IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EIE PART II: MEng, Beng and ACGI

FEEDBACK SYSTEMS

Wednesday, 4 June 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

Answer ALL questions. Question 1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : I.M. Jaimoukha
Second Marker(s) : S. Evangelou

1. a) Consider the mechanical system illustrated in Figure 1.1 where all the symbols have the standard interpretation. The input is the applied force $f(t)$ and the output is the displacement $y(t)$. Take $M = K_2 = D = 1$ in appropriate units.

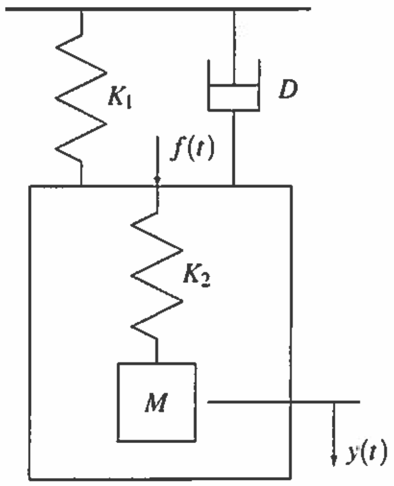

Figure 1.1

i) Determine the transfer function $G(s)$ relating y to f. [5]

ii) Use the Routh array to find the range of values of K_1 for stability. [5]

iii) Find the value of K_1 for which $G(s)$ is marginally stable. For this value of K_1, what are the poles of $G(s)$? [5]

iv) Let $f(t)$ be a unit step applied at $t = 0$. Use the final value theorem, which should be stated, to find the steady-state value y_{ss}, of $y(t)$ in terms of K_1. What is the value of K_1 for which $y_{ss} = 2$? [5]

b) In Figure 1.2 below, $G(s) = 2/(s^3 - 1)$ and $K(s)$ is a compensator.

i) Draw the Nyquist diagram of $G(s)$. [5]

ii) Let $K(s) = k$ be a constant compensator. Use the Nyquist criterion, which should be stated, to determine how many unstable or marginally stable poles the closed-loop has for all k. [5]

iii) Use the Routh-Hurwitz stability criterion to determine if the closed-loop can be stabilised using a PD compensator. [5]

iv) Show that the closed-loop can be stabilised using the compensator

$$K(s) = k\frac{s^2 + s + 1}{s^2 + 2s + 3}$$

for some $k > 0$. [5]

Figure 1.2

2. Consider the feedback control system in Figure 2.1 below. Here,

$$G(s) = \frac{1}{s^3 + as^2 + bs + c}$$

represents an uncertain model where it is only known that

$$a > 0, \qquad b > 0, \qquad c > 1, \qquad ab - c \geq 2. \tag{2.1}$$

$K(s)$ is the transfer function of a compensator.

a) Sketch a typical Nyquist diagram of $G(s)$, indicating the low and high frequency portions. Use the Routh array to find the real-axis intercepts. [8]

b) Let $K(s) = K$ be a nondynamic compensator. State the Nyquist stability criterion and use the Nyquist diagram to determine the number of unstable or marginally stable closed-loop poles for all values of K. [8]

c) What is the value of the gain margin for $G(s)$? [3]

d) Derive the minimum value of the gain margin for all a, b, c satisfying the relations in equation (2.1). [3]

e) Suppose that it is known that $G(s)$ has an adequate phase margin and that you have the option of either using a phase-lead or a phase-lag compensator. Which would you choose? Justify your choice. [8]

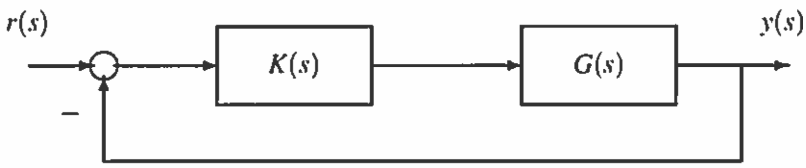

Figure 2.1

3. Consider the feedback loop shown in Figure 3.1 below.

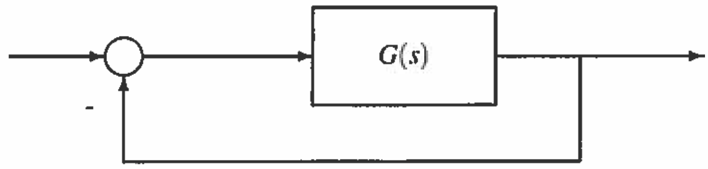

Figure 3.1

a) Suppose that

$$G(s) = \frac{-2s}{s^2 + ks + 1}$$

where $k > 0$ is a design parameter. It is required to find k such that closed–loop response to a step reference signal is critically damped with a settling time of $4s$.

i) Find the location of the closed–loop poles that achieves the design specification. [5]

ii) Derive the closed–loop characteristic equation. [5]

iii) Find the value of k that achieves the design specification. [5]

b) Suppose that

$$G(s) = \frac{-2s}{s^2 + k(s+z) + 1}$$

where $k > 0$ and $z \geq 0$ are design parameters. It is required to find k and z such that closed–loop response to a step reference signal achieves the following design specifications:

- The settling time is at most 4 seconds.
- The response is oscillatory with a maximum overshoot of 5%.

i) Find the location of the closed–loop poles that achieves the design specification. [5]

ii) Derive the closed–loop characteristic equation. [5]

iii) Find the values of k and z that achieve the design specifications. [5]