E4.23
CS2.1

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

MSc and EEE/ISE PART IV: MEng and ACGI

STABILITY AND CONTROL OF NON-LINEAR SYSTEMS

Tuesday, 11 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D. Angeli
Second Marker(s) : E.C. Kerrigan

1. Consider the following autonomous system with state variable $x = [x_1, x_2]' \in \mathbb{R}^2$:

$$\dot{x}(t) = \begin{cases} A_1 x(t) & \text{if } x_1(t) \geq 0 \\ A_2 x(t) & \text{if } x_1(t) < 0 \end{cases} \tag{1.1}$$

where A_1 and A_2 are defined according to:

$$A_1 = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 1 \\ -5 & -2 \end{bmatrix}.$$

a) Is the system linear or nonlinear ? Does it fulfill the conditions for existence and/or uniqueness of solutions ? (justify your answers). [3]

b) Compute the equilibria of the system. Is the system linearizable around equilibria ? [2]

c) Next we proceed to a detailed study of the phase portrait. Compute the eigenvalues and eigenvectors of A_1 and sketch the phase portrait of the linear system $\dot{x} = A_1 x$ (Hint: exploit the information obtained from the eigenvectors). [4]

d) Sketch the phase portrait of the linear system $\dot{x} = A_2 x$. [4]

e) Merge the two phase-portraits previously sketched in order to obtain the phase-portrait of system (1.1). [3]

f) Exploiting the previous graphical analysis, infer whether the system's equilibria are asymptotically stable or not (justify your response). [4]

2. Consider the following two-dimensional system:

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 - x_1^3 \\ \dot{x}_2 &= x_2 - x_1 - x_2^3 \end{aligned}$$

a) Find the equilibria of the system, linearize the system around each equilibrium and discuss the local phase-portrait around each equilibrium; [5]

b) Show that the set $\{x : x_1^2 + x_2^2 \leq M\}$ is forward invariant for all M sufficiently large; [5]

c) Show that the set $\{x : x_1^2 + x_2^2 \geq \varepsilon\}$ is forward invariant for all ε sufficiently small; [5]

d) Sketch a phase-portrait of the system, compatible with the dynamical properties highlighted in a),b) and c). [5]

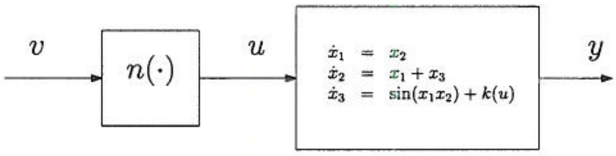

Figure 3.1 System with static input nonlinearity $n(\cdot)$

3. Consider the following nonlinear system with state x taking values in $\mathbb{R}^3$ and input $u \in \mathbb{R}$:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 + x_3 \\ \dot{x}_3 &= \sin(x_1 x_2) + k(u) \end{aligned} \tag{3.1}$$

where $k : \mathbb{R} \to \mathbb{R}$ is the following function $k(u) = (1 + |u|)u$.

a) A nonlinear control system $\dot{x} = f(x,u)$ is said to be affine if there exist $\tilde{f}$ and $\tilde{g}$ such that $f(x,u) = \tilde{f}(x) + \tilde{g}(x)u$. Show that by means of a static input nonlinearity it is possible to make (3.1) into an affine system from its new input variable v (see Fig. 3.1). [5]

b) Show that with $y = x_3$ the affine system obtained in question a) is globally Input-State feedback linearizable. [5]

c) Derive a controller that achieves global asymptotic stability at 0 for system (3.1) (Hint: exploit feedback linearizability) [5]

d) Could a similar method be employed if $k(u) = (|u| - 1)u$, (justify your answer). If not, can you think of an alternative solution to the problem of global feedback stabilization at 0 ? [5]

4. Consider the scalar nonlinear system of equations:

$$\dot{x} = -x^3 + u^3 \tag{4.1}$$

with input $u \in \mathbb{R}$.

a) Show that (4.1) is an Input-to-State Stable system. [4]

b) Find an estimate for the asymptotic gain of (4.1), namely a function γ of class $\mathscr{K}_\infty$ so that:

$$\limsup_{t \to +\infty} |x(t, x_0, u)| \leq \gamma(\|u\|_\infty).$$

[4]

c) Consider next the linear system:

$$\ddot{z} + \dot{z} + z = \alpha v \tag{4.2}$$

with output z and input v ($\alpha \in \mathbb{R}$ being an uncertain parameter). Find a state-space realization of the system. [2]

d) Show that the system is asymptotically stable by finding a suitable quadratic Lyapunov function. [3]

e) Use the previous Lyapunov function to prove ISS of the linear system and to estimate its asymptotic gain. [3]

f) Consider now the feedback interconnection of (4.1) and (4.2):

$$\begin{aligned} \dot{x} &= -x^3 + z^3 \\ \ddot{z} + \dot{z} + z &= \alpha x \end{aligned}$$

Is there a range of values of α for which global asymptotic stability at the origin of the closed-loop system holds ? (Hint: apply the "small gain theorem") [4]

5. Consider the following two-dimensional SISO linear system:

$$
\begin{aligned}
\dot{x}_1 &= x_2 + u \\
\dot{x}_2 &= -x_1 - x_2 \\
\\
y &= x_1
\end{aligned}
\tag{5.1}
$$

with state $x = [x_1, x_2]'$, input $u \in \mathbb{R}$ and output $y \in \mathbb{R}$.

a) Show that the system is passive from u to y. [5]

b) Connect the system in negative feedback with a PI controller with transfer function $K(s) = k_1 + \frac{k_2}{s}$. Find a suitable state-space realization of the PI controller and write down the state-space equations of the overall closed-loop system. [3]

c) Show that for all values of $k_1 > 0$ and $k_2 > 0$ the closed-loop system is globally asymptotically stable. [4]

d) Assume next that a saturation nonlinearity is acting on the input of (5.1); in particular that the equation for x_1 needs to be modified as follows:

$$\dot{x}_1 = x_2 + \operatorname{atan}(u)$$

Sketch the graphic diagram of the closed-loop system by using Input-Output blocks. [2]

e) Prove that the resulting closed-loop system is globally asymptotically stable at the origin, for all values of $k_1 > 0$ and $k_2 > 0$. (Hint: show that a PI controller in cascade with a saturation static nonlinearity, i.e. $\operatorname{atan}(\cdot)$, is a passive system). [6]

6. Consider the following two-dimensional nonlinear system:

$$\begin{aligned}\dot{x}_1 &= \cos(x_1)x_1 + x_2 \\ \dot{x}_2 &= -x_1x_2 + u\end{aligned}$$

a) Design a static state-feedback $u = k(x)$ which achieves global asymptotic stability of the closed-loop system at $x = (0,0)'$. Use the design technique that you find more suitable. [7]

b) Consider next the two-dimensional nonlinear system:

$$\begin{aligned}\dot{x}_1 &= \cos(x_2)x_1 + x_2 \\ \dot{x}_2 &= -x_1x_2 + u\end{aligned}$$

Notice that x_2 appears as the argument of the $\cos(\cdot)$ function, rather than x_1 as in part a). Design a static state-feedback that achieves global asymptotic stability of the origin $x = (0,0)'$. (Hint: use backstepping; in particular treat $\cos(x_2)$ as some kind of uncertain variable with values in $[-1,1]$, and use domination in order to design the feedback stabilizer for the x_1-subsystem). [7]

c) Let the output variable be defined as $y = x_1$. With this particular choice, is global asymptotic stabilization achievable by means of feedback linearization ? (justify your answer) [6]

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SOLUTIONS: STABILITY AND CONTROL OF NONLINEAR SYSTEMS MASTER IN CONTROL 2010

1. Exercise

a) The system is not linear; indeed for strictly positive x_1 we have, $f(x_1,0)+f(-x_1,0) = [-1,0]'x_1 - [2,-5]'x_1 = [-3,5]'x_1 \neq 0$ which contradicts linearity. The function $f(x_1,x_2)$ is (globally) Lipschitz continuous, so existence and unicity of solutions are guaranteed. To verify continuity just notice that:

$$\lim_{x_1\to 0^+} f(x_1,x_2) = \lim_{x_1\to 0^-} f(x_1,x_2) = \begin{bmatrix} x_2 \\ -2x_2 \end{bmatrix}.$$

b) Notice that $x \neq 0$ implies $A_1x \neq 0$ and $A_2x \neq 0$. Hence the only equilibrium is the origin. The system is not linearizable around the origin since $f(x_1,x_2)$ is not $\mathscr{C}^1$ (notice that $\partial f/\partial x$ is discontinuous in $x_1 = 0$).

c) Eigenvalues of A_1 are $\lambda_1 = -1$ and $\lambda_2 = -2$. The corresponding eigenvectors are: $v_1 = [1,0]'$ and $v_2 = [1,-1]'$. The phase-portrait is therefore a stable node, as in Fig. 1.1.

d) Eigenvalues of A_2 are purely imaginary. The origin is therefore a center. Solutions evolve around ellipses in a clockwise direction. See Fig. 1.2.

e) Merging the two plots yields a phase portrait of the type shown in Fig. 1.3.

f) It is clear that every solution $\varphi(t,x_0)$ for $x_0 \neq 0$, eventually enters the region $\{x : x_1 \geq 0\}$ and stays there everafter. Attractivity of solutions is therefore proved, since A_1 is an asymptotically stable matrix. Also Lyapunov stability follows from the same reason. Indeed, by Lyapunov stability of A_1 and A_2, we have:

$$\forall \varepsilon > 0 \exists \delta_\varepsilon > 0 : |x_0| \leq \delta_\varepsilon \Rightarrow |e^{A_1 t}x_0| \leq \varepsilon \quad \forall t \geq 0$$
$$\forall \varepsilon > 0 \exists \hat{\delta}_\varepsilon > 0 : |x_0| \leq \hat{\delta}_\varepsilon \Rightarrow |e^{A_2 t}x_0| \leq \varepsilon \quad \forall t \geq 0$$

Since only 2 switches can occur at most between system $\dot{x} = A_1x$ and $\dot{x} = A_2x$ (in particular starting in region $x_1 \geq 0$, then commuting to $x_1 \leq 0$ and back to $x_1 \geq 0$, we have that:

$$\forall \varepsilon > 0 \exists \Delta \doteq \delta_{\hat{\delta}_{\delta_\varepsilon}} > 0 : |x_0| \leq \Delta \Rightarrow |\varphi(t,x_0)| \leq \varepsilon \quad \forall t \geq 0$$

2. Exercise

a) Let us compute equilibrium points graphically. By plotting the nullclines of the system, equilibria are found at intersection points. This clearly shows that there exists a unique equilibrium, in $x = [0,0]$, (see Fig. 2.1). The linearized system, at the equilibrium reads:

$$\dot{\delta x} = \begin{bmatrix} 1-3x_1^2 & 1 \\ -1 & 1-3x_2^2 \end{bmatrix}_{x=0} \delta x = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \delta x := A\delta x$$

Eigenvalues of matrix A are $1 \pm j$, therefore the equilibrium is an *unstable focus*.

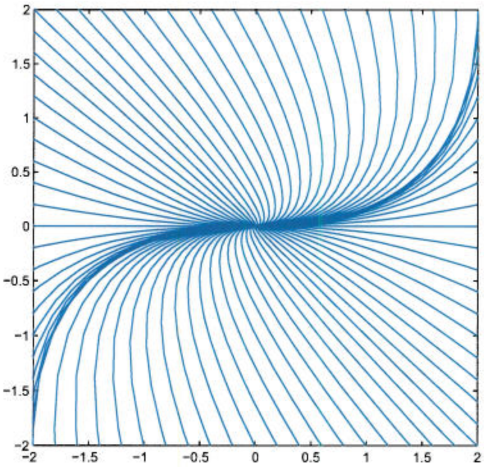

Figure 1.1 Phase portrait of $\dot{x} = A_1 x$

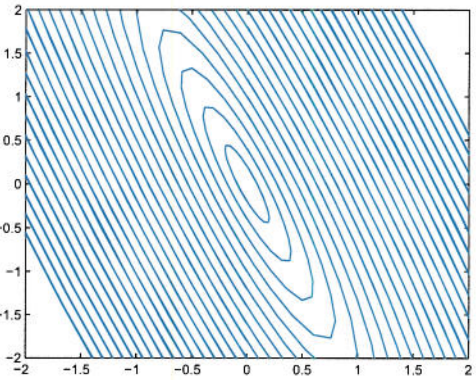

Figure 1.2 Phase portrait of $\dot{x} = A_2 x$

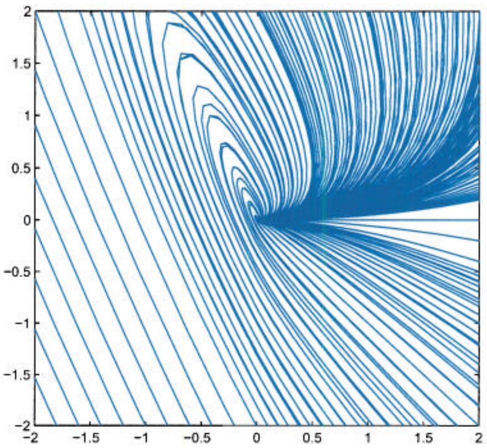

Figure 1.3 Phase portrait of $\dot{x} = A_2 x$

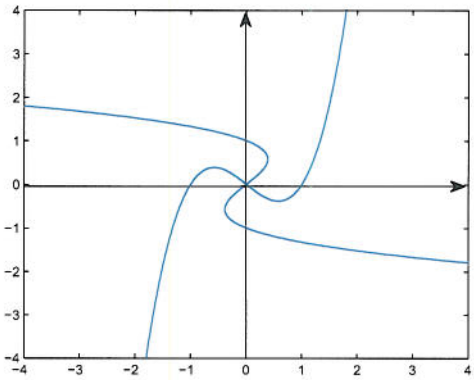

Figure 2.1 Graphic computation of equilibrium: nullclines

b) Define $V(x) = x_1^2 + x_2^2$. Computing derivatives of V along solutions of the system yields:

$$\frac{\partial V}{\partial x}(x) \cdot f(x) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2 = 2x_1^2 - 2x_1^4 + 2x_2^2 - 2x_2^4.$$

Obviously:

$$x_1^2 + x_2^2 = M \Rightarrow x_1^2 \geq M/2 \text{ or } x_2^2 \geq M/2.$$

Hence, for $x_1^2 + x_2^2 = M$ we have the following inequalities fulfilled:

$$\dot{V} = 2x_1^2 - 2x_1^4 + 2x_2^2 - 2x_2^4 = 2M - 2x_1^4 - 2x_2^4 \leq 2M - 2M^2/4$$

Hence, for $M \geq 4$ we obtain $\dot{V} \leq 0$, which shows forward invariance of $\{x : V(x) \geq M\}$ for all $M \geq 4$.

c) Similarly,

$$x_1^2 + x_2^2 = \varepsilon \Rightarrow x_1^2 \leq \varepsilon \text{ and } x_2^2 \leq \varepsilon$$

Hence, computing derivatives of V along solutions yields:

$$\dot{V} = 2(x_1^2 + x_2^2) - 2(x_1^4 + x_2^4) = 2\varepsilon - 2(x_1^4 + x_2^4) \geq 2\varepsilon - 4\varepsilon^2$$

Hence $0 \leq \varepsilon \leq 1/2$ implies $\dot{V} \geq 0$, which proves forward invariance of $\{x : x_1^2 + x_2^2 \geq \varepsilon\}$ for all $\varepsilon \leq 1/2$.

d) By the results in b) and c) we can identify a forward invariant annular region which does not contain equilibria. Hence, by the Poincarè-Bendixson Theorem there exists at least one periodic solution. Its direction of rotation around the equilibrium can be inferred by looking at nullclines. The resulting phase-portrait is shown in Fig. 2.2.

3. Exercise

a) Notice that $k(\cdot)$ is an invertible function. In particular, it is enough to let $u = k^{-1}(v)$ in order to obtain the system:

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & x_1 + x_3 \\ \dot{x}_3 & = & \sin(x_1 x_2) + v \end{array} \tag{3.1}$$

which is affine in the new input variable v. Explicit computation of $k^{-1}(v)$ yields:

$$k^{-1}(v) = \begin{cases} \frac{-1+\sqrt{1+4v}}{2} & \text{if } v \geq 0 \\ \frac{1-\sqrt{1-4v}}{2} & \text{if } v \leq 0 \end{cases}$$

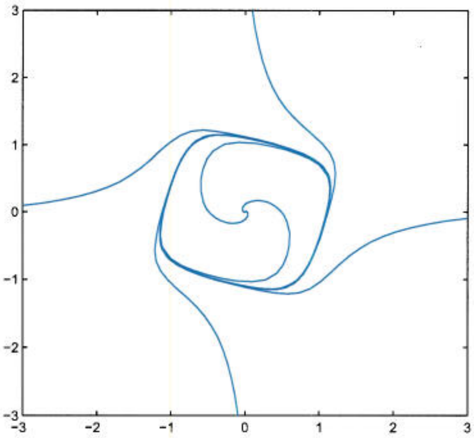

Figure 2.2 Phase portrait of Exercise 2

b) Letting $y = x_3$ be the system's output yields:

$$\dot{y} = \sin(x_1 x_2) + v.$$

Hence, picking $v = -\sin(x_1 x_2) + \tilde{v}$ yields $\dot{y} = \tilde{v}$ which is linear. We may complete the state vector with x_1 and x_2, so that overall the system reads:

$$\begin{aligned} \dot{y} &= \tilde{v} \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 + y \end{aligned}$$

c) Letting $y = x_1$ be the system's output yields:

$$\begin{aligned} \dot{y} &= x_2 \\ \ddot{y} &= x_1 + x_3 \\ y^{(3)} &= x_2 + \sin(x_1 x_2) + v. \end{aligned}$$

Hence, the relative degree of the system is 3 and since this is also equal to the system's dimension Input-to-State feedback linearization is possible simply by letting the new state variable be $X = [y, \dot{y}, \ddot{y}]$ and $v = -x_2 - \sin(x_1 x_2) + \tilde{v}$. Under such coordinate and feedback transformations, the systems equations read:

$$\dot{X} = \begin{bmatrix} X_2 \\ X_3 \\ \tilde{v} \end{bmatrix}$$

A global feedback stabilizer can be obtained simply by letting:

$$\tilde{v} = -X_1 - 3X_2 - 3X_3$$

In original state and input coordinates this reads:

$$u = k^{-1}(-x_2 - \sin(x_1 x_2) - x_1 - 3x_2 - 3(x_1 + x_3))$$

d) Notice that the newly defined function $k(u)$ is not invertible. Hence, at least apparently the previous approach cannot be straightforwardly implemented.

However, the function k admits a right-inverse, that is a function $k^\dagger(\cdot)$ such that $k \circ k^\dagger(v) = v$ for all $v \in \mathbb{R}$. In particular, we may define $k^\dagger$ as follows:

$$k^\dagger(v) = \operatorname{sign}(v)\frac{1+\sqrt{1+4v}}{2}.$$

Notice that this is a discontinuous function; however, the closed-loop system

$$u = k^\dagger(-x_2 - \sin(x_1 x_2) - x_1 - 3x_2 - 3(x_1 + x_3))$$

is smooth, so that existence and unicity of solutions is still guaranteed.

4. Exercise

a) Consider the candidate Lyapunov function $V(x) = x^2/2$. Taking derivatives of V along solutions of $\dot{x} = -x^3 + u^3$ we obtain:

$$\dot{V}(x,u) = x(-x^3 + u^3).$$

Henceforth, for $\varepsilon > 0$:

$$|x| \geq (1+\varepsilon)|u| \Rightarrow \dot{V}(x,u) = -x^4 + x^3 u \leq -x^4 + |x|^3|u| \leq -x^4 + \frac{x^4}{1+\varepsilon} = -\frac{\varepsilon}{1+\varepsilon}x^4.$$

This proves Input-to-State stability of the system.

b) Moreover, the asymptotic gain γ can be computed as $\alpha_1^{-1} \circ \alpha_2 \circ (1+\varepsilon)r$, where $\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|)$. Notice that in this case $\alpha_1(r) = \alpha_2(r) = r^2/2$. Therefore, $\gamma(r) = (1+\varepsilon)r$ for any positive value of ε.

c) A state-space realization can be computed by choosing as state variable $\tilde{x} = [z, \dot{z}]'$. This results in the following equations:

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} \tilde{x}_2 \\ -\tilde{x}_1 - \tilde{x}_2 + \alpha v \end{bmatrix}$$

d) Consider the following candidate Lyapunov function: $V(\tilde{x}_1, \tilde{x}_2) = (\tilde{x}_1^2 + \tilde{x}_1\tilde{x}_2 + \tilde{x}_2^2)/2$. It holds:

$$\frac{1}{4}\tilde{x}'\tilde{x} \leq V(\tilde{x}) \leq \frac{3}{4}\tilde{x}'\tilde{x}.$$

Moreover, taking derivatives of $\tilde{V}$ along solutions we have:

$$\dot{\tilde{V}} = -V(\tilde{x}) + \alpha(x_1/2 + x_2)v.$$

Therefore if $3\alpha < K/2$ for some positive $K > 0$, we obtain the estimate:

$$|\tilde{x}| \geq K|v| \Rightarrow \dot{\tilde{V}} \leq -V(\tilde{x}) + \frac{3}{2K}\alpha|\tilde{x}|^2 < 0$$

This shows (global) asymptotic stability and Input-to-State stability of the considered linear system.

e) We can estimate the asymptotic gain of this system by computing $\frac{r^2}{4}^{-1} \circ \frac{3r^2}{4} \circ Kr = \sqrt{3}Kr$. As a function of α we can upper-bound the gain as $6\sqrt{3}\alpha$.

f) As the gain of the first subsystem can be taken to be arbitrary close to the identity mapping, by applying the Small Gain Theorem we may prove global asymptotic stability for all $\alpha > 0$ such that $6\sqrt{3}\alpha < 1$.

5. Exercise

a) Consider the Lyapunov function $V(x) = \frac{x_1^2 + x_2^2}{2}$. Taking derivatives along solutions of the linear system yields:

$$\frac{\partial V}{\partial x}(x) f(x,u) = x_1\dot{x}_1 + x_2\dot{x}_2 = -x_2^2 + x_1 u = yu - x_2^2 \leq yu.$$

This proves passivity of the system from u to y.

b) A PI controller with input v and output w can be realized as follows:

$$\dot{z} = v \qquad w = k_1 v + k_2 z.$$

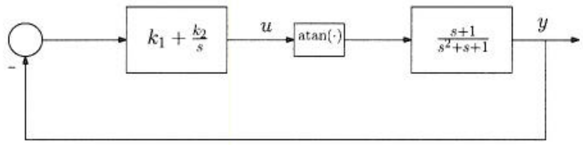

Figure 5.1 Block diagram of closed-loop system

The negative feedback interconnection $u = -w$, $v = y$ has equations:

$$\begin{aligned} \dot{x}_1 &= x_2 - k_1 x_1 - k_2 z \\ \dot{x}_2 &= -x_1 - x_2 \\ \dot{z} &= x_1 \end{aligned}$$

c) The previous equations define a linear system whose associated matrix A has a characteristic polynomial:

$$\chi_A(s) = s^3 + (1+k_1)s^2 + (1+k_1+k_2)s + k_2.$$

The polynomial $\chi_A(s)$ has all roots with negative real-part, regardless of $k_1 > 0$ and $k_2 > 0$ as can be verified by the Routh-Hurwitz criterion.

d) The diagram of the closed-loop interconnection is shown if Fig. 5.1.

e) Consider the PI controller in cascade with a saturation nonlinearity:

$$\dot{z} = v \qquad w = \operatorname{atan}(k_1 v + k_2 z).$$

We prove that this is a passive system from input v to output w. In particular:

$$\begin{aligned} wv &= \operatorname{atan}(k_1 v + k_2 z)v \\ &\geq \operatorname{atan}(k_2 z)v = \operatorname{atan}(k_2 z)\dot{z} = \frac{d}{dt}\int_0^z \operatorname{atan}(k_2 r)dr \end{aligned}$$

Hence choosing $W(z) = \int_0^z \operatorname{atan}(k_2 r)dr$ yields $\dot{W} \leq wv$ as requested. Notice that W is positive definite as atan is a continuous odd function. As predicated by the Passivity Theorem we take $V(x_1, x_2) + W(z)$ as a candidate Lyapunov function. Taking derivatives along solutions yields:

$$\dot{V} + \dot{W} \leq -x_2^2 + yu + wv = -x_2^2.$$

Notice that $x_2(t) \equiv 0$ implies $\dot{x}_2(t) \equiv 0$, and hence $x_1(t) \equiv 0$. As a consequence, also $\dot{x}_1(t) \equiv 0$ which gives $z(t) \equiv 0$. For this reason, the largest invariant set contained in $\{x : x_2 = 0\}$ is the origin. Hence, by Lasalle's invariance principle the system is globally asymptotically stable.

6. Exercise

a) We may design a feedback stabilizer by feedback linearization. To this end we define $y = x_1$. This yields:

$$\begin{aligned} \dot{y} &= \cos(x_1)x_1 + x_2 \\ \ddot{y} &= (\cos(x_1) - \sin(x_1)x_1) \cdot (\cos(x_1)x_1 + x_2) - x_1 x_2 + u \end{aligned}$$

Notice that the relative degree is 2. Hence, by letting:

$$u = -(\cos(x_1) - \sin(x_1)x_1) \cdot (\cos(x_1)x_1 + x_2) + x_1 x_2 - y - \dot{y}$$

we obtain the closed-loop system:

$$\ddot{y} = -y - \dot{y}$$

which is globally asymptotically stable at $y = 0, \dot{y} = 0$. In x coordinates, we have global asymptotic stability at $x = [0, 0]$.

b) We may stabilize the x_1 subsystem using x_2 as a virtual control variable. In particular, letting $x_{2v} = -2x_1$. Indeed, defining the error variable $\xi = x_2 - x_{2v} = x_2 + 2x_1$, yields the following expressions:

$$\dot{\xi} = -x_1 x_2 + u + 2(\cos(x_2)x_1 + x_2) = -x_1 x_2 + u + 2((\cos(x_2) - 2)x_1 + \xi)$$

Taking the Lyapunov function $V(x_1, \xi) = \frac{1}{2}x_1^2 + \frac{1}{2}\xi^2$ then yields:

$$\begin{aligned} \dot{V} &= x_1[(\cos(x_2) - 2)x_1 + \xi] + \xi[-x_1 x_2 + u + 2((\cos(x_2) - 2)x_1 + \xi)] \\ &\leq -x_1^2 + \xi[x_1 - x_1 x_2 + u + 2((\cos(x_2) - 2)x_1 + \xi)] \end{aligned}$$

Hence, global asymptotic feedback stabilization can be achieved simply by letting:

$$x_1 - x_1 x_2 + u + 2((\cos(x_2) - 2)x_1 + \xi) = -\xi$$

c) Let us attempt to design a global stabilizer by means of feedback linearization; set the output variable $y = x_1$. Thenm we compute the relative degree as follows:

$$\begin{aligned} &\dot{y} = \cos(x_2)x_1 + x_2 \\ &\ddot{y} = -x_1 x_2 + u + \cos(x_2)[\cos(x_2)x_1 + x_2] - \sin(x_2)[-x_1 x_2 + u]x_1 \end{aligned}$$

Notice that the coefficient of u in $\ddot{y}$ is $[1 - \sin(x_2)x_1]$. In particular $[1 - \sin(x_2)x_1] > 0$ in a neighborhood of $x = [0, 0]$. This implies that relative degree is 2 (locally). Hence local asymptotic stabilization is possible by means of feedback linearization. Global relative degree, however, is undefined since $\{x : [1 - \sin(x_2)x_1] = 0\}$ is a non-empty set. Hence, global asymptotic stabilization is not achievable by means of feedback linearization from the output $y = x_1$.