

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2010

MSc and EEE PART IV: MEng and ACGI

Thursday, 20 May 10:00 am

Time allowed: 3:00 hours

**There are SIX questions on this paper.**

**Answer FOUR questions.**

*All questions carry equal marks*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      S. Lucyszyn  
Second Marker(s) :      A.S. Holmes

## **Special instructions to candidates**

**Special instructions for invigilators:** This is a Closed Book examination.

**Information for candidates:** This is a Closed Book examination.

## The Questions

1.

- a) Briefly explain, with the aid of simple mathematical expressions, why IMD products are generated within a FET amplifier. Suggest how their generation can be avoided and a method by which they can be suppressed. [4]
- b) Show that the IMD values quoted using the Electronic Industries Association (EIA) standards are inflated by 6 dB, when compared to US Military Standard (1131 A-2204B). [6]
- c) With the Universal Mobile Telecommunications System (UMTS):
  - (i) What level of output back-off is required at the base station final stage power amplifier and why is this level required? What happens to the power-added efficiency in this case? [3]
  - (ii) If the base station's final-stage power amplifier has a 50 dBm peak output power capability, using the value in (i), what will the average output power level be? [2]
  - (iii) Assuming linear operation, if the overall input power drops by the amount given in (i), what will happen to the  $I_3$  power level and  $IMD_3$ ? [2]
- d) For a mixer within a receiver, what is the 2<sup>nd</sup> order intermodulation component frequency better known as and how does it relate to the other frequencies? From first principles, prove that the 2<sup>nd</sup> order intermodulation component log-power gain slope is twice that of the  $(G_{conversion} P_{RF})$  log-power slope for a mixer, given the following:

$$IMD_2 \approx (IP_2 - G_{conversion} P_{RF}) [dBc] \quad (1.1)$$

[3]

2.

- a) Briefly describe the various steps needed to derive a transfer function using the Mason's non-touching loop rule and state the general form of the corresponding equation for a transfer function.

[6]

- b) Two amplifiers are connected in cascade, with the final output stage terminated with a load impedance having an arbitrary value. The amplifiers can be defined by the scattering (S)-parameter matrices  $[S_a]$  and  $[S_b]$ , and the load impedance can be expressed by its voltage-wave reflection coefficient  $\rho_L$ . Draw the corresponding signal flow graph and determine the exact expression for the overall input voltage-wave reflection coefficient  $\rho_{IN}$ .

[8]

- c) For simplicity, phase angle is ignored and the amplifiers in 2(b) are assumed to be identical. Given the following:

$$[S_a] = [S_b] = \begin{bmatrix} 0.1 & 0.1 \\ 10 & 0.1 \end{bmatrix} \quad \text{and} \quad \rho_L = 0.5 \quad (2.1)$$

Calculate the overall input return loss for the circuit in 2(b) and comment on how this has changed by introducing the amplifiers.

[6]

3.

a) Write the well-known expressions for quality (Q)-factor, in terms of:

(i) Energy stored and dissipated. [1]

(ii) Energy, power and frequency. [1]

(iii) The component values and lossless angular resonant frequency  $\omega_o$  of a series *RLC* tuned circuit. [1]

(iv) Centre-frequency and -3dB bandwidth [1]

b) For a series *RLC* tuned circuit, from first principles, derive the following expression for the associated complex angular resonant frequency  $\tilde{\omega}_o = \omega_o' + j\omega_o''$  in terms of unloaded Q-factor at the lossless angular resonant frequency  $Q_u(\omega_o)$ :

$$\tilde{\omega}_o = \omega_o \sqrt{1 - \left( \frac{1}{2Q_u(\omega_o)} \right)^2} + j \frac{\omega_o}{2Q_u(\omega_o)} \quad (3.1)$$

[Hint: Set the driving-point impedance to zero and solve the quadratic equation]

c) Using (3.1), independently confirm that: [8]

(i)  $\omega_o = |\tilde{\omega}_o|$  [1]

(ii)  $\omega_o'' = \frac{\omega_o}{2Q_u(\omega_o)}$  [1]

d) Given a series *RLC* tuned circuit with ideal lossless resonant frequency of 2.450 GHz, inductance of 1 nH and equivalent loss resistance of 5  $\Omega$ , calculate the resonant frequency and corresponding Q-factor for the lossy tuned circuit. Comment on the effect of loss on the resonant frequency and corresponding unloaded Q-factor. [3]

e) The series *RLC* tuned circuit in 3(d) is terminated at both ends with a 1  $\Omega$  impedance. Assuming that there is negligible frequency de-tuning, calculate the:

(i) Loaded Q-factor [1]

(ii) -3 dB bandwidth [1]

(iii) Insertion loss at the resonant frequency [1]

4. With a given filter order  $n$ , the element values for the Butterworth lowpass prototype can be calculated using equation (4.1):

$$\begin{aligned} g_o &= 1.0 \\ g_i &= 2 \sin\left(\frac{(2i-1)\pi}{2n}\right) \quad \text{for } i = 1 \text{ to } n \\ g_{n+1} &= 1.0 \end{aligned} \quad (4.1)$$

- a) Design a 5-pole  $LC$  Butterworth bandpass filter with a passband from 2.4 to 2.5 GHz and terminating impedance of 50  $\Omega$ . The first and last components should be series. Explain why there is a large variation in component values and suggest a way to avoid this. [10]
- b) Show mathematically, from first principles, how a shunt  $RLC$  tuned circuit can replace a series  $RLC$  tuned circuit by employing admittance ( $J$ )-inverters. How can the admittance inverters be implemented with lumped-element components? [6]
- c) Using a capacitive admittance inverter:
  - (i) What would be an appropriate value for  $J^2$  and why? For the example in 4(a), calculate the appropriate modulus value for the capacitance in the admittance inverter. [2]
  - (ii) What happens to the negative capacitances? [1]
  - (iii) For the example in 4(a), what action can be taken to avoid unwanted resonances at high frequencies caused by parasitics? [1]



5. Figure 5.1 represents the generic topology of an FET oscillator

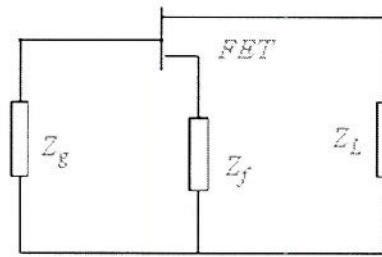


Figure 5.1

- Briefly describe the principal role for each of the impedances identified in Figure 5.1. [3]
- Replace the symbolic representation of the FET in Figure 5.1 with the equivalent circuit model of a MESFET, having the following intrinsic elements:  $R_i$ ,  $C_{gs}$ ,  $i_{ds}$ ,  $g_m$  and  $Z_{ds}$  (all variables have their usual meaning). [2]
- Reconfigure the circuit topology from 5(b) to that of the more conventional feedback oscillator topology. Clearly indicate the amplification and frequency-selective feedback network sections. With the conventional feedback topology, what are the two main conditions for oscillation? [5]
- Using linear circuit analysis, from first principles, prove that the condition for oscillation satisfies (5.1) below:

$$Z_{ds} + Z_i + Z_L = -g_m \alpha Z_{ds} Z_i \quad (5.1)$$

$$\text{where } \alpha = \frac{1}{1 + j\omega C_{gs}(R_i + Z_g)} \quad \text{and} \quad Z_i = Z_f // (R_i + 1/j\omega C_{gs} + Z_g)$$

and  $Z_i$  represents the impedance looking into the source of the MESFET.

- By derivation, or otherwise, give the equation for the value of negative resistance looking into the gate of a MESFET that has a feedback capacitance  $C_f$  connected between its source and ground. Calculate this resistance at 2.450 GHz, with  $C_f = 300$  fF and a MESFET having  $C_{gs} = 81$  fF and  $g_m = 15$  mS. [5]

6.

- a) Draw the circuit diagram of a simple single-ended class-B power amplifier with resistive load and state the role of each component. Describe the performance characteristics of a typical class-B power amplifier with resistive load and state where it is employed in practice.

[8]

- b) Given a MESFET with the following specifications:

$$V_{gd|BD} = 20 \text{ V}; V_p = -3 \text{ V}; V_k = 1 \text{ V}; I_{dss} = 1 \text{ A}; \phi = 0.5 \text{ V}$$

All variables have their usual meaning.

For a class-B power amplifier, using first-order approximations, calculate the load resistance required for maximum linear output power, and the output power in this case .

[6]

- c) Design a simple network to impedance match the load resistance in 6(b) to a  $50 \Omega$  load, using only  $50 \Omega$  coaxial cables.

[2]

- d) What simple change to the circuit drawn in 6(a) can be made to remove the unwanted harmonics? Give a disadvantage of this technique.

[2]

- e) Describe how the push-pull class-B power amplifier compares in performance with the single-ended class-B power amplifier with resistive load.

[2]



**Model answer to Q 1(a): Bookwork**

Intermodulation distortion (IMD) products are generated within a FET wherever signals, having at least two frequency components, are applied to bias dependent elements that exhibit nonlinear behaviour, e.g.  $g_m(V_{gs})$  &  $C_{gs}(V_{gs})$ .

$$g_m[V_{gs}] \text{ and / or } C_{gs}[V_{gs}] = a + bV_{gs} + cV_{gs}^2 + \dots \text{ nonlinear transfer function}$$

In simple calculations,  $g_m$  is assumed to be independent of  $V_{gs}$ . For this to occur, the FET has to have a non-uniform doping profile. Unfortunately, FETs with near uniform doping profiles are easier & cheaper to manufacture and are, thus, much more common. As a result,  $g_m$  has a  $|V_{gs}|^{-1/2}$  dependency and, therefore, the FETs inherently generate IMD components in the output spectrum – even in Class-A operation and under relatively small-signal conditions. For simplicity, however,  $g_m$  is more usually modelled as linearly decreasing with  $V_{gs}$ .

IMD components are also generated because  $C_{gs}$  has a  $|V_{gs}|^{-1/2}$  dependency.

IMD products can be avoided by employing FETs having appropriate non-uniform doping concentration profiles, so that  $g_m$  remains constant as  $V_{gs}$  is varied.

IMD products can be minimised with the use of a lineariser.

[4]

**Model answer to Q 1(b): Tutorial Derivation**

EIA standards use the peak envelope power (PEP) as the reference, while the military specification uses the average power of one of the two tones (e.g.  $I_1$ ). PEP occurs at the instance in time when the voltage peaks of the two tones coincide.

$$IMD_{EIA} = C[dBm] - PEP[dBm]$$

$$IMD_{MIL} = C[dBm] - I_1|_{RMS} [dBm]$$

$$PEP = I_1|_{pk} + I_2|_{pk} = 2I_1|_{pk}$$

$$I_1|_{pk} = \frac{(V_1|_{pk})^2}{R_{Load}} \text{ where } V_1|_{pk} = \sqrt{2}V_1|_{RMS}$$

$$\therefore I_1|_{pk} = \frac{2(V_1|_{RMS})^2}{R_{Load}} = 2I_1|_{RMS}$$

$$\therefore PEP = \frac{4(V_1|_{RMS})^2}{R_{Load}} = 4I_1|_{RMS}$$

$$\therefore PEP[dBm] = I_1|_{RMS} [dBm] + 6[dB]$$

$$\therefore IMD_{EIA} = IMD_{MIL} + 6[dB]$$

[6]

**Model answer to Q 1(c): Computed Example**

(i) 10 dB of output backoff is required at the base station power amplifier. This is because a peak to average power ratio of between 6 and 12 dB is required for UMTS (unlike GSM). Such a large backoff will have a dramatic reduction in the power amplifier's power added efficiency, which peaks at saturation.

[3]

(ii) A 50 dBm peak output power corresponds to 100 W and so 10 dB backoff results in a 10W average output power level.

[2]

(iii) The  $I_3$  power level will drop by 20 dB and  $IMD_3$  will improve by 20 dB.

[2]

### Model answer to Q 1(d): Extended Textbook Derivation

For a mixer within a receiver, the 2<sup>nd</sup> order intermodulation component frequency is better known as the intermediate frequency ( $F_{IF} = F_{RF} - F_{LO}$ ), where  $F_{RF}$  is the input signal frequency and  $F_{LO}$  is the input local oscillator frequency (generally, but not always  $F_{RF} > F_{LO}$ ).

$$IMD_2 = \frac{(G_{conversion} P_{RF})}{I_2} \quad \text{and} \quad IMD_2 \approx (IP_2 - G_{conversion} P_{RF})[dBc] \rightarrow IMD_2 = \frac{IP_2}{G_{conversion} P_{RF}}$$

$$\therefore I_2 \sim \frac{(G_{conversion} P_{IN})^2}{IP_2} \quad \therefore \frac{\partial I_2 [dBm]}{\partial Pin [dBm]} = 2 \frac{\partial (G_{conversion} P_{RF}) [dBm]}{\partial Pin [dBm]} - \frac{\partial IP_2 [dBm]}{\partial Pin [dBm]} \quad \text{where, } \frac{\partial IP_2 [dBm]}{\partial Pin [dBm]} = 0$$

In other words, the second - order intermodulation component log power gain slope is twice that of the  $(G_{conversion} P_{RF})$  log power gain slope. Therefore,  $IMD_2$  improves as input power decreases.

[3]

### Model answer to Q 2(a): Bookwork

Mason's Non-Touching Loop Rule has the following steps:

First identify the input and output nodes you are interested in. For example, to calculate  $S_{11}$  of a network you need to concentrate on the nodes  $a1$  (input) and  $b1$  (output)

Second, identify all the PATHS going from the input to output nodes.

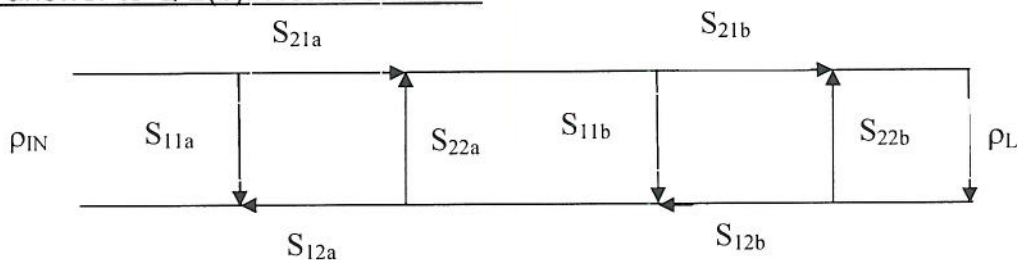
Third, identify any loops in the network, and note whether they touch any of your paths.

Fourth, apply the following equation:

$$\text{Transfer Function} = \frac{\text{Output Variable}}{\text{Input Variable}} = \frac{\sum_{n=1,2,3}^{\infty} P_n \cdot \left( \begin{array}{l} 1 - \Sigma 1\text{st order loops not touching } P_n \\ + \Sigma 2\text{nd order loops not touching } P_n \\ - \Sigma 3\text{rd order loops not touching } P_n \\ + \dots \end{array} \right)}{1 - \Sigma 1\text{st order loops} \\ + \Sigma 2\text{nd order loops} \\ - \Sigma 3\text{rd order loops} \\ + \dots}$$

[6]

### Model answer to Q 2(b): New Derivation



Paths (P) from nodes a1 to b1

P1 =  $S_{11a}$

P2 =  $S_{21a} S_{21b} \rho_L S_{12b} S_{12a}$

### Loops (L)

L1 = S22b  $\rho_L$  not touching P1

L2 = S21b  $\rho_L$  S12b S22a not touching P1

$$\rho_{IN} = b1/a1 = \frac{S11a [1-(S22b \rho_L + S21b \rho_L S12b S22a)] + S21a S21b \rho_L S12b S12a [1-0]}{[1-(S22b \rho_L + S21b \rho_L S12b S22a)]}$$

[8]

### Model answer to Q 2(c): Computed Example

$$\rho_{IN} = \frac{S11 - S11^2 \rho_L (1 + S21 S12) + S21^2 \rho_L S12^2}{[1 - S11 \rho_L (1 + S21 S12)]}$$

$$[S]_{transistor} = \begin{bmatrix} 0.1 & 0.1 \\ 10 & 0.1 \end{bmatrix} \quad \text{and} \quad \rho_L = 0.5$$

$\rho_{IN} = 0.6555$  and overall Input Return Loss is -3.668 dB. The introduction of the amplifiers has degraded the overall Input Return Loss from the value of Load Return Loss = -6 dB.

[6]

### Model answer to Q 3(a): Bookwork

$$Q(\omega) = 2\pi \frac{\text{Time - average Energy Stored}}{\text{Energy Dissipated per Cycle } (T)}$$

[1]

$$Q(\omega) = \omega \frac{\text{Time - average Energy Stored}}{\text{Power Dissipated}}$$

$$\omega = 2\pi / T$$

[1]

For a series RLC tuned circuit at the lossless resonant frequency  $\omega_o$  the unloaded Q-factor is:

$$Q_u(\omega_o) = \frac{\omega_o L}{R} \quad \text{where} \quad \omega_o = \frac{1}{\sqrt{LC}}$$

[1]

The loaded Q-factor is:

$$Q_L(\omega_o) = \frac{f_o}{-3dB \text{ bandwidth}} = \frac{1}{\text{fractional bandwidth}} = \frac{\omega_o L}{R + 2Z_o} \quad \text{where } Z_o \text{ is the load resistance}$$

[1]

### Model answer to Q 3(b): New Derivation

The driving-point impedance  $\tilde{Z}(\omega)$  of a series RLC tuned circuit is given by:

$$\tilde{Z}(\omega) = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\therefore \tilde{Z}(\omega) = \frac{\omega_o L}{Q_u(\omega_o)} + j\left(\omega L - \frac{\omega_o^2 L}{\omega}\right)$$

An Eigenmode solver determines the complex resonant frequency  $\tilde{\omega}_o$  of a resonant structure, within an electromagnetic modelling package. Alternatively, the complex frequency of the pole,  $\tilde{\omega}_o$ , in the driving-point admittance can be determined by setting  $\tilde{Z}(\omega) \equiv 0$  and solving the quadratic equation:



$$\therefore \frac{1}{Q_u(\omega_o)} + j\left(\frac{\tilde{\omega}_o}{\omega_o} - \frac{\omega_o}{\tilde{\omega}_o}\right) \equiv 0$$

multiply through by  $-j\omega_o\tilde{\omega}_o$

$$[1]\tilde{\omega}_o^2 + \left[-j\frac{\omega_o}{Q_u(\omega_o)}\right]\tilde{\omega}_o + [-\omega_o^2] \equiv 0$$

$$c.f. \quad a\tilde{\omega}_o^2 + b\tilde{\omega}_o + c \equiv 0 \rightarrow \tilde{\omega}_o \equiv \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \tilde{\omega}_o = j\frac{\omega_o}{2Q_u(\omega_o)} \pm \sqrt{\omega_o^2 - \left(\frac{\omega_o}{2Q_u(\omega_o)}\right)^2}$$

$$\therefore \tilde{\omega}_o = \omega_o \sqrt{1 - \left(\frac{1}{2Q_u(\omega_o)}\right)^2} + j\frac{\omega_o}{2Q_u(\omega_o)}$$

[8]

### Model answer to Q 3(c): New Derivation

$$\omega_o = |\tilde{\omega}_o| \text{ and only this satisfies } |\tilde{\omega}_o| = \sqrt{(\omega_o')^2 + (\omega_o'')^2}$$

[1]

$$Q_u(\omega_o) = \frac{\omega_o}{2\omega_o''} \text{ and only this satisfies } \omega_o' = \omega_o \sqrt{1 - \left(\frac{1}{2Q_u(\omega_o)}\right)^2}$$

[1]

### Model answer to Q 3(d): Computed Example

$f_o = 2.450 \text{ GHz}$ ,  $L = 1 \text{ nH}$  and  $R = 5 \text{ Ohm}$

$Q_u(f_o) = 3.07876$  at lossless resonant frequency

$$f_o' = 0.986724 f_o = 2.417 \text{ GHz}$$

[1]

$Q_u(f_o') = 3.03789$  at lossy resonant frequency

[1]

Loss has the effect of reducing both the resonant frequency and unloaded Q-factor.

[1]

### Model answer to Q 3(e): Computed Example

$$\text{Loaded Q-factor is: } Q_L(\omega_o) = \frac{f_o}{-3\text{dB bandwidth}} = \frac{1}{\text{fractional bandwidth}} = \frac{\omega_o L}{R + 2Z_o}$$

Therefore, with  $Z_o = 1 \text{ Ohm}$  the loaded Q-factor  $Q_L(\omega_o) = 2.19911$

[1]

The -3dB bandwidth is  $f_o / Q_L(\omega_o) = 1.114 \text{ GHz}$

[1]

$$\text{Insertion Loss} = 20 \log |S_{21}(\omega_o)| = 20 \log \left| 1 - \frac{Q_L(\omega_o)}{Q_U(\omega_o)} \right| = -10.88 \text{ dB}$$

[1]

### Model answer to Q 4(a): Computed Example

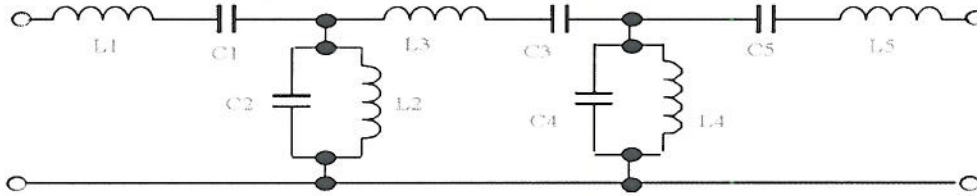
For a 5-pole Butterworth lowpass prototype the normalised elements are:

$$g_0 = 1;$$

$$g_1 = L_{n1} = 0.618; g_2 = C_{n2} = 1.618; g_3 = L_{n3} = 2; g_4 = C_{n4} = 1.618; g_5 = L_{n5} = 0.816;$$

$$g_6 = 1$$

#### Bandpass de-normalising:



$$\begin{aligned} L_{nS} &= \frac{\omega L_S}{Z_o} = \frac{g}{\Delta} \\ C_{nS} &= Z_o \omega C_S = \frac{\Delta}{g} \\ L_{nP} &= \frac{\omega L_P}{Z_o} = \frac{\Delta}{g} \\ C_{nP} &= Z_o \omega C_P = \frac{g}{\Delta} \end{aligned}$$

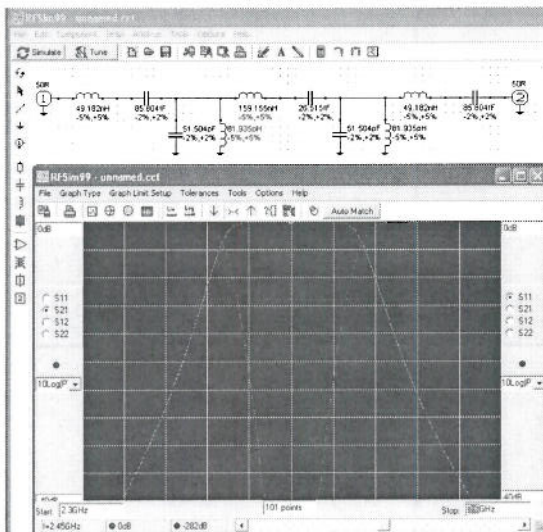
$$\begin{aligned} L_S &= \frac{g Z_o}{\omega \Delta} \\ C_S &= \frac{\Delta}{g Z_o \omega} \\ L_P &= \frac{\Delta Z_o}{\omega g} \\ C_P &= \frac{g}{\Delta Z_o \omega} \end{aligned}$$

with  $f_1 = 2.4$  GHz,  $f_2 = 2.5$  GHz, and  $R_L = 50 \Omega$ :

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_o} = \frac{f_2 - f_1}{f_o} = \frac{f_2 - f_1}{\sqrt{f_1 f_2}} = 0.0408$$

$$\omega_o = 2\pi f_o = 15.3938 \times 10^9 \text{ rad/s}$$

$$Z_o = 50 \Omega$$



$$\begin{aligned} L_{S1} = L_{S5} &= \frac{0.618 \times 50}{15.3938 \times 10^9 \times 0.0408} = 49.2 \text{ nH} \\ C_{S1} = C_{S5} &= \frac{\Delta}{g Z_o \omega} = \frac{0.0408}{15.3938 \times 10^9 \times 0.618 \times 50} = 85.8 \text{ fF} \\ L_{P2} = L_{P4} &= \frac{\Delta Z_o}{\omega g} = \frac{0.0408 \times 50}{15.3938 \times 10^9 \times 1.618} = 81.9 \text{ pH} \\ C_{P2} = C_{P4} &= \frac{g}{\Delta Z_o \omega} = \frac{1.618}{15.3938 \times 10^9 \times 0.0408 \times 50} = 51.5 \text{ pF} \\ L_{S3} &= \frac{2 \times 50}{15.3938 \times 10^9 \times 0.0408} = 159.2 \text{ nH} \\ C_{S3} &= \frac{\Delta}{g Z_o \omega} = \frac{0.0408}{15.3938 \times 10^9 \times 2 \times 50} = 26.5 \text{ fF} \end{aligned}$$

This filter has a fractional bandwidth of around 4% and so it will suffer from large variations in component values. From the values given above it can be seen that there is almost a 2000:1 variation in the inductance values and a 600:1 variation in capacitance values. To avoid such extremes impedance or admittance inverters are used.

[10]

### Model answer to Q 4(b): Extension of Bookwork

A series  $RLC$  circuit has the following impedance:

$$Z_S = R_S + j\omega L_S + \frac{1}{j\omega C_S}$$

This can be converted into a shunt  $RLC$  tuned circuit by using J-inverters. The corresponding admittance will be:

$$Y_P = G_P + j\omega C_P + \frac{1}{j\omega L_P} \equiv J^2 Z_S$$

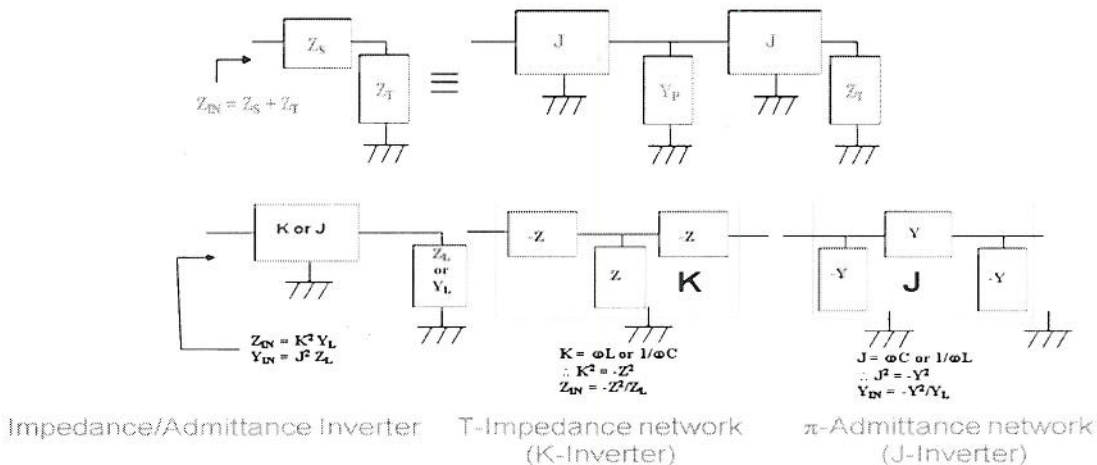
Using discrete inductance values to realise an impedance inverter with a  $-C/+C/-C$   $\pi$ -network:  $J = \omega C$

$$Y_P = R_S(\omega C)^2 + j\omega L_S(\omega C)^2 + \frac{(\omega C)^2}{j\omega C_S}$$

$$\therefore G_P = R_S(\omega C)^2 \quad ; \quad C_P = L_S(\omega C)^2 \quad ; \quad L_P = \frac{C_S}{(\omega C)^2}$$

For convenience, we can choose to set  $(\omega C)^2 = 1 \times 10^{-3}$  and, therefore, it can be easily seen that the series inductance value in  $nH$  will be equal to the parallel capacitance value in  $pF$ . Likewise, the series capacitance value in  $pF$  will be equal to the parallel inductance value in  $nH$ .

A series connected series R-L-C tuned circuit can be "synthesized" using a shunt connected parallel R-L-C tuned circuit having two admittance inverters.



Two identical inverters connected in cascade represents a zero inversion

[6]

### Model answer to Q 4(c): Computed Example

When using a capacitive admittance inverter:

- (i) For convenience, we can choose to set  $J^2 = 1 \times 10^{-3}$  because a capacitance in  $[pF]$  will transform to the same value in  $[nH]$ , and vice versa. Therefore, the capacitive admittance inverter has  $(\omega_o C)^2 = 1 \times 10^{-3}$  and  $\therefore C = 2.054 \text{ pF}$ . [2]
- (ii) The negative values of  $C$  are either absorbed into the adjacent shunt capacitance or the  $C/+C$  arrangement performs an impedance transformation of the terminating impedances. The latter can be avoided here if the first and last components are shunt. [1]
- (iii) To avoid resonances at high frequencies, due to unwanted parasitics, the lumped element shunt  $LC$  components are replaced by distributed-element transmission line stubs. [1]

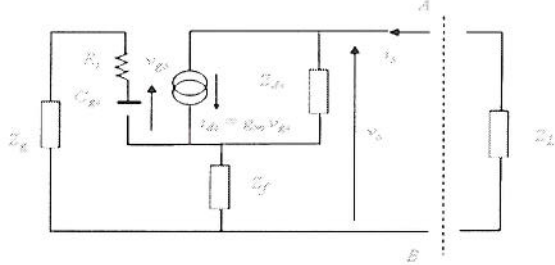


### Model answer to Q 5(a): Bookwork

The principal role of  $Z_g$  is to represent the lossy resonator (i.e. tank) circuit. The principal role of  $Z_f$  is to represent the feedback circuit that controls the negative resistance presented to the lossy resonator circuit. The negative resistance must exceed the equivalent positive loss resistance of the resonator. The principal role of  $Z_L$  is to represent the load that the oscillator feeds.

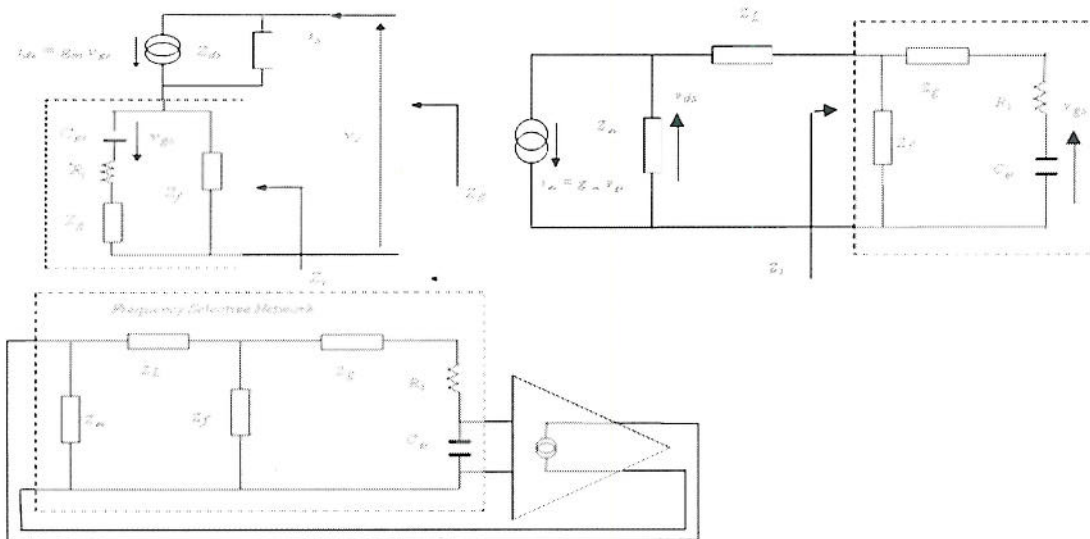
[3]

### Model answer to Q 5(b): Bookwork



[2]

### Model answer to Q 5(c): Bookwork



For oscillation, the gain of the amplifier must be sufficiently large to compensate for the losses in the frequency-selective network and the electrical phase shift around the loop must be an integer multiple of  $360^\circ$ .

[5]

### Model answer to Q 5(d): Bookwork

$$v_{ds} = -i_{ds} \times \frac{Z_{ds}(Z_i + Z_L)}{Z_{ds} + (Z_i + Z_L)}$$

$$v_{gs} = v_{ds} \times \frac{Z_i}{Z_i + Z_L} \alpha \quad \text{where} \quad \alpha = \frac{1}{1 + j\omega C_{gs}(R_i + Z_g)}$$

Now combining the above two equations:

$$Z_{ds} + Z_i + Z_L = -g_m \alpha Z_{ds} Z_i$$

[5]

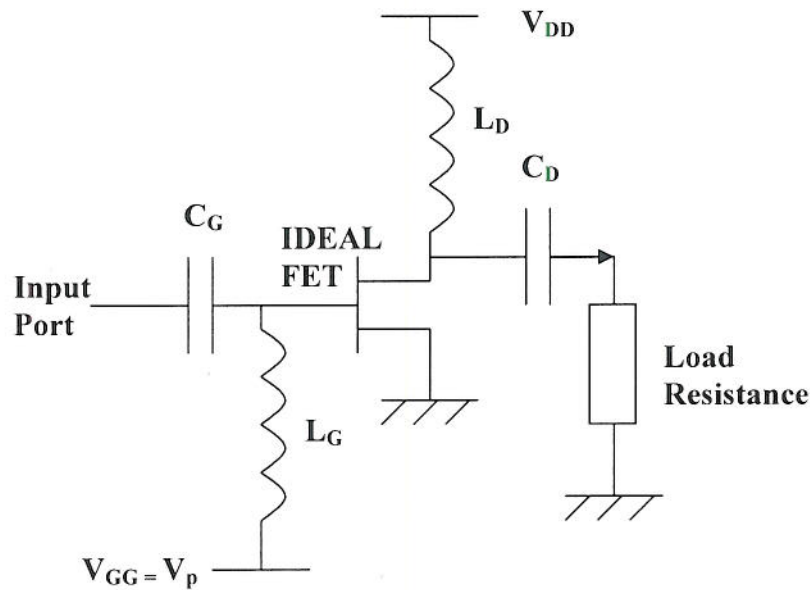
### Model answer to Q 5(e): Bookwork and Computed Example

$$R = -\frac{g_m}{\omega^2 C_{gs} C_f} = \frac{15 \times 10^{-3}}{(2\pi 2.45 \times 10^9)^2 \times 81 \times 10^{-15} \times 300 \times 10^{-15}} = -2.605 k\Omega$$

[5]

### Model answer to Q 6(a): Bookwork

The topology for a class-B amplifier with resistive load is given below:



$L_D$  and  $L_G$  are Radio Frequency Chokes (RFCs), ideally having infinite inductance.  $C_D$  and  $C_G$  are DC blocking capacitors, ideally having infinite capacitance. Since the metal-semiconductor field-effect transistor (MESFET) is also ideal, there is no need for any impedance matching networks. In practice, all these inductors and capacitors will be finite in value and lossy. They may also be absorbed into any impedance matching networks.

Class-B power amplifiers have the following characteristics, when compared to other classes:

- Output current flows for only half the period of the input voltage cycle.
- High even order harmonics are generated and, therefore, output signal distortion (in the time-domain) is found. As a result, it is only used as part of a push-pull amplifier.
- This gives poor  $P_{out}$  to  $P_{in}$  linearity compared to class-A.
- Poor dynamic range compared to class-A.
- Non-ideal for non-CW applications (e.g. AM, multi-carrier and band-filtered CW signals).
- No dissipated power (as heat) when there is no input signal, relaxing the thermal management system
- Low theoretical efficiency (<57.6%), but better than class-A and worse than class-C.

[8]

### Model answer to Q 6(b): Computed Example

$$V_{ds}|_{\max} = V_{gd}|_{BD} - V_{gs}|_{\min}$$

$$V_{gs}|_{\min} = -|V_p| - (|V_p| + \phi) = -6.5V \quad \text{from negative half cycle}$$

$$\therefore V_{ds}|_{\max} = 20 - 6.5 = 13.5V$$

$$R_{Load} = \frac{V_{ds}|_{\max} - V_k}{I_{dss}} = 12.5\Omega$$

$$P_{out}|_{\max} = (I_{ds}|_{\max RMS})^2 R_{Load} = \frac{I_{dss}}{2\sqrt{2}} \cdot \frac{V_{ds}|_{\max} - V_k}{2\sqrt{2}}$$

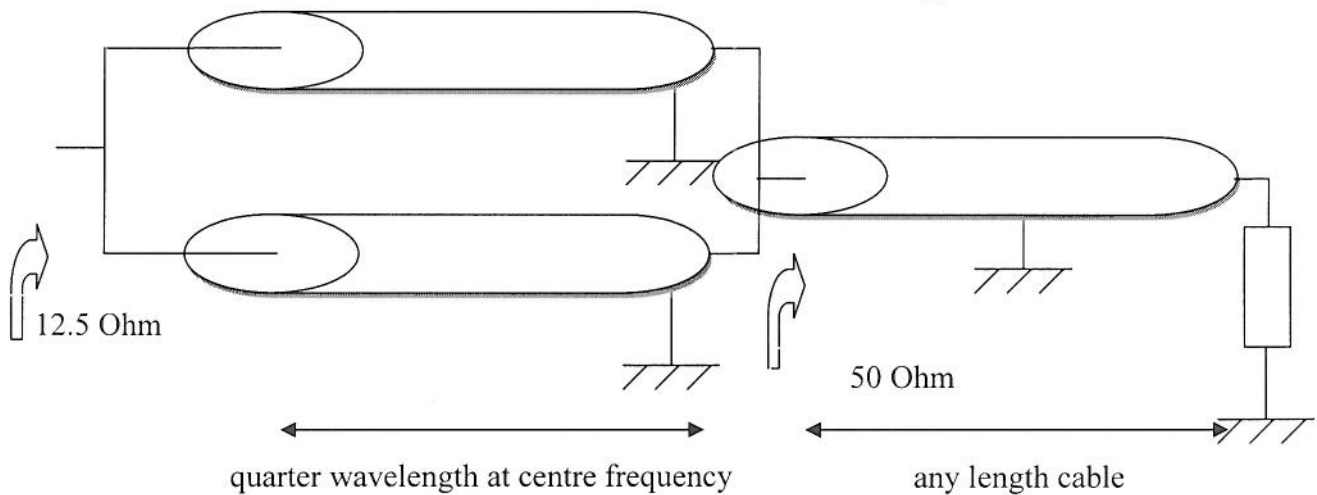
$$I_{ds}|_{\max RMS} = \frac{I_{dss}}{2\sqrt{2}} \quad @ \omega_o$$

$$\therefore P_{out}|_{\max} = \frac{I_{dss}}{8} (V_{ds}|_{\max} - V_k) = 1.5625W \rightarrow +32dBm$$

[6]

Model answer to Q 6(c): Bookwork

Two 50 Ohm transformers is equivalent to a 25 Ohm transformer:  $Z_{TX} = \sqrt{12.5 \times 50} = 25\Omega$



[2]

Model answer to Q 6(d): Bookwork

The solution is to simply add a resonant parallel tuned circuit across the load. The disadvantage is that the bandwidth will be reduced significantly.

[2]

Model answer to Q 6(e): Bookwork

When compared to the single-ended class-B amplifier with resistive load, the push-pull amplifier can have:

- ultra-wide bandwidth performance
- almost twice the amount of output power
- the same load resistance
- reduced power-added efficiency

[2]