Data Structures and Algorithms [1]

BITS-Pilani K. K. Birla Goa Campus

[1] Material for the presentation taken from Cormen, Leiserson, Rivest and Stein, *Introduction to Algorithms, Fourth Edition*;

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Ch. 10 : Elementary Data Structures

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- Dynamic sets : must be able to add and delete items (i.e. must be dynamic).
- Max Priority Queue
  - EXTRACT-MAXIMUM
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  - Worst-case time complexity : $O(\lg n)$
- Data structures for dynamic sets : Stacks, Queues and Linked list.

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- There are many ways of implementing stacks.

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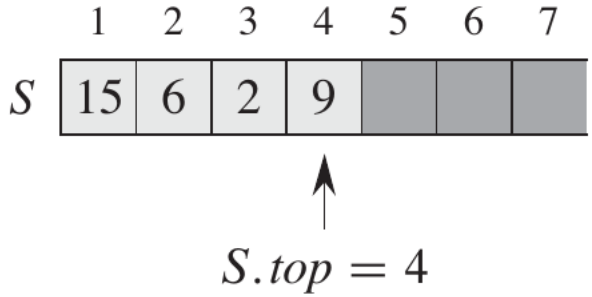

Elementary data structure: Stack

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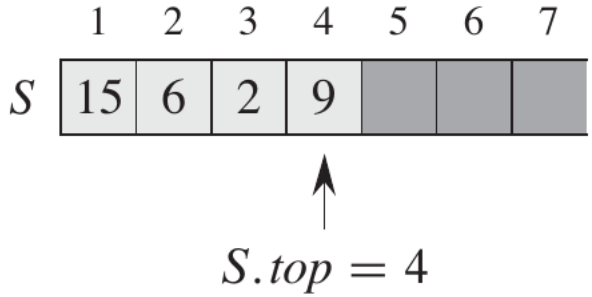

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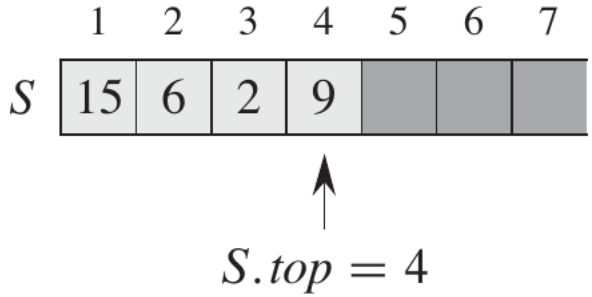

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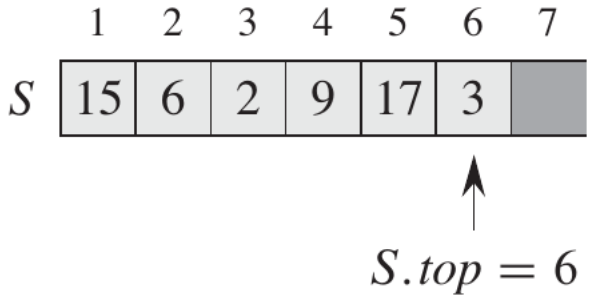

Elementary data structure: Stack

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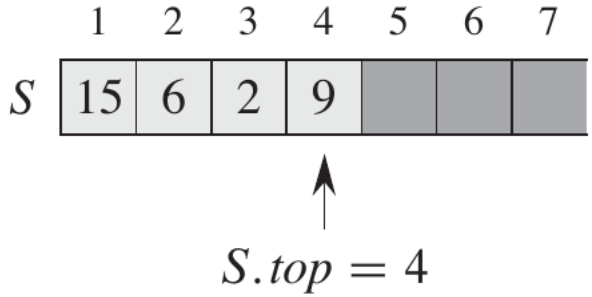

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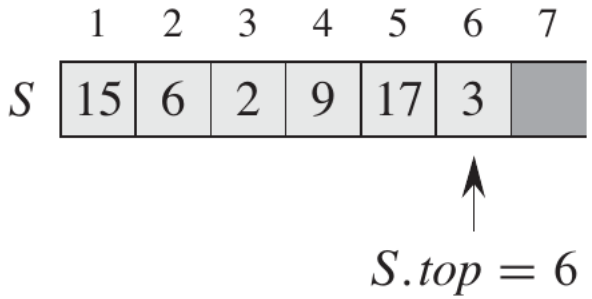
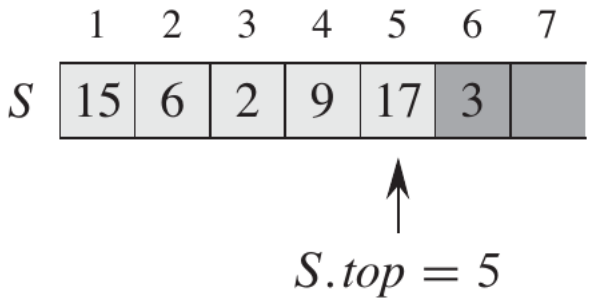


- POP(S)



Stack operation pseudocode

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Stack procedure pseudocode

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POP(S)
1  if STACK-EMPTY(S)
2       error "underflow"
3  else S.top = S.top − 1
4       return S[S.top + 1]
```

Stack operations

- Time complexity of stack operations?

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- Stack operations can lead to overflows and underflows.

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- Input : [( )] { } { [( ) ( )] ( ) }
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- Can we use a Stack to solve this problem?
- Use the `std::stack` container adapter or the `deque` template class.

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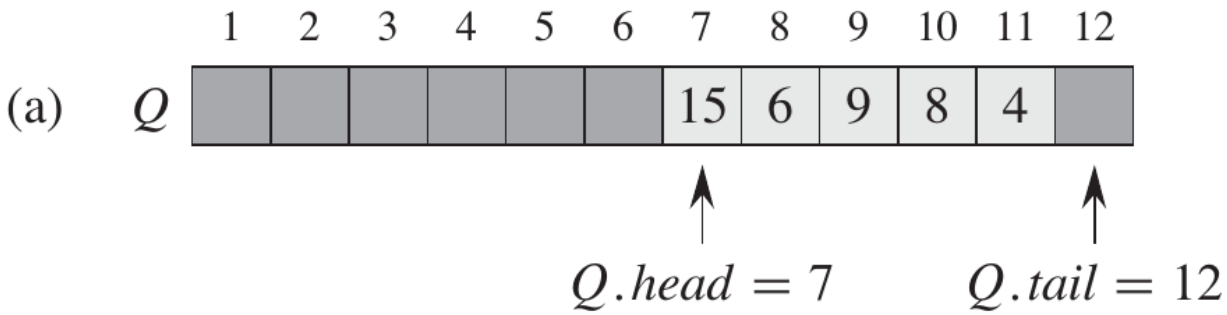
- ENQUEUE and DEQUEUE

Queue array implementation



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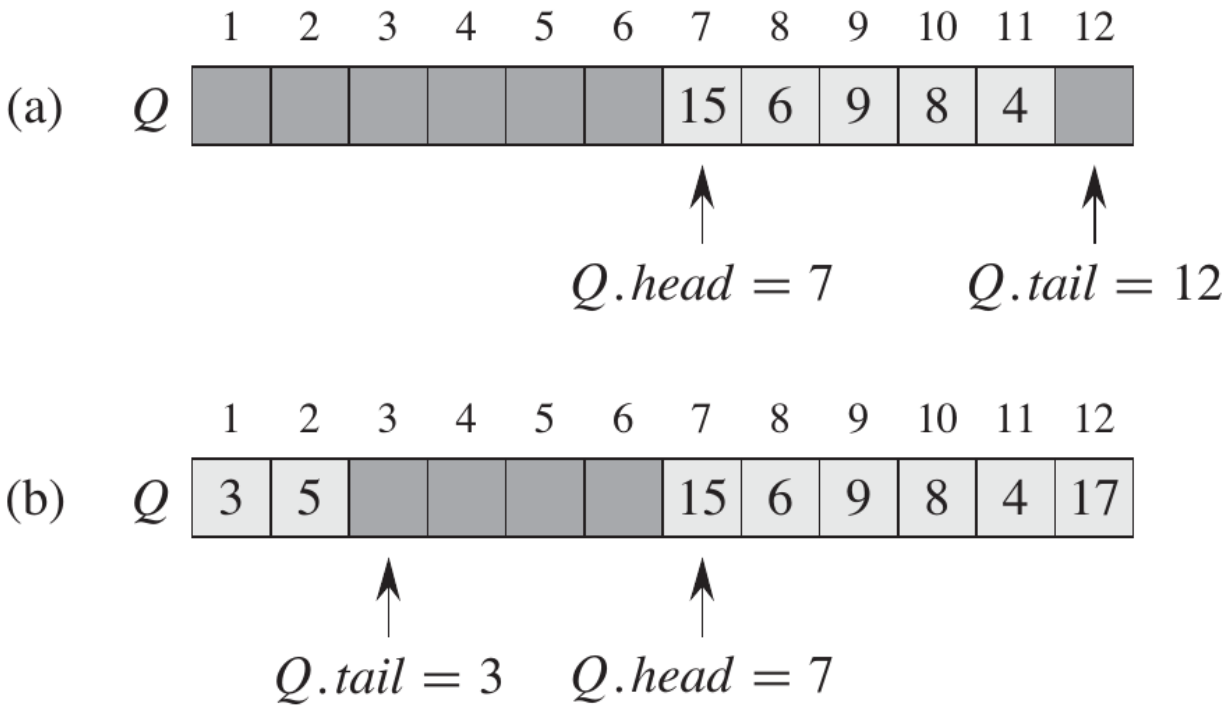

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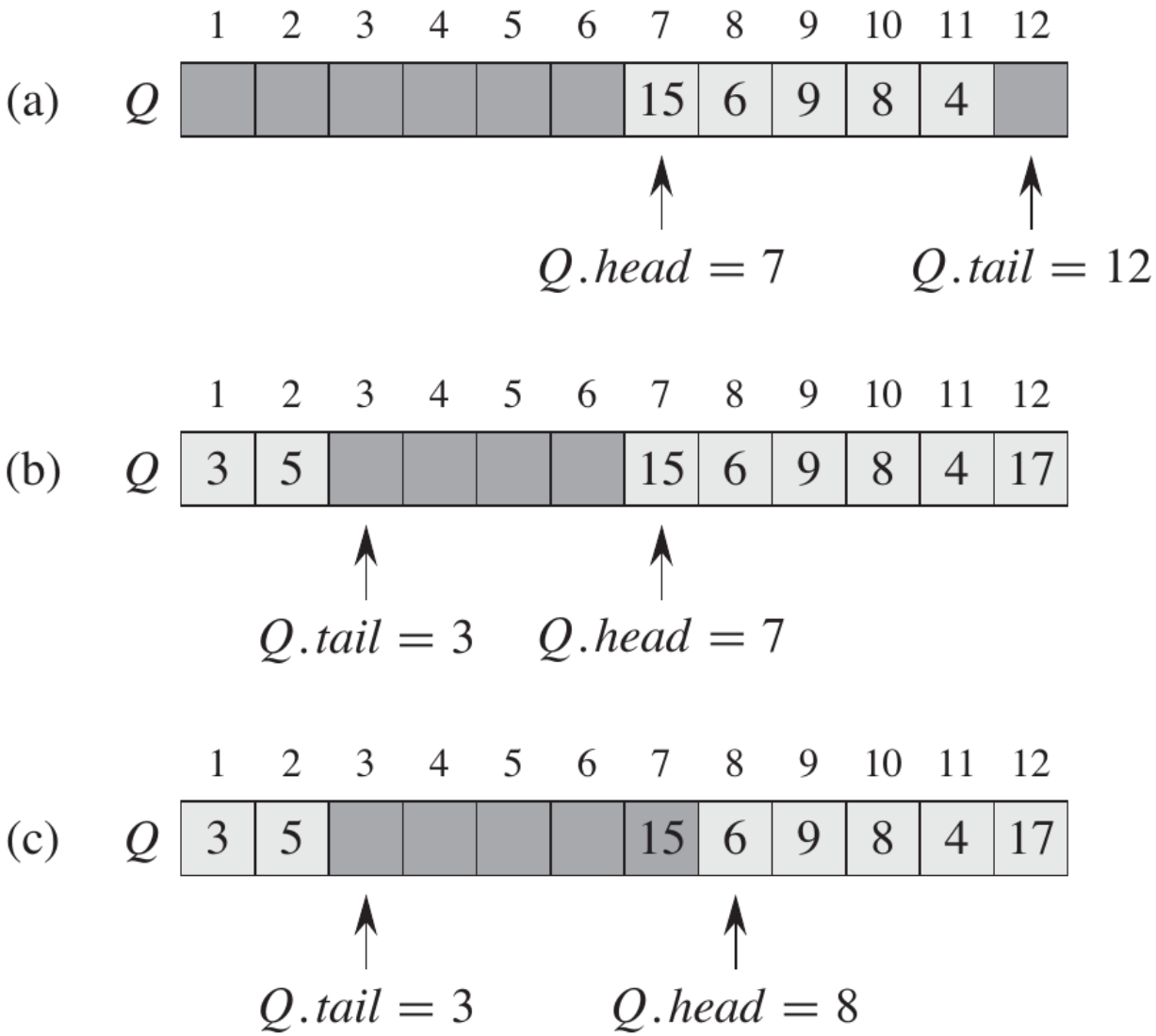

Enqueue and Dequeue

```
ENQUEUE(Q, x)
1  Q[Q.tail] = x
2  if Q.tail == Q.size
3        Q.tail = 1
4  else Q.tail = Q.tail + 1
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Enqueue and Dequeue

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ENQUEUE(Q, x)
1  Q[Q.tail] = x
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DEQUEUE(Q)
1  x = Q[Q.head]
2  if Q.head == Q.size
3        Q.head = 1
4  else Q.head = Q.head + 1
5  return x
```

Queue operations

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- Queue can be used to perform Breadth-first-search in Graphs.

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- We need to find the *minimum* number of operations needed to reach integer x starting from integer 1.

  Input : 31

  Output : 4

  Explanation : $(((1 \times 2) + 1) \times 10) + 1 = 31$.

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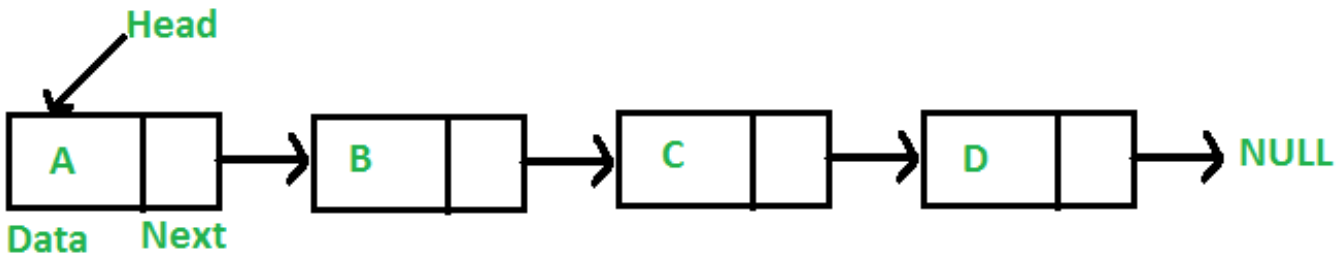

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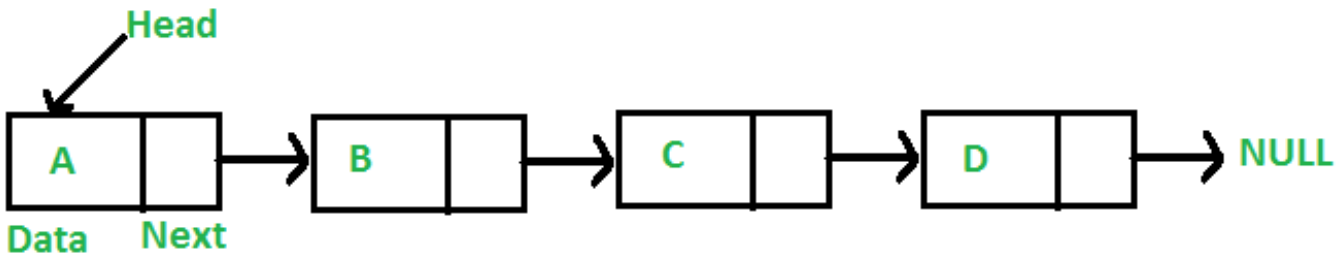

- Linked list can support all the operations on a dynamic set.

C++ program

- C++ program for singly linked list

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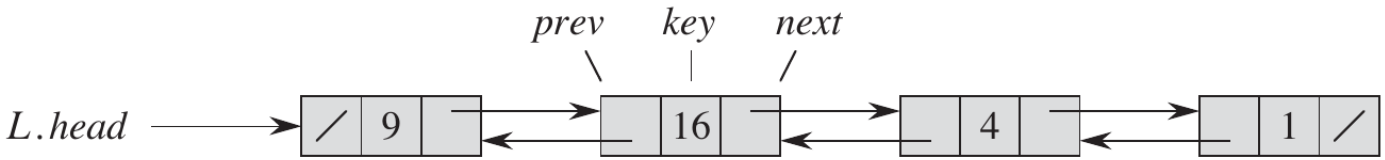
- insertNode function (L. 109)

Doubly Linked List



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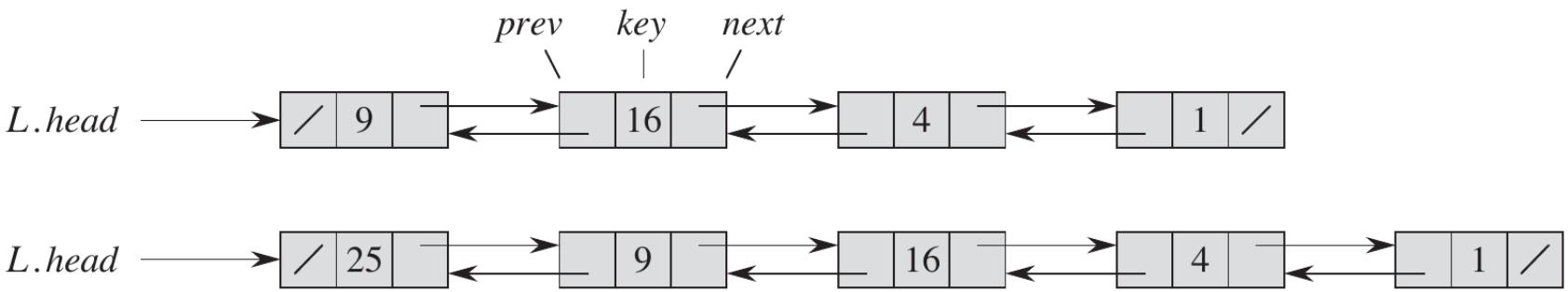

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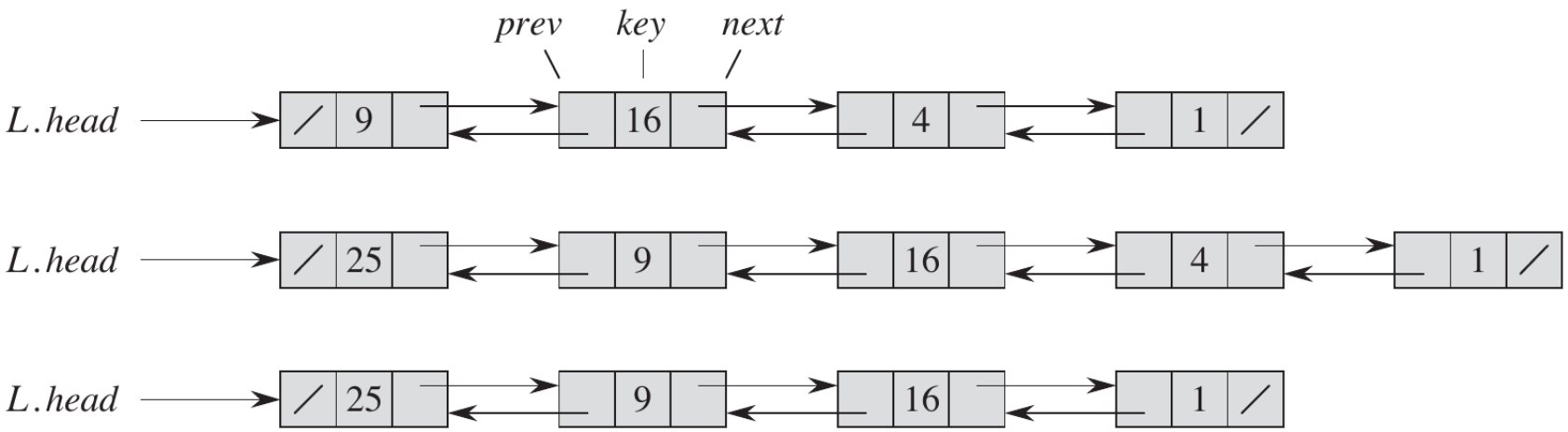

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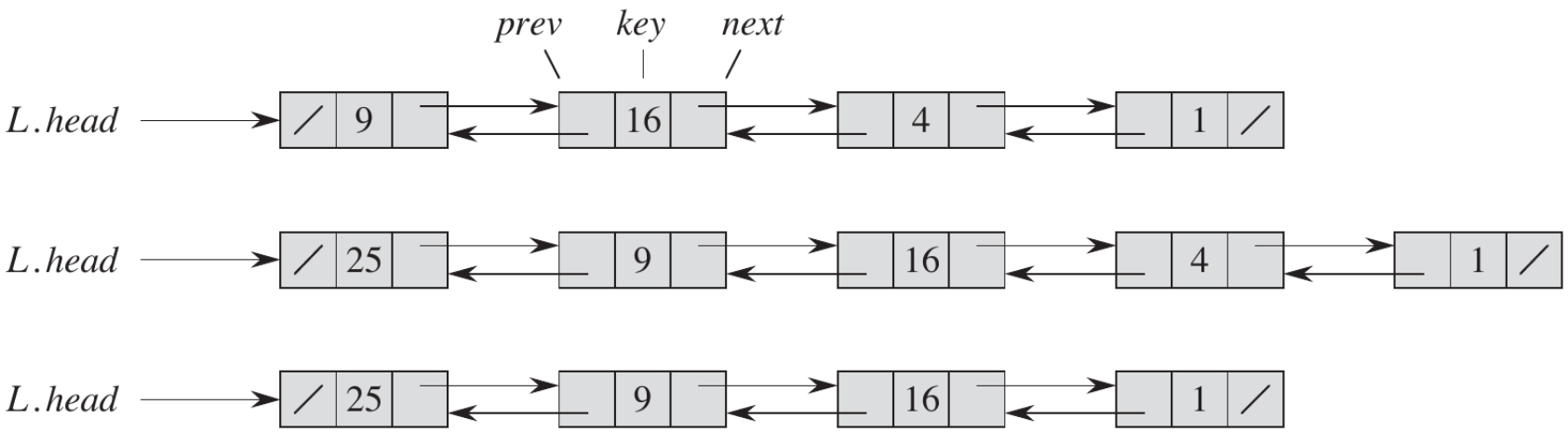

- NIL: prev of head element, and next of tail element

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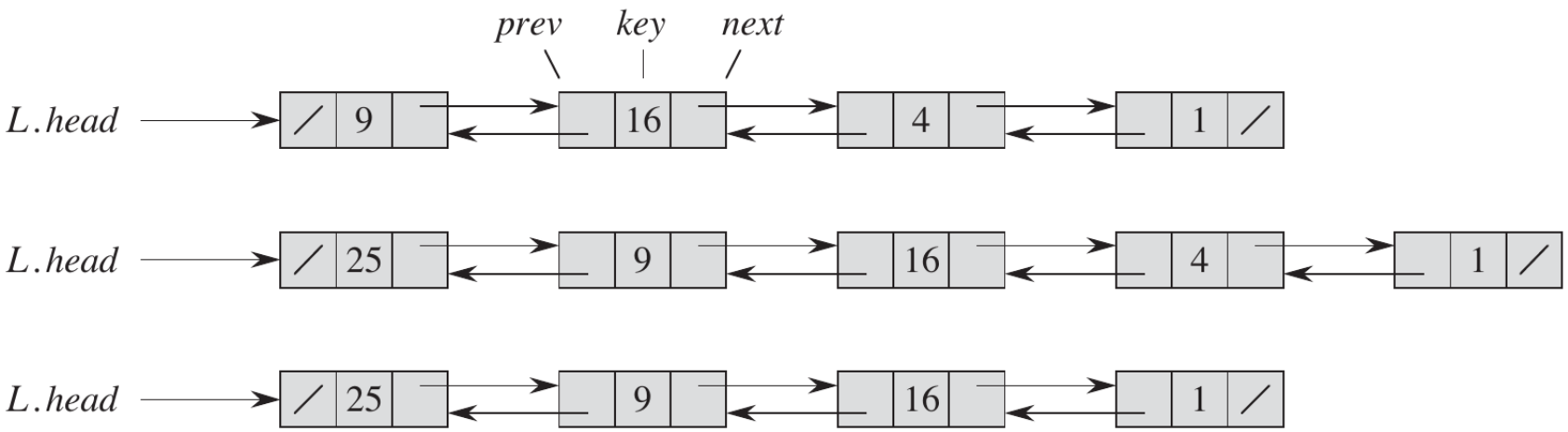

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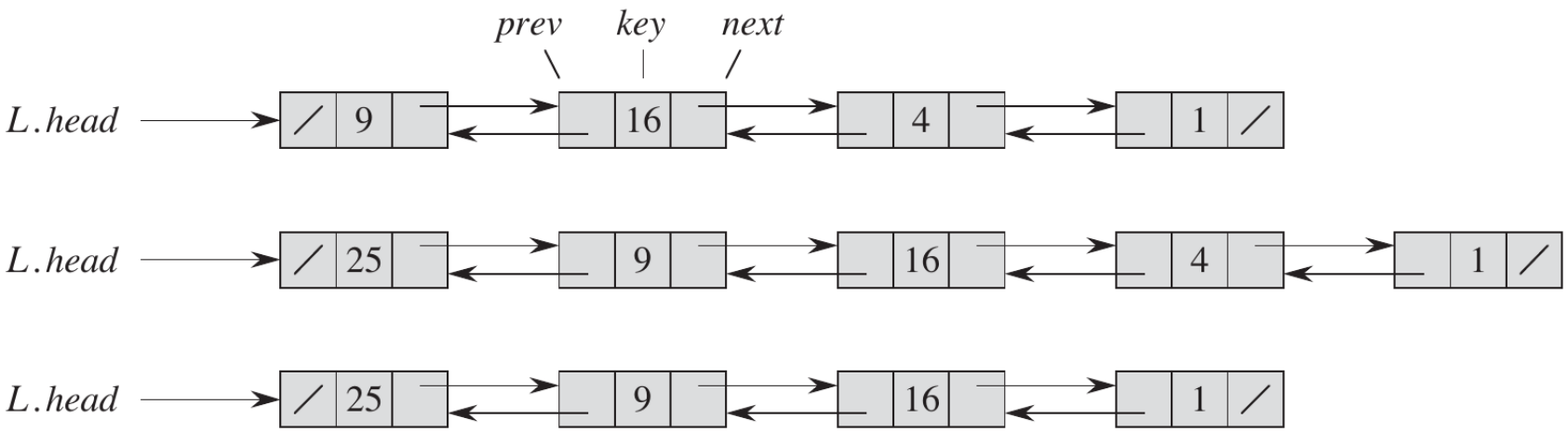

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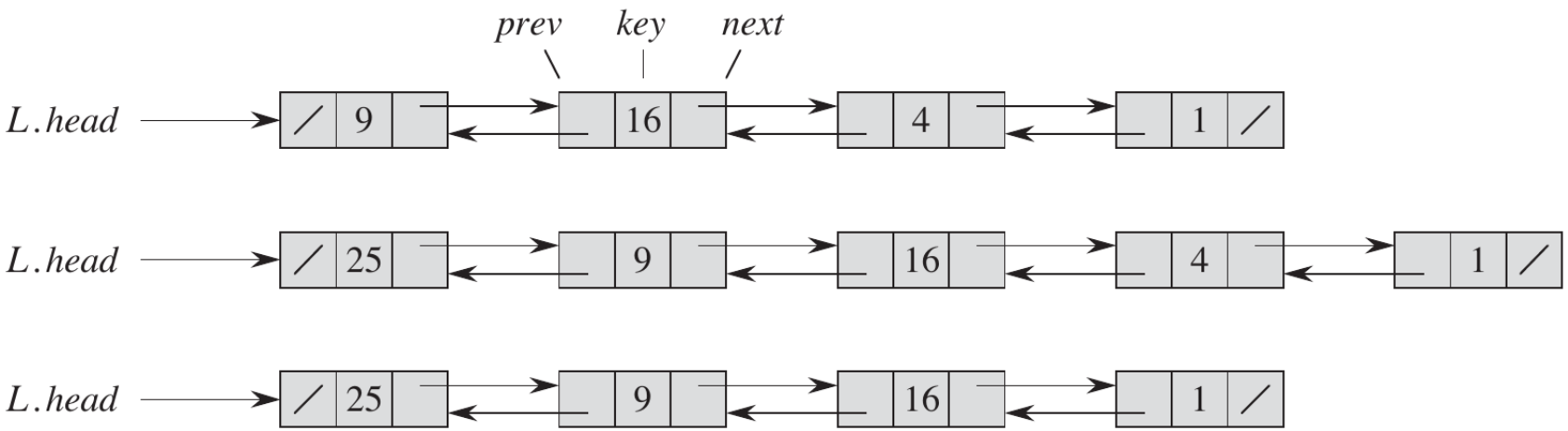

- NIL: prev of head element, and next of tail element
- Singly linked list does not have a prev pointer
- Circular linked list
- We will assume that we are working with an unsorted, doubly linked list.

List-Search

```
LIST-SEARCH(L, k)
1  x = L.head
2  while x ≠ NIL and x.key ≠ k
3        x = x.next
4  return x
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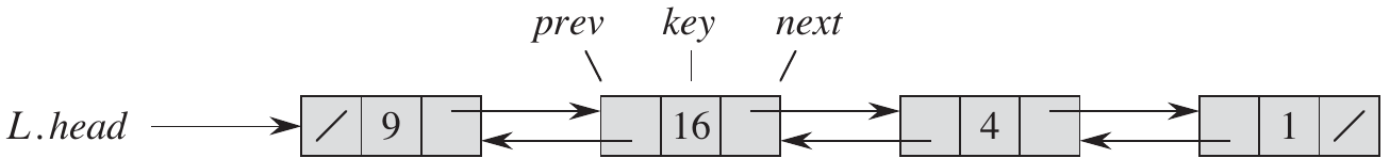
- List-Search takes $\Theta(n)$ time in the worst case.

List-Prepend



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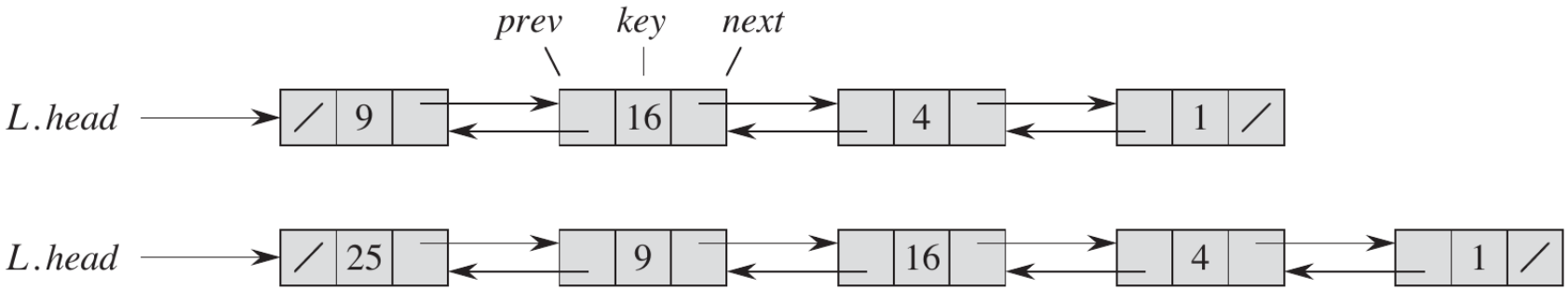

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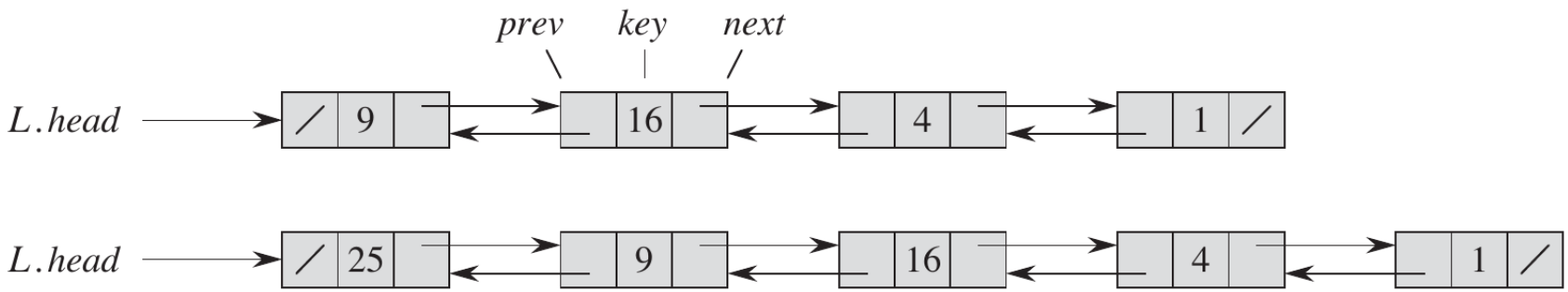
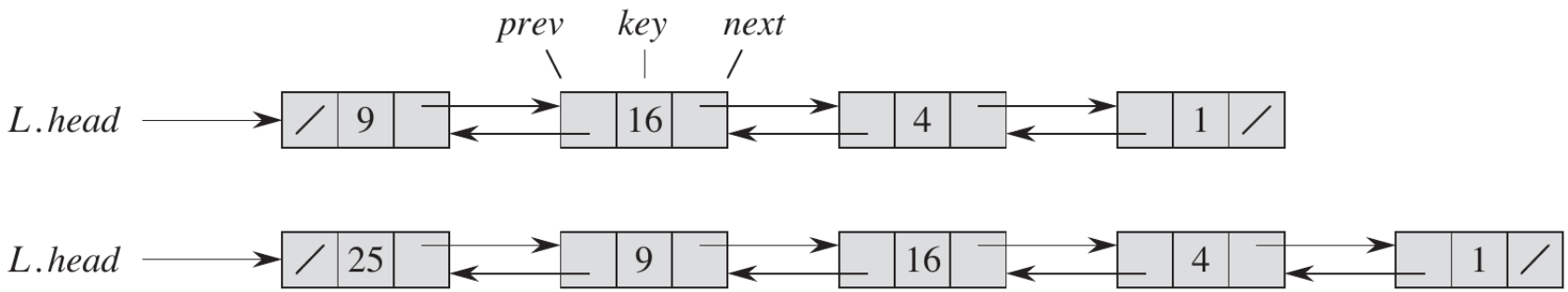



```
List-Prepend(L, x)
1   x.next = L.head
2   x.prev = NIL
3   if L.head ≠ NIL
4         L.head.prev = _____
5   L.head = _____
```

List-Prepend

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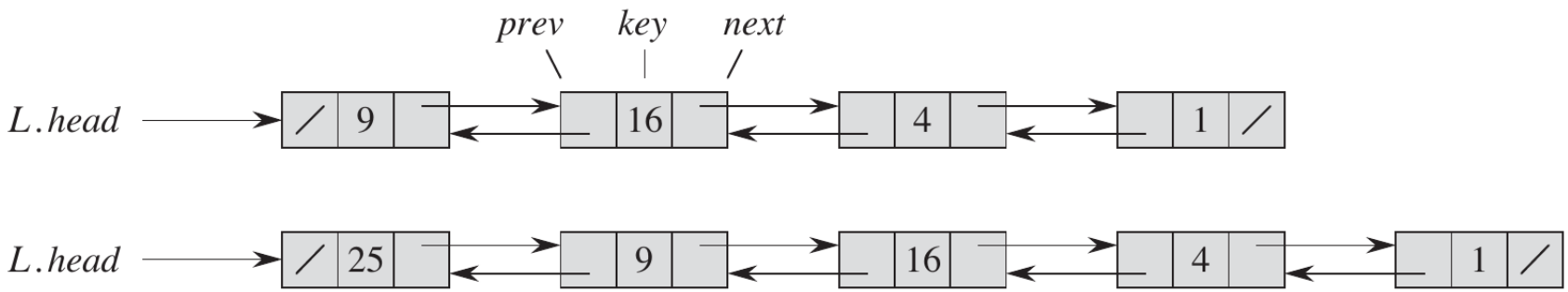


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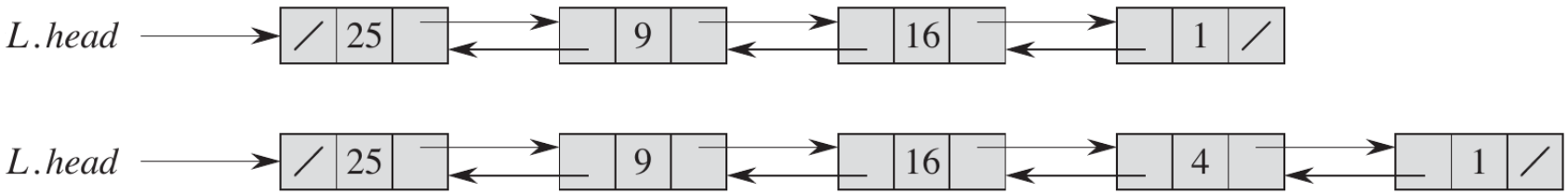
List-Prepend takes $O(1)$ time

List-Insert



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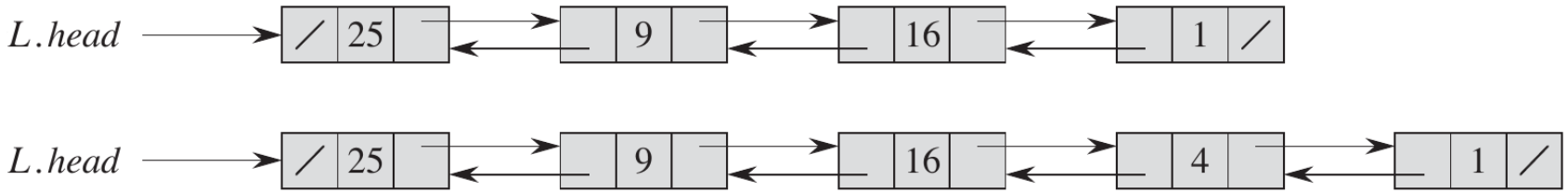

Insert node x after node y

```
LIST-INSERT(x, y)
1   x.next = y.next
2   x.prev = y
3   if y.next ≠ NIL
4         y.next.prev = _____
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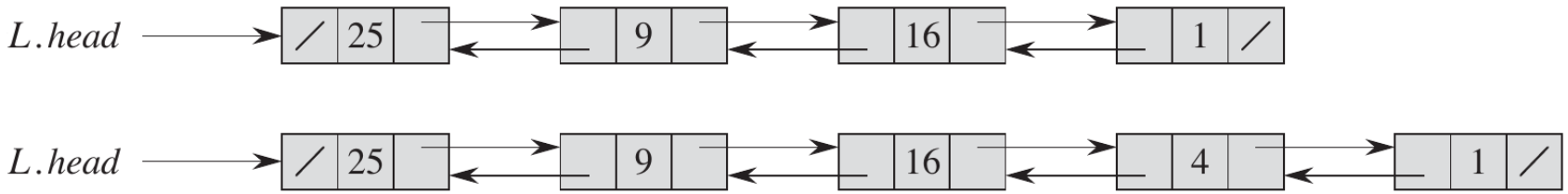
List-Insert



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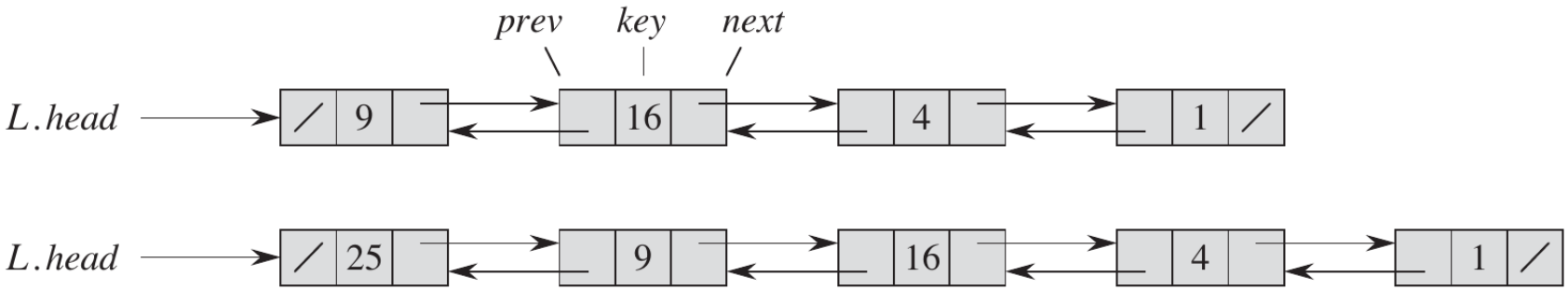
List-Delete



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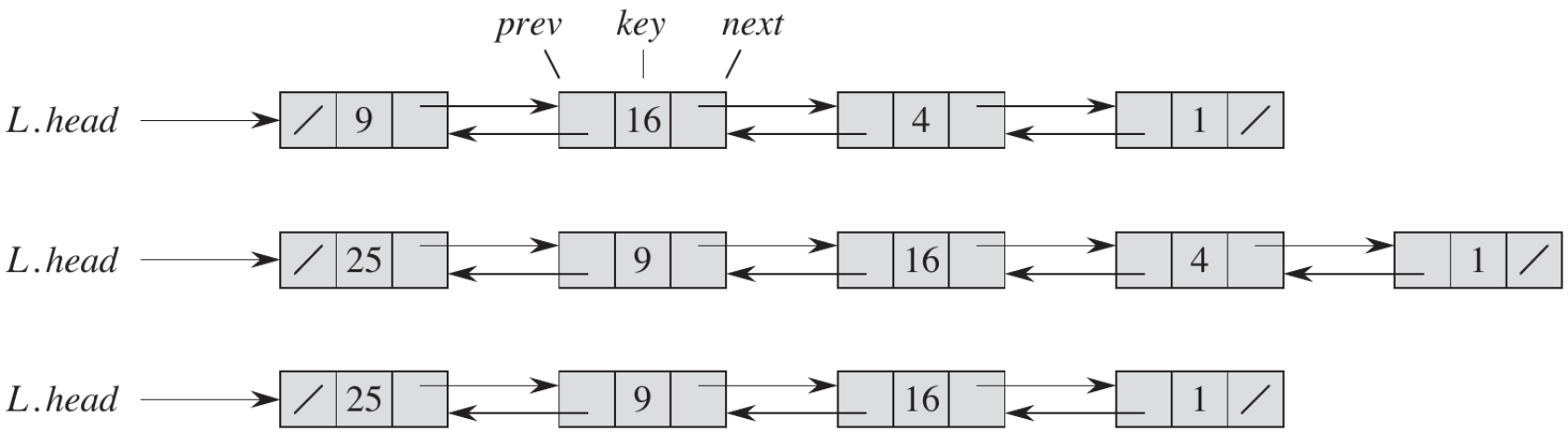

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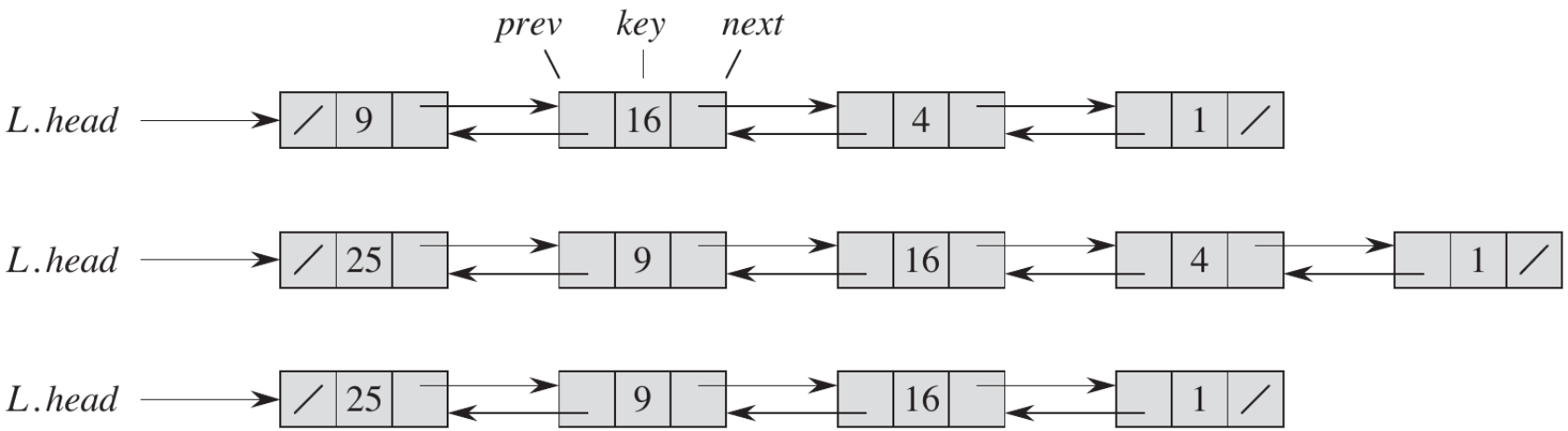
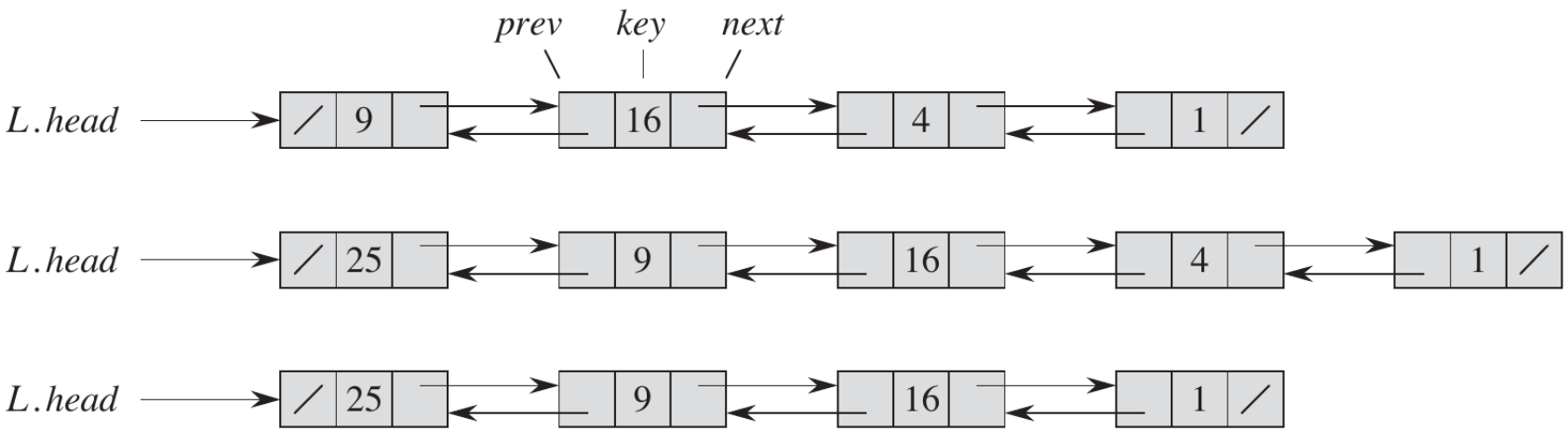


List-Delete(L, x)

1 **if** $x.prev \neq$ NIL
2 $\quad x.prev.next =$ _____
3 **else** $L.head = x.next$
4 **if** $x.next \neq$ NIL
5 $\quad x.next.prev =$ _____

List-Delete



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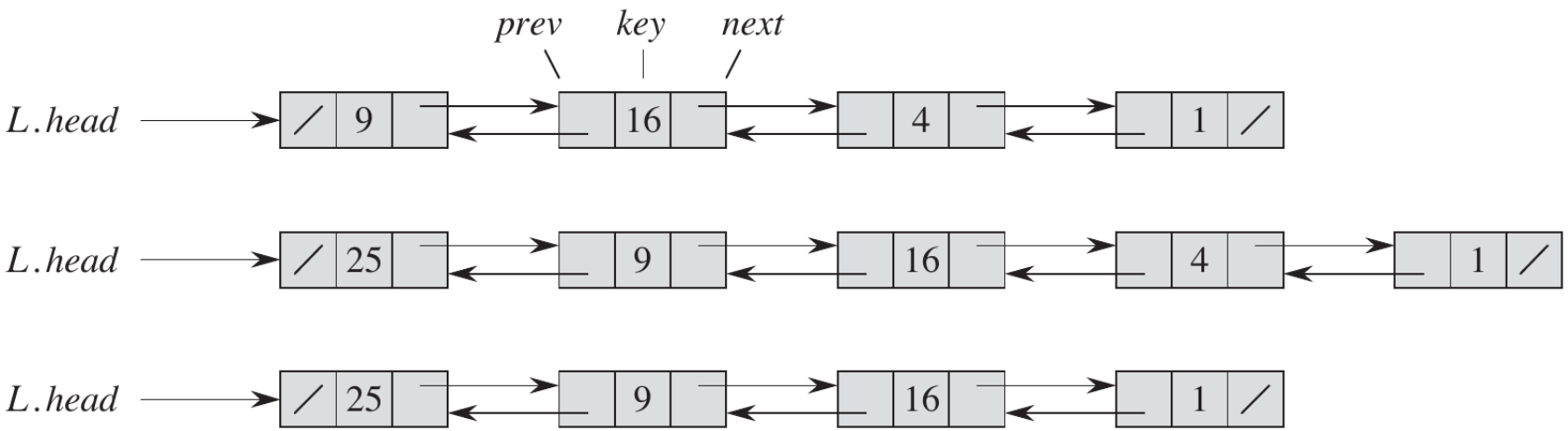

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List-Delete takes $O(1)$ time.

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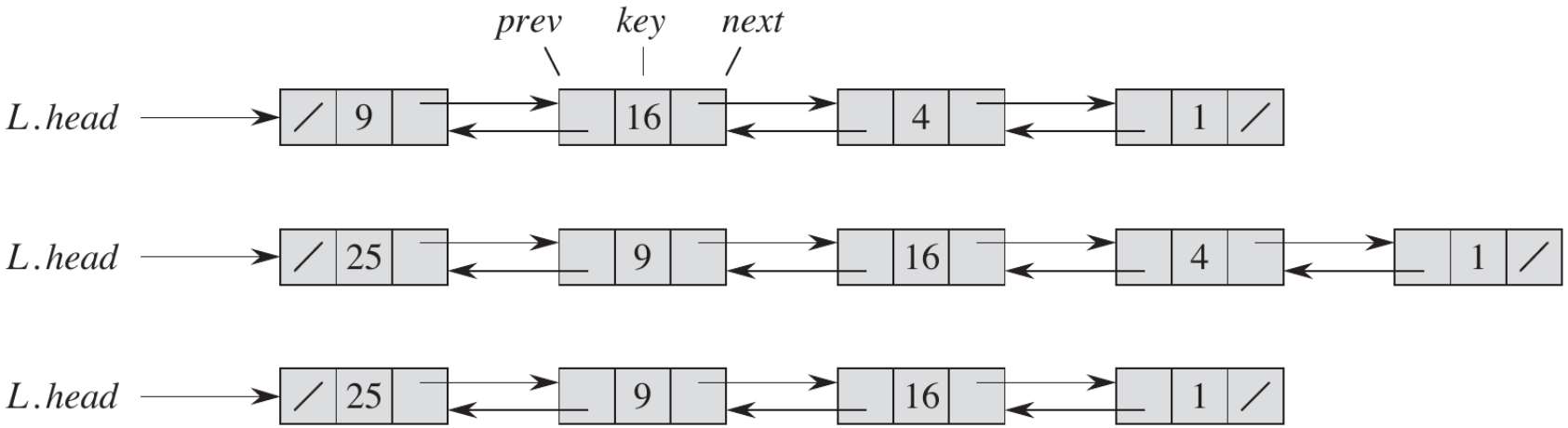

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List-Delete takes $O(1)$ time. Deleting an element with a given key would take $\Theta(n)$ time.