

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

ELECTRICAL ENERGY SYSTEMS

Time allowed: 3:00 hours

Answer FOUR questions.

Any special instructions for invigilators and information for candidates are on page 1.

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Electrical Energy Systems

Question 1

- (a) Consider a single phase AC circuit shown in Figure 1. Expressions for generator voltage and current are given by the following expressions:

$$v(t) = \sqrt{2}V \sin \omega t$$

$$i(t) = \sqrt{2}I \sin(\omega t - \phi)$$

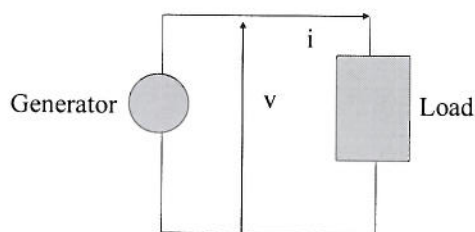


Figure 1. Single phase AC circuit

- (i) derive an expression for the instantaneous power, active and reactive powers. Sketch these three quantities on the same diagram. [3]
- (ii) Write the expression for instantaneous active power for a fully balanced three phase system (no need for a formal derivation, just a short description of the logic the expression itself). [2]
- (b) A 33/11kV 15MVA transformer has a leakage reactance of 5Ω as seen from the HV side
- (i) Calculate the leakage reactance as seen from the LV side [2]
- (ii) Calculate the pu impedance at the HV side and show that this is the same as the pu impedance as seen from the LV side [3]
- (c)
- (i) List three purposes of load flow calculations in a power system. What is the input data and what is the result of the load flow on a transmission system? [2]
- (ii) Explain why reactive power is supplied more locally than active power in case of a heavily loaded high voltage transmission network. [2]
- (iii) What makes the load flow problem non-linear? [2]

Question 1 is continued on next page

- (d) (i) Show that the positive and negative sequence components of a set of identical phasors are equal to zero. [2]

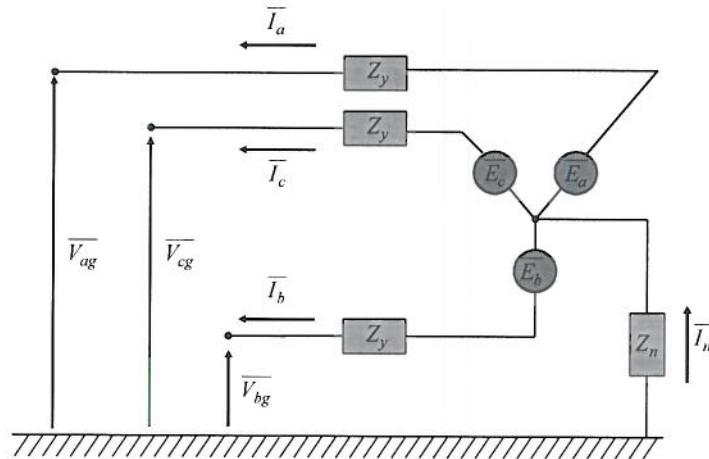


Figure 1.2. Generic unbalanced circuit

- (ii) for the unbalanced circuit shown in Figure 1.2, show that positive and negative sequence currents do not flow in the neutral circuit and if there is no neutral connection, there will be no zero sequence current. [2]

Question 2:

(a) Consider synchronous generator equivalent circuit presented in Figure 2

(i) Define the meaning of all the quantities in the diagram

[2]

(ii) draw the phasor diagram to show the link between \vec{V}_t and \vec{E}_i

[2]

(iii) demonstrate that the real power (P) and reactive power outputs (Q) are given by (angle δ is the power angle):

[6]

$$P = \frac{|V_t|}{R^2 + X_d^2} \{ |E_i| (R \cos \delta + X_d \sin \delta) - |V_t| R \}$$

$$Q = \frac{|V_t|}{R^2 + X_d^2} \{ X_d (|E_i| \cos \delta - |V_t|) - R |E_i| \sin \delta \}$$

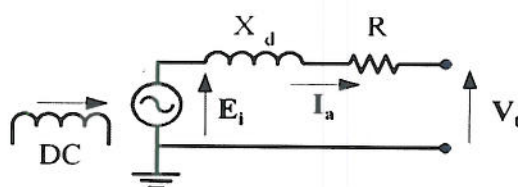


Figure 2. Synchronous generator equivalent circuit

(b) Explain how reactive power imports and exports are controlled

[2]

(c) A three phase round-rotor synchronous generator, rated 16 kV and 200 MVA, has armature resistance of 0.1 per unit and synchronous reactance of 1.65 per unit. The generator is connected to a very strong network that maintains the voltage at the generator terminals at 15 kV. The generator delivers 150 MVA at 0.8 power factor lagging at its terminal output.

(i) Determine E_i , the power angle δ , current I_a and losses.

[4]

(ii) If the field current of the machine is reduced by 10%, while the mechanical power input to the machine is maintained constant, determine the new value of δ

[4]

Question 3

A 132 kV network, supplied from a remote generator connected to busbar 1 (Figure 3) supplies an industrial site at busbar 2 and a town supplied from busbar 3. Network voltage is controlled by the generator (G) and Static-Var-Compensator (SVC) and is kept at 1 p.u. at busbar 1 and busbar 2. Lengths of circuits are shown in the Figure. Individual circuits all have the same unit reactance of $x = 0.2 \Omega/\text{km}$, while active power losses can be ignored.

- a) Identify the type of busbars for nodes 1, 2 and 3 (Slack, PV or PQ) [3]
 - b) Assuming a 100MVA base, calculate the per unit values of all system parameters [2]
 - c) Form the Ybus matrix for this system [2]
- Perform two iterations of the Gauss-Seidel power flow and calculate:
- i) Voltages (p.u.) and angles at all three busbars [8]
 - ii) Generator G active and reactive power output [2]
 - iii) Reactive power delivered by the Static Var Compensator [3]

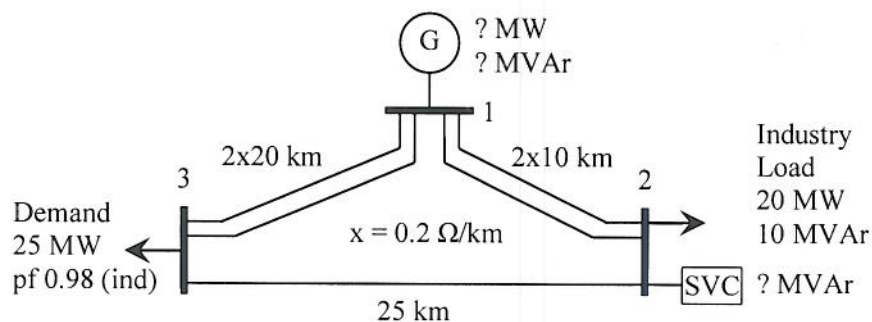
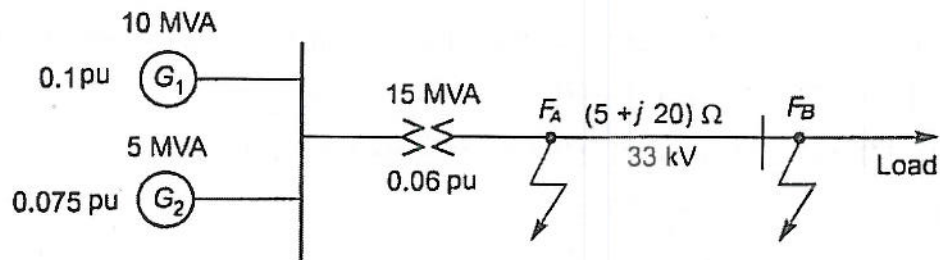


Figure 3: The 132 kV system

Question 4

- a) Mention two major factors that influence the fault level in an AC system. [2]
- b) Explain the theorem of constant flux linkage. [4]
- c) Use theorem of constant flux linkage to explain why the current under sub-transient condition is higher than that under transient condition. [5]
- d) A 33 kV three phase transmission circuit with resistance 5 ohms and reactance 20 ohms is connected to a generation station busbar through a 15 MVA step-up transformer with 0.06 pu reactance. Two generators, one 10 MVA with 0.10 pu reactance and another 5 MVA with 0.075 pu reactance, are connected to the busbar. Calculate the short circuit MVA and the fault current for a three-phase short circuit on the transmission line at the following locations:
 - i) near the high voltage terminal of the transformer (point F_A) [5]
 - ii) at the load end (point F_B) [4]



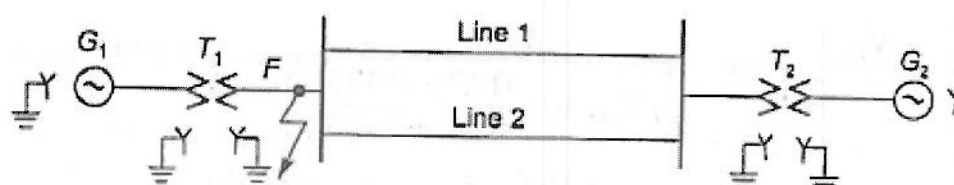
Question 5

- a) Define power systems stability? [2]
- b) Why and on what basis is the single phenomenon of power system stability classified into different types? [5]
- c) Use the standard power angle characteristics (neglecting resistance) and swing equation to show analytically that the steady-state stability limit corresponds to a load angle (δ) of 90 degrees. [5]
- d) A double-circuit corridor connects a synchronous generator to large network which can be approximated as an infinite bus. Steady-state stability limit for each circuit is 100 MW. If the total power flow through the corridor (out of the generator) is 80 MW, determine whether stability would be maintained or not after one of the circuits is suddenly switched out. [8]

Question 6

- a) Show that for a single-line to ground (LG) fault with zero fault impedance, the sequence networks are connected in series. [4]
- b) A single line diagram of a power system is shown in Fig 1 where the per unit (pu) positive (1), negative (2) and zero (0) sequence reactances referred to 100 MVA base are shown in Table 1. Draw the positive, negative and zero sequence networks for this system. [5]
- c) For the system in Fig. 1 calculate the fault currents in the three phases for the following fault types assuming 1.0 per unit (pu) voltage for both generators and zero fault impedance.
- i) Single line to ground (L-G) fault from phase A at point F [6]
- ii) Line-to-line (L-L) fault between phases B and C at point F [5]

	X_1	X_2	X_0
Generator G_1	0.30	0.20	0.05
Generator G_2	0.25	0.15	0.03
Line 1	0.30	0.30	0.70
Line 2	0.30	0.30	0.70
Transformer T_1	0.12	0.12	0.12
Transformer T_2	0.10	0.10	0.10



Solution to Question 1

(a)

(i)

$$P = VI \cos \phi$$

$$Q = VI \sin \phi$$

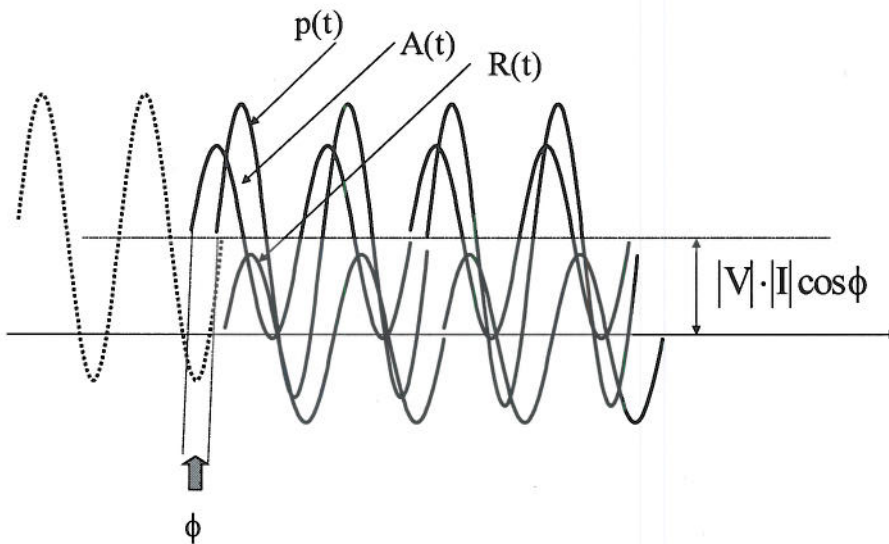
$$p(t) = v(t) \cdot i(t) = 2 VI \sin \omega t \sin (\omega t - \phi)$$

$$= 2 VI \sin \omega t \{ \sin \omega t \cos \phi - \cos \omega t \sin \phi \}$$

$$= VI \cos \phi (2 \sin^2 \omega t) - VI \sin \phi (2 \sin \omega t \cos \omega t)$$

$$= P (1 - \cos 2\omega t) - Q \sin 2\omega t$$

$$p(t) = \underbrace{P(1 - \cos 2\omega t)}_{A(t)} - \underbrace{Q \sin 2\omega t}_{R(t)}$$



[3]

(ii)

$$v_a(t) = \sqrt{2}V \sin \omega t$$

$$i_a(t) = \sqrt{2}I \sin(\omega t - \phi)$$

$$v_b(t) = \sqrt{2}V \sin(\omega t - 120^\circ)$$

$$i_b(t) = \sqrt{2}I \sin(\omega t - 120^\circ - \phi)$$

$$v_c(t) = \sqrt{2}V \sin(\omega t - 240^\circ)$$

$$i_c(t) = \sqrt{2}I \sin(\omega t - 240^\circ - \phi)$$

$$p_3(t) = v_a(t) \cdot i_a(t) + v_b(t) \cdot i_b(t) + v_c(t) \cdot i_c(t)$$

$$p_3(t) = 3VI \cos \phi = 3P$$

Instantaneous 3-phase power is constant

[2]

(b)

(i) $X_{lv} = X_{hv} \frac{U_{lv}^2}{U_{hv}^2} = 0.5556 \Omega$ [2]

(ii) $Z_{base\ hv} = \frac{U_{hv}^2}{S_b} = \frac{(33\ 10^3)^2}{(15\ 10^6)} = 72.6 \Omega$

$$Z_{base\ lv} = \frac{U_{lv}^2}{S_b} = \frac{(11\ 10^3)^2}{(15\ 10^6)} = 8.06 \Omega$$

$$X_{lv} = \frac{0.5556}{8.06} = 0.069\ p.u.$$

$$X_{hv} = \frac{5}{72.6} = 0.069\ p.u.$$

In per unit $\therefore X_{hv} = X_{lv}$ [3]

(c)

(i) Purposes of load flow calculation are:

1. to calculate voltage at each bus and power flows in each branch
 2. to calculate the effects of rearranging circuits and incorporating new circuits on system loading
 3. to calculate the effects of temporary loss of generation and transmission circuits on system loading
 4. to calculate the effects of injecting in-phase and quadrature voltages on system loading
 5. to find the optimum system running conditions and load distribution
 6. to calculate the improvement from change of conductor size and system voltages
- [2]

(ii) Unlike active power, reactive power cannot be transmitted across long distances

1. Transmitting Q requires a voltage drop that would become unacceptable for long distances
 2. Since $X \gg R$, the reactive losses are much larger than the active losses and the transmission of Q would be inefficient
- [2]

(iii) In power systems, sources and loads are defined in terms of power, not voltage, current or impedance. This makes power flow equations non-linear. [2]

(d)

(i)

Positive sequence component:

$$\overline{I}^1 = \overline{I}_a + a\overline{I}_b + a^2\overline{I}_c$$

Negative sequence component:

$$\overline{I}^2 = \overline{I}_a + a^2\overline{I}_b + a\overline{I}_c$$

If phasors are identical (i.e. $\overline{I}_a = \overline{I}_b = \overline{I}_c$), the above components are proportional to $(1 + a + a^2)$, which is equal to zero. [2]

(ii)

$$\begin{aligned}
\overline{I_n} &= \overline{I_a} + \overline{I_b} + \overline{I_c} \\
&= (\overline{I_a^0} + \overline{I_a^1} + \overline{I_a^2}) + (\overline{I_b^0} + \overline{I_b^1} + \overline{I_b^2}) + (\overline{I_c^0} + \overline{I_c^1} + \overline{I_c^2}) \\
&= (\overline{I_a^0} + \overline{I_b^0} + \overline{I_c^0}) + \underbrace{(\overline{I_a^1} + \overline{I_b^1} + \overline{I_c^1})}_0 + \underbrace{(\overline{I_a^2} + \overline{I_b^2} + \overline{I_c^2})}_0 \\
&= 3\overline{I^0} \\
\overline{I_a^1} + \overline{I_b^1} + \overline{I_c^1} &= \overline{I_a^1} + a\overline{I_a^1} + a^2\overline{I_a^1} = 0 \\
\overline{I_a^2} + \overline{I_b^2} + \overline{I_c^2} &= \overline{I_a^2} + a^2\overline{I_a^2} + a\overline{I_a^2} = 0 \\
\overline{I_a^0} + \overline{I_b^0} + \overline{I_c^0} &= \overline{I^0} + \overline{I^0} + \overline{I^0} = 3\overline{I^0}
\end{aligned}$$

If there is no current I_n , there would not be current I_0 .

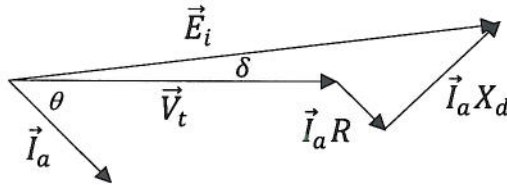
[2]

Solution to Question 2

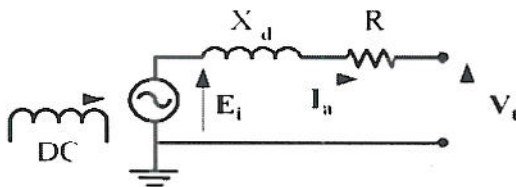
(a)

[ii]

phasor diagram to show the link between \vec{V}_t and \vec{E}_i



[iii] Deriving the formula to determine the real power (P) and reactive power outputs (Q)



$$\vec{I}_a = \frac{|E_i| \angle \delta - |V_t|}{R + jX_d}$$

$$\vec{I}_a^* = \frac{|E_i| \angle -\delta - |V_t|}{R - jX_d} \cdot \frac{R + jX_d}{R + jX_d}$$

$$\vec{I}_a^* = \frac{(|E_i| \angle -\delta - |V_t|) \cdot (R + jX_d)}{R^2 + X_d^2}$$

$$\vec{S} = |V_t| \vec{I}_a^* = \frac{(|E_i| \angle -\delta - |V_t|) \cdot (R + jX_d)}{R^2 + X_d^2}$$

$$\vec{S} = \frac{(|E_i| (\cos \delta - j \sin \delta) - |V_t|) \cdot (R + jX_d)}{R^2 + X_d^2}$$

$$\vec{S} = \frac{|V_t|}{R^2 + X_d^2} \{ |E_i| (R \cos \delta + X_d \sin \delta) - |V_t| R \} \\ + j \frac{|V_t|}{R^2 + X_d^2} \{ X_d (|E_i| \cos \delta - |V_t|) - R |E_i| \sin \delta \}$$

$$\therefore P = \frac{|V_t|}{R^2 + X_d^2} \{ |E_i| (R \cos \delta + X_d \sin \delta) - |V_t| R \}$$

$$\therefore Q = \frac{|V_t|}{R^2 + X_d^2} \{ X_d (|E_i| \cos \delta - |V_t|) - R |E_i| \sin \delta \}$$

(b) The field current is used to control the reactive power output from the generator. In order to export reactive power, the field current is over excited such that

$|E_i| \cos \delta > |V_t|$. To import reactive power, the field current is under excited such that $|E_i| \cos \delta < |V_t|$

(c) A number of constraints limit the maximum export that a synchronous generator can export.

The constraints are

- Prime mover limit
- Armature current limit
- Excitation current limit

(d)

[i] Determine the internal voltage E_i , the power angle δ , losses and the line current of the machine

MVA base = 200 MVA

kV base = 16 kV

S = 150 MVA

$P = 150 \cos \theta = 150 \times 0.8 = 120 \text{ MW} = 0.6 \text{ p.u.}$

$Q = 150 \sin \theta = 150 \times 0.6 = 90 \text{ MVAr} = 0.45 \text{ p.u.}$

$$P = \frac{|V_t|}{R^2 + X_d^2} \{ |E_i| (R \cos \delta + X_d \sin \delta) - |V_t| R \}$$

$$\Leftrightarrow 0.1 |E_i| \cos \delta + 1.65 |E_i| \sin \delta = 1.84257 \quad (1)$$

$$Q = \frac{|V_t|}{R^2 + X_d^2} \{ X_d (|E_i| \cos \delta - |V_t|) - R |E_i| \sin \delta \}$$

$$\Leftrightarrow 1.65 |E_i| \cos \delta - 0.1 |E_i| \sin \delta = 2.8585 \quad (2)$$

From (1) and (2)

$$|E_i| = 2.0573 \text{ p.u.}$$

$$\delta = 0.5120 \text{ rad}$$

Armature current

$$\vec{I}_a = \frac{|E_i| \angle \delta - |V_t|}{R + jX_d}$$

$$\vec{I}_a = \frac{2.0573 \angle 0.5120 - 0.9375}{0.1 + j1.65}$$

$$\vec{I}_a = 0.6399 - j 0.4799$$

Losses

$$Losses = |I|_a^2 R = 0.063993 \text{ p.u.}$$

[ii] If the field current of the machine is reduced by 10%, while the mechanical power input to the machine is maintained constant, determine the new value of δ

$$P = \frac{|V_t|}{R^2 + X_d^2} \{ |E_i| (R \cos \delta + X_d \sin \delta) - |V_t| R \}$$

$$\Leftrightarrow 0.1 |E_i| \cos \delta + 1.65 |E_i| \sin \delta = 1.84257 \quad (1)$$

$$|E_i| = 0.9 \times 2.0573 = 1.8516 \text{ p.u.}$$

From (1) we obtain

$$0.18516 \cos \delta + 3.05514 \sin \delta = 1.84257$$

$$\delta = 0.5855 \text{ rad}$$

Solution to Question 3

Busbar types: 1 – Slack, 2 – PV, and 3 – PQ

Per unit calculation:

$$S_b = 100 \text{ MVA}$$

$$V_b = 132 \text{ kV}$$

$$Z_b = 132^2 / 100 = 174.24 \Omega$$

$$\text{Industry } S_L = 0.2 + j*0.1 \text{ p.u.}$$

$$\text{Demand } S_D = 0.25 + j*0.25*\tan(\cos^{-1}(0.98)) = 0.25 + j*0.05 \text{ p.u.}$$

Circuits

$$z_{12} = j*0.2 * 10 / 2 / 174.24 = j*0.0057 \text{ p.u.}$$

$$z_{13} = j*0.2 * 20 / 2 / 174.24 = j*0.0115 \text{ p.u.}$$

$$z_{23} = j*0.2 * 25 / 174.24 = j*0.0287 \text{ p.u.}$$

Y bus matrix:

$$Y_{bus} = j * \begin{bmatrix} -261.36 & 174.24 & 87.12 \\ 174.24 & -209.09 & 34.85 \\ 87.12 & 34.85 & -121.97 \end{bmatrix} \text{ p.u.}$$

Two iterations of Gauss-Seidel power flow:

Bus input record

[Index]	[Type]	[V set]	[P]	[Q]
1.0000	3.0000	1.0000	0	0
2.0000	2.0000	1.0000	-20.0000	0
3.0000	0	0	-25.0000	-5.0765

Branches input record

[Index]	[From]	[To]	[R]	[X]
1.0000	1.0000	2.0000	0	0.0057
2.0000	1.0000	3.0000	0	0.0115
3.0000	2.0000	3.0000	0	0.0287

Iteration: 1

Buses record

[Index]	[Voltage]	[Angle]	[P]	[Q]	[DV]	[P Mism]	[Q Mism]
1.0000	1.0000	0	36.8290	4.4177	0	0	0.0000
2.0000	1.0000	-5.4639	-20.0000	1.6230	0.0953	-8.2874	0.0000
3.0000	0.9861	-13.6251	-25.0000	-5.0765	0.2360	0.1165	-6.1741

Branches record

[Index]	[From]	[To]	[P from]	[P to]	[P loss]	[Q from]	[Q to]	[Q loss]
1.0000	1.0000	2.0000	16.5909	-16.5909	0	0.7917	0.7917	1.5834
2.0000	1.0000	3.0000	20.2381	-20.2381	0	3.6260	1.2262	4.8523
3.0000	2.0000	3.0000	4.8784	-4.8784	0	0.8313	-0.1286	0.7028

Iteration: 2

Buses record

[Index]	[Voltage]	[Angle]	[P]	[Q]	[DV]	[P Mism]	[Q Mism]
1.0000	1.0000	0	43.5321	9.6312	0	0	0.0000
2.0000	1.0000	-7.7337	-20.0000	4.0100	0.0396	-0.2571	0.0000
3.0000	0.9365	-14.2518	-25.0000	-5.0765	0.0508	-1.2108	-0.5411

Branches record

[Index]	[From]	[To]	[P from]	[P to]	[P loss]	[Q from]	[Q to]	[Q loss]
1.0000	1.0000	2.0000	23.4473	-23.4473	0	1.5849	1.5849	3.1697
2.0000	1.0000	3.0000	20.0847	-20.0847	0	8.0464	-2.6728	5.3735
3.0000	2.0000	3.0000	3.7045	-3.7045	0	2.4251	-1.8626	0.5626

Not converged in two iterations

P mismatch: 1.2108 Q mismatch: 0.54108 V deviation: 0.050769

G: 43.5 MW, 9.6 MVar

SVS: 4.0 – (-10) = 14 MVar

Solution to Question 4

- a) Fault levels are influenced by :
- Proximity to generators
 - Number of generators connected to the system
 - Meshing of the network

- b) In a RL circuit fed by a voltage (emf) source

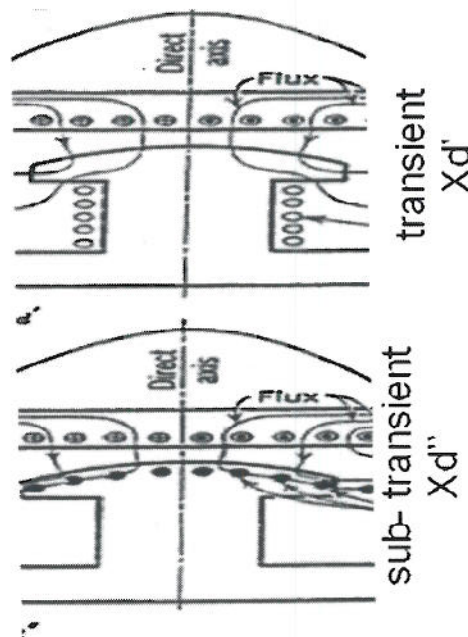
$\Sigma Ri(t) + d\Psi/dt = \Sigma e(t)$, where Ψ is the total flux linkage

Integrating over time interval Δt , $\Sigma R \int i(t) + \Delta\Psi = \Sigma \int e(t)$

For small time interval i.e. $\Delta t \rightarrow 0$, $\Delta\Psi \rightarrow 0$

i.e. flux linkage in any closed circuit with finite resistance and emf cannot change instantaneously. This is the theorem of constant flux linkage.

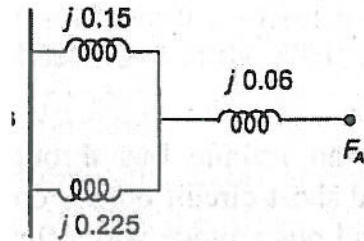
- c)



According to the constant flux linkage theorem, flux linking the field and damper winding cannot change instantaneously after the short circuit. From the flux paths shown in the figure above, it is clear that the reluctance offered is higher in the sub-transient condition than in the transient. Hence, the inductance or reactance is lower for the sub-transient. As a result, the current is higher for the sub-transient condition.

$$I' = E/X_d', \quad I'' = E/X_d'', \quad \text{As } X_d'' < X_d', \quad I'' > I'$$

- d) i) near the high voltage terminal of the transformer



Let the system base MVA be 15.

On 15 MVA base,

pu reactance of generator 1 is $j 0.10 \times 15/10 = j 0.15$ pu

pu reactance of generator 2 is $j 0.075 \times 15/5 = j 0.225$ pu

pu reactance of transmission line is $(5 + j 20) \times 15/33^2 = 0.06887 + j 0.27548$ pu

Equivalent impedance seen from the fault point F_A is:

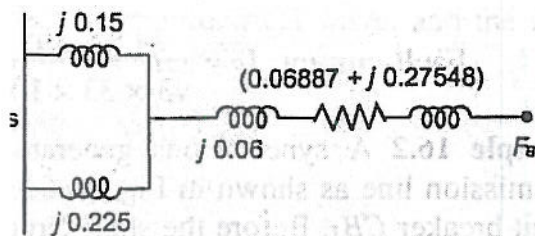
$$Z_e = (j 0.15 \parallel j 0.225) + j 0.06 = j 0.15 \text{ pu}$$

Short circuit MVA at F_A is

$$S_{SC} = S_b / |Z_e| = 15/0.15 = 100 \text{ MVA}$$

$$\text{Fault current } I_F = S_b / \sqrt{3}V = 100 \times 10^6 / (\sqrt{3} \times 33 \times 10^3) = 1749.5 \text{ A}$$

- ii) at the load end



Equivalent impedance seen from the fault point F_B is:

$$Z_e = (j 0.15 \parallel j 0.225) + j0.06 + j 0.06887 + j 0.27548 = 0.06887 + j 0.42548 \text{ pu}$$

$$|Z_e| = 0.431 \text{ pu}$$

Short circuit MVA at F_B is

$$S_{SC} = S_b / |Z_e| = 15/0.431 = 34.8 \text{ MVA}$$

$$\text{Fault current } I_F = S_b / \sqrt{3}V = 34.8 \times 10^6 / (\sqrt{3} \times 33 \times 10^3) = 608.8 \text{ A}$$

Solution to Question 5

- a) Power system stability is the ability of an electric power system to regain a state of operating equilibrium after being subjected to a physical disturbance with all system variables bounded and system integrity preserved.
- b) Power system stability is a single problem. However, it is not effective and practical to analyse it as a single problem due to the following reasons:
 - influence of components with different response rates and characteristics
 - high dimensionality and complexity of systems requiring simplifying assumptions retaining right degree of detail
- c) Classification of power system stability is based on the following considerations:
 - physical nature of resulting instability
 - size of disturbance
 - method of analysis
 - time span involved

A power-angle characteristic is given by:

$$P_e = \frac{|E||V|}{X_d} \sin \delta$$

For an incremental change in power output

$$\Delta P_e = \left[\frac{\partial P_e}{\partial \delta} \right]_0 \Delta \delta$$

Using swing equation

$$\frac{2H}{\omega_s} \frac{d^2 \Delta \delta}{dt^2} + \left[\frac{\partial P_e}{\partial \delta} \right]_0 \Delta \delta = 0$$

$$\frac{2H}{\omega_s} s^2 + \left[\frac{\partial P_e}{\partial \delta} \right]_0 = 0$$

The roots of the characteristic equation are:

$$s = \pm \left[-\frac{(\partial P_e / \partial \delta)_0 \omega_s}{2H} \right]^{\frac{1}{2}}$$

$(\partial P_e / \partial \delta)_0 > 0 \Rightarrow$ stability

$(\partial P_e / \partial \delta)_0 < 0 \Rightarrow$ instability

So stability limit corresponds to a load angle of 90 degrees

d) With both circuits in place, load angle is given by:

$$80 = 200 \sin \delta_1 \Rightarrow \delta_1 = 0.4115 \text{ rad}$$

With one circuit switched out load angle is given by:

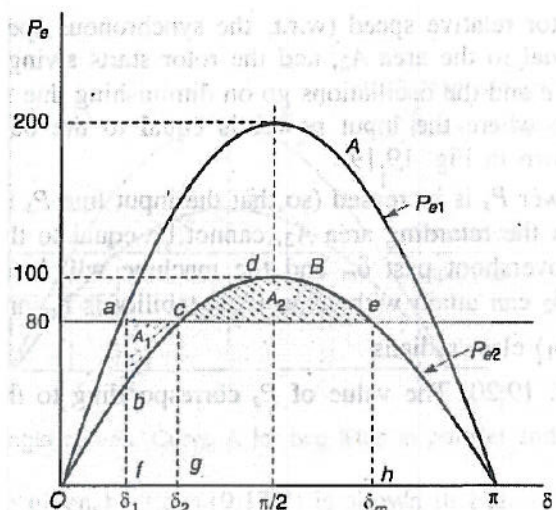
$$80 = 100 \sin \delta_2 \Rightarrow \delta_2 = 0.9273 \text{ rad}$$

Accelerating area A1 is:

$$\begin{aligned} \text{Area } A_1 &= \int_{\delta_{01}}^{\delta_2} (80 - 100 \sin \delta) d\delta \\ &= 9.6 \text{ MW rad} \end{aligned}$$

Decelerating area A2 is:

$$\begin{aligned} \text{Area } A_2 &= \int_{\delta_2}^{\pi - \delta_2} (100 \sin \delta - 80) d\delta \\ &= 17.04 \text{ MW rad} \end{aligned}$$



Since the decelerating area A2 is greater than the accelerating area A1, the system would be stable.

Solution to Question 6

- a) For a single line to ground (LG) fault in phase a with zero fault impedance, the boundary conditions at the fault point are:

$$I_b = I_c = 0, V_a = 0.$$

Corresponding symmetrical components of the currents would be

$$I_{a0} = (I_a + I_b + I_c)/3 = I_a/3$$

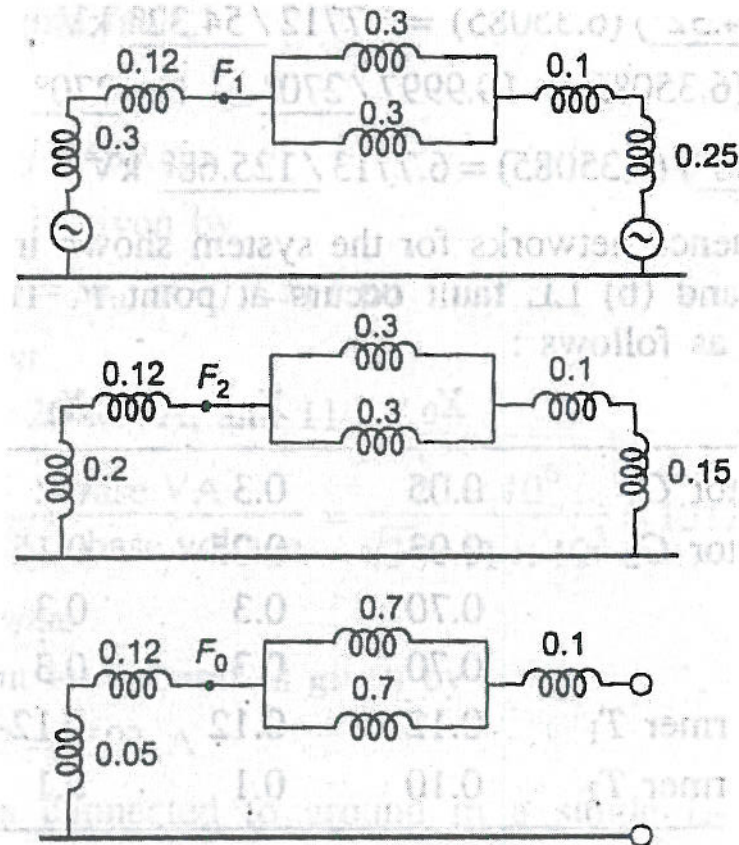
$$I_{a1} = (I_a + \alpha I_b + \alpha^2 I_c) = I_a/3$$

$$I_{a2} = (I_a + \alpha^2 I_b + \alpha I_c) = I_a/3$$

$$I_{a0} = I_{a1} = I_{a2} = I_a/3$$

As the current through each sequence network are equal, the three sequence networks are connected in series for a single line to ground (LG) fault.

- b)



- c) For the positive sequence network, equivalent impedance at the fault point F is:

$$Z_1 = j [(0.3+0.12) \parallel (0.25+0.1+0.3/2)] = j 0.22826 \text{ pu}$$

For the negative sequence network, equivalent impedance at the fault point F is:

$$Z_2 = j [(0.2+0.12) \parallel (0.15+0.1+0.3/2)] = j 0.1778 \text{ pu}$$

For the zero sequence network, equivalent impedance at the fault point F is:

$$Z_0 = j [0.05+0.12] = j 0.17 \text{ pu}$$

i) LG fault at F (zero fault impedance):

For single line to ground fault the sequence networks are connected in series. Hence,

$$I_{A1} = V_f / (Z_1 + Z_2 + Z_0) = 1 \angle 0^\circ / j (0.22826 + 0.1778 + 0.17) = -j 1.7359 \text{ pu}$$

$$\text{As } I_{A0} = I_{A1} = I_{A2}$$

Fault current in phase A is

$$I_A = I_{A0} + I_{A1} + I_{A2} = 3I_{A1} = -j 5.2078 \text{ pu}$$

Fault currents in the other two phases are zero i.e. $I_B = I_C = 0$

ii) LL fault at F (zero fault impedance):

For line to line (LL) fault between phases B and C

$$I_A = 0, I_B = -I_C$$

$$V_B - V_C = 0$$

Transforming from phase into sequence domain

$$I_0 = 0, V_1 = V_2 \text{ and } I_2 = -I_1$$

i.e. zero sequence network would not be present and the positive and negative sequence networks would be connected in parallel at the fault point F

$$I_{A1} = V_f / (Z_1 + Z_2) = 1 \angle 0^\circ / j (0.22826 + 0.1778) = -j 2.4627 \text{ pu}$$

$$I_{A2} = -I_{A1}$$

$$I_A = 0$$

$$I_B = I_{A0} + \alpha^2 I_{A1} + \alpha I_{A2} = -4.266 \text{ pu}$$

$$I_C = -I_B = 4.266 \text{ pu}$$