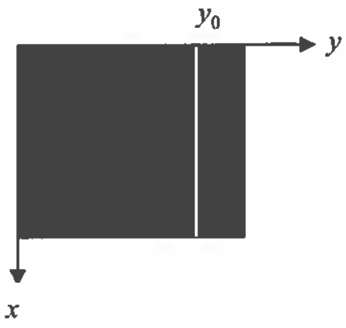
1. a) (i) Plot the image intensity.

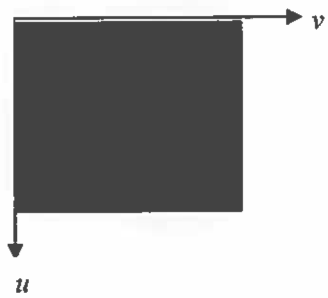


(ii) $F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j(\frac{2\pi}{M}ux+\frac{2\pi}{M}vy)}=\frac{1}{MN}\sum_{x=0}^{M-1}f(x,y_0)e^{-j(\frac{2\pi}{M}ux+\frac{2\pi}{M}vy_0)}$

$=\frac{1}{MN}\sum_{x=0}^{M-1}ce^{-j(\frac{2\pi}{M}ux+\frac{2\pi}{M}vy_0)}=\frac{1}{MN}ce^{-j\frac{2\pi}{M}vy_0}\sum_{x=0}^{M-1}e^{-j\frac{2\pi}{M}ux}$

$\sum_{x=0}^{M-1}e^{-j\frac{2\pi}{M}ux}=\frac{1-e^{-j\frac{2\pi}{M}uM}}{1-e^{-j\frac{2\pi}{M}u}}=\begin{cases}0 & u\neq 0\\ M & u=0\end{cases}$

$|F(u,v)|=\begin{cases}0 & u\neq 0\\ \frac{c}{N} & u=0\end{cases}$



$|F(u,v)|=\begin{cases}cN, & v=0\\ 0, & \text{otherwise}\end{cases}$

(iii) Compare the plots found in (i) and (ii) above.
As seen a straight line in space implies a straight line perpendicular to the original one in frequency.

b) Figure (c) is the right answer since it contains edges which are perpendicular to the edges of the original image. As we know, each image in space produces a perpendicular image in the amplitude of the DFT.

c) (i) The first image $f_1(x,y)$ has a solid horizontal edge. Its mean is $\frac{r_1+s_1}{2}$. The zero-mean version of it is $f_1(x,y)=\begin{cases}\frac{r_1-s_1}{2} & 1\le x\le M, 1\le y\le \frac{M}{2}\\ \frac{s_1-r_1}{2} & 1\le x\le M, \frac{M}{2}<y\le M\end{cases}$. The second image $f_2(x,y)$ has a solid vertical edge. Its mean is $\frac{r_2+s_2}{2}$. The zero-mean version of it is

$$f_2(x,y) = \begin{cases} \dfrac{r_2 - s_2}{2} & 1 \le x \le M, 1 \le y \le \dfrac{M}{2} \\ \dfrac{s_2 - r_2}{2} & 1 \le x \le M, \dfrac{M}{2} < y \le M \end{cases}$$

. The variance of $f_1(x,y)$ is $\frac{r_1^2 + s_1^2}{2}$. The variance of $f_2(x,y)$ is $\frac{r_2^2 + s_2^2}{2}$. The covariance between the two images is zero (this is the mean of the product of the two images). This is because $f_1(x,y)$ is of the form $\begin{bmatrix} a \\ \cdots \\ -a \end{bmatrix}$ and $f_2(x,y)$ is of the form $[b \;\vdots\; -b]$ therefore $f_1(x,y)f_2(x,y) = \begin{bmatrix} ab & \vdots & -ab \\ \cdots & \vdots & \cdots \\ -ab & \vdots & ab \end{bmatrix}$. So the mean of $f_1(x,y)f_2(x,y)$ is zero. In that case the covariance matrix of the population is $C = \begin{bmatrix} \frac{r_1^2 + s_1^2}{2} & 0 \\ 0 & \frac{r_2^2 + s_2^2}{2} \end{bmatrix}$. The eigenvalues of the covariance matrix are $\frac{r_1^2 + s_1^2}{2}$ and $\frac{r_2^2 + s_2^2}{2}$. The images $g_1(x,y)$ and $g_2(x,y)$ are simply the zero mean versions of the original images.

(ii) There is no point of using the KL transform since it is obvious visually that the images are uncorrelated.

Question 2 – Answer

(i) The intensities of the two inner squares are very similar and therefore the inner pattern is <u>not</u> visible. It basically looks like a single square instead of 

$$p(r_3) = \frac{64\times 64/2}{256\times 256} = \frac{1}{32}$$

$$p(r_2) = \frac{1}{32}$$

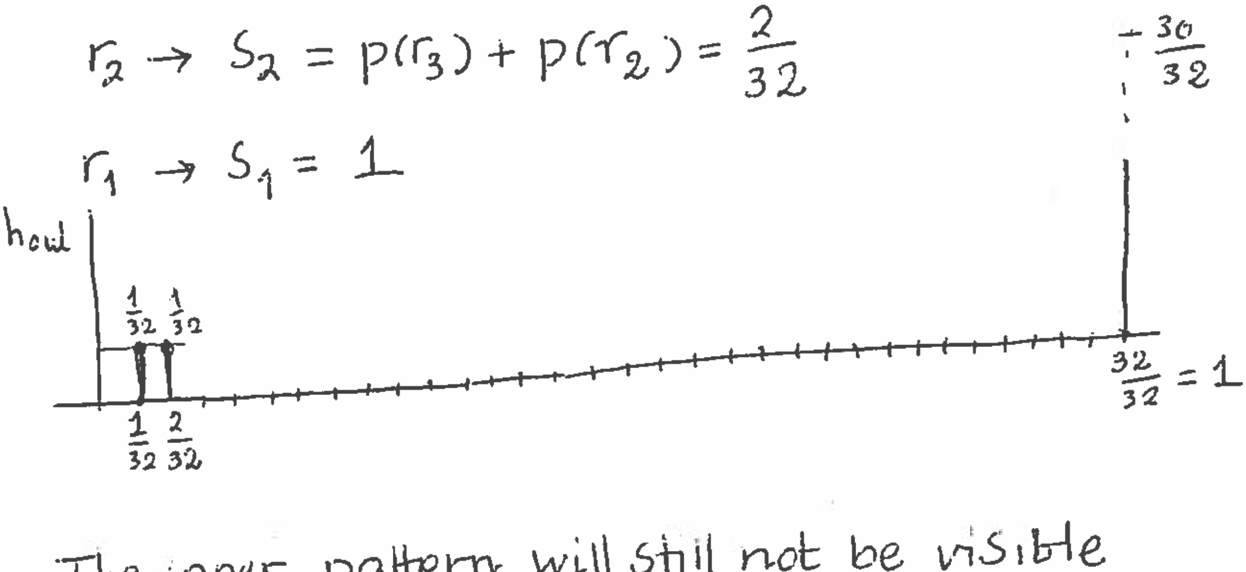
$$\Rightarrow p(r_1) = \frac{30}{32}$$

After histogram equalisation

$r_3 \rightarrow s_3 = p(r_1) = \frac{1}{32}$

$r_2 \rightarrow s_2 = p(r_3) + p(r_2) = \frac{2}{32}$

$r_1 \rightarrow s_1 = 1$



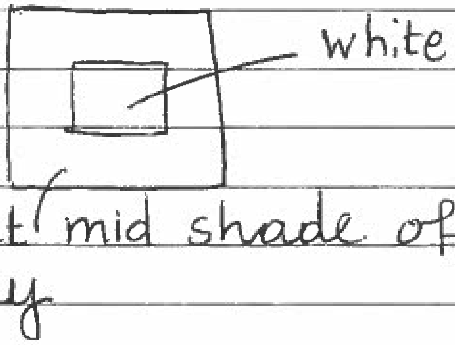
The inner pattern will still not be visible in the histogram equalised image

(ii) If we do local histogram equalisation the patch with the patter will perfectly fit in a scanning patch. For that patch we have

$$p(r_3) = \frac{1}{2}, \quad p(r_2) = \frac{1}{2}$$

$$r_3 \rightarrow s_3 = \frac{1}{2}$$

$$r_2 \rightarrow s_2 = 1$$



The rest of the image will turn white.

Therefore, the inner pattern will be visible $\Rightarrow$

(iii) Adaptive (local) HE is definitely more beneficial

b) We assume that the images are extended by zeros.

For the left image

borders
- black corners response : 0 (2 on total)
- white corners response : $\frac{4}{9}$ (2 on total

white non border pixels next to the edge have response $\frac{6}{9} = \frac{2}{3}$ (6 on total)

black non border pixels next to the edge have response $\frac{3}{9} = \frac{1}{3}$ (6 on total)

top and bottom white pixel next to the edge : $\frac{4}{9}$ (2 on total)

top and bottom ~~white~~ black pixel next to the edge $\frac{2}{9}$ (2 on total)

border white pixels $\frac{2}{3} = \frac{6}{9}$ (10 on total)

border black pixels 0 (10 on total)

rest of white pixels : 12 / rest of black pixels 12
response 1 response 0

~~Total number of border pixels 28 / rest 64 − 28 = 36~~
36/2 = 18

Intensities	Number of pixels	Probability
0	24	24/64
2/9	2	2/64
3/9	6	6/64
4/9	4	4/64
6/9	16	16/64
1	12	12/64

For the right image

internal while pixels response $\frac{5}{9}$ (18 pixels)

internal black pixels response $\frac{4}{9}$ (18 pixels)

corner while $\frac{2}{9}$ (2 pixels)

corner black $\frac{2}{9}$ (2 pixels)

border white $\frac{3}{9}$ (12 pixels)

border black $\frac{3}{9}$ (12 pixels)

Intensities	Number of pixels	Probability
2/9	4	4/64
3/9	24	24/64
4/9	18	18/64
5/9	18	18/64

$\Rightarrow$ Histograms are different

Question 3 – Answer

a)

(i) Book work

(ii) $h(x,y) = \begin{cases} 1 & x=0,\ y=0 \\ 2 & x=0\ \ y=1 \\ 1 & x=0\ \ y=2 \end{cases}$

$$H(u,v) = \frac{1}{M^2} \sum \sum h(x,y) e^{-j\frac{2\pi}{M}(ux+vy)}$$

$$= \frac{1}{M^2}\left(\cancel{e^{-j\frac{2\pi}{M}0}}^{1} + 2e^{-j\frac{2\pi}{M}v} + e^{-j\frac{2\pi}{M}2v} \right)$$

$$= \frac{1}{M^2} e^{-j\frac{2\pi}{M}v} \left(e^{j\frac{2\pi}{M}v} + 2 + e^{-j\frac{2\pi}{M}v} \right)$$

$$= \frac{1}{M^2} e^{-j\frac{2\pi}{M}v} \left(2\cos\frac{2\pi}{M}v + 2 \right)$$

$$H(u,v) = 0 \Rightarrow \cos\frac{2\pi}{M}v = -1$$

$$\frac{2\pi}{M}v = k\pi, \qquad k \text{ odd}$$

$$k=1 \quad \frac{2\pi}{M}v = \pi \Rightarrow v = \frac{M}{2}$$

$$k=3 \quad \frac{2\cancel{\pi}}{M}v = 3\cancel{\pi} \Rightarrow v = \frac{3M}{2} \quad \text{invalid}$$

since $v \in [0, M-1]$

(iii) book work

(iv) $C(u,v) = \frac{1}{M^2} e^{-j\frac{2\pi}{M}v} \left(2\cos\frac{2\pi}{M}v - 2 \right)$

$$\frac{2\pi}{M}v = k\pi, \quad k \text{ even} \Rightarrow v = 0$$

Therefore, $H(u,v)$ and $C(u,v)$ are never 0 at the same time and the restored image can always be estimated.

Question 4 – Answer

a) (i) 

(ii)  histogram of $g(x,y)$

(iii) Obviously $g(x,y)$

b) **One solution that does not always work! (extended Huffman)**

All the questions are answered here

Letter	Probability	Codeword
s_1	0.95	0
s_2	0.02	11
s_3	0.03	10

Table 1: Huffman code for three-letter alphabet; H = 0.335 bits/symbol; l_{avg} = 1.05 bits/symbol; redundancy = 0.715 bits/symbol or 213% of entropy.

Letter	Probability	Code
s_1s_1	0.9025	0
s_1s_2	0.0190	111
s_1s_3	0.0285	100
s_2s_1	0.0190	1101
s_2s_2	0.0004	110011
s_2s_3	0.0006	110001
s_3s_1	0.0285	101
s_3s_2	0.0006	110010
s_3s_3	0.0009	110000

Table 2: The Huffman code for the extended alphabet; l_{avg} = 1.222 bits/new symbol or l_{avg} = 0.611 bits/original symbol; redundancy = 72% of entropy; redundancy drops to acceptable values for N=8 (alphabet size = 6561).