Advanced Calculus with FE Application: Quiz 3

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Advisor: Dan Stefanica

Weiyi Chen

Problem 1

(i)

Since the covariance of Z_1 and Z_2 is 0.3, then

$$\begin{aligned} X &= 3Z_1 - Z_2 \\ &= (3\mu_1 - \mu_2) + \sqrt{(3\sigma_1)^2 + \sigma_2^2 - 2cov(3Z_1, Z_2)}Z \\ &= (3 \times 0 - 0) + \sqrt{(3^2 + 1^2 - 6 \times 0.3)} \\ &= \sqrt{8.2}Z \end{aligned} \tag{1}$$

Therefore the mean and the variance of X is

$$\begin{aligned} E[X] &= 0 \\ var[X] &= 8.2 \end{aligned} \tag{2}$$

(ii)

If Z_1 and Z_2 are independent, then

$$cov(Z_1, Z_2) = 0 \tag{3}$$

Using the similar process in part (i), we will have

$$X = \sqrt{10}Z \tag{4}$$

Therefore the probability density function of X is

$$\begin{aligned} f_X(x) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\ &= \frac{1}{\sqrt{20\pi}} e^{-\frac{x^2}{20}} \end{aligned} \tag{5}$$

where $\mu = 0, \sigma = \sqrt{10}$.

Problem 2

The probability that X is greater than 2 is

$$\begin{aligned} Pr(X > 2) &= Pr(3 + Z > 2) \\ &= Pr(Z > -1) \\ &= Pr(Z < 1) \\ &= N(1) \\ &= 0.8413 \end{aligned} \tag{6}$$

where $N(1) = 0.8413$ is given and $N(t)$ is the cumulative density of the standard normal variable.

Problem 3

Quiz's formula for CoN put

In the quiz, the value of cash-or-nothing put is defined as

$$P_{CoN} = Be^{-qT}N(-d_1) \tag{7}$$

Delta of the cash-or-nothing put is

$$\begin{aligned}\Delta(P_{CoN}) &= \frac{\partial P_{CoN}}{\partial S} \\ &= Be^{-qT}N'(-d_1)(-\frac{\partial d_1}{\partial S}) \\ &= Be^{-qT}(\frac{1}{\sqrt{2\pi}}e^{-d_1^2/2})(-\frac{1}{\sigma\sqrt{T}}\frac{K}{S}\frac{1}{K}) \\ &= -\frac{Be^{-qT-d_1^2/2}}{\sigma\sqrt{2\pi T}S}\end{aligned} \tag{8}$$

where

$$d_1 = \frac{\ln(\frac{S}{K}) + (r - q + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \tag{9}$$

Correct formula for CoN put

However, TA pointed out in the forum saying the value of cash-or-nothing put should be

$$P_{CoN} = Be^{-rT}N(-d_2) \tag{10}$$

Then in similar way, delta of the put is

$$\Delta(P_{CoN}) = -\frac{Be^{-rT-d_2^2/2}}{\sigma\sqrt{2\pi T}S} \tag{11}$$

where

$$d_2 = \frac{\ln(\frac{S}{K}) + (r - q - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \tag{12}$$

Problem 4

Using the Black-Scholes formula with input

$$\begin{aligned} S_1 &= K = 100 \\ T &= 1/2 \\ \sigma &= 0.3 \\ r &= 0.05 \\ q &= 0 \end{aligned} \tag{13}$$

we find that the value of one put options is

$$P_1 = 7.16586783128 \tag{14}$$

Therefore, $\$7,165.86143674$ must be paid for 1000 puts.

(i)

The Delta-hedging position for long 1000 puts is short

$$1000\Delta(P) = 1000(-e^{-qT}N(-d_1)) = -411.41 \tag{15}$$

units of the underlying. Therefore, 411 units of the underlying must be longed. I will have to borrow $41,100 to buy the underlying.

(ii)

The new spot price and muturity of the option are

$$S_2 = 102, T_2 = 125/252 \tag{16}$$

since there are 252 trading days in one year. The value of the put option is

$$P_2 = 6.35543075983 \tag{17}$$

and the value of the portfolio is

$$1000P_2 + 411S_2 - 411S_1e^{\frac{r}{252}} = 7169.27518887 \tag{18}$$

(iii)

If the long put position is not Delta-hedged, the loss incurred due to the increase in the spot price of the underlying asset is

$$1000(P_2 - P_1) = -\$810.43707145 \tag{19}$$

For the Delta-hedged portfolio, the loss incurred is

$$(1000P_2 + 411S_2 - 411S_1e^{\frac{r}{252}}) - (1000P_1 + 411S_1 - 411S_1) = \$3.40735758799 \tag{20}$$

As expected, this loss is much smaller than the loss incurred if the options positions is not hedged.