EE4-65
EE9-SO27

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

MSc and EEE PART IV: MEng and ACGI

WIRELESS COMMUNICATIONS

Monday, 13 May 10:00 am

Time allowed: 3:00 hours

There are THREE questions on this paper.

Answer THREE questions.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	B. Clerckx
	Second Marker(s) :	K.K. Leung

Important information for students

Notations:

(a) A $n_r \times n_t$ MIMO channel consists in n_r receive antennas and n_t transmit antennas.

(b) a, $\mathbf{a}$, $\mathbf{A}$ denote a scalar, vector and matrix respectively.

(c) $\mathbf{A}^H$ denotes conjugate transpose (Hermitian).

(d) $\mathbf{A}^*$ denotes conjugate.

(e) $\mathbf{A}^T$ denotes transpose.

(f) $|a|$ denotes the absolute value of scalar a.

(g) "i.i.d." means "independent and identically distributed".

(h) "CSI" means "Channel State Information".

(i) "CSIT" means "Channel State Information at the Transmitter".

(j) "CDIT" means "Channel Distribution Information at the Transmitter".

(k) $\mathscr{E}\{.\}$ denotes Expectation.

(l) $\text{Tr}\{.\}$ denotes the Trace of a matrix.

Assumptions:

(a) The CSI is assumed to be always perfectly known to the receiver.

(b) The receiver noise is a $n_r \times 1$ vector with i.i.d. entries modeled as zero mean complex additive white Gaussian noise with variance σ_n^2.

Some useful relationships:

(a) $\|\mathbf{A}\|_F^2 = \text{Tr}\{\mathbf{A}\mathbf{A}^H\} = \text{Tr}\{\mathbf{A}^H\mathbf{A}\}$

(b) $\text{Tr}\{\mathbf{AB}\} = \text{Tr}\{\mathbf{BA}\}$

(c) $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$

THE QUESTIONS

1. [40]

a) Consider the transmission of 2 independent streams using Spatial Multiplexing over a 4×2 MIMO channel $\mathbf{H}$. The Channel State Information (CSI) is unknown to the transmitter. The received signal is written as $\mathbf{y} = \mathbf{Hc} + \mathbf{n}$ where $\mathbf{c} = [c_1, c_2]^T$ is the vector of transmitted symbols. The channel matrix is given by

$$\mathbf{H} = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ -1 & 4 \\ 1 & -1 \end{bmatrix}.$$

At the receiver we would like to apply a combiner $\mathbf{G}$ with the lowest possible complexity such that $\mathbf{z} = \mathbf{Gy} = \mathbf{c} + \mathbf{Gn}$. Derive the expression of the receive combiner $\mathbf{G}$. What kind of combiner is this? Explain the result.

[6]

b) Figure 1.1 displays the error rate performance of one scheme (i.e. one transmission and reception strategy) vs. SNR for point-to-point channels with i.i.d. Rayleigh slow fading and two different antenna configurations, namely $n_r \times n_t = 2 \times 2$ and $n_r \times n_t = 4 \times 2$. The transmission strategy is known to be Spatial Multiplexing where two independent streams are transmitted using BPSK. The reception strategy is not mentioned. The CSI is unknown to the transmitter.

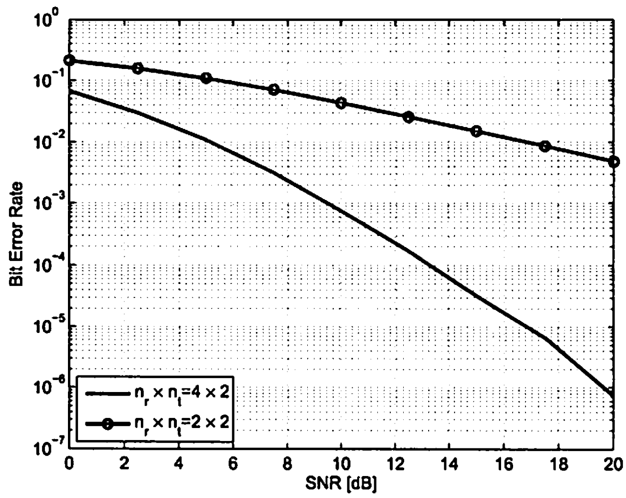

Figure 1.1 Error rate performance vs. SNR.

i) What is the diversity gain (at high SNR) achieved by that scheme in each antenna configuration? Provide your reasoning.

[4]

ii) Identify and explain one reception strategy that achieves such diversity gains in such antenna configurations.

[4]

c) Figure 1.2 displays the ergodic capacity of point-to-point i.i.d. Rayleigh fast fading channels with Channel Distribution Information at the Transmitter (CDIT) for five antenna ($n_r \times n_t$) configurations (denoted as (a), (b), (c), (d), (e)) satisfying $n_t + n_r = 6$.

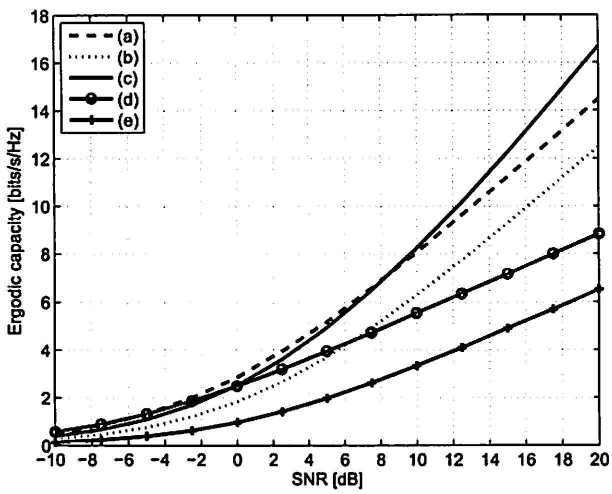

Figure 1.2 Ergodic capacity vs. SNR.

i) What is the achievable (spatial) multiplexing gain (at high SNR) for cases (a), (b), (c), (d) and (e)? Provide your reasoning.

[5]

ii) For (a), (b), (c), (d) and (e), identify the antenna configuration, i.e. n_t and n_r, satisfying $n_t + n_r = 6$ that achieves such multiplexing gain. Provide your reasoning.

[5]

d) Discuss the impact of the number of scatterers n_s on the mutual information of a $n_r \times n_t$ MIMO channel with identity input covariance matrix. Consider in particular the following two cases: 1) $n_s \geq \max\{n_t, n_r\}$, 2) $n_s \leq \min\{n_t, n_r\}$. For each case, derive the expression of the mutual information with identity input covariance matrix and discuss the behavior at high SNR. Provide your reasoning.
Hint: Use the finite scatterer MIMO channel representation with n_s Directions of Departure (DoD) and n_s Directions of Arrival (DoA).

[8]

e) Consider the two-user Gaussian SIMO Multiple Access Channel over the deterministic channels $\mathbf{h}_1$ and $\mathbf{h}_2$. The system model is written as $\mathbf{y} = \mathbf{h}_1 c_1 + \mathbf{h}_2 c_2 + \mathbf{n}$ where the transmit power constraint at transmitter i is given by $\mathcal{E}\{|c_i|^2\} \leq P_i$ for $i = 1, 2$.

i) What is the capacity region of such a channel? Explain its meaning.

[4]

ii) Determine the rates achievable at the corner points of the capacity region and explain the reception strategy to achieve those rates.

[4]

2. [30]

Discuss the validity of the following statements. Detail your argument, e.g. by giving a proof, example(s) and/or counter-example(s).

a) In a multi-user SISO Broadcast Channel with perfect CSIT, fading is detrimental to the average sum-rate capacity.

[6]

b) In a MIMO point-to-point communication system based on Spatial Multiplexing, a Zero-Forcing receiver outperforms a Matched Filter because it completely suppresses the interference between streams.

[6]

c) In a $n_r \times n_t$ point to point i.i.d. MIMO Rayleigh slow fading channel ($n_t \geq 1$ and $n_r \geq 1$), at asymptotically high SNR, we can simultaneously scale the transmission rate with the SNR as $R = \min\{n_t, n_r\} \log_2(\rho)$ (with ρ the SNR) and achieve a diversity gain of $n_t n_r$.

[6]

d) In a cellular system where users are uniformly dropped in a cell and have the same Quality of Service (QoS), the proportional fair (PF) scheduler is equivalent to the rate-maximization scheduler in the limit of very large scheduling time scale.

[6]

e) A narrowband transmission using a Linear Space-Time Block Code, characterized by codewords

$$\mathbf{C} = \frac{1}{2}\begin{bmatrix} c_1 + c_3 & c_2 + c_4 \\ c_2 - c_4 & c_1 - c_3 \end{bmatrix}$$

with c_1, c_2, c_3 and c_4 being the constellation symbols taken from a unit average energy QAM constellation, and a Maximum Likelihood (ML) receiver, achieves a diversity gain of $2n_r$ in i.i.d. slow Rayleigh fading channels with n_r receive antennas and 2 transmit antennas.

[6]

3. [30]

Assume a downlink narrowband transmission in a system consisting of n_c cells with one transmitter and two terminals (denoted as terminal 1 and 2) per cell. The transmitters are equipped with 4 antennas and the terminals equipped with 2 receive antennas. Each receiver perfectly estimates and reports its channel state information to its respective transmitter. In each cell, the transmitter schedules the two terminals at a time and sends using MU-MIMO linear precoding a total of 4 streams, with 2 streams per terminal. Each transmitter is subject to a total transmit power P.

a) Write an expression of the received signal of terminal 1 in cell 1 in terms of channel parameters, precoders and transmit symbol vectors. Clearly define each variable and identify the terms responsible for the intra-cell interference (also called multi-user interference) and inter-cell interference in your expression. [5]

b) Derive the expression of the MMSE combiner for stream 1 of terminal 1 in cell 1 and the SINR of that stream at the output of the MMSE combiner. Provide your reasoning. [5]

c) Ignore the inter-cell interference and focus exclusively on cell 1, i.e. transmitter 1 and its two terminals.

i) Derive the expression of a transmit precoder such that the two terminals do not experience any multi-user interference. Provide your reasoning.
Hint: Perform a block diagonalization. [5]

ii) Particularize your answer in i) to the case where the channel matrix between transmitter 1 and terminal i is given by the 2×4 matrix

$$\begin{bmatrix} a_1 & 0 & a_1 & 0 \\ 0 & b_1 & 0 & b_1 \end{bmatrix}$$

for $i = 1$ and

$$\begin{bmatrix} a_2 & a_2 & 0 & 0 \\ 0 & 0 & b_2 & b_2 \end{bmatrix}$$

for $i = 2$. Quantities a_1, b_1, a_2 and b_2 are complex scalars. [5]

iii) Improve the design of part i) such that the sum-rate is maximized under the constraint that the two terminals do not experience any multi-user interference. In particular, derive the expressions of the transmit precoding and power allocation strategy (assuming a fixed total power constraint) for transmitter 1 and a receive combiner for terminal 1 and 2 such that the sum-rate is maximized under the constraint that the two terminals do not experience any multi-user interference. Provide your reasoning. [10]

THE ANSWERS

Notations:

(a) B - Bookwork

(b) E - New example

(c) A - New application

1. a) Columns of $\mathbf{H}$ are orthogonal. Hence the MIMO channel is simply decoupled by applying a matched filter

$$\mathbf{G} = \mathbf{H}^H = \mathbf{H}^T = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 1 & 4 & -1 \end{bmatrix}.$$

[6 - E]

b) i) The diversity gain is the slope at high SNR of the error curve vs. the SNR on a log-log scale, i.e. $-\lim_{\rho\to\infty}\frac{\log(P_e(\rho))}{\log(\rho)}$ with ρ being the SNR. For $n_r \times n_t = 2\times 2$, the diversity gain is 1 as the error rate decreases by 10^{-1} when the SNR is increased from 10dB to 20dB.

[2 - E]

For $n_r \times n_t = 4\times 2$, the diversity gain is 3 as the error rate decreases by 10^{-3} when the SNR is increased from 10dB to 20dB.

[2 - E]

ii) Spatial Multiplexing with a ZF receiver achieves a diversity gain of $n_r - n_t + 1$ over i.i.d. Rayleigh fading at high SNR. $n_t = 2$ independent streams are encoded using AWGN capacity achieving codes and are each transmitted using one antenna. The receiver zero-forces the inter-stream interference by pseudo-inversing the MIMO channel, i.e. $\mathbf{G} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H$. At the output of the ZF filter, the two streams are decoupled and the SNR of each stream is $\chi^2_{2(n_r-n_t+1)}$ distributed leading to an error probability that decreases with a diversity gain of $n_r - n_t + 1$.

[4 - A]

c) i) The multiplexing gain is the pre-log factor of the ergodic capacity at high SNR, i.e. $g_s = \lim_{\rho\to\infty}\frac{C_{CDIT}}{\log_2(\rho)}$. Hence by increasing the SNR by 3dB (e.g. from 17dB to 20dB), the ergodic capacity increases by g_s bits/s/Hz.

(a) $g_s = 2$.

[1 - E]

(b) $g_s = 2$.

[1 - E]

(c) $g_s = 3$.

[1 - E]

(d) $g_s = 1$.

[1 - E]

(e) $g_s = 1$.

[1 - E]

ii) There are only 5 possible configurations that satisfy to $n_r + n_t = 6$, namely 3×3, 4×2, 2×4, 5×1 and 1×5. The matching between curves and antenna configurations is easily identified by using the following two arguments: 1) The multiplexing gain with CDIT at high SNR is given by $\min\{n_t, n_r\}$. 2) With CDIT only, the input covariance matrix in i.i.d. channel is $\mathbf{Q} = 1/n_t \mathbf{I}_{n_t}$. This implies that 4×2 and 5×1 outperform 2×4 and 1×5, respectively.

(a) $n_r \times n_t = 4 \times 2$

[1 - E]

(b) $n_r \times n_t = 2 \times 4$

[1 - E]

(c) $n_r \times n_t = 3 \times 3$

[1 - E]

(d) $n_r \times n_t = 5 \times 1$

[1 - E]

(e) $n_r \times n_t = 1 \times 5$

[1 - E]

d) The mutual information of a $n_r \times n_t$ MIMO channel with identity input covariance matrix is given by

$$\mathscr{I}_e = \log_2 \det\left(\mathbf{I}_{n_r} + \frac{\rho}{n_t}\mathbf{H}\mathbf{H}^H\right).$$

Using the finite scatterer MIMO channel representation, $\mathbf{H} = \mathbf{A}_r \mathbf{H}_s \mathbf{A}_t^T$ where the columns of $\mathbf{A}_r$ $[n_r \times n_s]$ contain the steering vectors related to the direction of arrival (DoA) from each scatterer. Similarly the columns of $\mathbf{A}_t$ $[n_t \times n_s]$ contain the steering vectors related to the direction of departure (DoD) of each scatterer. $\mathbf{H}_s$ entries are complex gain coupling each DoD and DoA.

$$\mathscr{I}_e = \log_2 \det\left(\mathbf{I}_{n_r} + \frac{\rho}{n_t}\mathbf{A}_r \mathbf{H}_s \mathbf{A}_t^T \mathbf{A}_t^* \mathbf{H}_s^H \mathbf{A}_r^H\right).$$

Let us denote the non-zero eigenvalues of $\mathbf{H}\mathbf{H}^H$ as λ_k.

If $n_s \geq \max\{n_t, n_r\}$, $\mathbf{A}_r \mathbf{H}_s \mathbf{A}_t^T \mathbf{A}_t^* \mathbf{H}_s^H \mathbf{A}_r^H$ has $\min\{n_t, n_r\}$ non-zero eigenvalues. At high SNR,

$$\mathscr{I}_e \approx \min\{n_t, n_r\} \log_2\left(\frac{\rho}{n_t}\right) + \sum_{k=1}^{\min\{n_t, n_r\}} \log_2(\lambda_k).$$

therefore leading to a multiplexing gain of $\min\{n_t, n_r\}$ at high SNR.

[4 - E]

If $n_s \leq \min\{n_t, n_r\}$, $\mathbf{A}_r \mathbf{H}_s \mathbf{A}_t^T \mathbf{A}_t^* \mathbf{H}_s^H \mathbf{A}_r^H$ only contain n_s non-zero eigenvalues. Therefore

$$\mathscr{I}_e = \log_2 \det\left(\mathbf{I}_{n_s} + \frac{\rho}{n_t}\mathbf{A}_r^H \mathbf{A}_r \mathbf{H}_s \mathbf{A}_t^T \mathbf{A}_t^* \mathbf{H}_s^H\right)$$

and in the limit of high SNR,

$$\mathscr{I}_e \approx n_s \log_2(\frac{\rho}{n_t}) + \sum_{k=1}^{n_s} \log_2(\lambda_k)$$

therefore leading to a multiplexing gain of n_s at high SNR.

[4 - A]

e) i) The capacity region is given by all the pairs (R_1, R_2) satisfying the following three inqualities

$$R_1 \leq \log_2\left(1 + \frac{P_1}{\sigma_n^2} \|\mathbf{h}_1\|^2\right),$$

$$R_2 \leq \log_2\left(1 + \frac{P_2}{\sigma_n^2} \|\mathbf{h}_2\|^2\right),$$

$$R_1 + R_2 \leq \log_2 \det\left(\mathbf{I}_{n_r} + \frac{P_1}{\sigma_n^2}\mathbf{h}_1\mathbf{h}_1{}^H + \frac{P_2}{\sigma_n^2}\mathbf{h}_2\mathbf{h}_2{}^H\right).$$

The first two inequalities come from the fact that the rates cannot be larger than the one achievable in a point to point scenario. The last inequality tells us the sum of rate cannot be larger than the one achievable when the two transmitters form a single big transmitter (with two transmit antennas) and send two independent streams subject to their respective transmit power constraints. Given the presence of a single transmit antenna at each transmitter, transmission is performed at the maximum transmit power.

[4 - B]

ii) The three inequalities define a prentagon with two corner points. The corner points $(R1, R_2')$ where transmitter 1 transmits at full rate and transmitter 2 transmits at rate R_2' can be computed from

$$R_2' = \log_2 \det\left(\mathbf{I}_{n_r} + \frac{P_1}{\sigma_n^2}\mathbf{h}_1\mathbf{h}_1{}^H + \frac{P_2}{\sigma_n^2}\mathbf{h}_2\mathbf{h}_2{}^H\right) - \log_2 \det\left(\mathbf{I}_{n_r} + \frac{P_1}{\sigma_n^1}\mathbf{h}_1\mathbf{h}_1{}^H\right)$$

or equivalently

$$R_2' = \log_2 \det\left(\mathbf{I}_{n_r} + \frac{P_2}{\sigma_n^2}\mathbf{h}_2\mathbf{h}_2{}^H\left(\mathbf{I}_{n_r} + \frac{P_1}{\sigma_n^2}\mathbf{h}_1\mathbf{h}_1{}^H\right)^{-1}\right),$$

$$R_2' = \log_2\left(1 + \frac{P_2}{\sigma_n^2}\mathbf{h}_2{}^H\left(\mathbf{I}_{n_r} + \frac{P_1}{\sigma_n^2}\mathbf{h}_1\mathbf{h}_1{}^H\right)^{-1}\mathbf{h}_2\right).$$

The argument of the log in the last equation is the SINR of a MMSE receiver where $\mathbf{I}_{n_r} + \frac{P_1}{\sigma_n^2}\mathbf{h}_1\mathbf{h}_1{}^H$ is the covariance matrix of the AWGN noise and interference from user 1. (R_1, R_2') is achieved by decoding transmitter 2's signal first using MMSE combiner by treating transmitter 1's signal as noise. Decode transmitter 2's signal and cancel it from the received signal using SIC and then decode transmitter 1's signal. (R_1', R_2) can be obtained similarly by changing the decoding order.

[4 - B]

2. Discuss the validity of the following statements. Give a proof, example(s) and/or couter-example(s) to support your argument.

a) The sum-rate capacity of a SISO BC with perfect CSIT is given by dynamic TDMA where the best user (with the highest instantaneous SNR) is selected at any time instant. The average sum-rate capacity therefore writes

$$\bar{C} = \mathscr{E}\left\{\log_2\left(1+\max_k \eta_k |h_k|^2\right)\right\}$$

with η_k the long term SNR of user k. Given the maximization term, the presence of fading is likely to be viewed as beneficial to the sum-rate capacity. Let us assume for simplicity that $\eta_k = \eta$. At low SNR,

$$\bar{C} \approx \eta\mathscr{E}\left\{\max_k |h_k|^2\right\}\log_2(e) \approx \mathscr{E}\left\{\max_k |h_k|^2\right\} C_{awgn}.$$

At high SNR,

$$\bar{C} \approx \log_2(\eta) + \mathscr{E}\left\{\log_2\left(\max_k |h_k|^2\right)\right\} \approx C_{awgn} + \mathscr{E}\left\{\log_2\left(\max_k |h_k|^2\right)\right\}.$$

Depending on the fading statistics, quantity $\mathscr{E}\left\{\max_k |h_k|^2\right\}$ and $\mathscr{E}\left\{\log_2\left(\max_k |h_k|^2\right)\right\}$ will vary. Nevertheless, assuming users have independent statistics, those quantities are likely to contribute positively to the sum-rate capacity. For i.i.d. Rayleigh fading across users and a large number of users K, $\mathscr{E}\left\{\max_k |h_k|^2\right\} \approx \log(K)$, leading to a significant increase of $\bar{C}$ compared to C_{awgn} at low SNR.

[6 - B]

b) In a MIMO point to point communication system based on Spatial Multiplexing, a Zero-Forcing receiver would outperform a Matched Filter only at high SNR when the inter-stream interference is dominating the noise. At low SNR, on the other hand, the inter-stream interference is negligible compared to the noise and the matched filter outperforms ZF as it maximizes the SNR. ZF would provide exactly the same performance as matched filter at low SNR if the columns of the channel matrix are orthogonal.

[6 - A]

c) If the rate is scaled as $R = \min\{n_t, n_r\}\log_2(\rho)$, the diversity gain is 0. Inversely, if the rate is fixed and therefore scales with a multiplexing gain of 0, the achievable diversity gain is $n_t n_r$. If is therefore not correct to simultaneously get a multiplexing gain of $\min\{n_t, n_r\}$ and a diversity gain of $n_t n_r$. A simple example proving that it is not possible is given by looking at a MISO channel where the optimal diversity multiplexing tradeoff of the MISO channel was shown to be $n_t(1-g_s)$ with $g_s \in [0,1]$.

Indeed, the outage probability of a MIMO channel writes as

$$P_{out} = \min_{\mathbf{Q}, \mathrm{Tr}\{\mathbf{Q}\}\leq 1} P\left(\log_2\det\left(\mathbf{I}_{n_r} + \rho\mathbf{H}\mathbf{Q}\mathbf{H}^H\right) \leq R\right).$$

The $\mathbf{Q}$ that minimizes the outage probability at high SNR is $\mathbf{Q} = \frac{1}{n_t}\mathbf{I}_{n_t}$ such that, for a MISO channel at high SNR,

$$P_{out} \approx P\left(\log_2\left(\frac{\rho}{n_t}\|\mathbf{h}\|^2\right) \leq R\right).$$

Let us write the total transmission rate as $R = g_s \log_2(\rho)$ with $0 \le g_s \le 1$, i.e. the rate scales with the SNR with a multiplexing gain of g_s. At high SNR,

$$P_{out} \approx P\left(\|\mathbf{h}\|^2 \le \rho^{g_s - 1}\right).$$

Given that $\|\mathbf{h}\|^2$ is a $\chi^2_{2n_t}$ random variable, at high SNR (when ρ^{g_s-1} is small) $P\left(\|\mathbf{h}\|^2 \le \rho^{g_s-1}\right) \approx \rho^{-n_t(1-g_s)}$. The diversity gain is obtained as the exponent of the SNR, therefore leading to the diversity-multiplexing tradeoff of $n_t(1-g_s)$ for $0 \le g_s \le 1$. A diversity gain n_t is achievable when the transmission rate is kept fixed and is not scaled with the SNR. When the rate is scaled with the SNR as $R = \log_2(\rho)$, the diversity gain is 0. Hence in a MISO slow fading channel, we cannot achieve simultaneously a diversity gain of n_t and a multiplexing gain of 1.

We could give an even simpler example based on SISO case and see that the multiplexing gain of 1 and the diversity gain of 1 cannot be achieved simultaneously.

[6 - A]

d) A PF scheduler aims at finding the scheduled user set $\mathbf{K}$ to maximize the weighted sum-rate $\sum_{k\in\mathbf{K}} \gamma_k \frac{R_k(t)}{\bar{R}_k(t)}$ where γ_k is the QoS of user k (which is the same for all users in the question), $R_k(t)$ is user k rate at time t and $\bar{R}_k(t)$ is the long-term average rate of user k at time t. $\bar{R}_k(t)$ is computed using a low-pass filter and is updated at every time instant based on the past estimates $R_k(t-1)$ and instantaneous rate $R_k(t-1)$, as

$$\bar{R}_k(t) = \begin{cases} (1-1/t_c)\bar{R}_k(t-1) + 1/t_c R_k(t-1), & q \in \mathbf{K}^\star \\ (1-1/t_c)\bar{R}_q(k-1), & q \notin \mathbf{K}^\star \end{cases}$$

where t_c is the scheduling time scale and $\mathbf{K}^\star$ refers to the scheduled user set at time k.

In the limit of very large scheduling time scale t_c, $\bar{R}_k(t) = \bar{R}_k(t-1) = \bar{R}_k$. If all users experience identical fading/path loss properties (leading to identical long term SNR), $\bar{R}_k = \bar{R}$ is the same for all users and the PF scheduler is equivalent to a rate maximization scheduler. Unfortunately, when users are uniformly distributed in a cell, they are not likely to experience the same long term average rate. Therefore the PF scheduler is not equivalent to a rate maximization scheduler even in the limit of large scheduling time scale.

[6 - A]

e) By the rank-determinant criterion, the diversity gain is given by $n_r \min r(\mathbf{C}-\mathbf{E})$ where $\min r(\mathbf{C}-\mathbf{E})$ refers to the minimum rank of the error matrix over all possible error matrices. The error matrix for this code writes as

$$\mathbf{C}-\mathbf{E} = \frac{1}{2}\begin{bmatrix} d_1+d_3 & d_2+d_4 \\ d_2-d_4 & d_1-d_3 \end{bmatrix}$$

where $d_k = c_k - e_k$ for $k = 1,\dots,4$. It is easily seen that by taking two codewords $\mathbf{C}$ and $\mathbf{E}$ such that $d_3 = d_4 = 0$ and $d_1 = d_2 = d$ (which is encountered for any QAM constellations), $r(\mathbf{C}-\mathbf{E}) = 1$, leading to a diversity gain of n_r only.

[6 - A]

3. a) The received signal of terminal 1 in cell 1 writes as

$$\mathbf{y}_1 = \mathbf{H}_{1,1}\mathbf{P}_{1,1}\mathbf{c}_{1,1} + \mathbf{H}_{1,1}\mathbf{P}_{1,2}\mathbf{c}_{1,2} + \sum_{i=2}^{n_c}\mathbf{H}_{1,i}\mathbf{P}_i\mathbf{c}_i + \mathbf{n}_1$$

where $\mathbf{y}_1$ is the $[2 \times 1]$ received signal at user 1 in cell 1, $\mathbf{H}_{1,i}$ is the channel between transmitter i and user 1, $\mathbf{P}_i = [\mathbf{P}_{i,1}, \mathbf{P}_{i,2}]$ is the $[4 \times 4]$ precoder at transmitter i made of two sub-precoder $\mathbf{P}_{i,1}$ and $\mathbf{P}_{i,2}$ of size $[4 \times 2]$ (precoding data for terminal 1 and 2 respectively), and $\mathbf{c}_i = [\mathbf{c}_{i,1}^T, \mathbf{c}_{i,2}^T]^T$ is the $[4 \times 1]$ transmit symbol vector whose entries are unit-average energy independent symbols. The transmit power at each transmitter writes as $\mathrm{Tr}\left\{\mathscr{E}\left\{\mathbf{P}_i\mathbf{c}_i(\mathbf{P}_i\mathbf{c}_i)^H\right\}\right\} = \mathrm{Tr}\left\{\mathbf{P}_i\mathbf{P}_i^H\right\} = P$. Hence the power allocation to each stream is naturally accounted for in the precoder. The first term is the intended signal, the second term refers to the intra-cell interference and the third term refers to the inter-cell interference.

[5 - A]

b) We can further expand the received signal as follows

$$\mathbf{y}_1 = \mathbf{H}_{1,1}\mathbf{p}_{1,1,1}c_{1,1,1} + \mathbf{n}_1'$$

with $\mathbf{n}_1' = \mathbf{H}_{1,1}\mathbf{p}_{1,1,2}c_{1,1,2} + \mathbf{H}_{1,1}\mathbf{P}_{1,2}\mathbf{c}_{1,2} + \sum_{i=2}^{n_c}\mathbf{H}_{1,i}\mathbf{P}_i\mathbf{c}_i + \mathbf{n}_1$ where $\mathbf{P}_{1,1} = [\mathbf{p}_{1,1,1}, \mathbf{p}_{1,1,2}]$ and $\mathbf{c}_{1,1} = [c_{1,1,1}, c_{1,1,2}]^T$. The term $\mathbf{n}_1'$ now expresses the combined noise plus interference (inter-stream, intra-cell and inter-cell interference) seen by stream 1. The covariance matrix of $\mathbf{n}_1'$ is given by $\mathbf{R}_{\mathbf{n}_1'} = \mathscr{E}\left\{\mathbf{n}_1'\mathbf{n}_1'^H\right\}$

$$\mathbf{R}_{\mathbf{n}_1'} = \sigma_n^2\mathbf{I}_2 + \mathbf{H}_{1,1}\mathbf{p}_{1,1,2}(\mathbf{H}_{1,1}\mathbf{p}_{1,1,2})^H + \mathbf{H}_{1,1}\mathbf{P}_{1,2}(\mathbf{H}_{1,1}\mathbf{P}_{1,2})^H + \sum_{i=2}^{n_c}\mathbf{H}_{1,i}\mathbf{P}_i(\mathbf{H}_{1,i}\mathbf{P}_i)^H.$$

[2 - A]

The receive combiner for stream 1 of terminal 1 in cell 1 writes as

$$\mathbf{g}_{MMSE,1} = (\mathbf{H}_{1,1}\mathbf{p}_{1,1,1})^H\mathbf{R}_{\mathbf{n}_1'}^{-1}$$

and the output SINR of stream 1 is given by

$$\rho_1 = (\mathbf{H}_{1,1}\mathbf{p}_{1,1,1})^H\mathbf{R}_{\mathbf{n}_1'}^{-1}\mathbf{H}_{1,1}\mathbf{p}_{1,1,1}.$$

[3 - A]

c) We ignore the inter-cell interference and focus exclusively on cell 1. We drop the cell index for simplicity and the received signal at user 1 and user 2 write as

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{P}_1\mathbf{c}_1 + \mathbf{H}_1\mathbf{P}_2\mathbf{c}_2 + \mathbf{n}_1,$$

$$\mathbf{y}_2 = \mathbf{H}_2\mathbf{P}_1\mathbf{c}_1 + \mathbf{H}_2\mathbf{P}_2\mathbf{c}_2 + \mathbf{n}_1.$$

i) The transmit precoder follows from the block diagonalization approach, i.e. the multi-user interference is completely zero-forced. $\mathbf{P}_2$ is designed such that $\mathbf{H}_1\mathbf{P}_2 = \mathbf{0}_2$ and $\mathbf{P}_1$ is designed such that $\mathbf{H}_2\mathbf{P}_1 = \mathbf{0}_2$, which is possible given that the number of transmit antennas is 4, 2 streams are transmitted per terminal and there are 2 receive antennas per terminal. Taking the svd of $\mathbf{H}_1 = \mathbf{U}_1\mathbf{S}_1[\ \mathbf{V}_1 \quad \mathbf{V}_1'\ ]^H$ where $\mathbf{V}_1$ contains the right singular vectors corresponding to the two non-zero singular values. Similarly, $\mathbf{H}_2 = \mathbf{U}_2\mathbf{S}_2[\ \mathbf{V}_2 \quad \mathbf{V}_2'\ ]^H$. By choosing $\mathbf{P}_2 = \mathbf{V}_1'\mathbf{P}_2'$ we transmit $\mathbf{c}_2$ in the null space of $\mathbf{H}_1$ and therefore create no multi-user interference. Similarly we take $\mathbf{P}_1 = \mathbf{V}_2'\mathbf{P}_1'$.

[5 - B]

ii) The first two dominant eigenvectors of $\mathbf{H}_1$ are easily computed as

$$\mathbf{V}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

given the orthogonality of the channel matrix $\mathbf{H}_1$. Similarly we find

$$\mathbf{V}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

given the orthogonality of the channel matrix $\mathbf{H}_2$. $\mathbf{V}_1'$ and $\mathbf{V}_2'$ are orthogonal to $\mathbf{V}_1$ and $\mathbf{V}_2$, respectively. Therefore we can easily derive

$$\mathbf{V}_1' = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathbf{V}_2' = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}.$$

Hence a simple example of precoder that would zero-force the multi user interference is given by $\mathbf{P}_1 = \mathbf{V}_2'$ and $\mathbf{P}_2 = \mathbf{V}_1'$.

[5 - A]

iii) With $\mathbf{P}_1 = \mathbf{V}_2'\mathbf{P}_1'$ and $\mathbf{P}_2 = \mathbf{V}_1'\mathbf{P}_2'$, the received signals become

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{V}_2'\mathbf{P}_1'\mathbf{c}_1 + \mathbf{n}_1,$$

$$\mathbf{y}_2 = \mathbf{H}_2\mathbf{V}_1'\mathbf{P}_2'\mathbf{c}_2 + \mathbf{n}_1.$$

Each terminal therefore sees an equivalent 2×2 point to point MIMO channel given by $\mathbf{H}_1\mathbf{V}_2'$ and $\mathbf{H}_2\mathbf{V}_1'$, respectively for terminal 1 and 2. For user 1 (resp. 2), the rate can be maximized by decoupling the point to point MIMO channels by transmitting along the right singular vector of $\mathbf{H}_1\mathbf{V}_2'$ (resp. $\mathbf{H}_2\mathbf{V}_1'$) and choosing a receive combiner $\mathbf{G}_1$ (resp. $\mathbf{G}_2$) as the left singular vector of $\mathbf{H}_1\mathbf{V}_2'$ (resp. $\mathbf{H}_2\mathbf{V}_1'$) and allocating the power across stream following the water-filling solution.

[4-B]

Hence, let us write the svd $\mathbf{H}_1\mathbf{V}_2' = \tilde{\mathbf{U}}_1\tilde{\mathbf{S}}_1\tilde{\mathbf{V}}_1^H$ and $\mathbf{H}_2\mathbf{V}_1' = \tilde{\mathbf{U}}_2\tilde{\mathbf{S}}_2\tilde{\mathbf{V}}_2^H$. To decouple the channel of terminal 1, we take $\mathbf{G}_1 = \tilde{\mathbf{U}}_1^H$ and $\mathbf{P}_1' = \tilde{\mathbf{V}}_1\text{diag}\left\{\sqrt{p_{11}}, \sqrt{p_{12}}\right\}$ where p_{11} and p_{12} are the power allocated to stream 1 and 2 of terminal 1. For terminal 2, we take $\mathbf{G}_2 = \tilde{\mathbf{U}}_2^H$ and $\mathbf{P}_2' = \tilde{\mathbf{V}}_2\text{diag}\left\{\sqrt{p_{21}}, \sqrt{p_{22}}\right\}$ where p_{21} and p_{22} are the power allocated to stream 1 and 2 of terminal 2.

[3-B]

Writing $\tilde{\mathbf{S}}_1 = \text{diag}\left\{\sqrt{s_{11}}, \sqrt{s_{12}}\right\}$ and $\tilde{\mathbf{S}}_2 = \text{diag}\left\{\sqrt{s_{21}}, \sqrt{s_{22}}\right\}$, the power on each stream are chosen to maximize the sum-rate

$$\sum_{k=1}^{2}\sum_{l=1}^{2}\log_2\left(1 + \frac{s_{kl}p_{kl}}{\sigma_n^2}\right)$$

subject to $\sum_{k=1}^{2}\sum_{l=1}^{2} p_{kl} = P$. The optimal power allocation is given by $p_{kl}^{\star} = \left(\mu - \frac{\sigma_n^2}{s_{kl}}\right)^{+}$ where μ is chosen such that the pwoer constraint is satisfied.

[3-B]

Project: Performance Evaluation of LTE 4Tx MIMO Downlink

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1 Objective

Evaluate (using computer simulations) the performance of LTE 4Tx Single-User MIMO with quantized precoding using a simplified system level evaluation methodology.

2 Deployment

We consider a downlink transmission from one base station (BS) to multiple users (UE), as illustrated in Figure 1. The BS serves $K = 10$ UEs randomly and uniformly dropped (at a distance $> 35m$ and $< 250m$ from the BS) in a centre cell. The transmission is subject to interference from 6 neighboring base stations. The transmit power at each base station is fixed to 46dBm. The noise variance at the UE is fixed to -174dBm. The BSs are all equippped with $n_t = 4$ transmit antennas and the UEs are equipped with either $n_r = 2$ or 4 receive antennas. All deployments and channel model parameters are listed in Table 1

Table 1: Deployment and channel model parameters

Parameter	**Explanation/Assumption**
Transmit power	46dBm
Noise variance	-174dBm
Number of users K dropped in the centre cell	$K = 10$ (baseline). Other values to investigated to assess impact on performance.
Path Loss [dB]	$128.1 + 37.6\log_{10}(d)$ with d the BS-user distance [km]
Shadowing model	Log-normal shadowing with 8dB standard deviation.
Shadowing correlation	0 for links BS $\rightarrow$ UE_i $\forall i$, 0.5 for links $\mathrm{BS}_j \rightarrow$ UE $\forall j$.
Antenna configurations $n_r \times n_t$	2×4 (baseline), 4×4

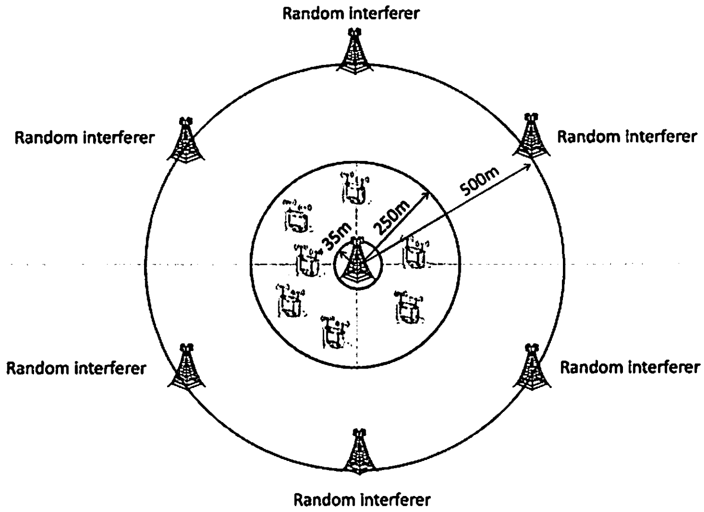

Figure 1: Deployment scenario: 1 centre BS/cell and 6 interfering BSs.

3 MIMO Channel Model

We assume for simplicity that the MIMO channel is flat fading and writes at time instant k between BS i and UE q as

$$\mathbf{H}_{k,q,i} = \tilde{\mathbf{H}}_{k,q,i}\mathbf{R}_{t,q,i}^{1/2} \tag{1}$$

where

- $\tilde{\mathbf{H}}_{k,q,i} \in \mathbb{C}^{n_r \times n_t}$ represents a spatially uncorrrelated Rayleigh flat fading channel matrix, whose entries are i.i.d. according to $\mathcal{CN}(0,1)$. The evolution of $\tilde{\mathbf{H}}_{k,q,i}$ is modeled by a first-order Gauss-Markov process

$$\tilde{\mathbf{H}}_{k,q,i} = \epsilon\tilde{\mathbf{H}}_{k-1,q,i} + \sqrt{1-\epsilon^2}\mathbf{N}_{k,q,i}. \tag{2}$$

  $\mathbf{N}_{k,q,i} \in \mathbb{C}^{n_r \times n_t}$ has i.i.d. entries with distribution $\sim \mathcal{CN}(0,1)$ and

$$\mathcal{E}\left\{\text{vec}\left(\tilde{\mathbf{H}}_{k-1,q,i}\right)\text{vec}\left(\mathbf{N}_{k,q,i}\right)^H\right\}. \tag{3}$$

  The time correlation ϵ represents the correlation between entries of $\tilde{\mathbf{H}}_{k,q,i}$ and $\tilde{\mathbf{H}}_{k-1,q,i}$.

- $\mathbf{R}_{t,q,i}$ is the transmit correlation matrix of BS i-user q link. Denoting by $i = 0$ the centre BS and by $i = 1, \ldots, 6$ the interfering BS, a simplified model is given by the exponential structure

$$\mathbf{R}_{t,q,i} = \begin{bmatrix} 1 & t_{q,i} & t_{q,i}^2 & t_{q,i}^3 \\ t_{q,i}^* & 1 & t_{q,i} & t_{q,i}^2 \\ t_{q,i}^{*2} & t_{q,i}^* & 1 & t_{q,i} \\ t_{q,i}^{*3} & t_{q,i}^{*2} & t_{q,i}^* & 1 \end{bmatrix}. \tag{4}$$

  Assume for simplicity that $t_{q,i} = 0\ \forall q$ for $i = 1, \ldots, 6$ and $t_{q,0} = te^{\phi_q}\ \forall q$ where t is the magnitude of the correlation coefficient (same for all users) and ϕ_q is a user-specific phase of the correlation coefficient, randomly distributed between 0 and 2π.

Further details on the MIMO channel assumption are provided in Table 2.

Table 2: Deployment and channel model parameters

Parameter	Explanation/Assumption
Time correlation ϵ	$\epsilon = 0.85$ (baseline). Other values to be investigated to assess the impact of time correlation on performance.
Spatial correlation t	$t = 0.5$ (baseline). Other values to be investigated to assess the impact of spatial correlation on performance.

4 Transmission Scheme

We aim at investigating the performance of LTE Single-User MIMO, consisting in Spatial Multiplexing with quantized precoding (as defined by LTE specifications). In the centre cell, the BS schedules one UE at a time (using Spatial Multiplexing with quantized precoding) so as to maximize a proportional fairness metric. To do so, the UEs report a Precoding Matrix Indicator (PMI), a Channel Quality Indicator (CQI) and a Rank Indicator (RI). It is assumed that there is no delay between the feedback and the transmission and no feedback errors on the uplink. The precoder is chosen in a codebook of precoders defined for RI = 1 to 4. LTE 4Tx codebook is provided in the appendix [1]. The RI and PMI together determines the index of the preferred precoder in the codebook. RI refers to the number of streams (denoted as Number of Layers in LTE specifications) that are transmitted to the UE and PMI is the index of the preferred precoder in the codebook corresponding to RI. The UE selects the RI and PMI that maximizes its rate. The CQI refers to the rate achievable by the selected RI and PMI. Various UE receivers can be considered but MMSE is considered as the baseline receiver. It could be designed differently depending on the assumptions on the knowledge of the inter-cell interference characteristics. Further details on the system-level assumptions are provided in Table 3.

The performance is measured in terms of the cumulative distribution function (CDF) of the user average throughput.

5 Tasks

The following tasks should be performed

1. Write the system model.
2. Show the cumulative distribution function (CDF) of the user long term SINR. The long term SINR is calcluated by ignoring the MIMO fading channel and only accounts for path loss and shadowing.
3. Show and discuss the influence of the number of receive antennas (2×4 vs. 4×4) on the performance.
4. Show and discuss the influence of the proportional fair scheduler parameters on the performance.
5. Show and discuss the influence of velocity/time correlation ϵ and the number of users K on the performance.
6. Show and discuss the influence of the receiver architecture on the performance. Example of receivers: MRC, ZF, MMSE, ZF-SIC, MMSE-SIC, ML.

Table 3: System-level assumptions

Parameter	Explanation/Assumption
Interference modeling	Random precoding, i.e. the precoder is randomly generated at each interfering BS and is selected from the LTE codebook
Scheduling	Proportional fair in time domain
Transmission mode	Single-user MIMO
Codebook of precoder	4Tx LTE codebook (see Appendix)
Feedback information	RI (2-bit), PMI (4-bit), CQI (unquantized)
Feedback delay/error	no delay, no error
Receiver	MMSE (baseline), MRC, ZF, ZF-SIC, MMSE-SIC, ML
Rank adaptation	the UE dynamically selects the preferred tranmission rank (RI)
Power allocation among streams	uniform power allocation among streams for a given transmission rank
Coding and modulation, link adaptation/abstraction	Shannon capacity expression, i.e. it is assumed that rate achievable with the transmission of a stream is given by $\log_2(1 + SINR)$ where $SINR$ is the instantaneous signal to interference plus noise ratio.

7. Show and discuss the influence of the spatial correlation t on the performance. Detail the expressions of the achievable rate and receiver combiner in your report.

To distinguish yourself,

8. Improve LTE codebook by replacing the LTE rank-1 codebook with an adaptive codebook designed using Lloyd Algorithm. Assume that the channel statistics (e.g. spatial correlation matrix) is known. Show and discuss the performance gain of the improved codebook vs LTE codebook when $K = 1$.

9. Show and discuss the influence on the performance of an ON-OFF coordinated power control scheduler across base stations that dynamically turns off the dominant interferer.

6 Deliverable

The project is conducted **individually** using Matlab. All Matlab files must have been written by yourself. Each student is requested to submit (on Blackboard)

1. A pdf **report** (conference paper-like) detailing the results. Format: Font size 10 pt, maximum 10 pages, single-spacing.

2. **All Matlab files** with comments. The files should be self-explanatory and the examiner should be able to run the code and get the same results as those provided in the report. Explain how to run the code.

Deadline for report submission: **29 March 2013, 6pm** (London time).

A Appendix: LTE 4Tx codebook [1]

Table 6.3.4.2.2-1: Large-delay cyclic delay diversity.

Number of layers υ	U	$D(i)$
2	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & e^{-j2\pi/2}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & e^{-j2\pi i/2}\end{bmatrix}$
3	$\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & e^{-j2\pi/3} & e^{-j4\pi/3}\\ 1 & e^{-j4\pi/3} & e^{-j8\pi/3}\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0\\ 0 & e^{-j2\pi i/3} & 0\\ 0 & 0 & e^{-j4\pi i/3}\end{bmatrix}$
4	$\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & e^{-j2\pi/4} & e^{-j4\pi/4} & e^{-j6\pi/4}\\ 1 & e^{-j4\pi/4} & e^{-j8\pi/4} & e^{-j12\pi/4}\\ 1 & e^{-j6\pi/4} & e^{-j12\pi/4} & e^{-j18\pi/4}\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & e^{-j2\pi i/4} & 0 & 0\\ 0 & 0 & e^{-j4\pi i/4} & 0\\ 0 & 0 & 0 & e^{-j6\pi i/4}\end{bmatrix}$

6.3.4.2.3 Codebook for precoding and CSI reporting

For transmission on two antenna ports, $p \in \{0,1\}$, and for the purpose of CSI reporting based on two antenna ports $p \in \{0,1\}$ or $p \in \{15,16\}$, the precoding matrix $W(i)$ shall be selected from Table 6.3.4.2.3-1 or a subset thereof. For the closed-loop spatial multiplexing transmission mode defined in [4], the codebook index 0 is not used when the number of layers is $\upsilon = 2$.

Table 6.3.4.2.3-1: Codebook for transmission on antenna ports $\{0,1\}$ and for CSI reporting based on antenna ports $\{0,1\}$ or $\{15,16\}$.

Codebook index	Number of layers υ: 1	Number of layers υ: 2
0	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
1	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
2	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ j\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1 & 1\\ j & -j\end{bmatrix}$
3	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -j\end{bmatrix}$	-

For transmission on four antenna ports, $p \in \{0,1,2,3\}$, and for the purpose of CSI reporting based on four antenna ports $p \in \{0,1,2,3\}$ or $p \in \{15,16,17,18\}$, the precoding matrix W shall be selected from Table 6.3.4.2.3-2 or a subset thereof. The quantity $W_n^{\{s\}}$ denotes the matrix defined by the columns given by the set $\{s\}$ from the expression $W_n = I - 2u_n u_n^H / u_n^H u_n$ where I is the 4×4 identity matrix and the vector u_n is given by Table 6.3.4.2.3-2.

Table 6.3.4.2.3-2: Codebook for transmission on antenna ports $\{0,1,2,3\}$ and for CSI reporting based on antenna ports $\{0,1,2,3\}$ or $\{15,16,17,18\}$.

Codebook index	u_n	Number of layers υ			
		1	2	3	4
0	$u_0=\begin{bmatrix}1 & -1 & -1 & -1\end{bmatrix}^T$	$W_0^{\{1\}}$	$W_0^{\{14\}}/\sqrt{2}$	$W_0^{\{124\}}/\sqrt{3}$	$W_0^{\{1234\}}/2$
1	$u_1=\begin{bmatrix}1 & -j & 1 & j\end{bmatrix}^T$	$W_1^{\{1\}}$	$W_1^{\{12\}}/\sqrt{2}$	$W_1^{\{123\}}/\sqrt{3}$	$W_1^{\{1234\}}/2$
2	$u_2=\begin{bmatrix}1 & 1 & -1 & 1\end{bmatrix}^T$	$W_2^{\{1\}}$	$W_2^{\{12\}}/\sqrt{2}$	$W_2^{\{123\}}/\sqrt{3}$	$W_2^{\{3214\}}/2$
3	$u_3=\begin{bmatrix}1 & j & 1 & -j\end{bmatrix}^T$	$W_3^{\{1\}}$	$W_3^{\{12\}}/\sqrt{2}$	$W_3^{\{123\}}/\sqrt{3}$	$W_3^{\{3214\}}/2$
4	$u_4=\begin{bmatrix}1 & (-1-j)/\sqrt{2} & -j & (1-j)/\sqrt{2}\end{bmatrix}^T$	$W_4^{\{1\}}$	$W_4^{\{14\}}/\sqrt{2}$	$W_4^{\{124\}}/\sqrt{3}$	$W_4^{\{1234\}}/2$
5	$u_5=\begin{bmatrix}1 & (1-j)/\sqrt{2} & j & (-1-j)/\sqrt{2}\end{bmatrix}^T$	$W_5^{\{1\}}$	$W_5^{\{14\}}/\sqrt{2}$	$W_5^{\{124\}}/\sqrt{3}$	$W_5^{\{1234\}}/2$
6	$u_6=\begin{bmatrix}1 & (1+j)/\sqrt{2} & -j & (-1+j)/\sqrt{2}\end{bmatrix}^T$	$W_6^{\{1\}}$	$W_6^{\{13\}}/\sqrt{2}$	$W_6^{\{134\}}/\sqrt{3}$	$W_6^{\{1324\}}/2$
7	$u_7=\begin{bmatrix}1 & (-1+j)/\sqrt{2} & j & (1+j)/\sqrt{2}\end{bmatrix}^T$	$W_7^{\{1\}}$	$W_7^{\{13\}}/\sqrt{2}$	$W_7^{\{134\}}/\sqrt{3}$	$W_7^{\{1324\}}/2$
8	$u_8=\begin{bmatrix}1 & -1 & 1 & 1\end{bmatrix}^T$	$W_8^{\{1\}}$	$W_8^{\{12\}}/\sqrt{2}$	$W_8^{\{124\}}/\sqrt{3}$	$W_8^{\{1234\}}/2$
9	$u_9=\begin{bmatrix}1 & -j & -1 & -j\end{bmatrix}^T$	$W_9^{\{1\}}$	$W_9^{\{14\}}/\sqrt{2}$	$W_9^{\{134\}}/\sqrt{3}$	$W_9^{\{1234\}}/2$
10	$u_{10}=\begin{bmatrix}1 & 1 & 1 & -1\end{bmatrix}^T$	$W_{10}^{\{1\}}$	$W_{10}^{\{13\}}/\sqrt{2}$	$W_{10}^{\{123\}}/\sqrt{3}$	$W_{10}^{\{1324\}}/2$
11	$u_{11}=\begin{bmatrix}1 & j & -1 & j\end{bmatrix}^T$	$W_{11}^{\{1\}}$	$W_{11}^{\{13\}}/\sqrt{2}$	$W_{11}^{\{134\}}/\sqrt{3}$	$W_{11}^{\{1324\}}/2$
12	$u_{12}=\begin{bmatrix}1 & -1 & -1 & 1\end{bmatrix}^T$	$W_{12}^{\{1\}}$	$W_{12}^{\{12\}}/\sqrt{2}$	$W_{12}^{\{123\}}/\sqrt{3}$	$W_{12}^{\{1234\}}/2$
13	$u_{13}=\begin{bmatrix}1 & -1 & 1 & -1\end{bmatrix}^T$	$W_{13}^{\{1\}}$	$W_{13}^{\{13\}}/\sqrt{2}$	$W_{13}^{\{123\}}/\sqrt{3}$	$W_{13}^{\{1324\}}/2$
14	$u_{14}=\begin{bmatrix}1 & 1 & -1 & -1\end{bmatrix}^T$	$W_{14}^{\{1\}}$	$W_{14}^{\{13\}}/\sqrt{2}$	$W_{14}^{\{123\}}/\sqrt{3}$	$W_{14}^{\{3214\}}/2$
15	$u_{15}=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}^T$	$W_{15}^{\{1\}}$	$W_{15}^{\{12\}}/\sqrt{2}$	$W_{15}^{\{123\}}/\sqrt{3}$	$W_{15}^{\{1234\}}/2$

For the purpose of CSI reporting for eight CSI reference signals the codebooks are given in section 7.2.4 of [4].

6.3.4.3 Precoding for transmit diversity

Precoding for transmit diversity is only used in combination with layer mapping for transmit diversity as described in Section 6.3.3.3. The precoding operation for transmit diversity is defined for two and four antenna ports.

For transmission on two antenna ports, $p \in \{0,1\}$, the output $y(i) = \begin{bmatrix}y^{(0)}(i) & y^{(1)}(i)\end{bmatrix}^T$, $i = 0,1,\ldots,M_{\mathrm{symb}}^{\mathrm{ap}} - 1$ of the precoding operation is defined by

$$
\begin{bmatrix} y^{(0)}(2i) \\ y^{(1)}(2i) \\ y^{(0)}(2i+1) \\ y^{(1)}(2i+1) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & j & 0 \\ 0 & -1 & 0 & j \\ 0 & 1 & 0 & j \\ 1 & 0 & -j & 0 \end{bmatrix} \begin{bmatrix} \mathrm{Re}\left(x^{(0)}(i)\right) \\ \mathrm{Re}\left(x^{(1)}(i)\right) \\ \mathrm{Im}\left(x^{(0)}(i)\right) \\ \mathrm{Im}\left(x^{(1)}(i)\right) \end{bmatrix}
$$

for $i = 0,1,\ldots,M_{\mathrm{symb}}^{\mathrm{layer}} - 1$ with $M_{\mathrm{symb}}^{\mathrm{ap}} = 2M_{\mathrm{symb}}^{\mathrm{layer}}$.

For transmission on four antenna ports, $p \in \{0,1,2,3\}$, the output $y(i) = \begin{bmatrix}y^{(0)}(i) & y^{(1)}(i) & y^{(2)}(i) & y^{(3)}(i)\end{bmatrix}^T$, $i = 0,1,\ldots,M_{\mathrm{symb}}^{\mathrm{ap}} - 1$ of the precoding operation is defined by

References

[1] 3rd Generation Partnership Project (3GPP), "3GPP TS 36.211, Technical Specification Group Radio Access Network, Evolved Universal Terrestrial Radio Access (E-UTRA), Physical Channels and Modulation (Release 11)," Dec. 2012.