1.

a)

Hole concentration p = p_i = 1.45 10^{10} cm^{-3}
Electron concentration n = n_i = 1.45 10^{10} cm^{-3} [2]

No problem

b) There are two possible approaches.

1)

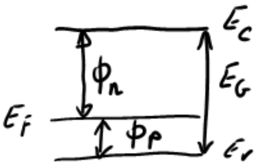
Use the equation for the list on p.2:

$$p = N_v e^{\frac{(E_v - E_F)}{kT}}$$

Rewrite to $E_F - E_v = kT \ln\left(\frac{N_v}{p}\right)$ use p = N_A = 10^{17} cm^{-3}

$$E_F - E_v = 0.026eV \ln\left(\frac{1.8\,10^{19}\ cm^{-3}}{10^{17}\ cm^{-3}}\right) = 0.135eV$$

Then use the band gap E_G = 1.12 eV
Based on the following energy band diagram:



We can derived that $E_G = \phi_n + \phi_p$ and thus $\phi_n = E_c - E_F = E_G - \phi_p$ = 1.12 eV – 0.135 eV = 0.985 eV

2) Start from the electron concentration:

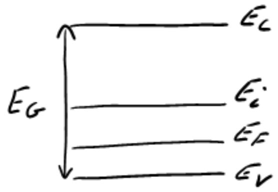
$$n = N_c e^{-\frac{(E_c - E_F)}{kT}} \text{ and } n = \frac{n_i^2}{N_A}$$

Rewrite to $E_c - E_F = kT \ln\left(\frac{N_c N_A}{n_i^2}\right)$

Fill in numbers $E_c - E_F = 0.026eV \ln\left(\frac{3.2\,10^{19}\ 10^{17}}{(1.45\,10^{10})^2}\right)$ = 0.969 eV [5]

Mainly silly mistakes or not realising this is p-type material

c) [5]



This question was really easy and answered correctly even though the previous one was incorrect. This is actually strange as it shows that the connection between the two questions was not spotted.

d) Formula to know by heart (A-level physics). [5]

$$R = \frac{\rho \times y}{x \times z} \text{ and } \rho = \frac{1}{\sigma} = \frac{1}{en\mu_n + ep\mu_p} \approx \frac{1}{ep\mu_p}$$

$$R = \frac{1}{ep\mu_p} \times \frac{y}{x \times z} = \frac{10^{-2}\ cm}{1.6\,10^{-19} C\ 10^{17} cm^{-3}\ 200\ cm^2/Vs\ 2000\,10^{-7} cm\ 0.5\,10^{-1} cm}$$

$$R = 312.50\Omega$$

This question was intended to be all about conversions of units, however a surprising number of students was unable to do this question, in general because they didn't know the expression of ρ. This is surprising in view of the fact that it is inherently available in the expression of the drift-diffusion equation given in the formulae sheet. It can be done as follows:

$V = R.I \rightarrow R = \frac{V}{I}$. $I = JA$ $\quad V = \mathcal{E} \times l$

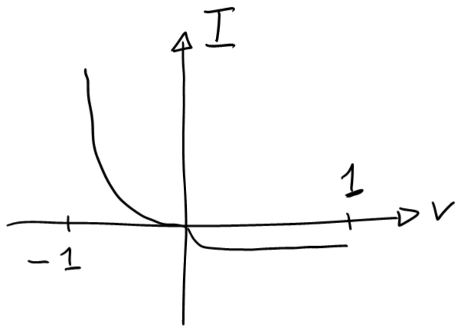
formulae sheet: $J_p = ep\mu_p \mathcal{E} - eD_p \frac{dp}{dx}$

only J_p because p-type material and only drift important.

$$R = \frac{\mathcal{E}\, l}{ep\mu_p \mathcal{E} A}$$

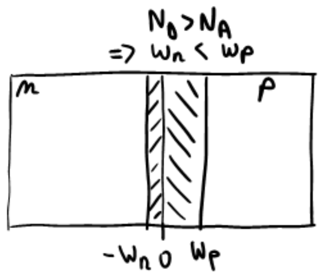
e)

i) [4]



Most graphs were flipped around the I axis. Note that the on diode is reverse biased for V>0 in the given biasing configuration.

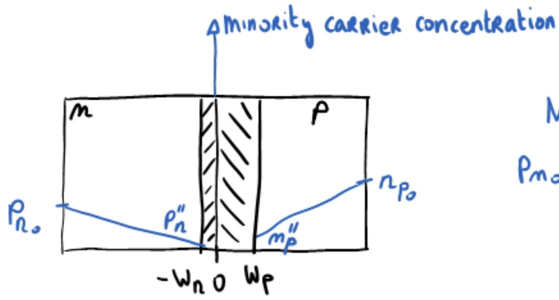
ii) [4]



cross hatched region are the depletion regions.

This sketch is independent of bias configuration. The key parts are where a depletion region is positioned and the $w_n < w_p$.

iii) [5]



$N_D > N_A$

$p_{n_o} = \frac{n_i^2}{N_D} < n_{p_o} = \frac{n_i^2}{N_A}$

Any bias configuration was acceptable. It is all about relative magnitudes.

f) $V_{eE} < V_{eB}$ (forward biased pn diode).
$V_{eB} < V_{eC}$ (reverse biased pn diode) [4]

Simple question, looked at during revision and basic knowledge for analogue electronics

g) $I_{C1} = I_{C2}$ [2]

The current density across the E-B pn diode is given by the formula:

$$J_{tot} = J_n + J_p = \frac{eD_n n_{p_0}}{W_B}\left(e^{\frac{eV}{kT}} - 1\right) + \frac{eD_p p_{n_0}}{X_n}\left(e^{\frac{eV}{kT}} - 1\right)$$

With W_B the base width and X_n the emitter width. Rewriting in function of doping:

$$J_{tot} = J_n + J_p = \frac{eD_n n_i^2}{N_A W_B}\left(e^{\frac{eV}{kT}} - 1\right) + \frac{eD_p n_i^2}{N_D X_n}\left(e^{\frac{eV}{kT}} - 1\right)$$

In an npn BJT the collector current is determined by the minority carrier diffusion current in the base, thus by J_n. Since the doping in the base is not changing between BJT 1 and BJT 2, J_n does not change and thus the collector current does not change. What changes is the base current I_B and thus the current gain β. [6]

A surprising number of students verify their answer with words rather than doing some simple maths. Most students who wrote the expression for the collector current were able to spot that the minority carrier concentration in the base does not change with emitter doping. And they got to the right answer. Arguments based on current gain β all led to the wrong conclusion because it actually starts from the assumption that I_c is changing while what is changing are I_B and thus I_E.

2.

a)

i) x = 0 [2]

ii) Under the depletion approximation we assume that the free carrier concentration in the depletion region is zero. Thus only the ionised charge density remains.

$$\rho(x) = -eN_A$$

$$Q(x) = -eN_A w_p A$$

Both have been accepted because of the misleading text in the question calling $_r$(x) charge instead of charge density.

[2]

iii) [6]

Poisson equation from formulae list:

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon}$$

Solving in the depletion region in the p-section.

$$\frac{dE}{dx} = \frac{-eN_A}{\varepsilon}$$

Integrate once:

$E = \frac{-eN_A}{\varepsilon} x + C_1$ with C_1 an integration constant. C_1 can be found from the boundary condition at the edge of the p-section depletion region. In x = -w_p, E(x) = 0.

$0 = \frac{eN_A}{\varepsilon} w_p + C_1$ thus $C_1 = \frac{-eN_A}{\varepsilon} w_p$

Thus $E(x) = \frac{-eN_A}{\varepsilon}\left(x + w_p\right)$. From the formulae list we have:

$w_p = \left[\frac{2\varepsilon V_{bi} N_D}{e(N_A + N_D)N_A}\right]^{1/2}$ since $N_D >> N_A$ we can simplify this expression to:

$w_p \approx \left[\frac{2\varepsilon V_{bi}}{eN_A}\right]^{1/2}$ thus

$$E(x) = \frac{-eN_A}{\varepsilon}\left(x + \left[\frac{2\varepsilon V_{bi}}{eN_A}\right]^{1/2}\right)$$

Mistake made here is making the following integration:

$\int_0^{E_{max}} dE = \int_{-w_p}^{0} \frac{-eN_A}{\varepsilon} dx$ which gives a value rather than a function (expression).

Second mistake was forgetting that V=0. No bias applied.

b)

i) $n(x) = 10^{17} - 5\times 10^{18} x$ (x in cm)

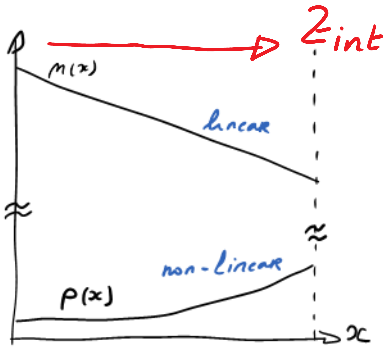
$$p(x) = \frac{n_i^2}{N(x)} = \frac{\left(1.45\times 10^{10}\right)^2}{10^{17} - 5\times 10^{18} x} = \frac{2.1\times 10^{20}}{10^{17} - 5\times 10^{18} x} = \frac{2.1\times 10^{3}}{1 - 50\, x}$$ [4]

Doping cannot become negative thus:

$$0 = 10^{17} - 5\times10^{18}x$$
$$x = 0.02cm$$

n(0) = 10^{17} cm^{-3}
n(0.02) = 0 cm^{-3}

p(0) = 2100 cm^{-3}
p(0.019) = 42000 cm^{-3} (infinite at 0.02 is not realistic)



Most important mistake is to draw p(x) linear. p(x) is non-linear when n(x) varies linearly. The other mistake made is to draw the doping concentration rather than n(x) the carrier concentration.

ii) Start from the drift-diffusion equation for electrons in the formulae sheet:

$$J_n(x) = e\mu_n n(x)E(x) + eD_n\frac{dn(x)}{dx}$$

Since the voltage is zero, the total current density has to be zero.

$$0 = e\mu_n n(x)E(x) + eD_n\frac{dn(x)}{dx}$$

$$E(x) = -\frac{D_n}{\mu_n n(x)}\frac{dn(x)}{dx}$$

Using Einstein's equation and differentiating $n(x) = 10^{17} - 5\times10^{18}x$:

$$E(x) = \frac{5\times10^{18}\times kT}{e\left[10^{17} - 5\times10^{18}x\right]} = \frac{5\times10^{18}\times0.026}{\left[10^{17} - 5\times10^{18}x\right]}$$

$$E(x) = \frac{13\times10^{16}}{\left[10^{17} - 5\times10^{18}x\right]} = \frac{13}{\left[10 - 500\,x\right]}$$

[4]

Students seem to have forgotten that bias applied is zero. Also answers mainly involved the depletion region, while here the answer is requested in the n-doped region, thus is charge neutral. As a consequence the poison equation is zero. The poison equation cannot be used to find the electric field.

iii) The internal electric field points to +x to cause drift opposite to diffusion. Eint $\rightarrow$

[2]

Note that you do not need the solution to ii) to find the solution to iii), you need to be aware of the physics that if you have diffusion in one direction you need an electric field that opposes this movement.

c) Charge in p-region: $\rho = e\left(p - n - N_A\right)$ [2]

Charge neutrality is required in the p-region. $0 = \left(p - n - N_A\right)$ [1]

Law of mass action: $n \times p = n_i^2$ [2] [10]

$$\begin{cases} n \times p = n_i^2 \\ p - n - N_A = 0 \end{cases}$$

$$p - \frac{n_i^2}{p} - N_A = 0 \quad (p \neq 0)$$

$$p^2 - N_A p - n_i^2 = 0$$ [5]

$$p = \frac{N_A \pm \sqrt{N_A^2 - 4n_i^2}}{2}$$

$$p = \frac{N_A + \sqrt{N_A^2 - 4n_i^2}}{2}$$

For $N_A >> n_i$ $\longrightarrow$ $N_A^2 - 4n_i^2 \approx N_A^2$

$$p \approx \frac{N_A + N_A}{2} = N_A$$

A major error made in the solution of this question is stating:

$p = \frac{n_i^2}{N_A}$ this cannot be done as this is actually what has to be proven.

$N_D = 0$ as there are no donor doping atoms in the p-type region (as given).

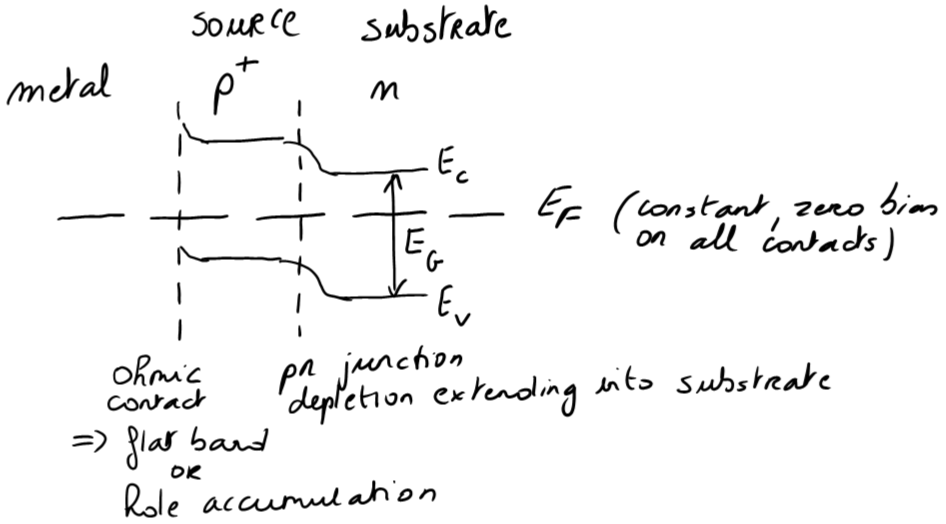
3. General remark: marks were lost by forgetting to write the correct units with the solutions in this question.

a)

i) p-channel or pMOS [2]

ii) source & drain p-type (heavily doped)
substrate region n-type [2]

iii) Should be an Ohmic contact on a heavily doped p-type region. Thus bend bending upwards towards metal. [6]



Mistakes were made by ignoring the metal-p+ contact junction and only drawing the pn junction. The other mistake made was drawing the metal-p+ contact in depletion rather than accumulation or explicitly writing that this contact is a Schottky contact. Source and drain contacts in ordinary MOSFETs are Ohmic.

b)

i) The oxide capacitance can be extracted from the maximum measured capacitance:

$$C_{\max} = C_{ox} \times W_G \times L_G$$

$$C_{ox} = \frac{C_{\max}}{W_G \times L_G} = \frac{0.885 \times 10^{-12} F}{100 \times 10^{-4} \times 5 \times 10^{-4}} = 1.77 \times 10^{-7} F / cm^2$$

The oxide thickness comes from:

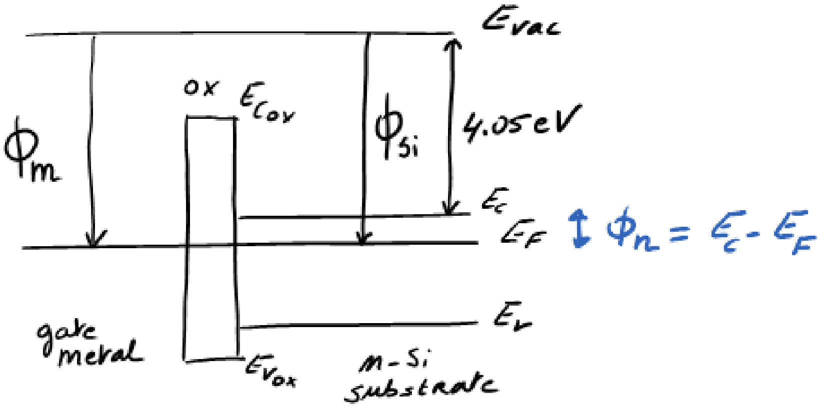
$$C_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{t_{ox}} \rightarrow t_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{C_{ox}}$$

$$t_{ox} = \frac{8.85 \times 10^{-14} F / cm \times 4}{1.77 \times 10^{-7} F / cm^2} = 2 \times 10^{-6} cm = 20\ nm$$

[2]

This is a study group question so mistakes were surprising. Common mistake made was forgetting the Cox is a capacitance per area and that the capacitance-voltage characteristic was given in F.

ii) A sketch of the flat band situation. [4]



From E_c-E_F we can find the doping.

E_c-E_F = ϕ_m – 4.05 eV= 4.259 eV – 4.05 eV=0.209 eV

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$N_D = 3.2\times 10^{19} cm^{-3} \exp\left(-\frac{0.209}{0.026}\right) = 1.03\times 10^{16} cm^{-3}$$

This was mostly correctly answered unless the student did not remember the definition of work function.

iii) C_{min} is the series connection of the oxide related capacitance and the depletion capacitance. Extract the maximum depletion width from the C-V measurements. [4]

$$\frac{1}{C_{\min}} = \frac{1}{C_{\max}} + \frac{1}{C_{depl_{\max}}}$$

$$\frac{1}{C_{depl_{\max}}} = \frac{1}{C_{\min}} - \frac{1}{C_{\max}}$$

$$C_{depl_{\max}} = \frac{\varepsilon_0 \varepsilon_r A}{W_{depl_{\max}}}$$

$$W_{depl_{\max}} = \frac{\varepsilon_0 \varepsilon_r \times L_G \times W_G}{C_{depl_{\max}}} = \frac{\varepsilon_0 \varepsilon_r \times L_G \times W_G}{\left(\frac{1}{C_{\min}} - \frac{1}{C_{\max}}\right)^{-1}} = \frac{\varepsilon_0 \varepsilon_r \times L_G \times W_G \left(C_{\max} - C_{\min}\right)}{C_{\max} C_{\min}}$$

This is based on a study group question. The key part is to see that when the capacitance is minimum, then the depletion width is maximum and a series connection of two capacitors is obtained. Main problem was the area in the final expression.

c)

i) V_{th} = -0.7 V. [2]

ii) Majority carriers in the channel are holes, thus mobility is approximately: μ_p = 410 cm^2/Vs. [2]

Main mistake is not to choose the hole mobility only, which is what the question asked, but to give both hole and electron mobility. Since it can't be both in the channel a mark was deducted in this case.

iii) Output characteristic is I_{DS} versus V_{DS}. [1] [6]

A p-channel MOSFET has a negative bias on the drain and "negative" current if S is at x=0 and D is at +x. [1]

Pinch-off is found for $V_{DS} = V_{GS} - V_{th}$ = -1 – (-0.7) = -0.3 V[1]

Thus the IV characteristic is linear up to ~ -0.3 V and then saturates.
The saturation current is derived from the expression for the drain current found in the formulae list:

$$I_{DS} = \frac{\mu C_{ox} W}{L}\left((V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Re-written for saturation

$$I_{DS}^{sat} = \frac{\mu C_{ox} W}{2L}\left((V_{GS} - V_{th})^2\right)$$

[1]

$$I_{DS}^{sat} = \frac{410 \times 1.77 \times 10^{-7} \times 100}{2 \times 5}\left((-0.3)^2\right) = 6.53 \times 10^{-5} A$$

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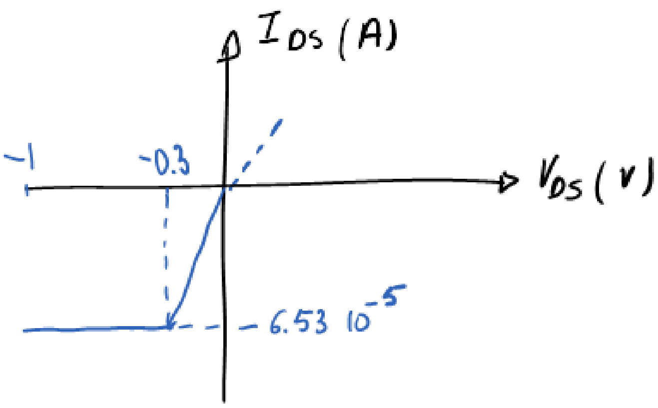

[2]

The main problem with answering this question was the lack of realisation that voltages and currents in a p-channel MOSFET are negative. Another problem was the lack of expression and/or calculation of I_{DS}^{sat}.