Solutions 2008
E2.3B

Question 1 is compulsory.
Answer question 1 and any two of questions 2-4
Question 1 caries 40% and the other questions carry equal weighting of 30%.

1.

(a) Explain why very high voltages are used for electricity transmission and lower voltages for distribution. [4]

Because the predominant loss mechanism is series resistance, a combination of low current and high voltage is the most efficient way to transport power. However, EHV (e.g. 400 kV) equipment is very expensive. Only for long distance bulk transmission is the capital cost justified by the lifetime efficiency savings. For local distribution the distances are short, the average loadings lower and the number of take-off points higher. Here, the best economic solution is HV (e.g. 33kV) where the capital cost is much lower and the efficiency still reasonable.

(b) Explain why a frequency of 50 Hz (or 60 Hz) was chosen for electricity generation and supply. [4]

- Lower frequencies (~20 Hz) would give flicker in lights
- Lower frequencies would require physically larger transformers and generators for the same apparent power
- Higher frequencies put greater stress on mechanical elements of generators
- Higher frequencies drive higher shunt currents through cable capacitance and cause higher voltage drops in line inductances.
- 50 / 60 Hz is a good compromise for land-based systems.

(c) A switch-mode power supply is more efficient than a linear regulator but still has some power losses. Explain why those power losses arise. [4]

- Conduction losses in diode and Mosfet
- Switching loss traversing linear region between on- and off-states of the Mosfet
- Conduction loss in inductor and to some extent in capacitor.
- Power consumption by control electronics

(d) Figure Q1 shows the output voltage of a buck switch-mode power supply as a function of output current when operated in open-loop with a set duty-cycle. Explain its shape. [4]

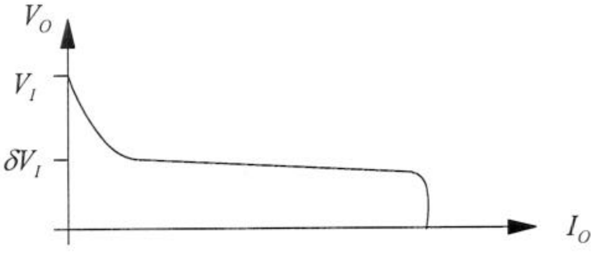



**Figure Q1**

Three regions are evident.
At low currents: the circuit is in discontinuous inductor current mode. The output voltage is an approximately reciprocal function of output current.
At moderate currents: the output voltage is almost constant at δVI as derived for the ideal circuit. It has a small negative slope which occurs because of voltage drops across the resistive part of the inductor and the mosfet and the voltage drop across the diode.

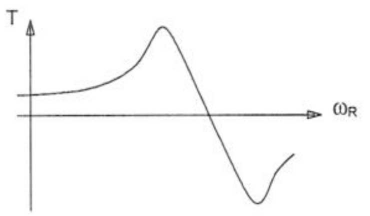
At high currents, a current limit circuit has intervened and reduced the duty-cycle to reduce the output voltage and limit the current.

(e) Explain how a rotating magnetic field is created by a three-phase winding in the stator of an induction machine. (You may assume the machine has one pole-pair.) [4]

The three phase windings are arranged to be displaced by one third of the circumference of the stator structure and so when supplied with current in the positive sense, they produce a magnetic field across the machine mutually displaced by 120°. If a positive current was switched between the windings, a magnetic field that jumped 120° at each switching instance would result. Supply the windings with a balanced sinusoidal set of currents (that is, equal magnitude and phase displaced by 120°) gives a smooth movement of the flux vector around the stator.

(f) Sketch the torque against speed graph of an induction machine, marking the point at which torque is zero and explain why torque is zero there. [4]

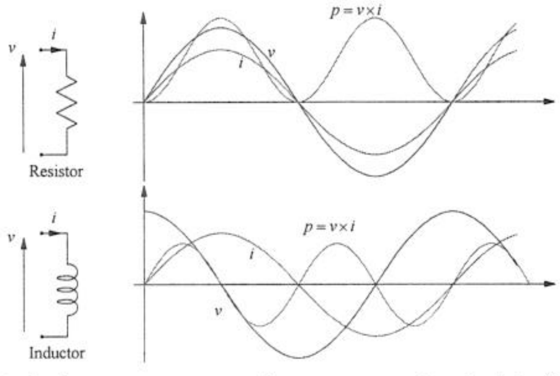
Sketch from notes




Torque is zero when the rotor rotates at synchronous speed. The rotor bars move with the stator filed and the absence of relative motion means no voltage, current or torque is developed in the rotor.

(g) Sketch the instantaneous power of a resistor and an inductor when a sinusoidal current flows through them both and comment on the real power consumed by each. [4]

Diagram from notes




Both inst. power waveforms seen to be double frequency sinusoids; the difference is in the DC component. The inductor power has no DC term and averages zero. It has zero real power and all power flow is reactive. The resistor power has a DC term equal to the peak of the double frequency term. The average, and real, power is the DC term. The instantaneous power does not turn negative and so there is no reactive power.

(h) Why is the three-phase system so widely adopted in power systems? [4]

- Poly-phase required to create a rotating field in a motor's stator.
- Poly-phase required to overcome double frequency pulsation of instantaneous power in single phase systems
- More than two phases required before neutral line can be omitted from balanced systems
- Too high a phase number increases cost of providing insulation, circuit breakers etc.
- High phase number means less voltage cancelation between coils of adjacent slots that have been series connected
- Benefits of previous point are diminishing returns and most benefit already achieved at 3 phases, a number that satisfies all the other concerns.

(i) What is the role of a governor in a power station generator? [4]

A generator will run at constant speed if the power drawn off in electrical form matches that applied at the shaft by the turbine (etc.). The power drawn by the electrical system is subject to changes that need to be satisfied. A sudden rise in electrical power demand will increase the reaction toque on the generator and cause it to decelerate. The governor detects this and increase the inlet (of steam etc.) to the turbine in proportion to the drop in speed. The speed will reach a new steady-state when the powers are in balance again. The proportional action facilitates sharing between several generators.

(j) The electricity industry in the UK has been "unbundled" into four distinct functions with separate ownership. Describe the four functions. [4]

- Generation: production of electrical energy for sale in a forward market. Normal for market to be in half hour delivery slots and market closes one hour or one day ahead of delivery. Counter parties in market are supply businesses (wholesalers) or large end-use customers.
- Transmission: most generation is far from the load centres so bulk movement of energy required. Transmission system also provides the interconnection that allows economically efficient provision of reserve services and balancing services. Transmission is at EHV. This is normally a regulated monopoly that applies use-of-system charges.
- Distribution: Onward movement of energy from a bulk supply point near a load centre to individual consumers is the role of the distribution network. For the most part, the DN has consumers but no generation. This is normally a regulated (local) monopoly that applies use-of-system charges and connection charges.
- Supply: wholesalers who have bought in the main generation market sell on to domestic and other small consumers. Sometime regionally based but in the UK there is a competitive market in supply.

Section B

2. A boost switch-mode power supply, SMPS, as shown in figure Q2, has the following design properties.

| | |
|---|---|
| Input voltage | $V_I = 6.0$ V |
| Output voltage | $V_O = 18.0$ V |
| Maximum output current | $I_O^{max} = 0.5$ A |
| Switching frequency | f = 10 kHz |
| Inductor | $L = 500$ μH |
| Capacitor | $C = 1{,}000$ μF |
| MOSFET on resistance | $R_{DS(on)} = 0.25\ \Omega$ |
| MOSFET turn-on loss | $E_{on} = 13$ μJ when switched on at 1.5 A |
| MOSFET turn-off loss | $E_{off} = 7$ μJ when switched off at 1.5 A |

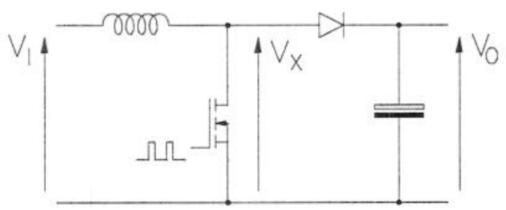



**Figure Q2**

(i) Calculate the duty-cycle at which the circuit should be operated assuming the circuit is in continuous conduction mode. [3]

$$\frac{V_O}{V_I} = \frac{1}{1-\delta}$$

$$\delta = 1 - \frac{V_I}{V_O} = 1 - \frac{1}{3} = \tfrac{2}{3}$$

(ii) Calculate the average value of the current through the diode and the average current through the inductor when the load draws the maximum output current. [4]

The average current through the capacitor (in steady-state) is zero and so the average value of the diode current equals the output current.
The diode carries the inductor current during the off-time and the average values are related by the duty-cycle. (Inductor current can also be found from balance of input and output powers.)

$$I_D^{Avg} = I_O = 0.5\ A$$

$$I_D^{Avg} = (1-\delta) I_L^{Avg}$$

$$I_L^{Avg} = \frac{0.5}{1-\tfrac{2}{3}} = 1.5\ A$$

(iii) Sketch the waveforms of the currents through the diode, capacitor and inductor for the conditions when the circuit operates at maximum output current. [5]

(iv) Calculate the voltage ripple across the capacitor. There is no need to consider the series resistance of the capacitor. [4]

Consider the discharge of the capacitor during the on-time of the switch (the off-time of the diode)

$$\Delta V = \frac{1}{C}\int i_L - I_O.dt$$
$$= \frac{I_L - I_O}{C}\cdot\frac{\delta}{f}$$
$$= \frac{(1.5-0.5)\times\frac{2}{3}}{10^{-3}\times 10^4} = 67\ mV$$

(v) Calculate the ripple component of the inductor current. [3]

$$\Delta I = \frac{V_I}{L}\frac{\delta}{f}$$
$$= \frac{6\times\frac{2}{3}}{5\times 10^{-4}\times 10^4} = 0.8A$$

(vi) Calculate the value of inductor current at which the SMPS enters discontinuous conduction mode and the value of output current to which this corresponds. [3]

$$I_L^{Critical} = \tfrac{1}{2}\Delta I = 0.4\ A$$
$$I_O^{Critical} = (1-\delta)I_L^{Critical} = 0.133\ A$$

(vii) Calculate the power losses in the MOSFET. [4]

$$P_{Loss} = \delta\, I_{DS}^2 R_{DS(on)} + (E_{On} + E_{Off})f$$
$$= \tfrac{2}{3}\times 1.5^2\times 0.25 + (20\times 10^{-6})\times 10^4$$
$$= 0.375 + 0.2 = 0.575\ W$$

(viii) It has been suggested that the switching frequency of this SMPS is too low. Discuss the advantages and disadvantages of raising the switching frequency. [4]

The calculations above make clear that raising the switching frequency would reduce the output voltage ripple proportionately and allow the SMPS to operate to lower powers while remaining in continuous mode. Alternatively, the ripple and critical current could be held the same and the size of the capacitor and inductor reduced in inverse proportion to frequency. Unfortunately, the switching power loss (a little less than half the total loss at present and about 2.2% efficiency loss) would increase with frequency.

3. A 2-pole-pair, 3-phase induction machine is connected in star to a 480 V (line), 50 Hz supply. The equivalent circuit parameters, referred to the stator, are:

| | |
|---|---|
| Stator resistance | 0.06 Ω |
| Referred rotor resistance | 0.08 Ω |
| Stator leakage reactance | 0.30 Ω |
| Referred rotor leakage reactance | 0.30 Ω |
| Magnetising reactance | 20.0 Ω |
| Magnetising resistance | 200 Ω |

(a) For operation at 1,450 r.p.m. calculate the following

(i) the slip [3]

$$\omega_S = \frac{2\pi f_E}{P} = 50\pi \quad or \quad n_S = 1{,}500\, r.p.m.$$

$$s = \frac{n_S - n_R}{n_S} = \frac{50}{1500} = 0.0\dot{3}$$

S=0.03 not accurate enough. S=0.0333 required.

(ii) the stator current [12]

First find the impedance

$$Z_R = \frac{R'_R}{s} + jX'_R = \frac{0.08}{0.0333} + j0.3 = 2.40 + j0.3\ \Omega = 2.419\angle 7.125°\ \Omega$$

$$Z_M = X_M \,//\, R_M = \frac{j20 \times 200}{200 + j20} = \frac{4000\angle 90°}{201.00\angle 5.711°} = 19.900\angle 84.29°\ \Omega = 1.980 + j19.801\ \Omega$$

$$Z_{In} = Z_S + Z_M \,//\, Z_R = 0.06 + j0.30 + \frac{19.900\angle 84.29° \times 2.419\angle 7.125°}{2.40 + j0.3 + 1.980 + j19.801} = 0.06 + j0.30 + \frac{48.138\angle 91.415°}{6.78 + j20.101}$$

$$= 0.06 + j0.30 + \frac{48.138\angle 91.415°}{21.214\angle 71.36°} = 0.06 + j0.3 + 2.269\angle 20.055° = 0.06 + j0.30 + 2.131 + j0.778$$

$$= 2.191 + j1.078\ \Omega$$

$$= 2.442\angle 26.20°$$

$$V_S = \tfrac{1}{\sqrt{3}} V_L = \frac{480}{\sqrt{3}} = 277.1\, V$$

$$I_S = \frac{V_S}{Z_{In}} = \frac{277.1}{2.442\angle 26.20°} = 113.47\angle -26.20°$$

(iii) the power losses [5]

Stator ohmic loss

$$P_{S-Loss} = 3 I_S^2 R_S = 3 \times 113.47^2 \times 0.06 = 2.32\ kW$$

Rotor ohmic loss

$$I_R = I_S \frac{Z_M}{Z_M + Z_R} = 113.47 \times \frac{19.900}{21.214} = 106.442\ A$$

$$P_{R-Loss} = 3I_R^2 R_R = 3 \times 106.442^2 \times 0.08 = 2.72\ kW$$

For the iron losses, find the voltage across RM first.

$$V_M = V_S \frac{\frac{Z_R Z_M}{Z_M + Z_R}}{Z_{In}} = 277.1 \times \frac{2.269}{2.442} = 257.47\ V$$

$$P_{Fe-Loss} = 3\frac{V_M^2}{R_M} = 3 \times \frac{257.47^2}{200} = 0.994\ kW$$

(iv) the mechanical power [5]

$$P_{Mech} = 3I'_R{}^2\ R'_R \left(\frac{1}{s} - 1\right) = 3 \times 106.442^2 \times 0.08 \left(\frac{1}{0.0333} - 1\right) = 78.86\ kW$$

(c) Without performing detailed calculations, comment on how the power losses, mechanical power and efficiency would change if the speed changed to 1,475 r.p.m., that is, the slip is halved. [5]

- The rotor branch impedance doubles
- The rotor current halves
- The rotor power loss reduces to a quarter
- The mechanical power reduces by a factor 4 because of the squared current but doubles because of the slip (i.e. it halves overall)
- Stator current approximately halves and stator loss reduced by factor of 4
- Iron loss is approximately unaffected (since dominated by stator voltage which is unchanged)
- Overall loss reduction is close to a factor of 4 because iron loss is smaller than ohmic loss but output power is halved. Overall the efficiency improves.
- (Machine was operating beyond peak efficiency point because conduction loss greater than iron loss)

4. A transformer has been designed as a step-down transformer for use in a 50 Hz distribution system. The design equations indicate that the following parameters will apply to the approximate equivalent circuit shown in figure Q4.

| | |
|---|---|
| Combined winding resistance referred to primary | $R_W = 0.55\ \Omega$ |
| Combined leakage reactance referred to primary | $X_W = 1.50\ \Omega$ |
| Magnetising resistance | $R_M = 5\ \text{k}\Omega$ |
| Magnetising reactance | $X_M = 800\ \Omega$ |
| Primary to secondary turns-ratio | $N_P{:}N_S = 11{:}1$ |

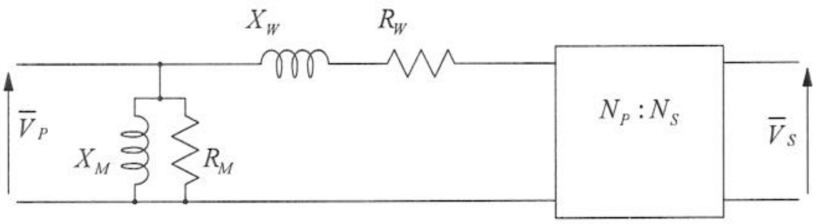



**Figure Q4**

(a) For the equivalent circuit in Figure Q4:

(i) Describe the physical effects in the real transformer that give rise to each component of the equivalent circuit; [5]

- The ideal transformer block represents perfectly coupled mutual inductors. Voltage imposed on one winding causes a changing flux that induces a voltage on the other in proportion to the turns-ratio. The imposed voltage dictates the flux and maintenance of the flux requires the ampere-turns of the two windings to balance.
- The windings are imperfect conductors and $R_W$ represents the resistance of the primary and the resistance of the secondary referred to the primary.
- The establishment of flux in the core requires a current flow which is the current flow through $X_M$, the magnetising reactance.
- The establishment of alternating flux in steel causes eddy current and hysteresis power losses which are modelled by $R_M$.
- Not all of the flux created by current flow in one winding links with the other. The voltage drops caused by this inductance are modelled by leakage reactances in series with each winding. Here the secondary winding reactance has been referred to the primary and lumped with the primary component.

(ii) Identify which components need to be considered when calculating the power losses in the transformer; [2]

- Reactances do not represent power losses.
- Power lost to eddy current and hysteresis is seen as power loss in $R_M$.
- Power lost to ohmic heating in the windings is seen as power loss in $R_W$.

(iii) Discuss any approximations in this equivalent circuit. [3]

- All non-linear and time-varying effects are neglected such as saturation of the magnetic core. Eddy current and hysteresis losses are functions of frequency and non-integer powers of flux density. They are assumed to be approximately proportional to the square of voltage at fixed frequency.

- Referring components from primary to secondary has no loss of accuracy but lumping in series components that are properly to either side of the magnetising branch is approximate.

(b) The primary of the transformer is to be supplied at 33.0 kV and the secondary connected to a load of 0.40 + j0.25 Ω.

(i) Calculate the secondary voltage and the voltage regulation this represents. [5]

Start by referring load impedance to primary

$$Z' = Z\left(\frac{N_P}{N_S}\right)^2 = (0.4 + j0.25)\times 11^2 = 48.4 + j30.25\ \Omega$$

$$V'_S = V_P \frac{Z'}{Z' + R_W + jX_W} = 33{,}000\times\frac{48.4 + j30.25}{48.4 + j30.25 + 0.5 + j1.5} = 33{,}000\times\frac{57.076\angle 32.01^\circ}{58.303\angle 32.995^\circ}$$

$$= 32.305\angle -0.98^\circ$$

$$V_S = V'_S \frac{N_S}{N_P} = 2.937\ kV$$

$$R = \frac{\frac{33}{11} - 2.937}{\frac{33}{11}} = 2.1\%$$

(ii) Calculate the real and reactive powers at the primary input and secondary output. [5]

$$I'_S = \frac{V_P}{Z'} = \frac{33000}{57.076\angle 32.01^\circ} = 578.2\angle -32.01^\circ$$

$$S_S = V'_S I'^*_S = 32.305\angle -0.98^\circ \times 578.2\angle +32.01^\circ$$

$$= 18.678\angle 31.03^\circ$$

$$P_S = 16.01\ MW$$

$$Q_S = 9.63\ MVAr$$

$$I_P = I'_S + \frac{V_P}{R_M} + \frac{V_P}{jX_M} = 490.3 - j306.5 + \frac{33000}{5000} - j\frac{33000}{800}$$

$$= 496.9 - j347.75$$

$$S_P = V_P I_P^* = 33000(496.9 + j347.75)$$

$$P_P = 16.40\ MW$$

$$Q_P = 11.48\ MVAr$$

(c) When built the transformer is tested to establish if the parameters of the equivalent circuit are close to the design values. Using the following test data, calculate the actual parameters. With 33.0 kV applied to the primary and the secondary open-circuit, the current was 41.0 A and the input power 210 kW.

With 1000 V applied to the primary and the secondary short circuit, the primary current was 635 A and the power 212 kW. [10]

Taking open-circuit test first.

$$R_M = \frac{V_P^{\ 2}}{P} = \frac{33{,}000^2}{210{,}000} = 5185\ k\Omega$$

$$Q^2 = (VI)^2 - P^2 = (33000 \times 41)^2 - 210000^2$$

$$Q = 1.336\ MVAr$$

$$X_M = \frac{V_P^{\ 2}}{Q} = \frac{33000^2}{1336000} = 814.8\ \Omega$$

Short-circuit neglecting the magnetising components.

$$R_W = \frac{P}{I^2} = \frac{212000}{635^2} = 0.526\ \Omega$$

$$Q^2 = (VI)^2 - P^2 = (1000 \times 635)^2 - 212000^2$$

$$Q = 599\ kVAr$$

$$X_W = \frac{Q}{I^2} = \frac{599000}{635^2} = 1.484\ \Omega$$