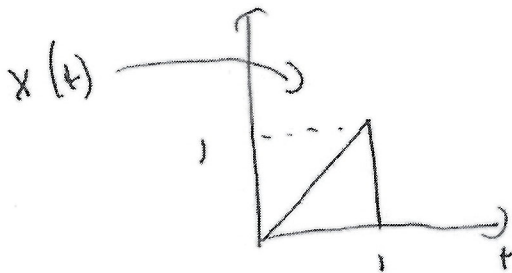


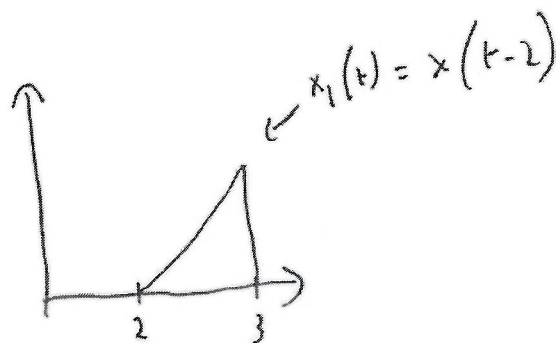
ANSWERS

QUESTION 1

(a)

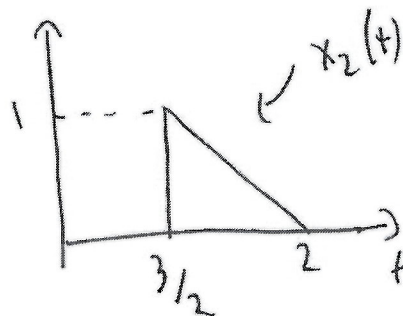


i.



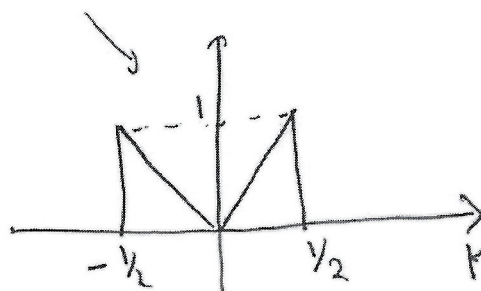
ii.

$$x_2(t) = x(-2(t-2))$$



iii.

$$x_3(t) = x(2t) + x(-2t)$$



(b)

2

$$i. \quad y(t) = x(t-2) + x(2-t)$$

COMMENT:
THIS IS
THE
CORRECT
WAY
TO
CHECK
LINEARITY

LET US CHECK LINEARITY

$$x_1(t) \rightarrow y_1(t) = x_1(t-2) + x_1(2-t)$$

$$x_2(t) \rightarrow y_2(t) = x_2(t-2) + x_2(2-t)$$

$$\begin{aligned} x_3(t) = ax_1(t) + bx_2(t) &\rightarrow y_3(t) = ax_1(t-2) + bx_2(t-2) \\ &\quad + ax_1(2-t) + bx_2(2-t) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

HENCE LINEAR

LET US CHECK TIME-INVARIANCE

$$x_1(t) \rightarrow y_1(t) = x_1(t-2) + x_1(2-t)$$

$$\begin{aligned} x_1(t-t_0) = x_2(t) &\rightarrow y_2(t) = x_2(t-2) + x_2(2-t) \\ &= x_1(t-t_0-2) + x_1(2-t-t_0) \\ &\neq y_1(t-t_0) \end{aligned}$$

NOTE THAT $y_1(t-t_0) = x_1(t-t_0-2) + x_1(2-t+t_0)$

HENCE TIME-VARIANT

~~NOTE THAT THE SYSTEM IS NOT~~
~~TIME-INVARIANT~~ ~~AND~~

COMMENT: MOST STUDENTS THOUGHT THIS WAS TIME-INVARIANT
THE TERM $x(2-t)$ MAKES IT TIME VARIANT
THINK OF THE SYSTEM $y(t) = x(-t)$
AND SHIFT THE OUTPUT WHEN THE INPUT
IS DELAYED BY t_0 .

ii.

$$y(t) = x(t) \cos 3t$$

THE SYSTEM IS LINEAR

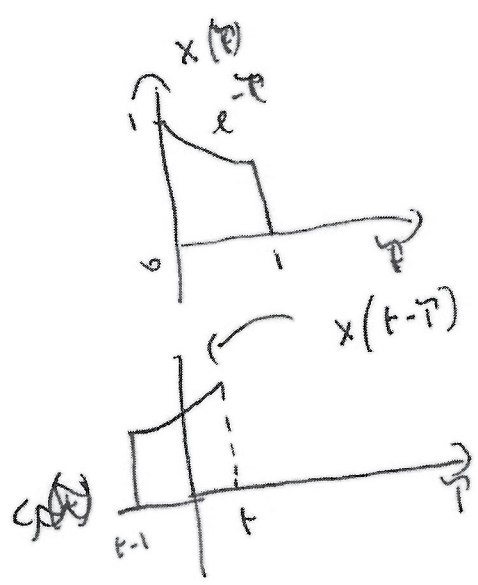
LET US CHECK FOR TIME-INVARIANCE

$$x_1(t) \rightarrow y_1(t) = x_1(t) \cos 3t$$

$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t) = x_1(t-t_0) \cos 3t \neq y_1(t-t_0)$$

HENCE TIME-VARIANT

(c)



INCE WE HAVE

$$x(t) * x(t) = \begin{cases} 0 & t \leq 0 \\ \int_0^t x(\tau) x(t-\tau) d\tau & 0 < t \leq 1 \\ \int_{t-1}^1 x(\tau) x(t-\tau) d\tau & 1 < t \leq 2 \\ 0 & t > 2 \end{cases}$$

$$\int_0^t x(\tau) x(t-\tau) d\tau = \int_0^t e^{-\tau} e^{-(t-\tau)} d\tau = t e^{-t} \quad 0 \leq t \leq 1$$

$$\int_{t-1}^1 x(\tau) x(t-\tau) d\tau = \int_{t-1}^1 e^{-\tau} e^{-(t-\tau)} d\tau = (2-t) e^{-t} \quad 1 < t \leq 2$$

HENCE

$$x(t) * x(t) = \begin{cases} 0 & t \leq 0 \\ t e^{-t} & 0 < t \leq 1 \\ (2-t) e^{-t} & 1 < t \leq 2 \\ 0 & t > 2 \end{cases}$$

COMMENTS:

MOST STUDENTS COULD NOT COMPUTE THIS CONVOLUTION. ~~BEFORE~~ WHEN THE TWO SIGNALS HAVE COMPACT SUPPORT IT IS HELPFUL TO USE THE GRAPHICAL METHOD TO SEE THE INTERVALS IN WHICH THE INTEGRAL SHOULD BE SPLIT

(ol)

5

$$y(t) = x(t-1) + x(-(t+1))$$

USING SHIFTING AND SCALING PROPERTY
OF THE FT YEDWS:

$$Y(\omega) = X(\omega) e^{-j\omega} + X(-\omega) e^{j\omega}$$

$$= \frac{1}{\omega^2} \left(1 - j\omega - e^{-j\omega} + 1 + j\omega - e^{j\omega} \right)$$

$$= \frac{1}{\omega^2} \left(2 - e^{j\omega} - e^{-j\omega} \right) = 4 \frac{\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}} \right)^2}{\left(j\frac{\omega}{2} \right)^2}$$

$$= \left(\text{sinc} \frac{\omega}{2} \right)^2$$

(2)

i. CHARACTERISTIC POLYNOMIAL

$$s^2 + 4s + 3$$

CHARACTERISTIC ROOTS

$$s_{1,2} = -2 \pm \sqrt{4-3} = \begin{matrix} -1 \\ -3 \end{matrix}$$

CHARACTERISTIC MODES

$$c_1 e^{-t} + c_2 e^{-3t}$$

ii.

6

$$y(t) = c_1 e^{-t} + c_2 e^{-3t} \quad t \geq 0$$

$$y(0) = c_1 + c_2 = 0$$

$$\Rightarrow c_1 = 1$$

$$\dot{y}(0) = -c_1 - 3c_2 = 2$$

$$c_2 = -1$$

HENCE

$$y(t) = (e^{-t} - e^{-3t}) u(t)$$

iii.

$$(s^2 + 4s + 5) Y(s) = X(s)$$

$$\text{SINCE } X(s) = \frac{1}{s}$$

WE HAVE THAT

$$Y(s) = \frac{1}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

USING COVERING METHOD WE FIND THAT

$$Y(s) = \frac{1}{3s} - \frac{1}{2(s+1)} + \frac{1}{6(s+3)}$$

\Downarrow

$$y(t) = \left(\frac{1}{3} - \frac{1}{2} e^{-t} + \frac{1}{6} e^{-3t} \right) u(t)$$

(vi).

BECAUSE OF LINEARITY
THE TOTAL ANSWER IS THE
SUM OF THE RESPONSES IN
PART ii. AND iii. THAT IS

$$y_{\text{TOT}}(t) = (e^{-t} - e^{-3t})u(t) + \left(\frac{1}{3} - \frac{1}{2}e^{-t} + \frac{1}{6}e^{-3t}\right)u(t)$$

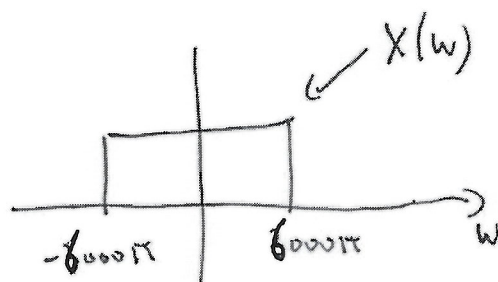
$$= \left(\frac{1}{3} + \frac{1}{2}e^{-t} - \frac{5}{6}e^{-3t}\right)u(t)$$

8

(8) USING FOURIER TABLES

i.

$$6000 \operatorname{sinc}(6000\pi t) \Leftrightarrow \operatorname{rect}\left(\frac{\omega}{12000\pi}\right)$$



COMMENT:
NORMALLY THE
BANDWIDTH IS
MEASURED IN Hz

ii)

$$\text{NYQUIST RATE FOR } x(t): f_s = 2 \cdot \frac{6000\pi}{2\pi} = 6 \text{ kHz}$$

(9)

$$\frac{X(z)}{z} = \frac{z+1}{z^2-5z+4} = \frac{z+1}{(z-4)(z-1)} = \frac{A}{z-4} + \frac{B}{z-1}$$

$$= \frac{5}{3} \frac{1}{z-4} - \frac{2}{3} \frac{1}{z-1}$$

HENCE

$$X(z) = \frac{5}{3} \frac{z}{z-4} - \frac{2}{3} \frac{z}{z-1}$$

USING z-TRANSFORM TABLE WE GET

$$x[n] = \left(\frac{5}{3} 4^n - \frac{2}{3} \right) u[n]$$

QUESTION 2

9

(a)

IN STEADY STATE THE INDUCTOR
IS A SHORT CIRCUIT AND THE CAPACITOR
IS AN OPEN CIRCUIT
THEREFORE

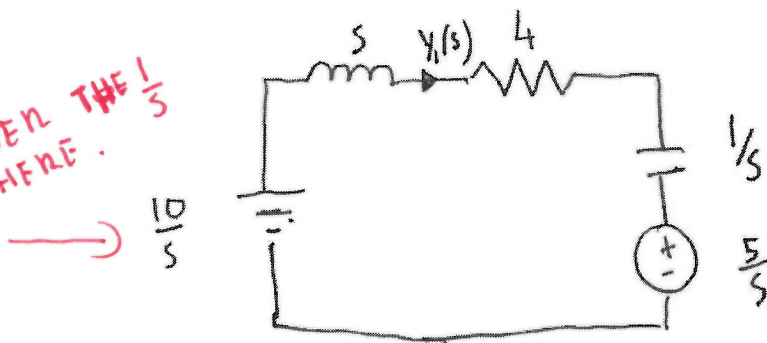
$$y_1(0^-) = 0$$

$$V_C(0^-) = 5V$$

COMMENTS : MOST STUDENTS
DID THIS QUESTION CORRECTLY
HOWEVER SOME ~~WAS~~
HAD $y_1(0^-) \neq 0$
AND OTHERS
ASSUMED $V_C(0^-) = 0$

(b) THE EQUIVALENT CIRCUIT IN THE
LAPLACE DOMAIN IS :

REMEMBER THE $\frac{1}{s}$
TERM HERE.



THUS THE LOOP EQUATION IN THE LAPLACE
DOMAIN IS :

$$\frac{10}{s} = s Y_1(s) + 4 Y_1(s) + \frac{Y_1(s)}{s} + \frac{5}{s} ;$$

$$Y_1(s) (s^2 + 4s + 1) = 5 ;$$

$$Y_1(s) = \frac{5}{s^2 + 4s + 1}$$

$$(c) \quad Y_1(s) = \frac{5}{s^2 + 4s + 1} = \frac{5}{(s+2-\sqrt{3})(s+2+\sqrt{3})} = \frac{5}{2\sqrt{3}} \left(\frac{1}{s+2-\sqrt{3}} - \frac{1}{s+2+\sqrt{3}} \right)$$

USING THE FACT THAT

$$e^{-\lambda t} u(t) \Leftrightarrow \frac{1}{s+\lambda}$$

WE OBTAIN

$$y_1(t) = \frac{5}{2\sqrt{3}} \left(e^{-(2-\sqrt{3})t} - e^{-(2+\sqrt{3})t} \right) u(t)$$

COMMENTS: THIS IS A PHYSICALLY REALIZABLE CIRCUIT SO $y_1(t)$ SHOULD BE REAL VALUED. THE EXPONENTS SHOULD BE NEGATIVE AND THE SOLUTION SHOULD BE CONSISTENT WITH THE INITIAL CONDITIONS, ~~AND~~ SPECIFICALLY IF $y_1(0^-) = 0$ YOU EXPECT $y_1(0^+) = 0$ AS WELL. YOU CAN HAVE A DISCONTINUITY IN \bullet ZERO ONLY WHEN THE GENERATOR IS A DIRAC.

QUESTION 3

11

(a)

IF WE DENOTE WITH $F(s)$ THE OUTPUT OF THE FEEDBACK SYSTEM WE HAVE THAT

$$Y(s) = (s+2) F(s)$$

$$F(s) = 1K (X(s) - (s+a)^2 F(s))$$

COMMENTS: MANY STUDENTS DID NOT GET THIS RIGHT.

THEREFORE

$$F(s) = \frac{1K}{1 + 1K(s+a)^2} X(s)$$

AND

$$Y(s) = \frac{1K(s+2)}{1 + 1K(s+a)^2} X(s)$$

THE TRANSFER FUNCTION IS:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1K(s+2)}{1 + 1K(s+a)^2}$$

(b)

USING FINAL VALUE THEOREM WE HAVE THAT

$$y(\infty) = \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s)$$

SINCE $X(s) = \frac{1}{s}$ AND $1K=1$

WE HAVE THAT

$$y(\infty) = \lim_{s \rightarrow 0} \frac{s+2}{1+(s+a)^2} = \frac{2}{1+a^2}$$

WE WANT $y(\infty) = \frac{2}{5}$ WITH $a > 0 \Rightarrow a = 2$.

(c) i.

$$\begin{aligned} Y(s) &= \frac{s+2}{1+(s+1)^2} \cdot \frac{1}{s} \\ &= \frac{C}{s} + \frac{As+B}{s^2+2s+2} \end{aligned}$$

~~(2/4)~~ MULTIPLY BOTH SIDES BY s AND SET $s=0$

$$1 = C$$

THEN MULTIPLY BOTH SIDES BY s AND LET $s \rightarrow \infty$

$$0 = 1 + A \Rightarrow A = -1$$

FINALLY SET $s = -1$ TO FIND

$$B = -1$$

WE HAVE

$$Y(s) = \frac{1}{s} - \frac{s+1}{(s+1)^2+1}$$

USING LAPLACE TABLES WE OBTAIN

$$y(t) = u(t) - e^{-t} \cos t \quad u(t)$$

(1.

FOR $t \geq 0$

$$y(t) = 1 - e^{-t} \cos t$$

$$\frac{dy}{dt} = e^{-t} \cos t + e^{-t} \sin t = 0$$

$$\Rightarrow e^{-t} (\cos t + \sin t) = 0$$

$$\Rightarrow \cos t = -\sin t \quad \Rightarrow \tan t = -1$$

THIS HAPPENS AT $t = \frac{\pi}{4} + \frac{\pi}{2}$.

THE OTHER POINT ARE EITHER LOCAL MINIMA OR LOCAL MAXIMA BECAUSE OF THE TERM e^{-t} IN THE EXPRESSION OF $y(t)$.