EE4-05
EE9-CS7-22
EE9-SO7

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

TRAFFIC THEORY & QUEUEING SYSTEMS

Thursday, 8 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : J.A. Barria
Second Marker(s) : D.P. Mandic

Special instructions for students

1. Erlang Loss formula recursive evaluation:

$$E_N(\rho) = \frac{\rho E_{N-1}(\rho)}{N + \rho E_{N-1}(\rho)}$$
$$E_0(\rho) = 1$$

2. Engset Loss formula recursive evaluation (for a fixed M and $p = \alpha/1+\alpha$):

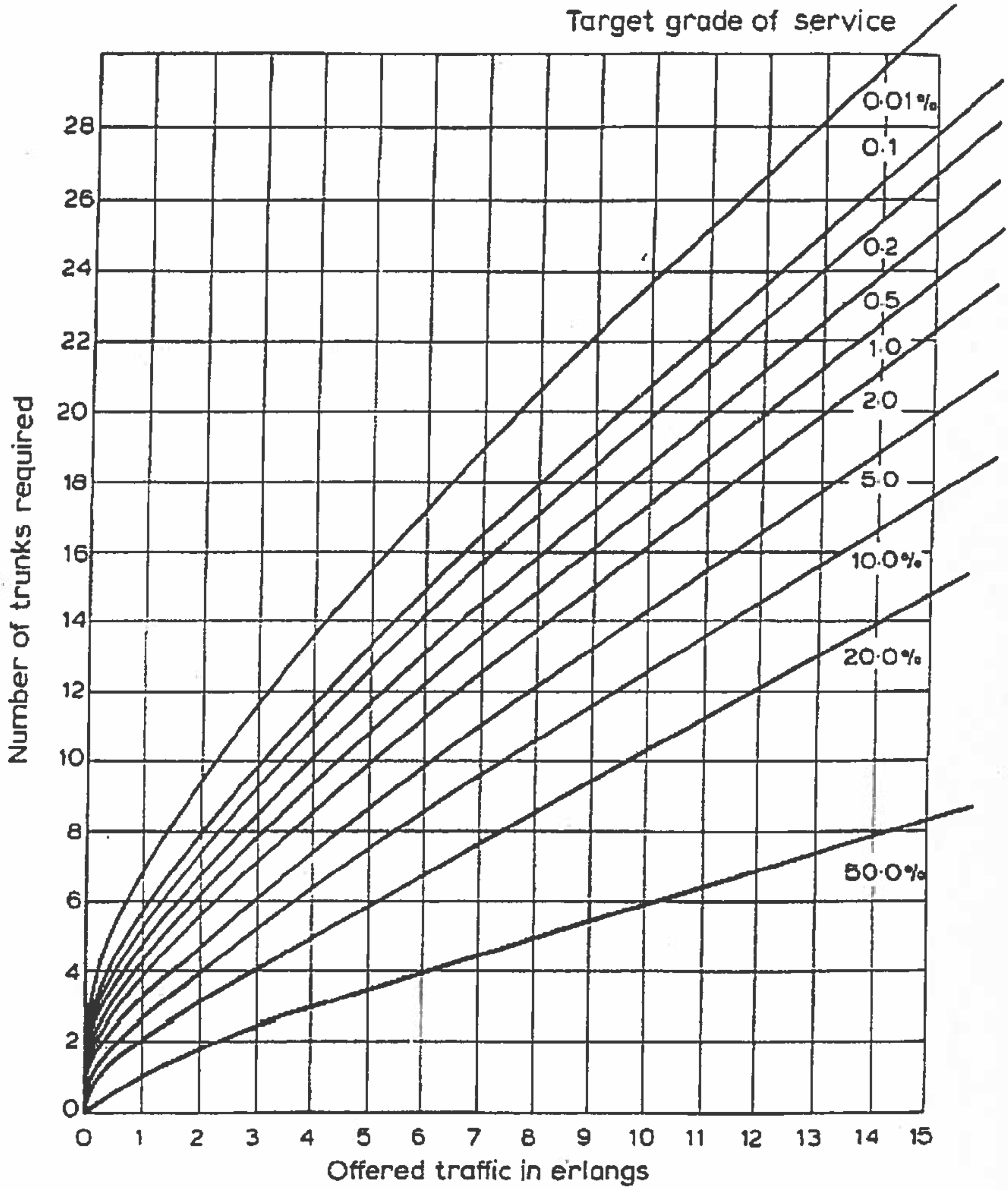
$$e_N = \frac{(M-N+1)\alpha e_{N-1}}{N + (M-N+1)\alpha e_{N-1}}$$
$$e_0 = 1$$
$$\alpha = \lambda/\mu$$

3. Traffic capacity on basis of Erlang B formula (next page).

Note: for large ρ, N is approximately linear: $N \approx 1.33\rho + 5$

4. Expected residual time

$$E[R] = \frac{1}{2}\lambda E[S^2]$$



Traffic capacity on basis of Erlang B. formula.

The Questions

1.

a) For the system shown in Fig. 1.1 the arrival stream is Poisson with rate λ. The holding time of the channels is independent and can be modelled as an exponential random variable with mean holding time $1/\mu$.

Assuming that there are i channels busy at time t derive:

i) The probability that any one busy channel will become idle in $(t, t+\Delta t)$. [3]

ii) The probability that exactly k channels will become idle in $(t, t+\Delta t)$. [5]

ii) The probability that one (1) channel will become idle if i channels are busy in $(t, t+\Delta t)$. [3]

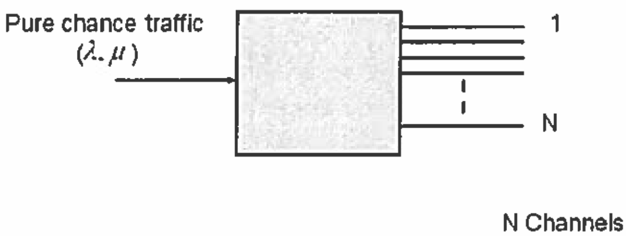


Figure 1.1

b) Figure 1.2. depicts an access switch which has automated alternative routing. Calls are only overflowed to the second choice link if all channels in first choice link are busy.

Assuming that the system is being offered pure chance traffic of 9.4 Erlangs and the mean holding time of a call is 12.5 seconds:

i) Determine the state transition diagram representing the activity seen by the second choice (overflow) link. Clearly identify all transition rates. [3]

ii) Estimate the mean ON and OFF periods for the overflow link when the size of the first choice link is twelve (12) channels and the size of the second choice link is six (6) channels. [6]

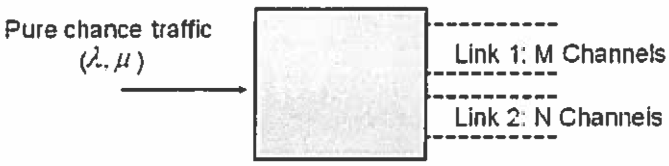


Figure 1.2

2.

a) A mixture of two types of pure chance traffic is offered to a five (5) channel communication link. Type 1 traffic requires one (1) channel to establish a communication, while type 2 traffic requires four (4) parallel channels to establish a communication.

Considering the parameters (λ_i, μ_i) for type traffic i:

i) Set up a 2-dimensional Birth/Death model for the traffic on the link. [3]

ii) Show that the equilibrium distribution for this system is given by:

$$\pi_{ij} = K\left(\frac{\rho_1^i}{i!}\right)\left(\frac{\rho_2^j}{j!}\right) \quad \text{for } (i+4j) \le 5$$ [4]

where, K = normalisation constant.

iii) Given that $\rho_1 = 1$ and $\rho_2 = 0.5$, calculate the value of K. [3]

iv) Calculate the blocking probabilities experienced by type 1 and type 2 traffic. [4]

b)

i) Explain why a Markov chain model would not be the most appropriate when modelling an automatic repeat attempt telephone system. [3]

ii) For an M/G/1 system the waiting time of the *i-th* arrival can be expressed by:

$$W_i = R_i + \sum_{j=1}^{Q_i} S_{i-j}$$

Define and explain all the variables in this expression, that is, R_i, S_{i-j} and Q_i. [3]

3.

a) Figure 3.1 represents the time sequence of a class k arrival to a pre-emptive resume priority system.

i) Using Fig. 3.1 as a reference, identify the random variable, V_k, that represents the work brought into the system during the effective service time, $\hat{S}_k$, by higher-priority arrivals. [2]

ii) Derive the expected value of V_k. [4]

iii) Derive the expected value of $\hat{S}_k$. [4]

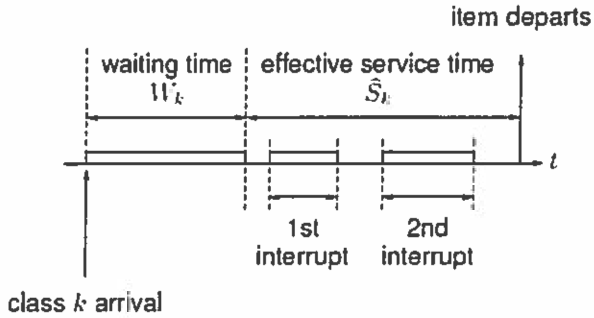


Figure 3.1.

b) Figure 3.2 represents a fluid flow model of a queueing system where the random variable x represents buffer occupancy. The system capacity is VC cells/s.

The units of x are defined to be the number of cells arriving during a talk spurt. A talk spurt has an average duration of $1/\alpha$ seconds and during a talk spurt each ON-OFF source generates V cells/s.

i) Derive the equivalent capacity of the system in terms of units of x. [3]

ii) Estimate how much the variable x will increase when there are i sources ON. [3]

iii) Using the results obtained in i) and ii), set up the equation for the probability that the buffer (queue) occupancy is less than or equal to x when there are i sources ON. [4]

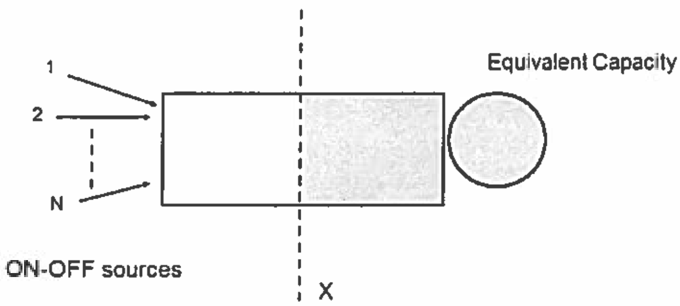


Figure 3.2.

4.

a) The system in Fig. 4.1 represents a finite capacity M/M/K/N system.

i) What is the buffer capacity of the system? [2]

ii) Calculate the probability that a typical arrival will have to wait. [5]

iii) Calculate the loss probability of the system. [5]

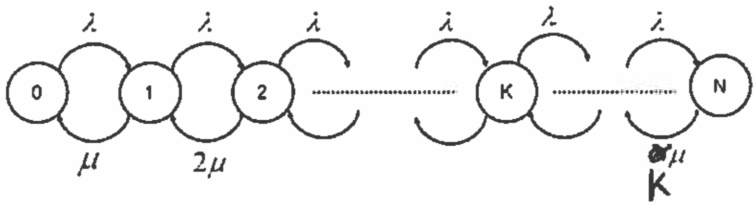


Figure 4.1

b) Figure 4.2 shows a communication network with two switches and one communication link. A switch fails at a constant rate of $\lambda = 2$ failures per year, and the link mean time to failure is $1/\gamma = 10$ years.

The access nodes can also fail at a rate of $\beta = 1$ failure per year. Access nodes have a mean time to repair, $1/\mu$, of 7.3 days (assume year = 365 days). There are ten (10) nodes in total.

The system is in a faulty condition state if:

- one switch fails, or

- one link fails, or

- there are three (3) or more access nodes which have failed at a time.

i) Define the state space of the system. [2]

ii) Derive the Markov chain representation of the system. [3]

iii) Derive the generator matrix of the Markov chain of the system. [3]

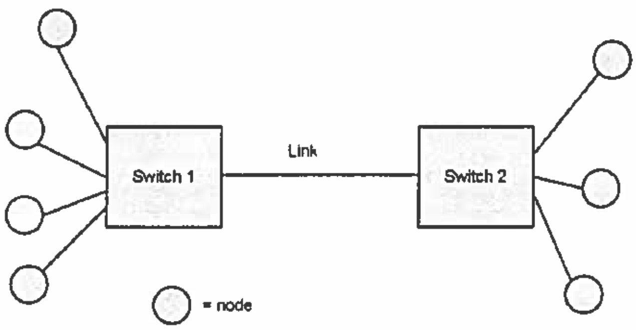


Figure 4.2