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IMPERIAL COLLEGE LONDON

EE2-3

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2012

EEE PART II: MEng, BEng and ACGI

POWER ENGINEERING

Thursday, 31 May 2:00 pm

Time allowed: 1:30 hours

There are THREE questions on this paper.

Answer ALL questions.

Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green
Second Marker(s) : B. Chaudhuri

Information for Candidates

Switch-Mode Power Supplies

Voltage ratio equations for buck SMPS $\frac{V_o}{V_i} = \delta$ or $\frac{V_O}{V_I} = \frac{1}{1 + \frac{2fLI_O}{V_I\delta^2}}$

Voltage ratio equations for boost SMPS $\frac{V_o}{V_i} = \frac{1}{1-\delta}$ or $\frac{V_o}{V_i} = \frac{1}{1 - \frac{V_i\delta^2}{2fLI_i}}$

Three-Phase Systems

Line Voltages and Current

Star $V_L = \sqrt{3}\,V_P \quad I_L = I_P$ Delta $V_L = V_P \quad I_L = \sqrt{3}\,I_P$

Power
$$P_{3\Phi} = 3\,V_P I_P \cos(\phi) = \sqrt{3}\,V_L I_L \cos(\phi)$$

Induction Machines

$$\omega_S = \frac{\omega_E}{P} \qquad s = \frac{\omega_S - \omega_R}{\omega_S} \qquad T_{em} = \frac{3\,I_R'^2\,R_R'}{\omega_R}\left(\frac{1-s}{s}\right)$$

Photovoltaic Systems

$$I_{PV} = I_{Ph} - I_{AK} - I_{Sh} \qquad I_{AK} = I_0\left[e^{\frac{V_{PV}+I_{PV}R_S}{K_I v_T}} - 1\right]$$

Power Flow in Lines and Cables

Cable Parameters

$$R'_{LF} = \frac{1}{\sigma_C \pi r_C^2} + \frac{1}{\sigma_C 2\pi r_O t_O} \quad L' = \frac{\mu_0}{2\pi}\ln\left(\frac{r_O}{r_C}\right) \quad C' = \frac{2\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(\frac{r_O}{r_C}\right)} \quad G' = \frac{2\pi\sigma_I}{\ln\left(\frac{r_O}{r_C}\right)}$$

OHL Parameters (approximate form)

$$R'_{LF} = \frac{2}{\sigma_C \pi r_C^2} \quad L' = \frac{\mu_0}{\pi}\ln\left(\frac{d}{r_C}\right) \quad C' = \frac{\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(\frac{d}{r_C}\right)} \quad G' = \frac{\pi\sigma_I}{\ln\left(\frac{d}{r_C}\right)}$$

Power Flow (full form)

$$P_S = \frac{V_S^2}{Z_{SR}}\cos(\theta) - \frac{V_S V_R}{Z_{SR}}\cos(\theta + \delta)$$

$$Q_S = \frac{V_S^2}{Z_{SR}}\sin(\theta) - \frac{V_S V_R}{Z_{SR}}\sin(\theta + \delta)$$

Voltage Drop (approximate form)

$$\Delta V = |V_S| - |V_R| \approx \frac{RP_S + XQ_S}{|V_S|}$$

1. This question covers several topics and all parts should be attempted.

a) A current of 150 A is drawn at an angle of 20° leading with respect to the 33 kV supply voltage. Calculate the real and reactive power. [5]

b) The expressions below give the resistance per unit length (R'), inductance per unit length (L') and capacitance per unit length (C') of an overhead line.

$$R'_{LF} = \frac{2}{\sigma_C \pi r_C^2} \qquad L' = \frac{\mu_0}{\pi}\ln\left(\frac{d}{r_C}\right) \qquad C' = \frac{\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(\frac{d}{r_C}\right)}$$

i) Explain why the inductance per unit length is large and the capacitance per unit length is small for an Extra High Voltage (EHV) overhead line compared to a cable. [5]

ii) Explain why the X:R ratios of overhead lines are different for different nominal voltages. [5]

iii) The equations below describe the real and reactive power flow in an overhead line assuming the line impedance is composed of a series impedance Z_{SR} which has an angle θ and has non-zero reactive and resistive parts. The angle difference between sending and receiving end voltages is δ. Explain why the real and reactive power flows through an EHV overhead line can be said to be proportional to the differences between the angle and magnitudes (respectively) between the sending and receiving end voltages. [5]

$$P_S = \frac{V_S^2}{Z_{SR}}\cos(\theta) - \frac{V_S V_R}{Z_{SR}}\cos(\theta+\delta)$$

$$Q_S = \frac{V_S^2}{Z_{SR}}\sin(\theta) - \frac{V_S V_R}{Z_{SR}}\sin(\theta+\delta)$$

c) Consider a photovoltaic (PV) panel used for capturing solar energy.

i) Describe the factors affecting the effectiveness of a PV panel. [5]

ii) Sketch the shape of the current against voltage curve of a PV panel for a low and a high irradiance and explain the existence of a maximum power point for the panel for each operating condition. [5]

d) The electricity supply industry is described in terms of four unbundled business activities of Generation, Transmission, Distribution and Supply. Describe these functions, highlighting differences between them and whether they operate as markets or regulated monopolies. [5]

e) The traditional electricity supply system uses AC at a frequency of 50 Hz or 60Hz. Explain why AC was chosen and why these frequencies were chosen. Explain why high voltage DC is used in certain circumstances. [5]

2.

a) Figure 2 shows a cut-away of an induction machine and a cross-section through the machine showing the shape of the laminations.

i) Name the parts labelled A, B, and C. [3]

ii) Why is magnetic core of the machine constructed from laminations? [1]

iii) Why is there a gap between parts L and M and why is it small? [1]

iv) What is the purpose of the shapes O and P cut out of the laminations? [1]

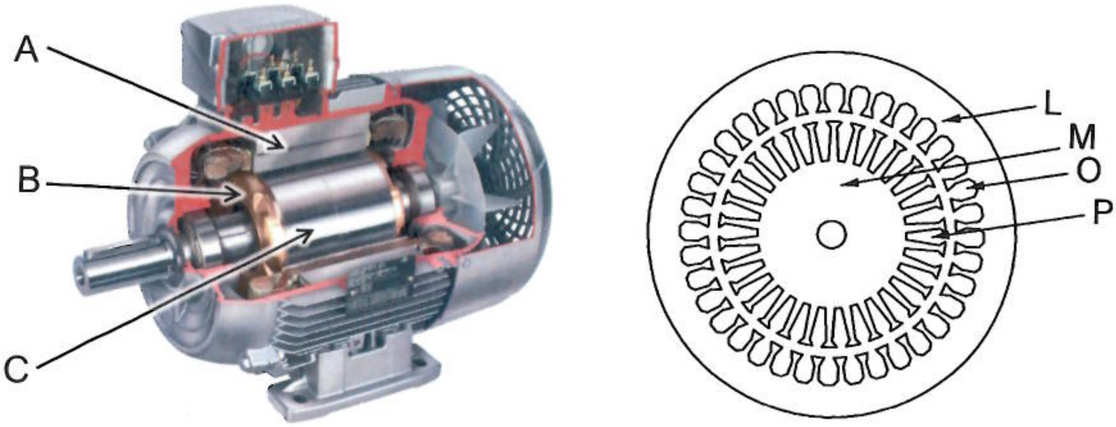


Figure 2. Cut-away and cross-section of an induction machine

b) A 3 pole-pair induction machine is rated as follows: 230 V phase; 50 Hz; 3-phase; star-connected. The equivalent circuit parameters, referred to the stator, are:

stator resistance	1.3 Ω,
referred rotor resistance	0.6 Ω,
stator leakage reactance	2.0 Ω,
referred rotor leakage reactance	0.7 Ω,
magnetising reactance	60 Ω,
magnetising resistance	negligible (infinite).

When driving a particular mechanical load, the machine is observed to spin at 960 rpm.

i) Calculate the slip. [3]

ii) Give an expression for the total input impedance of the machine and calculate its value in complex form. [7]

iii) Calculate the electro-magnetic torque. [10]

iv) Calculate the efficiency. [4]

3.

a) Sketch the circuit of a Buck Switch Mode Power Supply, SMPS. [3]

b) Explain the operation of a Buck SMPS and include sketches of graphs of the currents through the inductor, transistor and capacitor. [7]

c) A Buck SMPS is built with the following design choices: inductor, L = 100 μH; capacitor, C = 1,000 μF with ESR of 15 mΩ; switching frequency, f = 100 kHz. It is operated with an input voltage, V_I, of 20 V and an output voltage, V_O, of 10 V supplying an output current, I_O, of 3 A. Calculate the magnitude of the current ripple through the capacitor and estimate the voltage ripple appearing at the output. You may assume the converter is in continuous conduction. [12]

d) Consider the factors affecting the voltage ripple of a Buck SMPS.

i) Find analytical expressions for the resistive and capacitive portions of the voltage ripple in terms V_I, V_O, L, C, ESR and f with other variables eliminated. [4]

ii) Discuss the relative merits of following options for improving the output voltage ripple: increase f by a factor of 10; increase L by a factor of 10, increase C by a factor of 10 with and accompanying decrease in ESR of a factor of 3. [4]

1. This question covers several topics and all parts should be attempted.

a) A current of 150 A is drawn at an angle of 20° leading with respect to the 33 kV supply voltage. Calculate the real and reactive power. [5]

(i) [Calculation – 2 marks per correct answer, 1 mark for correct units]

$$P = \overline{V}.\overline{I} = VI\cos(\angle V - \angle I)$$

$$= 33{,}000 \times 150 \times \cos(0 - 20°) = +4.65MW$$

$$Q = VI\sin(\angle V - \angle I)$$

$$= 33{,}000 \times 150 \times \sin(0 - 20°) = -1.69MVAr$$

[A quite common mistake was to use a positive argument for the sine and cosine functions (+20 rather than -20) or retroactively make the reactive power positive. The sign of the angle and of P and Q are important so marks were lost for this mistake.]]

b) The expressions below give the resistance per unit length (R'), inductance per unit length (L') and capacitance per unit length (C') of an overhead line.

$$R'_{LF} = \frac{2}{\sigma_C \pi r_C^2} \qquad L' = \frac{\mu_0}{\pi}\ln\left(\frac{d}{r_C}\right) \qquad C' = \frac{\pi\varepsilon_0\varepsilon_{RI}}{\ln\left(\frac{d}{r_C}\right)}$$

i) Explain why the inductance per unit length is large and the capacitance per unit length is small for an Extra High Voltage (EHV) overhead line compared to a cable. [5]

[Book work]

Inductance is governed by loop area and a large separation of conductors gives a large inductance. Capacitance is governed by the separation of conductors and a small separation gives a large capacitance. Overhead lines, with air insulation, have large separation hence low capacitance and high inductance. Cables, which follow equations with similar but different logarithmic terms, with plastic insulation, have small separation hence low L' and high C'. {The C' is additionally increased by the relative permittivity of the plastic insulation.}

[It was rare for candidates to actually compare and contrast and instead talked about the OHL only. This lost marks.]

ii) Explain why the X:R ratios of overhead lines are different for different nominal voltages. [5]

[Book work]

Assuming that the current capacity stays approximately similar (at say 1, 000A) for OHL of different nominal voltages, the conductor radius and hence resistance stay approximately similar. The change in nominal voltage requires a change in insulation distances with longer insulator strings and greater separation between conductors with the higher voltage OHL having the greater distance, greater L, greater X and greater X:R ratio.

[The most common mistake was to deal with just the inductance and neglect any discussion of the resistance and/or neglecting to state that the inductance is dominant.]

iii) The equations below describe the real and reactive power flow in an overhead line assuming the line impedance is composed of a series impedance ZSR which has an angle θ and has non-zero reactive and resistive parts. The angle difference between sending and receiving end voltages is δ. Explain why the real and reactive power flows through an EHV overhead line can be said to be proportional to the differences between the angle and magnitudes (respectively) between the sending and receiving end voltages. [5]

$$P_S = \frac{V_S^2}{Z_{SR}}\cos(\theta) - \frac{V_S V_R}{Z_{SR}}\cos(\theta+\delta)$$

$$Q_S = \frac{V_S^2}{Z_{SR}}\sin(\theta) - \frac{V_S V_R}{Z_{SR}}\sin(\theta+\delta)$$

[Bookwork]

For an EHV OHL, the $X{:}R$ ratio is such that the impedance angle can be taken as approximately 90° and the equations simplify.

$$P_S \approx \frac{V_S V_R}{X_{SR}}\sin(\delta)$$

$$Q_S \approx \frac{V_S^2}{X_{SR}} - \frac{V_S V_R}{X_{SR}}\cos(\delta) = \frac{V_S}{X_{SR}}(V_S - V_R\cos(\delta))$$

If the voltage angle difference δ is small then the real power is proportional to δ. The reactive power becomes proportional to *VS-VR* under the same assumption.

[The small delta assumption was often not stated, especially in the equation for real power.]

c) Consider a photovoltaic (PV) panel used for capturing solar energy.

i) Describe the factors affecting the effectiveness of a PV panel [5]

[Bookwork]

Candidates need to list at least three factors and provide some detail on each

Choice of band-gap: a high band-gap material will extract more energy from a captured photon than a low band-gap material but fewer photons will have enough energy to be captured. The choice of band-gap is thus a compromise and 100% capture efficiency is not possible.

Masking and reflection: construction of the cells requires metallisation across the front surface which will prevent some light from reaching the semiconductor. The protective glass on the front surface will cause some reflection and absorption losses.

Orientation and air-mass: unless the panel has two-axis tracking, it will not be perpendicular to the incident light and so receives less light that the maximum possible. Light energy is absorbed by passaged through the atmosphere and this occurs more when the sun is low in the sky and light passes a greater distance through the atmosphere (a high air-mass number).

Cloud cover and shadows: clouds ill diffuse light and block light from reaching the panel. Other objects, trees and high buildings, may also cause shadows.

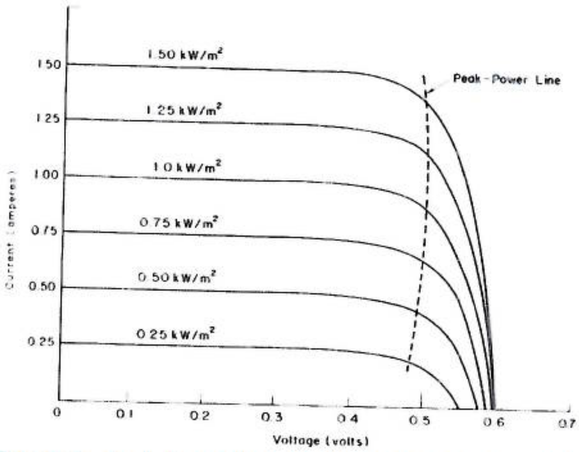
[Generally, the answers were good but candidates often resort to providing just lists of factors without explanation or giving only the physical and weather effects (essentially "more light, more power") and did cover factors in the construction of the cells.]

ii) Sketch the shape of the current against voltage curve of a PV panel for a low and a high irradiance and explain the existence of a maximum power point for the panel for each operating condition. [5]

[Bookwork]

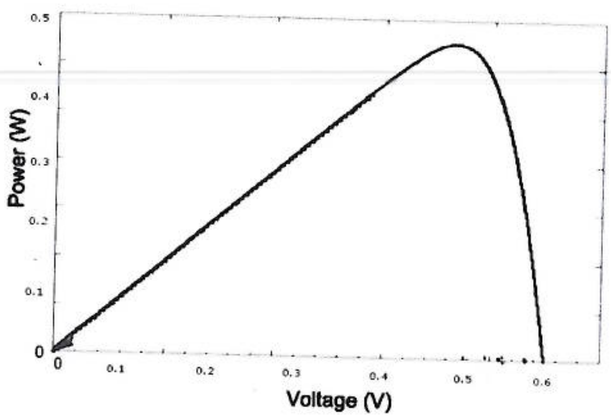
Graphs similar to two curves from this one is needed. [2 for shape and 1 for higher current and marginally high voltage at higher irradiance]

[The characteristics were often drawn with too little knee and with the current not flat enough to the left of the knee. This makes a difference to the shape of the peak in graph of power. Additionally, it was very common to show exaggerated differences in the short circuit currents; the short-circuit current is basically proportional to irradiance.]



Power is the I V product. In the region of almost flat current, the power is increased by operating at higher voltage until the knee point at which the current starts to collapse. Operation at the knee yields maximum power but the need is different for each irradiance (and temperature). [2 marks for description; graph can form part of answer]

[Explanations of the existence of the maximum power point were generally poor. Many candidates stated that maximum power is where I and V are maximum (without suggesting where that might be!) and simply stating P = IV without explaining the implications for the existence of a maximum. Some candidates wasted time discussing MPPT which isn't an answer to the question.



[It is incorrect to show this curve folding back on itself.]

d) The electricity supply industry is described in terms of four unbundled business activities of Generation, Transmission, Distribution and Supply. Describe these functions, highlighting differences between them and whether they operate as markets or regulated monopolies. [5]

[Bookwork]
[1 for proper distinction between four sectors; other marks for descriptions]

Generation: The function is generation is to produce electrical power by means of energy conversion. Typically in the UK this is done by a small number of large thermal or hydro plants. Each converts the mechanical energy at the output of a turbine into electrical energy. This conversion is achieved by use of a synchronous generator. The output of a synchronous generator is typically 10-20 kV which is stepped up via a transformer to transmission voltages typically 100 kV to 400 kV.

Electrical energy produced is sold in a forwards market, typically in half hour delivery slots. The market closes one hour or one day ahead of delivery. Counter parties in market are supply businesses (wholesalers) or large end-use customers.

[Generally answered well but occasionally incorrectly described as a monopoly.]

Transmission: The function of transmission is ship large amounts of power over long distances from generation sources to load centres. Transmission systems are generally interconnected in a mesh structure to allow economically efficient provision of reserve services, balancing services, security of supply and stability. Voltages are typically at 100 kV to 400 kV to reduce the I^2R losses of transmission lines of which are generally overhead pylons. The transmission network is normally highly meshed (nodes connect to several others) and routes are composed of double circuits to give a high degree of resilience of circuit outages. Transmission is normally a regulated (local) monopoly that applies use-of-system charges.

[Generally answered well.]

Distribution: The function of distribution is to take electrical energy from a bulk supply point and supply individual consumers. Distribution networks are usually a radial structure using underground cables and overhead lines with a normally unidirectional flow of power. The growth of distributed generation and renewable generation is changing this topology. Distribution voltages are typically 33kV, 11kV and 400V. For the most part most part, the DN has consumers but little generation. Distribution is again usually a regulated (local) monopoly that applies use-of-system charges.

[Some confusion arose here, mostly in describing "supply" as the low voltage (240V) consumer part of the network with "distribution" presumably ending at 11kV. Another common error was to describe distribution as a market partner to supply.]

Supply: Wholesalers who have bought in the main generation market sell onto domestic and other small consumers. Sometimes regionally based but in the UK there is a competitive market in supply.

[Often confused with the LV part of the distribution network.]

e) The traditional electricity supply system uses AC at a frequency of 50 Hz or 60Hz. Explain why AC was chosen and why these frequencies were chosen. Explain why high voltage DC is used is some limited circumstances. [5]

[Bookwork.]

A key to building an economically effective power system is to operate at EHV for long distance transmission (because power loss is proportional to I^2 and so current is minimised by increasing voltage) but to operate at lower voltages in distribution where the capital cost of numerous EHV substations is not justified by the saving in losses. Thus it is necessary to converter power between a variety of voltages. [1 mark]. Voltage conversion is most efficient through a transformer which requires AC to give a continuous rate-of-change of flux [1 mark]. A high frequency would enable transformers to be small but causes very high voltage drops in inductive lines and high shunt currents in capacitive cables and requires faster generators with problems for bearings etc. A low frequency eases the voltage drop and shunt current problems but requires large transformers and can cause lighting flicker. The best compromise between the factors (historically) is 50/60 Hz. [2 marks]. Long cables and very long OHL can have high shunt capacitive current that consumes all of the current rating of the conductors. Operation at lower frequency helps and DC avoids the problem. The cost of the AC/DC conversion to operate a DC line is borne when the cost of providing thick conductors or reactive compensation is to high. [1 mark].

[The basic case for AC was generally well understood but the argument was often very poorly laid out argument with steps in the argument skipped or an unjustified assumption on the requirement for transformers. The choice of frequency rang was remembered by most candidates but points were missed in the argument. In addressing the case for HVDC, some candidates gave the answer to a past paper question that discussed DC in ships rather than HVDC in general electricity networks.]

2.

[In the main this question was handled well by candidates, perhaps because it is of a familiar format.]

a) Figure 2 shows a cut-away of an induction machine and a cross-section through the machine showing the shape of the laminations.

i) Name the parts labelled A, B, and C. [3]

[Bookwork; 1 mark for each name]

A is the stator lamination / steel

B is the rotor end winding / shorting ring

C is the rotor lamination /steel

[Answers here tended to be too vague.]

ii) Why is magnetic core of the machine constructed from laminations? [1]

[Bookwork]

The stator core, and to a lesser extent the rotor core, is subjected to varying (rotating flux) and thus has voltages induced in the core that would cause eddy currents in the axial direction in the cores. To make these paths relatively high impedance and reduce eddy current power losses, the cores are laminated and the surfaces oxidised to provide isolation barriers.

[Note, lamination does not reuce hysteresis loss and candidates who said it did lost marks]

iii) Why is there a gap between parts L and M and why is it small? [1]

[Bookwork]

The gap provides clearance to allow the rotor to rotate and is small to keep the reluctance of the magnetic path small.

iv) What is the purpose of the shapes O and P cut out of the laminations? [1]

[Bookwork]

The punched holes, known as slots, provide the space for the rotor and stator windings while allowing the steel teeth of rotor and stator to be close together.

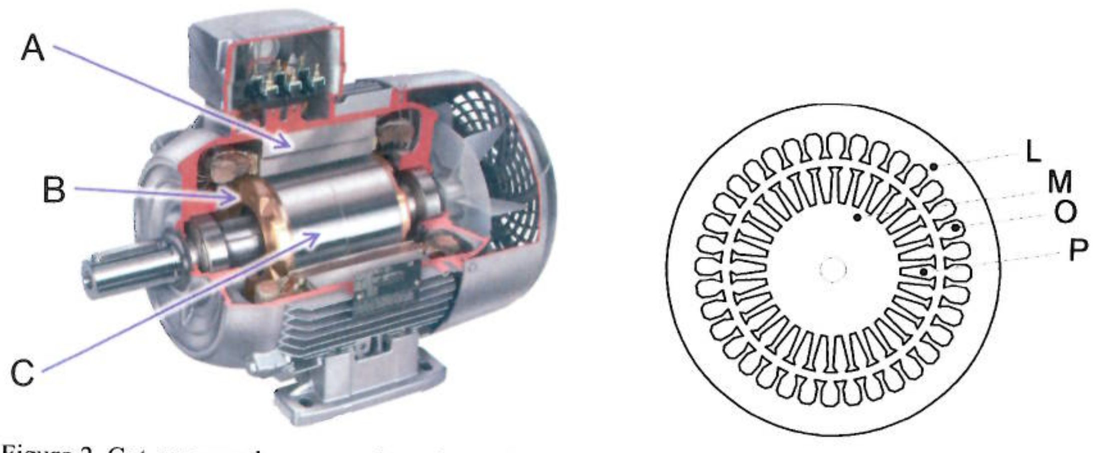


Figure 2. Cut-away and cross-section of an induction machine

b) A 3 pole-pair induction machine is rated as follows: 230 V phase; 50 Hz; 3-phase; star-connected. The equivalent circuit parameters, referred to the stator, are:

stator resistance	1.3 Ω,
referred rotor resistance	0.6 Ω,
stator leakage reactance	2.0 Ω,

referred rotor leakage reactance	0.7 Ω,
magnetising reactance	60 Ω,
magnetising resistance	negligible (infinite).

When driving a particular mechanical load, the machine is observed to spin at 960 rpm.

i) Calculate the slip. [3]

[Standard calculation]

$$\omega_S = \frac{2\pi f}{P} = \frac{2\pi 50}{3} \quad rad/s$$

$$n_S = \frac{60 \times 50}{2} = 1{,}000\ rpm$$

$$s = \frac{\omega_S - \omega_R}{\omega_S} = \frac{n_S - n_R}{n_S} = \frac{1000 - 960}{1000} = 0.04$$

ii) Give an expression for the total input impedance of the machine and calculate its value. [7]

[Standard calculation – 3 marks for expression; 4 marks for accurate answer]

[A small number of candidates neglected the 1/s term and this serious error lost 5 marks. An occasional error was to include the 60 Ω magnetising reactance as a real.]

[It should be noted that X=Im{Z} is a real number and a j is required in the expression for impedance, e.g. Z = R + jX. Z = R+ X is an error.}

$$\bar{Z}_T = R_S + jX_S + jX_M // \left({R'_R}/{s} + jX'_R \right)$$

$$\bar{Z}_T = 1.3 + j2.0 + j60 // \left({0.6}/{0.04} + j0.7 \right)$$

$$= 1.3 + j2.0 + \frac{j60 \times (15.0 + j0.7)}{15 + j60.7}$$

$$= 1.3 + j2.0 + \frac{60\angle 90° \times 15.02\angle 2.67°}{62.52\angle 76.12°}$$

$$= 1.3 + j2.0 + 14.41\angle 16.55°$$

$$= 15.11 + j6.10\ \Omega$$

$$= 16.30\angle 21.98°\ \Omega$$

iii) Calculate the electro-magnetic torque. [10]

[Standard calculation but several steps combined here without path to answer being given]

First find stator current [1 for method; 1 for answer]

$$\overline{I}_S = \frac{\overline{V}_S}{\overline{Z}_T} = \frac{230\angle 0^\circ}{16.30\angle 21.98^\circ}$$
$$= 14.11\angle -21.98^\circ\ A$$

Apply current divider rule to find rotor current [2 for method; 1 for answer]
(or calculate voltage across rotor branch and hence current)
[Note that the denominator of the current divider fraction is a sum not a parallel operation]

$$\overline{I}'_R = \overline{I}_S \frac{jX_M}{jX_M + {R'_R}/{s} + jX'_R}$$
$$= 14.11\angle -21.98^\circ \times \frac{60\angle 90^\circ}{62.52\angle 76.12^\circ}$$
$$= 13.54\angle -8.1^\circ$$

Use standard equation for torque [2 for eqn.; 2 for answer; 1 for unit]

[In this equation I^2 is the square of the magnitude which gives a real result (it is really **I•I** which is real because a dot product is always real). If I^2 is treated as a square of a complex number then a complex results is obtained which is meaningless in this context.]

$$T = 3\left|\overline{I}'_R\right|^2 R'_R \frac{\left(\frac{1}{s}-1\right)}{\omega_R}$$
$$= 3\left|\overline{I}'_R\right|^2 R'_R \frac{1}{s\omega_S}$$
$$= \frac{3\times 13.54^2 \times 0.6}{0.04\times 2/3\times 50\pi}$$
$$= 78.78\ Nm$$

iv) Calculate the efficiency [4]

[Standard calculation. Several approaches could be taken]

Find input power [1 mark]

$$P = 3\overline{V}_S \bullet \overline{I}_S = 3\times 230\times 14.11\times\cos(21.98^\circ) = 9.03kW$$

[A surprisingly large number of candidates did not include the cos() term.]

Find output power [1 mark]

$$P = 3I'^2_R R'_R\left(\tfrac{1}{s}-1\right) = 3\times 13.54^2\times 0.6\left(\frac{1}{0.04}-1\right) = 7.92kW$$

Take ratio [1 mark plus 1 mark for overall accuracy of answer]

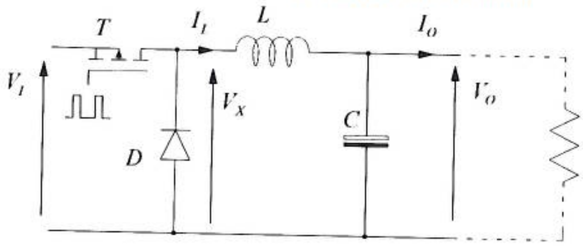
$$\eta = \frac{P_O}{P_I}\times 100\% = \frac{7.92k}{9.03k}\times 100\% = 87.7\%$$

3.

[Candidates performed well on this question notwithstanding the fact that the problem was not broken down into small steps and a new equation arrangement was needed.]

a) Sketch the circuit of a Buck Switch Mode Power Supply, SMPS. [3]

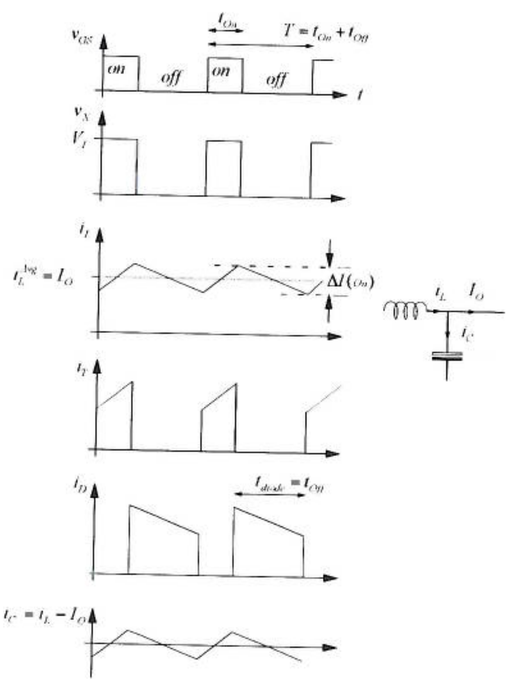
[Book work]
Mosfet and Diode must be of correct orientation;



b) Explain the operation of a Buck SMPS and include sketches of graphs of the currents through the inductor, transistor and capacitor. [7]

[Book work; 4 for explanation; 3 for 3 graphs]
Transistor T is pulsed on and off. During the on-time, positive voltage is applied across L and current builds up and energy is stored. During the off-time, the inductor current must be continue, initially and thus the diode is brought into forward bias. Negative voltage is applied across the inductor, decreasing the current and stored energy. Energy/charge is supplied to the capacitor in both states and energy/charge is continuously taken from the capacitor to the load. The output voltage is the DC component of the pulsed voltage marked V_x which is simply the input voltage multiplied by the transistor duty cycle in continuous mode.

[Book work; 3 for 3 graphs]
Graphs should align in time, i_T must show rising slope and i_C must be centred on zero. (i_D and v_T not needed)



c) A Buck SMPS is built with the following design choices: inductor, L = 100 μH; capacitor, C = 1,000 μF with ESR of 15 mΩ; switching frequency, f = 100 kHz. It is operated with an input voltage, V_I, of 20 V and an output voltage, V_O, of 10 V supplying an output current of 3 A. Calculate the magnitude of the current ripple through the capacitor and estimate the voltage ripple appearing at the output. You may assume the converter is in continuous conduction. [12]

[Standard calculation with one intermediate step noted.]

Duty cycle should be established first from voltage ratio. [1 marks for eqn; 1 for ans]

$$\delta = \frac{V_O}{V_I} = \frac{10}{20} = 0.5$$

Current ripple in the inductor is calculated from rate of change of current. [2 marks for eqn; 2 for ans]

$$\Delta i_L = \frac{V_I - V_O}{L} t_{On} = \frac{V_I - V_O}{L}\frac{\delta}{f} = \frac{(20-10)\times 0.5}{100\times 10^{-6}\times 100\times 10^{3}} = 0.5A$$

Current ripple in capacitor equals current ripple in inductor. [1 mark]

Voltage ripple across capacitor's resistive and capacitive components [1 for v_{ESR} ; 3 for v_C]

$$\Delta v_{ESR} = \Delta i_C R_{ESR} = 0.5\times 0.015 = 7.5mV$$

$$\Delta v_C = \frac{\Delta i_C}{8fC} = \frac{0.5}{8\times 1000\times 10^{-6}\times 100\times 10^{3}} = 0.625mV$$

A worse case estimate of the output voltage ripple is the sum of these two magnitudes [1mark]
8.125 mV

[Most common error was to only calculate one of the two terms. Some candidates confused the equations for Buck and Boost circuits.]

d) Consider the factors affecting the voltage ripple of a Buck SMPS.

i) Form analytical expressions for the resistive and capacitive portions of the voltage ripple in terms V_I, V_O, L, C, ESR and f with other variables eliminated. [4]

[New development: 2 for basic line of reasoning; 1 for each result]

$$\Delta i_L = \frac{V_I - V_O}{L}\frac{\delta}{f} = \frac{V_I - V_O}{fL}\frac{V_O}{V_I} = \frac{V_O - V_O^2/V_I}{fL}$$

$$\Delta v_{ESR} = \frac{V_O - V_O^2/V_I}{fL} R_{ESR}$$

$$\Delta v_C = \frac{\Delta i_C}{8fC} = \frac{\frac{V_O - V_O^2/V_I}{fL}}{8fC} = \frac{V_O - V_O^2/V_I}{8f^2LC}$$

ii) Discuss the relative merits of following options for improving the output voltage ripple: increase f by a factor of ten; increase L by a factor of 10, increase C by a factor of ten with the ESR decreasing by a factor of 3. [4]

[Judgement and interpretation of new problem]

Part (c) has shown a typical result with resistive component more significant that capacitive.

Inc f: Reductions in v_{ESR} and v_C are proportional and proportional to square respectively. v_C becomes negligible and so reduction dominated by change in v_{ESR}. and so a factor of a little over 10 reduction achieved A significant penalty in switching power loss occurs. [1 mark]

Inc L: Proportional reduction in both terms so factor of 10 reduction achieved. A penalty in volume/mass/cost arises. [1 mark]

Inc C and Dec ESR: ESR term reduces factor 3; C term reduces factor 10. Reduction in ESR term is limiting factor so factor of 3 reduction in ripple. A penalty in volume/mass/cost arises. [2 marks]

[Most common omission was to not reflect on the disadvantages of the options.]