Final

E3.12

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

MSc and EEE PART III/IV: MEng, BEng and ACGI

OPTOELECTRONICS

Tuesday, 4 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	R.R.A. Syms
	Second Marker(s) :	W.T. Pike

Fundamental constants

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ m kg/C^2

$c = 3 \times 10^8$ m/s

$h = 6.62 \times 10^{-34}$ Js

Maxwell's equations – integral form

$\iint_A \underline{\mathbf{D}} \cdot d\underline{a} = \iiint_V \rho \, dv$

$\iint_A \underline{\mathbf{B}} \cdot d\underline{a} = 0$

$\int_L \underline{\mathbf{E}} \cdot d\underline{L} = - \iint_A \partial\underline{\mathbf{B}}/\partial t \cdot d\underline{a}$

$\int_L \underline{\mathbf{H}} \cdot d\underline{L} = \iint_A [\underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t] \cdot d\underline{a}$

Maxwell's equations – differential form

$\mathrm{div}(\underline{\mathbf{D}}) = \rho$

$\mathrm{div}(\underline{\mathbf{B}}) = 0$

$\mathrm{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t$

$\mathrm{curl}(\underline{\mathbf{H}}) = \underline{\mathbf{J}} + \partial\underline{\mathbf{D}}/\partial t$

Material equations

$\underline{\mathbf{J}} = \sigma \, \underline{\mathbf{E}}$

$\underline{\mathbf{D}} = \varepsilon \, \underline{\mathbf{E}}$

$\underline{\mathbf{B}} = \mu \, \underline{\mathbf{H}}$

Vector calculus (Cartesian co-ordinates)

$\mathrm{grad}(\phi) = \partial\phi/\partial x \, \underline{i} + \partial\phi/\partial y \, \underline{j} + \partial\phi/\partial z \, \underline{k}$

$\mathrm{div}(\underline{\mathbf{F}}) = \partial \mathbf{F}_x/\partial x + \partial \mathbf{F}_y/\partial y + \partial \mathbf{F}_z/\partial z$

$\mathrm{curl}(\underline{\mathbf{F}}) = \underline{i} \, \{\partial \mathbf{F}_z/\partial y - \partial \mathbf{F}_y/\partial z\} + \underline{j} \, \{\partial \mathbf{F}_x/\partial z - \partial \mathbf{F}_z/\partial x\} + \underline{k} \, \{\partial \mathbf{F}_y/\partial x - \partial \mathbf{F}_x/\partial y\}$

$\mathrm{curl} \, \{\mathrm{curl}(\underline{\mathbf{F}})\} = \mathrm{grad} \, \{\mathrm{div}(\underline{\mathbf{F}})\} - \nabla^2 \underline{\mathbf{F}}$

$\iint_A \underline{\mathbf{F}} \cdot d\underline{a} = \iiint_V \mathrm{div}(\underline{\mathbf{F}}) \, dv$

$\int_L \underline{\mathbf{F}} \cdot d\underline{L} = \iint_A \mathrm{curl}(\underline{\mathbf{F}}) \cdot d\underline{a}$

1a) What three assumptions are normally made about materials for electromagnetic fields in transparent media, and how do these assumptions modify Maxwell's equations? What assumption is made about the fields themselves, and how does this allow the derivation of a single equation containing only (say) the electric field?

[8]

b) The identity curl{curl($\underline{\mathbf{F}}$)} = grad {div($\underline{\mathbf{F}}$)} - $\nabla^2\underline{\mathbf{F}}$ is often used to simplify the single equation above. Explain how this process leads to a vector wave equation. Derive a scalar wave equation for linearly polarised waves.

[6]

c) What key material property determines the propagation constant of the wave? How do the wavelength and the phase velocity depend on this property?

[6]

2a) Explain the meaning of the following terms and how they are defined in terms of the electric and magnetic fields: Poynting vector, irradiance, time-averaged power.

[6]

b) Two y-polarised optical waves of equal amplitude and identical angular frequency cross in free space as shown in Figure 1. Calculate the pattern of light seen on a screen placed normal to the x-axis.

[10]

c) Assuming that the wavelength is 0.633 μm, calculate the separation between adjacent intensity maxima for $\theta = 10^\circ$.

[4]

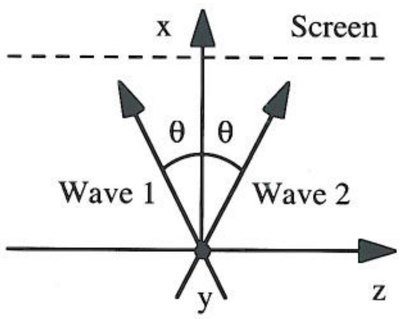


Figure 1.

3. The StarLight optical component company manufactures two planar lightwave circuits (PLCs), shown in Figure 2a. The transceiver contains a laser diode (LD), a receiver photodiode (PD) and a monitor photodiode (MON), linked by Y-junctions (Y). The splitter contains two back-to-back Y-junctions. By connecting these modules with optical fibres, Starlight constructs complete communication systems such as the 4-user network in Figure 2b. The channel guides used in the PLC are essentially loss-less. The laser diode has an output power of 1 mW. The fibre has a propagation loss of 1 dB/km and each fibre connection incurs a splice loss of 0.25 dB.

a) Explaining your reasoning, calculate i) the optical power that can be delivered by the transmitter, ii) the efficiency of the receiver, and ii) the efficiency of the splitter.

[8]

b) Assuming that each separate fibre link is 10 km long, calculate the power transmitted by Module 1 that is received at Module 3.

[6]

c) If StarLight chose to upgrade all its PLCs with 3 dB directional couplers, by how much could the laser power be lowered, without reducing the received signal?

[6]

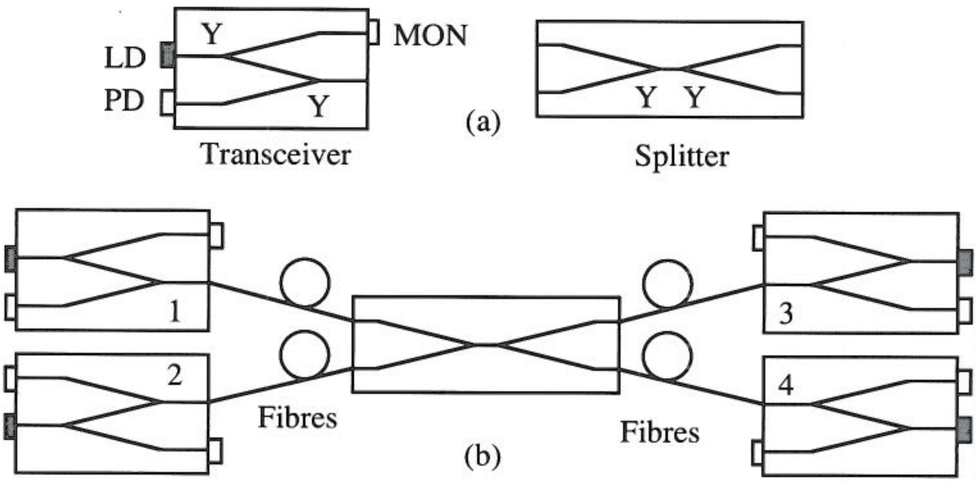


Figure 2.

4. Silica-based optical fibre is commonly used as a transmission medium in optical communication systems.

a) Sketch the spectral variation of propagation loss, describe the major mechanisms for loss and identify the wavelength for optimum performance.

[7]

b) The scalar waveguide equation for a parabolic-index medium in radial co-ordinates is:

$$d^2E/dr^2 + (1/r)\,dE/dr + \{n_0^2 k_0^2\,[1 - (r/r_0)^2] - \beta^2\}\,E = 0$$

What is the radial variation in refractive index, and how does this compare with the real variation in a fibre? Show that the Gaussian transverse mode $E(r) = E_0 \exp(-r^2/a^2)$ is a solution, and derive expressions for the mode radius and propagation constant. What are their values, if $n_0 = 1.5$, $r_0 = 50\ \mu m$ and $\lambda = 1.5\ \mu m$?

[13]

5. The lumped element rate equations for a light emitting diode (LED) are:

$$dn/dt = I/ev - n/\tau_e$$
$$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$$

a) Show how the DC and AC internal efficiencies of the LED are calculated, and find the frequency at which the optical output falls by 3 dB from its value at DC.

[8]

b) Derive an expression for the external efficiency of the LED, assuming that the medium external to the LED is air.

[6]

c) Hence calculate the 3 dB frequency, and the overall AC external efficiency at this frequency, for a LED fabricated in a material with equal radiative and non-radiative recombination lifetimes of 1 ns and a refractive index of 3.5.

[6]

6. Figure 3 shows an unlabelled sketch of an InGaAsP Fabry-Perot laser.

a) Copy the diagram and label the six layers, choosing from the compounds n-InP, p-InP, n^{+}-InP, u-InGaAsP, p^{+}-InGaAs and metal. Identify the double heterostructure and the active layer. What is the function of the etched ridge?

[10]

b) Sketch the band diagram across the double heterostructure, and explain its construction. How does it provide an effective arrangement for laser operation?

[10]

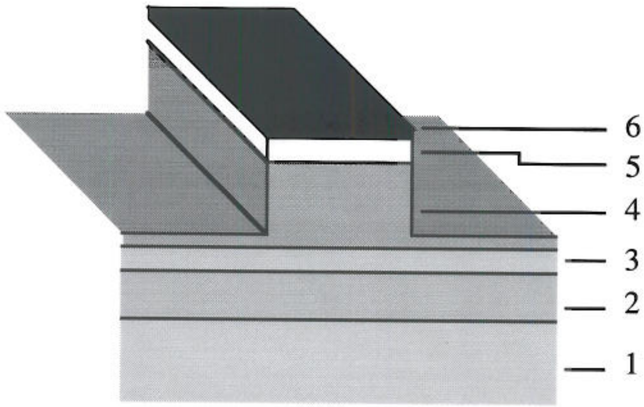


Figure 3.

E3.12

Optoelectronics 2010 – Solutions

1a) Assumptions for transparent media in the material equations

$\underline{\mathbf{J}} = 0$ (dielectric media; zero conductivity)
$\underline{\mathbf{D}} = \varepsilon_r \varepsilon_0 \underline{\mathbf{E}}$ (dielectric media; non-unity relative permittivity)
$\underline{\mathbf{B}} = \mu_0 \underline{\mathbf{H}}$ (dielectric media; unity relative permeability)

Corresponding assumptions in Maxwell's equations

$\text{div}(\underline{\mathbf{D}}) = 0$ (No charges present)
$\text{div}(\underline{\mathbf{B}}) = 0$ (Unchanged)
$\text{curl}(\underline{\mathbf{E}}) = -\partial\underline{\mathbf{B}}/\partial t = -\mu_0 \partial\underline{\mathbf{H}}/\partial t$ (From material equations)
$\text{curl}(\underline{\mathbf{H}}) = \varepsilon_r \varepsilon_0 \partial\underline{\mathbf{E}}/\partial t$ (Assuming ε_r is constant)

[5]

Assumptions made about the fields:
All oscillate at the same angular frequency ω, so that $\underline{\mathbf{F}} = \underline{\mathrm{F}} \exp(j\omega t)$ and $\partial\underline{\mathbf{F}}/\partial t = j\omega\underline{\mathbf{F}}$, where $\underline{\mathbf{F}}$ is a field quantity and $\underline{\mathrm{F}}$ is the corresponding time-independent field. Hence:

$\text{curl}(\underline{E}) = -j\omega\mu_0\underline{H}$
$\text{curl}(\underline{H}) = +j\omega\varepsilon_r\varepsilon_0\underline{E}$

Here all the exponential terms have been cancelled.

Taking the curl of both sides of the upper equation above, we get:
$\text{curl}\,\{\text{curl}(\underline{E})\} = -j\omega\mu_0\,\text{curl}\{\underline{H}\}$
Substituting using the lower equation, we get:
$\text{curl}\,\{\text{curl}(\underline{E})\} = -j\omega\mu_0 \times +j\omega\varepsilon_r\varepsilon_0\underline{E} = \omega^2\mu_0\varepsilon_r\varepsilon_0\underline{E}$

[3]

b) Given the vector identity provided for the electric field, we have:
$\text{curl}\,\{\text{curl}(\underline{E})\} = \text{grad}\,\{\text{div}(\underline{E})\} - \nabla^2\underline{E}$
Since $\text{div}(D) = \text{div}(\varepsilon_r\varepsilon_0\underline{E}) = 0$ and ε_r is constant, the equation above reduces to:
$\nabla^2\underline{E} + \omega^2\mu_0\varepsilon_r\varepsilon_0\underline{E} = 0$

[3]

Assuming that the field is linearly polarized in (say) the y-direction, we obtain the scalar equation:
$\nabla^2 E_y + \omega^2\mu_0\varepsilon_r\varepsilon_0 E_y = 0$
Assuming that the solution is an infinite plane wave, travelling in perpendicular to the direction of polarization (say, the z-direction) we must have $\partial E_y/\partial x = \partial E_y/\partial y = 0$. Hence;
$d^2E_y/dx^2 + \omega^2\mu_0\varepsilon_r\varepsilon_0 E_y = 0$

[3]

c) Assuming that $E_y = E_0 \exp(-jkz)$ and substituting into the equation above, we obtain:
$-k^2 + \omega^2\mu_0\varepsilon_r\varepsilon_0 = 0$
The propagation constant is then $k = \omega\sqrt{(\mu_0\varepsilon_r\varepsilon_0)}$
In free space, we must have $k = \omega\sqrt{(\mu_0\varepsilon_0)} = k_0$
So in general, $k = nk_0$, where $n = \sqrt{(\varepsilon_r)}$ is the <u>refractive index</u>.

[2]

The phase velocity is $v_{ph} = \omega/k = 1/\sqrt{(\mu_0\varepsilon_r\varepsilon_0)}$
In free space, we must have $v_{ph} = 1/\sqrt{(\mu_0\varepsilon_0)} = c$ (the velocity of light)
So in general, $v_{ph} = c/n$

[2]

The propagation constant can also be written as $k = 2\pi/\lambda$
Consequently $\lambda = 2\pi/k = 2\pi/nk_0$
In free space, we must have $\lambda = 2\pi/k_0 = \lambda_0$
So in general, we must have $\lambda = \lambda_0/n$

[2]

2a) The Poynting vector $\underline{\mathbf{S}} = \underline{\mathbf{E}} \times \underline{\mathbf{H}}$ represents the instantaneous magnitude and direction of power flow in an electromagnetic field.

[1]

The irradiance $\underline{S} = (1/T)\ {}_T\!\int \underline{\mathbf{S}}\ dT$ represents the time-average of the power flow, and is the quantity that is normally detected (for example, by a photodetector). For oscillating fields, $\underline{\mathbf{E}} = \underline{E}\exp(j\omega t)$ and $\underline{\mathbf{H}} = \underline{H}\exp(j\omega t)$, where the real part is implied. Consequently:

$\underline{S} = (1/T)\ {}_T\!\int \mathrm{Re}\{\underline{E}\exp(j\omega t)\} \times \mathrm{Re}\{\underline{H}\exp(j\omega t)\}\ dT$, or

$\underline{S} = (1/T)\ {}_T\!\int 1/2\ \{\underline{E}\exp(j\omega t) + \underline{E}^*\exp(-j\omega t)\} \times 1/2\ \{\underline{H}\exp(j\omega t) + \underline{H}^*\exp(-j\omega t)\}\ dT$

The terms above that vary as $\exp(j2\omega t)$ and $\exp(-j2\omega t)$ will average to zero, leaving:

$S = 1/4\ \{\underline{E} \times \underline{H}^* + \underline{E}^* \times \underline{H}\} = 1/2\ \mathrm{Re}\ \{\underline{E} \times \underline{H}^*\}$

[3]

The time-averaged power represents the total time average power flowing through a surface A, and is given by $P = {}_A\!\iint \underline{S} \cdot d\underline{a}$, where $d\underline{a}$ is a vector whose length represents the area of a small element of the surface and whose direction is normal to the element.

[2]

b) If both waves are polarised in the y-direction, travel in free space, and have equal amplitude E_0, their time-independent electric fields are given by:

$E_{y1} = E_0 \exp[-jk_0\{x\cos(\theta) + z\sin(\theta)\}]$ For wave 1, and:

$E_{y2} = E_0 \exp[-jk_0\{x\cos(\theta) - z\sin(\theta)\}]$ For wave 2, where $k_0 = 2\pi/\lambda$

The total field can be written as a linear superposition of these two fields, in the form:

$E_y = E_0\ \{\exp[-jk_0\{x\cos(\theta) + z\sin(\theta))] + \exp[-jk_0(x\cos(\theta) - z\sin(\theta))]\}$

Grouping together common exponential terms, the solution may be rearranged as:

$E_y = 2E_0 \exp\{-jk_0 x\cos(\theta)\}\cos\{k_0 z\sin(\theta)\}$

[4]

The magnetic field can be found using the Maxwell equation $\mathrm{curl}(\underline{E}) = -j\omega\mu_0\underline{H}$

Since $\underline{E}$ has only a y-component, $\mathrm{curl}(\underline{E})$ only has x- and z-components, given by:

$\mathrm{curl}(E) = -\partial E_y/\partial z\ \underline{i} + \partial E_y/\partial x\ \underline{k}$

Consequently, $\underline{H}$ has only x- and z-components, given by:

$H_x = (-j/\omega\mu_0)\ \partial E_y/\partial z$

$H_z = (+j/\omega\mu_0)\ \partial E_y/\partial x$

Consequently the irradiance must be given by:

$\underline{S} = 1/2\ \mathrm{Re}\ \{(E_y H_z^*)\underline{i} - (E_y H_x^*)\underline{k}\}$

The component of S that strikes the screen is then:

$S_x = 1/2\ \mathrm{Re}\ \{E_y H_z^*\} = 1/2\ \mathrm{Re}\{(-j/\omega\mu_0)\ E_y \partial E_y^*/\partial x\}$

[3]

Now $\partial E_y^*/\partial x = 2E_0\ \{+jk_0\cos(\theta)\}\exp\{+jk_0 x\cos(\theta)\}\cos\{k_0 z\sin(\theta)\}$ Hence:

$S_x = 1/2\ \mathrm{Re}[(-j/\omega\mu_0)\ \{+jk_0\cos(\theta)\}\ 4E_0^2\cos^2\{k_0 z\sin(\theta)\}]$ Or:

$S_x = \{2E_0^2 k_0\cos(\theta)/\omega\mu_0\}\cos^2\{k_0 z\sin(\theta)\}$

This result represents an interference pattern or fringe pattern, with alternating regions of light and dark.

[3]

c) Intensity maxima occur whenever $\cos^2\{k_0 z\sin(\theta)\} = 1$, i.e. when $k_0 z\sin(\theta) = \nu\pi$

The distance Λ between adjacent maxima is then given by:

$(2\pi\Lambda/\lambda)\sin(\theta) = \pi$

Hence $\Lambda = \lambda/\{2\sin(\theta)\}$

For $\lambda = 0.633\ \mu m$ and $\theta = 10^\circ$ we get $\Lambda = 1.823\ \mu m$

[4]

3a) Each Y-junction acts as a perfect 50 : 50 splitter in the forward direction, and radiates 50% of the power when a single input is excited in the reverse direction. These effects may be explained in terms of the excitation of symmetric and anti-symmetric super-modes.

[2]

i) Transmitter analysis: The first Y-junction will split off 50% of the power to the monitor diode, and pass the remaining 50% to the second Y-junction. The second Y-junction will radiate 50% of the remaining power, and transmit a further 50% to the output. Consequently, 25% of the power generated by the laser diode (-6dB) will be delivered to the output. If the laser generates 1 mW, this fraction corresponds to 0.25 mW

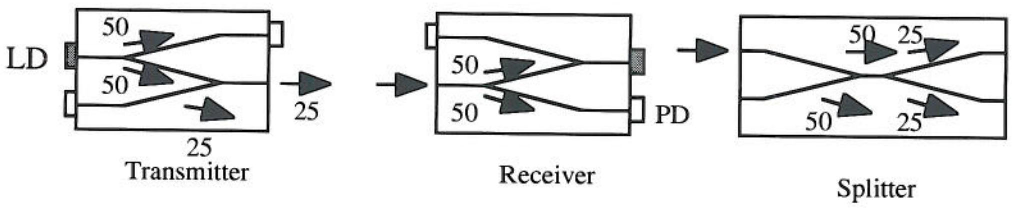
[2]

ii) Receiver analysis: The signal entering the receiver encounters a single Y-junction before the photodiode, so the receiver efficiency is 50% (-3 dB).

[2]

iii) Splitter analysis: Whichever port is used, the first Y-junction reached will always be excited in reverse, and 50% of the power will be lost at this point. The second Y-junction is 100% efficient, but splits the power equally between the two outputs so the transfer efficiency is 25% (-6 dB).

[2]



b) To calculate the transmission between Module 1 and 3, we assume the following losses:

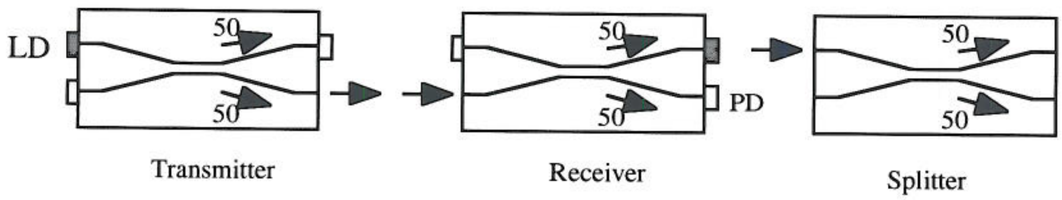
6 dB loss in the transmitter, from part a) above	6.0
1 x 10 x 0.1 dB propagation loss in the first fibre, plus	1.0
2 x 0.25 dB connection loss	0.5
6 dB loss in the splitter, again from part a)	6.0
1 x 10 x 0.1 dB propagation loss in the second fibre, plus	1.0
2 x 0.25 dB connection loss	0.5
3 dB loss in the receiver, again from part a)	3.0
Total	18.0 dB

18 dB = 6 x 3 dB. The power received by module 3 is then 1 mW x 0.5^6 mW = 15.6 μW

[6]

b) If the Y-junctions are replaced with directional couplers, the component arrangements and transfer efficiencies modify as shown below. Transmitter loss is reduced by 3 dB to 3 dB, receiver losses are unchanged and splitter efficiency is increased by 3 dB to 3dB. The overall gain is therefore 6 dB, and the laser power may therefore be decreased to 0.25 mW.

[3]

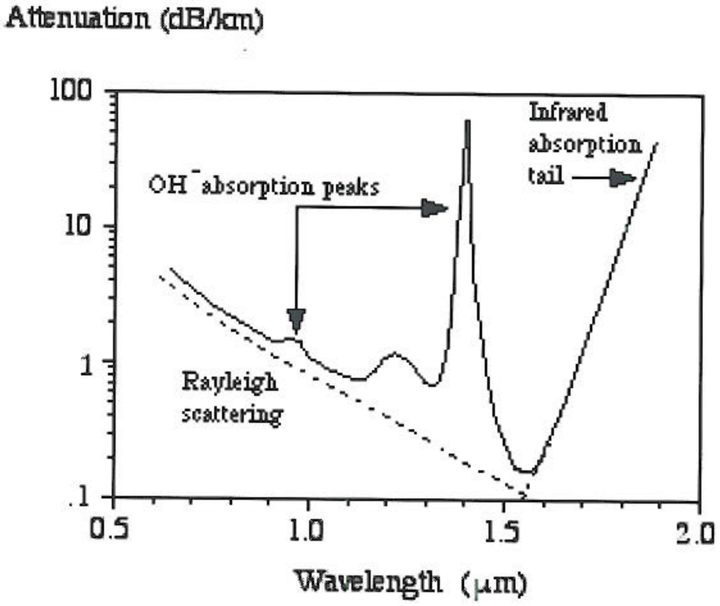


[3]

4. a) The figure below shows a typical plot of attenuation versus wavelength for silica fibre.

- At short wavelengths ($\lambda_0 < 1.6$ µm), attenuation is dominated by Rayleigh scattering loss, caused by small inhomogeneities and imperfections in the structure of the glass - compositional fluctuations on solidification, dopants and so on. The loss varies as $1/\lambda_0^4$, and is responsible for the rise in attenuation in at short wavelengths.
- At longer wavelengths, the attenuation is mainly caused by intrinsic absorption, arising from the excitation of lattice transitions at near-infrared wavelengths.
- There may also be significant attenuation near a number of discrete wavelengths. The most significant band lies at $\lambda_0 = 1.39$ µm, and is caused by the presence of residual hydroxyl (OH^-) ions, which also give rise to a number of smaller absorption peaks. The ions originate as water contamination, and must be removed by careful dehydration.
- Minimum attenuation is obtained when the Rayleigh scattering and infrared absorption curves cross, at around $\lambda_0 = 1.55$ µm.

[4]

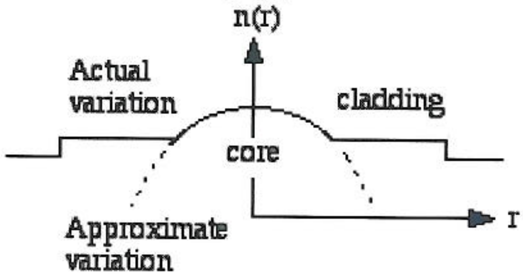


[3]

b) Start with the waveguide equation $d^2E/dr^2 + (1/r)\, dE/dr + \{n_0^2k_0^2\,[1 - (r/r_0)^2] - \beta^2\}\, E = 0$
The radial variation in index must be $n^2(r) = n_0^2\,[1 - (r/r_0)^2]$, or $n(r) = n_0\,\sqrt{[1 - (r/r_0)^2]}$

[1]

This variation corresponds to the following approximation to the real core shape:



[2]

To solve the waveguide equation, assume the Gaussian modal solution $E(r) = E_0 \exp(-r^2/a^2)$, where a is the mode radius

Differentiate to get:
$dE/dr = -(2r/a^2)\ E_0 \exp(-r^2/a^2)$
$d^2E/dr^2 = \{(4r^2/a^4) - (2/a^2)\}\ E_0 \exp(-r^2/a^2)$

Substitute into the waveguide equation to get:
$[\{(4r^2/a^4) - (2/a^2)\} - (2/a^2) + \{n_0^2 k_0^2 [1 - (r/r_0)^2] - \beta^2\}]\ E_0 \exp(-r^2/a^2) = 0$

Remove common terms and regroup to get:
$r^2\{4/a^4 - n_0^2 k_0^2/r_0^2\} + \{n_0^2 k_0^2 - 4/a^2 - \beta^2\} = 0$

[3]

To satisfy the equation for all r, the terms in each curly bracket must sum separately to zero. Hence:
$a^4 = 4r_0^2/n_0^2 k_0^2$ so that $a = \sqrt{(2r_0/n_0 k_0)}$

Similarly:
$\beta^2 = n_0^2 k_0^2 - 4/a^2 = n_0^2 k_0^2\{1 - 2/n_0 k_0 r_0\}$ so:
$\beta = n_0 k_0 \sqrt{\{1 - 2/(n_0 k_0 r_0)\}}$ hence
$\beta \approx n_0 k_0 \{1 - 1/(n_0 k_0 r_0)\}$ since $n_0 k_0 r_0 >> 1$

[3]

Assuming $n_0 = 1.5$, $r_0 = 50$ μm and $\lambda = 1.5$ μm we get:
$k_0 = 2\pi/(1.5 \text{ x } 10^{-6}) = 4.188 \text{ x } 10^6 \text{ m}^{-1}$
$n_0 k_0 r_0 = 1..5 \text{ x } 4.188 \text{ x } 10^6 \text{ x } 50 \text{ x } 10^{-6} = 314.1$

$a = \sqrt{\{2\text{x } 50 \text{ x } 10^{-6} /(1.5 \text{ x } 4.188 \text{ x } 10^6)\}} = 4$ μm (3.989)

[2]

$\beta = 1.5 \text{ x } 4.188 \text{ x } 10^6 \{1 - 1/314.1\}$
$\beta = 1.5 \text{ x } 4.188 \text{ x } 10^6 \text{ x } 0.9968 = 6.262 \text{ x } 10^6 \text{ m}^{-1}$

[2]

5a) Assume the rate equations for a light-emitting diode are:

$dn/dt = I/ev - n/\tau_e$
$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$

Assume that the current I is modulated at angular frequency ω, as $I = I' + I'' \exp(j\omega t)$, and that the electron and photon densities n and ϕ respond in similarly, as $n = n' + n'' \exp(j\omega t)$ and $\phi = \phi' + \phi'' \exp(j\omega t)$.

Substitute into the upper equation above to get:
$j\omega n'' \exp(j\omega t) = \{I' + I'' \exp(j\omega t)\}/ev - \{ n' + n'' \exp(j\omega t)\}/\tau_e$

Equate the coefficients of DC and AC terms separately, to get:
$I'/ev - n'/\tau_e = 0$
$I''/ev - n''\{1/\tau_e + j\omega\} = 0$

Re-arrange to get:
$n' = I'(\tau_e/ev)$
$n'' = I''(\tau_e/ev) \times 1/\{1 + j\omega\tau_e\}$

Assume that $d\phi/dt$ may be neglected by comparison with ϕ/τ_p since τ_p is so small. Hence:
$\phi/\tau_p = n/\tau_{rr} - \phi/\tau_p$ or $\{\phi' + \phi'' \exp(j\omega t)\}/\tau_p = \{n' + n'' \exp(j\omega t)\}/\tau_{rr}$

Equate coefficients of DC and AC terms separately:
$\phi'/\tau_p = n'/\tau_{rr} = (I'/ev)(\tau_e/\tau_{rr})$
$\phi''/\tau_p = n''/\tau_{rr} = (I''/ev)(\tau_e/\tau_{rr}) \times 1/\{1 + j\omega\tau_e\}$

Now, ϕ/τ_p represents the rate of escape of photons per unit volume. The total photon flux is $\Phi = \phi v/\tau_p$. Writing $\Phi = \Phi' + \Phi'' \exp(j\omega t)$, where $\Phi' = \phi' v/\tau_p$ and $\Phi'' = \phi'' v/\tau_p$, we get:
$\Phi' = (I'/e)(\tau_e/\tau_{rr})$
$\Phi'' = (I''/e)(\tau_e/\tau_{rr}) \times 1/\{1 + j\omega\tau_e\}$

Here I/e represents the rate of injection of electrons. If each electron generated one photon, we would expect $\Phi = I/e$. The term $\eta' = (\tau_e/\tau_{rr})$ must therefore represent the DC internal efficiency. Using a similar argument, the AC efficiency η'' must be:
$\eta'' = \eta'/\{1 + j\omega\tau_e\} = \eta'/\{1 + j\omega\eta'\tau_{rr}\}$

[6]

The light output will fall by 3 dB from the DC value when $\omega\eta'\tau_{rr} = 1$, or $\omega = 1/\eta'\tau_{rr}$.

[2]

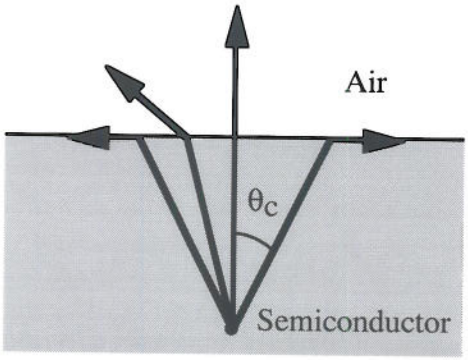
b) Light generated inside the LED will escape only if it is incident on the semiconductor-air interface at an angle less than the critical angle θ_c as shown below.

Refraction at a boundary between two media is governed by Snell's law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$, where n_1 and n_2 are the refractive indices of the two media and θ_1 and θ_2 are the angles of the incident and refracted waves.

When total internal reflection occurs, $\theta_2 = \pi/2$ and $\sin(\theta_1) = \sin(\theta_c) = n_2/n_1$.
In this case, $n_2 = 1$ (air) and n_1 is large (semiconductor) so we may write $\theta_c \approx 1/n_1$.

The fraction of the generated radiation lying within a cone with half-angle θ_c is then:

$F = \pi\theta_c^2R^2 / 4\pi R^2 = \theta_c^2/4 = 1/4n_1^2$



[3]

Not all this fraction is transmitted, since some must be reflected. At normal incidence, the (amplitude) reflection coefficient is $R = (n_2 - n_1)/(n_2 + n_1)$, and the power reflection coefficient is $P_R = R^2$. The power transmission coefficient is then $P_T = 1 - P_R$, or:
$P_T = 1 - \{(1 - n_1)^2/(1 + n_1)^2\} = 4n_1/(1 + n_1)^2$

The external efficiency η_e is then:
$\eta_e = FP_T = (1/4n_1^2)\ \{4n_1/(1 + n_1)^2\} = 1/\{n_1(1 + n_1)^2\}$

[3]

c) Assuming that $\tau_{nr} = \tau_{rr} = 1$ ns, τ_e can be found from $1/\tau_e = 1/\tau_{rr} + 1/\tau_{rr}$ as $\tau_e = 0.5$ ns.
The DC internal efficiency is then $\eta' = (\tau_e/\tau_{rr}) = 0.5$

[2]

The (angular) 3 dB frequency is $\omega = 1/\eta'\tau_{rr} = 1/(0.5 \times 10^{-9}) = 2 \times 10^9$ rad/sec, or 318 MHz
At this frequency, the AC internal efficiency is $|\eta''| = \eta'/2$

[2]

Assuming that $n_1 = 3.5$, the external efficiency is $\eta_e = 1/\{3.5(4.5)^2\} = 0.0141$
The AC external efficiency is then $|\eta''|\ \eta_e = 0.25 \times 0.0141 = 0.325\%$

[2]

6a) The available materials are: n-InP, p-InP, n^+-InP, u-InGaAsP, p^+-InGaAs and metal.
Layer 1 (substrate) must be a binary compound, heavily doped for ohmic contact (n^+-InP)
Layer 2 must be a binary compound with a large energy gap (n-InP
Layer 3 (active layer) must be a compound with a smaller energy gap (u-InGaAsP)
Layer 4 must be a compound with a large energy gap (p-InP)
Layer 5 must be a compound heavily doped for ohmic contact (p^+-InGaAs)
Layer 6 must be contact metal

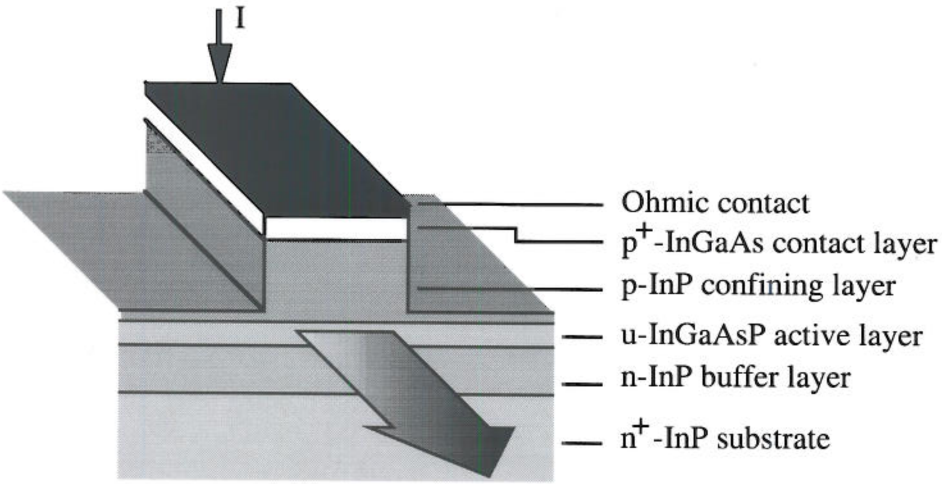
[6]

The layer structure must therefore be as shown below. Layers 2, 3 and 4 form the double heterostructure and light emission is from Layer 3.

[2]

The etched ridge is used to provide transverse confinement, so that light is emitted from a channel guide. It also localises the current flow to an active volume in the guide core.

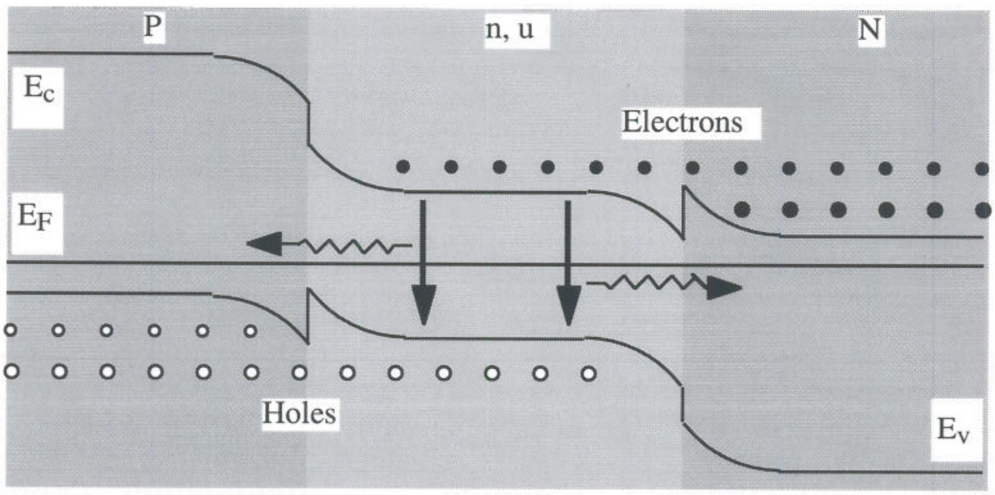
[2]



Optical output

b) A heterojunction is a p-n junction between two semiconductors with different energy gaps, and can provide energy barriers of different heights for electrons and holes. A double heterojunction is formed from cascaded junctions of different type, arranged back-to-back. In equilibrium, the band diagram is constructed by aligning the Fermi levels of the three layers, placing the conduction band edge χ from the vacuum level, where χ is the electron affinity. The variation of the vacuum level has the S-shape characteristic of a homojunction.

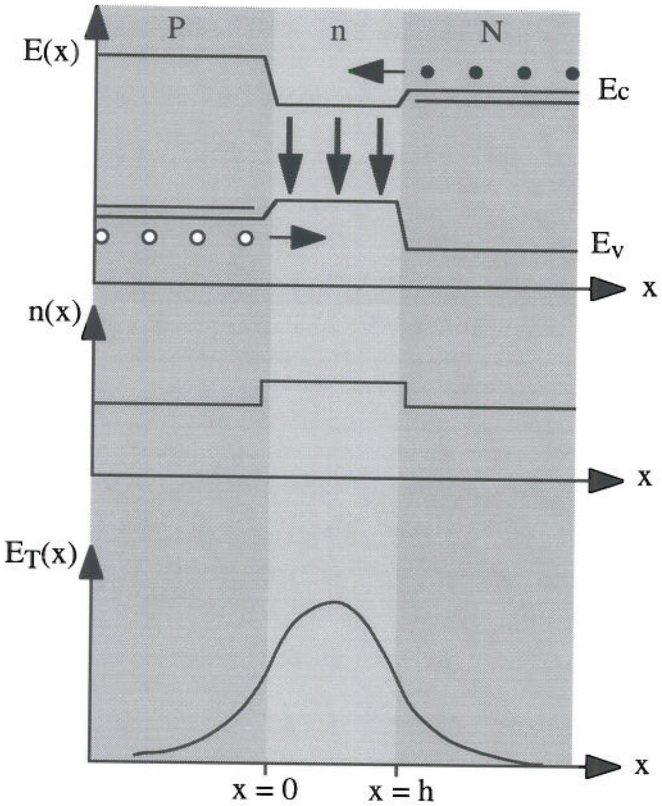
[2]



[3]

The double heterojunction can provide a large energy barrier for electrons at one of the junctions, and a large barrier for holes at the other. The spatial separation of the barriers then provides a region into which electrons and holes can be pumped, making a strongly inverted population available for generation of light by stimulated emission. In strong forward bias, this region effectively becomes a well, as shown below. Differences in refractive index between the three materials provide a waveguide in which the light may be guided, localising photons in the exact region where the densities of electrons and hole are both high and again enhancing stimulated emission.

[2]



[3]