1.

a)

Hole concentration $p = p_i = 1.45\ 10^{10}\ cm^{-3}$
Electron concentration $n = n_i = 1.45\ 10^{10}\ cm^{-3}$ [2]

b) There are two possible approaches.

1)

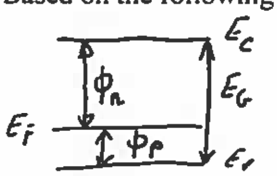
Use the equation for the list on p.2:

$$p = N_v e^{\frac{(E_v - E_F)}{kT}}$$

Rewrite to $E_F - E_v = kT \ln\left(\frac{N_v}{p}\right)$ use $p = N_A = 10^{17}\ cm^{-3}$

$$E_F - E_v = 0.026eV \ln\left(\frac{1.8\,10^{19}\ cm^{-3}}{10^{17}\ cm^{-3}}\right) = 0.135eV$$

Then use the band gap $E_G = 1.12$ eV
Based on the following energy band diagram:



We can derived that $E_G = \phi_n + \phi_p$ and thus $\phi_n = E_c - E_F = E_G - \phi_p = 1.12$ eV – 0.135 eV = 0.985 eV

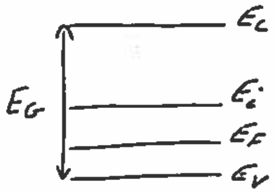
2) Start from the electron concentration:

$$n = N_c e^{\frac{(E_c - E_F)}{kT}} \text{ and } n = \frac{n_i^2}{N_A}$$

Rewrite to $E_c - E_F = kT \ln\left(\frac{N_c N_A}{n_i^2}\right)$

Fill in numbers $E_c - E_F = 0.026eV \ln\left(\frac{3.2\,10^{19}\ 10^{17}}{(1.45\,10^{10})^2}\right)$ = 0.969 eV [5]

c) [5]



d) Formula to know by heart (A-level physics). [5]

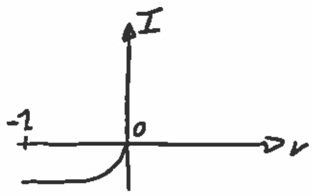
$$R = \frac{\rho \times y}{x \times z} \text{ and } \rho = \frac{1}{\sigma} = \frac{1}{en\mu_n + ep\mu_p} \approx \frac{1}{ep\mu_p}$$

$$R = \frac{1}{ep\mu_p} \times \frac{y}{x \times z} = \frac{100\,10^{-2}\ cm}{1.6\,10^{-19} C\ 10^{17} cm^{-3}\ 200\ cm^2 / Vs\ 2000\,10^{-7} cm\ 0.5\,10^{-1} cm}$$

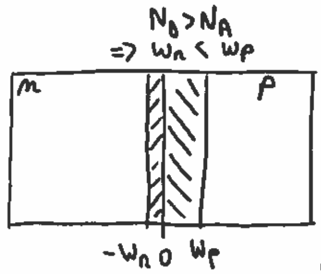
$$R = 31250\Omega$$

e)

i) [4]

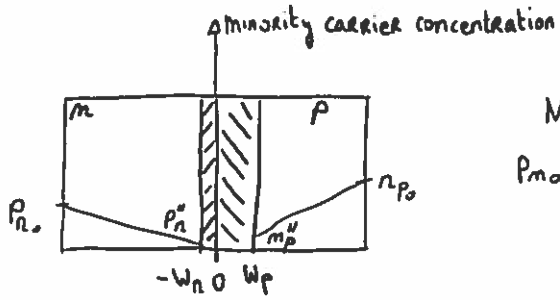


ii) [4]



cross hatched region are the depletion regions.

iii) [5]



$N_D > N_A$

$$p_{n_0} = \frac{n_i^2}{N_D} < n_{p_0} = \frac{n_i^2}{N_A}$$

f) $V_{cE} < V_{cB}$ (forward biased pn diode).
$V_{cB} < V_{cC}$ (reverse biased pn diode) [4]

g) $I_{C1} = I_{C2}$
The current density across the E-B pn diode is given by the formula:

$$J_{tot} = J_n + J_p = \frac{eD_n n_{p_0}}{W_B}\left(e^{\frac{eV}{kT}} - 1\right) + \frac{eD_p p_{n_0}}{X_n}\left(e^{\frac{eV}{kT}} - 1\right)$$

With W_B the base width and X_n the emitter width. Rewriting in function of doping:

$$J_{tot} = J_n + J_p = \frac{eD_n n_i^2}{N_A W_B}\left(e^{\frac{eV}{kT}} - 1\right) + \frac{eD_p n_i^2}{N_D X_n}\left(e^{\frac{eV}{kT}} - 1\right)$$

In an npn BJT the collector current is determined by the minority carrier diffusion current in the base, thus by J_n. Since the doping in the base is not changing between BJT 1 and BJT 2, J_n does not change and thus the collector current does not change. What changes is the base current I_B and thus the current gain β. [6]

2.

a)

i) x = 0 [2]

ii) Under the depletion approximation we assume that the free carrier concentration in the depletion region is zero. Thus only the ionised charge remains.

$\rho(x) = -eN_A$ [2]

iii) [6]

Poisson equation from formulae list:

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon}$$

Solving in the depletion region in the p-section.

$$\frac{dE}{dx} = \frac{-eN_A}{\varepsilon}$$

Integrate once:

$E = \frac{-eN_A}{\varepsilon}x + C_1$ with C_1 an integration constant. C_1 can be found from the boundary condition at the edge of the p-section depletion region. In x = $-w_p$, E(x) = 0.

$0 = \frac{eN_A}{\varepsilon}w_p + C_1$ thus $C_1 = \frac{-eN_A}{\varepsilon}w_p$

Thus $E(x) = \frac{-eN_A}{\varepsilon}\left(x + w_p\right)$. From the formulae list we have:

$w_p = \left[\frac{2\varepsilon V_{bi} N_D}{e(N_A + N_D)N_A}\right]^{1/2}$ since $N_D \gg N_A$ we can simplify this expression to:

$w_p \approx \left[\frac{2\varepsilon V_{bi}}{eN_A}\right]^{1/2}$ thus

$$E(x) = \frac{-eN_A}{\varepsilon}\left(x + \left[\frac{2\varepsilon V_{bi}}{eN_A}\right]^{1/2}\right)$$

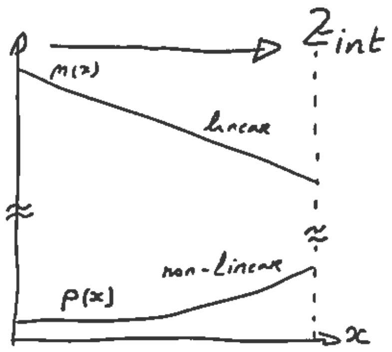
b)

i) $n(x) = 10^{17} - 2.5\times10^{18}x$

$$p(x) = \frac{n_i^2}{N(x)} = \frac{\left(1.45\times10^{10}\right)^2}{10^{17} - 2.5\times10^{18}x} = \frac{2.1\times10^{20}}{10^{17} - 2.5\times10^{18}x} = \frac{2.1\times10^{3}}{1 - 25\,x}$$ [4]

n(0) = 10^{17} cm^{-3}
n(0.035) = 1.25 10^{16} cm^{-3}

p(0) = 2100 cm^{-3}
p(0.02) = 4200 cm^{-3}
p(0.035) = 16800 cm^{-3}



ii) Start from the drift-diffusion equation for electrons in the formulae sheet:

$$J_n(x) = e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx}$$

Since the voltage is zero, the total current density has to be zero.

$$0 = e\mu_n n(x)E(x) + eD_n \frac{dn(x)}{dx}$$

$$E(x) = -\frac{D_n}{\mu_n n(x)} \frac{dn(x)}{dx}$$

Using Einstein's equation and differentiating $n(x) = 10^{17} - 2.5\times10^{18}x$:

$$E(x) = \frac{2.5\times10^{18}\times kT}{e\left[10^{17} - 2.5\times10^{18}x\right]} = \frac{2.5\times10^{18}\times0.026}{\left[10^{17} - 2.5\times10^{18}x\right]}$$

[4]

$$E(x) = \frac{6.5\times10^{16}}{\left[10^{17} - 2.5\times10^{18}x\right]} = \frac{6.5}{\left[10 - 250\,x\right]}$$

iii) The internal electric field points to +x to cause drift opposite to diffusion. $\xrightarrow{\mathcal{E}_{int}}$

[2]

c) Charge in p-region: $\rho = e\left(p - n - N_A\right)$ [2]

Charge neutrality is required in the p-region. $0 = \left(p - n - N_A\right)$ [1]

Law of mass action: $n\times p = n_i^2$ [2] [10]

$$\begin{cases} n\times p = n_i^2 \\ p - n - N_A = 0 \end{cases}$$

$$p - \frac{n_i^2}{p} - N_A = 0 \quad (p \neq 0)$$

$$p^2 - N_A p - n_i^2 = 0$$ [5]

$$p = \frac{N_A \pm \sqrt{N_A^2 - 4n_i^2}}{2}$$

$$p = \frac{N_A + \sqrt{N_A^2 - 4n_i^2}}{2}$$

For $N_A >> n_i$ $\longrightarrow$ $N_A^2 - 4n_i^2 \approx N_A^2$

$$p \approx \frac{N_A + N_A}{2} = N_A$$

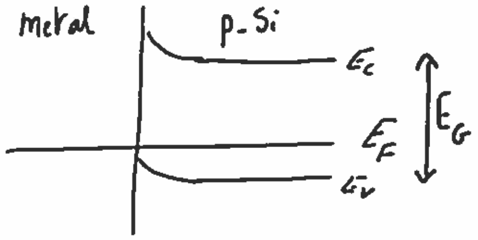
3.

a)

i) p-channel or pMOS [2]

ii) source & drain p-type (heavily doped)
substrate region n-type [2]

iii) Should be an Ohmic contact on a heavily doped p-type region. Thus bend bending upwards towards metal. [6]



b)

i) The oxide capacitance can be extracted from the maximum measured capacitance:

$$C_{max} = C_{ox} \times W_G \times L_G$$

$$C_{ox} = \frac{C_{max}}{W_G \times L_G} = \frac{0.885\times10^{-12} F}{100\times10^{-4}\times5\times10^{-4}} = 1.77\times10^{-7} F/cm^2$$

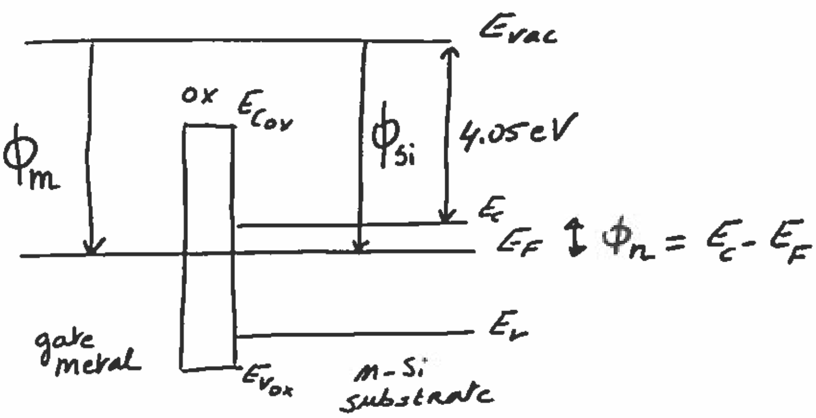
The oxide thickness comes from:

$$C_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{t_{ox}} \rightarrow t_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{C_{ox}}$$

$$t_{ox} = \frac{8.85\times10^{-14} F/cm \times 4}{1.77\times10^{-7} F/cm^2} = 2\times10^{-6} cm = 20\ nm$$

Extract the thickness of the oxide from the CV measurements. [2]

ii) A sketch of the flat band situation. [4]



From E_c-E_F we can find the doping.

E_c-E_F = ϕ_m – 4.05 eV= 4.259 eV – 4.05 eV=0.209 eV

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$N_D = 3.2\times10^{19} cm^{-3} \exp\left(-\frac{0.209}{0.026}\right) = 1.03\times10^{16} cm^{-3}$$

iii) C_{min} is the series connection of the oxide related capacitance and the depletion capacitance. Extract the maximum depletion width from the C-V measurements. [4]

$$\frac{1}{C_{\min}} = \frac{1}{C_{\max}} + \frac{1}{C_{depl_{\max}}}$$

$$\frac{1}{C_{depl_{\max}}} = \frac{1}{C_{\min}} - \frac{1}{C_{\max}}$$

Depletion capacitance per gate area:

$$C'_{depl_{\max}} = \frac{C_{depl_{\max}}}{L_G \times W_G}$$

The relationship between depletion width and depletion capacitance:

$$C'_{depl_{\max}} = eN_D W_{depl\,\max}$$

$$\frac{C_{depl_{\max}}}{L_G \times W_G} = eN_D W_{depl\,\max}$$

$$W_{depl\,\max} = \frac{C_{depl_{\max}}}{eN_D \times L_G \times W_G}$$

c)

i) $V_{th} = -0.7$ V. [2]

ii) Majority carriers in the channel are holes, thus mobility is approximately: μ_p = 410 cm^2/Vs. [2]

iii) Output characteristic is I_{DS} versus V_{DS}. [1] [6]
A p-channel MOSFET has a negative bias on the drain and "negative" current if S is at x=0 and D is at +x. [1]
Pinch-off is found for $V_{DS} = V_{GS} - V_{th} = -1 - (-0.7) = -0.3$ V[1]
Thus the IV characteristic is linear up to ~ -0.3 V and then saturates.
The saturation current is derived from the expression for the drain current found in the formulae list:

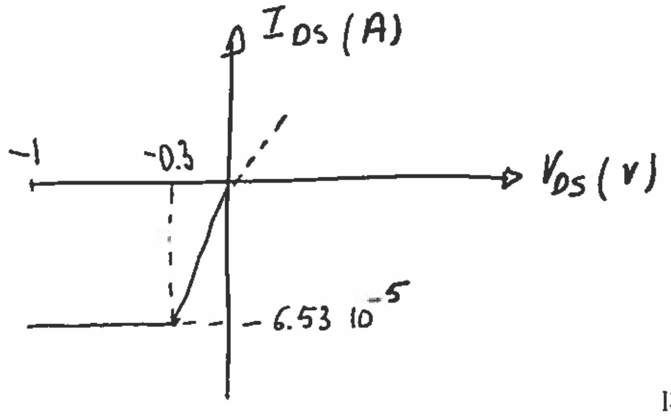
$$I_{DS} = \frac{\mu C_{ox} W}{L}\left(\left(V_{GS} - V_{th}\right)V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Re-written for saturation

$$I_{DS}^{sat} = \frac{\mu C_{ox} W}{2L}\left(\left(V_{GS} - V_{th}\right)^2\right)$$

[1]

$$I_{DS}^{sat} = \frac{410 \times 1.77 \times 10^{-7} \times 100}{2 \times 5}\left(\left(-0.3\right)^2\right) = 6.53 \times 10^{-5} A$$



[2]