Data Structures and Algorithms [1]

BITS-Pilani K. K. Birla Goa Campus

[1] Material for the presentation taken from Cormen, Leiserson, Rivest and Stein, *Introduction to Algorithms, Fourth Edition*;

Course plan

- Ch. 20, 21, 22.3 (Graph Algorithms)

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- Ch. 12, 13 (Binary Search Tree and Red-Black Tree)

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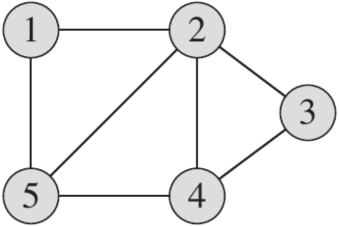
- Ch. 20, 21, 22.3 (Graph Algorithms)
- Ch. 12, 13 (Binary Search Tree and Red-Black Tree)
- Ch. 11 (Hash Tables)


1
2
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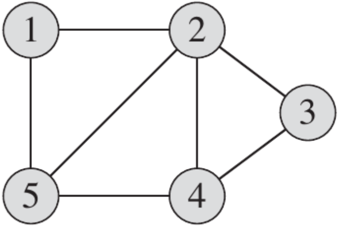
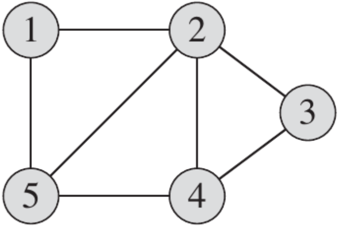


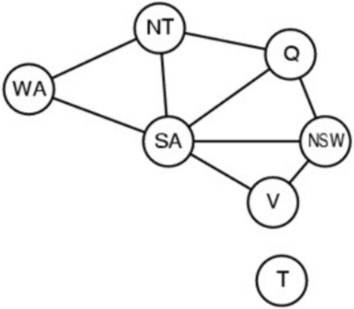
- Graphs are mathematical structures consisting of vertices and edges.



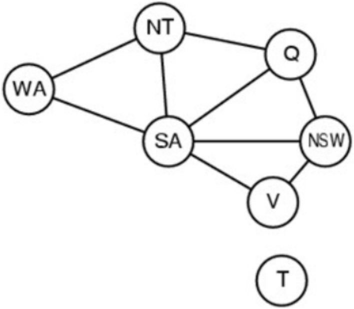
- Graphs are mathematical structures consisting of vertices and edges.
- The input to the graph algorithms will be a graph represented as $G = (V, E)$.

Map coloring problem

Vertex coloring problem



Vertex coloring problem



Module IV: Graph Algorithms

- Ch. 20: Elementary Graph Algorithms

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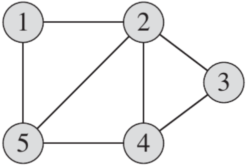
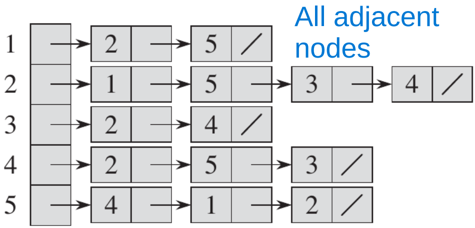
- Ch. 20: Elementary Graph Algorithms
  Breadth-first search, Depth-first search, Topological sort

Representing undirected graphs

- Adjacency-list representation of undirected graphs





Representing undirected graphs

- Suppose we sum the lengths of all the adjacency lists of an undirected graph. What will be the sum?

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  (i) V (ii) E (iii) V^2 (iv) VE
  (v) None of the above

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- Suppose we sum the lengths of all the adjacency lists of an undirected graph. What will be the sum? 2E

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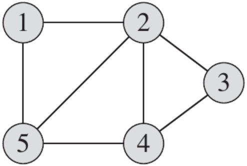
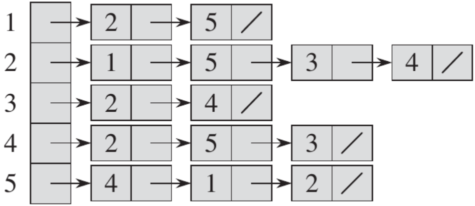
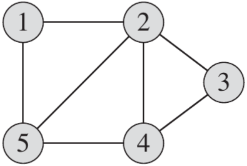
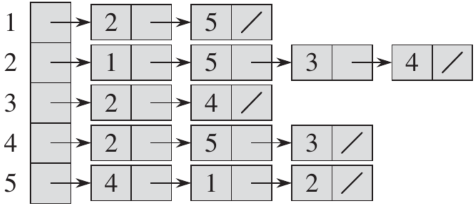




Representing undirected graphs

- Adjacency-matrix representation of undirected graphs



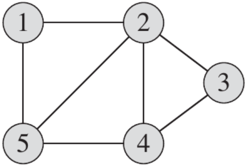


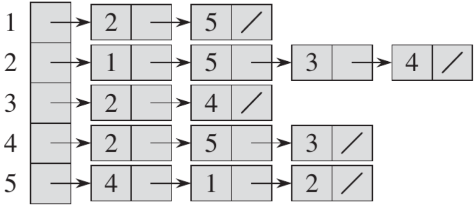
	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

1 if adjacent

Representing undirected graphs

- Will the Adjacency-matrix for an undirected graph be symmetric? Yes





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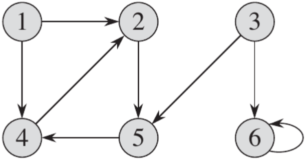
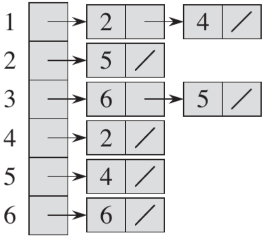
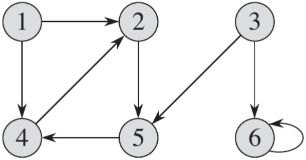
Representing directed graphs

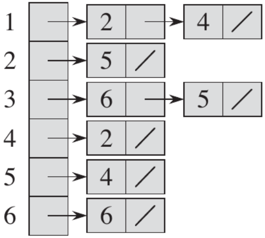




adjacency only 1-way not 2-way

Representing directed graphs





	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

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- Will the Adjacency-matrix for a directed graph be symmetric?

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- Sparse graphs: $|E|$ is much less than $|V|^2$.
- We may use adjacency-list to represent sparse graphs.
- Dense graphs: $|E|$ is close to $|V|^2$.

Graph Representation : Weighted graphs

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  Weight function $w : E \to \mathbb{R}$.

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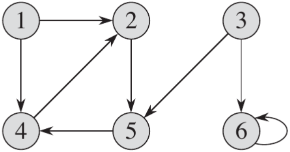

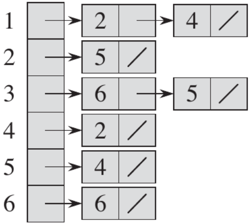

	1	2	3	4	5	6
1	0	1	0	1	0	0
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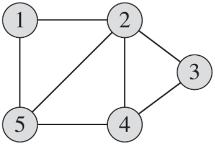
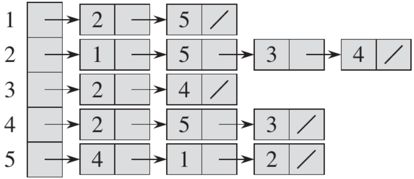
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

https://algorithmtutor.com/images/graph_representation_weighted.png

Graph Representation

- Can we have an edge which is a self-loop in an undirected graph?





$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 1 & 0 & 0 & 1 \\
2 & 1 & 0 & 1 & 1 & 1 \\
3 & 0 & 1 & 0 & 1 & 0 \\
4 & 0 & 1 & 1 & 0 & 1 \\
5 & 1 & 1 & 0 & 1 & 0 \\
\hline
\end{array}
$$

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- Adjacency-list representation requires $\Theta(V + E)$ memory compared to $\Theta(V^2)$ memory for Adjacency matrix.
- Suppose we wish to check whether a given edge (u, v) is present in a graph $G(V, E)$. Which representation will take more time to check this? Adjacency-list or adjacency-matrix?

Matrix better in this case, it will be o(2*v), must check for one vertex and then next once that is found. List you must check for one of the vertices and then follow that section of the linked list to find the edge.

Directed graph can double time but pretty much same complexity.

Breadth-first search algorithm

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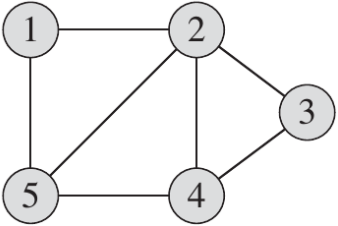
- Operation of Breadth-first search (P. 557)
- Breadth-first search pseudocode (P.556)

Breadth-first search : Undirected graph



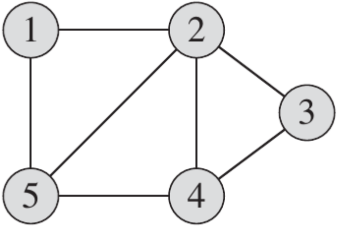
- Suppose we do a Breadth-first search starting from node 5. Which of the following orders of node visits is not possible under Breadth-first search?

a. 5,4,1,2,3

b. 5,1,2,4,3

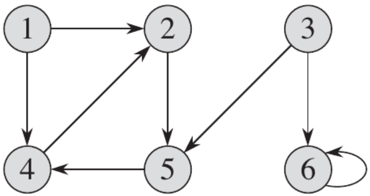
c. 5,1,2,3,4 c, since 4 is at less dist than 3

Breadth-first search : Undirected graph



- If we start from node 5, what will be the distance of node 3 found by the Breadth-first search?

Breadth-first search : Directed graph



- Suppose we do a Breadth-first search starting from node 1. Which of the following orders of node visits is not possible under Breadth-first search?

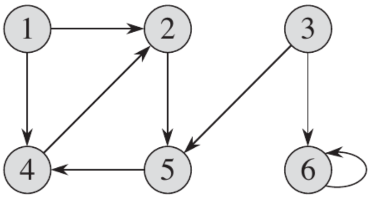
a. 1,2,4,5,3,6

b. 1,2,5,4

c. 1,2,4,5

Only c is possible

Breadth-first search : Directed graph



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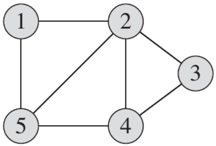
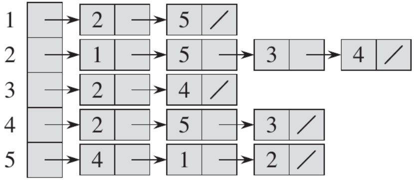
Breadth-first search





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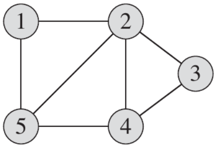
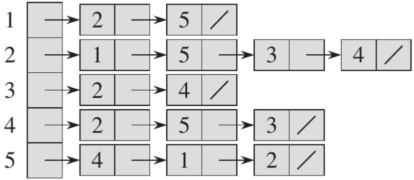
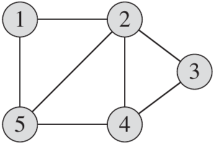
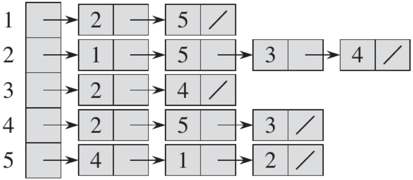




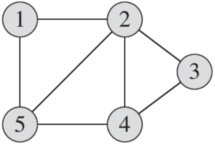
- Running time of BFS is

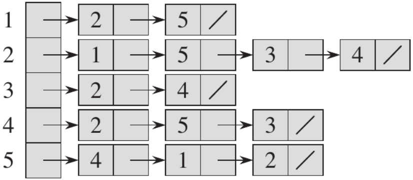
Breadth-first search





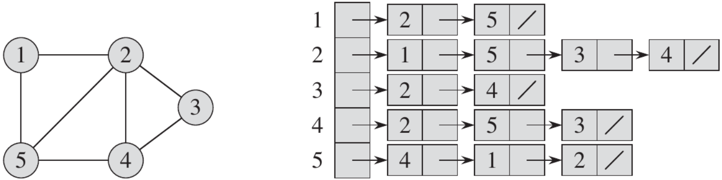
- Running time of BFS is $O(V + E)$.





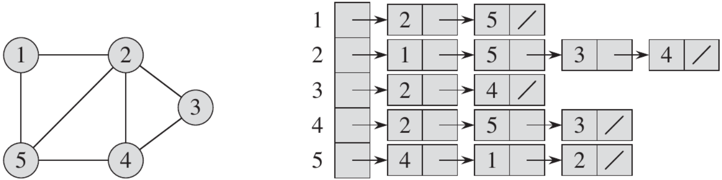
- Running time of BFS is $O(V + E)$. (Aggregate analysis)

Breadth-first search



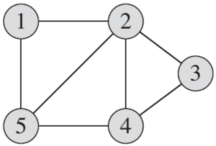
- Running time of BFS is $O(V + E)$. (Aggregate analysis)
- Why is it called Breadth-first search?

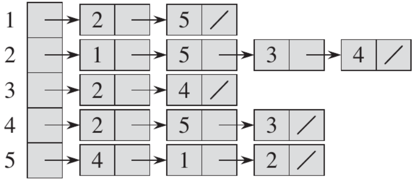
Breadth-first search



- Running time of BFS is $O(V + E)$. (Aggregate analysis)
- Why is it called Breadth-first search?
- When the BFS algorithm terminates, the attribute $v.d$ contains the shortest path (minimum number of edges) from node s to v.

Predecessor attribute π





- The attribute π stores the parent (predecessor) of each node in the breadth-first tree. We can use π attributes to find the shortest path from s to any vertex v.

Breadth-first search

Q. What will be the running time of BFS if we represent its input graph by an adjacency matrix and modify the algorithm to handle this form of graph input?

Breadth-first tree

- Predecessor subgraph $G_\pi = (V_\pi, E_\pi)$:

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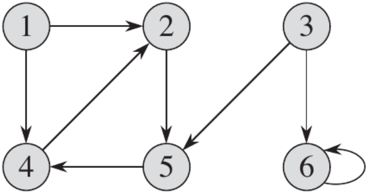
- Predecessor subgraph obtained after breadth-first search is called a breadth-first tree.

Breadth-first tree



- Suppose we do a Breadth-first search starting from node 1. Find the Breadth-First tree $G_\pi = (V_\pi, E_\pi)$ obtained?

Tree consists of parts of graph reachable from given node

Shortest path in a maze

$$\text{maze} = \begin{bmatrix} S & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & E \end{bmatrix}$$

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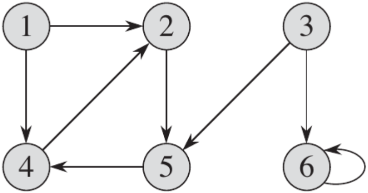
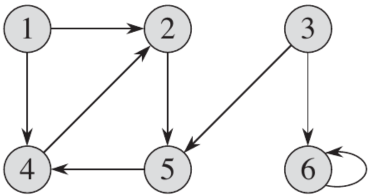
- Discovered and finished timestamps.
- DFS pseudocode (p. 565)



- Suppose that the first node to be discovered is 3. What are the possible finish timestamps for node 4?

Depth-first search : Directed graph



- Suppose that the first node to be discovered is 3. What are the possible finish timestamps for node 4?
- Which node remains white when node 3 is finished?

Predecessor subgraph

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- Running time of DFS is

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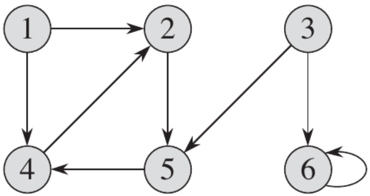
- Forest - set of trees
- DFS helps us generate a depth-first forest G_π.
- Depth-first forest depends on the order in which vertices are visited (Line 5 of DFS procedure).
- Running time of DFS is $\Theta(V + E)$.

Forest is set of trees obtained on performing DFS from various initial nodes

Depth-first Tree



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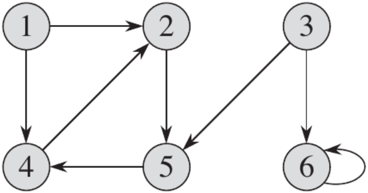
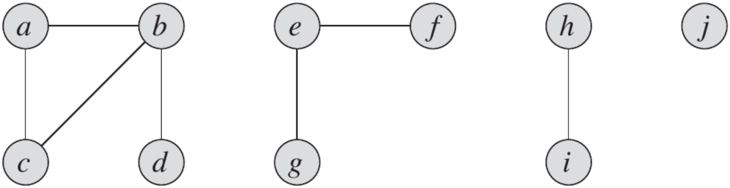


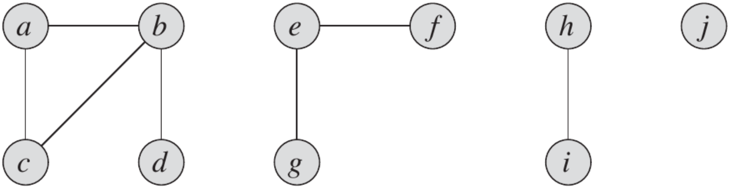
- DFS helps us generate a depth-first forest G_π.
- Suppose that the first node to be discovered is 3. What will the depth-first forest G_π look like?

Depth-first Forest



- Consider the undirected graph shown above.

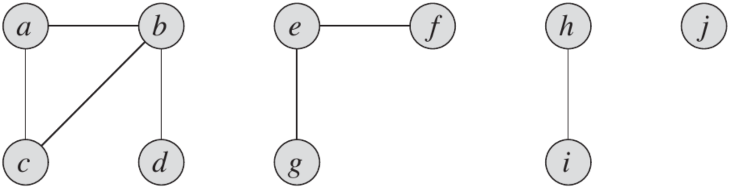
Depth-first Forest



- Consider the undirected graph shown above.

  $|V| = 10$, $|E| = 7$.

Depth-first Forest



- Consider the undirected graph shown above.
  $|V| = 10$, $|E| = 7$.
- How many trees will the Depth-first forest G_π contain?

Classification of Edges

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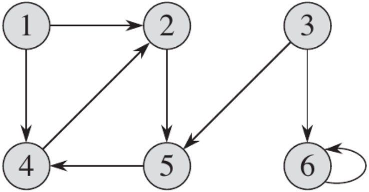

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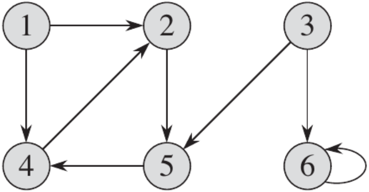

- What will be a back edge in the above graph?

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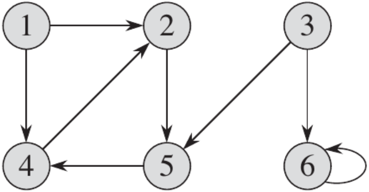

- What will be a back edge in the above graph? 4,2 is back edge
- Back edge helps us detect a cycle in a graph.

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- **Back edge** if v is GRAY.

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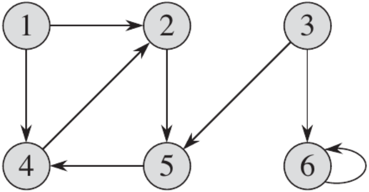

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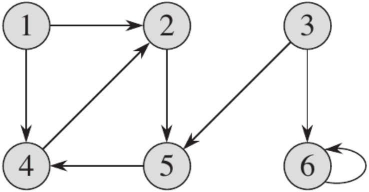

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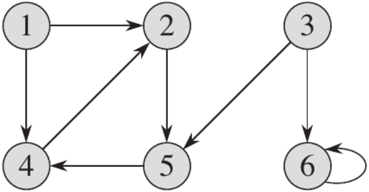

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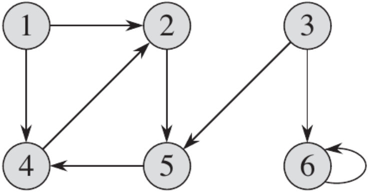

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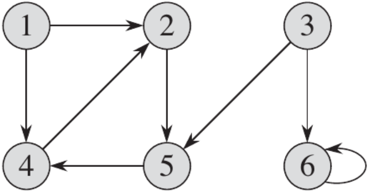

color implemented by map or hash table

- How can we detect a Forward edge using the *COLOR* attribute?
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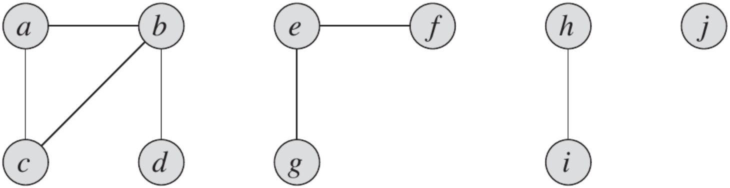
- All the remaining edges are Cross edges.
- **Cross edge** : if v is BLACK.

Connected components of an undirected graph



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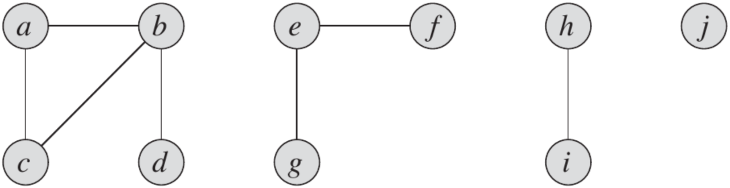
Connected components of an undirected graph



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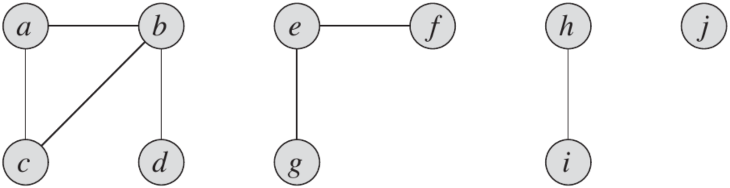
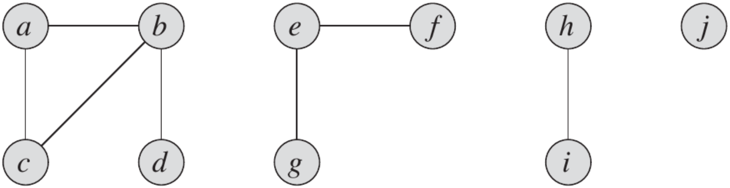


- Subgraph of a graph G is another graph formed from a subset of the vertices and edges of G.
- The vertex subset must include all endpoints of the edge subset, but may also include additional vertices.
- Connected component is a connected subgraph that is not part of any larger connected subgraph.

Counting the connected components



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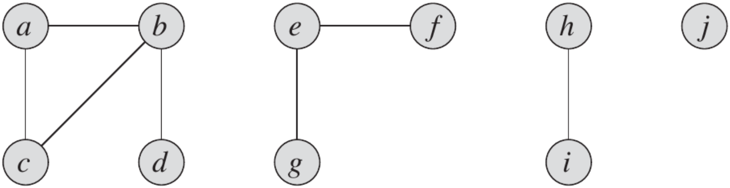

- How many connected components are there in the above graph?
- Can we modify the DFS algorithm to count the number of connected components?

Topological sort

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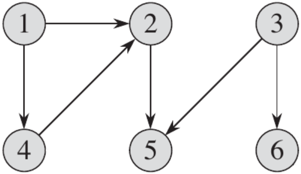

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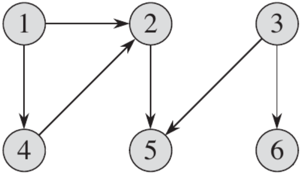

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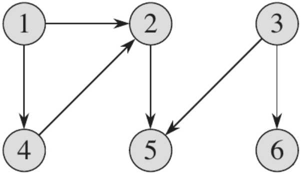

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- Which of the following is a topological sort?

a. 1, 4, 2, 5, 3, 6

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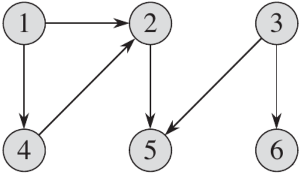

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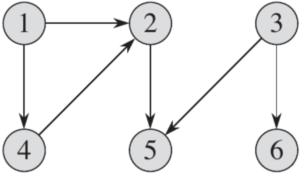

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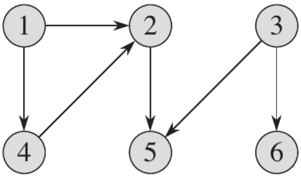

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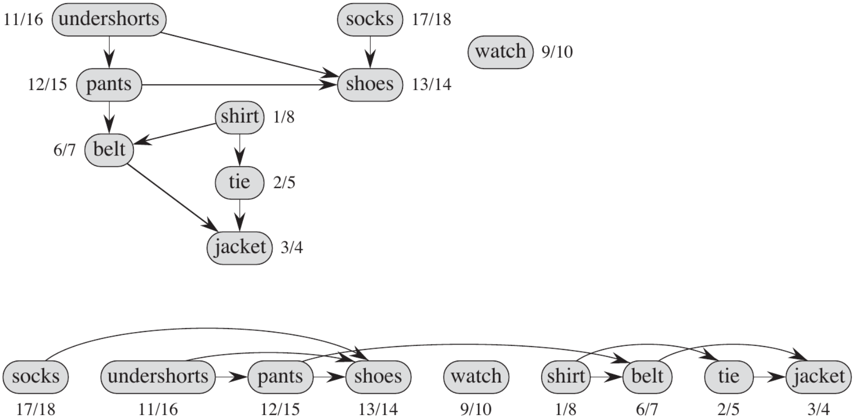
c. 3, 6, 1, 4, 2, 5

Topological sort example

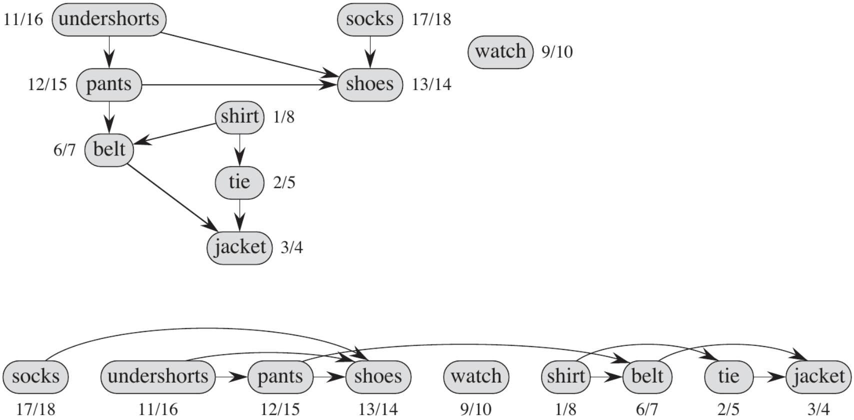


Topological sort pseudocode

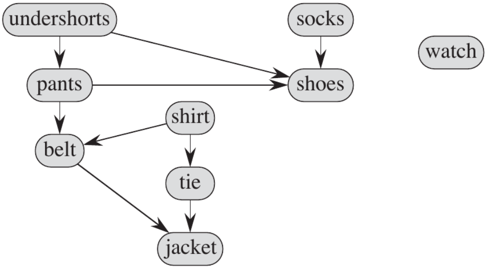
TOPOLOGICAL-SORT(G)

call DFS(G) to compute finishing times $v.f$ for each vertex v
as each vertex is finished, insert it onto the front of a linked list
return the linked list of vertices

Topological sort example



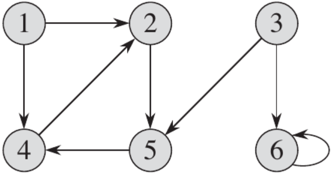
Topological sort example 2



- Let us say we consider nodes in the following order: Belt, Shoes, Shirt, Undershorts, Watch, Socks

Topological sort

- Topological sorting should fail if we detect a back edge.



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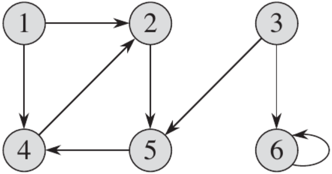

- Why does topological sort algorithm work?

Applications of DFS and Topological sort

- Operating system deadlock detection

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