IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

EEE PART IV: MEng and ACGI

CORRECTED COPY.
Q2

POWER SYSTEM ECONOMICS

Friday, 7 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : G. Strbac
Second Marker(s) : B.C. Pal

The Questions

Question 1

(a) List and discuss the key objectives of unbundling and liberalisation of electricity markets.

[3]

(b) Explain the notions of supply curve, demand curve, market clearing price and social welfare.

[2]

(c) How can option contracts be used for risk management? Explain the meaning of exercise price and option fee.

[3]

(d) Explain why electricity prices may vary with location. What is the difference between constraint costs and congestion costs (surplus)?

[3]

(e) The demand function of a transmission interconnector is given in the form of: $\pi_T = 7.2 - 0.015F$ (π_T is expressed in £/MWh and F is the capacity of the line in MW). Determine the capacity that would maximise the revenue to the transmission operator.

[3]

(f) If the annuitised investment cost of building the interconnector in (e) can be expressed as a linear function of its capacity: C = 13,140F [£], estimate the capacity that should be built to maximise the benefit for the entire system.

[3]

(g) Define Financial Transmission Rights and explain how these are used.

[3]

Question 2

Dimensionally incorrect BUT
0·02 has units...
Standard equ

p/kWh?

The electricity demand curve for a particular price-sensitive customer is given by the following expression: $\pi = -0.02Q + 6$, where Q is the quantity in kWh bought by the consumer in a given period, and π is the electricity price in p/kWh. The electricity producer's generation cost in the same period is given by $C = 0.015Q^2 + 0.8Q$ [p/h].

(a) Determine the expression for marginal production cost, and sketch the demand and supply functions. Then find the equilibrium price and demand at which the social welfare is maximised.

[4]

(b) For the situation in (a) determine the total production cost, average production cost, supplier's revenue and supplier's profit. Calculate the demand charges, gross demand benefit and consumer's surplus.

[4]

(c) If the price level is for some reason artificially fixed at $\boxed{\pi = 4.5 \text{ p/kWh}}$, calculate the level of consumption, consumer surplus, demand charges ~~and revenue~~ received by suppliers. What would be the percentage change in quantity bought if the price increased by 1%?

[3]

(d) For the price level in (c), calculate the producer's surplus and calculate the overall social welfare.

[2]

(e) If the government administratively decides to reduce the price to 3.6 p/kWh, determine the change in producer's generation output and the new revenue received by the producer. Find the producer's and consumer's surpluses. What is the social welfare?

[4]

(f) Calculate the social welfare at the equilibrium point (a), and welfare losses for cases (c) and (e). Discuss the differences between the social welfare in the two cases and the welfare at the equilibrium point.

[3]

Question 3

First Generation Company (FGC) has in its portfolio two generating units that have the following cost functions:

$$C_A = 10 + 1.2P_A + 0.05P_A^2 \text{ [£/h]}$$
$$C_B = 20 + 1.5P_B + 0.08P_B^2 \text{ [£/h]}$$

The demand that FGC needs to cover varies within a year. It can be assumed that the demand is constant within each of the two periods within the year. Demand levels as well as period durations are given in Table Q3.1.

Table Q3.1

	Period 1	*Period 2*
Demand (MW)	300	180
Period duration (% of year)	25%	75%

(a) How should the generators be dispatched if FGC needs to supply the demand at minimum cost in both periods of the year? What are the generation marginal prices in the two periods? What is the total annual profit made by FGC?

[8]

(b) How would the dispatch in (a) change if FGC, while supplying their demand, had the opportunity to buy or sell electricity on the spot market throughout the year at a price of 15 £/MWh? Calculate FGC's annual profit in this case.

[6]

(c) How would the dispatch in (a) change if the minimum output of unit B is 100 MW? Calculate the marginal prices in the two periods and calculate FGC's annual profit.

[6]

Question 4

a) What are the conditions for perfect competition? How can a company in an imperfect competition environment influence the market price? What is the relationship between the marginal cost of production and the market price in perfectly and imperfectly competitive markets?

[5]

b) Consider a market for electrical energy that is supplied by two generating companies whose costs are:

$C_A = 40\ P_A$ [£/h]
$C_B = 60\ P_B$ [£/h]

The inverse demand function of the market is given by: $\pi = 200 - 2D$ (π is the price in £/MWh and D is the demand in MW).

(i) If we assume a Bertrand model for the competition in this market, calculate the production of each of the companies, the resulting market price and profits made by both companies.

[4]

(ii) If we now assume that the companies compete with each other according to the Cournot model, calculate market price and demand for different combinations of production levels: P_A (20 MW, 25 MW, 30 MW, 35 MW) and P_B (15 MW, 20 MW, 25 MW) of the two companies. Determine their profits for each output combination. In which of these states will the market settle *i.e.* find equilibrium?

[8]

(iii) What is the principal difference between Bertrand and Cournot competition models?

[3]

Question 5

Consider the three-bus power system shown in Figure Q5. Demands at buses 2 and 3 and line reactance values are indicated in the figure.

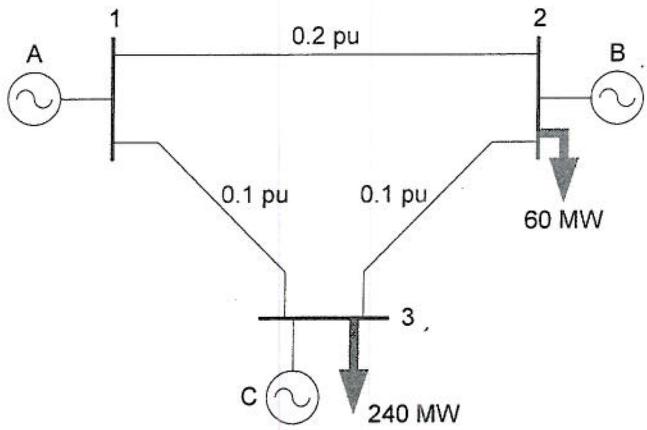

Figure Q5. A simple three-bus system

Assume that generating units A, B and C have the following marginal production costs:

$$MC_A = 12 + 0.1P_A \text{ [£/MWh]}$$
$$MC_B = 18 + 0.2P_B \text{ [£/MWh]}$$
$$MC_C = 45 \text{ [£/MWh]}$$

(a) Calculate the optimal unconstrained dispatch for these conditions, and the nodal prices. What is the hourly cost of this dispatch?

[3]

(b) Calculate the power flows in each line of the network if this dispatch was implemented.

[4]

(c) Assume that the flow in line 1-3 is limited to 160 MW for security reasons. How should the generators be re-dispatched in order to supply the demand at minimum cost without violating line 1-3 flow limit?

[5]

(d) Calculate the hourly cost of this constrained dispatch and the hourly cost of security.

[3]

(e) Calculate the marginal cost of energy at each node after the constraint on the flow on line 1-3 is taken into consideration. What is the total congestion surplus?

[5]

Question 6

(a) Consider two regions, A and B, of a small power system that are not connected. The marginal cost of generation in areas A and B can be modelled by the following expressions:

$MC_A = 10 + 0.02P_A$ [£/MWh]
$MC_B = 11 + 0.04P_B$ [£/MWh]

The load in regions A and B varies depending on the season, as specified in Table Q6.

Table Q6

	Winter	*Summer*
Demand A (MW)	100	70
Demand B (MW)	400	250
Duration (hours)	2500	6260

For both winter and summer, calculate the marginal costs in both regions and the corresponding generator outputs, hourly generator revenues and demand charges. What are the seasonal marginal values of transmission?

[4]

(b) If the capacity of the transmission link between the regions is never binding (*i.e.* it always exceeds the need), calculate the optimal generator dispatch, marginal costs in regions A and B, and generator revenues in both seasons. What is the marginal value of transmission?

[4]

(c) A well-paid consultant has proposed two schemes to be considered for construction of the 300 km long transmission line connecting regions A and B: (i) 80 MW and (ii) 160 MW link. The annuitised investment cost of transmission (including the allowable profit) is 37 £/MW.km.year. For each of the schemes calculate:

- Marginal prices in regions A and B for both seasons;
- Hourly generator payments, demand charges and congestion surpluses in both seasons;
- Annual network revenues (assuming that the transmission company charges for the use of link on the basis of short-run marginal cost) and the transmission company's annual profit.

[7]

(d) Explain which of the two schemes would be preferable from the viewpoint of:

(i) A regulated transmission company that maximises the benefit of transmission for the entire country;
(ii) A merchant transmission company that makes profit from buying electricity in region A and selling it in region B. [5]

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Power System Economics – Solutions 2010

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Solution to Question 1

Discussion along the following line is needed for full mark.

(i) Unbundling refers to the process of separating the activities of generation, transmission, distribution and retail of electricity, previously performed by a single vertically integrated utility, into different entities i.e. companies. This then allows for the competition to be introduced in the generation and retail business, while transmission and distribution are organised as regulated monopolies, ensuring fair and open network access to all market participants.
The reasons for liberalising electricity markets are the increased efficiency in the supply of electricity and the lower cost of electricity to consumers. This is achieved through introducing competition both at the wholesale level (generators compete to sell electrical energy) and the retail level (consumers choose from whom they buy electricity).

(ii) The *supply curve* indicates the value that the market price should take to make it worthwhile for the aggregated producers to supply a certain quantity of the commodity to the market. Similarly, the *demand curve* describes the quantity of products that the aggregated consumers are willing to purchase for a particular price in a given market. Demand curve is normally downward sloping, while the supply curve is normally upward sloping. *Market clearing price* or the *equilibrium price* is the price for which the quantity that the suppliers are willing to provide is equal to the quantity that the consumers wish to obtain. *Social welfare* is the sum of the net consumers' surplus and of the producers' profit, and it quantifies the overall benefit that arises from trading.

(iii) An *option contract* is an agreement between two parties, giving one party the right but not the obligation to buy goods from or sell goods to the other party at a certain price at a certain time in the future. A call option gives its holder the right to buy a given amount of a commodity at a price called the *exercise price*. A put option gives its holder the right to sell a given amount of a commodity at the exercise price. Whether the holder of an option decides to exercise its rights under the contract depends on the spot price for the commodity. When an option contract is agreed, the seller of the option receives a non-refundable *option fee* from the holder of the option. The buyer of the option on the other hand gets a guarantee that he/she will be able to buy/sell a commodity for at least the option exercise price, which can be used for risk management.

(iv) In situations where transmission network causes the generation dispatch to be constrained (i.e. the generators cannot be used in the most cost-efficient way), they segment the market and the price of electricity is not the same at each bus of the system. This is because the marginal generator is not the same in all parts of the network, but rather varies with location, as do the locational prices.
The *cost of constraints* is the cost of making the network secure and it is calculated as the difference between the cost of the constrained dispatch (dispatch when network constraints are taken into account) and the cost of the economic

dispatch (dispatch when network constraints are not taken into account). The *congestion cost (surplus)* is the difference between demand charges and generation payments, emerging from the congestion in the network. It is calculated as the sum of products of differences between the nodal prices at any two buses and the flow on the transmission line between those two buses:

$$S_{cong} = \sum_{i,j} (\pi_j - \pi_i) F_{i-j}$$

(v) The transmission operator's revenue is given by: $R = \pi_T \cdot F = 7.2F - 0.015F^2$. The capacity resulting in maximum revenue is obtained by differentiating this expression and making it equal to zero:

$$7.2 - 0.03F = 0 \Rightarrow F = 240 \text{ MW}$$

(vi) The optimal capacity from the perspective of global welfare satisfies the fact that:
Marginal value of transmission = Marginal investment cost of transmission
Value of transmission is given by: $\pi_T = 7.2 - 0.015F$
Marginal investment cost of transmission (hourly) = 13,140 / 8,760 = 1.5 £/MWh
Hence, $7.2 - 0.015F = 1.5 \Rightarrow F = 380$ MW.

(vii) FTRs are market instruments defined between any two nodes in the network, and they entitle their holders to a revenue equal to the product of the amount of transmission rights bought and the price differential between the two nodes: $R_{FTR} = F(\pi_1 - \pi_2)$. This enables a contract for difference between a producer at location 1 and a consumer at location 2 to be settled even in the presence of congestion in the network.

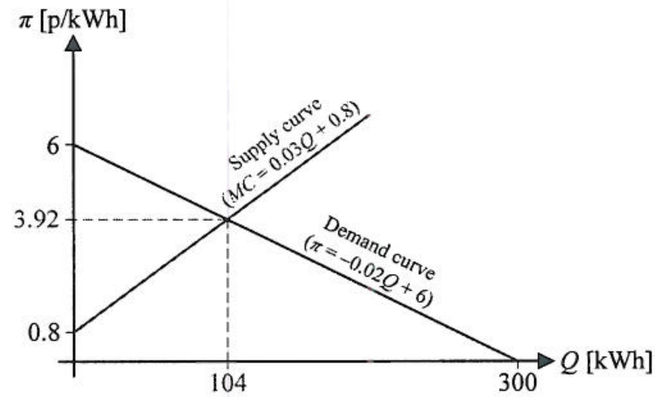
Solution to Question 2

(i) Marginal production cost: $MC = \dfrac{dC}{dQ} = 0.03Q + 0.8$ [p/kWh]

The equilibrium price and demand:

$$\pi = MC \Rightarrow -0.02Q + 6 = 0.03Q + 0.8$$
$$\Rightarrow \pi_{eq} = 3.92 \text{ p/kWh}, \; Q_{eq} = 104 \text{ kWh}$$

Sketch of the supply and demand functions:



(ii) Total production cost = $C(104) = 0.015 \cdot 105^2 + 0.8 \cdot 105 = 245.44$ p
Average production cost = Total production cost / 104 kWh = 2.36 p/kWh
Supplier's revenue = 104·3.92 = 407.68 p
Supplier's profit = Supplier's revenue – Production cost = 407.68 – 245.44 = 162.24 p

Demand charges = supplier's revenue = 407.68 p
Gross demand benefit = 0.5·(6 + 3.92)·104 = 515.84 p
Consumer's surplus = 0.5·(6 – 3.92)·104 = 108.16 p

(iii)Consumption: $Q = \dfrac{6-\pi}{0.02} = \dfrac{6-4.5}{0.02} = 75 \text{ kWh}$

Demand charges and suppliers' revenues are: $Q \cdot \pi = 75 \cdot 4.5 = 337.5 \text{ p}$

Consumer's surplus $= \dfrac{1}{2} \cdot 75 \cdot (6 - 4.5) = 56.25 \text{ p}$

The percentage change in quantity in reaction to a 1% increase in price is quantified as the *price elasticity*, and is calculated as:

$$\varepsilon = \frac{\pi}{Q} \cdot \frac{dQ}{d\pi} = \frac{4.5}{75} \cdot \frac{d}{d\pi}\left(\frac{6-\pi}{0.02}\right) = \frac{4.5}{75} \cdot (-50) = -3$$

Thus, the quantity bought would drop by 3%.

(iv)Producer's surplus = Producer's revenue – production cost
$= 337.5 - C(75) = 337.5 - 144.375 = 193.125 \text{ p}$

Social welfare = Producer's surplus + Consumer's surplus
= 193.125 + 56.25 = 249.375

(v) The quantity that would be purchased by the consumer is:

$$Q_D = \frac{6-3.6}{0.02} = 120 \text{ kWh}$$

However, the producer will only produce up to the level where this price equals its marginal cost: $MC = 0.03Q + 0.8 = 3.6 \Rightarrow Q_S = 93.33\text{kWh}$

Producer's revenue = 93.33 kWh · 3.6 p/kWh = 336 p
Producer's surplus = 336 – C(93.33) = 336 – 205.33 = 130.67 p
Price that the consumer would be willing to pay for the quantity of 93.33 kWh is:
$\pi = -0.02 \cdot 93.33 + 6 = 4.13$ p
Consumer's surplus is given by the area of the trapezoid defined by the following (π,Q) pairs: (3.6, 0), (3.6, 93.33), (4.13, 93.33) and (6, 0). Its area is equal to:

$$\frac{(6-3.6)+(4.13-3.6)}{2} \cdot 93.33 = 136.89\,\text{p}$$

Finally, social welfare is the sum of producer's and consumer's surpluses, and is equal to 267.56 p, which is 18.18 p higher than in (ii).

(vi) At the equilibrium point we have:
Demand payments = Producer's revenue = 3.92 p/kWh · 104 kWh = 407.68 p
Producer's surplus = 407.68 – C(104) = 407.68 – 245.44 = 162.24 p

Consumer's surplus = $\dfrac{104 \cdot (6-3.92)}{2}$ = 108.16 p

Social welfare = Consumer's surplus + Producer's surplus = 270.40 p
Social welfare at the equilibrium point is larger than welfares obtained in cases (iii) and (v). The welfare loss in (ii) is 21.03 p, and in (iv) it is 2.84 p. This illustrates that the social welfare is maximised when the price is determined by the intersection of supply and demand curves, and that any administrative price fixing tends to reduce the global welfare.

5

Solution to Question 3

(i) We first find the marginal cost functions for the two units by differentiating their total cost functions:

$MC_A = 1.2 + 0.1P_A$ [£/MWh]
$MC_B = 1.5 + 0.16P_B$ [£/MWh]

At the optimal dispatch the two units will have the same marginal costs. In addition, the sum of their outputs needs to be equal to the load in a given period.

$$MC_A = MC_B \rightarrow 1.2 + 0.1P_A = 1.5 + 0.16P_B$$
$$P_A + P_B = D$$

Rearranging the equations yields:

$$P_A = \frac{0.16D + 0.3}{0.26},\ P_B = \frac{0.1D - 0.3}{0.26},\ \pi = \frac{0.016D + 0.342}{0.26}$$

By inserting the demand levels in the two periods into the above results, we obtain:

Period 1: $P_{A1} = 185.77$ MW, $P_{B1} = 114.23$ MW, $\pi_1 = 19.78$ £/MWh
Period 2: $P_{A2} = 111.92$ MW, $P_{B2} = 68.08$ MW, $\pi_2 = 12.39$ £/MWh

Hourly profit earned by FGC is: $\pi \cdot (P_A - P_B) - C_A(P_A) - C_B(P_B)$, which amounts to 2739.4 £/h in Period 1, and 967.1 £/h in Period 2. Total annual profit is calculated by multiplying these values with period duration:

$$\Omega_{annual} = (2739.4 \cdot 0.25 + 967.1 \cdot 0.75) \cdot 8760 = £12.35\,\text{m}$$

(ii) If FGC had access to the market with a constant price of 15 £/MWh, it would reduce the output of its units in Period 1 (when its marginal cost is higher than 15 £/MWh), and increase it in Period 2 (when its marginal cost is below the market price). In both periods FGC would adjust the unit outputs so that their marginal costs are exactly equal to 15 £/MWh.
This means that unit outputs will be the same in both periods:
$P_A = 138$ MW, $P_B = 84.375$ MW
Since $P_A + P_B = 222.375$ MW, this means that FGC will need to buy additional 77.625 MW in the market in Period 1, while in Period 2 it will sell an excess of 42.375 MW in the market.
Hourly profits are now the same in both periods: 1491.7 £/h, which yields an annual profit of £13.07m.

(iii) Obviously, the dispatch in Period 1 will not be affected by the minimum output constraint, and therefore the outputs in Period 1 are again: $P_{A1} = 185.77$ MW, $P_{B1} = 114.23$ MW, with the price of $\pi_1 = 19.78$ £/MWh.

In Period 2 FGC now needs to increase the output of unit B in order to satisfy its operating constraint. This means that the output of unit A will have to be reduced in order to balance generation and demand:

$$P_{A2} = 80 \text{ MW}, P_{B2} = 100 \text{ MW}$$

The price in Period 2 will now be determined by the marginal cost of unit A:

$$\pi_2 = MC_A = 1.2 + 0.1P_{A2} = 9.2 \text{ £/MWh}$$

Now the hourly profits are: Period 1 – 2739.4 £/h, Period 2 – 260 £/h.
The annual profit in this case is:

$$\Omega_{annual} = (2739.4 \cdot 0.25 + 260 \cdot 0.75) \cdot 8760 = \text{£}7.71\text{m}$$

Solution to Question 4

a) In a perfectly competitive market no participant has the ability to influence the market price through its individual actions (i.e. all participants act as price takers). This assumption is valid if the number of market participants is large and if none of them controls a large proportion of the production or consumption. Perfect competition is a highly desirable goal because it encourages efficient market behaviour.
Strategic market players can manipulate the prices either by withholding quantity (physical withholding) or by raising the asking price (economic withholding), compared to the perfectly competitive case.
In the presence of imperfect competition market prices tend to be higher than producers' marginal costs (which determine the price in perfectly competitive markets).

b)

(i) In the Bertrand competition model, Company A will set its price marginally below the generation cost of Company B in order to squeeze it out of the market. The market price will therefore be: $\pi = 60$ £/MWh.

Demand for this price will be: $D = \dfrac{200-\pi}{2} = 70\,\text{MW}$. All demand will be supplied by Company A, i.e. $P_A = 70$ MW, $P_B = 0$ MW.
Profit made by Company A = $(\pi - 40)\cdot P_A = 140$ £/h.

(ii) The results of the Cournot competition for different output levels of generators A and B is given in the following table.

		P_A (MW)							
		20		25		30		35	
P_B (MW)	15	35	1800	40	2000	45	2100	50	2100
		1050	130	900	120	750	110	600	100
	20	40	1600	45	1750	50	1800	55	1750
		1200	120	1000	110	800	100	600	90
	25	45	1400	50	1500	55	1500	60	1400
		1250	110	1000	100	750	90	500	80

Each element of the table (corresponding to one output combination) is subdivided into four cells with the following meaning:

Demand	Profit of A
Profit of B	Price

The red line marks the Nash equilibrium, i.e. the state where the market will settle according to Cournot competition model. In that state none of the companies can change its output without diminishing its profit.

(iii) The key difference between Bertrand and Cournot competition models is that the first models imperfect competition by introducing competition on prices, and the latter assumes competition takes place through companies choosing which quantity to produce in order to maximise profits.

Solution to Question 5

a. In the unconstrained case marginal costs of all active generators should be the same. Unit C has a constant marginal cost which does not depend on its output level. If units A and B reached the output corresponding to 45 £/MWh, we would have:

$$12 + 0.1P_A = 45 \rightarrow P_A^* = 330 \text{ MW}$$
$$18 + 0.2P_B = 45 \rightarrow P_B^* = 135 \text{ MW}$$

This suggests that the total output would need to be at least 465 MW in order for the marginal cost to reach 45 £/MWh, and the total load in our three-bus system is only 300 MW. Hence, the marginal cost will be lower than 45 £/MWh and unit C will not be used. Dispatch of units A and B is found from the following system of equations:

$$MC_A = MC_B \rightarrow 12 + 0.1P_A = 18 + 0.2P_B$$
$$P_A + P_B = 240 + 60 = 300$$

This yields: $P_A = 220$ MW, $P_B = 80$ MW, $P_C = 0$ MW. All nodal prices are equal to: $\pi_1 = \pi_2 = \pi_3 = 12 + 0.1 \cdot 220 = 34$ £/MWh.
The cost functions of the two units are found by integrating the marginal cost functions (assuming the constant to be zero):

$$C_A(P_A) = 12P_A + 0.05P_A^2,\ C_B(P_B) = 18P_B + 0.1P_B^2$$

The hourly costs are therefore: $C_A = 5060$ £/h, $C_B = 2080$ £/h, and the total hourly cost is 7140 £/h.

b. The line flows are obtained from the following system of linear equations:

$$F_{12} + F_{13} = P_A = 220$$
$$F_{23} - F_{12} = P_B - D_B = 20$$
$$F_{12} \cdot X_{12} + F_{23} \cdot X_{23} - F_{13} \cdot X_{13} = 0$$

The solution to this system is the following: $F_{12} = 50$ MW, $F_{23} = 70$ MW, $F_{13} = 170$ MW.

c. With the 160 MW limit on line 1-3, we have a 10 MW overload in the unconstrained case. In order to relieve the overloading, it is necessary to decrease output from unit A and increase from unit B by the same amount. The change in outputs needs to result in a decrease of flow in line 1-3 by 10 MW. It will depend on the network topology, as specified in the following system of equations:

$$\Delta P_A + \Delta P_B = 0$$

$$\Delta P_A \cdot \frac{X_{12} + X_{23}}{X_{12} + X_{23} + X_{13}} + \Delta P_B \cdot \frac{X_{23}}{X_{12} + X_{23} + X_{13}} = 0.75\Delta P_A + 0.25\Delta P_B = -10$$

The solution to this system is: $\Delta P_A = -20$ MW, $\Delta P_B = 20$ MW. This means that the new dispatch will be: $P_A = 200$ MW, $P_B = 100$ MW, $P_C = 0$ MW.
Line loadings for this case will be: $F_{12} = 40$ MW, $F_{23} = 80$ MW, $F_{13} = 160$ MW.

d. The hourly cost of this dispatch, using the same approach as in a), is: C_A = 4400 £/h, $C_B = 2800$ £/h, and the total hourly cost is 7200 £/h.
The hourly cost of security is the difference between generation costs in constrained and unconstrained case: $C_{sec} = 7200 - 7140 = 60$ £/h.

e. Marginal cost of electricity at every node is the cost of supplying an additional MWh at that node. Marginal generator at bus 1 is unit A, and its marginal cost is:

$$\pi_1 = 12 + 0.1 \cdot 200 = 32 \text{ £/MWh}$$

Marginal generator at bus 2 is unit B, as unit A cannot increase its output due to the flow limit on line 1-3. Therefore, nodal price at bus 2 is:

$$\pi_2 = 18 + 0.2 \cdot 100 = 38 \text{ £/MWh}$$

For the bus 3, we cannot simply increase outputs of units A or B by 1 MW as this would overload the line 1-3. Due to network topology, an increase of output of unit A by 1 MW causes an overloading of line 1-3 by 0.75 MW, while the same increase by unit B causes an overload of 0.25 MW. Therefore, in order to supply 1 MW of electricity to bus 3 without increasing the loading of line 1-3, we need to solve the following equations:

$$\delta P_A + \delta P_B = 1$$
$$0.75 \cdot \delta P_A + 0.25 \cdot \delta P_B = 0$$

Solving this yields: $\delta P_A = -0.5$ MW, $\delta P_B = 1.5$ MW, which means that unit A needs to reduce its output by 0.5 MW, and unit B needs to increase output by 1.5 MW in order to supply additional 1 MW to bus 3. The nodal price for bus 3 is therefore:

$$\pi_3 = 1.5MC_B - 0.5MC_A = 41 \text{ £/MWh}$$

An alternative approach would be to supply the additional 1 MW by unit C, but its marginal cost is higher than the above price, so it will not be used.
The sum of congestion surpluses across all three lines is obtained as follows:

$$\text{Surplus} = (\pi_2 - \pi_1)F_{12} + (\pi_3 - \pi_2)F_{23} + (\pi_3 - \pi_1)F_{13} = 1920 \text{ £/h}$$

Solution to Question 6

a. Since there is no interconnection, generator outputs are equal to local demand levels:

$$P_A^w = D_A^w = 100 \text{ MW},\ P_A^s = D_A^s = 70 \text{ MW}$$
$$P_B^w = D_B^w = 400 \text{ MW},\ P_B^s = D_B^s = 250 \text{ MW}$$

Marginal costs are obtained from generators' costs:

$$\pi_A = MC_A \rightarrow \pi_A^w = 10 + 0.02 \cdot 100 = 12 \text{ £/MWh},\ \pi_A^s = 10 + 0.02 \cdot 70 = 11.4 \text{ £/MWh}$$
$$\pi_B = MC_B \rightarrow \pi_B^w = 11 + 0.04 \cdot 400 = 27 \text{ £/MWh},\ \pi_B^s = 11 + 0.04 \cdot 250 = 21 \text{ £/MWh}$$

Generator revenues are equal to demand charges:

$$R_A = \pi_A \cdot D_A \rightarrow R_A^w = 12 \cdot 100 = 1200 \text{ £/h},\ R_A^s = 11.4 \cdot 70 = 798 \text{ £/h}$$
$$R_B = \pi_B \cdot D_B \rightarrow R_B^w = 27 \cdot 400 = 10{,}800 \text{ £/h},\ R_B^s = 21 \cdot 250 = 5250 \text{ £/h}$$

Marginal value of transmission is found as the difference between the two nodal prices:

$$\pi_T^w = \pi_B^w - \pi_A^w = 15 \text{ £/MWh},\ \pi_T^s = \pi_B^s - \pi_A^s = 9.6 \text{ £/MWh}$$

b. The optimal dispatch for the unconstrained case is obtained from the following pair of equations:

$$MC_A = MC_B$$
$$P_A + P_B = D_A + D_B$$

This system yields the following solutions:

$$P_A = \frac{1 + 0.04(D_A + D_B)}{0.06},\ P_B = \frac{0.02(D_A + D_B) - 1}{0.06}$$

The optimal unconstrained dispatch is:

Winter: $P_A = 350$ MW, $P_B = 150$ MW
Summer: $P_A = 230$ MW, $P_B = 90$ MW

Marginal costs: $\pi_A^w = \pi_B^w = 17$ £/MWh
$\pi_A^s = \pi_B^s = 14.6$ £/MWh

Generator revenues: $R_A^w = 5950$ £/h, $R_B^w = 2550$ £/h
$R_A^s = 3358$ £/h, $R_B^s = 1314$ £/h

Marginal value of transmission equals zero in both seasons (as the capacity is unconstrained).

c.

(i) $F = 80$ MW
Marginal prices are found using: $\pi_A = 10 + 0.02(P_A + F)$, $\pi_B = 11 + 0.04(P_B - F)$. This yields:

$$\pi_A^w = 10 + 0.02\cdot 180 = 13.6 \text{ £/MWh},\ \pi_A^s = 10 + 0.02\cdot 150 = 13 \text{ £/MWh}$$
$$\pi_B^w = 11 + 0.04\cdot 320 = 23.8 \text{ £/MWh},\ \pi_B^s = 11 + 0.04\cdot 170 = 17.8 \text{ £/MWh}$$

Hourly generator payments / demand charges:

$$R_A = \pi_A\cdot D_A \rightarrow R_A^w = 13.6\cdot 180 = 2448 \text{ £/h},\ R_A^s = 13\cdot 150 = 1950 \text{ £/h}$$
$$R_B = \pi_B\cdot D_B \rightarrow R_B^w = 23.8\cdot 320 = 7616 \text{ £/h},\ R_B^s = 17.8\cdot 170 = 3026 \text{ £/h}$$

Hourly congestion surpluses:

$$S_{cong} = (\pi_A - \pi_B)F \rightarrow S_{cong}{}^w = (23.8 - 13.6)\cdot 80 = 816 \text{ £/h},$$
$$S_{cong}{}^s = (17.8 - 13)\cdot 80 = 384 \text{ £/h}$$

Annual investment cost: $I_{ann} = k\,\lambda\, F = 37\cdot 300\cdot 80 = 888{,}000$ £/yr.

Annual revenue is calculated from hourly congestion surpluses and period durations d_w and d_s:
$$R_{ann} = S_{cong}{}^w d_w + S_{cong}{}^s d_s = 816\cdot 2500 + 384\cdot 6260 = 4{,}443{,}840 \text{ £/yr}$$

Finally, the annual profit is found as the difference between the congestion revenue and investment cost:
$$\Omega_{ann} = R_{ann} - I_{ann} = 3{,}555{,}840 \text{ £/yr}$$

(ii) $F = 160$ MW
Marginal prices:

$$\pi_A^w = 10 + 0.02\cdot 260 = 15.2 \text{ £/MWh},\ \pi_A^s = 10 + 0.02\cdot 230 = 14.6 \text{ £/MWh}$$
$$\pi_B^w = 11 + 0.04\cdot 240 = 20.6 \text{ £/MWh},\ \pi_B^s = 11 + 0.04\cdot 90 = 14.6 \text{ £/MWh}$$

Hourly generator payments / demand charges:

$$R_A = \pi_A\cdot D_A \rightarrow R_A^w = 15.2\cdot 260 = 3952 \text{ £/h},\ R_A^s = 14.6\cdot 230 = 3358 \text{ £/h}$$
$$R_B = \pi_B\cdot D_B \rightarrow R_B^w = 20.6\cdot 240 = 4944 \text{ £/h},\ R_B^s = 14.6\cdot 90 = 1314 \text{ £/h}$$

Hourly congestion surpluses:

$$S_{cong} = (\pi_A - \pi_B)F \rightarrow S_{cong}{}^w = (20.6 - 15.2)\cdot 160 = 864 \text{ £/h},$$
$$S_{cong}{}^s = (14.6 - 14.6)\cdot 160 = 0 \text{ £/h}$$

Annual investment cost: $I_{ann} = k\,\lambda\, F = 37\cdot 300\cdot 160 = 1{,}776{,}000$ £/yr.

Annual revenue:
$$R_{ann} = S_{cong}{}^w d_w + S_{cong}{}^s d_s = 864\cdot 2500 + 0\cdot 6260 = 2{,}160{,}000 \text{ £/yr}$$

13/13

Annual profit:

$$\Omega_{ann} = R_{ann} - I_{ann} = 384{,}000 \text{ £/yr}$$

d.

(i) A merchant transmission company (making money from buying electricity in one region and selling it in another) seeks to maximise its annual profit, and would therefore prefer the first option (F = 80 MW), where the profit is larger by an order of magnitude.

(ii) A regulated transmission company normally wants to maximise the system-level benefits of the transmission network. This means that the congestion surplus collected by the network operator needs to be offset by the investment cost of building the interconnecting line. Ideally, since allowable profits are already calculated into the investment cost, the goal would be to build the capacity at the level which yields zero profits. Out of the two cases, clearly the second one (F = 160 MW) is closer to the ideal situation, since the over-recovery of cost is much smaller than for the 80 MW link.