The Answers

(comments on student mistakes in blue)

1.

a) iii) diffusion of minority carriers through the different device regions

Majority carriers are injected across the junction in forward bias because the **potential barrier decreases** in forward bias. After injection, they become minority carriers. Since the voltage drops occurs mainly across the junction, the movement of these minority carriers in the neutral regions is due to diffusion to the contacts. The contacts have infinite generation and recombination capabilities and thus keep the minority carrier concentration there at the bulk value, creating the **minority carrier gradient**. For all normal pn diode operations the variation in majority carrier concentration can be neglected. [5]

Mistakes are often made against the carrier type that is injected across the potential barrier. It are **majority** carriers that diffuse (injected) across the lowered potential barrier. It are **minority** carriers that diffuse in the neutral regions.

b) First calculate the minority carrier diffusion length. From the formulae list:

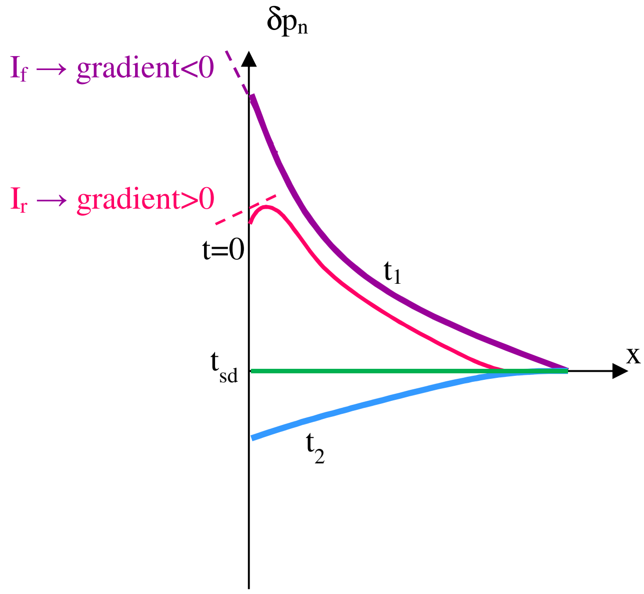
$$L = \sqrt{D\tau} \text{ and } D = V_T \mu = 0.026V \times 120\, cm^2 V^{-1} s^{-1} = 3.12\, cm^2 s^{-1}$$

$$L = \sqrt{3.12 \times 2.88\, 10^{-10}}\, cm \approx 3\, 10^{-5}\, cm = 300\ nm$$

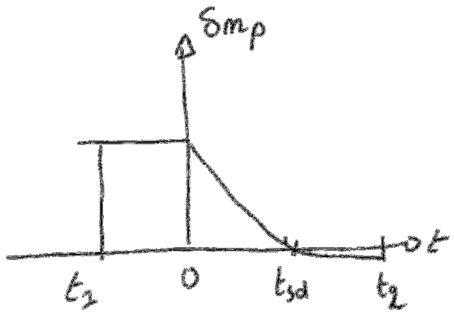
The minority carrier diffusion length is equal to the length of the material. In that case the error due to the long material and short material approximation is exactly the same. [5]

Many students cannot see that 3 10^{-5} cm = 300 nm or forget to write the unit and then get confused. Units and conversions between them is something an engineer in the second year should not make any errors against.

c) [5]

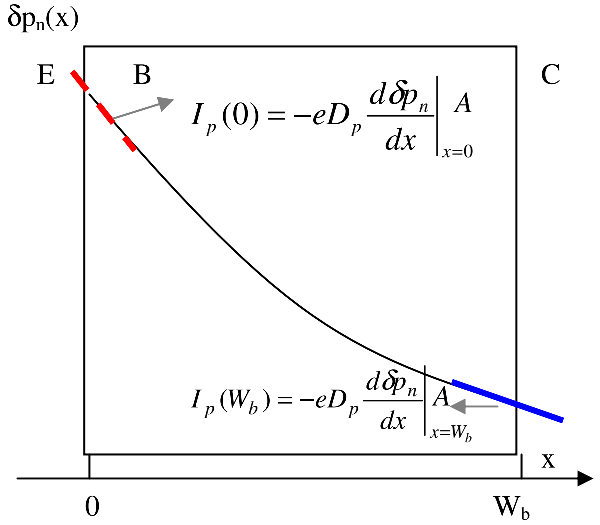


Often the following solution was given:



Although this solution doesn't answer the question, as a sketch in function of **distance** has been asked for, the above solution indicates understanding in what is happening. This solution received 4/5 because it does not show understanding of the non-linear minority carrier variation nor of the change in gradient of the minority carrier concentration at the depletion region edge.

d) [5]



The hole current injected across the EB junction at x=0 is proportional to the slope of the excess minority carrier concentration at x=0. The hole current injected across the BC junction is given by the slope of the minority carrier concentration at x=W_b. These slopes are different in the case recombination is taken into account. The difference between these two hole currents is the hole current recombined in the base layer.

Comment: [1] for having the minority carrier type correct
[1] relation of current to slope
[3] the correct formulae with the correct slopes and position thereof

Common mistakes:
Relating the current to the absolute value of the minority carrier concentration at a position in x.
Giving the equations for electron diffusion current in the base. This is not correct because it are holes that diffuse.
Giving the diode current expressions from the formulae list. These are not valid in the base of the BJT when recombination has to be taken into account.
Stating $I_B = I_E - I_C$ is correct but I_B is in this case $I'_B + I''_B$. Only I''_B is asked for. Some answers explicitly stated that $I_B = I_E - I_C = I''_B$ **when γ=1**. That is accepted as correct answer.
Alternative solution used:
$I''_B = Q_p/\tau_p$ with Q_p the area underneath the δp_n graph and τ_p the lifetime of the carriers. This is also correct. Note, it is not needed in this case to make the approximation γ=1.

2.

Table 2.1: all parameters in same unit - cm.

Material parameters	n-region	p-region
$N_{D,A}$ (cm^{-3})	8×10^{19}	2×10^{17}
$X_{n,p}$ (cm)	2.5×10^{-5}	5×10^{-2}
$\tau_{p,n}$ (s)	9×10^{-10}	1×10^{-6}
$L_{p,n}$ (cm)	3×10^{-5}	5×10^{-3}
$D_{p,n}$ ($cm^2\ s^{-1}$) = L^2/τ	1	25
A (cm^2)	1×10^{-6}	1×10^{-6}

a)

n-region is short, p-region is long.

Hole current in n-region:

$$\mathrm{I_p} = \frac{e\,D_p\left(p'_n - p_{n_0}\right)A}{X_n} \approx \frac{e\,D_p\,p'_n\,A}{X_n} = \frac{e\,D_p\,p_{n_0}\exp\left(\frac{V_d}{V_T}\right)A}{X_n} = \frac{e\,D_p n_i^2\exp\left(\frac{V_d}{V_T}\right)A}{N_D X_n}$$

$$\mathrm{I_p} = \frac{1.6\times10^{-19}\,C\,1\frac{cm^2}{s}\left(1.45\times10^{10}\right)^2 cm^{-6}\exp\left(\frac{0.5}{0.026}\right)10^{-6}\,cm^2}{8\times10^{19}\,cm^{-3}\,2.5\times10^{-5}\,cm} = 3.78\times10^{-12}\,A$$

Electron current in p-region:

$$\mathrm{I_n} \approx \frac{e\,D_n n_{p_0}\exp\left(\frac{V_d}{V_T}\right)A}{L_n} = \frac{e\,D_n n_i^2\exp\left(\frac{V_d}{V_T}\right)A}{N_A L_n}$$

$$\mathrm{I_n} \approx \frac{1.6\times10^{-19}\,C\,25\frac{cm^2}{s}\left(1.45\times10^{10}\right)^2 cm^2\exp\left(\frac{0.5}{0.026}\right)10^{-6}\,cm^2}{2\times10^{17}\,cm^{-3}\;5\times10^{-3}\,cm} = 1.89\times10^{-10}\,A$$ [5]

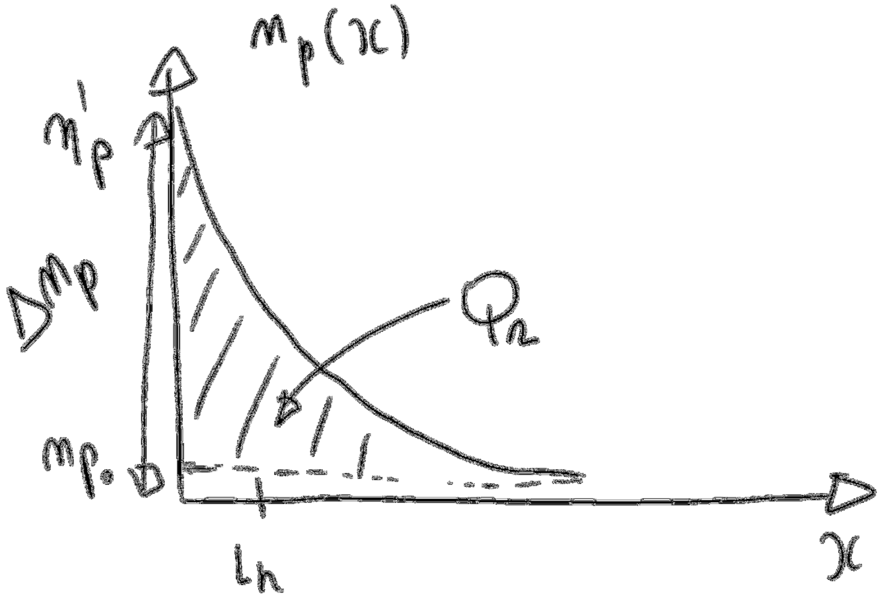
$$\mathrm{I_{tot}} = I_p + I_n = 1.93\times10^{-10}\,A \approx I_n = 1.89\times10^{-10}\,A$$

Comment: the diffusion length in the n-region is that of the holes and in the p-region that of the electrons. Many errors on this.
Note that the p-region is short, thus the diffusion length in the diode diffusion current in the formulae list must be replace by the actual length of the material.
Note, there is no point extracting the diffusion current equations, those in the formulae list can be used, you just have to remember that in the short layer, the length of the material is smaller than the diffusion length and thus the gradient will be higher (change length).
Plenty of errors against units (mixing of μm and cm) and forgetting the area.
In calculated via Q_n/τ_n is good with Q_n minority carrier electron charge in p-region. This approach cannot be taken for the hole current unless the correct equation for $p_n(x)$ is used.

b)

In the calculation of the forward bias current we see that the electron current in the n region is 100× larger than the hole current. Thus we assume the largest time delay is associated to the largest current component as this holds the largest stored minority carrier charge. The switching experiment is to open circuit, thus charges are removed via recombination. We also see that the recombination time of the electrons in the p-region is much larger than that of holes in the n-region thus the assumption that recombination in the p-region is determining the delay seems appropriate.

Plotting the minority carrier variation in the p-region:



Assume that the long material approximation is acceptable then the excess carrier concentration in the p-layer is given by the formula from the formulae sheet:

$$\delta n_p = \Delta n_p \exp\left(\frac{-x}{L_n}\right)$$

The charge in the p-type region can be calculated by integrating the above equation. However, since we have assumed the long material approximation, we need to put the contact at infinite in the integration.

$$Q_n = -e \times A \times \int_0^\infty \Delta n_p \exp\left(\frac{-x}{L_n}\right) dx = -e \times A \times \Delta n_p \times \int_0^\infty \exp\left(\frac{-x}{L_n}\right) dx$$

$$Q_n = e \times A \times \Delta n_p \times L_n \times \exp\left(\frac{-x}{L_n}\right)\Bigg|_0^\infty = e \times A \times \Delta n_p \times L_n \times (0-1) = -e \times A \times \Delta n_p \times L_n$$

from the formulae list:

$$\Delta n_p = n_p^{'} - n_{p0} = n_{p0}\left(\exp\left(\frac{eV}{kT}\right) - 1\right)$$

If we neglect the change in slope @ x=0 in $n_p(x)$, we can write:

$$Q_n(t) = -e \times A \times L_n \times n_{p0}\left(\exp\left(\frac{e\,v(t)}{kT}\right) - 1\right)$$

On the other hand, the time variation of the charge is given by:

$$Q_n(t) = I_n \tau_n \exp\left(\frac{-t}{\tau_n}\right)$$

$$I_n \tau_n \exp\left(\frac{-t}{\tau_n}\right) = e \times A \times L_n \times n_{p0}\left(\exp\left(\frac{e\,v(t)}{kT}\right) - 1\right)$$

$$\exp\left(\frac{e\,v(t)}{kT}\right) = \frac{I_n \tau_n}{e \times A \times L_n \times n_{p0}} \exp\left(\frac{-t}{\tau_n}\right) + 1$$

$$\frac{e\,v(t)}{kT} = \ln\left(\frac{I_n \tau_n}{e \times A \times L_n \times n_{p0}} \exp\left(\frac{-t}{\tau_n}\right) + 1\right)$$

$$v(t) = V_T \ln\left(\frac{I_n \tau_n}{e \times A \times L_n \times n_{p0}} \exp\left(\frac{-t}{\tau_n}\right) + 1\right)$$

c) Calculate the time needed for the voltage across the diode to become 0. [3]

In principle this could be calculated from:

$$0 = V_T \ln\left(\frac{I_n \tau_n}{e \times A \times L_n \times n_{p0}} \exp\left(\frac{-t_{OFF}}{\tau_n}\right) + 1\right)$$

However ln(y) only becomes zero for y = -∞ this for $t_{OFF} \to +\infty$

Correct assumption -> [1]

Correct statement on voltage not disappearing immediately and reason why -> [2]

Solving for Q(t) which is in fact already given receives [2] when done correctly and no other faults are made.

Solution can also be found on p26 and 27 of the course notes. The only advice that can be given here is probably that just solving past exam papers is not sufficient to cover the whole course contents. Any calculation in which current is used after switch should be immediately seen as not appropriate as i = 0 after switch as it is an open circuit.

3.

a) The term bipolar means that both holes and electrons (thus both carrier types) play a role in the description of the current of the device. [3]

Note: you have to guarantee that your statement ensures there is a clear difference between a MOSFET (unipolar device) and a BJT (bipolar device). Thus just stating junctions and voltage differences across junctions are not sufficient because they occur in both devices.

b) $\gamma = 1$ means that there is no current from base into emitter. The base current is then only composed of recombination re-supply current. In an npn BJT the base current are holes, the collector and emitter current electrons.

$$I_C = I_n = \frac{Q_n}{\tau_t}$$

$$I_B = I_p = \frac{Q_n}{\tau_n}$$

Q_n is the minority carrier excess charge (electrons) in the base.

τ_t is the transit time – the time it takes the injected electron to cross the base from emitter to collector junction.

τ_n is the lifetime of the minority carriers in the base. Is the average time the minority carriers spend in the base before recombining. [5]

Note: If taking I_{CB0} into account here, this current needs to be introduced in both I_B as well as I_C. Signs are important though. However it should be realised that in forward active mode this current is negligible.

$$I_C = I_n = \frac{Q_n}{\tau_t} + I_{CB0}$$

$$I_B = I_p = \frac{Q_n}{\tau_n} - I_{CB0}$$

c) Once it is established that collector and base current can be expressed in simple expressions as given in b), the current gain $\beta = \dfrac{I_C}{I_B} = \dfrac{Q_n/\tau_t}{Q_n/\tau_n} = \dfrac{\tau_n}{\tau_t}$.

The transit time formula is given in the formulae sheet:

$$\tau_t = \frac{W_B^2}{2D_n}$$

The effective base width will be the metallurgic base width minus the depletion width extending into the base region. The depletion region expressions are given in the formulae sheet:

$$w_p = \sqrt{\frac{2\varepsilon}{e}\frac{N_D}{N_A N_D + N_A^2}\left(V_0 - V\right)}$$

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.026V \ln\left(\frac{10^{17} \times 5\times10^{16}}{(1.45\times10^{10})^2}\right) = 0.80V$$

$$w_p = \sqrt{\frac{2\varepsilon}{e}\frac{N_D}{N_A N_D + N_A^2}(V_0 - V)} = \sqrt{\frac{2\times11\times8.85\times10^{-14}}{1.6\times10^{-19}}\frac{5\times10^{16}}{5\times10^{16}\times10^{17}+10^{34}}(0.8-(-1))}$$

$$w_p = 8.5\times10^{-6}\,cm$$

The base width becomes: $W_B = 10^{-4} - 8.5\times10^{-6} = 9.15\times10^{-5}\,cm$

The transit time:

$$\tau_t = \frac{W_B^2}{2D_n} = \frac{(9.15\times10^{-5})^2}{2\times6} = 6.98\times10^{-10}\,s$$

The current gain:

$$\beta = \frac{\tau_n}{\tau_t} = \frac{10^{-6}}{6.98\times10^{-10}} = 1433$$

Table 3.1 Material parameters of the bipolar junction transistor in cm.

Material parameters	emitter	base	collector
Doping density (cm^{-3})	10^{20}	10^{17}	5×10^{16}
Region length (cm)	0.1×10^{-4}	1×10^{-4}	5×10^{-2}
Diffusion constant ($cm^2\,s^{-1}$)	2	6	20
Minority carrier lifetime (s)	1×10^{-10}	1×10^{-6}	1×10^{-2}
Cross sectional area (cm^2)	1×10^{-6}	1×10^{-6}	1×10^{-6}

Calculation of the current in the base using the long diode equation is not acceptable because the long diode approximation assumes the layer is much longer than the minority carrier diffusion length.
If "taking the effect of the BC junction into account" is interpreted as taking I_{CBO} into account, one needs to calculate this current in the collector where the long diode approximation is acceptable. Doing this does not give you the correct solution because in forward active mode I_{CBO} is effectively negligible but the influence of the depletion width is not (e.g. base width modulation in forward active mode).