Final

E4.55
AO11

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

MSc and EEE PART IV: MEng and ACGI

MEMS AND NANOTECHNOLOGY

Monday, 17 May 10:00 am

Time allowed: 3:00 hours

There are FIVE questions on this paper.

Answer Question 1.
Answer Question 2 OR Question 3.
Answer Question 4 OR Question 5.

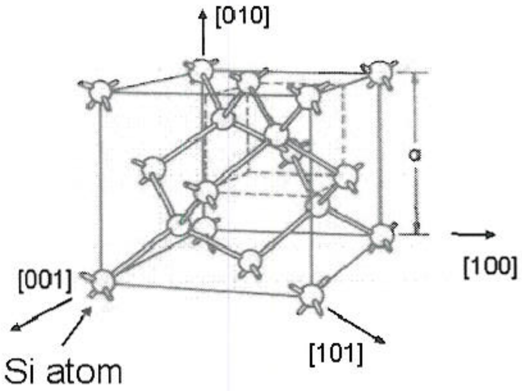
Question 1 carries 40% of the marks. Remaining questions carry 30% each.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	Z. Durrani, A.S. Holmes, Z. Durrani
	Second Marker(s) :	A.S. Holmes, Z. Durrani, A.S. Holmes

This question is compulsory

1. a) The diagram below shows the unit cell in a Si crystal, where side a = 0.54 nm. Using this diagram, calculate the concentration of atoms in crystalline Si, per cubic metre.



[5]

b) Calculate the wavelength λ of a photon emitted from a spherical Si nanocrystal 2 nm in diameter, when a conduction band electron of minimum energy recombines with a valance band hole of maximum energy. The effective masses of electrons and holes in crystalline Si are 0.33 and 0.55 respectively. The rest mass of an electron m_0 = 0.91×10^{-30} kg, Planks' constant $h = 6.625\times10^{-34}$ J.s, the speed of light $c = 3 \times 10^{8}$ m/s and the band-gap in bulk Si, E_g = 1.1 eV. [5]

c) Assuming the wavefunction of an electron of mass m travelling in a potential V is given by $\Psi = \mathrm{A.exp}[\mathrm{i}(kx - \omega t)]$, derive the 1-D time-dependent form of Schrödinger's Equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$ [5]

d) Using suitable diagrams, explain briefly the operation of a resonant tunnelling diode. [5]

Question 1 continues on the next page.

Question 1 continued.

e) The resolution limit R of a projection lithography system is typically expressed in the form:

$$R = k_1 \frac{\lambda}{NA} \quad \text{where} \quad NA = n \sin \theta_m$$

Explain briefly the physical significance of each of the parameters in these equations and, for each parameter, state whether its value depends on the light source, the illumination conditions, the projection lens, and/or on other factors. [5]

f) Derive a scaling law for the stiffness of an elastic structure, and hence for the natural frequency of a micromechanical resonator. If the damping is dominated by viscous losses in the air gap between the moving part and the substrate, and the viscosity of the air is constant, how will the Q-factor of the resonator scale with its size? [5]

g) List the main actuation mechanisms used in MEMS devices, and briefly compare them in terms of speed, force and ease of implementation. [5]

h) Write down the bending equation for a buckled beam that is built in at both ends. By solving the equation subject to appropriate boundary conditions, derive an expression for the critical axial load at which buckling will occur. [5]

2. An electron travelling in the positive x-direction is incident from the region $x < 0$ on a one-dimensional potential barrier of height V_0 and width L. The potential energy $V(x)$ is given by:

$$V(x) = V_0, \quad \text{for } 0 \le x \le L$$
$$V(x) = 0, \quad \text{elsewhere}$$

a) Write down the general form of the time-independent wavefunctions $\psi(x)$ in the regions $x < 0$, $0 \le x \le L$, and $x > 0$. [6]

b) What are the wavevectors for the regions $x < 0$, $0 \le x \le L$, and $x > 0$, and the boundary conditions at $x = 0$ and $x = L$? [6]

c) Sketch the form of the wavefunctions, with matching of the wavefunctions, for energy $E < V_0$, in the regions $x < 0$, $0 \le x \le L$, and $x > 0$. [4]

d) Hence, derive an expression for the transmission coefficient for wavefunction amplitude t across the barrier. [14]

3. The n-channel Si MOSFET shown in Figure 3 has gate length L = 1 μm, gate width W = 10 μm, and gate oxide thickness t_{ox} = 50 nm. The source and drain doping concentration $N_D = 10^{25}$ /m^3, and the bulk doping concentration $N_A = 10^{23}$ /m^3. The maximum value of the drain voltage V_d, and gate voltage, V_g, applied during device operation is 5 V.

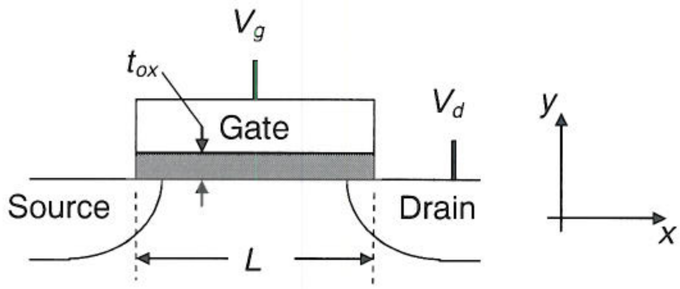


Figure 3

a) The MOSFET dimensions are to be scaled down by a factor of 5, without increasing electric fields in the device. For this condition, estimate new values for the doping concentrations, the maximum value of applied voltages and corresponding inversion layer charge density, and the gate capacitance. For the new applied voltages, demonstrate that the electric fields remain constant. [12]

b) Determine by what factor the current in the saturation and sub-threshold region changes. What is the consequence of this? [6]

c) If the scaled device is used in an integrated circuit, determine by what factors the circuit delay, transistor density, and power density change. [6]

d) The threshold voltage V_{th} of the MOSFET is given by:

$$V_{th} = \varphi_{ms} - \frac{Q_f}{C_{ox}} + 2\psi_B + \frac{\sqrt{2e\varepsilon_{Si}\varepsilon_0 N_A(2\psi_B + |V_{sub}|)}}{C_{ox}}$$

where φ_{ms} is the work function difference between the gate and Si regions, Q_f is fixed charge at the gate/Si interface, C_{ox} is the gate capacitance, V_{sub} is the substrate bias, and ψ_B is the difference between the intrinsic and Fermi levels in the Si substrate.

Discuss how V_{th} changes as the MOSFET is scaled by a factor of 5. [6]

You may use $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m. The dielectric constant for SiO_2 is $\varepsilon_{ox} = 4$.

4. a) Figure 4.1 shows the electromechanical model for an out-of-plane electrostatic actuator, comprising a suspended moveable plate and a fixed plate covered by a dielectric layer. Derive an expression for the total force acting on the moveable plate, and sketch the variation of this force with the gap g for several values of V in order to illustrate the origin of the *snap-down* instability. Also sketch the variation of g with applied voltage. [8]

b) State the conditions that apply at the point of snap-down, and hence show that the snap-down voltage is given by:

$$V_P = \sqrt{\frac{8k(g_0 + t_d/\varepsilon_r)^3}{27\varepsilon_0 A}}$$ [8]

c) Assuming the moveable plate has snapped down, to what level will the applied voltage need to be reduced before the moveable plate will spring back up again? [4]

d) Figure 4.2 shows an electrostatically actuated, capacitive shunt RF MEMS switch comprising a gold bridge suspended over a transmission line. The bridge is 400 μm long, 100 μm wide and 1 μm thick, and the signal line beneath is 100 μm wide. The dielectric spacer is a 100 nm-thick layer of silicon nitride. The initial gap between the bridge and the dielectric is 2 μm.

Calculate the stiffness of the gold bridge, and hence estimate the pull-down voltage, assuming that the electrostatic force is a point load at the centre of the bridge. How would residual stress in the gold affect the actual pull-down voltage, if at all? [6]

What advantages do RF MEMS switches of the kind shown in Figure 4.2 offer over more traditional solid state devices? [4]

You should assume E = 80 GPa for gold, and ε_r = 7.6 for silicon nitride.

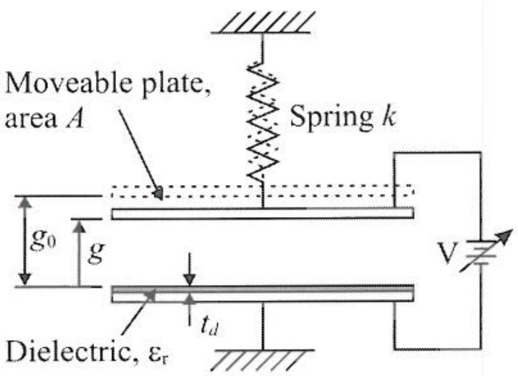


Figure 4.1

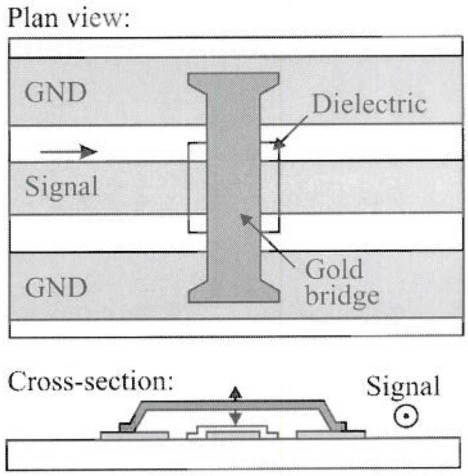


Figure 4.2

5. a) Briefly describe the piezoelectric and piezoresistive effects and how they are used in MEMS devices. [6]

b) Figure 5 shows a simple accelerometer comprising a silicon cantilever with a thin zinc oxide layer covering its top surface. The ZnO layer is oriented as shown, and is sandwiched between two aluminium electrodes which are connected to a charge amplifier. Outline a possible fabrication sequence for the device, assuming that the starting material is a BSOI wafer, and that the ZnO can be deposited by sputtering and patterned by reactive ion etching. [6]

c) Show that the following is an approximate expression for the sensitivity S_q of the accelerometer:

$$S_q = \frac{V_{out}}{a} = Gd_{31}\rho\frac{E_p}{E}\frac{bL^3}{h}$$

where G is the gain of the charge amplifier, L and b are the length and width of the cantilever, h is the thickness of the mechanical layer, ρ is the average density of the cantilever, d_{31} is the relevant piezoelectric coefficient for ZnO, and E and E_p are the Young's moduli of the silicon and ZnO respectively. State clearly any assumptions used in obtaining this result. [8]

Calculate the expected sensitivity for an accelerometer having L = 1 mm, b = 50 μm, and h = 5 μm if the ZnO layer is 0.5 μm-thick and the amplifier has a gain of 1 V/pC. Assume $E_p \approx E$ = 160 GPa, and d_{31} = 2.3 pC/N. The densities of silicon and ZnO are 2330 kgm^{-3} and 5605 kgm^{-3} respectively. You may ignore the mass of the aluminium electrodes. [4]

d) An alternative device is proposed, where the cantilever has the same dimensions but the transduction is performed using a p-type piezoresistive bridge. Assuming the silicon is (100)-oriented, and that the cantilever is <110>-aligned, show on a sketch where, and in what orientation, the bridge should be placed for maximum sensitivity. Also estimate the sensitivity of this device at the bridge output, assuming the bridge has a 5 V supply. You may assume the following values for the reduced piezoresistive coefficients for silicon: π_L = 72 x 10^{-11} Pa^{-1}; π_T = -66 x 10^{-11} Pa^{-1}. [6]

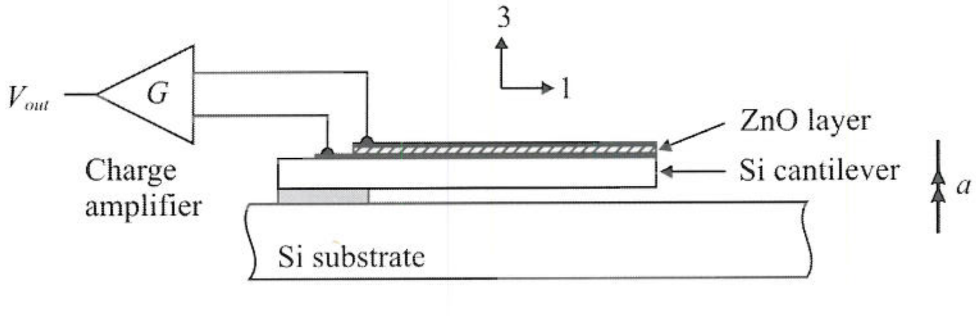


Figure 5

MEMS and Nanotechnology E4.55
2010 AO11

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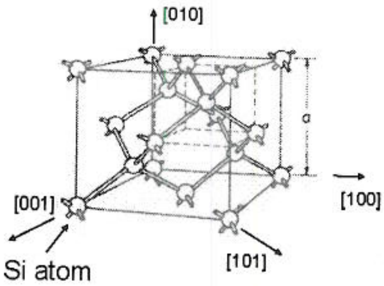
Answer, Question 1:

a) Inspecting the unit cell:



Total number of atoms in the unit cell $N_t = 18$. However, in the crystal a group of only $n = 8$ atoms repeats, the remaining 10 atoms lie in the next unit cells in the x, y and z directions. Volume of unit cell $V = a^3 = 1.57 \times 10^{-28}$ m^3. Therefore, atomic concentration $D = n/V = 8/1.57 \times 10^{-28} = 5 \times 10^{28}$ atoms/m^3.

[Marks: 5]

b) Quantum confinement in the spherical nanocrystal will raise the bottom of the conduction band and lower the top of the valance band by the corresponding confinement energy $E_k = [\pi^2\hbar^2]/[2m_{eff}m_0(d/2)^2]$, where $\hbar = h/2\pi$ is reduced Planck's constant, d is the diameter, m_0 is the electron rest mass, and m_{eff} is the relevant effective mass. This increases the band gap E_g. Confinement of electron within a small volume, from Heisenberg's Uncertainty Principle, will remove the requirement for momentum conservation in the e-h recombination process.

The energy E_p of the emitted photon is then given by:

$$E_p = \hbar\omega_p = E_g + [\pi^2\hbar^2]/[2m_e m_0(d/2)^2] + [\pi^2\hbar^2]/[2m_h m_0(d/2)^2]$$
$$= 1.1 + 1.17 + 0.69 = 2.96 \text{ eV} = 4.74 \times 10^{-19} \text{ J}$$

The wavelength of the emitted photon is then given by:

$\lambda = c/f = 2\pi c/\omega_p = 2\pi c\hbar/E_p = 420$ nm.

[If the answer uses the confinement energy $E = [\pi^2\hbar^2]/[2m_{eff}m_0d^2]$ in 1-D, then 1 mark may be deducted.]

[Marks: 5]

c) Assume the wave-function Ψ is represented by a travelling wave:

$$\Psi = A\exp i(kx - \omega t)\ , \quad \text{where } k = 2\pi/\lambda$$

For the free particle in a region with potential energy V:

Total energy = kinetic energy + potential energy

$$\Rightarrow \qquad E = \frac{p^2}{2m} + V$$

where p is the momentum. Furthermore, $p = \hbar k$.

Using the wave-function Ψ, we see that:

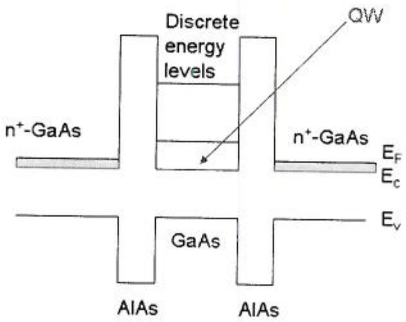
$$\frac{\partial \Psi}{\partial t} = -i\omega\Psi \quad \text{and} \quad \frac{\partial^2 \Psi}{\partial x^2} = -k^2\Psi$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

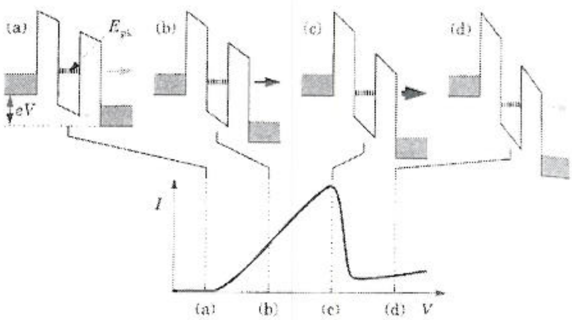
where $\Psi\Psi^*$ represents the probability density.

[Marks: 5]

d) Resonant tunnelling diodes typically use double-barrier heterostructures to form a potential well along one dimension. For example, in a n^+GaAs/AlAs/i-GaAs/AlAs/n^+GaAs heterostructure, the AlAs layers form tunnel barriers and the central GaAs region forms a potential well. A simplified band diagram is shown below:



The discrete states in the well lead to current peaks in the I-V characteristics, as they drop below the conduction band edge in the source. Current flows when a state overlaps with filled states in the source. The process is shown below diagrammatically, for a single current peak/state in the well:



[Marks: 5]

e) λ is the optical wavelength which is determined by the light source.
NA is the numerical aperture of the projection lens, which is a measure of the maximum off axis angle for rays forming the image. θ_m is the maximum ray angle, which is set by the lens, while n is the refractive index of the medium between the lens and the wafer. So the NA depends on both the lens and the medium in the image space.
k_1 is a dimensionless parameter of order unity which depends primarily on the illumination conditions, but also on the resist contrast and the type of mask used (normal/phase-shift/proximity-corrected). [5]

f) Considering the axial extension/compression of a cantilever (simplest case), the force P will be AEv/L where A is the cross-section, L is the length, v is the axial displacement at the free end, and E is Young's modulus. The stiffness is therefore $k = P/v = AE/L$ which will scale as L^1.

The resonant frequency for a mass-spring system is $\omega_0 = (k/m)^{1/2}$. Since k scales as L^1, and m scales as L^3, it follows that ω_0 scales as L^{-1}.

The viscous force due to Couette flow of the air in the gap is $F = \mu A\dot{x}/d$, where μ is the viscosity, A is the plate area, $\dot{x}$ is the velocity and d is the gap. The damping constant is therefore $c = \mu A/d$, and the Q is given by $Q = m\omega_0/c = m\omega_0 g/\mu A$. It follows that Q will scale as L^1 if μ is constant. [5]

g) The main actuation mechanisms are: electrostatic (ES), electrothermal (ET), piezoelectric (PE) and electromagnetic (EM).

ES and ET actuators are easiest to implement because they can be fabricated in a single mechanical layer. EM actuators are most difficult because they require magnetic materials and tend to be geometrically more complex if they include an efficient magnetic circuit.

ES, PE and EM actuators are all fast. ET actuators are slower because they are limited by the thermal time constant of the structure.

ES and EM are limited to relatively low force levels by breakdown and Joule heating respectively. PE and ET actuators can generally develop larger forces. [5]

h) The bending equation for the deflection $v(x)$ of a buckled beam that is built-in at both ends and subject to an axial end load is:

$$EI\frac{d^2v}{dx^2} = C - Pv$$

where E is Young's modulus, I is the second moment of area of the beam, P is the axial load and C is the couple required at the supports to prevent end rotation. The general solution is of the form:

$$v = A\cos\kappa x + B\sin\kappa x + C/P \quad , \quad \kappa = \sqrt{\frac{P}{EI}}$$

The boundary conditions are $v = 0$, $v' = 0$ at $x = 0$ and $x = L$, from which it follows that:

$$B = 0,\ A = -C/P \text{ and } \kappa = 2n\pi/L$$

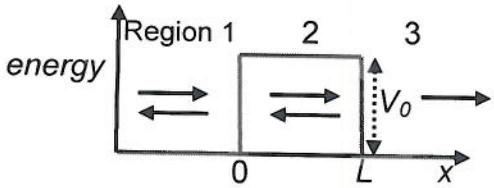
The lowest order solution, where n = 1, is the only one that occurs in practice, and for this solution we have $k = 2\pi/L$. The end load on the buckled beam, which is also the critical load for buckling is therefore:

$$P = EI\kappa^2 = \frac{4\pi^2 EI}{L^2} \qquad [5]$$

Answer, Question 2:

Here, the potential energy $V(x)$ given by:

$V(x) = V_0,\ 0 \le x \le L$
$V(x) = 0,\ x < 0,\ x > L$



(a) Space-dependent wave-functions in region 1, 2 and 3 are:

Region 1:

$$\psi_1 = a_{1i}\exp(ik_1x) + a_{1r}\exp(-ik_1x) = \exp(ik_1x) + r\exp(-ik_1x)$$

where we assume $a_{1i} = 1$
and r = amplitude reflection coeff.

Region 2:

$$\psi_2 = a_{2i}\exp(ik_2x) + a_{2r}\exp(-ik_2x)$$

Region 3:

$$\psi_3 = a_{3i}\exp(ik_1x)$$

[Marks: 2 +2 + 2]

(b) Wave-vectors in regions 1 and 2, for energy E, are as follows:

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} \qquad k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

Region 3 is identical to region 1, therefore $k_3 = k_1$

The boundary conditions at $x = 0$ are:

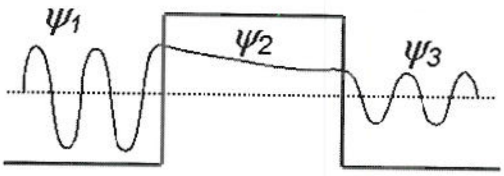
$$\psi_1 = \psi_2 \qquad \text{and} \qquad \frac{d\psi_1}{dx} = \frac{d\psi_2}{dx}$$

The boundary conditions at $x = L$ are:

$$\psi_2 = \psi_3 \qquad \text{and} \qquad \frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$$

[Marks: 3 + 3]

(c) For $E < V_0$, the wavefunctions have the form shown below:



[Marks: 4]

6

(d)

Boundary conditions:

At x = 0: $\psi_1 = \psi_2 \quad \Rightarrow \quad 1 + r = a_{2i} + a_{2r}$ (1)

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \quad \Rightarrow \quad k_1(1-r) = k_2(a_{2i} - a_{2r}) \qquad (2)$$

Similarly, at x = L: $a_{2i}e^{ik_2L} + a_{2r}e^{-ik_2L} = a_{3i}e^{ik_1L} = t$ (3)

$$k_2\left(a_{2i}e^{ik_2L} - a_{2r}e^{-ik_2L}\right) = k_1 t \qquad (4)$$

where t = amplitude transmission coeff.

Dividing Eq. 4 by k_2 and adding to Eq. 3 $\Rightarrow$

$$a_{2i} = \left(1 + \frac{k_1}{k_2}\right)\frac{te^{-ik_2L}}{2} \qquad (5)$$

Substituting Eq. 5 into Eq. 3 $\Rightarrow$

$$a_{2r} = \left(1 - \frac{k_1}{k_2}\right)\frac{te^{ik_2L}}{2} \qquad (6)$$

Dividing Eq. 2 by k_1 and adding to Eq. 1 $\Rightarrow$

$$2 = a_{2i}\left(1 + \frac{k_2}{k_1}\right) + a_{2r}\left(1 - \frac{k_2}{k_1}\right) \qquad (7)$$

Finally, substituting Eq. 5 and 6 into Eq. 7 $\Rightarrow$

$$\boxed{t = \frac{4k_1k_2}{(k_1+k_2)^2 e^{-ik_2L} - (k_1-k_2)^2 e^{ik_2L}}} \qquad (8)$$

[Marks: 14]

Answer, Question 3:

(a) We scale all the device dimensions by a factor $K = 5$. We then t_{ox}, L, and W changing to t_{ox}/K, L/K, and W/K. This gives us the following new values for the dimensions:

$t_{ox} = 50/5 = 10$ nm, $L = 1000/5 = 200$ nm, and $W = 10000/5 = 2000$ nm.

In order to keep the electric fields F constant, the following changes must be made:

(i) Doping concentration:

In depletion region, Poisson's Eq. $\Rightarrow \dfrac{\partial F_x}{\partial x} + \dfrac{\partial F_y}{\partial y} = \dfrac{-eN_A}{\varepsilon_{Si}\varepsilon_o}$

So, to keep F_x, F_y constant when the dimensions are scaled by $1/K$, we must scale N_A and N_D by K:

$\Rightarrow N_A, N_D \rightarrow KN_A, KN_D$

This give us new values of doping concentrations, $N_A = 5 \times 10^{23}$ /m^3, and $N_D = 5 \times 10^{25}$ /m^3.

(ii) Voltages:

As $F \sim V/L$, reducing L by $L/K \Rightarrow V \rightarrow V/K$. Therefore, the new maximum values of gate and drain voltage are: $V_g = V_d = 5/5 = 1$ V.

(iii) Gate capacitance:

As $C = \varepsilon_{ox}\varepsilon_0 A/t_{ox}$, and $A \rightarrow A/K^2$, $t_{ox} \rightarrow t_{ox}/K \Rightarrow C \rightarrow C/K$. Therefore, the new value of the gate capacitance is:
$C = \varepsilon_{ox}\varepsilon_0 A/Kt_{ox} = (4\times8.854\times10^{-12}\times10^{-6}\times10^{-5})/(5\times50\times10^{-9}) = 1.42 \times 10^{-15}$ F.

(iv) Inversion layer charge:

$Q_n \sim CV/A$ and $C \rightarrow C/K$, $V \rightarrow V/K$, $A \rightarrow A/K^2$. This $\Rightarrow Q_n$ remains constant. The value for $Q_n \sim CV/A = 7.1\times 10^{-4}$ C/m^2.

This has assumed that V_{th} has scaled. In practice, this is difficult to achieve.

The answer may use the more accurate version for inversion layer charge = surface charge – depletion layer charge, i.e. $Q_n = CV/A + \sqrt{(2\varepsilon_{si}\varepsilon_0\, eN_A(V_g+2\psi_B))} = 5.1\times 10^{-3}$ C/m^2. However, this expression also does not change with scaling as in the second term, KN_A and V_g/K cancel each other's effect. Both versions are acceptable here.

[Marks: 3+3+3+3]

(b) For the saturation and sub-threshold currents:

(i) Drift current (linear and saturation region current):

$I_{drift}/W = Q_n v$, where $v = \mu F$ is the carrier velocity. As both Q_n and v are constant, $\Rightarrow I_{drift}/W$ remains constant.

But, as $W \rightarrow W/K$, we have $I_{drift} \rightarrow I_{drift}/K$

(ii) Diffusion current (sub-threshold current):

$I_{diff} / W = D_n(dQ_n /dx) = (mkT/e)(dQ_n /dx)$. As Q_n is constant, and $x \rightarrow x/K$, $\Rightarrow I_{diff}/W \rightarrow K(I_{diff}/W)$, i.e. I_{diff}/W increases by factor K. But, as $W \rightarrow W/K$, we have I_{diff} is constant.

(i) and (ii) combined imply that the MOSFET 'on/off' ratio is degraded, and relatively, it tends not to turn 'off'.

[Marks: 3+3]

(c) For circuit delay, transistor density and power density:

(i) Circuit delay:

Here, $t \sim RC$. The channel resistance $R \sim V/I_{drift}$. As $V \rightarrow V/K$ and $I_{drift} \rightarrow I_{drift}/K \Rightarrow R$ remains constant. Therefore, as $C \rightarrow C/K$ and $R \rightarrow$ constant $\Rightarrow t \rightarrow t/K$, i.e. the MOSFET is faster.

(ii) Transistor density:

Here, $D_c \propto 1/A$. As $A \rightarrow A/K^2$, $\Rightarrow D_c \rightarrow K^2 D_c$, i.e. more transistors on the chip.

(iii) Power density:

The power per transistor $P \sim VI_{drift}$. As $V \rightarrow V/K$ and $I_{drift} \rightarrow I_{drift}/K$, $\Rightarrow P \rightarrow P/K^2$. Therefore, $P_{den} \sim P/A$ is constant.

[Marks: 2+2+2]

(d) For the threshold voltage:

$$V_{th} = \varphi_{ms} - \frac{Q_f}{C_{ox}} + 2\psi_B + \frac{\sqrt{2e\varepsilon_{Si}\varepsilon_0 N_A(2\psi_B + |V_{sub}|)}}{C_{ox}}$$

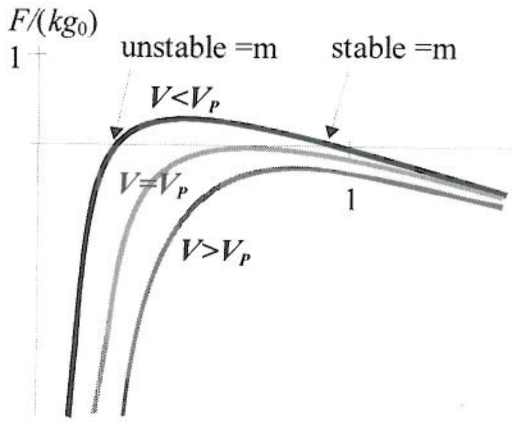
$V_{FB} = \varphi_{ms} - Q_f/C_{ox}$, and ψ_B are mainly material dependent. The last term will scale as $K^{3/2}$ as $N_A \rightarrow KN_A$, $C_{ox} \rightarrow C_{ox}/K$. Therefore, $\underline{V_{th}}$ tends not to inherently scale down, and for constant V_{FB} has a part that scales-up. This will tend to increase the 'off' current, and limit the down-scaling of gate and drain voltages.

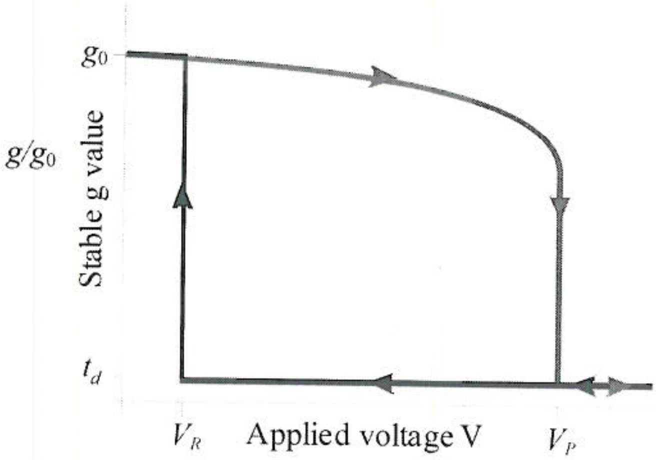
[Marks: 6]

Answer, Question 4:

a) The electrostatic force on the moveable plate is $F_e = -A\varepsilon_0 E^2/2$ where $E = V/(g + t_d/\varepsilon_r)$ is the electric field in the gap. The only other force acting on the moveable plate is the spring force $F_k = k(g_0 - g)$, so the total force is:

$$F = F_e + F_k = k(g_0 - g) - \frac{\varepsilon_0 A V^2}{2(g + t_d/\varepsilon_r)^2} \qquad (1)$$





LH graph shows variation of force with gap for different values of V. When $V < V_P$ there are two equilibrium points (i.e. points where $F = 0$). One is stable, and the other is unstable, but the moveable plate naturally settles at the stable one (and never reaches the unstable one) if the applied voltage is increased from zero. For $V > V_P$ there is no equilibrium point, and the total force is always negative so the plate snaps down. Once this has occurred, the voltage must be reduced to the point where the electrostatic force at $g = 0$ falls below the spring force. This results in hysteretic behaviour as shown in RH graph. [8]

b) The conditions at the point of snap-down are $F = 0$ and $\partial F/\partial g = 0$. Differentiating the force equation gives:

$$\frac{\partial F}{\partial g} = -k + \frac{\varepsilon_0 A V^2}{(g + t_d/\varepsilon_r)^3} \qquad (2)$$

So, when $\partial F/\partial g = 0$ we have: $\quad \varepsilon_0 A V^2 = k(g + t_d/\varepsilon_r)^3 \qquad (3)$

Substituting (3) into (1), and setting $F = 0$, we find that $g = \dfrac{2g_0}{3} - \dfrac{t_d}{3\varepsilon_r}$ at the point of snap-down. Substituting this value of g into (3) gives the quoted result. [8]

c) The release voltage is obtained by setting the total force to zero at g = 0. From (1) this gives:

$$0 = kg_0 - \frac{\varepsilon_0 A V^2}{2(t_d/\varepsilon_r)^2} \quad \Rightarrow \quad V = \sqrt{\frac{2kg_0}{\varepsilon_0 A}}\,\frac{t_d}{\varepsilon_r} \qquad [4]$$

d) The stiffness of a built-in beam subject to a point load at the centre is $k = 192EI/L^3$, where $I = bd^3/12$ is the second moment of area. (This result is easily derived, either from first principles or from the cantilever result. It may also be quoted from memory.) For the gold bridge we have E = 80 GPa, b = 100 μm, d = 1 μm and L = 400 μm, so we expect the stiffness to be k = 2 N/m.

With k = 2 N/m, g_0 = 2 μm, A = 100 x 100 μm^2, the snap-down voltage is obtained as V_P = 7.3 V. This ignores the term t_d/ε_r which makes negligible difference to the answer.

Compressive residual stress in the bridge will tend to lower its stiffness and hence reduce the snap-down voltage. Tensile residual stress will have the opposite effect. [6]

Compared to solid-state RF switches, which are normally based on pin-diodes or FETs, RF MEMS switches generally offer lower on-state insertion loss, better off-state isolation, lower power consumption (assuming electrostatic actuation), and better linearity. [4]

Answer, Question 5:

a) In a piezoelectric material, applied stress will generate dielectric polarization and hence surface charge. This is the direct piezoelectric effect. A given stress component may generate polarisation in more than one axis, and the general relationship between the stress and the polarisation is of the form:

$$P_i = d_{ij}\sigma_j$$

where d_{ij} is a 3 x 6 matrix of piezoelectric coefficients. (In the presence of electric field the above equation will also include a normal dielectric response term.) The effect is reversible, so an applied electric field will generate mechanical strain. The direct effect is used for transduction, while the inverse effect is used for actuation. The piezoelectric effect is observed only in non-centrosymmetric crystals, so silicon is not piezoelectric.

In a piezoresistive material, applied stress leads to a change in electrical resistivity. In general both the resistivity and the piezoresistive response will be anisotropic, and the generalised relation between electric field and current density is of the form:

$$\boldsymbol{E} = (\boldsymbol{\rho}_0 + \boldsymbol{\Pi}\cdot\boldsymbol{\sigma})\cdot\boldsymbol{J}$$

where ρ_0, Π and σ are all tensors. Usually symmetries allow these equations to be vastly simplified. [6]

b) A possible sequence would be:

sputter deposit Al for bottom electrode, followed by ZnO and Al for top electrode
deposit and pattern photoresist to define top electrode
transfer pattern through Al and ZnO by wet etching and RIE
deposit and pattern photoresist in shape of cantilever (with larger anchor that will not be completely undercut during sacrificial etch)
transfer pattern through Al by wet etching and mechanical layer by RIE
remove oxide from underneath cantilever by wet etching to release [6]

c) The applied acceleration leads to a uniformly distributed transverse load of $p = -\rho bha$ per unit length on the cantilever. The bending equation is therefore:

$$M = EIv'' = p(L-x)\cdot\frac{(L-x)}{2}$$

where $v(x)$ is the deflection profile of the beam. The local strain in the beam is $\varepsilon = -v''y$ where y is the distance above the neutral axis. The N.A. is half-way up the beam if the deposited layers are assumed to have no effect on the bending behaviour. The strain at the top surface, and hence in the piezoelectric layer, is therefore $\varepsilon_p = -v''h/2$. The resulting stress component $\sigma_1 = E_p\varepsilon_p$ in the ZnO will generate a polarisation component $P_3 = d_{31}\sigma_1$ normal to the layer, and hence surface charge densities of $\pm P_3$ on the upper and lower surfaces.

The total surface charge (+/-)Q on either electrode is obtained by integrating the charge density over the width and length of the cantilever i.e.

12/12

$$Q = \int_0^L P_3 \cdot bdx = \int_0^L d_{31} E_p \frac{h}{2} \rho bha \frac{(L-x)^2}{2EI} bdx = d_{31} \rho \frac{E_p}{E} \frac{h^2 b^2 L^3}{12I} a = d_{31} \rho \frac{E_p}{E} \frac{bL^3}{h} a$$

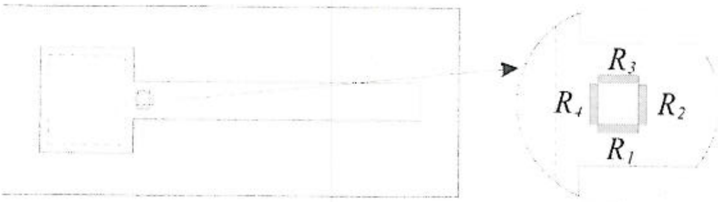
where we have used $I = bh^3/12$ for the second moment of area of the cantilever. Multiplying by G, to account for the charge amplifier, the output voltage is obtained as:

$$V_{out} = G d_{31} \rho \frac{E_p}{E} \frac{bL^3}{h} a$$

from which the required result follows. [8]

Ignoring the aluminium, the average density is (0.5x5605 + 5x2330)/5.5 = 2628 kgm^{-3}. With $\rho = 2628$ kgm^{-3}, $L = 1$ mm, b = 50 μm, h = 5 μm, $E_p \approx E = 160$ GPa, $d_{31} = 2.3$ pC/N and G = 1 V/pC, we obtain $S_q = 60$ μV/(ms^{-2}). [4]

d) The piezoresistive bridge should be placed at the position of highest stress which is at the root of the cantilever. The response in p-type material is dominated by the shear stress terms, and these will be maximised when the piezoresistors are aligned to <110> directions i.e. parallel and perpendicular to the cantilever:



Resistors R_1 and R_3 will have $\sigma_L = \sigma_{max}$, $\sigma_T \approx 0$, while R_2 and R_4 will have $\sigma_T = \sigma_{max}$, $\sigma_L \approx 0$. Assuming all resistors have the same nominal value R, their values will change to:

$R_1 = R_3 = R(1 + \pi_L \sigma_{max})$; $R_2 = R_4 = R(1 + \pi_T \sigma_{max})$

The bridge output will then be:

$$V_{out} = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} V_S = \frac{(1+\pi_L \sigma_{\max})^2 - (1+\pi_T \sigma_{\max})^2}{(2 + \pi_L \sigma_{\max} + \pi_T \sigma_{\max})^2} V_S$$

The maximum stress is given by:

$$\sigma_{\max} = -(Ev''h/2)\big|_{x=0} = -(Mh/2I)\big|_{x=0} = 3\rho L^2 a/h$$

With $\rho = 2628$ kgm^{-3}, $L = 1$ mm, h = 5 μm, and at unit acceleration, we get $\sigma_{max} = 1577$ Pa, so that $\pi_L \sigma_{max} = 1.13 \times 10^{-6}$ and $\pi_T \sigma_{max} = -1.04 \times 10^{-6}$. These values are small so we can expand the above expression to give:

$$V_{out} \approx \frac{\pi_L \sigma_{\max} - \pi_T \sigma_{\max}}{2} V_S$$

With $V_S = 5$V, this gives $V_{out}|_{a=1} = S_q = 5.4$ μV/(ms^{-2}). [6]