

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2014

EEE PART III/IV: MEng, BEng and ACGI

**OPTOELECTRONICS**

Friday, 17 January 2:30 pm

Time allowed: 3:00 hours

Corrected Copy

**There are SIX questions on this paper.**

**Answer FOUR questions.**

*All questions carry equal marks*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      R.R.A. Syms  
Second Marker(s) :      O. Sydoruk

### Fundamental constants

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ m kg/C}^2$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.62 \times 10^{-34} \text{ Js}$$

### Maxwell's equations – integral form

$$\oint \oint_A \underline{D} \cdot d\underline{a} = \int \int \int_V \rho \, dv$$

$$\oint \oint_A \underline{B} \cdot d\underline{a} = 0$$

$$\oint_L \underline{E} \cdot d\underline{L} = - \int \int_A \frac{\partial \underline{B}}{\partial t} \cdot d\underline{a}$$

$$\oint_L \underline{H} \cdot d\underline{L} = \int \int_A [\underline{J} + \frac{\partial \underline{D}}{\partial t}] \cdot d\underline{a}$$

### Maxwell's equations – differential form

$$\text{div}(\underline{D}) = \rho$$

$$\text{div}(\underline{B}) = 0$$

$$\text{curl}(\underline{E}) = -\frac{\partial \underline{B}}{\partial t}$$

$$\text{curl}(\underline{H}) = \underline{J} + \frac{\partial \underline{D}}{\partial t}$$

### Material equations

$$\underline{J} = \sigma \underline{E}$$

$$\underline{D} = \epsilon \underline{E}$$

$$\underline{B} = \mu \underline{H}$$

### Vector calculus (Cartesian co-ordinates)

$$\text{grad}(\phi) = \frac{\partial \phi}{\partial x} \underline{i} + \frac{\partial \phi}{\partial y} \underline{j} + \frac{\partial \phi}{\partial z} \underline{k}$$

$$\text{div}(\underline{F}) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\text{curl}(\underline{F}) = \underline{i} \{ \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \} + \underline{j} \{ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \} + \underline{k} \{ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \}$$

$$\text{curl} \{ \text{curl}(\underline{F}) \} = \text{grad} \{ \text{div}(\underline{F}) \} - \nabla^2 \underline{F}$$

$$\oint \oint_A \underline{F} \cdot d\underline{a} = \int \int \int_V \text{div}(\underline{F}) \, dv$$

$$\oint_L \underline{F} \cdot d\underline{L} = \int \int_A \text{curl}(\underline{F}) \cdot d\underline{a}$$

1. Layered structures such as optical waveguides are often analysed by first finding the solution for each layer as if it were infinite, and then matching the solutions together at the boundaries between layers.

a) Draw a labelled sketch on the  $x$ - $z$  plane of a symmetric three-layer slab waveguide of thickness  $h$  with propagation in the  $z$ -direction. Sketch the transverse variation in refractive index, and explain any condition that must be satisfied if the guide is to operate properly.

[5]

b) Assuming the electric field is polarized in the  $y$ -direction, write down the scalar wave equation that must be satisfied in each of the three layers. Assuming a modal solution, derive the corresponding waveguide equations. What are the different possible solutions to the waveguide equations?

[5]

c) Explain the boundary conditions that must be satisfied, in terms of the transverse electric field.

[4]

d) Show how boundary matching leads to an eigenvalue equation for the modal propagation constant.

[6]

2. a) Briefly describe the fabrication of graded-index silica optical fibre. [4]
- b) The theoretical refractive index profile  $n(r) = n_0 \{ 1 - (r/r_0)^2 \}$  is used to model a parabolic index fibre. Sketch the variation, and explain how it relates to variation in a real fibre. What happens in the model when the index is less than unity? [4]
- c) Assuming a simpler one-dimensional index profile  $n(r) = n_0 \{ 1 - (x/x_0)^2 \}$ , develop an equation for ray trajectories, and show that the trajectories are sinusoids. [8]
- d) Hence explain the operation of a quarter-pitch graded index (GRIN) rod lens. [4]

3. a) Sketch the arrangement used for end-fire coupling into an optical fibre using a free space beam. What factors are important in achieving high efficiency? [4]

b) Explain what is meant by i) an overlap integral, and ii) modal orthogonality. How can modal amplitudes be found in waveguide excitation problems? How can excitation efficiency be calculated? [9]

c) The lowest order mode of parabolic index fibre has the transverse field variation  $E_1 = E_0 \exp(-r^2/a^2)$ . Calculate the overlap between this field and itself, and between this field and a similar variation  $E_2 = E'_0 \exp(-r^2/b^2)$ . Hence or otherwise, calculate the coupling efficiency that can be achieved with a Gaussian beam of mode field radius  $b$ . [7]

4. a) Explain the difference between a Y-junction and a directional coupler. [4]

b) Sketch the layout of a symmetric Mach-Zehnder interferometer (MZI) based on a pair of Y-junctions and a single electro-optic phase shifter. Explain its operation. What is such a device generally used for, and why? [10]

c) A 10 mm long phase shifter can provide a phase shift of  $\pi$  radians at 5 V drive voltage when constructed using a particular material and electrode geometry. Assuming the phase shifter in a MZI is 5 mm long, sketch the time variation of transmission obtained using the three different drive signals shown in Figure 4.1. [6]

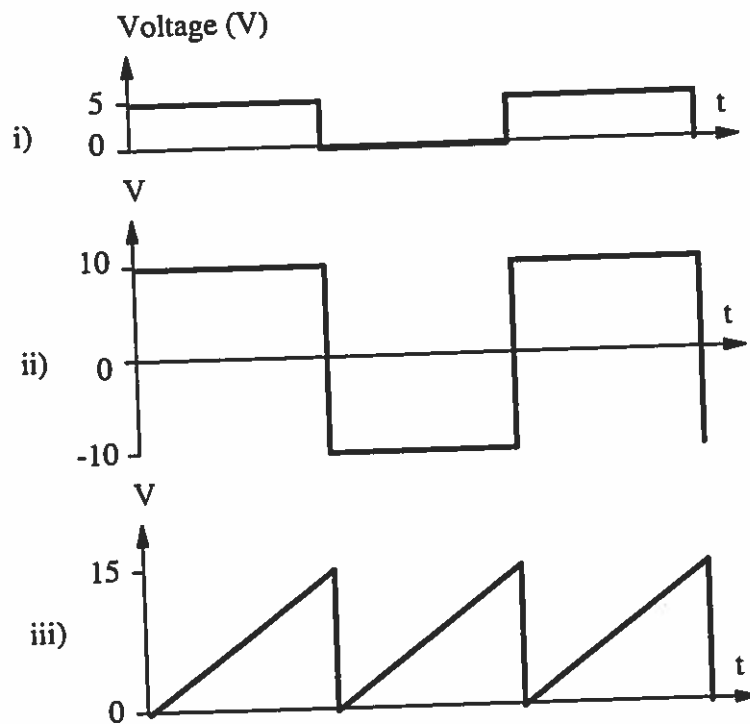


Figure 4.1.

5. a) Describe two different methods of generating light in optoelectronics, and explain which one is preferred. [6]
- b) Explain why direct-gap material is used to construct semiconductor emitters. [6]
- c) Describe the construction and operation of the double heterostructure used in semiconductor lasers. [8]

6. The rate equations for a light emitting diode are:

$$dn/dt = I/eV - n/\tau_e$$

$$d\phi/dt = n/\tau_{\pi} - \phi/\tau_p$$

- a) What quantities are involved in the equations? Explain the significance of the two terms on the right-hand side of each equation. How are the three time-constants  $\tau_e$ ,  $\tau_{\pi}$  and  $\tau_p$  related? [8]

- b) How should the equations be modified to allow modelling of a semiconductor laser? Explain how the resulting equations can be solved in the steady state, above threshold. How do the electron density and the emitted optical power vary with current during lasing? What is the threshold current? [8]

- c) A semiconductor laser has an active channel measuring  $1 \mu\text{m} \times 0.1 \mu\text{m}$ , and a cavity length of  $250 \mu\text{m}$ . Its threshold current is 20 mA. Assuming the electron lifetime is 1 nsec, what is the electron density during lasing? If the wavelength is  $1.55 \mu\text{m}$ , how much power is emitted at a drive current of 30 mA? [4]