

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2012

MSc and EEE PART III/IV: MEng, BEng and ACGI

**MICROWAVE TECHNOLOGY**

Friday, 18 May 2:30 pm

Time allowed: 3:00 hours

**There are SIX questions on this paper.****Answer FOUR questions.***All questions carry equal marks***Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      S. Lucyszyn  
   Second Marker(s) :      E. Shamonina

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**Special instructions for invigilators**

*This is a closed book examination.*

**Special instructions for students**

*Permeability of free space,  $\mu_o = 4\pi \times 10^{-7} [H / m]$*

*Permittivity of free space,  $\epsilon_o \approx 8.854 [pF / m]$*

## The Questions

1. Consider an electromagnetic wave propagating inside a normal metal.
- a) Show mathematically the simplest forms of the classical skin-effect model and relaxation-effect model for intrinsic conductivity. Define all variables used. [2]
  - b) From the result in 1(a), what physical conditions would require the use of the relaxation-effect model? [2]
  - c) Define the 'material quality factor',  $Q_m$ , for a metal in terms of the real and imaginary parts of the following parameters:  
Loss tangent  
Effective relative permittivity  
Refractive index  
Propagation constant  
Surface impedance [3]
  - d) Define the 'component quality factor',  $Q_c$ , for a metal in terms of the real and imaginary parts of the following parameters:  
Component impedance  
Refractive index  
Propagation constant  
Surface impedance  
Complex skin effect [3]
  - e) Sketch the trends of both material and component quality factors for a metal at room temperature, as frequency increases through the THz gap, for both the classical skin-effect and relaxation-effect models. [2]
  - f) Determine a simple expression for the power attenuation (i.e. absorption) per unit normal skin depth within a normal metal and calculate this value for 15.3 normal skin depths. [2]
  - g) Derive a simple expression for the power attenuation (i.e. absorption) per unit wavelength within a normal metal in terms of the component quality factor. [3]
  - h) Using the derived expression in 1(g), if the component quality factor is 2.41, calculate the increase in power attenuation per unit wavelength for the classical relaxation-effect model, when compared to the value calculated using the classical skin-effect model. [3]

2. Consider an electromagnetic wave propagating inside a normal metal. Gold at room temperature has an intrinsic conductivity  $\sigma_0 = 4.517 \times 10^7 \text{ [S/m]}$ , phenomenological relaxation time  $\tau = 27.135 \text{ [fs]}$  and a component quality factor  $Q_{cr} = 2.41$  at the relaxation frequency.
- a) Given that the complex skin depth is the reciprocal of the propagation constant for an electromagnetic wave propagating into a normal metal, for the relaxation-effect model, derive an expression for the wavelength inside the metal in terms of the normal skin depth and 'component quality factor'. [6]
- b) Ignoring displacement current, for the relaxation-effect model unless otherwise stated, calculate the following at the relaxation frequency:
- i) Relaxation frequency. [2]
- ii) Normal skin depth. [2]
- iii) Wavelength inside the metal. [2]
- iv) Complex skin depth using the classical skin-effect model. [2]
- vi) Unloaded Q-factor for the  $TE_{101}$  mode of a rectangular cavity resonator having internal dimensions of  $a = d = 2b = 300 \text{ [\mu m]}$  using the classical skin-effect model. [2]
- v) Complex skin depth. [2]
- vi) Unloaded Q-factor for the  $TE_{101}$  mode of a rectangular cavity resonator having internal dimensions of  $a = d = 2b = 300 \text{ [\mu m]}$ . [2]

3.

- a) State the vector Helmholtz equation for an E-field in a homogeneous medium and then expand this for an orthogonal rectangular coordinate system. Determine the plane wave solution and give the E-field component in the y direction, as it propagates along the z direction. [6]
- b) Define what is meant by the term transverse electromagnetic wave, and draw a diagram showing the E-field, H-field and Poynting vector for such a wave. Describe the orientation of a stack of metal plates spaced an arbitrary distance apart that may be introduced into this TEM wave without violating any boundary conditions. [4]
- c) The largest parabolic antenna in the world is at Arecibo Observatory, Puerto Rico. With a diameter of 305 m and 41.6% efficiency, calculate its maximum power gain at 2.38 GHz. [Hint: *Directivity*  $D = 4\pi (A/\lambda^2)$  and for a parabolic antenna, capture area,  $A = \pi (\text{diameter}/2)^2$ ]. [3]
- d) From first principles, derive the basic Friis' Link equation. With the aid of a diagram, show the meaning of all variables used. [4]
- e) The closest the moon comes to the earth is 357,460 km. If the Arecibo antenna described in Q3(c) was perfectly aligned to an identical one on the moon, using the Friis' link equation from Q3(d), calculate the minimum transmit power needed to obtain a received power level of -120 dBm at 2.38 GHz. [3]



4.

- a) Draw cross-sectional sketches for both the conventional microstrip transmission line and its equivalent having a uniform E-field distribution across its signal track width, highlighting any relevant parameters. From the latter, write down the simple expression for the characteristic impedance of a microstrip transmission line, using the quasi-TEM 'magnetic wall' model, in terms of the effective relative permittivity and effective width.

[5]

- b) Using the simple expression from 4(a), explain the concept of TFMS transmission lines and briefly comment on its application.

[5]

- c) An expression for effective relative permittivity is given in equation (4.1), with all the variables having their usual meaning. Sketch this function as the physical width of the line,  $w$ , increases from zero to a large value. Deduce the limiting values of effective relative permittivity from an infinitesimal to infinite width line. Also, describe how the E-field lines behave with both width extremes and how this relates to the dielectric constant of the substrate.

$$\epsilon_{r\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ \left( 1 + 12 \left( \frac{h}{w} \right) \right)^{-0.5} + 0.4 \left( 1 - \left( \frac{h}{w} \right) \right)^2 \right] \quad \text{for } \left( \frac{w}{h} \right) < 1$$

(4.1)

[5]

- d) Using the expression in (4.1) and that for the characteristic impedance in equation (4.2), calculate the associated values for a microstrip transmission line having a 635  $\mu\text{m}$  thick alumina substrate with a dielectric constant of 9.8 and a track width of 620  $\mu\text{m}$ . If the simple expression in 4(a) was used to calculate the characteristic impedance of the line, by assuming that the real width of the line was the same as the effective width, what kind of error would you get? To get the correct value of characteristic impedance, what is the value of  $(w_{\text{eff}}/w)$ .

$$Z_o = \frac{60}{\sqrt{\epsilon_{r\text{eff}}}} \ln \left\{ 8 \left( \frac{h}{w} \right) + 0.25 \left( \frac{w}{h} \right) \right\} [\Omega] \quad \text{for } \left( \frac{w}{h} \right) < 1$$

(4.2)

[5]

5.

- a) Draw the equivalent circuit of the MMIC shown in Figure 5.1 and identify the overall circuit and its topology. There are 12 probe pads that have been identified and these must be shown on the circuit. From the circuit, identify a major design flaw in the design of the MMIC.

[8]

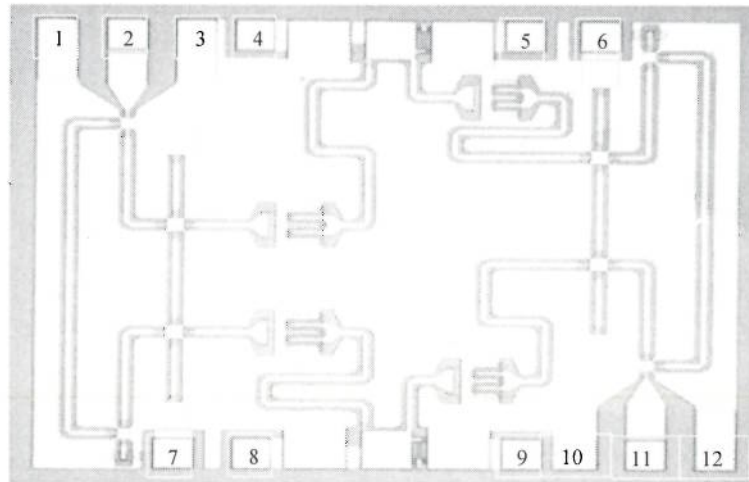


Figure 5.1 Photograph of an MMIC

- b) State five main advantages of CPW transmission lines, when compared to microstrip lines.
- c) Briefly explain why CBCPW transmission lines are created, instead of pure-CPW lines. In detail, describe how multi-modeing occurs in CBCPW lines and explain how some of the unwanted parasitic modes can be suppressed and what the resulting guided-wave structures are referred to as.

[5]

[7]

6.

- a) Describe the key characteristics, including advantages and disadvantages, of ohmic contact RF MEMS switches. [4]
- b) If an ohmic contact switch has an effective closed-state series resistance of  $R_{on} = 1.5 \Omega$  and an effective open-state isolation capacitance of  $C_{off} = 5 \text{ fF}$ , calculate the resulting performance figure-of-merit. [4]
- c) Describe the key characteristics, including advantages and disadvantages, of capacitive membrane (or switch capacitance) RF MEMS switches. [4]
- d) Write the well-known equation for the capacitance of a parallel-plate capacitor and calculate an approximate value for the down-state capacitance for a capacitive membrane switch, given the following variables:
- Plate Length,  $L = 500 \mu\text{m}$   
Plate Width,  $W = 300 \mu\text{m}$   
Plate separation distance,  $d = 2 \mu\text{m}$   
Separation dielectric constant,  $\epsilon_r = 3.4$
- [2]
- e) What would the separation distance of the membrane switch specified in 6(d) need to be so that it has the open-state capacitance given in 6(b)? Assume that the  $2 \mu\text{m}$  thick separation dielectric can be ignored in the open state. [4]
- f) In practice, can the separation distance calculated in 6(e) be realised using electrostatic actuation? Which of the variables in 6(d) does the designer have any control over? [2]



The Solutions for E3.18, 2012

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Model answer to Q 1(a): Bookwork

Show mathematically the simplest forms of the classical skin-effect model and relaxation-effect model for intrinsic conductivity. Define all variables used.

$$\text{Classical Relaxation - Effect Model} \quad \sigma_R = \frac{\sigma_o}{(1 + j\omega\tau)}$$

$$\text{Classical Skin - Effect Model} \quad \sigma_o$$

$\sigma_R$  = intrinsic relaxation-effect conductivity;  $\sigma_o$  = intrinsic skin-effect conductivity;  $\omega$  = angular frequency of the driving electromagnetic field propagating inside the metal;  $\tau$  = phenomenological temperature-dependent scattering relaxation time for the free electrons (i.e., mean time between collisions);  $j$  = complex operator.

[2]

Model answer to Q 1(b): Bookwork

From the result in 1(a), what physical conditions would require the use of the relaxation-effect model?

The relaxation-effect model should be used for THz frequencies at room temperature or microwave frequencies at cryogenic temperatures (e.g. liquid helium temperatures for radio astronomy can be at around 4K), since  $\omega\tau > 1$  for both and so intrinsic conductivity will be a complex number.

[2]

Model answer to Q 1(c): Bookwork

Define the 'material quality factor',  $Q_m$ , for a metal in terms of the real and imaginary parts of the following parameters:

- Loss tangent
- Effective relative permittivity
- Refractive index
- Propagation constant
- Surface impedance

$$Q_m = \frac{1}{\tan\delta} = \frac{|\Re\{\epsilon_{r \text{ effective}}\}|}{\Im\{\epsilon_{r \text{ effective}}\}} \quad Q_m = \frac{\Re\{n^2\}}{\Im\{n^2\}} = \frac{\Re\{\gamma^2\}}{\Im\{\gamma^2\}} = \frac{\Im\{\sigma_{\text{equivalent}}\}}{\Re\{\sigma_{\text{equivalent}}\}} = \frac{-\Re\{Z_s^2\}}{\Im\{Z_s^2\}}$$

[3]

Model answer to Q 1(d): Bookwork

Define the 'component quality factor',  $Q_c$ , for a metal in terms of the real and imaginary parts of the following parameters:

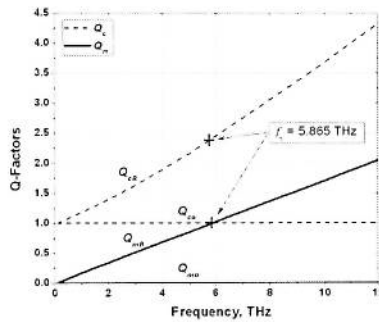
- Component impedance
- Refractive index
- Propagation constant
- Surface impedance
- Complex skin effect

$$Q_c = \frac{|X_c|}{R_c} \rightarrow \frac{\Im\{n\}}{\Re\{n\}} = \frac{\Re\{\gamma\}}{\Im\{\gamma\}} = \frac{\Im\{Z_s\}}{\Re\{Z_s\}} \quad Q_c = \frac{\Re\{\delta_c\}}{\Im\{\delta_c\}}$$

[2]

### Model answer to Q 1(e): Bookwork

Sketch the trends of both material and component quality factors for a metal at room temperature, as frequency increases through the THz gap, for both the classical skin-effect and relaxation-effect models.



$$Q_m \Rightarrow \begin{cases} \equiv 0 & \text{for } Q_{mo} \\ > 0 & \text{for } Q_{mR} \end{cases}$$

$$Q_c \Rightarrow \begin{cases} \equiv 1 & \text{for } Q_{co} \\ > 1 & \text{for } Q_{cR} \end{cases}$$

[2]

### Model answer to Q 1(f): Bookwork and calculated example

Determine a simple expression for the power attenuation (i.e. absorption) per unit normal skin depth within a normal metal and calculate this value for 15.3 normal skin depths.

$$A_{dB} = -20 \log_{10} |e^{-\gamma_R T}| = 20 \log_{10} (e^{T/\delta_{SR}}) \rightarrow 20 \log_{10} (e^{a_R}) \cong 8.686 a_R = 133 \text{ dB}$$

[2]

### Model answer to Q 1(g): Bookwork

Derive a simple expression for the power attenuation (i.e. absorption) per unit wavelength within a normal metal in terms of the component quality factor.

$$A_{dB} = 10 \log_{10} (e^{-2\alpha\lambda}) [dB/\lambda]$$

$$\alpha\lambda [dB/\lambda] = 8.686 \cdot 2\pi \left( \frac{\alpha}{\beta} \right) \Rightarrow \begin{cases} \cong 55 [dB/\lambda_o] \text{ Classical Skin Effect} \\ \cong 55 Q_{cR} [dB/\lambda_R] \text{ Classical Relaxation Effect} \end{cases}$$

[3]

### Model answer to Q 1(h): Bookwork and calculated example

Using the derived expression in 1(g), if the component quality factor is 2.41, calculate the increase in power attenuation per unit wavelength for the classical relaxation-effect model, when compared to the value calculated using the classical skin-effect model.

$$A_{dB} \cong 55 Q_{cR} = 133 [dB/\lambda_R] \text{ Classical Relaxation Effect}$$

$$A_{dB} \cong 55 [dB/\lambda_o] \text{ Classical Skin Effect}$$

Therefore, the increase is 78 dB/wavelength

[3]

Model answer to Q 2(a): Bookwork derivation

Given that the complex skin depth is the reciprocal of the propagation constant for an electromagnetic wave propagating into a normal metal, for the relaxation-effect model, derive an expression for the wavelength inside the metal in terms of the normal skin depth and 'component quality factor'.

$$\delta_c = \frac{1}{\gamma} = \frac{1}{\alpha + j\beta} = \frac{\alpha}{\alpha^2 + \beta^2} - j \frac{\beta}{\alpha^2 + \beta^2}$$

$$\text{But, } Q_c = \frac{\alpha}{\beta}$$

$$\therefore \delta_c'' = \frac{\beta}{Q_c^2 \beta^2 + \beta^2} = \frac{1}{\beta(1 + Q_c^2)}$$

$$\text{Now, } \lambda = \frac{2\pi}{\beta} \Rightarrow 2\pi\delta_c''(1 + Q_c^2)$$

$$\text{But, } \delta_{cR} = \Re\{\delta_{co}\} \left( \sqrt{Q_{cR}} - j \frac{1}{\sqrt{Q_{cR}}} \right) \text{ and } \delta_{co} = \frac{\delta_{so}}{2}(1 - j)$$

$$\therefore \lambda_R = 2\pi \frac{\delta_{so}}{2} \left( \frac{1 + Q_{cR}^2}{\sqrt{Q_{cR}}} \right)$$

$$\text{But, } \lambda_0 = \frac{2\pi}{\beta_o} = \frac{2\pi}{\alpha_o} = 2\pi\delta_{so}$$

$$\therefore \lambda_R = \frac{\lambda_0}{2} \left( \frac{1 + Q_{cR}^2}{\sqrt{Q_{cR}}} \right) \equiv 2\pi\delta_{sR}Q_{cR}$$

[6]

Model answer to Q 2(b): Bookwork and calculated example

Gold at room temperature has an intrinsic conductivity  $\sigma_o = 4.517 \times 10^7 \text{ [S/m]}$ , phenomenological relaxation time  $\tau = 27.135 \text{ [fs]}$  and a component quality factor  $Q_{cR} = 2.41$  at the relaxation frequency. Ignoring displacement current, for the relaxation-effect model unless otherwise stated, calculate the following at the relaxation frequency:

- i) Relaxation frequency.

$$f\tau = \frac{1}{2\pi\tau} = 5.865 \text{ THz}$$

[2]

- ii) Normal skin depth.

$$\delta_{so} = \sqrt{\frac{1}{\pi f \tau \mu_o \sigma_o}} = 30.9 \text{ nm}$$

[2]

- iii) Wavelength inside the metal.

$$\lambda_R = \pi\delta_{so} \left( \frac{1 + Q_{cR}^2}{\sqrt{Q_{cR}}} \right) = 425.7 \text{ nm}$$

[2]



iv) Complex skin depth using the classical skin-effect model.

$$\delta_{co} = \frac{\delta_{so}}{2} (1 - j) = 15.45(1 - j) \text{ nm} \quad [2]$$

v) Unloaded Q-factor for the  $TE_{101}$  mode of a rectangular cavity resonator having internal dimensions of  $a = d = 2b = 300 [\mu\text{m}]$  using the classical skin-effect model.

$$Qu|_{TE_{101}} = \frac{\text{Volume}}{\text{Area} \times \text{Im}\{\delta_{co}\}} = \frac{a}{8 \times \text{Im}\{\delta_{co}\}} = 2,427 \quad [2]$$

vi) Complex skin depth.

$$\delta_{cR} = \Re\{\delta_{co}\} \left( \sqrt{Q_{cR}} - j \frac{1}{\sqrt{Q_{cR}}} \right) = 23.98 - j9.95 \text{ nm} \quad [2]$$

vii) Unloaded Q-factor for the  $TE_{101}$  mode of a rectangular cavity resonator having internal dimensions of  $a = d = 2b = 300 [\mu\text{m}]$ .

$$Qu|_{TE_{101}} = \frac{\text{Volume}}{\text{Area} \times \text{Im}\{\delta_{cR}\}} = \frac{a}{8 \times \text{Im}\{\delta_{cR}\}} = 3,769 \quad [2]$$

#### Model answer to Q 3(a): Bookwork

State the vector Helmholtz equation for an E-field in a homogeneous media and then expand this for an orthogonal rectangular coordinate system. Determine the plane wave solution and give the E-field component in the y direction, as it propagates along the z direction.

The E-field wave (propagation) equation can be obtained from:

$$\nabla^2 \hat{E} + k_m^2 \hat{E} = 0 \quad \text{vector Helmholtz equation for an E-field in a homogeneous media}$$

where, modified wavenumber,  $k_m = \omega \sqrt{\mu\epsilon}$

Therefore, in a rectangular coordinate system:

$$\nabla^2 \hat{E} = \frac{\partial^2 \hat{E}}{\partial x^2} + \frac{\partial^2 \hat{E}}{\partial y^2} + \frac{\partial^2 \hat{E}}{\partial z^2} = -\omega^2 \mu\epsilon \hat{E}$$

In an orthogonal rectangular coordinate system:

$$\nabla^2 E_x = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu\epsilon E_x$$

$$\nabla^2 E_y = \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = -\omega^2 \mu\epsilon E_y$$

$$\nabla^2 E_z = \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu\epsilon E_z$$

With a plane wave, there is a variation in the field quantities only in one dimension, e.g. the z direction of propagation:

$$\therefore \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial y^2} = 0 \quad \text{and} \quad \nabla^2 \hat{E} = \frac{\partial^2 \hat{E}}{\partial z^2} = -\omega^2 \mu \epsilon \hat{E}$$

and

$$\nabla^2 E_x = \frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x$$

$$\nabla^2 E_y = \frac{\partial^2 E_y}{\partial z^2} = \gamma^2 E_y$$

$$\nabla^2 E_z = \frac{\partial^2 E_z}{\partial z^2} = \gamma^2 E_z$$

Now, considering only the E-field component in the  $y$  direction, as it propagates along the  $z$  direction:

$$E_y = Ae^{-\gamma z} + Be^{+\gamma z} \quad \text{where} \quad \gamma = jk_m \quad \text{and} \quad k_m = \omega \sqrt{\mu \epsilon}$$

$A$  represents the amplitude of the forward travelling wave, while  $B$  represents the amplitude of the backward travelling wave.

This solution is identical to the voltage waves on a transmission line. In general, further analysis is usually confined to just the forward wave. When time dependency is also considered, the forward wave of the field is represented as:

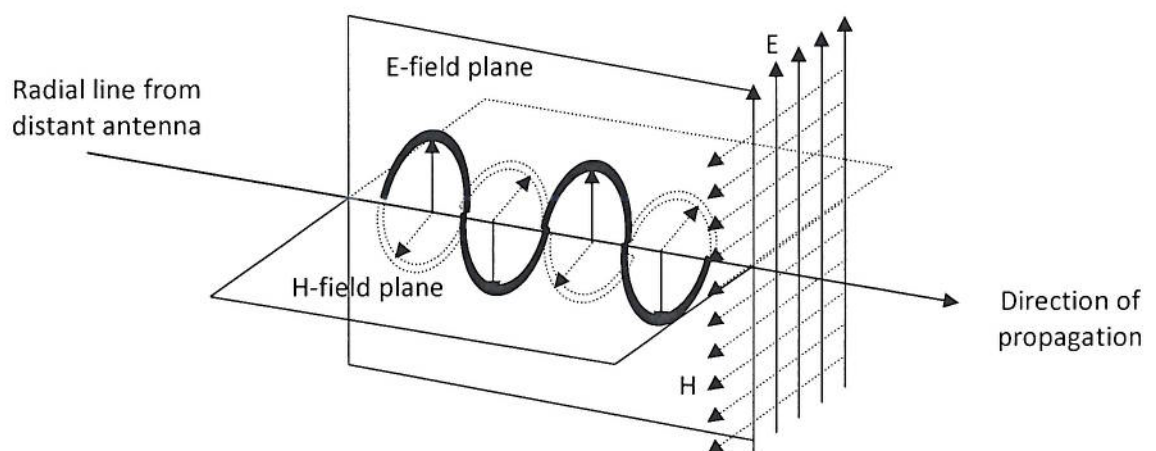
$$E_y = E_o e^{(j\omega t - \gamma z)}$$

[6]

#### Model answer to Q 3(b): Bookwork

Define what is meant by the term transverse electromagnetic wave, and draw a diagram showing the E-field, H-field and Poynting vector for such a wave. Describe the orientation of a stack of metal plates spaced an arbitrary distance apart that may be introduced into this TEM wave without violating any boundary conditions.

In a transverse electromagnetic (TEM) wave, the E-field vector and H-field vector are both at right angles to each other and to the direction of propagation.



The wavefront is the plane that is mutually orthogonal to the E- & H-planes



A stack of metal sheets may be placed so that the E-field is at right angles to the surface of the plates, and the H-field parallel to the surface of the plates, without violating the boundary conditions.

[4]

#### Model answer to Q 3(c): Bookwork and calculated example

The largest parabolic antenna in the world is at Arecibo Observatory, Puerto Rico. With a diameter of 305 m and 41.6% efficiency, calculate its maximum power gain at 2.38 GHz.

[Hint: Directivity  $D = 4\pi \text{ Effective Capture Area, } A/\lambda_o^2$  and for a parabolic antenna,

$$A = \pi \left( \frac{\text{diameter}}{2} \right)^2]$$

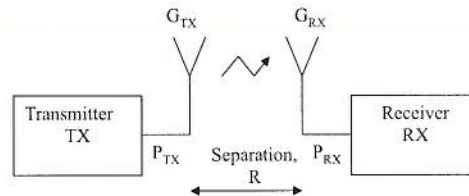
$$G = 10 \log \left[ \text{efficiency} \left( \frac{\pi \text{ diameter}}{\lambda_o} \right)^2 \right] = 73.8 \text{ dBi at } 2.38 \text{ GHz}$$

[3]

#### Model answer to Q 3(d): Bookwork

From first principles, derive the basic Friis' Link equation. With the aid of a diagram, show the meaning of all variables used.

#### Friis' Link Equation



It is assumed that the separation distance,  $R$ , is greater than the combined far-field distance,  $R_{ff}$ , for both the transmit and receive antennas (for the gain of both antennas to be known):

$$R_{ff} > \begin{cases} \frac{2(L_{TX}^2 + L_{RX}^2)}{\lambda_o} & \text{for electrically long antennas} \\ 2 \times 2\lambda_o & \text{for electrically short antennas} \end{cases}$$

where,  $L$  = Largest Dimension of the Antenna's Aperture

and,  $\lambda_o$  = Free-Space Wavelength

$$P_{RX} = EIRP \times \text{Spreading Loss} \times A_{RX}$$

where, Effective Isotropically Radiated Power,  $EIRP = P_{TX} \cdot G_{TX}$

$$\text{and Spreading Loss} = \frac{1}{4\pi R^2}$$

and Effective Aperture of the Receiving Antenna,  $A_{RX} \sim G_{RX} \cdot \frac{\lambda_o^2}{4\pi}$

$$\therefore P_{RX} = P_{TX} \cdot \frac{G_{TX} \cdot G_{RX} \cdot \lambda_o^2}{(4\pi R)^2} \quad \text{Friis' Range Equation}$$

It can be seen from the spreading loss that the received power decreases by:  
6 dB when range is doubled and  
20 dB for every order of magnitude increase in range.

[4]

#### Model answer to Q 3(e): Bookwork and calculated example

The closest the moon comes to the earth is 357,460 km. If the Arecibo antenna described in Q3(c) was perfectly aligned to an identical one on the moon, using the Friis' link equation from Q3(d), calculate the minimum transmit power needed to obtain a received power level of -120 dBm at 2.38 GHz.

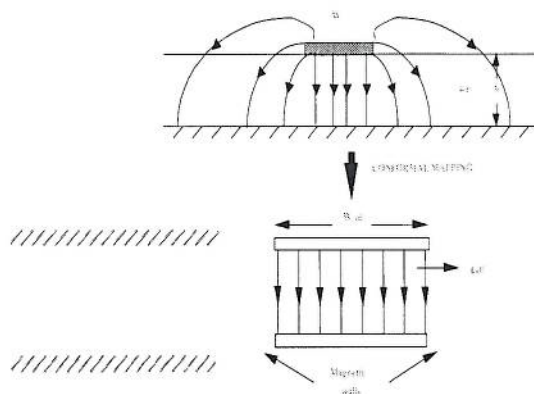
$$P_{TX} = P_{RX} \left( \frac{4\pi \text{Range}}{G\lambda_o} \right)^2 = 2.2 \text{ nW} = -56 \text{ dBm}$$

with  $P_{RX} = 1 \text{ fW}$  and antenna gain  $G = 23,988,000$

[3]

#### Model answer to Q 4(a): Bookwork

An expression for effective relative permittivity is given in equation (4.1), with all the variables having their usual meaning. Sketch this function as the physical width of the line,  $w$ , increases from zero to a large value. Deduce the limiting values of effective relative permittivity from an infinitesimal to infinite width line. Also, describe how the E-field lines behave with both width extremes and how this relates to the dielectric constant of the substrate.

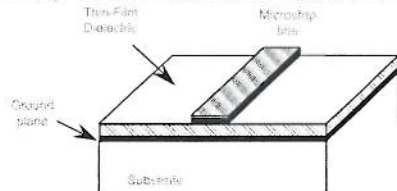


$$Z_o(f) = \frac{\eta_o}{\sqrt{\epsilon_{r\text{eff}}(f)}} \cdot \frac{h}{W_{\text{eff}}(f)}$$

[5]

#### Model answer to Q 4(b): Bookwork

Using the simple expression from 4(a), explain the concept of TFMS transmission lines and briefly comment on its application.



Instead of the substrate acting as the dielectric medium between the microstrip line and the ground plane, a new ground plane metallic layer and dielectric layer are deposited on top of the substrate and compact thin-film microstrip (TFMS) can maintain its characteristic impedance if the new  $(h/w_{\text{eff}})$  is maintained, assuming the effective dielectric constant of the dielectric layer is the same as that for the substrate.

[5]

#### Model answer to Q 4(c): Bookwork

An expression for effective relative permittivity is given in equation (4.1), with all the variables having their usual meaning. Sketch this function as the physical width of the line,  $w$ , increases from zero to a large value. Deduce the limiting values of effective relative permittivity from an infinitesimal to infinite width line. Also, describe how the E-field lines behave with both width extremes and how this relates to the dielectric constant of the substrate.

$$\epsilon_{r\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ \left( 1 + 12 \left( \frac{h}{w} \right) \right)^{-0.5} + 0.4 \left( 1 - \left( \frac{h}{w} \right) \right)^2 \right] \quad \text{for } \left( \frac{w}{h} \right) < 1 \quad (4.1)$$

\* Narrow lines: fields are equally in substrate and air and so the effective relative permittivity is approximately one have the value of a high dielectric constant substrate.

\* Wide lines: fields are confined inside the substrate, similar to a parallel-plate capacitor.

[5]

#### Model answer to Q 4(d): Calculated example

Using the expression in (4.1) and that for the characteristic impedance in equation (4.2), calculate the associated values for a microstrip transmission line having a 635  $\mu\text{m}$  thick alumina substrate with a dielectric constant of 9.8 and a track width of 620  $\mu\text{m}$ . If the simple expression in 4(a) was used to calculate the characteristic impedance of the line, by assuming that the real width of the line was the same as the effective width, what kind of error would you get? To get the correct value of characteristic impedance, what is the value of  $(w_{\text{eff}}/w)$ .

$$Z_o = \frac{60}{\sqrt{\epsilon_{r\text{eff}}}} \ln \left\{ 8 \left( \frac{h}{w} \right) + 0.25 \left( \frac{w}{h} \right) \right\} [\Omega] \quad \text{for } \left( \frac{w}{h} \right) < 1 \quad (4.2)$$

$$\epsilon_{r\text{eff}} = 6.61 \text{ and } Z_o = 50 [\Omega]$$

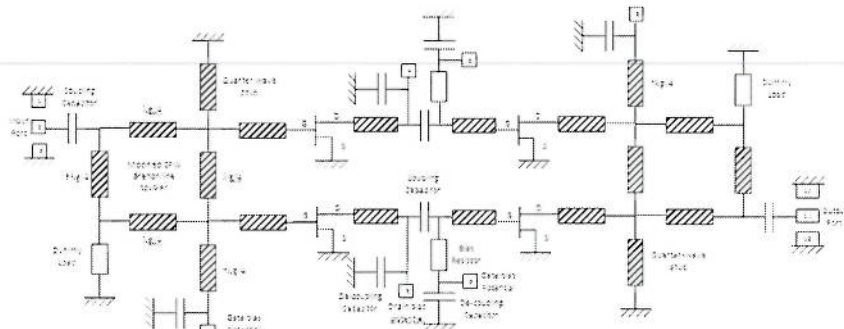
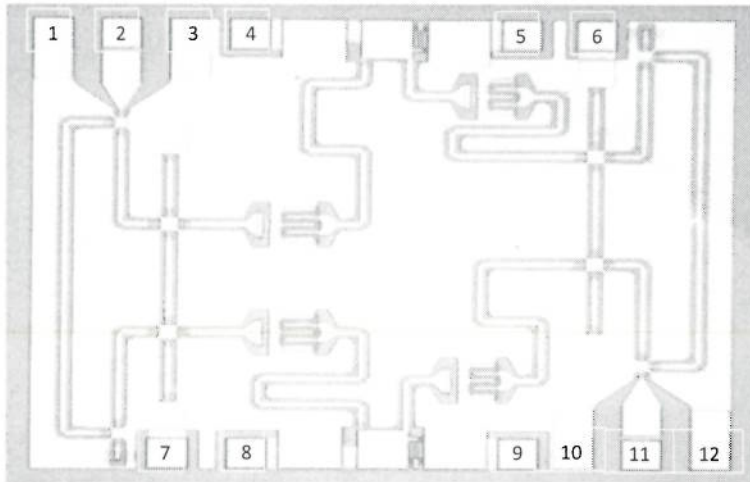


Using, the expression in 4(a) the calculated characteristic impedance  $Z_o = 150 [\Omega]$ , which is completely wrong!!! To get the correct value,  $w_{eff} = 3w = 1,860 \mu m$  and so a 3:1 ratio is needed.

[5]

Model answer to Q 5(a): Bookwork and discussion in class example

Draw the equivalent circuit of the MMIC shown in Figure 5.1 and identify the overall circuit and its topology. There are 12 probe pads that have been identified and these must be shown on the circuit. From the circuit, identify a major design flaw in the design of the MMIC.



This is a 2-stage CPW balanced amplifier. The major design flaw is that there is a decoupling capacitor connected to the output signal path of first transistor. As a result, the microwave signal will be shunted down to ground and so there will be almost no signal at the output of the chip!

[8]

Model answer to Q 5(b): Bookwork

State five main advantages of CPW transmission lines, when compared to microstrip lines.

[5]

1. Devices and components can be grounded without via-holes.
2. It suffers from much less frequency dispersion than microstrip, making it suitable for millimetre-wave circuits.
3. A given characteristic impedance can be realized with almost any track width and gap combination.
4. A considerable increase in packing density is possible because the ground planes provide shielding between adjacent CPW lines.

5. With the back-face ground plane removed, lumped-elements exhibit less parasitic capacitance.

Model answer to Q 5(c): Bookwork

Briefly explain why CBCPW transmission lines are created, instead of pure-CPW lines. In detail, describe how multi-modeing occurs in CBCPW lines and explain how some of the unwanted parasitic modes can be suppressed and what the resulting guided-wave structures are referred to as.

Conductor-backed coplanar waveguide transmission lines are created because of the manufacturing/assembly process being used. Ideally, only pure-CPW is wanted, but the presence of a parasitic metal sheet may be unavoidable.

**Multi-Modeing in Conductor-Backed CPW Lines**

With an ideal CPW line, only the pure-CPW (quasi-TEM) mode is considered to propagate. In the case of a grounded-CPW (GCPW) line, where the backside metallization is at the same potential as the two upper-ground planes (through the use of through-substrate vias), a microstrip like mode can also co-exist with the pure-CPW mode. With the conductor-backed CPW (CBCPW) line, where the backside metallization has a floating potential, parallel-plate line (PPL) modes can also co-exist. The significant PPL modes that are associated with CBCPW lines include the fundamental TEM mode (designated  $TM_0$ ) found at frequencies from DC to infinity and the higher order  $TM_n$  modes that can only be supported above their cut-off frequency,  $f_{c0} \sim \pi c / (2h\sqrt{\epsilon_r})$ . By inserting a relatively thick dielectric layer (having a lower dielectric constant than that of the substrate), between the substrate and the lower ground plane, the pure CPW mode can be preserved. This is because the capacitance between the upper and lower conductors will be significantly reduced and, therefore, there will be less energy associated with the parasitic modes. Alternatively, the parallel-plate line modes can also be suppressed by reducing the width of the upper-ground planes, resulting in finite ground CPW (or FGC). Finally, in addition to all the modes mentioned so far, the slot-line mode can also propagate if there is insufficient use of air-bridges/underpasses to equalise the potentials at both the upper-ground planes.

PURE-CPW + SLOT-LINE + MICROSTRIP + PARALLEL-PLATE (TEM +  $TM_n$ )



[7]

Model answer to Q 6(a): Bookwork

Describe the key characteristics, including advantages and disadvantages, of ohmic contact RF MEMS switches.

Ohmic contact switch has:

- \* high open-state isolation
- \* low closed-state insertion loss
- \* considerable force is required to create a good contact
- \* microscopic bonding of the metal surfaces
- \* highly susceptible to corrosion and stiction

[4]

Model answer to Q 6(b): Computed Example

If an ohmic contact switch has an effective closed-state series resistance of  $R_{on} = 1.5 \Omega$  and an effective open-state isolation capacitance of  $C_{off} = 5 \text{ fF}$ , calculate the resulting performance figure-of-merit.

Performance figure-of-merit,  $f_c = 1/(2\pi R_{on} C_{off}) = 21 \text{ THz}$

[4]

Model answer to Q 6(c): Bookwork

Describe the key characteristics, including advantages and disadvantages, of capacitive membrane (or switch capacitance) RF MEMS switches.



Switched capacitance switch has:

- \* compromise is made between insertion loss and isolation
- \* insertion loss is independent of the contact force
- \* electrode separation need to be maximised
- \* higher lifetime (typically several orders of magnitude)

[4]

Model answer to Q 6(d): Computed Example

Write the well-known equation for the capacitance of a parallel-plate capacitor and calculate an approximate value for the down-state capacitance for a capacitive membrane switch, given the following variables:

Plate Length,  $L = 500 \mu m$   
 Plate Width,  $W = 300 \mu m$   
 Plate separation distance,  $d = 2 \mu m$   
 Separation dielectric constant,  $\epsilon_r = 3.4$

$$\text{Capacitance} \approx C_{\text{fringe}} + \epsilon_o \epsilon_r \times (L \times W) / d \sim \epsilon_o \epsilon_r \times (L \times W) / d = 2.25 pF$$

[2]

Model answer to Q 6(e): Computed Example

What would the separation distance of the membrane switch specified in 6(d) need to be so that it has the open-state capacitance given in 6(b)? Assume that the  $2 \mu m$  thick separation dielectric can be ignored in the open state.

$$\text{Separation distance needs to be, } d \sim \epsilon_o \epsilon_r \times (L \times W) / 5 fF = 902 \mu m$$

[4]

Model answer to Q 6(f): Bookwork

In practice, can the separation distance calculated in 6(e) be realised using electrostatic actuation? Which of the variables in 6(d) does the An expression for effective relative permittivity is given in equation (4.1), with all the variables having their usual meaning. Sketch this function as the physical width of the line,  $w$ , increases from zero to a large value. Deduce the limiting values of effective relative permittivity from an infinitesimal to infinite width line. Also, describe how the E-field lines behave with both width extremes and how this relates to the dielectric constant of the substrate.designer have any control over?

The separation distance calculated in 6(e) is way too big for conventional electrostatic actuation. The chips designer only has control over  $L$  and  $W$ .

[2]