Solutions - 2003 E4.38

Modelling & Control in Power Engineering C3.3

1.

(a) Describe the principle of state-space averaged models and explain why they are useful in designing control systems for power electronic systems. [4]

- *Power electronic circuits are switch-mode and are therefore not LTI*
- *However, each mode (or state) of the circuit is normally LTI and so linear state-space models can be formed*
- *Provided that the switching frequency is high enough, the state-transition can be approximated by the time-average of the state-transition due to each mode.*
- *Perturbation analysis can then yield a linearised model of the circuit for perturbations around an operating point.*

(b) Consider a circuit has two conduction modes: mode "On" with a duty-cycle of δ and mode "Off" with a duty-cycle of $(1-\delta)$.

(i) Give the equations for the state-space averaged model in terms of the stead-state and perturbation components of the signals defined as:

$$\mathbf{u}=\mathbf{U}+\tilde{\mathbf{u}} \quad \mathbf{x}=\mathbf{X}+\tilde{\mathbf{x}} \quad \mathbf{y}=\mathbf{Y}+\tilde{\mathbf{y}} \quad \delta=\Delta+\tilde{\delta}$$ [2]

$$\begin{aligned}\dot{\mathbf{X}}+\dot{\tilde{\mathbf{x}}} \cong & \left(\left(\Delta+\tilde{\delta}\right)\mathbf{A}_{\mathbf{On}}\left(\mathbf{X}+\tilde{\mathbf{x}}\right)+\left(1-\left(\Delta+\tilde{\delta}\right)\right)\mathbf{A}_{\mathbf{Off}}\left(\mathbf{X}+\tilde{\mathbf{x}}\right)\right)\\ &+\left(\left(\Delta+\tilde{\delta}\right)\mathbf{B}_{\mathbf{On}}\left(\mathbf{U}+\tilde{\mathbf{u}}\right)+\left(1-\left(\Delta+\tilde{\delta}\right)\right)\mathbf{B}_{\mathbf{Off}}\left(\mathbf{U}+\tilde{\mathbf{u}}\right)\right)\end{aligned}$$

$$\begin{aligned}\mathbf{Y}+\tilde{\mathbf{y}} \cong & \left(\Delta+\tilde{\delta}\right)\mathbf{C}_{\mathbf{On}}\left(\mathbf{X}+\tilde{\mathbf{x}}\right)+\left(1-\left(\Delta+\tilde{\delta}\right)\right)\mathbf{C}_{\mathbf{Off}}\left(\mathbf{X}+\tilde{\mathbf{x}}\right)\\ &+\left(\left(\Delta+\tilde{\delta}\right)\mathbf{D}_{\mathbf{On}}\left(\mathbf{U}+\tilde{\mathbf{u}}\right)+\left(1-\left(\Delta+\tilde{\delta}\right)\right)\mathbf{D}_{\mathbf{Off}}\left(\mathbf{U}+\tilde{\mathbf{u}}\right)\right)\end{aligned}$$

(iii) Separate the model into its steady-state and small-signal portions for the case where the circuit input, $\mathbf{u}$, is constant. [7]

Steady-state

$$\dot{\mathbf{X}}=\left(\Delta\mathbf{A}_{\mathbf{On}}+\left(1-\Delta\right)\mathbf{A}_{\mathbf{Off}}\right)\mathbf{X}+\left(\Delta\mathbf{B}_{\mathbf{On}}+\left(1-\Delta\right)\mathbf{B}_{\mathbf{Off}}\right)\mathbf{U}$$
$$\mathbf{Y}=\left(\Delta\mathbf{C}_{\mathbf{On}}+\left(1-\Delta\right)\mathbf{C}_{\mathbf{Off}}\right)\mathbf{X}+\left(\Delta\mathbf{D}_{\mathbf{On}}+\left(1-\Delta\right)\mathbf{D}_{\mathbf{Off}}\right)\mathbf{U}$$

define:

$$\mathbf{A}=\left(\Delta\mathbf{A}_{\mathbf{On}}+\left(1-\Delta\right)\mathbf{A}_{\mathbf{Off}}\right)$$
$$\mathbf{B}=\left(\Delta\mathbf{B}_{\mathbf{On}}+\left(1-\Delta\right)\mathbf{B}_{\mathbf{Off}}\right)$$
$$\mathbf{C}=\left(\Delta\mathbf{C}_{\mathbf{On}}+\left(1-\Delta\right)\mathbf{C}_{\mathbf{Off}}\right)$$
$$\mathbf{D}=\left(\Delta\mathbf{D}_{\mathbf{On}}+\left(1-\Delta\right)\mathbf{D}_{\mathbf{Off}}\right)$$

then

$$\dot{\mathbf{X}}=0$$
$$\mathbf{X}=-\mathbf{A}^{-1}\mathbf{B}\mathbf{U}$$
$$\mathbf{Y}=\mathbf{C}\mathbf{X}+\mathbf{D}\mathbf{U}$$

Small-signal

$$\dot{\tilde{\mathbf{x}}} \cong \left(\Delta\,\mathbf{A}_{\mathbf{On}} + (1-\Delta)\mathbf{A}_{\mathbf{Off}} \right)\tilde{\mathbf{x}}$$
$$+ \left(\left(\mathbf{A}_{\mathbf{On}} - \mathbf{A}_{\mathbf{Off}} \right)\mathbf{X} + \left(\mathbf{B}_{\mathbf{On}} - \mathbf{B}_{\mathbf{Off}} \right)\mathbf{U} \right)\tilde{\delta}$$

$$\tilde{\mathbf{y}} \cong \left(\Delta\mathbf{C}_{\mathbf{On}} + (1-\Delta)\mathbf{C}_{\mathbf{Off}} \right)\tilde{\mathbf{x}}$$
$$+ \left(\left(\mathbf{C}_{\mathbf{On}} - \mathbf{C}_{\mathbf{Off}} \right)\mathbf{X} + \left(\mathbf{D}_{\mathbf{On}} - \mathbf{D}_{\mathbf{Off}} \right)\mathbf{U} \right)\tilde{\delta}$$

$$\tilde{\mathbf{w}} = \left[\tilde{\delta}\right] \quad \tilde{\mathbf{x}} = \left[\tilde{i}_L \quad \tilde{v}_C\right]^T \quad \tilde{\mathbf{y}} = \left[v_O\right]$$

$$\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{A}}\,\tilde{\mathbf{x}} + \tilde{\mathbf{B}}\,\tilde{\mathbf{w}}$$
$$\tilde{\mathbf{y}} = \tilde{\mathbf{C}}\,\tilde{\mathbf{x}} + \tilde{\mathbf{D}}\,\tilde{\mathbf{w}}$$

$$\tilde{\mathbf{A}} = \mathbf{A}$$
$$\tilde{\mathbf{B}} = \left(\mathbf{A}_{\mathbf{On}} - \mathbf{A}_{\mathbf{Off}} \right)\mathbf{X} + \left(\mathbf{B}_{\mathbf{On}} - \mathbf{B}_{\mathbf{Off}} \right)\mathbf{U}$$
$$\tilde{\mathbf{C}} = \mathbf{C}$$
$$\tilde{\mathbf{D}} = \left(\mathbf{C}_{\mathbf{On}} - \mathbf{C}_{\mathbf{Off}} \right)\mathbf{X} + \left(\mathbf{D}_{\mathbf{On}} - \mathbf{D}_{\mathbf{Off}} \right)\mathbf{U}$$

(c) Figure Q1 shows the circuit flyback diagram of the flyback switch-mode power supply together with indications of the circuit elements in conduction in the on and off states. The state-space models of the two states have been established as:

On-State

$$\mathbf{x} = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$\mathbf{A}_{\mathbf{On}} = \begin{bmatrix} -\dfrac{R_L}{L} & 0 \\ 0 & -\dfrac{1}{C(R_O + R_C)} \end{bmatrix} \quad \mathbf{B}_{\mathbf{On}} = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \quad \mathbf{C}_{\mathbf{On}} = \begin{bmatrix} 0 & \dfrac{R_O}{R_O + R_C} \end{bmatrix} \quad \mathbf{D}_{\mathbf{On}} = [0]$$

Off- State

$$\mathbf{x}=\begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$\mathbf{A}_{\mathbf{Off}}=\begin{bmatrix} -\dfrac{R_L+R_C}{L}+\dfrac{R_C^{\,2}}{L(R_O+R_C)} & \dfrac{1}{L}-\dfrac{R_C}{L(R_O+R_C)} \\ -\dfrac{1}{C}+\dfrac{R_C}{C(R_O+R_C)} & -\dfrac{1}{C(R_O+R_C)} \end{bmatrix}$$

$$\mathbf{B}_{\mathbf{Off}}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \mathbf{C}_{\mathbf{Off}}=\begin{bmatrix} -\dfrac{R_O R_C}{R_O+R_C} & \dfrac{R_O}{R_O+R_C} \end{bmatrix} \quad \mathbf{D}_{\mathbf{Off}}=[0]$$

Show that the solution to the steady-state portion of the model yields the relationship $\dfrac{V_O}{V_I}=\dfrac{-\Delta}{1-\Delta}$ when the parasitic resistances R_C and R_L are ignored [7]

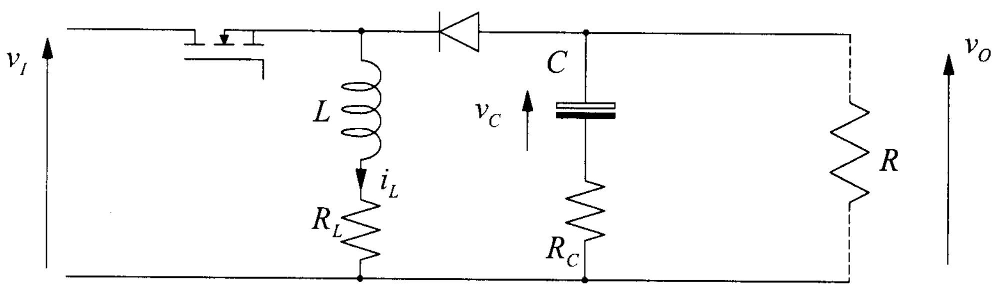


Figure Q1 A Flyback switch-mode power supply

$$\mathbf{A}=\left(\Delta\mathbf{A}_{\mathbf{On}}+(1-\Delta)\mathbf{A}_{\mathbf{Off}}\right)=\begin{bmatrix} -\dfrac{R_L}{L}-\dfrac{(1-\Delta)R_C}{L}+\dfrac{(1-\Delta)R_C^{\,2}}{L(R_O+R_C)} & \dfrac{(1-\Delta)R_O}{L(R_O+R_C)} \\ -\dfrac{(1-\Delta)R_O}{C(R_O+R_C)} & -\dfrac{1}{C(R_O+R_C)} \end{bmatrix}$$

$$\mathbf{B}=\left(\Delta\mathbf{B}_{\mathbf{On}}+(1-\Delta)\mathbf{B}_{\mathbf{Off}}\right)=\begin{bmatrix} \dfrac{\Delta}{L} \\ 0 \end{bmatrix}$$

$$\mathbf{C}=\left(\Delta\mathbf{C}_{\mathbf{On}}+(1-\Delta)\mathbf{C}_{\mathbf{Off}}\right)=\begin{bmatrix} -(1-\Delta)\dfrac{R_O R_C}{R_O+R_C} & \dfrac{R_O}{R_O+R_C} \end{bmatrix}$$

$$\mathbf{D}=\left(\Delta\mathbf{D}_{\mathbf{On}}+(1-\Delta)\mathbf{D}_{\mathbf{Off}}\right)=[0]$$

Then ignoring parasitic resistances

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{(1-\Delta)}{L} \\ -\frac{(1-\Delta)}{C} & -\frac{1}{CR_O} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \frac{\Delta}{L} \\ 0 \end{bmatrix}$$

$$\mathbf{C} = [0 \quad 1]$$

$$\mathbf{X} = -\mathbf{A}^{-1}\,\mathbf{BU} = -\begin{bmatrix} 0 & \frac{(1-\Delta)}{L} \\ -\frac{(1-\Delta)}{C} & -\frac{1}{CR_O} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\Delta}{L} \\ 0 \end{bmatrix} [V_I]$$

$$= -\frac{LC}{(1-\Delta)^2} \begin{bmatrix} -\frac{1}{CR_O} & -\frac{(1-\Delta)}{L} \\ \frac{(1-\Delta)}{C} & 0 \end{bmatrix} \begin{bmatrix} \frac{\Delta V_I}{L} \\ 0 \end{bmatrix}$$

$$= -\frac{LC}{(1-\Delta)^2} \begin{bmatrix} -\frac{\Delta V_I}{LCR_O} \\ \frac{(1-\Delta)\Delta V_I}{LC} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\Delta V_I}{(1-\Delta)^2 R_O} \\ -\frac{\Delta V_I}{(1-\Delta)} \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{CX} + \mathbf{DU} = [0 \quad 1]\mathbf{X}$$

$$V_O = -\frac{\Delta}{(1-\Delta)} V_I$$

2.

(a) Explain why it is important that the three-phase abc to $\alpha\beta\gamma$ transformation matrix T has the property $T^T T = I$. [4]

We wish the transform to be power invariant and the transformed terms to be orthogonal such that:

$$p = \left[\bar{u}'\right]^T \cdot \left[\bar{u}'\right]^* = \left(\left[T\right]\left[\bar{u}\right]\right)^T \cdot \left(\left[T\right]\left[\bar{u}\right]^*\right) = \left[\bar{u}\right]^T \cdot \left[T\right]^T \left[T\right] \cdot \left[\bar{u}\right]^*$$

Therefore the matrix transpose should equal the matrix inverse. The power calculated from the transformed variables will involve no mixed-product terms (provided that was true for the original variables – as would be case for a three independent phase circuits)

(b) Show that an inductive voltage drop in abc quantities, $L\dfrac{di_{abc}}{dt}$ transforms to two terms in $dq\gamma$ quantities when transformed by the transformation matrices:

$$[T] = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad [T_R] = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and explain the physical interpretation of the two terms. [6]

The multiplication by T to transform to $\alpha\beta\gamma$ quantities is straightforward because T is time-invariant

$$[T][L]\frac{d}{dt}[i_{abc}] = [T][L][T]^{-1}[T]\frac{d}{dt}[i_{abc}] = [L]\frac{d}{dt}([T][i_{abc}]) = [L]\frac{d}{dt}[i_{\alpha\beta\gamma}]$$

because $[T][L][T]^{-1} = [L]$ if L is diagonal

The multiplication by T_R requires a product rule differentiation because T_R is time-varying

$$\begin{aligned}
[T_R][L]\frac{d}{dt}[i_{\alpha\beta\gamma}] &= [T_R][L][T_R]^{-1}[T_R]\frac{d}{dt}[i_{\alpha\beta\gamma}] \\
&= [L][T_R]\left(\frac{d}{dt}\left([T_R]^{-1}[i_{dq\gamma}]\right)\right) \\
&= [L][T_R]\left([T_R]^{-1}\frac{d}{dt}[i_{dq\gamma}] + \frac{d}{dt}\left([T_R]^{-1}\right)[i_{dq\gamma}]\right) \\
&= [L]\frac{d}{dt}[i_{dq\gamma}] + [L][T_R]\frac{d}{dt}\left([T_R]^{-1}\right)[i_{dq\gamma}]
\end{aligned}$$

$$[T_R]^{-1} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
[T_R]\frac{d}{dt}[T_R]^{-1} &= \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) & 0 \\ \sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\omega\sin(\omega t) & \omega\cos(\omega t) & 0 \\ -\omega\cos(\omega t) & -\omega\sin(\omega t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}$$

$$[T_R][L]\frac{d}{dt}[i_{\alpha\beta\gamma}]=[L]\frac{d}{dt}[i_{dq\gamma}]+\begin{bmatrix}0 & \omega L & 0\\ -\omega L & 0 & 0\\ 0 & 0 & 0\end{bmatrix}[i_{dq\gamma}]$$

The first term is a voltage drop that occurs when the magnitude or phase-angle of the current changes; the second term is the steady voltage drop caused by the steady rotation of the three-phase set. The rotational voltage is proportional to ωL as expected of an inductive voltage drop and also has a leading quadrature relationship to the current (the voltage drop in the d-axis is proportional to the q-axis current)

(c) Figure Q2 shows a three-phase circuit. The circuit is balanced, *i.e.*, the circuit parameters for the three phases are the same. Write circuit equation for this circuit and transform them into $dq\gamma$ terms (using T and T_R as defined in part (b)) and draw a circuit diagram to represent the transformed system. [10]

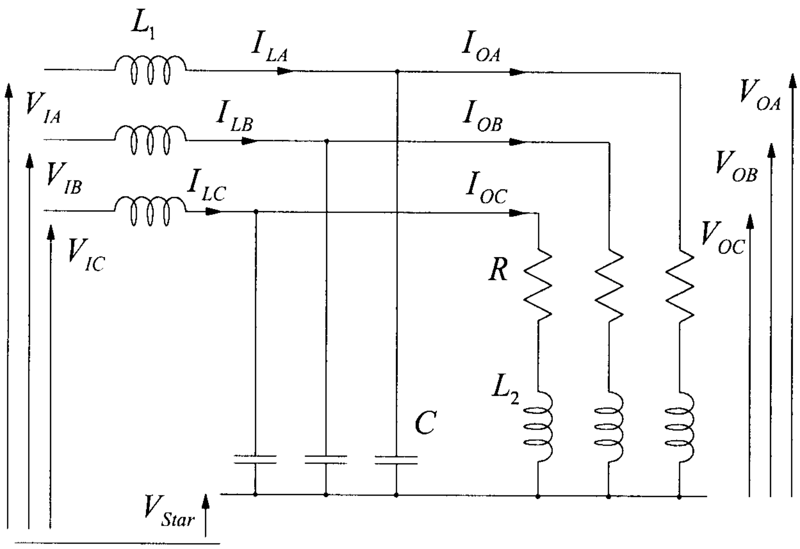


Figure Q2

$$v_{1abc}=L_1\frac{di_{1abc}}{dt}+v_{2abc}+v_{Star}$$

$$v_{2abc}=L_2\frac{di_{2abc}}{dt}+R\,i_{2abc}$$

$$i_{1abc}-i_{2abc}=C\frac{dv_{2abc}}{dt}$$

$$v_{1dq\gamma} = L_1 \frac{di_{1dq\gamma}}{dt} + X_1\, i_{1dq\gamma} + v_{2dq\gamma} + v_{Star--\gamma}$$

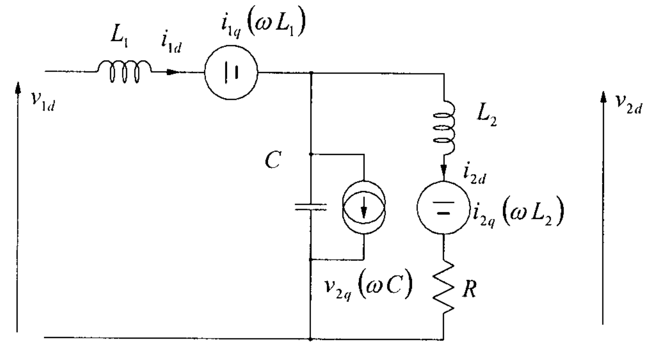
$$\text{where } X_1 = \begin{bmatrix} 0 & \omega L_1 & 0 \\ -\omega L_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad v_{Star--\gamma} = \begin{bmatrix} 0 \\ 0 \\ 3v_{Star} \end{bmatrix}$$

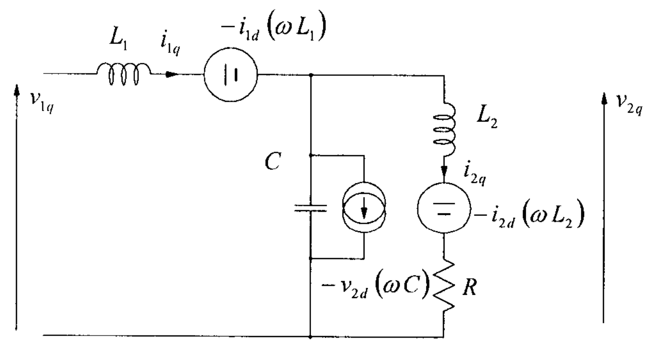
$$v_{2dq\gamma} = L_2 \frac{di_{2dq\gamma}}{dt} + X_2\, i_{2dq\gamma} + R\, i_{2dq\gamma}$$

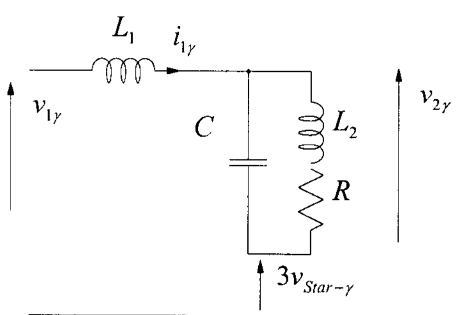
$$\text{where } X_2 = \begin{bmatrix} 0 & \omega L_2 & 0 \\ -\omega L_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$i_{1dq\gamma} - i_{2dq\gamma} = C \frac{dv_{2dq\gamma}}{dt} + B_C\, v_{2dq\gamma}$$

$$\text{where } \mathrm{B}_C = \begin{bmatrix} 0 & \omega C & 0 \\ -\omega C & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$







3.

(a) Starting from the voltage equation for the referred model of an induction machine,

$$[v'_{DQ}] = [R'_{DQ}][i'_{DQ}] + [X'_{DQ}][i'_{DQ}] + [L'_{DQ}]\frac{d}{dt}[i'_{DQ}]$$

show that the torque produced by the machine is related to the product of stator current and rotor current. The rotational voltage matrix, X'_{DQ} is:

$$[X'_{DQ}] = \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M' \\ \omega L_S & 0 & \omega M' & 0 \\ 0 & -P\omega_{slip} M' & 0 & -P\omega_{slip} L'_R \\ P\omega_{slip} M' & 0 & P\omega_{slip} L'_R & 0 \end{bmatrix}$$

[10]

The power equation is formed by multiplying by i^T.

$$p = [i'_{DQ}]^T [R'_{DQ}][i'_{DQ}] + [i'_{DQ}]^T [X'_{DQ}][i'_{DQ}] + [i'_{DQ}]^T [L'_{DQ}]\frac{d}{dt}[i'_{DQ}]$$

The second term represents the power conversion to mechanical form.

$$p_{EM} = [i'_{DQ}]^T [X'_{DQ}][i'_{DQ}]$$

$$= \begin{bmatrix} i_{SD} \\ i_{SQ} \\ i'_{RD} \\ i'_{RQ} \end{bmatrix}^T \begin{bmatrix} 0 & -\omega L_S & 0 & -\omega M' \\ \omega L_S & 0 & \omega M' & 0 \\ 0 & -P\omega_{slip} M' & 0 & -P\omega_{slip} L'_R \\ P\omega_{slip} M' & 0 & P\omega_{slip} L'_R & 0 \end{bmatrix} \begin{bmatrix} i_{SD} \\ i_{SQ} \\ i'_{RD} \\ i'_{RQ} \end{bmatrix}$$

$$= \begin{bmatrix} i_{SD} & i_{SQ} & i'_{RD} & i'_{RQ} \end{bmatrix} \begin{bmatrix} -\omega L_S i_{SQ} - \omega M' i'_{RQ} \\ +\omega L_S i_{SD} + \omega M' i'_{RD} \\ -P\omega_{slip} M' i_{SQ} - P\omega_{slip} L'_R i'_{RQ} \\ +P\omega_{slip} M' i_{SD} + P\omega_{slip} L'_R i'_{RD} \end{bmatrix}$$

$$\begin{aligned} &= \omega L_S(-i_{SD} i_{SQ} + i_{SQ} i_{SD}) \\ &\quad + \omega M'(-i_{SD} i'_{RQ} + i_{SQ} i'_{RD}) \\ &\quad + P\omega_{slip} M'(-i'_{RD} i_{SQ} + i'_{RQ} i'_{SD}) \\ &\quad + P\omega_{slip} L'_R(-i'_{RD} i'_{RQ} + i'_{RQ} i'_{RD}) \\ &= (\omega - P\omega_{slip}) M'(i_{SQ} i'_{RD} - i_{SD} i'_{RQ}) \end{aligned}$$

$$T = \frac{p_{EM}}{\omega_R} = \frac{(\omega - P\omega_{slip})}{\left(\frac{\omega}{P} - \omega_{slip}\right)} M'(i_{SQ} i'_{RD} - i_{SD} i'_{RQ})$$

$$= PM'(i_{SQ} i'_{RD} - i_{SD} i'_{RQ})$$

(b) Within the context of a filed orientation controller of an induction machine, explain why it is desirable to orientate the *d*-axis to the rotor flux linkage. [4]

The torque equation can be re-expressed in terms of flux linkage:

If the model is oriented so that the d-axis is aligned to the rotor flux linkage, then there is no q-axis component of rotor flux linkage and the torque equation becomes a simple product. If one of the terms is held constant then the torque becomes proportional to the other and a simple, linear torque controller has been formed.

(c) Give a reason why, in a field orientation controller, the flux linkage magnitude is controlled to be constant and the torque is set via the q-axis current (rather than adopting the opposite approach). [2]

Because the rotor flux linkage is established by d-axis current flowing in the magnetising inductance which is a long time constant circuit (formed with the rotor resistance). Thus changes in flux linkage are slow to effect. Fast response is obtained by keeping the flux linkage constant and varying torque through the q-axis current which does not flow in the magnetising inductance but instead in a low time constant circuit involving only the leakage inductances.

(c) An induction machine is normally supplied from a voltage source inverter whereas the toque equation from (a) is a function of current. Explain what steps are taken to enable field orientation control to be achieved with a voltage source rather than a current source. [4]

- *Current feedback is applied*
- *Decoupling (feed-forward) terms are applied to counter-act coupling of currents in one axis to voltages in the other.*
- *Slip calculator must operate from measured not demand currents.*

4.

[*Bookwork*]

(a) Angle stability in a power system is the ability of interconnected synchronous machines to remain in step with each other. The rotor angle stability problem involves the study of electro-mechanical oscillations. The fundamental factor of rotor angle stability is the manner in which power output of a synchronous machine varies with rotor oscillation. The rotor angle stability problem can be defined in two different ways depending on the nature and the location of the disturbances in the system;

(i) Small signal stability is the ability of power system to maintain synchronism under small disturbances.

(ii) Transient stability is the ability of power system to maintain synchronism when subjected to severe transient disturbance. The resulting system response involves large excursions of generator rotor angles and is influenced by non-linear power angle relationship.

Voltage stability is the ability of power system to maintain steady acceptable voltage at all the busses in the system under normal operating conditions and after busses have been subjected to disturbances. The system enters into a state of voltage instability when an increased demand in load or a change in the system condition causes progressive decrease in voltage. The main factor causing the voltage instability is the inability of the power system to meet the demand for reactive power.

In short, angle stability related issues are encountered when the balance between real power generation and loads is not zero. Voltage stability related problems are encountered in the system when the balance between the reactive power generation and consumption is not zero. [5 marks]

(b) [*Book work*] Oscillatory stability is the dynamic behaviour of the system subject to small disturbances or the behaviour of the system following the clearing of a large disturbance such as a fault. The term *oscillatory stability* and *small signal stability* are interchangeably used. In small signal stability, the following types of oscillations are of concern.

(i) **Intra-plant modes:** Here the machines within a power plant oscillate. Here frequency range of oscillation is 2-3Hz.

(ii) **Local modes:** Local modes are associated with swinging of units at a generating station with respect to the rest of the power system. The oscillations are localised at one station. Here frequency range of oscillation is 1-2 Hz.

(iii) **Inter-area modes:** Inter-area modes are associated to the swings of many machines in one part of the system against a group of machines in the other parts. As the number of machines involved here is large, frequency of oscillation is lower compared to local modes. Here frequency range of oscillation is 0.1-0.9 Hz.

(iv) **Control modes:** Control modes are associated with generating units and other control units like poorly tuned exciters, speed governors, HVDC converters and SVC. The nonlinear interaction between exciter and loads leads to oscillatory response in bus voltage.

(v) **Torsional modes:** Torsional modes relate to oscillation of various stages of a steam turbine shaft with the electrical network. Instability in torsional modes may be caused by interaction with excitation controls, speed governors, HVDC controls, and series capacitor compensated lines. [5 marks]

(c) [*application of theory*] In this model, machine angle δ and machine speed ω are the state variables. In order to obtain a linear state space model, linearization of the dynamic equations needs to be carried out. Let me give a small perturbation of $\Delta\delta, \Delta\omega$ and ΔP_m around an initial δ_0, ω_0 and P_{m0} and rewrite the dynamic equation as follows. The synchronous speed ω_s is a fixed quantity and so disappears from the equations. This is worked out as follows.

$$\frac{d(\delta_0 + \Delta\delta)}{dt} = (\omega_0 + \Delta\omega) \tag{4.1}$$

$$M\frac{d}{dt}(\omega_0 + \Delta\omega) = P_{m0} + \Delta P_m - P_{\max}\sin(\delta_0 + \Delta\delta) - K_D(\omega_0 + \Delta\omega) \tag{4.2}$$

Where $P_{m0} = P_{\max}\sin\delta_0$. The DC terms on both sides balance out. With the following approximation, the small signal model is written as:

$$\frac{d\Delta\delta}{dt} = \Delta\omega \tag{4.3}$$

$$M\frac{d\Delta\omega}{dt} = \Delta P_m - P_{\max}\cos\delta_0 - K_D\Delta\omega \tag{4.5}$$

$$y = \Delta\omega \tag{4.6}$$

[7marks]

Approximation: for small $\Delta\delta$

$$\cos\Delta\delta \approx 1.0;$$

$$\sin\Delta\delta \approx \Delta\delta$$

Taking $\Delta\delta, \Delta\omega$ as X, ΔP_m as input (u) and $\Delta\omega$ as output (y) the above can be expressed in standard state-space form (A,B,C,D)

$$\begin{bmatrix}\Delta\dot{\delta}\\ \Delta\dot{\omega}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{1}{M}P_{\max}\cos\delta_0 & -\frac{K_D}{M}\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{M}\end{bmatrix}[\Delta P_m] \tag{4.7}$$

$$y = \Delta\omega \tag{4.8}$$

Where

$$A = \begin{bmatrix}0 & 1\\ -\frac{1}{M}P_{\max}\cos\delta_0 & -\frac{K_D}{M}\end{bmatrix}, B = \begin{bmatrix}0\\ \frac{1}{M}\end{bmatrix}$$

[3 marks]

$$C = \begin{bmatrix}0 & 1\end{bmatrix} \quad \text{and } D[0]$$

5 (a) (i) **Damper Winding**: The stability problem in power system dates back to early 1920's. Examples arose from a group of synchronous machines interconnected or a single machine used to supply variable induction motor loads in mining industry or pulsation in input torque of diesel engine used as prime mover. The nature of the variation was such that low frequency oscillations known in those days as hunting used to persist over around synchronous speed. This phenomenon was studied in detail by many power engineers and remedy was suggested in the form of closed winding on the rotor periphery of low speed generator driven by water wheel or hydro turbine. These are thick winding that induces eddy currents of hunting frequency (these days known oscillatory modes 0.2-3.0 Hz). The eddy currents induced in these windings interact with main air gap flux produced by stator and developed a torque in antiphase with rotor oscillations. Thus the oscillations are damped out. That is how they are known as damper winding because of their function of damping oscillations. It is also known as amortisseurs winding named after the famous engineer Amortisseurs who worked in bringing this concept into synchronous machine stability. In turbo-generator, the speed is very high (3000/3600), the rotor field winding is distributed over the rotor periphery. The slots contains damper bar. The rotor itself is made of solid construction that provides significant amount of damping action.

[5 marks]

(ii) **Inter-area oscillations**: The inter-area oscillations are low frequency oscillations of interconnected system. Under these conditions, synchronous machine rotors of one area oscillate with the rotors of other areas. The frequency of oscillations is in the range of 0.2 to 1.0 Hz. The phenomenon is very complex as it involves several electromechanical oscillatory subsystems often comprising several groups of machines distributed over neighbouring utilities. This historical problem has been faced by many power utilities across the world for several decades. Inadequate damping restricts the maximum power transfer across the tie lines. The very general nature of the problem experienced by several interconnected power utilities can be described very briefly as follows:

- Inter-area oscillations are associated with weak transmission links and heavy power transfers.
- They are due to a natural mode of the system and therefore, cannot be eliminated. However, their damping and frequency can be modified.
- As power systems evolve, the frequency and damping of existing modes change and new ones may emerge.
- Inter-area oscillations often involve more than one utility and may require the co-operation of all to arrive at the most effective and economical solution.
- The generator amortisseurs winding is no longer effective as the damping produced at very low-frequency is reduced in approximately inverse proportion to the square of the effective external impedance plus the stator impedance. Therefore, the damping torque practically disappears.

[5 marks]

(iii) **Eigen-value sensitivity in small signal stability**: This a quantitative measure of variation of eigen-value with an individual element of system state matrix. This has great significance in power system oscillatory stability as this gives rise to a very useful method of designing power system stabiliser (PSS) and other damping devices. It can be described in detail as follows:

Let us examine the sensitivity of eigenvalues to the elements of the state matrix. We know

$$A\,\phi_i = \lambda_i\,\phi_i$$

When ϕ_i is the eigenvector corresponding to λ_i. Differentiating with respect to a_{kj} (the element of A in the k^{th} row and j^{th} column)

$$\frac{\partial A}{\partial a_{kj}}\varphi_i + A\frac{\partial \varphi_i}{\partial a_{kj}} = \frac{\partial \lambda_i}{\partial a_{kj}}\varphi_i + \lambda_i \frac{\partial \varphi_i}{\partial a_{kj}} \qquad (5.1)$$

Premultiplying by ψ_i, and noting that $\psi_i \phi_i = 1$ for normalised eigenvector and $\psi_i (A - \lambda_i I) = 0$, we see that the above equation simplifies to

$$\psi_i \frac{\partial A}{\partial a_{kj}}\phi_i = \frac{\partial \lambda_i}{\partial a_{kj}} \qquad (5.2)$$

All elements of $\frac{\partial A}{\partial a_{kj}}$ are zero, except for the element in the k^{th} row and j^{th} column witch is equal to 1. Hence,

$$\frac{\partial \lambda_i}{\partial a_{kj}} = \psi_{ik}\,\phi_{ji}$$

Thus the sensitivity of the Eigen-value λ_i to the element a_{kj} of the state matrix is equal to the product of the left eigenvector element ψ_{ik} and the right eigenvector element ϕ_{ji} . This sensitivity information is used in stabiliser design employing output feedback. [5marks]

(iv) **FACTS Devices**: It is established that power flow in HVAC system is limited by voltage and stability considerations. For long distance transmission, voltage drop in the series inductance of the line is too high preventing adequate power transfer through the line without compensations. High power electronic devices when installed in key locations of the system can compensate for voltage drop by providing local reactive power support. This moves up the power practical power transfer capacity of the lines towards thermal capacity. The devices are either installed at bus (known as shunt device), such as Static VAr compensator (SVC), or in the line (known as series devices), such as Thyristor Controlled Series Capacitor (TCSC), Thyristor Controlled Phase Shifter (TCPS) and off late hybrid configuration is being introduced. The functions of these devices are to compensate for line voltage, phase shift and bus voltage magnitudes. In the process, they control real and reactive power flow through lines. The added benefits they bring in are greater transient and small signal stability margin of the system with very fast and effective control. [5 marks]

(v) **Effect of Automatic Voltage Regulator (AVR) on power system stability**: Excitation system provides desired rotor field current. When it is in manual mode it has very slow control over bus voltage variation and cannot follow or influence any system changes quickly because of continuous load change or disturbance in the system. AVR is a device that regulates the system voltage to desired level within in defined region of system variation. During a large disturbance, when generator output voltage collapses, AVR exercises field-forcing option through the exciter to arrest the collapse in voltage so that the transient stability is ensured. The AVR for large generators are of high gain and fast acting. This is chosen deliberately for better transient stability performance. But this has an adverse effect on the small signal stability where the frequency of oscillations is in the range of 0.1 to 2.0 Hz. It is established that AVR introduces negative damping torque and thus contributes negatively to the low frequency oscillations of the system. This is taken care of by the excitation stabiliser circuit or transient gain reduction circuit in the AVR control loop. The best way to deal with this problem is to install power system stabiliser as supplementary block to AVR system. [5 marks]

(vi) **Mid-term and long-term stability** deals with the problems associated with dynamic response of power system to severe upsets. Severe system upset results in large excursion of voltage, frequency and power flows and invokes the action of slow processes, controls and protections not modelled in conventional transient stability studies. The characteristics times of the processes and devices activated by large voltage and frequency shifts will range from seconds to several minutes.

In long-term stability problem, the focus is on slower and larger duration phenomena, those accompany large system upsets and sustained mismatch between generation and consumption of active and reactive power. These phenomena include boiler dynamics of thermal units, penstock and conduit dynamics of hydro units, automatic generation control, power plant and transmission system protection/ controls, transformer saturation and off-nominal frequency effects on the loads and the network. In the mid-term stability studies, the focus is on synchronising power oscillations between machines, including the effect of some slower phenomena, and possibly large voltage and frequency excursions. The time period importance in mid term stability is between 10 seconds to few minutes and for the long term: it is few minutes to 10 minutes. The distinction between mid-term and long-term stability is primarily based on the phenomenon being analysed and system representation being used, particularly with regard to fast transients and intermediate oscillations, rather than time involved [5 marks]

6 (a) [*Book work*] It is an established fact that the high gain and fast acting excitation control system , designed for better large signal stability performance of the system deteriorates the small signal stability performance of the system. This results in undue low frequency electromechanical oscillation following small disturbance or after fault clearing in the post-fault recovery system. These oscillations are damped out through an additional controller, known as supplementary excitation controller or power system stabiliser (PSS). The voltage reference point of the excitation system is modulated by the action of the PSS. In the process, any negative damping introduced by the AVR is compensated for plus additional damping is introduced in the desired frequency range to provide an adequate damping to the system. As a results, any oscillations that could have persisted otherwise, settle down quickly. [5 marks]

The common input signals to a power system stabiliser are rotor speed, bus frequency, rotor accelerating power etc. These are local signals. There is growing practice of using some remote or synthesised signals. These are area frequency errors, apparent impedance etc. Each of these signals has its advantage, as well as disadvantage. Whilst local signals are more reliable, remote signals are more effective in visualising the system dynamics. [3 marks]

(b) *[Bookwork]*The basic function of a governor is to control speed and/or load. The general requirement of load/frequency control is that any change in total system loads are shared by the units equitably. This is realised by introducing a slope in turbine output power versus speed characteristic. This is known as droop. A 5% droop means that a drop of 5% in no-load speed results when the gate or valve opens from fully closed position (no output) to fully open position (rated output). The model of a hydro turbine is a non-minimum phase one, i.e a right half plane zero is present. This imposes a limit on the upper value of gain (reciprocal of droop) for stable operation. This means that the droop setting has to be larger. [5]

(c) [*Computed example*] The closed-loop characteristic equation of the turbine-governor system in Fig 6.1 of this question as function of governor droop R is:

$$1+\frac{1}{R}\left(\frac{1-T_W s}{1+0.5T_W s}\right)\left(\frac{1}{T_M s+K_D}\right)=0 \tag{6.1}$$

For $T_W=2.0$, $T_M=10.0$ and $K_D=0$ this simplifies to

$$10Rs^2+(10R-2)s+1=0\,. \tag{6.2}$$

For stable operation, the roots of (6.2) should be in the left half of eigen plane. This requires that:

$$R>0$$

$$10R-2>0$$

That means R > 0.2 or the droop should be more than 20%

[7]