

Special instructions for invigilators

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Special instructions for students

If both sections here are omitted (the normal case) the whole page may be deleted.

Questions

Question 2

Consider a proposed 132kV/11kV substation is to be supplied from a very strong 132kV network.

- (i) The substation will supply an 11kV network through one 40 MVA transformer. Show that the minimum value of the per unit reactance on rating of the transformer must be at least 0.133 p.u. if the 11kV equipment planned to be installed is rated at 300MVA short-circuit power. [4]
- (ii) calculate the three phase fault level at busbar B. [4]

Table 1.1

	Positive	Negative	Zero
Transformer	0.13	0.13	0.13
Line	0.5	0.5	0.8

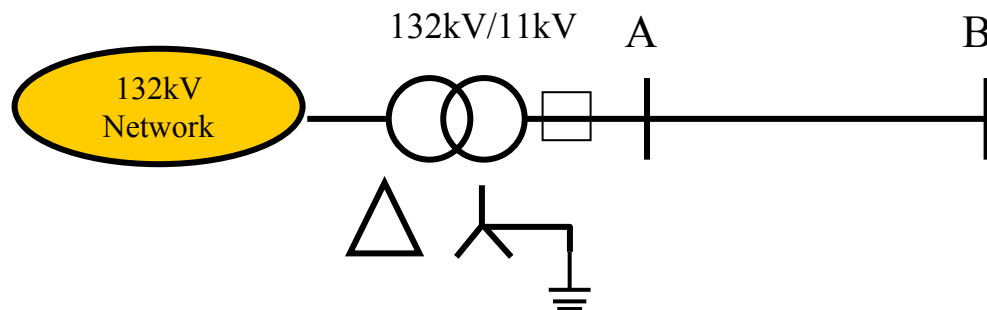


Figure 1.3. A simple power system composed of a generator, transformer and transmission line

- (iii) Determine the loading condition that would lead to a maximum voltage drop across the transformer. What will be the value of the maximum voltage drop? [4]
- (iv) This transformer will supply 80 distribution 11/0.4 kV substations of two types. Type A (30 substations), with expected peak demand of 400kW and Type B (50 substations) with peak demand of 650kW. Coincidence coefficients for an infinite number of Type A and Type B substations are 0.6 and 0.8 respectively. Assuming that peaks of both types of distribution substations coincide, calculate the peak demand of the proposed 132kV/11kV substation. [4]
- (v) Estimate the voltage drop across 132kV/11kV transformer during the demand condition determined in (iv). [4]

Solution to Question 2

(i) the minimum value of the per unit reactance

$$S_B = 40 \text{ MVA}$$

$$V_B = 11 \text{ kV}$$

$$I_B = \frac{S_B}{\sqrt{3} \cdot V_B}$$

$$i_F = \frac{1}{x}$$

$$I_F = i_F \cdot I_B$$

$$S_{3p} = \sqrt{3} \cdot V \cdot I_F \leq 300 \text{ MVA}$$

$$\sqrt{3} \cdot V \cdot \frac{1}{x} \cdot \frac{S_B}{\sqrt{3} \cdot V_B} \leq 300 \text{ MVA}$$

$$V \approx V_B$$

$$x \geq \frac{S_B}{300} = 0.133 \text{ p.u.}$$

(ii) the three phase fault level at busbar B

$$i_F = \frac{1}{0.13 + 0.5} = 1.59 \text{ p.u.}$$

(iii) Given that $r \approx 0$, voltage drop can be estimated from the following equation

$$\Delta V \approx Q \cdot x$$

Maximum voltage drop would be when reactive power is at maximum: $Q = 40 \text{ MVar}$, $P = 0$.

$$\Delta V \approx 1 \cdot 0.13 = 0.13 \text{ p.u.}$$

(iv)

$$j_{A30} = 0.6 + \frac{1 - 0.6}{\sqrt{30}} = 0.673$$

$$j_{B50} = 0.8 + \frac{1 - 0.8}{\sqrt{50}} = 0.828$$

$$P_{max} = 0.673 \cdot 30 \cdot 400 \cdot 10^{-3} + 0.828 \cdot 50 \cdot 650 \cdot 10^{-3} = 35 \text{ MW}$$

(v)

$$\Delta V \approx P_{max} \cdot r = 0$$

Question 3

a) Explain briefly why the power flow problem is non-linear? [3]

(b) Active power demand of the 132 kV system shown in Figure below is supplied by two generators G1 and G2. System voltage is supported by generator G2 and a large synchronous compensator SC (see Figure below), which both maintain the voltage at 1 pu at their respective nodes. Generator G1, connected at node 1, has no reactive power capacity available for voltage control.

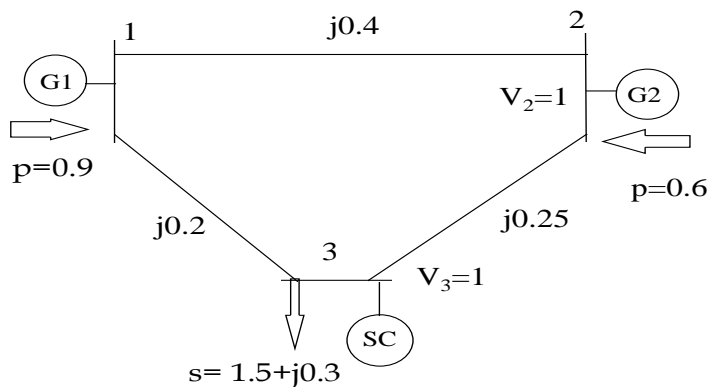


Figure 132 kV system - all values are in p.u with

- Identify the type of busbars for nodes 1, 2 and 3 (Slack, PV or PQ)
- Form the Y_{bus} matrix for this system.
- Perform two iterations of the Gauss – Seidel load flow.

[14]

Solution to Question 3

(a) In power systems, sources and loads are defined in terms of power, not voltage, current or impedance. Therefore power flow equations are non-linear.

(b)

(i) Types of busbars are as follows: 1 – PQ, 2 – Slack, and 3 – PV.

(ii) Y_{bus} matrix

$$z_{12} = 0 + j0.4 \text{ p.u.} \quad z_{13} = 0 + j0.2 \text{ p.u.} \quad z_{23} = 0 + j0.25 \text{ p.u.}$$

$$y_{12} = \frac{1}{z_{12}} = -j2.5 \text{ p.u.} \quad y_{13} = \frac{1}{z_{13}} = -j5 \text{ p.u.} \quad y_{23} = \frac{1}{z_{23}} = -j4 \text{ p.u.}$$

$$Y_{11} = y_{12} + y_{13} = -j2.5 + (-j5) = -j7.5 \text{ p.u.}$$

$$Y_{22} = y_{12} + y_{23} = -j2.5 + (-j4) = -j6.5 \text{ p.u.}$$

$$Y_{33} = y_{13} + y_{23} = -j5 + (-j4) = -j9 \text{ p.u.}$$

$$Y_{12} = Y_{21} = -y_{12} = j2.5 \text{ p.u.}$$

$$Y_{13} = Y_{31} = -y_{13} = j5 \text{ p.u.}$$

$$Y_{23} = Y_{32} = -y_{23} = j4 \text{ p.u.}$$

$$Y = j \begin{bmatrix} -7.5 & 2.5 & 5 \\ 2.5 & -6.5 & 4 \\ 5 & 4 & -9 \end{bmatrix}$$

(iii) Two iterations of the Gauss-Seidel load flow

- Initialisation

$$V_1^{(0)} = 1 + j0 \quad \text{PQ bus}$$

$$V_2^{(0)} = V_2^{spec} = 1 + j0 \quad \text{Slack bus}$$

$$V_3^{(0)} = V_3^{spec} = 1 + j0 \quad \text{PV bus}$$

$$Q_3^{(0)} = -\text{Im}\{V_3^{(0)*} \cdot (Y_{31} \cdot V_1^{(0)} + Y_{32} \cdot V_2^{(0)} + Y_{33} \cdot V_3^{(0)})\}$$

$$= -\text{Im}\{(1 - j0) \cdot [j5 \cdot (1 + j0) + j4 \cdot (1 + j0) + (-j9) \cdot (1 + j0)]\} = 0 \text{ p.u.}$$

$$S_3^{(0)} = -\text{Re}(s_3) + jQ_3^{(0)} = -\text{Re}(1.5 + j0.3) + j0 = -1.5 \text{ p.u.}$$

Note: SC does not produce real power

- The first iteration

$$V_1^{(1)} = \frac{1}{Y_{11}} \cdot \left(\frac{S_1^*}{V_1^{(0)*}} - Y_{12} \cdot V_2^{(0)} - Y_{13} \cdot V_3^{(0)} \right)$$

$$= \frac{1}{-j7.5} \cdot \left(\frac{0.9 - j0}{1 - j0} - j2.5 \cdot (1 + j0) - j5 \cdot (1 + j0) \right)$$

$$= 1 + j0.12 \text{ p.u.}$$

$$\tilde{V}_3^{(1)} = \frac{1}{Y_{33}} \cdot \left(\frac{S_3^{(0)*}}{V_3^{(0)*}} - Y_{31} \cdot V_1^{(1)} - Y_{32} \cdot V_2^{(0)} \right)$$

$$= \frac{1}{-j9} \cdot \left(\frac{-1.5}{1 - j0} - j5 \cdot (1 + j0.12) - j4 \cdot (1 + j0) \right) = 1 - j0.1 \text{ p.u.}$$

$$V_3^{(1)} = \left| V_3^{spec} \right| \frac{\tilde{V}_3^{(1)}}{\left| \tilde{V}_3^{(1)} \right|} = 1 \cdot \frac{1 - j0.1}{|1 - j0.1|} = 0.995 - j0.0995 \text{ p.u.}$$

$$\begin{aligned} S_2^{(1)} &= V_2^{(0)} \cdot (Y_{21} \cdot V_1^{(1)} + Y_{22} \cdot V_2^{(0)} + Y_{23} \cdot V_3^{(1)})^* \\ &= (1 + j0) \cdot (j2.5 \cdot (1 + j0.12) + (-6.5) \cdot (1 + j0) + j4 \cdot (0.995 - j0.0995))^* \\ &= 0.0980 + j0.0199 \text{ p.u.} \end{aligned}$$

$$Q_3^{(1)} = -\text{Im}(V_3^{(1)*} \cdot (Y_{31} \cdot V_1^{(1)} + Y_{32} \cdot V_2^{(0)} + Y_{33} \cdot V_3^{(1)})) = 0.1044 \text{ p.u.}$$

$$S_3^{(1)} = \text{Re}(S_3^{(0)}) + jQ_3^{(1)} = -1.5 + j0.1044 \text{ p.u.}$$

$$S_{12} = V_1^{(1)} \cdot ((V_1^{(1)} - V_2^{(0)}) \cdot y_{12})^* = 0.3 + j0.036 \text{ p.u.}$$

$$S_{21} = V_2^{(0)} \cdot ((V_2^{(0)} - V_1^{(1)}) \cdot y_{12})^* = -0.3 \text{ p.u.}$$

$$S_{13} = V_1^{(1)} \cdot ((V_1^{(1)} - V_3^{(1)}) \cdot y_{13})^* = 1.0945 + j0.1565 \text{ p.u.}$$

$$S_{31} = V_3^{(1)} \cdot ((V_3^{(1)} - V_1^{(1)}) \cdot y_{13})^* = -1.0945 + j0.0845 \text{ p.u.}$$

$$S_{23} = V_2^{(0)} \cdot ((V_2^{(0)} - V_3^{(1)}) \cdot y_{23})^* = 0.3980 + j0.0199 \text{ p.u.}$$

$$S_{32} = V_3^{(1)} \cdot ((V_3^{(1)} - V_2^{(0)}) \cdot y_{23})^* = -0.3980 + j0.0199 \text{ p.u.}$$

$$\Delta S_1^{(1)} = S_1 - S_{12} - S_{13} = -0.4945 - j0.1925 \text{ p.u.}$$

$$\Delta S_2^{(1)} = S_2^{(1)} - S_{23} - S_{21} = 0 \text{ p.u. (no mismatch at slack bus)}$$

$$\Delta S_3^{(1)} = S_3^{(1)} - S_{32} - S_{31} = -0.0074 + j0 \text{ p.u.}$$

$$\Delta V_1^{(1)} = |V_1^{(1)} - V_1^{(0)}| = |(1 + j0.12) - (1 + j0)| = 0.12 \text{ p.u.}$$

$$\Delta V_3^{(1)} = |V_3^{(1)} - V_3^{(0)}| = |(0.995 - j0.0995) - (1 + j0)| = 0.0996 \text{ p.u.}$$

- The second iteration

$$\begin{aligned} V_1^{(2)} &= \frac{1}{Y_{11}} \cdot \left(\frac{S_1^*}{V_1^{(1)*}} - Y_{12} \cdot V_2^{(0)} - Y_{13} \cdot V_3^{(1)} \right) \\ &= \frac{1}{-j7.5} \cdot \left(\frac{0.9 - j0}{1 - j0.12} - j2.5 \cdot (1 + j0) - j5 \cdot (0.995 - j0.0995) \right) \\ &= 0.9825 + j0.0520 \text{ p.u.} \end{aligned}$$

$$\begin{aligned} \tilde{V}_3^{(2)} &= \frac{1}{Y_{33}} \cdot \left(\frac{S_3^{(1)*}}{V_3^{(1)*}} - Y_{31} \cdot V_1^{(2)} - Y_{32} \cdot V_2^{(0)} \right) \\ &= \frac{1}{-j9} \cdot \left(\frac{-1.5 - j0.1044}{0.995 + j0.0995} - j5 \cdot (0.9825 + j0.0520) - j4 \cdot (1 + j0) \right) \\ &= 0.9852 - j0.1381 \text{ p.u.} \end{aligned}$$

$$V_3^{(2)} = \frac{\tilde{V}_3^{(2)}}{\left| \tilde{V}_3^{(2)} \right|} = \frac{0.9852 - j0.1381}{|0.9852 - j0.1381|} = 0.9903 - j0.1388 \text{ p.u.}$$

$$\begin{aligned}
S_2^{(2)} &= V_2^{(0)} \cdot (Y_{21} \cdot V_1^{(2)} + Y_{22} \cdot V_2^{(0)} + Y_{23} \cdot V_3^{(2)})^* \\
&= (1 + j0) \cdot (j2.5 \cdot (0.9825 + j0.0520) + (-j6.5) \cdot (1 + j0) + j4 \cdot (0.995 - j0.0995)) \\
&= 0.4255 + j0.0825 \text{ p.u.} \\
Q_3^{(2)} &= -\text{Im}(V_3^{(2)*} \cdot (Y_{31} \cdot V_1^{(2)} + Y_{32} \cdot V_2^{(0)} + Y_{33} \cdot V_3^{(2)})) = 0.2099 \text{ p.u.} \\
S_3^{(2)} &= \text{Re}(S_3^{(1)}) + jQ_3^{(2)} = -1.5 + j0.2099 \text{ p.u.} \\
S_{sc} &= S_3^{(2)} + s_3 = (-1.5 + j0.2099) + (1.5 + j0.3) = j0.5099 \text{ p.u.} \\
S_{12} &= V_1^{(2)} \cdot ((V_1^{(2)} - V_2^{(0)}) \cdot y_{12})^* = 0.1299 - j0.0362 \text{ p.u.} \\
S_{21} &= V_2^{(0)} \cdot ((V_2^{(0)} - V_1^{(2)}) \cdot y_{12})^* = -0.1299 + j0.0438 \text{ p.u.} \\
S_{13} &= V_1^{(2)} \cdot ((V_1^{(2)} - V_3^{(2)}) \cdot y_{13})^* = 0.9393 + 0.0112 \text{ p.u.} \\
S_{31} &= V_3^{(2)} \cdot ((V_3^{(2)} - V_1^{(2)}) \cdot y_{13})^* = -0.9393 + 0.1712 \text{ p.u.} \\
S_{23} &= V_2^{(0)} \cdot ((V_2^{(0)} - V_3^{(2)}) \cdot y_{23})^* = 0.5554 + j0.0387 \text{ p.u.} \\
S_{32} &= V_3^{(2)} \cdot ((V_3^{(2)} - V_2^{(0)}) \cdot y_{23})^* = -0.5554 + j0.0387 \text{ p.u.} \\
\Delta S_1^{(2)} &= S_1 - S_{12} - S_{13} = -0.1692 + j0.0251 \text{ p.u.} \\
\Delta S_2^{(2)} &= S_2^{(2)} - S_{23} - S_{21} = 0 \text{ p.u.} \\
\Delta S_3^{(2)} &= S_3^{(2)} - S_{31} - S_{32} = -0.0053 \text{ p.u.} \\
\Delta V_1^{(2)} &= |V_1^{(2)} - V_1^{(1)}| = |(0.9825 + j0.0520) - (1 + j0.12)| = 0.0703 \text{ p.u.} \\
\Delta V_3^{(2)} &= |V_3^{(2)} - V_3^{(1)}| = |(0.9903 - j0.1388) - (0.995 - j0.0995)| = 0.0396 \text{ p.u.}
\end{aligned}$$

Question 4

Figure below shows a 66 kV transformer feeder supplying an 11 kV busbar. The load on the busbar B is 12 MW and 5 MVar. The 20 MVA, 66/11 kV transformer has a leakage reactance of 15% on rating and is equipped with an on-load tap changer with tap-steps of 1.25%. The 66 kV overhead line is 30 km long with an impedance of $0.25+j0.5 \Omega/\text{km}$. Busbar A is held at 63 kV.

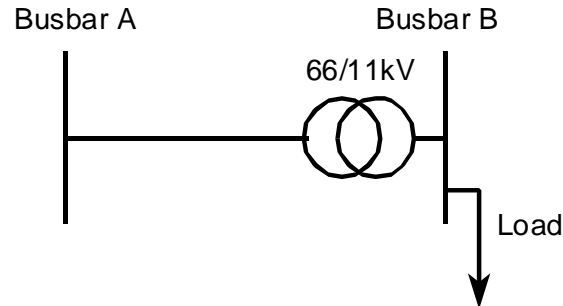


Figure – The simple power system

- (i) Explain why for determining the exact value of voltage at Busbar B an iterative procedure would be required. Write an non-linear complex equation that links voltages at Busbar A and B. [3]
- (ii) Without carrying out the iterative procedure, calculate an approximate value of the voltage at Busbar B (use a 20 MVA base) [6]
- (iii) Calculate then approximate value of losses in the system [4]
- (iv) Calculate the size of the shunt capacitor bank required at busbar B to bring the voltage to 11 kV [6]
- (v) Explain why loses will reduce after the capacitor bank is installed and calculate this reduction. [4]

Solution to Question 4

- (i) Complex power and voltage are specified at different busses, Magnitude of the losses is unknown, Losses affect the voltage drop
- (ii)

$$S_{\text{base}} = 20 \text{ MVA}$$

$$V_{\text{base}} = 66 \text{ kV}$$

$$Z_{\text{base}} = V_{\text{base}}^2 / S_{\text{base}} = (66 \cdot 10^3)^2 / (20 \cdot 10^6) = 217.8 \text{ Ohm}$$

$$P_L = 12/20 = 0.6 \text{ p.u.}$$

$$Q_L = 5/20 = 0.25 \text{ p.u.}$$

For 66 kV circuit:

$$r = (0.25 \times 30) / 217.8 = 0.0344 \text{ p.u.}$$

$$x = (0.5 \times 30) / 217.8 = 0.0688 \text{ p.u.}$$

For 66/11kV transformer $x = 0.15 \text{ p.u.}$

Tapchanger is at the HV side

$$\text{Voltage at A} = 63/66 = 0.9545$$

$$\begin{aligned} \text{Voltage at B} &\approx V_A - \Delta V \\ &\approx 0.9545 - \{ 0.6 \times 0.0344 + 0.25 \times (0.0688 + 0.15) \} \\ &\approx 0.8792 \text{ p.u.} \end{aligned}$$

(iii)

$$\begin{aligned} S_L &= \sqrt{0.6^2 + 0.25^2} = 0.65 \text{ p.u.} \\ i &= \frac{0.65}{0.8792} = 0.74 \text{ p.u.} \\ p_{Loss} &= 0.0344 \cdot 0.74^2 = 0.019 \text{ p.u.} \\ P_{Loss} &= 0.019 \cdot 20 = 0.38 \text{ MW} \end{aligned}$$

(iv)

The size of capacitor bank required at busbar B to bring the voltage at B to 11 kV:

$$V_B = 11 \text{ kV} = 1 \text{ p.u. (VBase = 11 kV)}$$

$$V_B \approx V_A - \Delta V$$

$$\begin{aligned} \Leftrightarrow 1 &\approx 0.9545 - (0.6 \times 0.0344 + Q \times 0.2188) \\ \Leftrightarrow Q &\approx -0.3023 \text{ p.u.} \\ \Leftrightarrow Q &\approx -0.3023 \times 20 = -6.046 \text{ MVar (the Q flows from B to A)} \end{aligned}$$

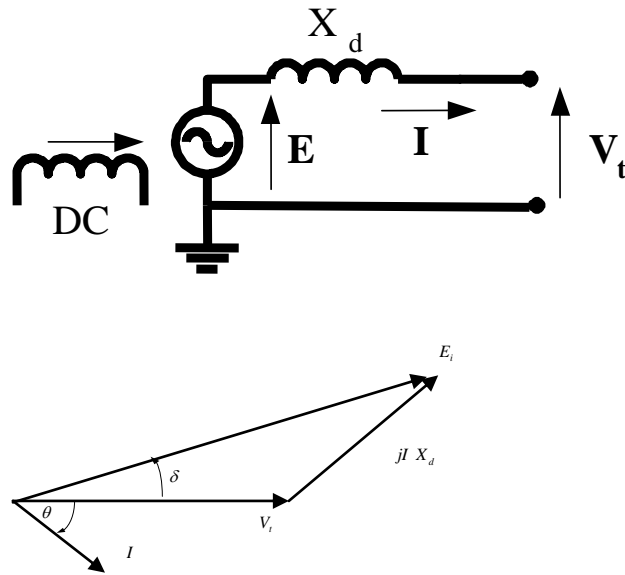
So the size of capacitor bank should be around $6.046 + 5 = 11.046 \text{ MVar}$

(v) After installing capacitor bank the voltage at busbar B will increase and therefore the current will decrease and, at the end, losses will decrease. The losses reduction is as follows

$$\begin{aligned} i &= \frac{0.65}{1} = 0.65 \text{ p.u.} \\ p_{Loss} &= 0.0344 \cdot 0.65^2 = 0.0145 \text{ p.u.} \\ \Delta p_{Loss} &= 0.019 - 0.0145 = 0.0045 \text{ p.u.} \\ \Delta P_{Loss} &= 0.0045 \cdot 20 = 0.09 \text{ MW} \end{aligned}$$

Question 5

Consider a synchronous generator model given in Figure below.



(i) Generator is delivering power and a certain angle δ exists between the terminal voltage V_t and the internal voltage of the machine E . Show that the expressions for active and reactive power are given by:

$$P = \frac{|V_t||E|}{X_d} \sin \delta$$

$$Q = \frac{|V_t|}{X_d} (|E| \cos \delta - |V_t|)$$

[4]

(ii) Given that the generator is feeding into a very strong network that maintains the voltage at the terminals of the generator and system frequency, what would be the effects of changing (a) excitation voltage and (b) turbine torque? [4]

(iii) The generator parameters are: $S=150$ MVA, $P=130$ MW, $V=11.5$ kV and $X_s = 1.3$ p.u. on rating, $E = 2.2$ p.u. max. If the generator operates at 100 MW, what is the value of internal voltage at which the generator delivers no reactive power to the system? [4]

(iv) what is the maximum reactive power that the generator can deliver to the system while exporting 100 MW? [4]

(v) what is the maximum reactive power that the generator can absorb while delivering 100 MW, assuming that the machine angle δ , due to the stability limits should not exceed 75 degrees? [4]

Solution to Question 5

(i)

$$\left. \begin{aligned} V_t &= |V_t| \\ E_i &= |E_i|(\cos \delta + j \sin \delta) \end{aligned} \right\} \begin{aligned} I &= \frac{|E_i|(\cos \delta + j \sin \delta) - |V_t|}{jX_d} \\ I^* &= \frac{|E_i|(\cos \delta - j \sin \delta) - |V_t|}{-jX_d} \end{aligned}$$

Complex power delivered to the system at the terminals of the generator is:

$$S = P + jQ = V_t I^* = \frac{|V_t||E_i|(\cos \delta - j \sin \delta) - |V_t|^2}{-jX_d}$$

$$P = \frac{|V_t||E_i|}{X_d} \sin \delta$$

$$Q = \frac{|V_t|}{X_d} (|E_i| \cos \delta - |V_t|)$$

(ii)

(a) The effects of changing excitation voltage would be change of angle delta and reactive power output

(b) The effects of changing turbine torque would be change of angle delta and active power output

(iii)

Assumed $V = 1$ p.u.

$$Q = 0 \Rightarrow E \cdot \cos \delta - V = 0 \Rightarrow E = \frac{V}{\cos \delta}$$

$$P = \frac{V \cdot E}{X} \cdot \sin \delta = \frac{V^2}{X} \cdot \tan \delta \Rightarrow \tan \delta = \frac{P \cdot X}{V^2} = \frac{100}{150} \cdot 1.3 = 0.866 \Rightarrow \delta = 40.91^\circ$$

$$E = \frac{1}{\cos(40.91^\circ)} = 1.32 \text{ p.u.}$$

(iv)

Export limited by excitation limit

$$E = 2.2 \text{ p.u.}$$

$$\sin \delta = \frac{100}{150} \cdot 1.3 = 0.866 \Rightarrow \delta = 40.91^\circ$$

$$Q = \frac{1}{1.3} \cdot (2.2 \cdot \cos(40.91^\circ) - 1) = 0.786 \text{ p.u.}$$

Export limited by MVA limit

$$S^2 = P^2 + Q^2 \Rightarrow Q = \sqrt{S^2 - P^2} = \sqrt{1^2 - \left(\frac{100}{150}\right)^2} = 0.745 \text{ p.u.}$$

Maximum reactive power that the generator can deliver is 0.745 p.u. i.e. 112 MVar.

(v)

Import limited by stability angle

$$\delta = 75^\circ$$

$$E = \frac{P \cdot X}{V \cdot \sin \delta} = \frac{\left(\frac{100}{150}\right) \cdot 1.3}{1 \cdot \sin(75^\circ)} = 0.8972 \text{ p.u.}$$

$$Q = \frac{1}{1.3} \cdot (0.8972 \cdot \cos(75^\circ) - 1) = -0.59 \text{ p.u.}$$

$$Q = -0.59 \cdot 150 = -88.6 \text{ MVar}$$

Check whether MVA limits are satisfied

$$S = \sqrt{\left(\frac{100}{150}\right)^2 + (-0.59)^2} = 0.89 \text{ p.u.} < 1 \text{ p.u.} \Rightarrow \text{limit satisfied}$$

Maximum reactive power that the generator can absorb is 88.6 MVar.

Question 6

Consider the one-line diagram of a simple power system shown in Figure 6.1. System data in per-unit (p.u.) on appropriate MVA base are given as follows:

Synchronous generators:

G1:	400 MVA	16 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.05$
G2:	400 MVA	16 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.05$

Transformers:

T1:	400 MVA	16/400 kV	$X_1 = X_2 = X_0 = 0.08$
T2:	400 MVA	16/400 kV	$X_1 = X_2 = X_0 = 0.08$

Transmission lines:

TL12:	250 MVA	400 kV	$X_1 = X_2 = 0.05$	$X_0 = 0.15$
TL13:	250 MVA	400 kV	$X_1 = X_2 = 0.025$	$X_0 = 0.075$
TL23:	250 MVA	400 kV	$X_1 = X_2 = 0.025$	$X_0 = 0.075$

The neutral of each generator is grounded through a current limiting reactor of 0.03 p.u. on 100 MVA base. All transformer neutrals are solidly grounded. The generators are operating at no-load with a voltage of 1.05 p.u.

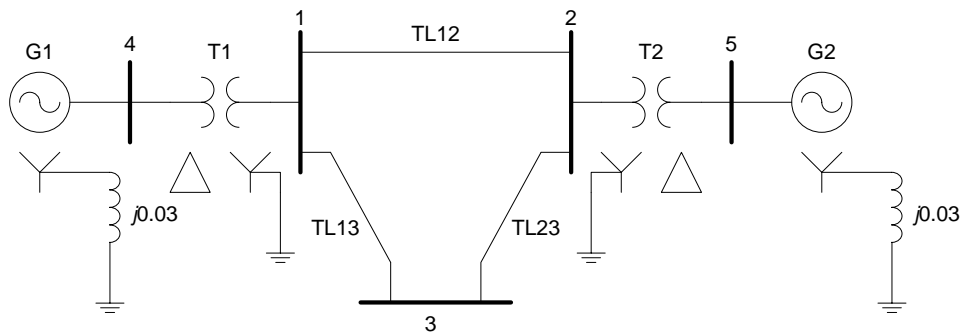


Figure 6.1. A simple power system

- Draw the positive, negative and zero sequence networks after expressing all the parameters in 100 MVA base. [5]
- Calculate the per unit and absolute value of the current flowing in the fault for a 1-ph to earth fault at busbar 1 [5]
- For this condition calculate the current flowing in the faulted phase of the transformers T1 and T2. [8]

Solution to Question 6

a) For G1:

$$X_1 = X_2 = 0.15 \text{ p.u.} \cdot \frac{100\text{MVA}}{400\text{MVA}} = 0.038 \text{ p.u.}$$

$$X_0 = 0.05 \text{ p.u.} \cdot \frac{100\text{MVA}}{400\text{MVA}} = 0.013 \text{ p.u.}$$

For G2:

$$X_1 = X_2 = 0.15 \text{ p.u.} \cdot \frac{100\text{MVA}}{400\text{MVA}} = 0.038 \text{ p.u.}$$

$$X_0 = 0.05 \text{ p.u.} \cdot \frac{100\text{MVA}}{400\text{MVA}} = 0.013 \text{ p.u.}$$

For T1:

$$X_1 = X_2 = X_0 = 0.08 \text{ p.u.} \cdot \frac{100\text{MVA}}{400\text{MVA}} = 0.02 \text{ p.u.}$$

For T2:

$$X_1 = X_2 = X_0 = 0.08 \text{ p.u.} \cdot \frac{100\text{MVA}}{400\text{MVA}} = 0.02 \text{ p.u.}$$

For TL1:

$$X_1 = X_2 = 0.05 \text{ p.u.} \cdot \frac{100\text{MVA}}{250\text{MVA}} = 0.002 \text{ p.u.}$$

$$X_0 = 0.15 \text{ p.u.} \cdot \frac{100\text{MVA}}{250\text{MVA}} = 0.01 \text{ p.u.}$$

For TL2:

$$X_1 = X_2 = 0.025 \text{ p.u.} \cdot \frac{100\text{MVA}}{250\text{MVA}} = 0.01 \text{ p.u.}$$

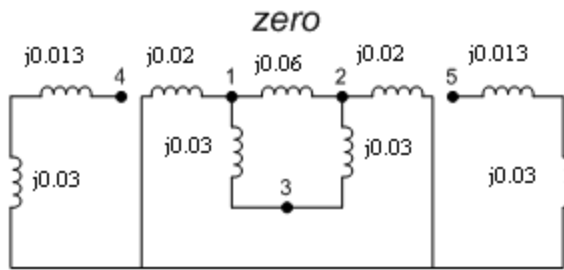
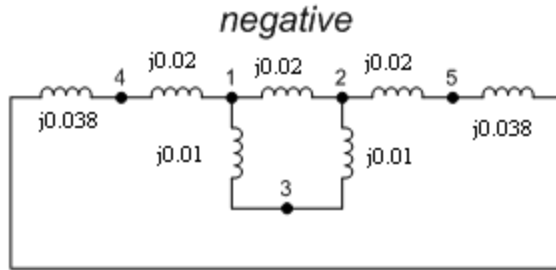
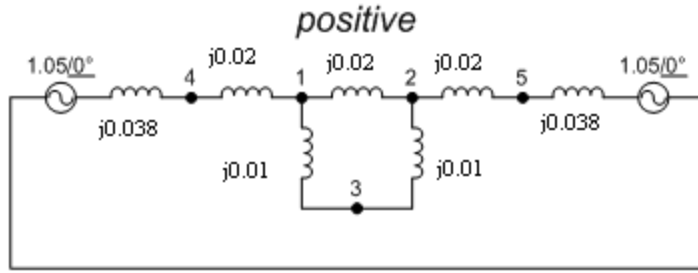
$$X_0 = 0.075 \text{ p.u.} \cdot \frac{100\text{MVA}}{250\text{MVA}} = 0.03 \text{ p.u.}$$

For TL3:

$$X_1 = X_2 = 0.025 \text{ p.u.} \cdot \frac{100\text{MVA}}{250\text{MVA}} = 0.01 \text{ p.u.}$$

$$X_0 = 0.075 \text{ p.u.} \cdot \frac{100\text{MVA}}{250\text{MVA}} = 0.03 \text{ p.u.}$$

Resulting circuit diagrams:



b) 1-phase earth fault at busbar 1 (note that \parallel is for parallel):

$$Z_1 = (j0.038 + j0.02) \parallel [(j0.02 \parallel (j0.01 + j0.01)) + j0.02 + j0.038] = j0.031 \text{ p.u.}$$

$$Z_2 = j0.031 \text{ p.u.}$$

$$Z_0 = j(0.02) \parallel [((j0.06) \parallel (j0.03 + j0.03)) + j0.02] = j0.014 \text{ p.u.}$$

$$I_{\text{fault}} = I_{\text{fault, pu}} \cdot I_{\text{base}}$$

$$I_{\text{base}} = \frac{S_{\text{base}}}{\sqrt{3} \cdot V_{\text{base}}} = \frac{100 \text{ MVA}}{\sqrt{3} \cdot 400 \text{ kV}} = 144.338 \text{ A}$$

$$I_{\text{fault, pu}} = 3 \cdot \frac{V_{\text{bus1, pu}}}{Z_1 + Z_2 + Z_0} = 3 \cdot \frac{1.05 \text{ p.u.}}{j(0.031 + 0.031 + 0.014) \text{ p.u.}} = 41.448 \text{ p.u.}$$

$$I_{\text{fault}} = (41.448 \text{ p.u.}) \cdot (144.38 \text{ A}) = 5.982 \text{ kA}$$

Calculate the current flowing in the faulted phase of transformers T1 and T2:

$$A = [(j0.02 \parallel (j0.01 + j0.01)) + j0.02 + j0.038]$$

$$B = (j0.038 + j0.02)$$

$$C = [(j0.06 \parallel (j0.03 + j0.03)) + j0.02]$$

$$D = j(0.02)$$

Positive sequence:

$$I_1^{TOTAL} = \frac{V_{bus1}}{Z_1 + Z_2 + Z_0} = -j13.816 \text{ p.u.}$$

$$I_1^{T1} = I_1^{TOTAL} \cdot \frac{A}{A+B} = -j7.456 \text{ p.u.}$$

$$I_1^{T2} = I_1^{TOTAL} \cdot \frac{B}{A+B} = -j6.360 \text{ p.u.}$$

Negative sequence:

$$I_2^{TOTAL} = I_1^{TOTAL} = -j13.816 \text{ p.u.}$$

$$I_2^{T1} = I_2^{TOTAL} \cdot \frac{A}{A+B} = -j7.456 \text{ p.u.}$$

$$I_2^{T2} = I_2^{TOTAL} \cdot \frac{B}{A+B} = -j6.360 \text{ p.u.}$$

Zero sequence:

$$I_0^{TOTAL} = I_1^{TOTAL} = -j13.816 \text{ p.u.}$$

$$I_0^{T1} = I_0^{TOTAL} \cdot \frac{C}{C+D} = -j9.869 \text{ p.u.}$$

$$I_0^{T2} = I_0^{TOTAL} \cdot \frac{D}{C+D} = -j3.947 \text{ p.u.}$$

$$I_{\alpha,pu}^{T1} = I_0^{T1} + I_1^{T1} + I_2^{T1} = -j24.781 \text{ p.u.} \Rightarrow I_{\alpha}^{T1} = I_{\alpha,pu}^{T1} \cdot I_{base} = (-j24.781 \cdot 144.338)$$

$$\Rightarrow |I_{\alpha}^{T1}| = 3.577 \text{ kA}$$

$$I_{\alpha,pu}^{T2} = I_0^{T2} + I_1^{T2} + I_2^{T2} = -j16.667 \text{ p.u.} \Rightarrow I_{\alpha}^{T2} = I_{\alpha,pu}^{T2} \cdot I_{base} = (-j16.667 \cdot 144.338)$$

$$\Rightarrow |I_{\alpha}^{T2}| = 2.406 \text{ kA}$$