IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

EEE/ISE PART II: MEng, BEng and ACGI

COMMUNICATIONS 2

Wednesday, 15 June 2:00 pm

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : C. Ling
Second Marker(s) : J.A. Barria

EXAM QUESTIONS

1. a) i) Explain the terms "noise", "external noise", and "internal noise". [3]

ii) Explain the terms "white noise", "Gaussian noise", and "additive white Gaussian noise". [3]

iii) Consider bandpass noise $n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$ which has the power spectral density shown in Fig. 1.1. Draw the power spectral density (PSD) of baseband noise $n_c(t)$ or $n_s(t)$ if the center frequency is chosen as: [4]

- $f_c = 7$ Hz
- $f_c = 10$ Hz

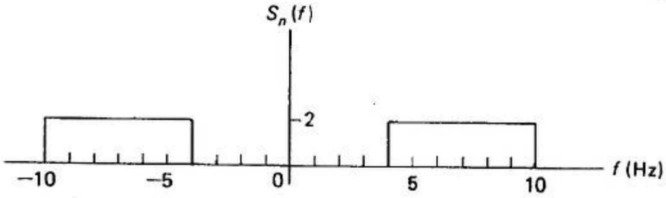


Figure 1.1 PSD of bandpass noise.

b) i) Explain the advantages and disadvantages of synchronous detection and envelope detection respectively for standard amplitude modulation (AM). [5]

ii) With help of a diagram, explain the operation of coherent detection for frequency shift keying (FSK). [5]

c) i) Name two primary resources in communications. [2]

ii) Write down the expression of the capacity of a Gaussian channel with bandwidth W. [2]

iii) What does the channel coding theorem say about the relation between transmission rate R and channel capacity C? [3]

iv) Consider a telephone line channel. If the signal to noise ratio (SNR) is 20 dB and the bandwidth available is 4 kHz, calculate the corresponding channel capacity. [3]

d) Consider an information source generating the random variable X with probability distribution

x_k	x_1	x_2	x_3	x_4
$P(X = x_k)$	0.4	0.2	0.25	0.15

i) Calculate the entropy of this source. [3]

ii) Construct a binary Huffman code. [5]

iii) Compute the average codeword length. [2]

2. a) Figure 2.1 shows the diagram of the FM receiver. The bandpass filter has bandwidth B_T, while the baseband low-pass filter has bandwith W. Let the FM signal be $s(t) = A\cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau]$ and assume the bandpass noise $w(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$ has single-sided power spectral density N_0.

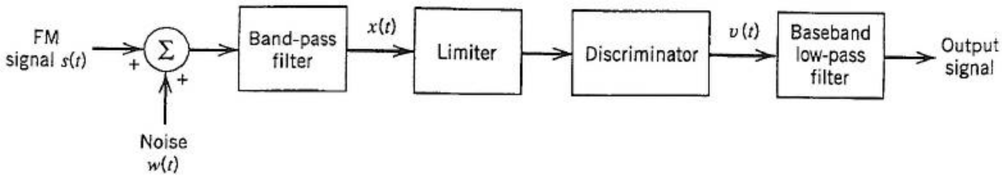


Figure 2.1 Block diagram of the FM receiver.

i) Explain the function of each of the blocks in Figure 2.1. [5]

ii) Given the noise at the output of the discriminator

$$f_i(t) = \frac{1}{2\pi A}\frac{dn_s(t)}{dt},$$

derive an expression for its power spectral density. [5]

iii) Sketch power spectral densities of $n_s(t)$, $f_i(t)$, and the noise at the output of the lowpass filter. [5]

b) Consider pre-emphasis and de-emphasis.

i) Show that in order to achieve flat noise power spectral density at the output of the FM receiver, the ideal de-emphasis filter has a transfer function $H_{de}(f) = 1/f$ within the message bandwidth. [5]

ii) Discuss why an FM transmitter with a corresponding pre-emphasis filter $H_{pre}(f) = f$ is essentially phase modulation. [5]

c) The signal-to-noise ratio (SNR) improvement factor is defined as

$$I = \frac{\text{Noise power without pre/de-emphasis}}{\text{Noise power with pre/de-emphasis}}.$$

Derive an expression of the improvement factor I for the scaled ideal de-emphasis filter $H_{de}(f) = f_0/f$, then compute the gain in dB for the parameter $W = 15$ kHz and $f_0 = 3$ kHz. [5]

3. a) A uniform quantizer for PCM has 2^n levels. The input signal is

$$m(t) = [A + A\cos(\omega_1 t)]\cos(\omega_2 t)$$

where $\omega_1 \neq \pm\omega_2$. Assume the dynamic range of the quantizer matches that of the input signal.

i) Work out the signal power. [3]

ii) Write down the probability density function of the quantization noise and the quantization noise power. [3]

iii) Work out the SNR in dB at the output of the quantizer. [3]

iv) Determine the minimum value of n such that the output SNR is no less than 60 dB. [3]

v) What can be done to increase the output SNR? [3]

b) An (n,k) linear block code has the following parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad (3.1)$$

i) What are the values of n and k? [2]

ii) Give a systematic generator matrix G of this code. [3]

iii) Compute the syndrome table for a single error. [5]

iv) The vector $y = [1000001]$ is received. Find the syndrome and hence the most likely data bits. [5]

2011

ANSWERS

B — Bookwork
E — New examples
A — New applications

EE2-4 COMMUNICATIONS 2

1. a) i) Noise refers to unwanted waves that disturb communications. [3 B]

External noise: interference from nearby channes, human-made noise, natural noise for external.

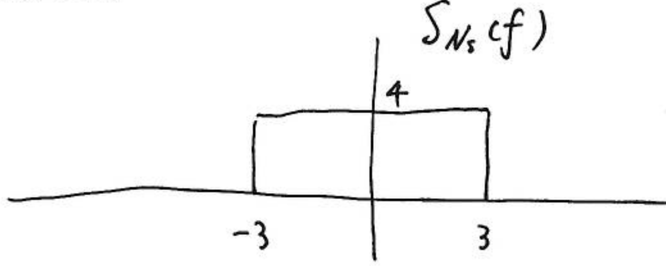
Internal noise: noise from within electronic devices, such as thermal noise.

ii). White noise: the PSD is constant

Gaussian noise: the PDF is Gaussian. [3 B]

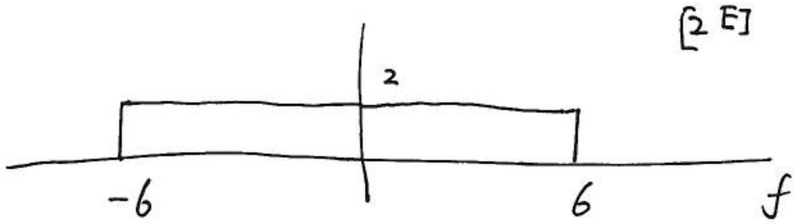
additive white Gaussian noise: noise is additive, ~~white~~ white, and Gaussian.

iii). $f_c = 7\,Hz$



[2 E]

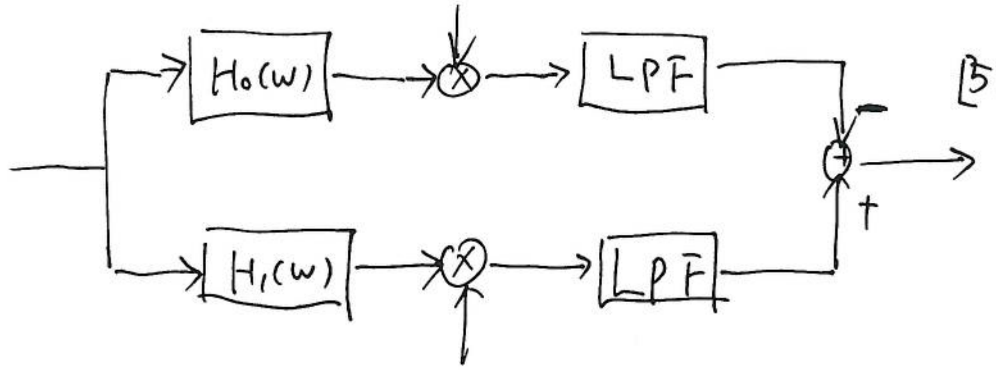
$f_c = 10\,Hz$



[2 E]

b) i) Synchronous detection: good noise performance. complex sync circuit. [2 B]

envelope detection: simple circuit, good performance at high SNR, but suffers from threshold effect. [3 B] ii)



[3 B]

The BPFs $H_0(w)$ and $H_1(w)$ have central frequency f_0 and f_1, respectively.

c) i) power, bandwidth [2 B]

ii)
$$C = W \log_2(1 + SNR)$$ [2 B]

iii) Channel coding theorem: As long as $R \le C$, we can achieve reliable communication over a noisy channel (i.e., with arbitrarily small probability of error); conversely, it is impossible to transmit messages without error if $R > C$. [3 B]

iv) $20\text{dB} \Rightarrow SNR = 100$ [3 E]

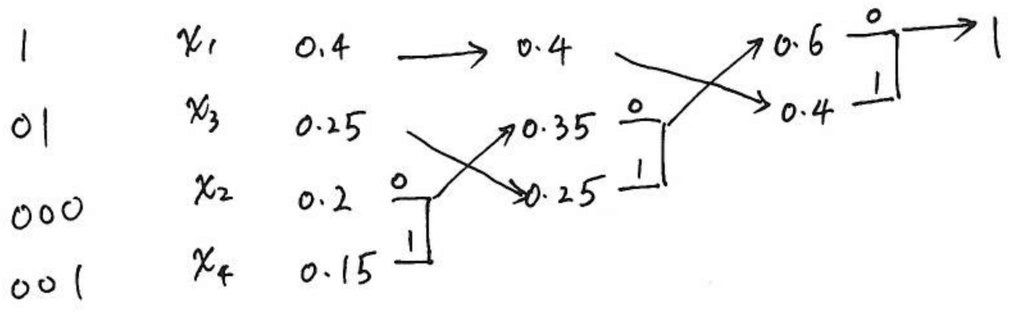
$$C = 4k \times \log_2(1+100) = 26.6 \text{ kbps}$$

d) i) $H(X) = -\sum p(x_k) \log p(x_k)$ [3 E]

$= -0.4 \times \log 0.4 - 0.2 \times \log 0.2 - 0.25 \times \log 0.25 - 0.15 \times \log 0.15$

$= 1.9$

ii)



[5E]

iii) $L = 1 \times 0.4 + 2 \times 0.25 + 3 \times 0.35$ [2E]

$= 1.95$

2\. a) i) The bandpass filter removes out-of-band noise. [5B]

The Limiter results in a constant envelope.

The discriminator outputs the deviation in the instantaneous frequency, i.e., it recover the message signal.

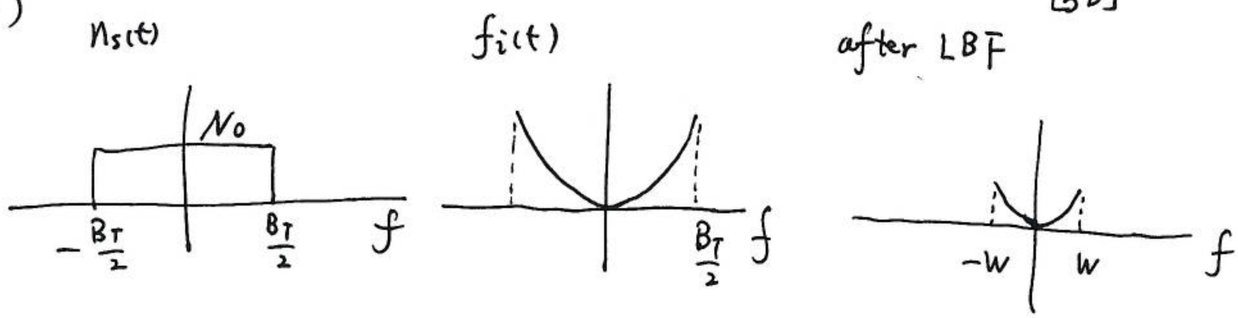
Low-pass filter: has a bandwidth W. It passes the message and removes out-of-band noise.

ii) This can be viewed as a linear system with transfer function $\frac{1}{2\pi A} j2\pi f = j\frac{f}{A}$. [5B]

Therefore, PSD for $f_i(t)$ is

$$\frac{f^2}{A^2} N_0, \qquad |f| \le \frac{B_T}{2}.$$

iii) [5B]



b) i) It's clear that to equalize the PSD $\frac{f^2}{A^2} N_0$, we need a de-emphasis filter $H_{de}(f) = \frac{1}{f}$. [5E]

ii) $H_{pre}(f) = f$ is a differentiation circuit. Thus the signal becomes $s(t) = A\cos[2\pi f_c t + k_f m(t)]$, which is PM. [5A]

c) Without deemphasis, the noise power is

$$P_N = \int_{-W}^{W} \frac{f^2}{A^2} N_0 \, df = \frac{2}{3} \frac{W^3 N_0}{A^2}. \qquad [1B]$$

With deemphasis, the noise power is

$$P_N = \int_{-W}^{W} \frac{f^2}{A^2} N_0 \cdot \frac{f_0^2}{f^2} \, df$$

$$= \int_{-W}^{W} \frac{f_0^2 N_0}{A^2} \, df \qquad [2A]$$

$$= \frac{2 W f_0^2 N_0}{A^2}.$$

Then,

$$I = \frac{\frac{2 W^3 N_0}{3 A^2}}{\frac{2 W f_0^2 N_0}{A^2}} = \frac{W^2}{2 f_0^2} \qquad [1A]$$

When $W = 15$ kHz, $f_0 = 2.1$ kHz

$$I = \frac{15^2}{2 \cdot 2.1^2} = 25.5. \qquad [1A]$$

In dB,

$$I\,(\mathrm{dB}) = 14 \text{ dB}.$$

3. a) i) $P_S = \frac{A^2}{2} + \frac{A^2}{4} = \frac{3}{4}A^2$ [3E]

ii) The quantization noise has a uniform PDF

$$f(q) = \frac{1}{\Delta}, \quad |q| < \frac{\Delta}{2}$$

Power: $P_N = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x^2 \cdot \frac{1}{\Delta}\, dx$ [3E]

$$= \frac{2}{\Delta} \cdot \frac{1}{3}\left(\frac{\Delta}{2}\right)^3$$

$$= \frac{\Delta^2}{12}$$

The signal range is $[-A, A]$.

$$\Delta = \frac{2A}{2^n} = \frac{A}{2^{n-1}}$$

iii) $SNR = \frac{P_S}{P_N} = \frac{\frac{3}{4}A^2}{\frac{(A/2^{n-1})^2}{12}} = \frac{9}{4} \cdot 2^{2n}$ [3E]

$SNR(dB) = 6n + 3.5$ dB

iv) $n = 10$ so that $SNR \geq 60\,dB$ [3E]

v) increase n. [3B]

b) i) $n = 7 \qquad k = 4$ [2 E]

ii) $$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$ [3 E]

iii) $S = e \cdot H^T$ [5 A]

Syndrome table

S	e
1 1 1	1 000 000
1 1 0	0 1 00 000
1 0 1	0 0 1 0 0 0 0
0 1 1	0 0 0 1 0 0 0
1 0 0	0 0 0 0 1 0 0
0 1 0	0 0 0 0 0 1 0
0 0 1	0 0 0 0 0 0 1

iv) [5 A]

$$S = y \cdot H^T$$
$$= (1\,1\,0)$$

$\Rightarrow e = (0\,1\,0\,0\,0\,0\,0)$

$\Rightarrow$ the second bit is wrong

$\Rightarrow$ sent codeword $x = [1\,1\,0\,0\,0\,0\,1]$

$\Rightarrow$ data bits $= [1\,1\,0\,0]$