IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

EEE PART I: MEng, BEng and ACGI

SEMICONDUCTOR DEVICES

Wednesday, 5 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.

Question One carries 40% of the marks. Questions Two and Three each carry 30%.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : K. Fobelets
Second Marker(s) : S. Lucyszyn

Constants

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
intrinsic carrier concentration in Si:	$n_i = 1.45 \times 10^{10}$ cm^{-3} at $T = 300$K
dielectric constant of Si:	$\varepsilon_{Si} = 11$
dielectric constant of SiO_2:	$\varepsilon_{ox} = 4$
thermal voltage:	$V_T = kT/e = 0.026$V at $T = 300$K
charge of an electron:	$e = 1.6 \times 10^{-19}$ C
Planck's constant:	$h = 6.63 \times 10^{-34}$ Js
Bandgap Si:	$E_G = 1.12$ eV at $T = 300$K
Effective density of states of Si:	$N_C = 3.2 \times 10^{19}$ cm^{-3} at $T = 300$K $N_V = 1.8 \times 10^{19}$ cm^{-3} at $T = 300$K

Formulae

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x)$$

Schrödinger's equation in one dimension

$$f(E)=\frac{1}{1+\exp\left(\frac{E-E_f}{kT}\right)}$$

Fermi distribution

$$n_i=\sqrt{N_V N_C}\exp\left(\frac{-E_G}{2kT}\right)$$

Intrinsic carrier concentration

$$n=N_c e^{-\frac{(E_c-E_F)}{kT}}$$

Concentration of electrons

$$p=N_v e^{\frac{(E_v-E_F)}{kT}}$$

Concentration of holes

$$\frac{dE}{dx}=\frac{\rho(x)}{\varepsilon}$$

Poisson equation in 1 dimension

$$\left.\begin{aligned}J_n(x)&=e\mu_n n(x)E(x)+eD_n\frac{dn(x)}{dx}\\ J_p(x)&=e\mu_p p(x)E(x)-eD_p\frac{dp(x)}{dx}\end{aligned}\right\}$$

Drift and diffusion current densities in a semiconductor

$$I_{DS}=\frac{\mu C_{ox}W}{L}\left((V_{GS}-V_{th})V_{DS}-\frac{V_{DS}^2}{2}\right)$$

Current in a MOSFET

$$\left.\begin{aligned}J_n&=\frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}}-1\right)\\ J_p&=\frac{eD_p p_{n_0}}{L_p}\left(e^{\frac{eV}{kT}}-1\right)\end{aligned}\right\}$$

Current densities for a pn-junction with lengths L_n & L_p

$$V_0=\frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in voltage

$$c=c_0\exp\left(\frac{eV}{kT}\right)\text{ with }\begin{cases}c=p_n\text{ or }n_p\\ c_0\text{ bulk minority carrier concentration}\end{cases}$$

Minority carrier injection under bias V

$$D=\frac{kT}{e}\mu$$

Einstein relation

$$W_{depl}(V)=\left[\frac{2\varepsilon(V_{bi}-V)}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\right]^{1/2}$$

Total depletion width under bias V

1.

a) Give the definition for the Fermi Level, E_F. [4]

b) In Fig.1.1 the energy band diagram of a doped semiconductor is given at room temperature. Draw the Fermi distribution function $f(E)$ for this semiconductor on this graph and indicate 3 values on the $f(E)$ axis. [6]

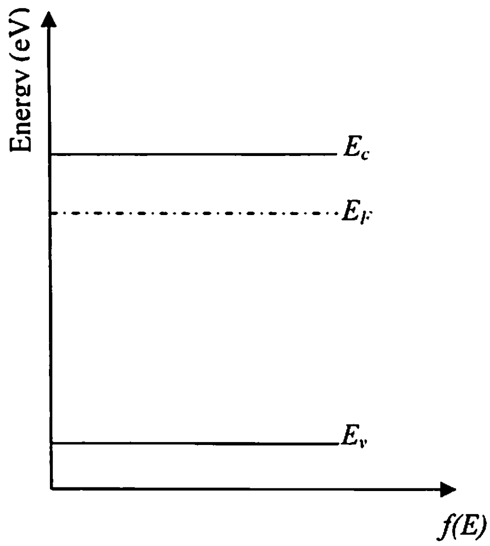


Figure 1.1: Energy band diagram for a doped semiconductor at room temperature

c) Sketch the energy band diagram (E_c, E_v, E_F, E_G) for a Schottky contact on n-type Si when the potential on the metal is negative with respect to the potential on the semiconductor. [6]

d) Which two possible changes can be made to the structure in the previous question (1.c) to make the contact Ohmic? [4]

e) Sketch the majority and minority carrier concentrations in both n and p regions of a forward biased, short n^+p junction. The doping concentration in the n region is higher than the doping concentration in the p-region. Ensure that the relative magnitudes of the carrier concentrations at both sides of the junction are correct. Label the graphs and the axes with the correct parameters. Take the depletion width into account. [4] marks for each neutral region and [2] marks for the accuracy of the depletion region. [10]

f) Draw the transfer characteristic I_{DS}-V_{GS} for an n-channel enhancement mode MOSFET for one drain voltage in the saturation region ($V_{DS} > V_{DS}^{sat}$). [5]

g) The electron and hole current flowing across the emitter-base junction of a short npn bipolar junction transistor in forward active mode is given by I_{EBn} and I_{EBp} respectively.
Give the expressions for emitter, base and collector current (I_E, I_B, I_C) in terms of these current I_{EBn} and I_{EBp}. [5]

2.

a) The intrinsic level, E_i is defined as the position of the Fermi level, E_F for the intrinsic semiconductor. Rewrite the expression of the electron concentration $n = N_c e^{-\frac{(E_G - E_F)}{kT}}$ as a function of (E_i - E_F) and the intrinsic carrier concentration n_i. [5]

b) Calculate the position of the intrinsic level E_i with respect to the Fermi level E_F for p-type Si with $N_A = 10^{16}$ cm^{-3} at room temperature and using the formula derived in question (2.a). [5]

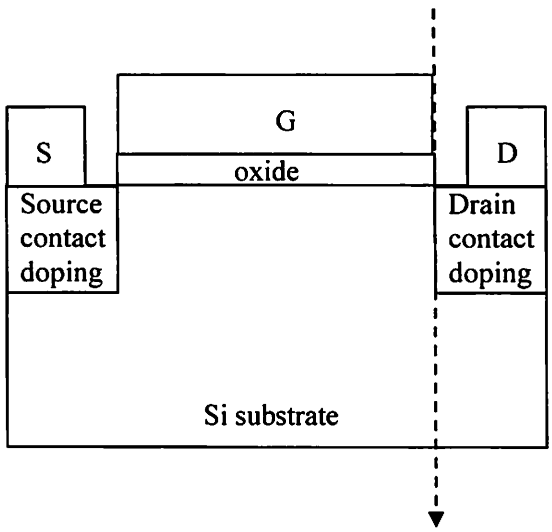


Figure 2.1: Material cross section of a MOSFET. The dashed line with arrow indicates where the energy band diagrams for this MOSFET must be drawn – through the gate, oxide, channel region into the substrate, left of the drain contact doping.

c) Sketch the energy band diagram (E_c, E_v, E_i, E_F, E_G) of an enhancement mode n-channel Si MOSFET for $V_{GS} = V_{DS} = 0$ V along the dashed line indicated in Fig. 2.1. The workfunction of the gate contact is equal to the workfunction of the substrate. Include the intrinsic level E_i in your drawing. [5]

d) Sketch the energy band diagram (E_c, E_v, E_i, E_F, E_G) of the MOSFET of question (2.c) when the MOSFET is in inversion and at the start of saturation. Your sketch must be along the dashed line indicated in Fig. 2.1. Include the intrinsic level E_i in your drawing. [5]

e)

i) Add the position and relative magnitude of the parameters in the list below to the energy band diagram of question 2.d by drawing arrows: $\updownarrow$ and writing the name of the parameter.
- the gate voltage V_{GS}
- the voltage drop across the oxide V_{ox}
- the voltage drop across the semiconductor V_s [6]

ii) Compare to magnitude of V_s to the difference E_i - E_F in the substrate far away from the junction. What can you conclude? [4]

3.

a) Knowing that for a bipolar junction transistor in forward active mode, the emitter-base junction is forward biased and the base-collector junction reverse, draw the common **emitter** biasing circuit for the n^+pn BJT in Fig. 3.1. [5]

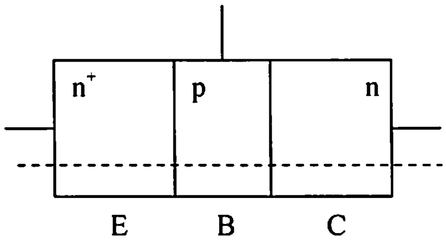


Figure 3.1: A cross-section of a short n^+pn BJT in forward active mode.

b) Draw the energy band diagram (E_c, E_v, E_F, E_G) of the short n^+pn BJT of question (3.a) along the dashed line in Fig. 3.1. [3] marks for each region + [1] mark for accurateness. [10]

c) Do the minority carriers travel through the base due to an electric field across the base region? Explain your answer briefly. [5]

d)

i) The n^+pn Si BJT of Fig. 3.1 has a uniform area of 10^{-4} cm^2, an emitter length of 200 nm, a base width of 0.5 μm and a collector length of 0.5 mm. The emitter doping is 10^{18} cm^{-3} and the base and collector doping are 10^{16} cm^{-3}. Calculate I_C for $V_{EB} = 0.26$ V and $V_{BC} = -1$ V at $T = 300$K. The mobility of minority carrier electrons is 200 cm^2/Vs and for minority carrier holes is 150 cm^2/Vs. [6]

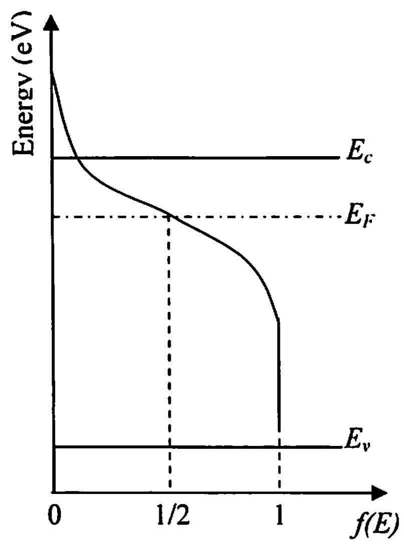
ii) Give two possible changes that can be applied to the emitter to increase the current gain $\beta = I_C/I_B$ of this BJT? [4]

The answers

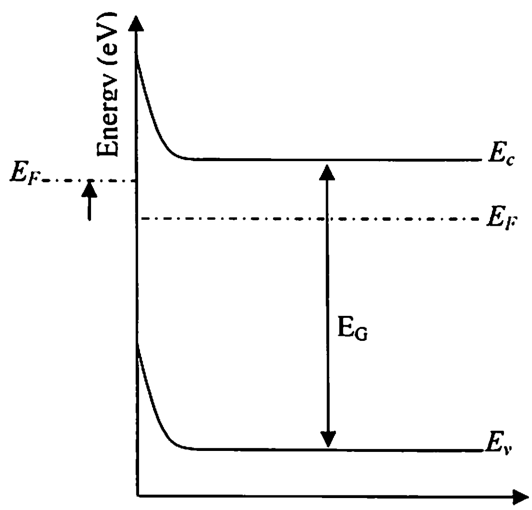
1.

a) The Fermi Level, E_F. is the energy at which the **probably** to find an **electron** is ½. [4]

b) [6]

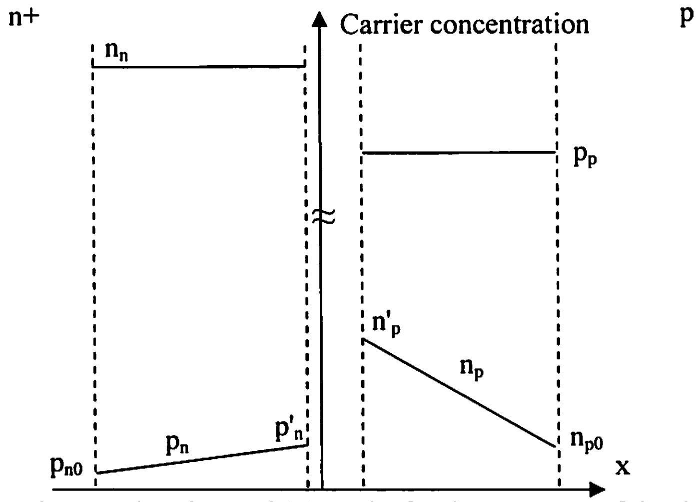


c) [6]



d) Change the workfunction of the metal or change the doping type. [4]

e) Depletion width in p region larger than in n. [10]



[4] marks for each neutral region and [2] marks for the accuracy of the depletion region.

Key information: linear minority carrier variation. Concentrations at contacts must be the equilibrium concentrations. Majority carrier concentrations must be must higher than minority carrier concentrations and n'p must be higher than p'n. For the depletion width: it should extend most in the lowest doped region and the minority carrier sketches should start from the edges of the depletion width.

f) [5]

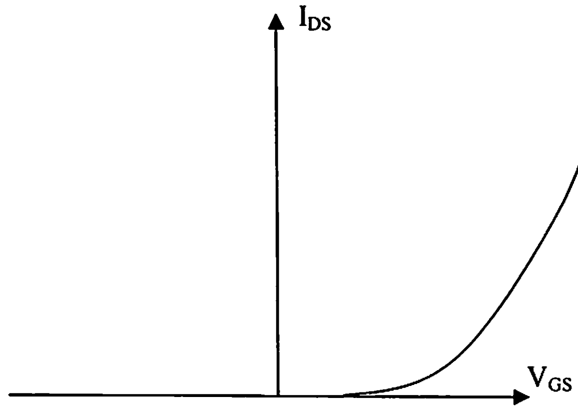
$$I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2} \right)$$

In saturation this becomes:

$$V_{DS} = (V_{GS} - V_{th})$$

$$I_{DS} = \frac{\mu C_{ox} W (V_{GS} - V_{th})^2}{2L}$$

Thus graph must be quadratic. nMOS enhancement has positive threshold voltage.



g) npn BJT: output current is due to electrons. $I_E = I_{EBn} + I_{EBp}$. $I_B = I_{EBp}$. $I_C = I_{EBn}$. [5]

2.

a) From the formulae list:

$$n = N_c e^{-\frac{(E_c - E_F)}{kT}}$$

Definition of E_i gives rewrites (1) for an intrinsic material $n = n_i$ and $E_F = E_i$ to:

$$n_i = N_c e^{-\frac{(E_c - E_i)}{kT}} \ .$$

Combining (1) and (2) gives:

$$n = N_c e^{-\frac{(E_c - E_F)}{kT}} \ \& \ n_i = N_c e^{-\frac{(E_c - E_i)}{kT}}$$

$$\frac{n}{n_i} = \frac{N_c e^{-\frac{(E_c - E_F)}{kT}}}{N_c e^{-\frac{(E_c - E_i)}{kT}}} = e^{-\frac{(E_c - E_F) + (E_c - E_i)}{kT}} = e^{-\frac{(E_i - E_F)}{kT}}$$ [5]

$$n = n_i e^{-\frac{(E_i - E_F)}{kT}}$$

b) $p_p = N_A = 10^{16}$ cm^{-3}

Thus the electron concentration is: $n_p = n_i^2/N_A = (1.45 \times 10^{10}\ \text{cm}^{-3})^2/10^{16}\ \text{cm}^{-3}$ $= 21\ 10^3\ \text{cm}^{-3}$. [5]

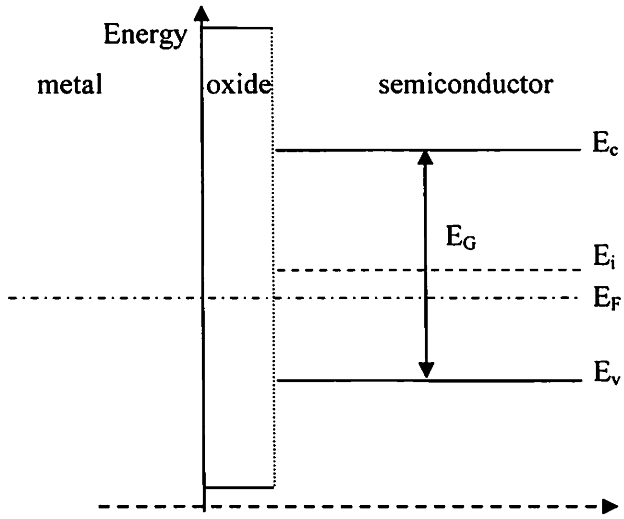
$$n_p = n_i e^{-\frac{(E_i - E_F)}{kT}}$$

$$\frac{n_p}{n_i} = e^{-\frac{(E_i - E_F)}{kT}}$$

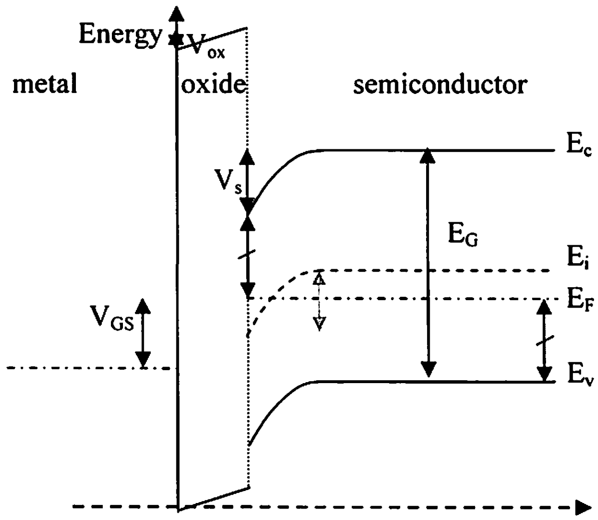
$$\ln\left(\frac{n_p}{n_i}\right) = -\frac{(E_i - E_F)}{kT}$$

$$E_F - E_i = kT \ln\left(\frac{n_p}{n_i}\right) = kT \ln\left(\frac{n_i}{N_A}\right) = 0.026eV \ln\left(\frac{1.45 \times 10^{10}}{10^{16}}\right) = -0.35eV$$

c) . [5]



d) Note that at the given conditions the channel is pinched off at the drain side and thus we find the onset of inversion condition there, see red arrows. [5]



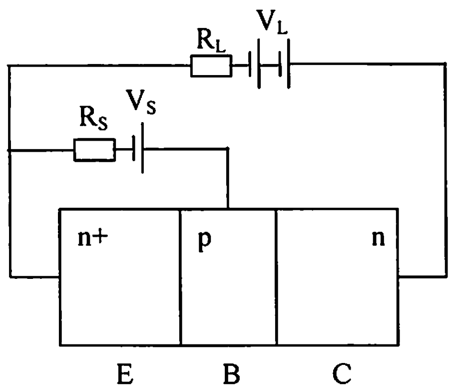
e)

i) See blue (on-line) arrows in previous : ↕ [6]

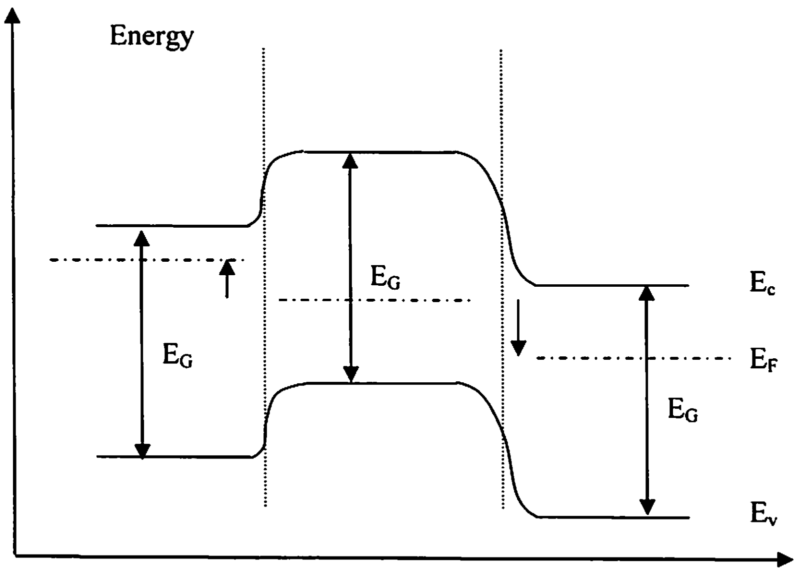
ii) $V_s = 2 \times (E_i - E_F)$. We conclude that for the onset of inversion the voltage drop across the semiconductor is given by $V_s = 2 \times (E_i - E_F)$. [4]

3.

a) [5]



b) [10]



Key information:

The position of the Fermi level in each region must be consistent with the doping type. Fermi level in emitter must be closer to conduction band that that in collector. The lower doping in the base must be reflected by the position of the Fermi level with respect to the valence band in comparison to the position of the Fermi level in the emitter with respect to the conduction band. Fermi level should not be constant as voltages are applied. EF in emitter higher than EF in base (EB forward bias thus the potential barrier smaller than that of BC). EF in base higher than EF in collector because reverse bias (potential barrier larger than that of EB). Ec and Ev should be horizontal outside the depletion regions (must not be drawn as not requested). If drawn the relative magnitude difference between EB and BC must be apparent.

[3] marks for each region + [1] mark for accurateness.

c) No, there is no potential drop across the base, the voltage is dropped mainly across the depletion regions surrounding the junction. The minority carriers in the base move due to diffusion because the concentration of minority carriers at the emitter junction is larger than at the collector junction. [5]

d)

i) I_C is due to the electrons injected into the base by the emitter that then diffusion through the base. Thus it is the electron current of the emitter-base diode. Looking at the expression of the electron diode diffusion current density in the formulae list we find:

$$J_n = \frac{eD_n n_{p_0}}{L_n}\left(e^{\frac{eV}{kT}} - 1\right)$$

However this electron diffusion current is determined by the base width and not the diffusion length. The voltage is the emitter-base voltage. The diffusion constant is given via the Einstein equation in the formulae list. Thus the expression of the electron diffusion current through the base is given by:

$$I_C = I_n = \frac{eD_n n_{p_0}}{W_B}\left(e^{\frac{eV_{EB}}{kT}} - 1\right)A = \frac{ekT\mu_n n_{p_0}}{eW_B}\left(e^{\frac{eV_{EB}}{kT}} - 1\right)A = \frac{ekT\mu_n n_i^2}{eW_B N_{A_B}}\left(e^{\frac{eV_{EB}}{kT}} - 1\right)A$$

$$I_C = \frac{0.026V \times 1.6\,10^{-19}C \times 200cm^2/Vs \times \left(1.45\,10^{10}\,cm^{-3}\right)^2}{500 \times 10^{-4}\,cm \times 10^{16}\,cm^{-3}}\left(e^{\frac{0.26}{0.026}} - 1\right) \times 10^{-4}\,cm^2$$

$$I_C = 7.71 \times 10^{-13}\,A$$

[6]

ii) increase the doping density and increase the emitter width. [4]