Special instructions for invigilators

This exam consists of **2 sections**. **Section A: Devices** and **Section B: Fields**. Each section has to be solved in their respective answer books. Check that 2 different answer books are available for the students.

Special instructions for students

Use different answers books for each section:

Devices: answer book **A**
Fields: answer book **B**

Constants and Formulae for section A: Devices

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at $T = 300$K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$kT/e = 0.026$V at $T = 300$K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C

$$\left.\begin{aligned} J_n(x) &= e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx} \\ J_p(x) &= e\mu_p p(x)E(x) - eD_p \frac{dp(x)}{dx} \end{aligned}\right\}$$ Drift-diffusion current equations

$$I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2} \right)$$ MOSFET current

$$\left.\begin{aligned} J_n &= \frac{eD_n n_p}{L_n}\left(e^{\frac{eV}{kT}} - 1 \right) \\ J_p &= \frac{eD_p p_n}{L_p}\left(e^{\frac{eV}{kT}} - 1 \right) \end{aligned}\right\}$$ Diode diffusion currents

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2} \right)$$ Built-in voltage

$$c = c_0 \exp\left(\frac{eV}{kT} \right) \text{ with } \begin{cases} c = p_n \text{ or } n_p \\ c_0 \text{ bulk minority carrier concentration} \end{cases}$$ Minority carrier injection under bias *V*

$$\delta c = \Delta c \exp\left(\frac{-x}{L} \right) \text{ with } \begin{cases} \delta c = \delta p_n \text{ or } \delta n_p \\ \Delta c \text{ the excess carrier concentration} \\ \text{at the edge of the depletion region} \end{cases}$$ Excess carrier concentration as a function of distance

$$L = \sqrt{D\tau}$$ Diffusion length

$$D = \frac{kT}{e}\mu$$ Einstein relation

$$W_{depl} = \left[\frac{2\varepsilon V_0}{e} \frac{N_A + N_D}{N_A N_D} \right]^{1/2}$$ Depletion width in pn diode

$$C_{diff} = \frac{e}{kT} I\tau$$ Diffusion capacitance

Constants and Formulae for section B: Fields

Vector calculus (Cartesian co-ordinates)

$\nabla = \underline{i}\, \partial/\partial x + \underline{j}\, \partial/\partial y + \underline{k}\, \partial/\partial z$

$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$

$\text{grad}(\phi) = \nabla\phi = \underline{i}\, \partial\phi/\partial x + \underline{j}\, \partial\phi/\partial y + \underline{k}\, \partial\phi/\partial z$

$\text{div}(\underline{\mathbf{F}}) = \nabla \cdot \underline{\mathbf{F}} = \partial\mathbf{F}_x/\partial x + \partial\mathbf{F}_y/\partial y + \partial\mathbf{F}_z/\partial z$

$\text{curl}(\underline{\mathbf{F}}) = \nabla \times \underline{\mathbf{F}} = \underline{i}\, \{\partial\mathbf{F}_z/\partial y - \partial\mathbf{F}_y/\partial z\} + \underline{j}\, \{\partial\mathbf{F}_x/\partial z - \partial\mathbf{F}_z/\partial x\} + \underline{k}\, \{\partial\mathbf{F}_y/\partial x - \partial\mathbf{F}_x/\partial y\}$

Where ϕ is a scalar field and $\underline{\mathbf{F}}$ is a vector field

Maxwell's equations – integral form

$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho\, dv$

$\iint_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = -\iint_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

Where $\underline{\mathbf{D}}$, $\underline{\mathbf{B}}$, $\underline{\mathbf{E}}$, $\underline{\mathbf{H}}$, $\underline{\mathbf{J}}$ are time-varying vector fields

Maxwell's equations – differential form

$\text{div}(\underline{\mathbf{D}}) = \rho$

$\text{div}(\underline{\mathbf{B}}) = 0$

$\text{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\text{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

Material equations

$\underline{\mathbf{J}} = \sigma\, \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon\, \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu\, \underline{\mathbf{H}}$

Theorems

$\iint_A \underline{\mathbf{F}} \cdot d\underline{a} = \iiint_V \text{div}(\underline{\mathbf{F}})\, dv$ – Gauss' theorem

$\int_L \underline{\mathbf{F}} \cdot d\underline{L} = \iint_A \text{curl}(\underline{\mathbf{F}}) \cdot d\underline{a}$ – Stokes' theorem

$\text{curl}\,\{\text{curl}(\underline{\mathbf{F}})\} = \text{grad}\,\{\text{div}(\underline{\mathbf{F}})\} - \nabla^2\underline{\mathbf{F}}$

The Questions

SECTION A: SEMICONDUCTOR DEVICES

1.

a. What are the minority carriers in an n-type semiconductor device? [2]

b. Calculate the minority and majority carrier concentration of n-type Si doped with a donor concentration of N_D=5×10^{17} cm^{-3} at room temperature. [4]

c. There exist two types of currents, caused by different physical processes. Give the physical process that causes each current. [4]

d. In fig. 1 electrons are injected at a constant rate (steady-state) into intrinsic Si at x=0. Sketch the *excess* electron concentration from x=0 to x=L when recombination of carriers occurs. L_n is the diffusion length of electrons and L>>L_n. [6]

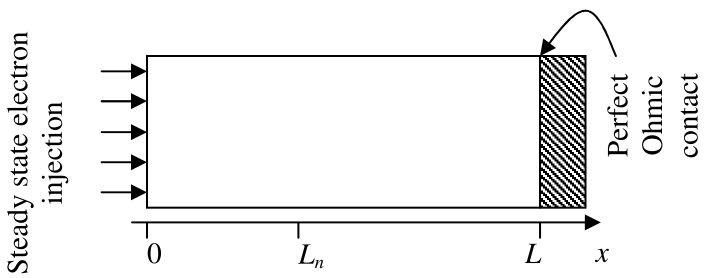

Figure 1: Steady-state electron injection into intrinsic Si at x=0. The ohmic contact x=L is ideal.

e. Explain briefly the cause for the switch-off delay in a bipolar junction transistor (BJT) with respect to charges in the different layers. [4]

2. pn diode

Given: a Si n^+p diode. The doping concentrations are $N_D = 10^{19}$ cm^{-3} and $N_A = 10^{16}$ cm^{-3}. The width of the layers are $x_n = 200$ nm (n^+ layer) and $x_p = 1$ µm (p layer). Thus the n-layer is "short", the p-layer "long". The diffusion length of the minority carrier holes is $L_p = 400$ nm and minority carrier electrons is $L_n = 500$ nm. The Ohmic contacts are ideal.

a. Sketch the energy band diagram including E_c, E_v, E_G and E_F and the built-in voltage V_0 across the n^+p diode when no external bias is applied.
Ensure that the relative distances between the energy levels are consistent with the doping density. Label all parameters in your graph. [5]

b. Sketch the *excess* minority carrier concentration in all layers of the diode in forward bias. Included the depletion region.
Ensure that the relative distances between the excess carrier levels at the junctions are consistent with the doping density. Label all parameters in your graph. [8]

c. Based on the equations given in the formulae sheet, give the expression of the stored excess charge Q_n and Q_p in each layer of the forward biased diode as a function of the material parameters: doping densities, layer widths and diffusion constants. [8]

d. Give a reasonable approximation for the value of the current density flowing through the diode when you know that the excess carrier concentration at the edge of the depletion width in the p region is 2×10^{12} cm^{-3}. The lifetime of the minority carriers in the p region is $\tau = 0.1$ µs.
State all approximations you make. [5]

e. When the given diode is switched from on to open circuit (off) at time t_{off}. What is the sign (positive, negative or zero) of the:
   i. the current through the diode immediately after t_{off}?
   ii. the voltage across the diode immediately after t_{off}? [4]

   Give a *brief* reason for each case.

SECTION B: ELECTROMAGNETIC FIELDS

3.

a. State the key contributions to electromagnetism and its applications made by the following: [7]

   i. Wilhelm Roentgen and Heinrich Hertz
   ii. Alexander Graham Bell, Guglielmo Marconi and John Logie Baird
   iii. John Tyndall, and Charles Kao and George Hockham

b. Figure 2 shows the entire electromagnetic spectrum. On a copy, indicate the ranges of: [7]

   i. Gamma rays
   ii. X-rays
   iii. Ultraviolet light
   iv. Visible light
   v. Infrared waves
   vi. Microwaves
   vii. Radio waves

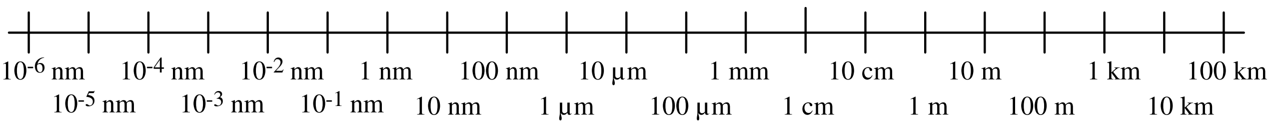

Figure 2: The electromagnetic spectrum.

c. Free-space terrestrial communications can operate using radio waves, microwaves or light waves. Point out any difficulties that each might suffer, and how these might be overcome. [6]

4.

a. Explain the difference between phase velocity and group velocity. Find the group velocity of a signal consisting of a pair of sinusoidal travelling waves of equal amplitude, with frequencies $\omega + \Delta\omega$ and $\omega - \Delta\omega$. [6]

b. The relationship between the angular frequency ω and the propagation constant k for electromagnetic waves in the ionosphere may be shown to have the form:

$$\omega = \sqrt{[\omega_p^2 + c^2k^2]}$$

Here c is the velocity of light and ω_p is the plasma frequency.

i. Derive expressions for the variation of the phase velocity and the group velocity with ω.
[6]

ii. What are the corresponding results for the atmosphere (which has $\omega_p = 0$)? [6]

iii. Sketch the dispersion diagram for the ionosphere and find the value of the velocity of the envelope (v_g) when ω tends to ω_p. What is the significance of this result for information propagation? [6]

iv. Find a solution for k when $\omega < \omega_p$. What form of wave does this solution describe? Explain the significance of this result for waves incident on the ionosphere from the atmosphere. [6]