General comments on EE2-21 Feedback Systems paper 2012

1. The students have done relatively well on this question, scoring approximately 78%.
   (a) This is a mechanical modeling question and is a somewhat typical study group question. Figure 1.1 has come up in questions before.
      i. Typical study group question.
      ii. Typical study group question.
      iii. Typical study group question.
      iv. A bit tricky, since it requires the steady-state value of the derivative of a variable (rather than the variable itself).
      v. A bit tricky since it uses all the results above. It also asks for a physical interpretation.
   (b) This is a Nyquist diagram/Routh-Hurwitz question and is mostly typical of study group questions. Figure 1.2 has come up in questions before.
      i. Typical study group question.
      ii. Typical study group question.
      iii. Typical study group question, however, can be done much more quickly if the student uses the answer to Part (1.b.ii) above.
      iv. Typical study group question.
      v. Typical study group question, however, it can be done more quickly if the students use Parts (1.b.iii) and (1.b.iv) above.
      vi. Typical study group question.
2. This question combines knowledge about Nyquist analysis and the Routh-Hurwitz criterion in a slightly non-standard way for compensator design. The students did less well on this question, scoring an average mark of 61%.
   (a) Standard study group question.
   (b) The Nyquist diagram can be more easily drawn if the students make use of Part (2.a) above.
   (c) This uses the extended Nyquist stability criterion in that it requires the determination of closed-loop stability for all possible gains. The students tend to make elementary mistakes in signs, inversions and inequalities.
   (d) This part is quite tricky since there are two ways of achieving the specifications of a compensated gain margin of 2. In the first, $K = 0.25$, which results in an infinite phase-margin (since the resulting Nyquist diagram is within the unit circle). For the second, we can take $K = -0.5$ (typically the students discount compensators with negative gain), which results in a phase-margin of 180°. Many students expect phase-margins between 0° and 90°.
3. This is a a relatively straightforward design question that uses basic concepts of stability from a system's characteristic equation. It turned out to be less tricky than I expected, and the students did relatively well scoring approximately 69%.
   (a) Most students got the closed-loop poles right, but many did not give the correct comment on the closed-loop stability.
   (b) Most students got the closed-loop poles right, but many did not give the correct comment on the closed-loop stability.
   (c) Most students seem to have understood the concept of PD compensator design.
      i. This is standard study group question, and most students did well.
      ii. This was from the notes, and most students got it right.
      iii. This is also mostly from the notes, and most students got it right.
      iv. A standard question, although a few students couldn't evaluate the limit.

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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

ISE PART II: MEng, BEng and ACGI

Corrected Copy

FEEDBACK SYSTEMS

Friday, 1 June 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

Answer ALL questions. Question 1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : I.M. Jaimoukha
Second Marker(s) : S. Evangelou

1. a) Figure 1.1 shows a mass-spring system where K, D and M have the standard interpretation. The signal $u(t)$ represents an applied force and $y(t)$ the displacement from the rest position.

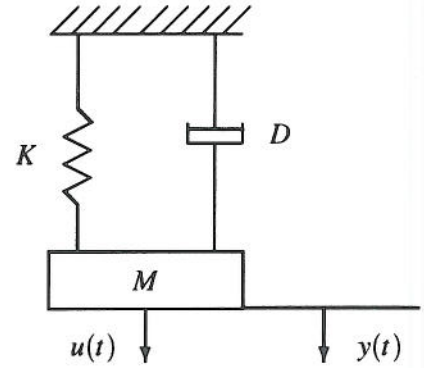


Figure 1.1

i) Derive the differential equation relating $u(t)$ to $y(t)$. [3]

ii) Evaluate the transfer function relating $u(s)$ to $y(s)$. [3]

iii) Let $u(t)$ be a unit impulse applied at $t=0$. For this part of the question, take $M=1$, $D=3$ and $K=2$ in appropriate units. Evaluate $y(t)$. [3]

iv) Take $K=0$ and let $u(t)$ be a unit step applied at $t=0$. Find the terminal velocity $v_{ss} = \lim_{t\to\infty} v(t)$ where $v(t) = \dot{y}(t)$. [3]

v) Take $K=0$, $M=75kg$ and let $u(t) = 75 \times g$ where $g = 10ms^{-2}$. Find the value of D for which the terminal velocity as defined above is $2ms^{-1}$. Comment on your answer. [4]

b) In Figure 1.2 below, $G(s) = 4/(s+1)^3$ and K is a gain.

i) Determine the steady-state error for a unit step reference signal assuming the closed-loop is stable. [4]

ii) Use the Routh Hurwitz criterion to determine the range of values of K for closed-loop stability. [4]

iii) Determine the value of $K > 0$ for which the closed-loop is marginally stable. What is the frequency of the resulting oscillations? [4]

iv) Sketch the Nyquist diagram of $G(s)$, indicating the low and high frequency portions. [4]

v) Let $K=1$. Use the Nyquist criterion, which should be stated, to show that the closed-loop is stable. Find the gain margin. [4]

vi) Let $K=10$. Use the Nyquist criterion to show that the closed-loop is unstable. How many unstable poles does the closed-loop have? [4]

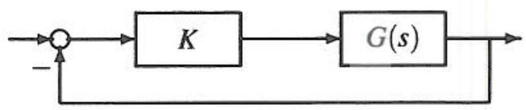


Figure 1.2

2. Consider the feedback control system in Figure 2.1 below. Here,

$$G(s) = \frac{2(s-1)}{(s+1)^2}$$

and $K(s)$ is the transfer function of a compensator.

a) Let $K(s)$ be a constant compensator $K(s) = K$. Construct a Routh array to find the values of K, call them K_1 and K_2, such that the closed-loop is marginally stable with $K_1 < K_2$. [6]

b) Sketch the Nyquist diagram of $G(s)$, clearly indicating the low and high frequency portions. Use the Routh array above to find the real–axis intercepts.[8]

c) Let $K(s)$ be a constant compensator $K(s) = K$. State the Nyquist stability criterion and use the Nyquist diagram to determine the number of unstable close–loop poles when:

i) $-\infty < K < K_1$, [2]

ii) $K_1 < K < 0$, [2]

iii) $0 < K < K_2$. [2]

iv) $K_2 < K < \infty$. [2]

d) Design a constant compensator $K(s) = K$ so that the closed-loop is stable and the gain margin of the compensated system is equal to 2. Comment on the phase-margin of the compensated system. [8]

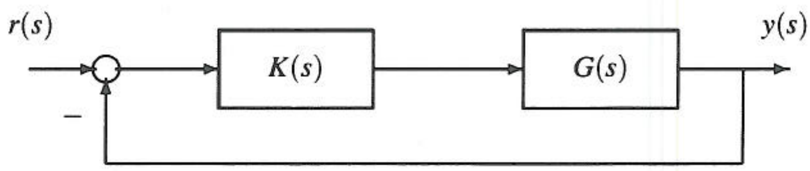


Figure 2.1

3. Let

$$G(s) = \frac{1}{s^2}$$

and consider the feedback loop shown in Figure 3.1 below.

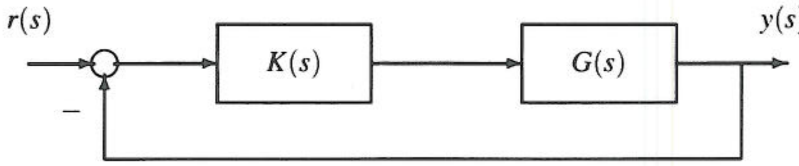


Figure 3.1

a) Let $K(s) = K$ be a constant compensator with $K > 0$. Find the closed-loop characteristic equation in terms of K. Comment on the closed-loop stability as K varies from 0 to ∞. [5]

b) Let $K(s) = -K$ be a constant compensator with $K > 0$. Find the closed-loop characteristic equation in terms of K. Comment on the closed-loop stability as K varies from 0 to ∞. [5]

c) A PD compensator $K(s) = K_P + sK_D$, with $K_P > 0$ and $K_D > 0$ is required such that the following design specifications are satisfied:

- The closed–loop is stable.
- The closed–loop system has a damping ratio $\zeta = 1/\sqrt{2}$.
- The closed–loop step response has a settling time of 4 seconds.

i) Derive the location of the closed–loop poles that satisfy the design specifications. [5]

ii) Write down the closed-loop characteristic equation in terms of K_P and K_D. [5]

iii) Derive the values of the parameters K_P and K_D that achieve the design specifications. [5]

iv) For the compensated system, use the final value theorem to evaluate the steady-state error when $r(t) = t$. [5]

1. a) i) Applying Newton's laws on the mass,

$$u(t) = M\ddot{y}(t) + D\dot{y}(t) + Ky(t).$$

ii) Taking Laplace transforms,

$$\frac{y(s)}{u(s)} = \frac{1}{Ms^2 + Ds + K}.$$

iii) Since $u(t)$ is a unit impulse, $u(s) = 1$. Putting in the numbers,

$$y(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

and so

$$y(t) = e^{-t} - e^{-2t}.$$

iv) Taking $v(t) = \dot{y}(t)$, the differential equation satisfied by $v(t)$ is

$$u(t) = M\dot{v}(t) + Dv(t)$$

and the transfer function is

$$\frac{v(s)}{u(s)} = \frac{1}{Ms + D}.$$

Since $u(s) = 1/s$, using the final value theorem,

$$v_{ss} = \lim_{s \to 0} sv(s) = 1/D.$$

v) Putting in the numbers, $u(s) = 750/s$ and so $v_{ss} = 750/D$. Therefore $D = 375$. This answer could represent the evaluation of a damping value for safe landing for, e.g. a parachutist.

b) i) The error signal is given by $e(s) = \frac{r(s)}{1+KG(s)}$ and so, using the final value theorem, $e_{ss} = \lim_{s\to 0} \frac{1}{1+KG(0)} = \frac{1}{1+4K}$.

ii) The characteristic equation for the closed-loop is

$$1+KG(s) = 1+\frac{4K}{(s+1)^3} = 0 \Rightarrow s^3+3s^2+3s+1+4K = 0$$

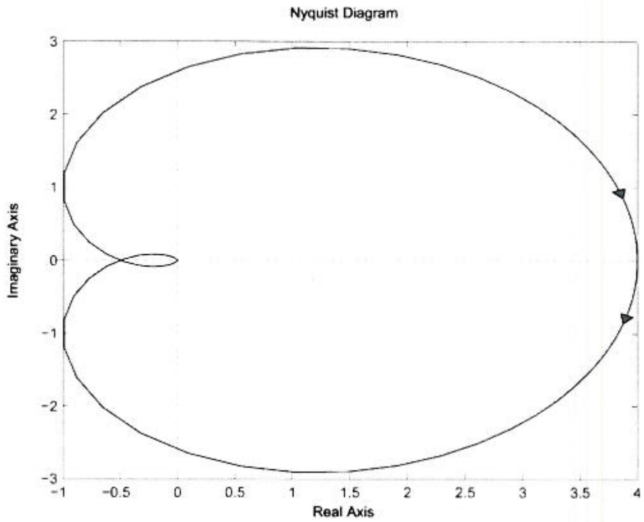
The Routh array is:

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 3 & 1+4K \\ s & 0.75(2-K) & \\ 1 & 1+4K & \end{array}$$

For stability we need the first column to be positive, so $-0.25 < K < 2$.

iii) When $K = 2$ the third row is zero and so the closed-loop is marginally stable. The auxiliary equation is given by $3(s^2+3) = 0$ and so the resulting frequency of oscillations is $\sqrt{3}$ rad/s.

iv) The Nyquist diagram is shown below.



v) When $K = 1$, we need the real-axis intercept. This can be obtained from Part (iii) above as -0.5. The Nyquist criterion states that $N = Z - P$, where N is the number of clockwise encirclements by the Nyquist diagram of the point $-K^{-1} = -1$, P is the number of unstable open–loop poles and Z is the number of unstable closed–loop poles. Since $G(s)$ is stable, $P = 0$. From the diagram, $N = 0$ and so $Z = 0$ and the closed-loop is stable.

vi) When $K = 10$, $N = 2$ and so $Z = 2$. Therefore there are two unstable closed-loop poles.

2. a) The characteristic equation for the closed-loop is

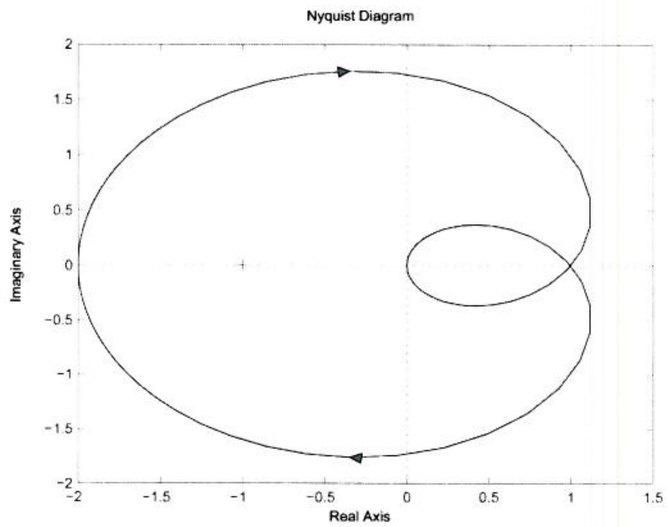
$$1+KG(s) = 1+\frac{2K(s-1)}{(s+1)^2} = 0 \Rightarrow s^2+2(1+K)s+(1-2K) = 0$$

The Routh array is:

$$\begin{array}{c|cc} s^2 & 1 & 1-2K \\ s & 2(1+K) & \\ 1 & 1-2K & \end{array}$$

Therefore $K_1 = -1$ and $K_2 = 0.5$.

b) The Nyquist diagram is shown below. The real–axis intercepts can be found as $-1/K_1,\ -1/K_2$, or $1,\ -2$ as well as 0.



c) When $K(s) = K$, we have $N = Z-P$, where N is the number of clockwise encirclements by the Nyquist diagram of the point $-K^{-1}$, P is the number of unstable open–loop poles and Z is the number of unstable closed–loop poles. Here, $P = 0$.

i) When $-\infty < K < -1, N = 2$ so $Z = 2$.

ii) When $-1 < K < 0, N = 0$ so $Z = 0$.

iii) When $0 < K < 0.5, N = 0$ so $Z = 0$.

iv) When $0.5 < K < \infty, N = 1$ so $Z = 1$.

d) For closed-loop stability we need $-1 < K < 0.5$. An inspection of the Nyquist diagram shows that for a gain margin of 2, the compensated system must have a real-axis intercept at -0.5. This implies that $K = 0.25$. Since the Nyquist diagram of the compensated system $KG(s)$ lies within the unit circle centred at the origin, the phase margin is infinite.

3. a) The closed-loop characteristic equation is

$$1 + KG(s) = 0$$

or

$$s^2 + K = 0.$$

It follows that the closed-loop poles are given as

$$s = \pm j\sqrt{K}$$

and so the closed-loop is marginally stable for all K.

b) The closed-loop characteristic equation is

$$1 - KG(s) = 0$$

or

$$s^2 - K = 0.$$

It follows that the closed-loop poles are given as

$$s = \pm\sqrt{K}$$

and so the closed-loop is unstable for all K.

c) i) For $\zeta = 1/\sqrt{2}$, the real and imaginary parts of the pole are equal. For a settling time of 4 seconds, the real part must be equal to -1. Thus the closed–loop poles must be equal to

$$s_1,\ \bar{s}_1 = -1 \pm j.$$

ii) The characteristic equation is

$$1 + \frac{K_P + sK_D}{s^2} = 0$$

or

$$s^2 + sK_D + K_P = 0.$$

iii) Since the closed-loop poles must be equal to $s_1,\ \bar{s}_1 = -1 \pm j$, it follows that $K_D = 2$ and $K_P = 2$.

iv) The error signal for the compensated system is

$$e(s) = \frac{r(s)}{1 + K(s)G(s)}$$

with

$$G(s) = 1/s^2,\ K(s) = 2(s+1),\ r(s) = 1/s^2.$$

Using the final value theorem

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} se(s) = \lim_{s\to 0} \frac{s}{s^2 + 2(s+1)} = 0.$$