

Dynamics ME534 Project Memo

Cody Greener

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1 Equations of Motion (EOMs)

$$[M] \{\ddot{q}\} = [F]$$

$$[M] = \begin{bmatrix} m_p & 0 & 0 & 0 \\ 0 & m_p & 0 & 0 \\ 0 & 0 & I_{AO} + m_w d_w^2 & m_w d_w d_g \sin(\theta - \phi) \\ 0 & 0 & m_w d_w d_g \sin(\theta - \phi) & I_{WG} + m_w d_g^2 \end{bmatrix}$$

$$\{q\} = \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \end{pmatrix}$$

$$\{F\} = \begin{bmatrix} -\mu N \\ -m_p g \\ m_w d_w d_g \dot{\phi}^2 \cos(\theta - \phi) + g \cos(\theta) (m_w d_w - m_A d_A) \\ -m_w d_w d_g \dot{\theta}^2 \cos(\theta - \phi) - m_w g d_g \sin(\phi) \end{bmatrix}$$

2 Kinematic Constraints

Phase:	Sliding
n:	2
m:	4
p:	2
Kinematic Constraints (configuration form):	$(x - d_R \cos \theta)^2 + (y - d_R \sin \theta)^2 - l^2 = 0$ $y + h = 0$
Kinematic Constraints (velocity form):	$(x - d_R \cos \theta)\dot{x} + (y - d_R \sin \theta)\dot{y}$ $+ (d_R x \sin \theta - d_R y \cos \theta)\dot{\theta} = 0$ $\dot{y} = 0$
Constraint Coefficients [a]:	$\begin{bmatrix} x - d_R \cos \theta & y - d_R \sin \theta & d_R x \sin \theta - d_R y \cos \theta & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
Constraint Coefficients [da/dt]:	$\begin{bmatrix} \dot{x} + d_R \dot{\theta} \sin \theta & \dot{y} - d_R \dot{\theta} \cos \theta & d_R(\dot{x} \sin \theta + x \dot{\theta} \cos \theta - \dot{y} \cos \theta + y \dot{\theta} \sin \theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Phase:	Swinging
n:	3
m:	4
p:	1
Kinematic Constraints (configuration form):	$(x - d_R \cos \theta)^2 + (y - d_R \sin \theta)^2 - l^2 = 0$
Kinematic Constraints (velocity form):	$(x - d_R \cos \theta)\dot{x} + (y - d_R \sin \theta)\dot{y}$ $+ (d_R x \sin \theta - d_R y \cos \theta)\dot{\theta} = 0$
Constraint Coefficients [a]:	$\begin{bmatrix} x - d_R \cos \theta & y - d_R \sin \theta & d_R x \sin \theta - d_R y \cos \theta & 0 \end{bmatrix}$
Constraint Coefficients [da/dt]:	$\begin{bmatrix} \dot{x} + d_R \dot{\theta} \sin \theta & \dot{y} - d_R \dot{\theta} \cos \theta & d_R(\dot{x} \sin \theta + x \dot{\theta} \cos \theta - \dot{y} \cos \theta + y \dot{\theta} \sin \theta) & 0 \end{bmatrix}$
Phase:	Flying
n:	4
m:	4
p:	0
Kinematic Constraints (configuration form):	N/A
Kinematic Constraints (velocity form):	N/A
Constraint Coefficients [a]:	N/A
Constraint Coefficients [da/dt]:	N/A

3 Constraint Forces

$$T = -\lambda_1 \sqrt{(\dot{x} + d_R \dot{\theta} \sin \theta)^2 + (\dot{y} - d_R \dot{\theta} \cos \theta)^2}$$

$$N = \lambda_2$$

4 Plots & Tables of Results

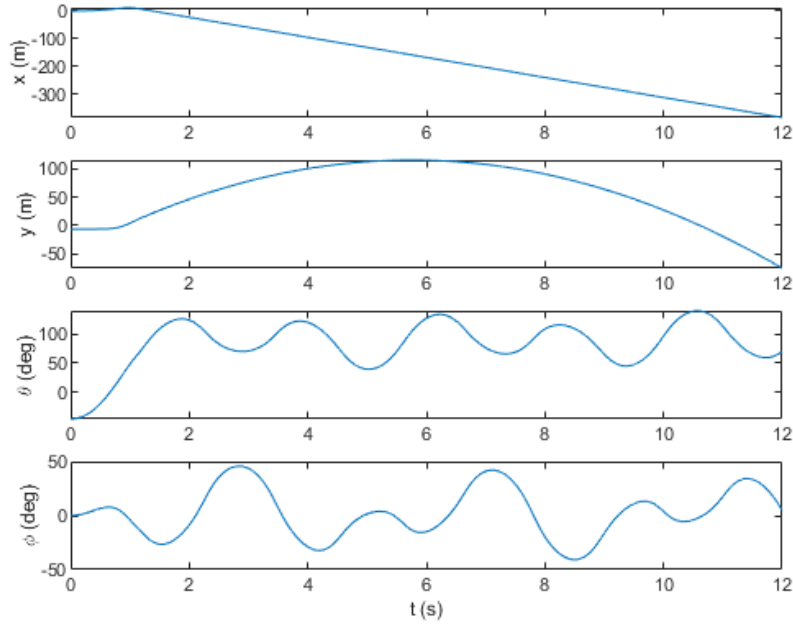


Figure 1: A plot of the four generalized coordinates as functions of time

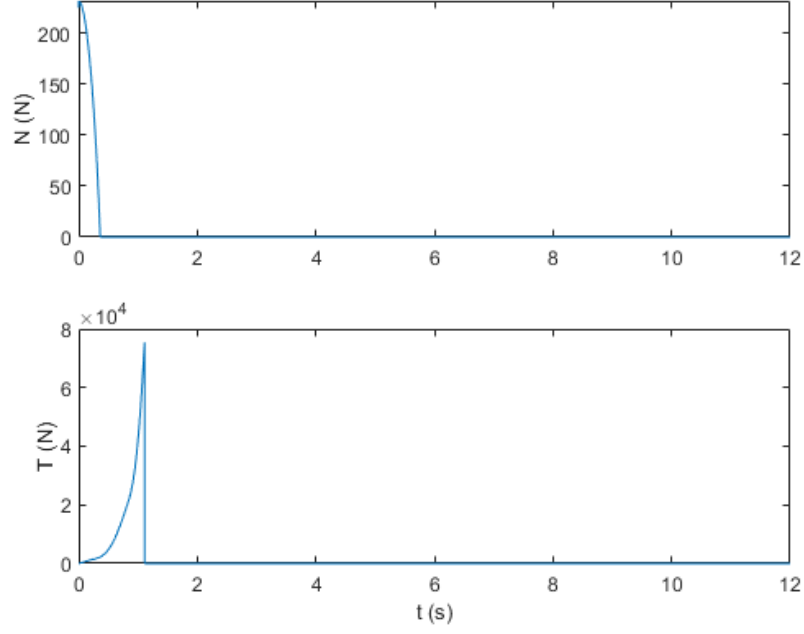


Figure 2: A plot of the two constrain forces (T and N) as functions of time

m_p (kg)	20	30	40
Range (m)	-494.4	-338.6	-234.6
Time of Impact (s)	10.48	10.74	10.22
Max Height (m)	117.3	121.9	108.7
Time Leaves Track (s)	0.3430	0.3634	0.3846
Time Leaves Sling (s)	1.083	1.113	1.154