Data Structures and Algorithms [1]

BITS-Pilani K. K. Birla Goa Campus

[1] Material for the presentation taken from Cormen, Leiserson, Rivest and Stein, *Introduction to Algorithms, Third Edition*;

Searching in constant time

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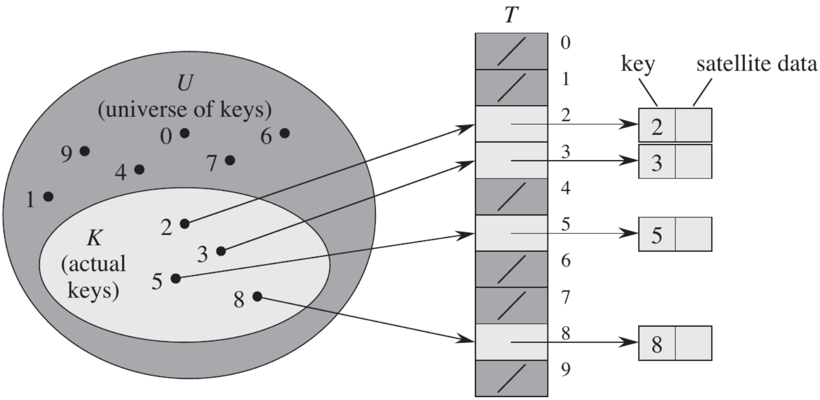
- Hash tables is a datastructure that allows us to perform SEARCH operation in $O(1)$ time.
- Average time to perform any operation (INSERT, SEARCH and DELETE) should be $O(1)$.

Satellite Data related to keys

- Roll : **1023**
  Name : Ravi
  YOB : 2000
- Roll : **0912**
  Name : Lata
  YOB : 2000
- Roll : **1756**
  Name : Sagar
  YOB : 1999
- Roll : **1504**
  Name : Mohit
  YOB : 1999
- Roll : **1393**
  Name : Maya
  YOB : 1999

Direct-address Table



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1. The universe U of keys is small. $U = 0, 1, \dots, m-1$
2. No two elements in the dynamic set have the same key.

Direct-address Table

DIRECT-ADDRESS-SEARCH(T, k)

return $T[k]$

DIRECT-ADDRESS-INSERT(T, x)

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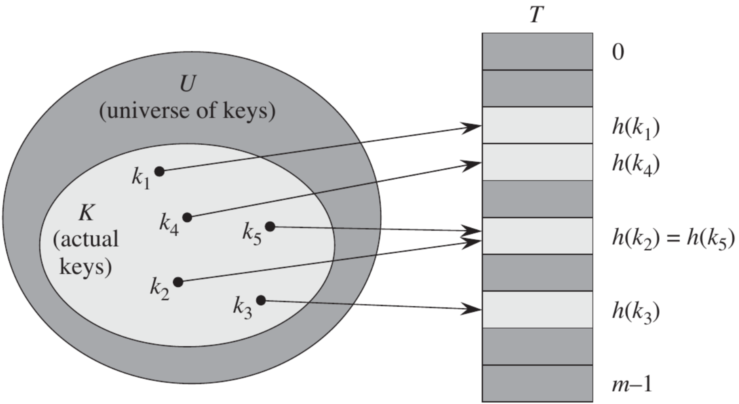
Hash tables



T
U
(universe of keys)
K
(actual keys)
k_1
k_4
k_5
k_2
k_3
0
$h(k_1)$
$h(k_4)$
$h(k_2) = h(k_5)$
$h(k_3)$
$m-1$

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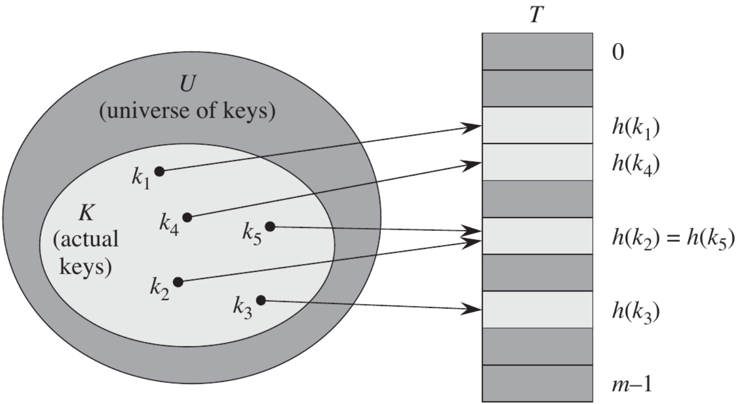

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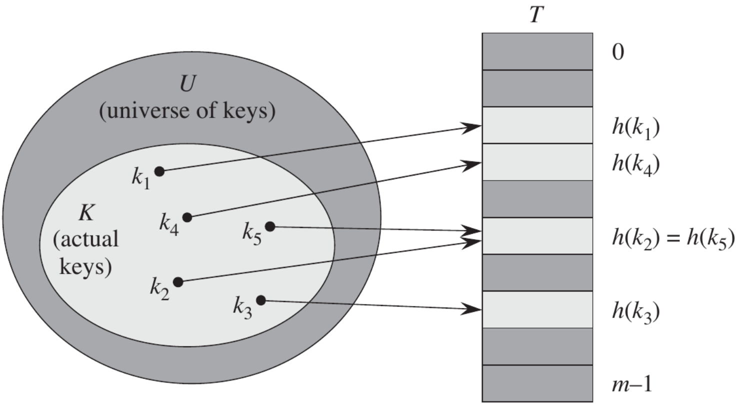

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- $h(k)$ is called the hash value of key k

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- Storage requirements of Hash table is only $\Theta(m)$, but collisions may occur.

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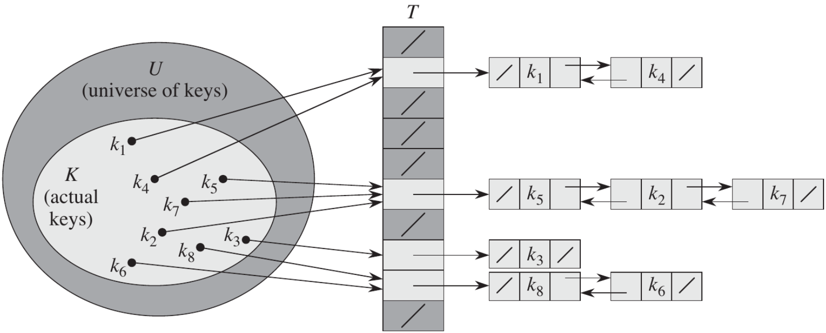
   In chaining, we place all the elements that hash to the same slot into the same linked list.

Collision resolution by chaining



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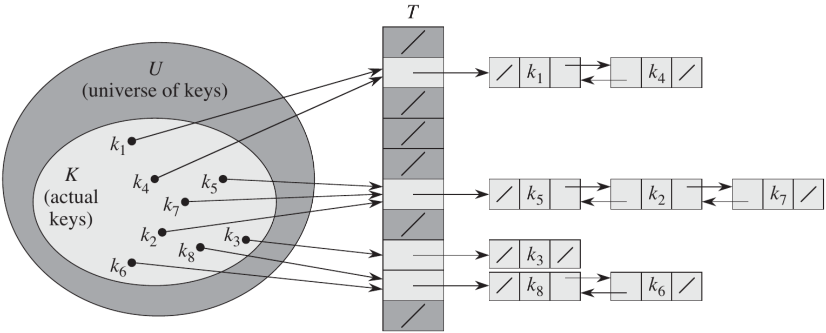

- Each slot contains pointer to the head of the linked list.

Operations on Hash table with chaining

CHAINED-HASH-INSERT(T, x)
1 LIST-PREPEND$(T[h(x.key)], x)$

CHAINED-HASH-SEARCH(T, k)
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- Worst-case running time $\Theta(n)$.

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- Time required to search an element with key k depends linearly on the length $n_{h(k)}$ of the list $T[h(k)]$.

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Theorem: (Unsuccessful search)

In a hash table in which collisions are resolved by chaining, an unsuccessful search takes average-case time $\Theta(1 + \alpha)$, under the assumption of independent uniform hashing.

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- Goal : Come up with heuristic methods to achieve independent uniform hashing.

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- The computed hash value should be independent of any pattern that might exist in the keys.

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- In our discussion, we will assume that keys are natural numbers.

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- A good choice for m : A prime number that is not too close to any power of 2.

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  $$\frac{\sqrt{5}-1}{2} = 0.6180339887\ldots$$

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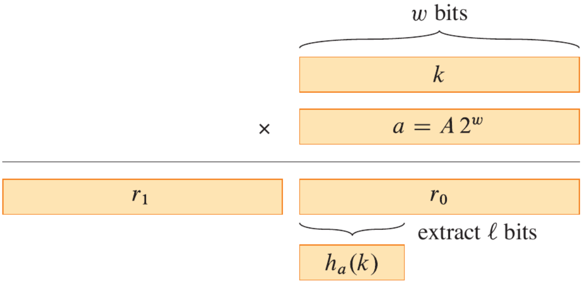
- When A is an irrational number, the pattern in the fractional part of kA becomes less predictable (there is no recurring pattern).

Multiply-shift Method



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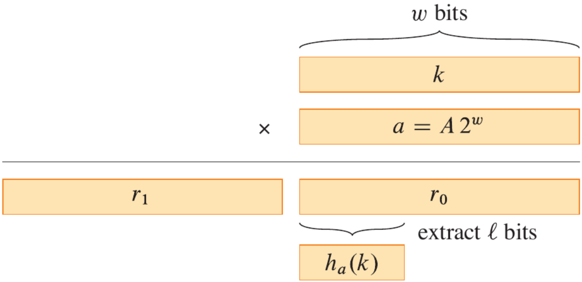

- Product (2-w bit value) $= r_1\,2^w + r_0$
- $h_a(k) = (k\,a \mod 2^w) \ggg (w - l)$

Static Hashing vs. Random Hashing

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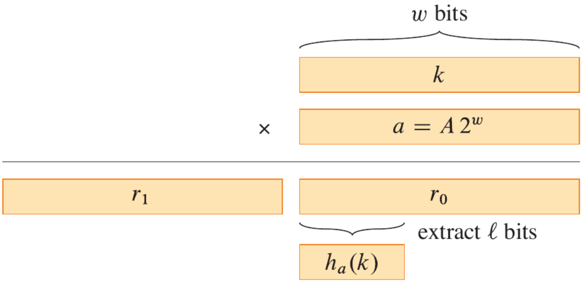
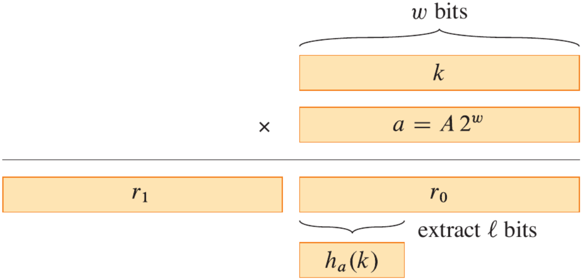
- Multiplication method : $h(k) = \lfloor m(kA \mod 1) \rfloor$
- Multiply-shift method :



Static Hashing vs. Random Hashing

- Division method : $h(k) = k \mod m$
- Multiplication method : $h(k) = \lfloor m(kA \mod 1) \rfloor$
- Multiply-shift method :



- $h(k) = (k(2^w A) \mod 2^w) \ggg (w - l)$

Take first l bits of remainder(result of mod)

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- **Theorem :** The family $\mathcal{H}_{pm}$ of hash functions defined below is universal.

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- Proof:

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