Data Structures and Algorithms [1]

BITS-Pilani K. K. Birla Goa Campus

[1] Material for the presentation taken from Cormen, Leiserson, Rivest and Stein, *Introduction to Algorithms, Fourth Edition*;

Course plan

- Ch. 20, 21, 22.3 (Graph Algorithms)

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- Ch. 12, 13 (Binary Search Tree and Red-Black Tree)

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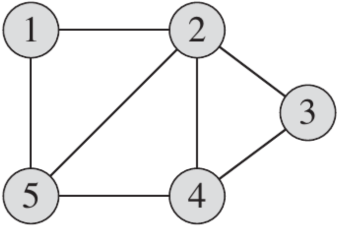
- Ch. 20, 21, 22.3 (Graph Algorithms)
- Ch. 12, 13 (Binary Search Tree and Red-Black Tree)
- Ch. 11 (Hash Tables)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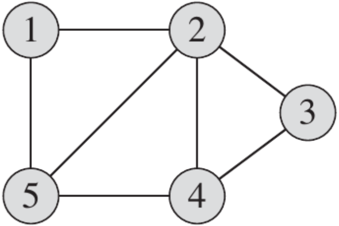
Module IV: Graph Algorithms



- Graphs are mathematical structures consisting of vertices and edges.

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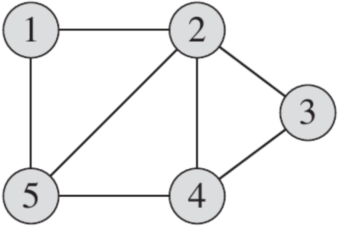
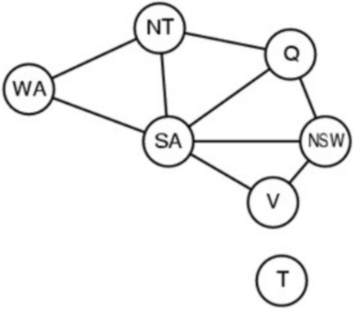


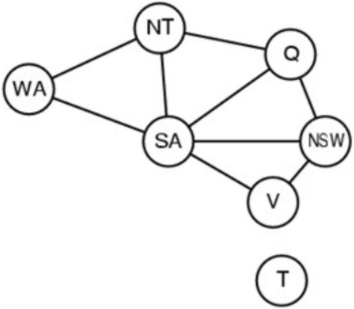
- Graphs are mathematical structures consisting of vertices and edges.
- The input to the graph algorithms will be a graph represented as $G = (V, E)$.

Map coloring problem

Vertex coloring problem



Vertex coloring problem



Module IV: Graph Algorithms

- Ch. 20: Elementary Graph Algorithms

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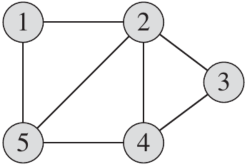
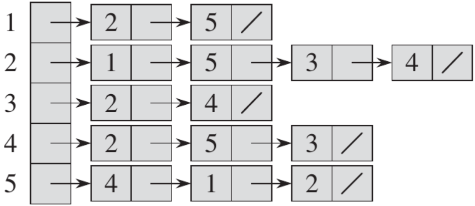
  Breadth-first search, Depth-first search, Topological sort

Representing undirected graphs

- Adjacency-list representation of undirected graphs





Representing undirected graphs

- Suppose we sum the lengths of all the adjacency lists of an undirected graph. What will be the sum?

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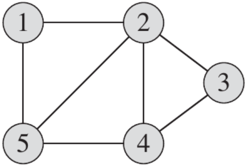
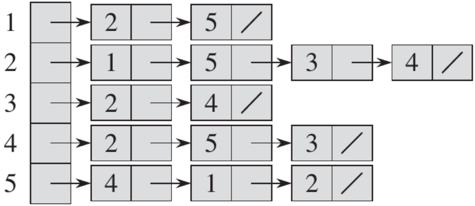
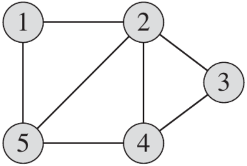
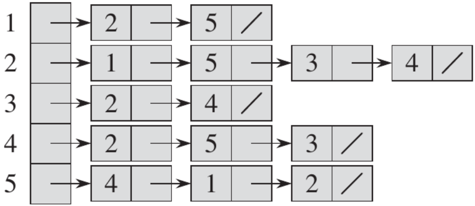




Representing undirected graphs

- Adjacency-matrix representation of undirected graphs





	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
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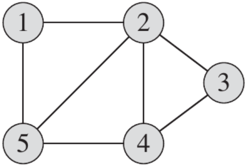
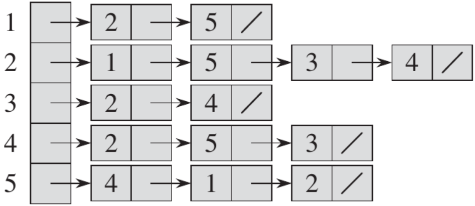
5	1	1	0	1	0

- Will the Adjacency-matrix for an undirected graph be symmetric?





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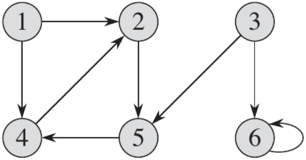

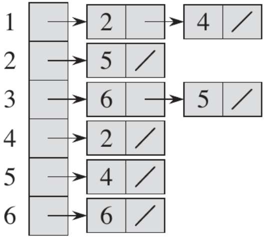

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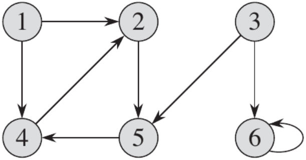

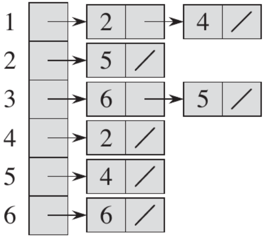

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

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Elementary Graph Algorithms

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- Dense graphs: $|E|$ is close to $|V|^2$.

Graph Representation : Weighted graphs

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  Weight function $w : E \rightarrow \mathbb{R}$.

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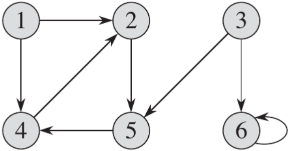

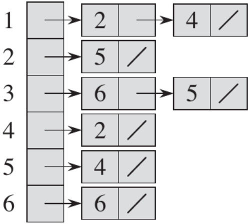

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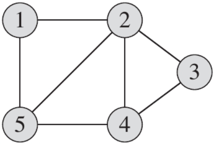
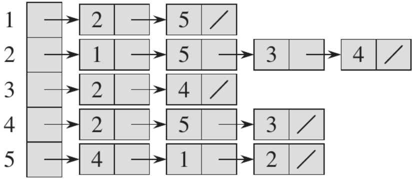
- Can we have an edge which is a self-loop in an undirected graph?





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- Adjacency-list representation requires $\Theta(V + E)$ memory compared to $\Theta(V^2)$ memory for Adjacency matrix.
- Suppose we wish to check whether a given edge (u, v) is present in a graph $G(V, E)$. Which representation will take more time to check this? Adjacency-list or adjacency-matrix?

Breadth-first search algorithm

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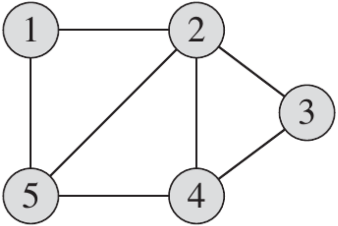
- Operation of Breadth-first search (P. 557)
- Breadth-first search pseudocode (P.556)

Breadth-first search : Undirected graph



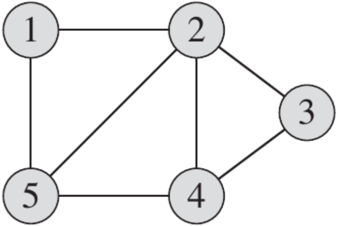
- Suppose we do a Breadth-first search starting from node 5. Which of the following orders of node visits is not possible under Breadth-first search?

a. 5,4,1,2,3

b. 5,1,2,4,3

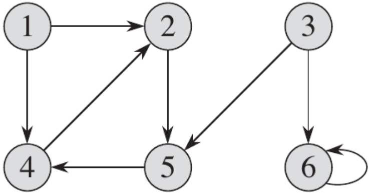
c. 5,1,2,3,4

Breadth-first search : Undirected graph



- If we start from node 5, what will be the distance of node 3 found by the Breadth-first search?

Breadth-first search : Directed graph



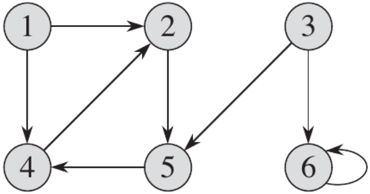
- Suppose we do a Breadth-first search starting from node 1. Which of the following orders of node visits is not possible under Breadth-first search?

a. 1,2,4,5,3,6

b. 1,2,5,4

c. 1,2,4,5

Breadth-first search : Directed graph



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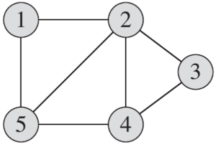
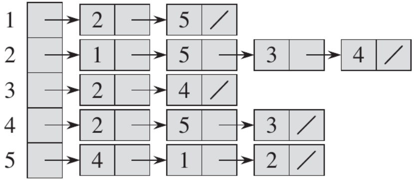
Breadth-first search





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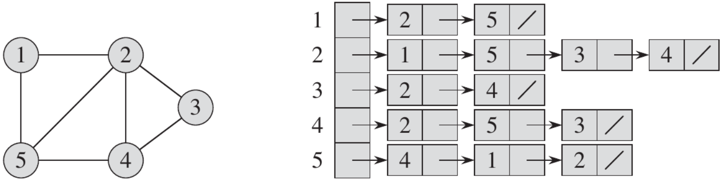
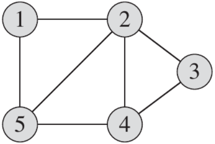
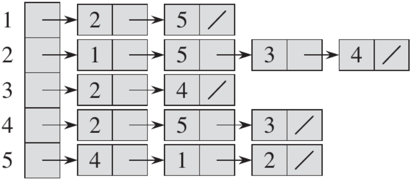


- Running time of BFS is

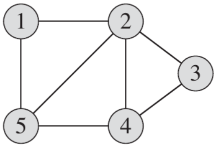
Breadth-first search

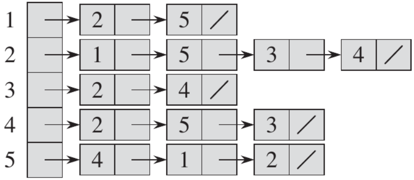




- Running time of BFS is $O(V + E)$.

Breadth-first search





- Running time of BFS is $O(V + E)$. (Aggregate analysis)

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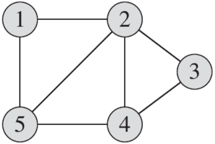

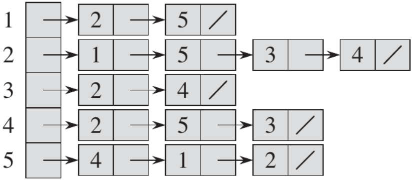

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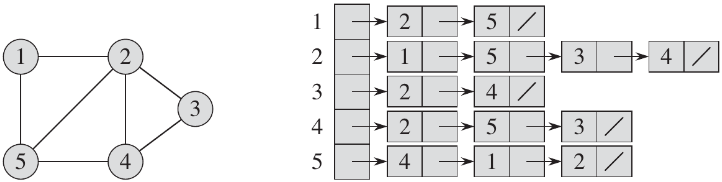
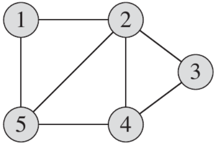
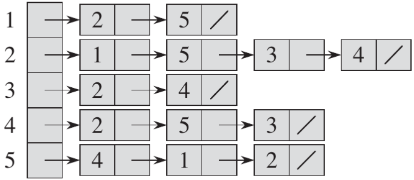
Breadth-first search



- Running time of BFS is $O(V + E)$. (Aggregate analysis)
- Why is it called Breadth-first search?
- When the BFS algorithm terminates, the attribute $v.d$ contains the shortest path (minimum number of edges) from node s to v.

Predecessor attribute π





- The attribute π stores the parent (predecessor) of each node in the breadth-first tree. We can use π attributes to find the shortest path from s to any vertex v.

Breadth-first search

Q. What will be the running time of BFS if we represent its input graph by an adjacency matrix and modify the algorithm to handle this form of graph input?

Breadth-first tree

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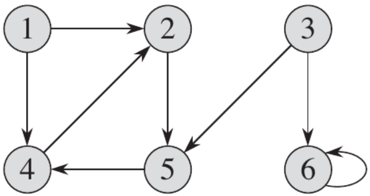
- Predecessor subgraph obtained after breadth-first search is called a breadth-first tree.

Breadth-first tree



- Suppose we do a Breadth-first search starting from node 1. Find the Breadth-First tree $G_\pi = (V_\pi, E_\pi)$ obtained?

Shortest path in a maze

$$\text{maze} = \begin{bmatrix} S & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & E \end{bmatrix}$$

Depth-first search

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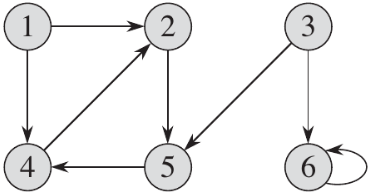
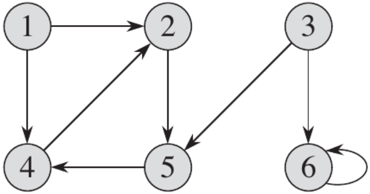
- Discovered and finished timestamps.
- DFS pseudocode (p. 565)



- Suppose that the first node to be discovered is 3. What are the possible finish timestamps for node 4?

Depth-first search : Directed graph



- Suppose that the first node to be discovered is 3. What are the possible finish timestamps for node 4?
- Which node remains white when node 3 is finished?

Predecessor subgraph

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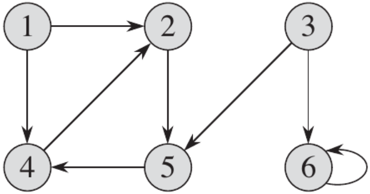
Depth-first Tree



- DFS helps us generate a depth-first forest G_π.

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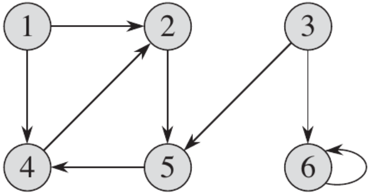
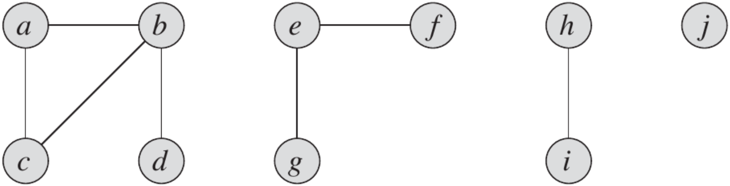


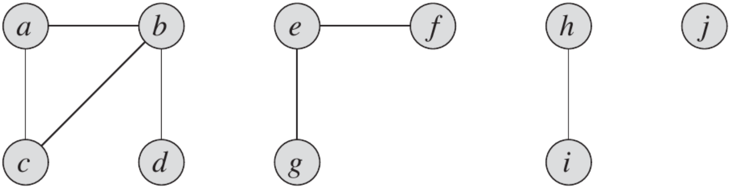
- DFS helps us generate a depth-first forest G_π.
- Suppose that the first node to be discovered is 3. What will the depth-first forest G_π look like?

Depth-first Forest



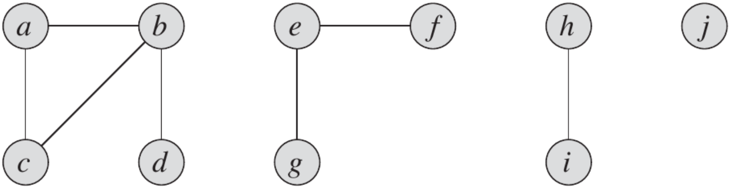
- Consider the undirected graph shown above.

Depth-first Forest



- Consider the undirected graph shown above.
  $|V| = 10$, $|E| = 7$.

Depth-first Forest



- Consider the undirected graph shown above.
  $|V| = 10$, $|E| = 7$.
- How many trees will the Depth-first forest G_π contain?

Classification of Edges

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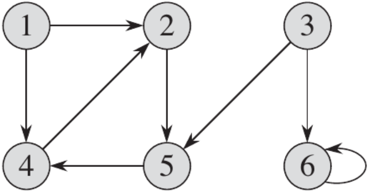

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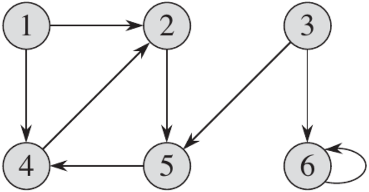

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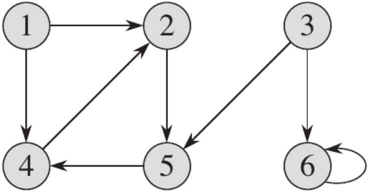

- What will be a back edge in the above graph?
- Back edge helps us detect a cycle in a graph.

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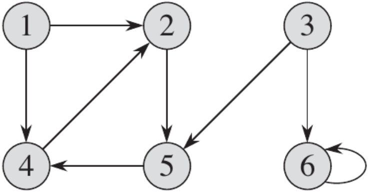

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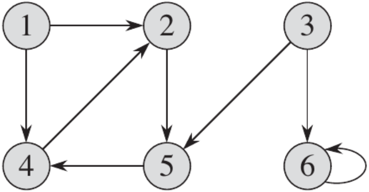

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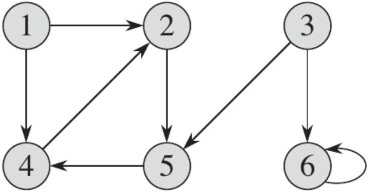

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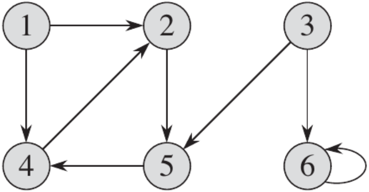

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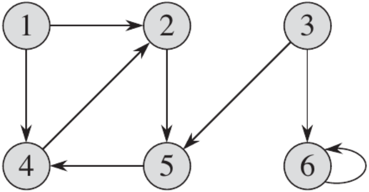

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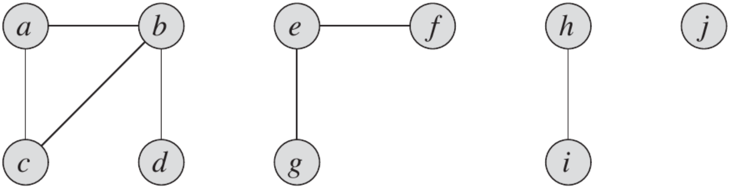
- **Cross edge** : if v is BLACK.

Connected components of an undirected graph



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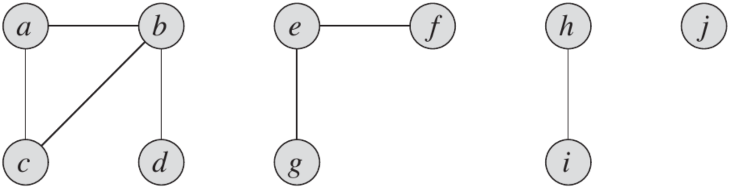
Connected components of an undirected graph



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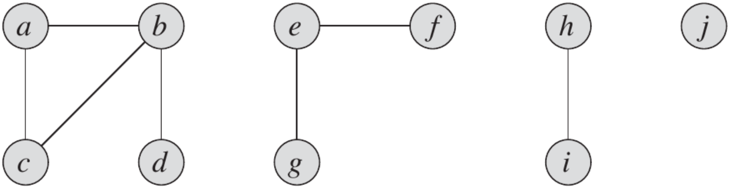

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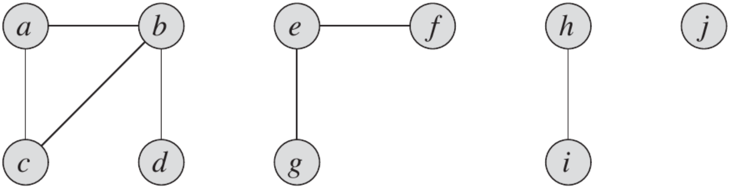
- The vertex subset must include all endpoints of the edge subset, but may also include additional vertices.
- Connected component is a connected subgraph that is not part of any larger connected subgraph.

Counting the connected components



- How many connected components are there in the above graph?

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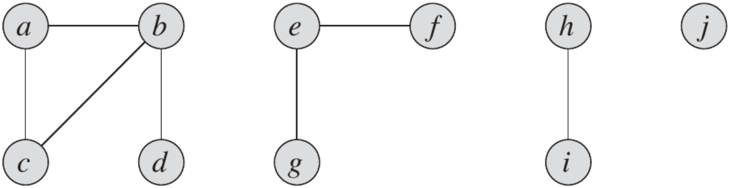

- How many connected components are there in the above graph?
- Can we modify the DFS algorithm to count the number of connected components?

Topological sort

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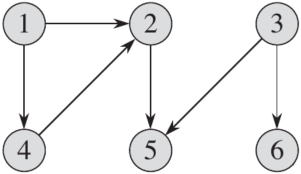

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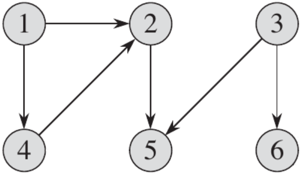

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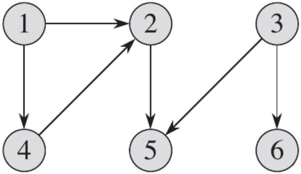

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- Which of the following is a topological sort?

a. 1, 4, 2, 5, 3, 6

b. 1, 4, 2, 3, 5, 6

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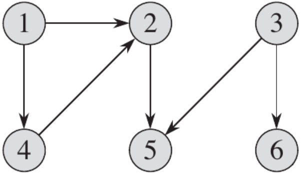

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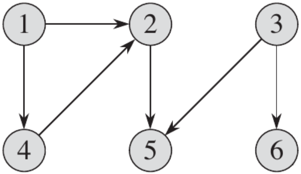

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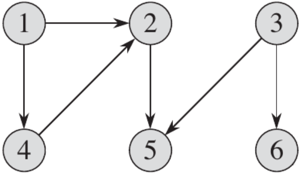

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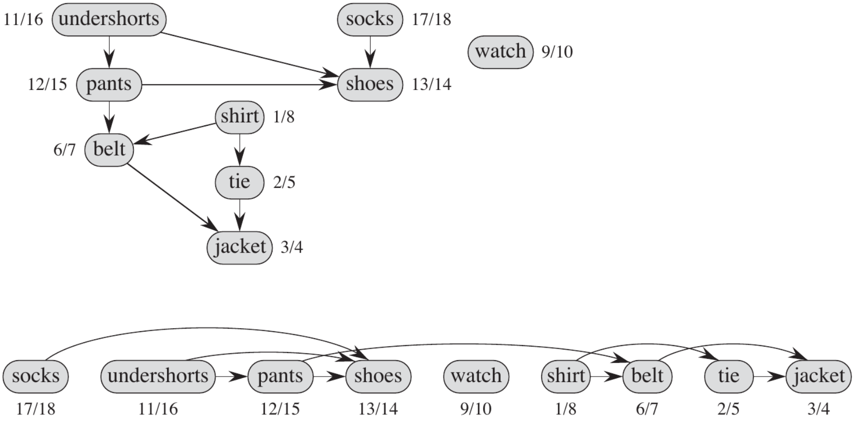
c. 3, 6, 1, 4, 2, 5

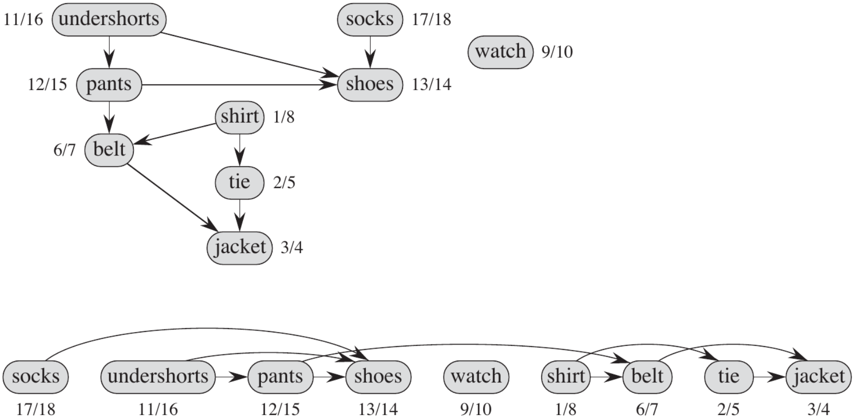
Topological sort example



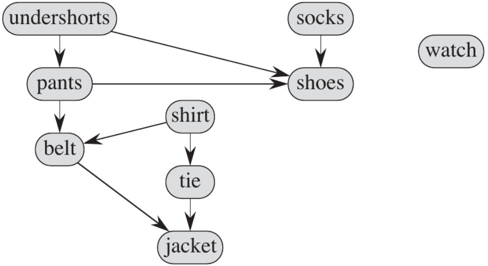
Topological sort pseudocode

```
TOPOLOGICAL-SORT(G)
    call DFS(G) to compute finishing times v.f for each vertex v
    as each vertex is finished, insert it onto the front of a linked list
    return the linked list of vertices
```

Topological sort example



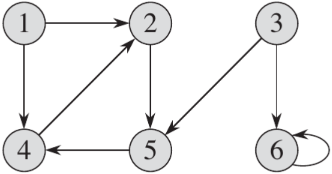
Topological sort example 2



- Let us say we consider nodes in the following order: Belt, Shoes, Shirt, Undershorts, Watch, Socks

Topological sort

- Topological sorting should fail if we detect a back edge.



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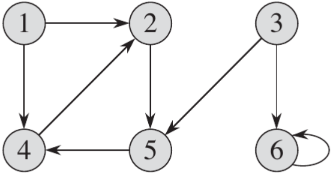

- Why does topological sort algorithm work?

Applications of DFS and Topological sort

- Operating system deadlock detection

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