivity shocks. Higher values of ν indicate that individuals are more willing to adjust their labor supply in response to economic conditions, making labor choices more elastic.

1.8.1. LCPs for different gamma (preference on leisure)

We simulate life-cycle profiles across three levels of risk aversion, $\gamma = 1.00, 3.00$ and 5.00, to understand its implications. The plots are shown in Figure 19:

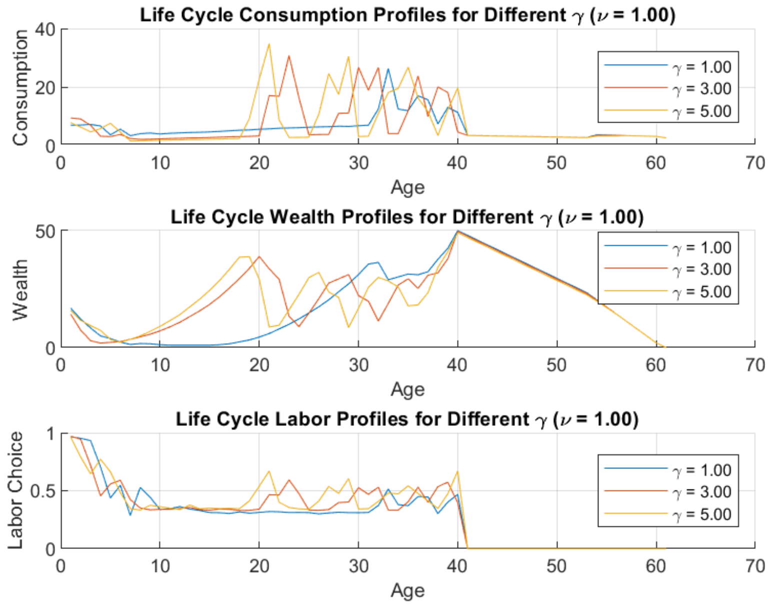


Figure 19: Life cycle profiles for consumption (*upper*), wealth (*middle*) and labor choice (*lower*) for different values of γ.

From the *top panel* (consumption), we observe that higher γ values lead to more volatile consumption paths during working years, with peaks occurring around middle age. This is somewhat counterintuitive since high γ is associated with greater aversion to consumption fluctuations. However, in our model, this volatility arises from the interplay between uncertain income, endogenous labor supply, and precautionary motives. Households with higher γ tend to delay consumption and build savings more aggressively to guard against future shocks, resulting in elevated midlife consumption once adequate buffers are secured. The *middle panel* shows the corresponding wealth profiles. Higher risk aversion generally increases the savings rate early in life, leading to faster wealth accumulation, but also more erratic patterns depending on labor and income realization.

Labor supply responses (*bottom panel*) also vary with γ. A higher value of γ induces households to work more early in life to insure against future consumption risk, but this effect diminishes with age. In particular, for $\gamma = 5.00$, labor participation is higher during youth and falls sharply as households approach retirement and accumulate sufficient assets. In contrast, with $\gamma = 1.00$, labor supply is relatively flatter across age, reflecting a lower precautionary motive and more willingness to smooth effort. An initially counterintuitive result emerges when we increase the coefficient of relative risk aversion γ, as observed in the descriptive statistice table 12.

Gamma	Mean	StdDev	Min	Max	Median
1.00	0.27067	0.24426	0	0.96479	0.31115
3.00	0.29362	0.24773	0	0.97065	0.33715
5.00	0.30472	0.25415	0	0.96325	0.34622

Table 12: Descriptive Statistics for Labor Choice for different values of γ

Typically, we expect that higher γ would lead individuals to smooth consumption more aggressively over time and potentially reduce labor effort to enjoy more leisure. However, in the simulated life-cycle labor profiles, we do not consistently observe a monotonic decline in labor supply as γ rises. In some cases, individuals with higher γ appear to work more since the marginal utility earned from an additional unit of labor is large enough that they do not need as much to maximize their lifetime value function.

This behavior stems from an important trade-off embedded in the model: the intertemporal substitution between leisure and labor. A high γ individual places more weight on maintaining stable consumption and enjoying leisure today. To preserve leisure in the current period, such an individual may choose to work more in the future to fund desired consumption, especially under borrowing constraints.

However, this dynamic trade-off is not directly visible in static graphs of labor supply over age. The plotted labor profiles show average effort across time but not the substitution decision across periods (i.e., leisure today vs. labor tomorrow). As a result, the underlying economic mechanism—where individuals with high γ prefer to defer labor effort—is obscured.

1.8.2. LCP for different nu (Frisch Elasticity)

we simulate outcomes under $\nu = 0.5, 1.5$ and 2.5, keeping the risk aversion parameter $\gamma = 2.00$ fixed.

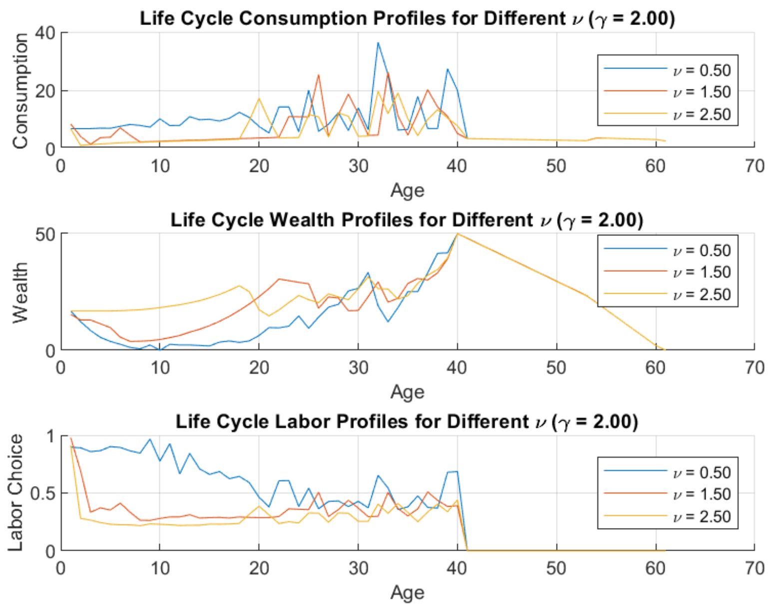


Figure 20: Life cycle profiles for consumption (*upper*), wealth (*middle*) and labor choice (*lower*) for different values of ν.

The top panel of Figure 20 shows that increasing ν results in more jagged and dispersed consumption profiles. At higher Frisch elasticity, households are more willing to vary labor supply in response to wage shocks, which translates into greater income variation and hence more volatile consumption. Although consumption still rises over the life cycle, the peaks are sharper and the timing more irregular at higher ν, suggesting a reduction in consumption smoothing.

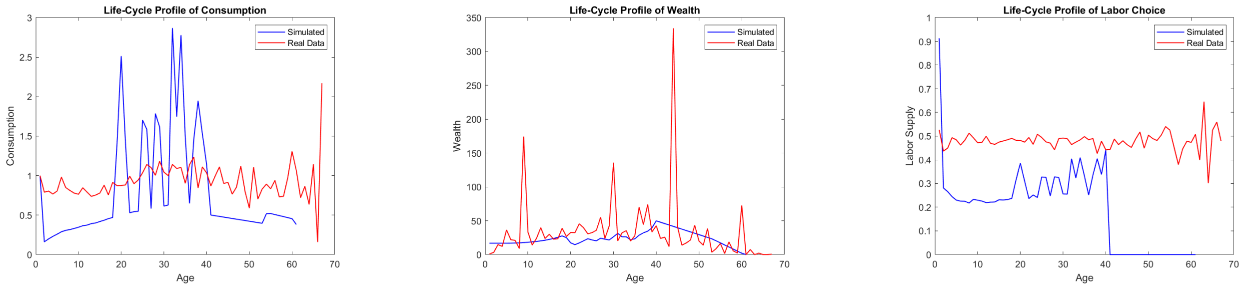
In terms of wealth accumulation (*middle panel*), we find that higher ν results in greater variation in asset holdings across the life cycle. Households with high labor elasticity accumulate more wealth in early and middle adulthood but also show more erratic drawdown behavior. These fluctuations mirror the underlying labor choices (*bottom panel*), which become substantially more volatile at higher ν.

For example, individuals with $\nu = 2.50$ sharply reduce work hours during low-productivity years and work more when productivity is high, leading to significant variation in both earnings and savings. In contrast, at $\nu = 0.50$, labor supply is smoother and more consistent, and both consumption and wealth paths exhibit less fluctuation.

1.9. Real vs. Simulated Data Comparison

We compare the model-generated (simulated) data with the real-world observations extracted from household survey data. The real dataset provides **average household consumption, household wealth, and labor supply** (as a share of total possible annual work hours: 260 days $\times$ 16 hours = 4,160 hours per year) **by age group**. These values were normalized and aggregated accordingly.

1.9.1. Plot the comparison



(a) LCP of Consumption (b) LCP of Wealth (c) LCP of Labor choice

Figure 21: Comparison of Actual vs. Simulated Macroeconomic Indicators

1.9.2. Descriptive Statistics: Real vs Actual data

Tables 13, 14, and 15 present a comparative analysis of the distribution and variability between real and simulated data, complementing the graphical representations in Figure 21.

Consumption. As shown in Figure 21a, both the real and simulated consumption profiles exhibit a relatively flat trend across most of the life cycle. However, the simulated consumption pattern is considerably more volatile, with large spikes and drops, especially during the working years. These fluctuations may arise from the stochastic income process in the model, where productivity shocks and borrowing constraints cause significant variation in consumption smoothing. The real data, by contrast, are smoother and more stable.

The descriptive statistics in Table 13 confirm this: although the mean consumption is broadly comparable (0.927 for real vs. 0.778 for simulated), the standard deviation is much higher in the simulated case (0.648 vs. 0.236), highlighting excessive volatility in the model relative to observed behavior.

Table 13: Descriptive Statistics: Consumption (by age group)

Scenario	Mean	StdDev	Min	Max
"Real Data"	0.92727	0.23613	0.1595	2.1702
"Simulated"	0.77815	0.64849	0.16172	2.8687

Wealth. Figure 21b shows the life-cycle profile of wealth. Both profiles show a familiar hump shape, where individuals accumulate wealth during working years and draw it down during retirement. However, real data show significantly more dispersion and higher peaks. For instance, large spikes in the real data (e.g., around age 40 and 60) may result from outliers or specific cohort behavior, whereas the simulated data cap wealth at a more constrained and symmetric peak. Table 14 shows that the real average wealth is about 33.18 with a high standard deviation of 46.94, while the model underpredicts the level (mean of 23.97) and variance (standard deviation of 10.37). This discrepancy suggests the model might be too conservative in predicting wealth accumulation, potentially due to limited heterogeneity or missing bequest motives.

Table 14: Descriptive Statistics: Wealth (by age group)

Scenario	Mean	StdDev	Min	Max
"Real Data"	33.177	46.943	0	334.33
"Simulated"	23.968	10.369	0	49.898

Labor Supply. The labor supply profiles in Figure 21c reveal stark differences. In the real data, labor effort is quite stable throughout the working life, hovering around 0.48 of the total available time, reflecting consistent work habits. In contrast, the simulated profile shows a declining trend, with sharp drops and eventual retirement (zero labor supply) by the terminal working age. This suggests that the model's assumptions regarding the utility cost of labor and its tradeoff with leisure may overly penalize work, especially as individuals age. Table 15 supports this interpretation: the real mean labor supply is approximately 0.479 with minimal fluctuation, whereas the simulated mean is much lower at 0.196 and more volatile (standard deviation of 0.172).

Table 15: Descriptive Statistics: Labor Choice (by age group)

Scenario	Mean	StdDev	Min	Max
"Real Data"	0.47912	0.041758	0.30096	0.64543
"Simulated"	0.19566	0.17167	0	0.91315

Conclusion. Overall, while the model broadly replicates the expected shapes of life-cycle behavior, it fails to capture the stability and smoother dynamics of real-world consumption, wealth, and labor patterns.

2. Part II: Modeling Firms in Vietnam

$$
\begin{aligned}
V\left(A_t, K_t, p_t\right) = & \max_{x_t} \Big\{ R\left(A_t, K_t, x_t\right) - w_t x_t \\
& - \frac{\gamma}{2} C\left(K_{t+1}, A_t, K_t\right) - p_t \left(K_{t+1} - (1-\delta) K_t\right) \\
& + \beta \mathbb{E}_{A_{t+1}, p_{t+1} | A_t, p_t} V\left(A_{t+1}, K_{t+1}, p_{t+1}\right) \Big\}, \\
\text{s.t. } \log A_{t+1} & = \rho^A \log A_t + \varepsilon_{t+1}^A, \\
\log p_{t+1} & = \rho^p \log p_t + \varepsilon_{t+1}^p,
\end{aligned}
$$

where $R(\cdot)$ is the revenue function, A_t is productivity, K_t is the stock of physical capital, x_t are variable inputs, w_t are variable costs, p_t is the price of investments in new capital, $C(\cdot)$ is the adjustment cost for capital, $\left|\rho^A\right| < 1, |\rho^p| < 1, \varepsilon_t^A \sim \mathcal{N}\left(0, \sigma_A^2\right)$, and $\varepsilon_t^p \sim \mathcal{N}\left(0, \sigma_p^2\right)$.

2.1. ES data processing

We will process on the panel dataset provided `ES_Vietnam_2005_2009_2015`.

2.1.1. Re-categorizing firm size

After loading the ES panel dataset `ES_Vietnam_2005_2009_2015.csv`. The original categorization for firm size has 3 sizes: small (labelled as **1**), medium (**2**) and large (**3**). We re-categorize the firms into two groups: Small (1) and Large (2). The conditions for large firms are:

- If the firm's original size is either medium or large (`a6a` in [2, 3]), AND
- If the number of employees (`l1`) is greater than the median number of employees (`l1_median`).

After applying the conditions, a new column `firm_size` is created, with: **1** indicating a small firm, **2** indicating a large firm.

2.1.2. Firm Identifiers

In the dataset, firm IDs are used as identifiers for the firms in both years to merge and compare data from 2005 and 2015. We use the most recent update rule, which is year 2015. There are two establishment identifiers used for tracking each firm:

- `idstd2015`: A global unique identifier used for 2015 data.
- `id2015`: A country-specific identifier for 2015.
- `idstd2005`: A global unique identifier used for 2005 data.
- `id2005`: country-specific identifier for 2005.

Thus, we have the following column identifiers are used to select the relevant firm data for analysis:
`col_id = ['idstd2015', 'id2015', 'idstd2005', 'id2005', 'firm_size']`

2.1.3. Selecting the Variables

From the dataset, we focus on key variables to assess the firm size, production inputs, and financial information for 2005 and 2015, which are the years being analyzed. We will utilize these variables to capture information about the firms' capital, labor, investment, and debt.

Year 2015:

We summarize the selected variables from the questionaire in table 16:

Inputs	Variables	Description
Cost Variables	`c9b`	Annual losses due to power outages.
	`d2`	Total annual sales for all products and services (in VND).
	`n2a`	Total annual cost of labor.
	`n2b`	Total annual cost of electricity.
	`n2i`	Total annual cost of sales (for retail).
	`i2b`	Total annual cost of security.
Production Input Variables	`l1`	Permanent, full-time workers end of last fiscal year
	`n5a`	Purchases of new or used machinery, vehicles, and equipment
	`n5b`	Purchases of land and buildings
	`h8`	Cost of formal research and development activities.
Financing Variables	`k1c`	Percentage of purchases on credit (loans).
	`k2c`	Percentage of sales on credit (receivables).

Table 16: Selected Cost Variables, Production Input Variables, and Financing Variables in 2015

Year 2005:

We summarize the selected variables from the questionaire in table 17:

Inputs	Variables	Description
Cost Variables	`q86a1`	Total sales.
	`q86a2`	Total costs.
	`q86a3`	Total purchases of raw materials and intermediate goods.
	`q86a4`	Total cost of labor.
	`q86a5`	Depreciation.
Production Input Variables	`q87a`	Net profits (after tax)
	`q87d`	% of net profits (after tax) were reinvested in the establishment
	`q91a1`	Total fixed assets
	`q91a7`	Total current assets (excluding receivables).
Financing Variables	`q91a12`	Receivables

Table 17: Selected Cost Variables, Production Input Variables, and Financing Variables in 2005

2.1.4. Processing The Input Variables

Year 2015:

The calculation of the 2015 variables in the dataset involves converting percentage-based variables into actual values using total sales (`d2`) as the base reference for proportions. We use the following code lines to calculate the values:

1. Main Activity Cost (`d1a3`):

```
df['2015_cost_main_activity'] = df['d1a3'] * df['d2'] / 100
```

2. Total Variable Costs (n5b): `df['2015_total_vrcost'] = df['n5b']`

3. Investment Costs (n5a and _2015_h8): Investment costs are calculated as the sum of two components:

   - n5a: Purchase of new or used machinery, vehicles, and equipment.
   - _2015_h8: The cost of formal research and development activities (innovation activities).

   The total investment cost is then calculated as:

   `df['2015_investment_cost'] = df['n5a'] + df['_2015_h8']`

4. Finance Based on Credit Purchases (k1c):

   `df['2015_finance_on_credit'] = (df['k1c'] / 100) * df['2015_total_vrcost']`

5. Finance Based on Receivables (texttk2c):

   `df['2015_finance_from_receivables'] = (df['k2c'] / 100) * df['2015_total_vrcost']`

Year 2005:

1. Total Reinvestment in the Establishment (q87a and q87d): The total reinvestment by multiplying the net profits by the percentage of those profits reinvested into the firm:

   `df['2005_total_reinvestment'] = df['_2005_q87a'] * (df['_2005_q87d'] / 100)`

2. Total Assets Excluding Receivables (q91a1 and q91a12): In the 2005 dataset, the total assets are split into fixed assets and current assets, which included receivables. Thus, we want to exclude from the total assets calculation.

   `df['2005_total_assets'] = df['_2005_q91a1'] - df['_2005_q91a12']`

3. Receivables (q91a12): The variable _2005_q91a12, which represents receivables, is renamed to 2005_receivables for clarity and easier reference within the dataset.

4. Total Cost Calculation (q86a3 to q86a11): To calculate the total cost of a firm in 2005, we sum up several cost components. These cost components represent various operational and production costs incurred by the firm. The relevant columns include _2005_q86a3, _2005_q86a4, _2005_q86a5, _2005_q86a6, _2005_q86a8, _2005_q86a9, _2005_q86a10, _2005_qq86a11.

2.2. Calculate Inputs for Quantitative Model

The following steps calculate w_t, which are variable costs; x_t as variable inputs and p_t as the price of investments in new capital, which involves the following steps for both years:

1. Calculate reinvestment and costs

2. Assign input prices

3. Compute quantities $x_t = w_t/e_t$, with e_t being the price of the input variables and w_t the total expenditures for the inputs

4. Split firms (small/large) (for both years and in the same data frame)

5. Average the costs to obtain $\bar{w}_t$ and $\bar{p}_t$ by firm size (small/large)

Algorithm 1 Calculation of Input Costs and Prices

For 2005:
Compute total reinvestment: $\text{Reinvestment}_{2005} = \text{Net Profits} \times \left(\frac{\%\text{Reinvested}}{100}\right)$.
Calculate total assets excluding receivables: $\text{Assets}_{2005} = \text{Fixed Assets} - \text{Receivables}$.
Sum selected cost items to obtain Total Cost_{2005}.
For 2015:
Sum labor, electricity, material, and security costs to obtain Total Cost_{2015}.
Assign input prices from external sources (wages, electricity, rent, PPI).
Input Quantities:
For each input t, compute quantity:

$$x_t = \frac{w_t}{e_t}.$$

Aggregation:
Classify firms by size (small or large).
For each group, compute average costs and reinvestments:

$$\text{Average} = \frac{\sum \text{Total Cost or Investment}}{N}.$$

Cost Aggregation (2015): Compute total cost per firm as the sum of labor, electricity, materials, and security expenses
Cost Aggregation (2005): Compute total cost and reinvestment per firm based on reported values.
Averaging: Calculate mean costs and reinvestments by dividing summed values by the number of firms.
Results:

$$\text{Small Firms:} \quad \begin{aligned} \text{Avg. 2015 Investment Cost} &= 1.6825 \times 10^{12} \\ \text{Avg. 2015 Total Cost} &= 5.0182 \times 10^{9} \\ \text{Avg. 2005 Total Cost} &= 0.0 \\ \text{Avg. 2005 Total Reinvestment} &= 0.0 \end{aligned}$$

$$\text{Large Firms:} \quad \begin{aligned} \text{Avg. 2015 Investment Cost} &= 6.9893 \times 10^{11} \\ \text{Avg. 2015 Total Cost} &= 3.2978 \times 10^{10} \\ \text{Avg. 2005 Total Cost} &= 0.0 \\ \text{Avg. 2005 Total Reinvestment} &= 0.0 \end{aligned}$$

Combined Averaging: For each group, compute average between 2015 and 2005 values:

$$\text{Average Combined Investment} = \frac{\text{Avg. 2015 Investment Cost} + \text{Avg. 2005 Total Reinvestment}}{2},$$

$$\text{Average Combined Total Cost} = \frac{\text{Avg. 2015 Total Cost} + \text{Avg. 2005 Total Cost}}{2}.$$

Final Results:

$$\text{Small Firms:} \quad \begin{aligned} \text{Average Combined Investment Cost} &= 8.4126 \times 10^{11} \\ \text{Average Combined Total Cost} &= 2.5091 \times 10^{9} \end{aligned}$$

$$\text{Large Firms:} \quad \begin{aligned} \text{Average Combined Investment Cost} &= 3.4946 \times 10^{11} \\ \text{Average Combined Total Cost} &= 1.6489 \times 10^{10} \end{aligned}$$

Hence, we obtain the mean values of variable cost w_t and price of investments in new capital p_t:

- Small Firms ($n_S = 1106$)

$$w_t^S = 2.5091 \times 10^9; \quad p_t^S = 8.4126 \times 10^{11}$$

- Large Firms ($n_L = 936$)

$$w_t^L = 1.6489 \times 10^{10}; \quad p_t^L = 3.4946 \times 10^{11}$$

2.3. Empirical relationships between debt, production inputs, finances

We explore the relationships between debt, production inputs, and financial factors by examining the correlations between several key variables in both the 2015 and 2005 datasets.

2.3.1. Description of method

2015 Data Analysis: Variables Observed

- Total Revenue (`total_revenue`)
- Total Variable Cost (`total_vrcost`)
- Main Activity Cost (`cost_main_activity`)
- Finance Based on Credit Purchases (`finance_on_credit`)
- Finance Based on Sales on Credit (`finance_from_receivables`)

2005 Data Analysis: Variables Observed

- Total Assets (`total_assets`)
- Total Cost (`total_cost`)
- Total Reinvestment (`total_reinvestment`)
- Receivables (`receivables`)

Further Analysis Through Regression After identifying significant correlations from the matrix, we conduct regression analyses to quantify these relationships further by fitting ordinary least squares (OLS) regression models.

2.3.2. Correlation results

Year 2015 results

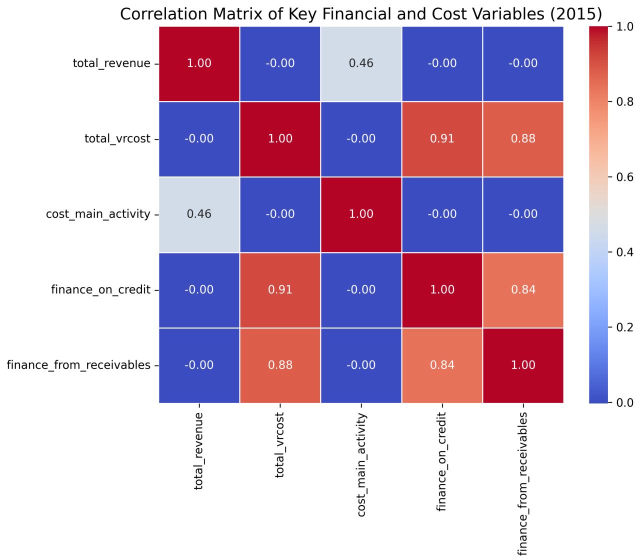


Figure 22: Correlation Matrix of Key Financial and Cost Variables (2015)

From the correlation matrix for 2015 in figure 22, we observe that

- Finance Based on Credit Purchases (`finance_on_credit`) and Total Variable Cost (`total_vrcost`) have a very high positive correlation of 0.9141.
- Finance from Receivables (`finance_on_receivables`) and Total Variable Cost (`total_vrcost`) also show a high positive correlation of 0.8795.

Given these high correlation values, both `finance_on_credit` and `finance_on_receivables` are strongly correlated with `total_vrcost`, indicating that as the total variable cost increases, both credit purchases and receivables financing tend to increase as well.

This strong relationship suggests that firms with higher total variable costs tend to rely more on credit & receivables financing. Given this high correlation, it is valuable to further analyze this relationship using regression analysis to better understand how finance on credit & receivables influences total variable costs. If the regression confirms the strong relationship observed in the correlation, it would indicate that credit & receivables financing play a crucial role in covering variable costs.

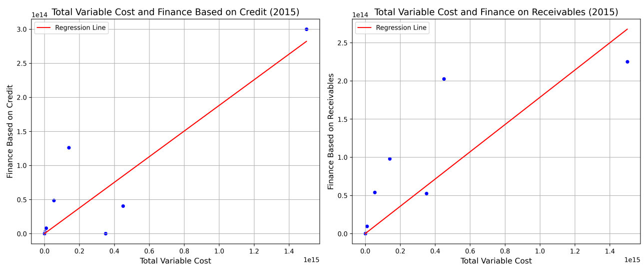


Figure 23: (a) Total Variable Cost and Finance on Credit (2015), (b) Total Variable Cost and Finance on Receivables (2015)

The two scatterplots in Figure 23 show significant positive correlations between credit & receivables financing and variable cost, which aligns with the conclusion in the correlation matrix discussed above. The OLS regression results with `finance_on_credit` and `finance_on_receivables` as independent variables (X) and `total_vrcost` as the dependent variable (Y):

$$\text{total\_vrcost} = \beta_0 + \beta_1 \cdot \text{finance\_on\_credit} + \beta_2 \cdot \text{finance\_from\_receivables} + \varepsilon$$

where:

- β_0 is the constant (intercept),
- β_1 is the coefficient for `finance_on_credit`,
- β_2 is the coefficient for `finance_from_receivables`,
- ε is the error term.

Table 18: Regression Results

Dep. Variable:	total_vrcost	**R-squared:**	0.880
Model:	OLS	**Adj. R-squared:**	0.879
Method:	Least Squares	**F-statistic:**	1.167e+04
Date:	Mon, 28 Apr 2025	**Prob (F-statistic):**	0.00
Time:	23:31:41	**Log-Likelihood:**	-1.0026e+05
No. Observations:	3199	**AIC:**	2.005e+05
Df Residuals:	3196	**BIC:**	2.005e+05
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> $\|t\|$**	**[0.025**	**0.975]**
const	-7.839e+10	1.75e+11	-0.448	0.654	-4.21e+11	2.65e+11
finance_on_credit	2.8880	0.054	53.036	0.000	2.781	2.995
finance_from_receivables	1.8850	0.055	34.140	0.000	1.777	1.993

Omnibus:	6608.125	**Durbin-Watson:**	2.000
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1.743e+08

Interpreting the Coefficients

- **Intercept** (β_0): The intercept value is -7.839×10^{10}.
- **Finance on Credit** (β_1): The coefficient for `finance_on_credit` is 2.8880. This means that for every unit increase in `finance_on_credit`, `total_vrcost` increases by 2.8880 units, holding other factors constant. The high t-value (53.036) and p-value (0.000) indicate that this variable has a statistically significant impact on `total_vrcost`.
- **Finance from Receivables** (β_2): The coefficient for `finance_from_receivables` is 1.8850. This means that for every unit increase in `finance_from_receivables`, `total_vrcost` increases by 1.8850 units, holding other factors constant. The t-value (34.140) and p-value (0.000) indicate that this variable also has a statistically significant impact on `total_vrcost`.

Model Fit

- **R-squared (0.880)**: This indicates that approximately 88% of the variation in `total_vrcost` is explained by `finance_on_credit` and `finance_from_receivables`. This suggests that the model fits the data well.
- **F-statistic (1.167e+04)**: The F-statistic tests the overall significance of the regression model. With a very small p-value (0.000), it indicates that the model is statistically significant.

Year 2005

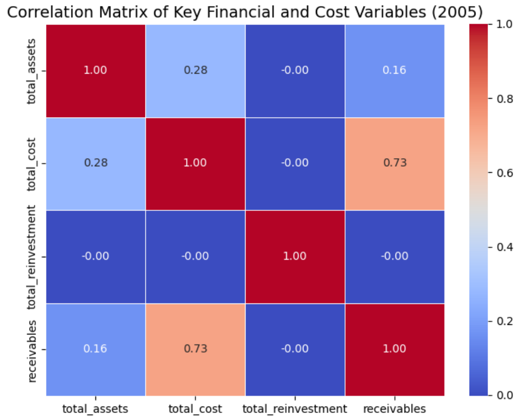


Figure 24: Correlation Matrix of Key Financial and Cost Variables (2005)

From the correlation matrix in Figure 24, a strong positive correlation between `total_cost` and `receivables` (0.725555) indicates a significant relationship between these two variables. As the company incurs more costs, it likely extends more credit or experiences higher receivables. This relationship is essential to analyze, especially when studying financial strategies related to credit management and cost control.

Thus, given the high correlation, the relationship between total costs and receivables might reveal insights into how a company's operational costs are linked to its credit practices.

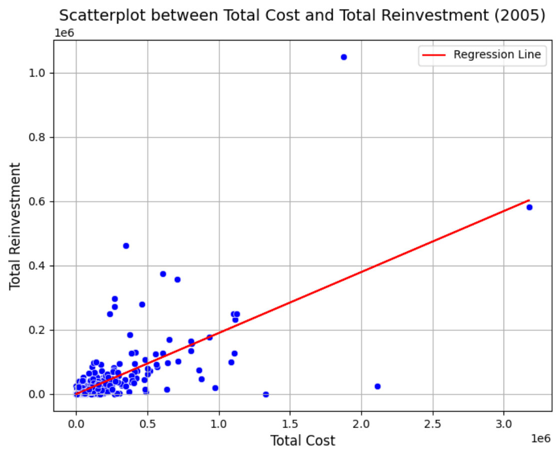


Figure 25: Scatterplot between Total Cost and Total Reinvestment (2005)

Figure 25 shows a strong positive correlation between the two inputs. The OLS table presents the regression results:

Dep. Variable:	receivables	**R-squared:**	0.526
Model:	OLS	**Adj. R-squared:**	0.526
Method:	Least Squares	**F-statistic:**	3554.
Date:	Mon, 28 Apr 2025	**Prob (F-statistic):**	0.00
Time:	23:39:23	**Log-Likelihood:**	-36349.
No. Observations:	3199	**AIC:**	7.270e+04
Df Residuals:	3197	**BIC:**	7.271e+04
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
const	100.7450	375.992	0.268	0.789	-636.464	837.954
total_cost	0.1898	0.003	59.614	0.000	0.184	0.196

Omnibus:	6015.422	**Durbin-Watson:**	2.011
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	32632766.655
Skew:	13.504	**Prob(JB):**	0.00

- **R-squared:** `0.526` — The model explains about 52.6% of the variation in `receivables`. This suggests a moderate fit, meaning that other factors not included in the model also influence `receivables`.
- **P-value of F-statistic:** `0.00` — The p-value for the F-statistic is less than 0.05, suggesting that the model is statistically significant.
- **Coefficients:**
  - **Intercept (const):** `100.7450` — The intercept represents the value of `receivables` when `total_cost` is zero. This value is not statistically significant, as indicated by the high p-value of `0.789`, which suggests that the intercept alone does not have predictive power.
  - **Coefficient for `total_cost`:** `0.1898` — For every unit increase in `total_cost`, `receivables` is expected to increase by 0.1898 units, assuming all other factors are constant. This relationship is statistically significant, as the p-value is `0.000`, well below the typical threshold of 0.05.

2.4. Describe the method

We provide a pseudo code to explain the implementation of model of firm investment as follows:

Algorithm 2 Dynamic Firm Problem: VFI, Simulation, and Visualization

Input: Parameter struct `par`, grid dimensions, transition matrices
Output: Policy functions `sol`, simulation outputs `sim`, plots

— Phase 1: Solve the Model (Value Function Iteration) —
Initialize value function $V_0(k, A, p) = 0$ on grid
repeat
    for each capital state k **do**
        for each productivity state A **do**
            for each price state p **do**
                Compute optimal input: $x^* \leftarrow \left(\frac{(1-\alpha)Ak^\alpha}{w}\right)^{1/\alpha}$
                Compute revenue $y = Ak^\alpha x^{1-\alpha}$
                Compute cost (input, investment, adjustment)
                Compute profit $\pi = y - \text{cost}$
                Compute expected future value EV via Π
                Solve Bellman update: $V(k, A, p) = \max_{k'}\{\pi + \beta EV\}$
                Store optimal k', x, π, cost, V
            end for
        end for
    end for
    Update convergence criterion $\delta \leftarrow \max |V_1 - V_0|$
    $V_0 \leftarrow V_1$
until $\delta < \varepsilon$
Output: Value and policy functions `sol`

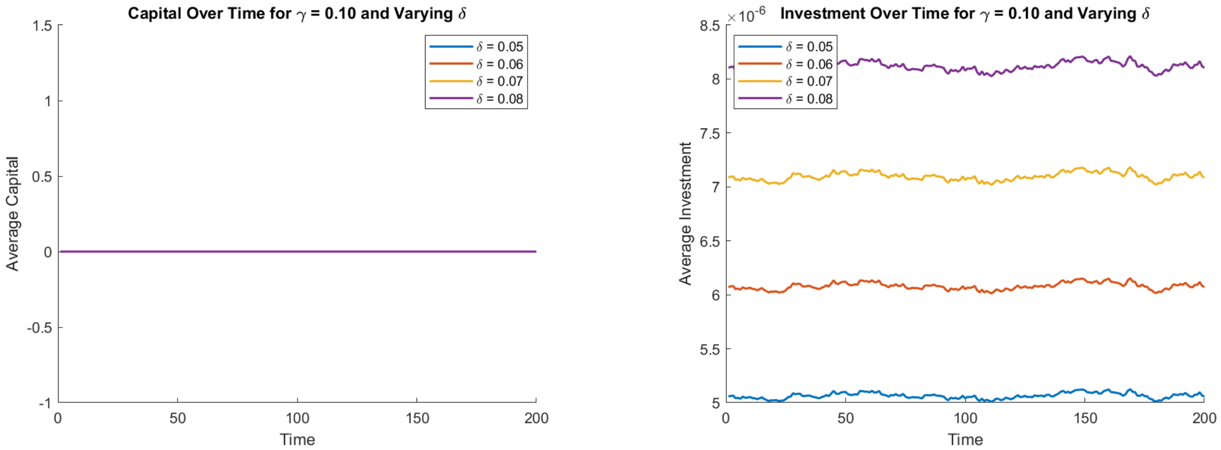
— Phase 2: Simulate the Model Over Time —
Initialize initial states k_0, A_0, p_0 for each firm
for each time $t = 2$ to T **do**
    for each firm i **do**
        Observe current state $(k_{t-1}, A_{t-1}, p_{t-1})$
        Apply policy function $\Rightarrow k_t$, compute x_t, π_t, y_t
        Draw A_t, p_t using transition probabilities
        Store k_t, x_t, π_t, y_t, cost, value
    end for
end for
Output: Simulated panel data `sim`

— Phase 3: Aggregate and Plot Simulated Results —
for each period t **do**
    Compute averages: $\bar{k}_t$, $\bar{x}_t$, $\bar{\pi}_t$, $\bar{v}_t$, etc.
end for
for each variable **do**
    Plot $\bar{z}_t$ over t, label axes, save figure
end for
Plot 3D policy surfaces (e.g., $k, p \rightarrow k'$) for selected A

2.5. Simulation Results by Parameter Variation

2.5.1. Fixed gamma = 0.10, varying delta

Effect of Varying γ (Adjustment Cost) on Investment Figure 26b illustrates the dynamic behavior of average investment over time across different values of the capital adjustment cost parameter γ, holding the depreciation rate δ constant at 0.08. Despite higher γ implying greater costs for adjusting capital, the simulation reveals that average investment increases slightly as γ rises from 0.10 to 0.25. This suggests that when adjustment costs are higher, firms may respond more aggressively to productivity or price shocks to optimize their long-term capital paths, potentially leading to more consistent or elevated investment behavior. The investment paths across all γ values display cyclical fluctuations driven by underlying shocks but remain relatively stable in their separation.



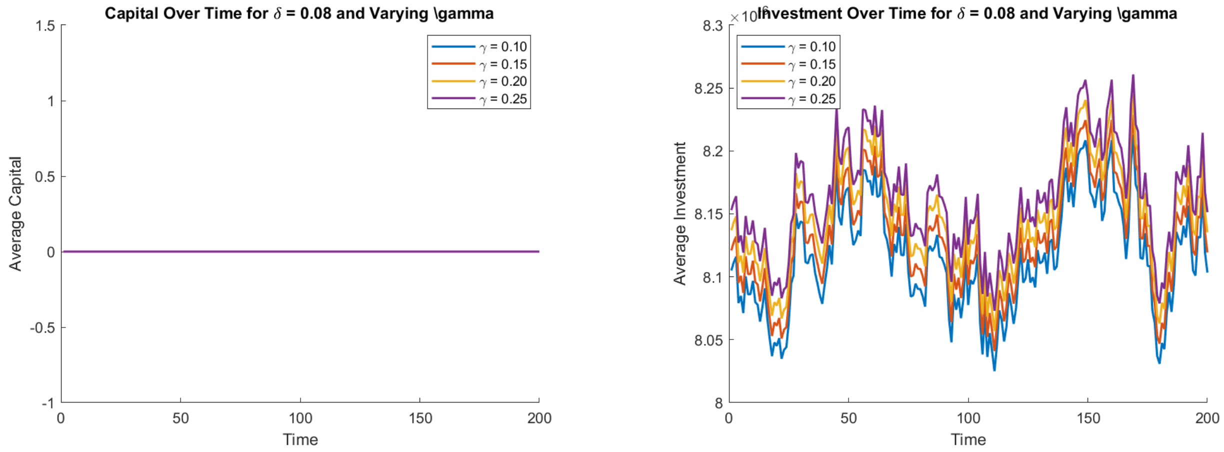
(a) Capital over time

(b) Investment over time

Figure 26: Simulation results for $\gamma = 0.10$ and varying $\delta \in \{0.05, 0.06, 0.07, 0.08\}$

2.5.2. Fixed delta = 0.08, varying gamma

Effect of Varying δ (Depreciation) on Investment Figure 27b shows the time series of average investment for a fixed adjustment cost $\gamma = 0.10$ while varying the depreciation rate δ from 0.05 to 0.08. The simulation results align well with economic intuition: as δ increases, firms experience greater capital erosion and must invest more to sustain their productive capacity. Consequently, the average investment rises systematically with δ, resulting in clearly separated and stable curves over time. These findings highlight the strong role that depreciation plays in determining the investment behavior of firms in the model, with higher depreciation translating directly into greater capital replenishment needs.



(a) Capital over time

(b) Investment over time

Figure 27: Simulation results for $\delta = 0.08$ and varying $\gamma \in \{0.10, 0.15, 0.20, 0.25\}$

2.5.3. Heatmap of Average Final-Period Capital

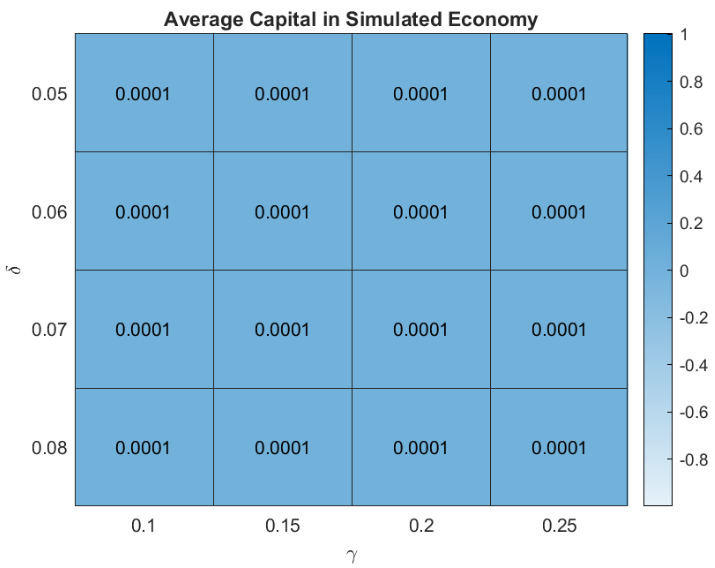


Figure 28: Average capital across all (γ, δ) combinations in the final period

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