2E Electromagnetic Fields 2012 – Formula sheet

- Vector calculus (Cartesian co-ordinates)

$\nabla = \underline{i}\ \partial/\partial x + \underline{j}\ \partial/\partial y + \underline{k}\ \partial/\partial z$

$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$

$\text{grad}(\phi) = \nabla\phi = \underline{i}\ \partial\phi/\partial x + \underline{j}\ \partial\phi/\partial y + \underline{k}\ \partial\phi/\partial z$

$\text{div}(\underline{\mathbf{F}}) = \nabla \cdot \underline{\mathbf{F}} = \partial \mathbf{F}_x/\partial x + \partial \mathbf{F}_y/\partial y + \partial \mathbf{F}_z/\partial z$

$\text{curl}(\underline{\mathbf{F}}) = \nabla \times \underline{\mathbf{F}} = \underline{i}\ \{\partial \mathbf{F}_z/\partial y - \partial \mathbf{F}_y/\partial z\} + \underline{j}\ \{\partial \mathbf{F}_x/\partial z - \partial \mathbf{F}_z/\partial x\} + \underline{k}\ \{\partial \mathbf{F}_y/\partial x - \partial \mathbf{F}_x/\partial y\}$

Where ϕ is a scalar field and $\underline{\mathbf{F}}$ is a vector field

- Maxwell's equations – integral form

$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho\ dv$

$\iint_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = -\iint_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

Where $\underline{\mathbf{D}}$, $\underline{\mathbf{B}}$, $\underline{\mathbf{E}}$, $\underline{\mathbf{H}}$, $\underline{\mathbf{J}}$ are time-varying vector fields

- Maxwell's equations – differential form

$\text{div}(\underline{\mathbf{D}}) = \rho$

$\text{div}(\underline{\mathbf{B}}) = 0$

$\text{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\text{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

- Material equations

$\underline{\mathbf{J}} = \sigma\ \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon\ \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu\ \underline{\mathbf{H}}$

- Theorems

$\iint_A \underline{\mathbf{F}} \cdot d\underline{a} = \iiint_V \text{div}(\underline{\mathbf{F}})\ dv$ – Gauss' theorem

$\int_L \underline{\mathbf{F}} \cdot d\underline{L} = \iint_A \text{curl}(\underline{\mathbf{F}}) \cdot d\underline{a}$ – Stokes' theorem

$\text{curl}\ \{\text{curl}(\underline{\mathbf{F}})\} = \text{grad}\ \{\text{div}(\underline{\mathbf{F}})\} - \nabla^2\underline{\mathbf{F}}$

2E Electromagnetic Fields 2012 – Formula sheet (continued)

- Electromagnetic waves (pure dielectric media)

Time dependent vector wave equation $\nabla^2\underline{\mathbf{E}} = \mu_0\varepsilon\ \partial^2\underline{\mathbf{E}}/\partial t^2$

Time independent scalar wave equation $\nabla^2\underline{E} = -\omega^2\mu_0\varepsilon_0\varepsilon_r\ \underline{E}$

For z-going, x-polarized waves $d^2E_x/dz^2 + \omega^2\mu_0\varepsilon_0\varepsilon_r\ E_x = 0$

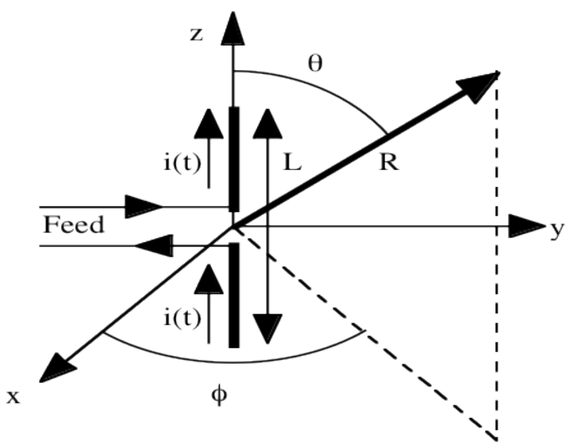
Where $\underline{E}$ is a time-independent vector field

- Antenna formulae

Far-field pattern of half-wave dipole

$E_\theta = j\ 60I_0\ \{\cos[(\pi/2)\cos(\theta)] / \sin(\theta)\}\ \exp(-jkR)/R;\ H_\phi = E_\theta/Z_0$

Here I_0 is peak current, R is range and $k = 2\pi/\lambda$



Power density $\underline{S} = 1/2\ \mathrm{Re}\ (\underline{E} \times \underline{H}^*) = S(R, \theta)$

Normalised radiation pattern $F(\theta, \phi) = S(R, \theta, \phi) / S_{max}$

Directivity $D = 1/\ \{1/4\pi \iint_{4\pi} F(\theta, \phi) \sin(\theta)\ d\theta\ d\phi\}$

Gain $G = \eta D$ where η is antenna efficiency

Effective area $A_e = \lambda^2 D/4\pi$

Friis transmission formula $P_r = P_t\ (\ \eta_t\eta_r A_t\ A_r\ /R^2\lambda^2)$

Fields 2012 – Questions

1. a) Discuss the implications for electromagnetic waves of **no more than two** of the following, illustrating your answer with diagrams, graphs or formulae where appropriate.

   i) Characteristic impedance
   ii) Dispersion diagram
   iii) Group velocity

[2 x 5]

b) What is Snell's Law? State the conditions under which total internal reflection can occur, and find the critical angle for an interface between glass (for which n = 1.5) and air.

[5]

c) Write down an expression for the electric field of a plane electromagnetic wave with wavelength λ travelling in the (x, z) plane in air at an angle $+\theta$ to the z-axis.

Two plane waves now travel at angles $\pm\theta$ to the z-axis. Show that an interference pattern will be generated, and sketch the waves and the pattern.

Assume polarization in the y-direction throughout.

[5]

2. a) Explain in words the meaning of the 'directivity' and 'effective area' of an antenna?

[5]

b) A transmitting station has an isotropic antenna and a transmitter with an average power of 20 kW. Calculate the power received at a distance of 10 km by an antenna with effective area of 10 m^2.

[5]

c) A broadside antenna consists of two dipoles separated by a distance of d. Sketch the array, from the side and from the top.

[5]

d) Assuming that the two dipoles are fed in-phase, find an expression for the normalised radiation pattern at wavelength λ.

[7]

e) Assuming that d = 1 m, sketch the normalised radiation pattern at frequencies of a) 1.5 MHz and b) 150 MHz.

[8]