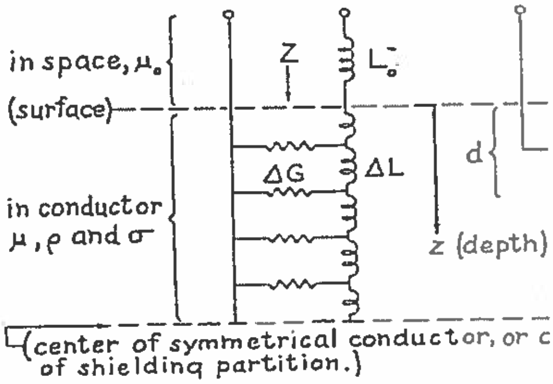
The Solutions for E3.18 and AO12, 2014

Model answer to Q 1(a): Bookwork

Consider room temperature copper, having an intrinsic bulk DC conductivity of 5.8×10^7 S/m.

a) With the classical skin-effect model, the electromagnetic fields within the metal decay exponentially from their surface values.

i) Illustrate the transmission circuit model that would account for this type of exponential decay in the fields as they propagate into the metal. What is the normal name given to the internal impedance of a metal (i.e. equivalent input impedance into this circuit)?

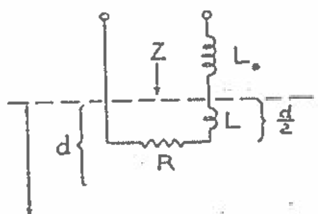


Here, $Lo = \Delta L = \mu_o$and $\Delta G = \sigma = \sigma_o$ and d = δ_o, with all variables having their usual meaning.

The normal name given to the internal impedance of a metal is surface impedance.

[3]

ii) Illustrate the equivalent lumped-element circuit model for the internal impedance of the metal.



Here, $Lo = \mu_o$ and R = Rs and $L = Xs/\omega$ and d = δ_o, with all variables having their usual meaning.

[2]

Model answer to Q 1(b): Bookwork Derivation and Calculated Example

a) Given that intrinsic impedance of a normal material is the square root of the ratio of permeability over permittivity as the starting point:

i) Give an expression for the effective relative permittivity (also known as the dielectric function) of the metal. Define all variables.

For a normal metal, $\sigma_{effective} = j\omega\varepsilon_{effective} = \sigma + j\omega\varepsilon_o$

Therefore, the effective relative permittivity or dielectric function is given by

$$\varepsilon_{effective\_r} = 1 - j\frac{\sigma}{\omega\varepsilon_o}$$

for the normal skin-effect model. Here, σ = bulk DC conductivity, ω = angular frequency, ε_o = permittivity of free space.

[2]

ii) From 1(b)(i), derive the expression for the intrinsic impedance of the metal. Clearly define all variables and state any approximations.

For a normal metal, the internal or surface impedance is given by

$$Zs = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_o\varepsilon_{effective\_r}}} = \sqrt{\frac{j\omega\mu_0\mu_r}{\sigma}} \text{ with } \varepsilon_{effective\_r} = 1 - j\frac{\sigma}{\omega\varepsilon_o} \cong -j\frac{\sigma}{\omega\varepsilon_o}$$

where μ_o = permeability of free space and μ_r = relative permeability.

[2]

iii) From 1(b)(ii), derive the expression for classical skin depth from the propagation constant and calculate its value at 2.45 GHz. Clearly define all variables. How does the order of magnitude of this calculated value compare with the usual thickness of copper-cladding in printed circuit boards and with its general ability to provide screening at this frequency?

Propagation constant, $\gamma = \frac{j\omega\mu_0\mu_r}{Zs}$ and skin depth,

$$d = \frac{1}{\mathrm{Re}\{\gamma\}} = \sqrt{\frac{2}{\omega\mu_0\mu_r\sigma}} = 1.34\mu m$$

with relative permeability of unity for copper.

The usual thickness of copper-cladding in printed circuits boards is about an order of magnitude larger and so this provides excellent screening.

[4]

Model answer to Q 1(c): Calculated Example

For the two lumped elements drawn in 1(a)(ii):

i) Calculate their values at 2.45 GHz and show their approximate dependency on skin depth and also frequency, using the expression derived in 1(b)(iii).

with relative permeability of unity for copper

The surface resistance, $R = \mathrm{Re}\{Zs\} = \sqrt{\frac{\omega\mu_0\mu_r}{2\sigma}} = 0.0129\Omega / square = \frac{1}{d\sigma} \propto \sqrt{\omega}$

The surface Inductance, $L = \frac{\mathrm{Im}\{Zs\}}{\omega} = \sqrt{\frac{\mu_o\mu_r}{2\omega\sigma}} = 0.839 pH / square = \mu_o\mu_r\left(\frac{d}{2}\right) \propto \frac{1}{\sqrt{\omega}}$

[5]

ii) State their effects on the frequency resonance curve of a high-Q resonator that is implemented using copper transmission lines.

The surface inductance will shift the lossless resonance frequency down in value from the perfect electrical conductor case. The surface resistance will effectively broaden out the resonance frequency curve, while reducing the quality (Q)-factor, and further reduce the natural resonance frequency when the resonator is excited with a direct impulse excitation.

[2]

Model answer to Q 2(a): Calculated Example

Human body tissue can be modelled as a microwave dielectric material. At 2.45 GHz, the following material properties have been reported:

Fat has $\varepsilon_r' = 12$ and $\sigma = 0.82\ S/m$

Muscle has $\varepsilon_r' = 49.6$ and $\sigma = 2.56\ S/m$

The above variables have their usual meaning.

a) For both body fat and muscle, calculate:

i) Intrinsic impedance.

$$\varepsilon_r'' = \frac{\sigma}{\omega\varepsilon_o} = 6.02 \quad and \quad 18.78$$

$$\eta = \frac{377}{\sqrt{\varepsilon_r' - j\varepsilon_r''}} = 100.12 + j23.71\Omega \quad and \quad 50.92 + j9.32\Omega$$

[3]

ii) Propagation constant.

$$\gamma = \frac{j\omega\mu_o}{\eta} = 43.3 + j183 \quad and \quad 67.3 + j367.6$$

[2]

iii) Skin depth.

$$\delta = \frac{1}{\mathrm{Re}(\gamma)} = 23mm \quad and \quad 14.5mm$$

[2]

iv) Power attenuation in dB per unit wavelength.

$$\alpha = 44.3 \quad and \quad 67.3$$

$$\beta = 183 \quad and \quad 367.6$$

$$\lambda = \frac{2\pi}{\beta} = 34.3mm \quad and \quad 17.1mm$$

$$Power\ Attenuation = e^{-2\alpha\lambda}\ Np/\lambda$$

$$Power\ Attenuation = 8.686\alpha\lambda\ dB/\lambda = 13.2 \quad and \quad 10.0 \quad dB/\lambda$$

[3]

v) Power flux density given an RMS electric field intensity of 4 V/cm.

$$P_D = \frac{|E|^2}{\mathrm{Re}(\eta)} = 160 \quad and \quad 314 \quad mW/cm^2$$

[2]

Model answer to Q 2(b): Calculated Example

From the results in 2(a), calculate the voltage-wave reflection coefficient and power reflectance at the interface between the fat and muscle.

$$\rho = \frac{\eta_{fat} - \eta_{muscle}}{\eta_{fat} + \eta_{muscle}} = 0.33076 + j0.02294$$

$$\Gamma = |\rho|^2 = 0.11$$

[4]

Model answer to Q 2(c): Calculated Example

Based on the results calculated in 2(a) and 2(b), compare and contrast the different materials and suggest how it may be possible to detect the presence of fat using microwave techniques.

It can be seen from the results in 2(a) that the two different body tissue types have very different microwave properties. Fat is much less effective at attenuating/absorbing microwave power, when compared to muscle. Therefore, it may be possible to detect the fat by transmitting microwave pulsed energy into the human body and observing the transmission and reflected properties. With transmission measurements, high levels of attenuation and longer delays will be found with muscle. With reflection measurements, measureable reflected waves will be observed at the interface between the different body tissue types.

[4]

Model answer to Q 3(a): Bookwork Derivation

From first principles, derive the well-known expression for the normalised input impedance to a lossless transmission line of arbitrary electrical length θ that is terminated with arbitrary load impedance.

$$V(z) = V_+ e^{-\gamma z} + V_- e^{+\gamma z}$$
$$I(z) = I_+ e^{-\gamma z} + I_- e^{+\gamma z}$$

$$where,\ V_\pm(I_\pm)\ \textbf{represents voltage (current) waves at z = 0}$$
$$and,\quad e^{\mp\gamma z}\ \textbf{represents wave propagation in the} \pm \textbf{z direction}$$

$$Voltage\ Wave\ Reflection\ Coefficient,\ \rho(z) = \frac{V_- e^{+\gamma z}}{V_+ e^{-\gamma z}} = \frac{-I_- e^{+\gamma z}}{I_+ e^{-\gamma z}}$$

$$\therefore \rho(z) = \rho(0) e^{+2\gamma z} \equiv \rho(0) e^{+j2\beta z}\ \ for\ a\ lossless\ line$$
$$where,\ \rho(0) = \frac{V_-}{V_+} = \frac{-I_-}{I_+} \quad and \quad Zo = \frac{V_+}{I_+}$$

The impedance looking into a transmission line terminated with load impedance Z_T is given by:

$$\mathbf{Zin} = \frac{V(l)}{I(l)} = Zo\frac{\left(e^{+\gamma l} + \rho(0)e^{-\gamma l}\right)}{\left(e^{+\gamma l} - \rho(0)e^{-\gamma l}\right)} = \frac{\left((Z_T + Zo)e^{+\gamma l} + (Z_T - Zo)e^{-\gamma l}\right)}{\left((Z_T + Zo)e^{+\gamma l} - (Z_T - Zo)e^{-\gamma l}\right)}$$

$$\mathbf{Zin} = Zo\frac{\left(Z_T(e^{+\gamma l} + e^{-\gamma l}) + Zo(e^{+\gamma l} - e^{-\gamma l})\right)}{\left(Zo(e^{+\gamma l} + e^{-\gamma l}) + Z_T(e^{+\gamma l} - e^{-\gamma l})\right)}$$

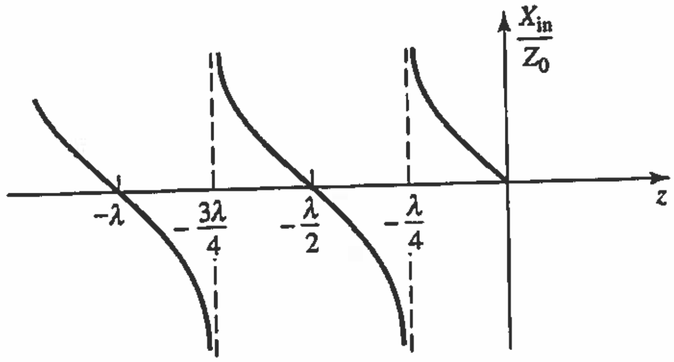
$$\textit{Therefore, } zin = \frac{\mathbf{Zin}}{Zo} = \frac{z_T + \tanh(\gamma l)}{1 + z_T \tanh(\gamma l)} \Rightarrow \frac{z_T + j\tan\theta}{1 + jz_T \tan\theta} \textbf{ for a lossless line}$$

[8]

Model answer to Q 3(b): Bookwork and Computed Example

From the expression derived in 3(a), for a short-circuit terminating load impedance, sketch the normalized input impedance against physical length *l* as it increases through a complete wavelength. If the lengths of the transmission line are λg/10 and 3λg/10, calculate the effective lumped-element component values at 2.45 GHz, for 50 Ω transmission lines. If these two stubs were then connected at the same point along a transmission line, in a stunt arrangement, what would be the resulting circuit?

If $Z_T = 0$ then $Zin = jZo\tan\theta$ and the impedance is always reactive and periodic along the line, which takes a value from 0 to j∞ to and -j∞ to 0 as *l* increases from 0 to λg/4 and λg/4 to λg/2. This is useful for realising any value of "effective" inductance or capacitance over a narrow bandwidth.



If the lengths of the transmission line are λg/10 and 3λg/10, the effective lumped-element component values at 2.45 GHz, for 50 Ω transmission lines are:

$$Zin = j50\tan(36^o)\ \Omega = j0.7265x50\ \Omega = 36.3\ \Omega \equiv j\omega L \rightarrow L = 2.35\ nH$$

$$Zin = j50\tan(36^o) = -j3.0777x50\ \Omega = -j153.9\ \Omega \equiv \frac{1}{j\omega C} \rightarrow C = 422\ fF$$

If these two stubs were then connected at the same point along a transmission line, there would be an equivalent shunt parallel *L-C* tuned circuit having a resonant frequency at 5.05 GHz.

[6]

Model answer to Q 3(c): Bookwork

State one advantage of an open-circuit stub, when compared to a short-circuit stub. Also, give a common practical application for the open-circuit stub and state any assumptions about its electrical length.

Unlike a short-circuit stub, an open-circuit stub does not require any through-substrate plated via holes. These can be very difficult and/or expensive to implement. One common application is a simple band-stop filter, if the open-circuit stub is a quarter-wavelength long, since the short circuit at the input will reflect the incident wave.

[3]

State one advantage of a short circuit stub, when compared to an open circuit stub. Also, give a common practical application for the short circuit stub and state any assumptions about its electrical length.

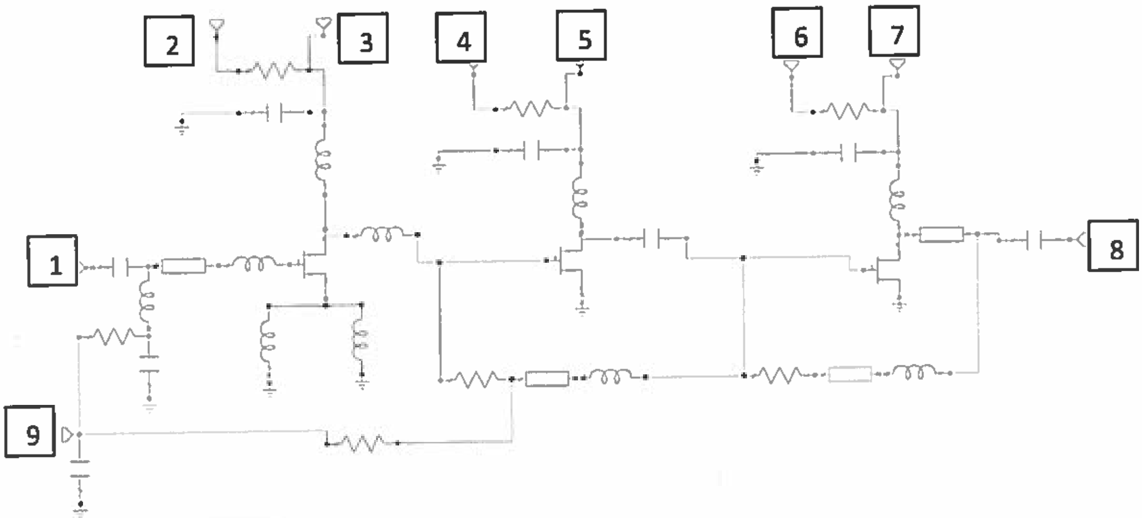
Unlike the open-circuit stub, short-circuit stubs do not suffer from radiation losses, although they can still excite unwanted substrate modes. One common application is in DC biasing networks, where the RF short-circuit at the termination end is transformed into a RF open-circuit that will choke off any RF signal.

[3]

Model answer to Q 4(a): Application of Discussions in Class

The photograph in Figure 4.1 is of a MMIC.

Draw the basic equivalent circuit model for the MMIC shown in Figure 4.1, and mark the RF and DC bias ports with the corresponding probe pad numbers shown. Describe the type of amplifier circuit. Hint: if you are uncertain about a component then state any assumptions used.



This is a three-stage low noise amplifier with parallel feedback in the last two stages.

[10]

Model answer to Q 4(b): Application of Discussions in Class

Briefly describe the different range of component technologies used for the transistors, inductors and capacitors, and also state the advantages and disadvantages of these technologies. State what compromises have to be made with the design of MMICs, when compared to HMICs.

This amplifier used metal-semiconductor field-effect transistor (MESFET) technologies. This is the workhorse of microwave GaAs ICs. They have good general performance in gain, power-added efficiency, output power and noise. However, they do not have superior performance over HEMT devices. The inductors are all of the planar 2D spiral type. This gives high inductance per unit area but poor noise and bandwidth performance. The capacitors are all of the metal-insulator-metal type. These can give high capacitance values per unit area, but are prone to low yield due to pinhole short circuits.

Most MMIC devices have to be tailored to volume production and tend not to give state-of-the-art performance. This introduces compromises for the MMIC designer: (i) this can be a serious problem for LNA and PA design; (ii) special devices (e.g. Gunn diodes, PIN switches, and hyperabrupt varactor diodes) are rarely used in MMIC processes; (iii) FET switch is a poor substitute for the PIN diode and the HEMT millimetre-wave; (iv) oscillator will have a low output power compared with a Gunn diode; (v) most compromises can be absorbed into the specifications of the system design, and good communications between the circuit designer and systems designer are very beneficial to the final product

[5]

<u>Model answer to Q 4(c): Application of Discussions in Class</u>

Briefly comment as to why the complexity of the full equivalent circuit model is much more than the circuit drawn in 4(a) and explain why circuit modelling alone is not sufficient if a significant reduction in the chip area is required.

In practice, most of the components identified in 4(a) should be represented by many equivalent circuit model elements. As a result, while there are only 3 MESFETs, 8 capacitors, 10 spiral inductors, 7 resistors, 4 microstrip interconnections identifies, the circuit could be represented by hundreds of individual parasitic elements. In order to reduce the chip size the components have to be placed closer together. However, by doing this, the electromagnetic coupling between components cause adverse interactions that can only be modelled sufficiently using 3D electromagnetics simulation packages.

[5]

<u>Model answer to Q 5(a): Derivations and Computed Example</u>

An ideal air-filled rectangular waveguide has a guided-wavelength given by the following expression:

$$\lambda_g = \frac{\lambda_o}{\sqrt{1-\left(\frac{f_c}{f_o}\right)^2}} \qquad (5.1)$$

All variables have their usual meaning.

In addition, for the TE_{101} mode, the unloaded Q-factor for an air-filled rectangular waveguide resonant cavity is given by the following expression:

$$Q_u\Big|_{TE101} \cong \frac{Volume}{Area \times (\delta_o/2)} \qquad (5.2)$$

All variables have their usual meaning.

For a half-height waveguide (i.e. its height dimension b is half that of the width dimension a:

(i) Using (5.1), derive an expression for the length l of the cavity in terms of a and the frequency dependence term.

$$for \quad half-height: b = \frac{a}{2}$$

$$\lambda_c = 2a \quad \therefore f_c = \frac{c}{2a} \quad \therefore a = \frac{\lambda_c}{2} = \frac{c}{2f_c} \quad also \quad \lambda_o = \frac{c}{f_o} \quad \therefore a = \frac{\lambda_o}{2}\left(\frac{f_o}{f_c}\right)$$

$$for \quad TE_{101} \text{ mode}: l \equiv \frac{\lambda g}{2} = \frac{\lambda_o/2}{\sqrt{1-\left(\frac{f_c}{f_o}\right)^2}} = \frac{a}{\left(\frac{f_o}{f_c}\right)\sqrt{1-\left(\frac{f_c}{f_o}\right)^2}} = \frac{a}{\sqrt{\left(\frac{f_o}{f_c}\right)^2-1}}$$

[4]

(ii) Using (i), derive an expression for the internal volume of the cavity.

$$Volume \equiv a\, b\, l = \frac{a^3}{2\sqrt{\left(\frac{f_o}{f_c}\right)^2-1}}$$

[2]

(iii) Using (i), derive an expression for the internal area of the cavity.

$$Area \equiv 2(a\,l + a\,b + b\,l) = 2a^2\left[\frac{1}{2} + \frac{1}{\sqrt{\left(\frac{f_o}{f_c}\right)^2-1}} + \frac{1}{2\sqrt{\left(\frac{f_o}{f_c}\right)^2-1}}\right] = a^2\left[1 + \frac{3}{\sqrt{\left(\frac{f_o}{f_c}\right)^2-1}}\right]$$

[3]

(iv) Using (5.2) and assuming that $f_o/f_c = \sqrt{2}$, derive an expression for the unloaded Q-factor in terms of a and classical skin depth.

$$Volume = \frac{a^3}{2\sqrt{\left(\frac{f_o}{f_c}\right)^2-1}} \to \frac{a^3}{2} \quad and \quad Area = a^2\left[1 + \frac{3}{\sqrt{\left(\frac{f_o}{f_c}\right)^2-1}}\right] \to 4a^2$$

$$\therefore Q_u\big|_{TE101} \equiv \frac{2\,Volume}{\delta_o\, Area} \to \frac{a}{4\delta_o}$$

[4]

(v) Using (iv), calculate the unloaded Q-factor for a 15.5 GHz resonant cavity made with copper walls having a DC bulk conductivity of 5.8 x 10^7 S/m.

$$\lambda_o = \frac{c}{f_o} = 19.355mm \quad and \quad a = \frac{\lambda_o}{2}\left(\frac{f_o}{f_c}\right) = 13.686mm$$

$$\delta_o = \sqrt{\frac{2}{\omega_o \mu_o \sigma_o}} = \sqrt{\frac{2}{2\pi 15.5\times10^9 \times 4\pi\times10^{-7} \times 5.8\times10^7}} = 0.531\mu m$$

$$\therefore Q_u\big|_{TE101} \cong \frac{a}{4\delta_o} = 6,446$$

[4]

Model answer to Q 5(b): Derivation and Computed Example

For a cubic cavity (i.e. all internal dimensions are equal) derive an expression for the unloaded Q-factor in terms of *a* and classical skin depth and show that this has a 33.333% higher unloaded Q-factor when compared to the half-height case.

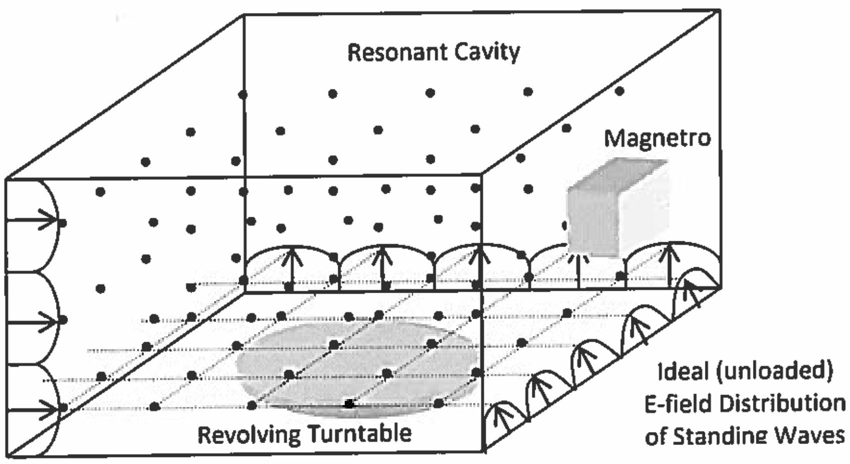
$$Volume = a^3 \quad and \quad Area = 6a^2$$

$$\therefore Q_u\big|_{TE101} \cong \frac{2\,Volume}{\delta_o\,Area} \to \frac{a}{3\delta_o} \quad which\ is\ 4/3 = 1.3333333\ higher\ than\ with\ half-height$$

[3]

Model answer to Q 6(a): Bookwork and Simple Computed Example

State the electric and magnetic field boundary conditions for a perfect electrical conductor (PEC) wall. A microwave oven operates at 2.45 GHz. If the internal cavity dimensions of the oven are 30 cm x 30 cm x 20 cm, illustrate the electric field energy distribution inside an empty oven having PEC walls. What happens to this distribution when food is placed inside the oven and what is the role of the turntable?



For a PEC, the boundary conditions are such that there is no tangential electric field component at the surface of the wall. Also, there is no magnetic field component normal to the surface of the wall.

At 2.45 GHz the wavelength in free space is 122.4 mm. Therefore, there are approximately 5 x 5 x 4 = 100 points of E-field maxima (not shown accurately in the figure below).

The E-field distribution illustrated above will be completely altered at all points other than at the cavity walls. Because there are both E-field maxima and minima distributed within the cavity, there will be localised hot-spots and cold-spots in any food. With time, thermal conduction will help to even out these hot and cold spots. In an attempt to avoid hot and sold spots, a turntable below the food or metallic stirring fan above the food "sees" a time and space variant E-field distribution.

[5]

Model answer to Q 6(b): Computed Example

Calculate the loss tangent for a solid block of fat having a dielectric constant $\varepsilon_r' = 12$ and conductivity $\sigma = 0.82\, S/m$. With a peak magnetic field intensity of 265 mA/m, estimate the average power dissipated per unit volume within the fat.

With fat having $\varepsilon_r' = 12$ and conductivity of $\sigma = 0.82$ S/m thee $\tan\delta = \varepsilon_r''/\varepsilon_r'$ and $\sigma = \omega\varepsilon_o\varepsilon_r' \tan\delta$. Therefore, $\tan\delta = \sigma/(0.1363\, \varepsilon_r') = 0.5$.

The peak H-field intensity is 265 mA/m. Therefore the peak E-field intensity is $E_{pk} = 377\ \Omega \times 0.265$ A/m= 100 V/m.

$\therefore$ Power absorbed per unit volume, $P_v = \sigma E_{RMS}^2 |_{dielectric\ interface} \sim 4.1$ kW/m^3

[5]

Model answer to Q 6(c): Computed Example

Using the power density calculated in 6(b), if the block of fat has a mass of 250 g, dimensions of 4 cm x 6 cm x 9 cm, specific heat capacity of 1.67 kJ/kg C and melting temperature of 184 C, estimate the approximate number of days needed to melt this fat when taken from a refrigerator that is cooled down to 3 C. For this simple calculation, what assumptions have to be made with respect to the spatial distribution of power density within the block of fat and also the solid-liquid phase states of the fat during heating?

Power dissipated (i.e. absorbed) within the food, $P = \dfrac{m S_p \Delta T}{t}$ [W]

m = mass [g]
S_p = specific heat capacity [kJ/kg K]
ΔT = raise in temperature [K or °C]
t = time [s]

- Power dissipated per unit volume, $P_V = \dfrac{\rho S_p \Delta T}{t} \left[W/m^3\right]$

ρ = mass/volume [g/m^3]

If the fat has dimensions of 4 x 6 x 9 cm^3 then using the result from 4(b):

$P_v = 4.1$ kW/m^3 and volume = 216×10^{-6} m^3, $\therefore P = P_v \times volume = 0.8856$ W

With m = 0.25 kg, S_p = 1.67 kJ/kg C, ΔT = 181°C

$$\therefore\ t = \frac{mS_p\Delta T}{P} = \frac{0.25\times1670\times181}{0.8856} = 85329\ s \cong 24\ hours = 1\ day$$

All of these calculations assume that there is a uniform power density with the whole block of fat and that the fat remains in the solid state until the melting point is reach, at which point there is an unrealistic step change to a liquid phase.

[7]

Model answer to Q 6(d): Application of Discussions in Class

What energy source technology is employed in traditional microwave ovens? What are the three main advantages for microwave ovens that employ a solid-state solution to providing the energy source? What methods can be employed to create time-variant spatial fields within the cavity for both the traditional and solid-state solutions?

Traditional microwave ovens employs large and heavy magnetron sources. With a solid-state solution, there are 3 main advantages to using an RF transistor as the energy source: (1) increased longevity (mean time to failure of approximately 20 years, versus 500 hours for magnetron-based designs), (2) ruggedness (for high-vibration applications, including mobile homes, airplanes, and other vehicles), and (3) the ability to create alternative oven form factors that can be slightly bigger than the size of the cavity. The advantage of the solid-state-solution is that it is easier to implement phase array beam forming, to scan the beam through the food, as an alternative to using the more traditional methods of employing a mode stirring fan or turntable.

[3]