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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

EEE PART II: MEng, BEng and ACGI

POWER

Friday, 3 June 2:00 pm

Corrected Copy Q1 ✓

Time allowed: 1:30 hours

There are THREE questions on this paper.

Answer ALL questions.

Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : T.C. Green

Second Marker(s) : B. Chaudhuri

1.

a) A current of 120 A is drawn at an angle of 15° lagging with respect to a 11 kV supply voltage.

i) Calculate the real and reactive power. [5]

ii) Determine what combination of resistance and capacitance or inductance forms the simplest impedance that would draw this current if the supply frequency is 50 Hz. [5]

b) Figure 1.1 shows a transformer equivalent circuit annotated with various voltage and current measurements.

i) Describe what is meant by flux leakage and identify the components of the equivalent circuit that model this effect. [3]

ii) Explain what physical effects in the transformer are modelled by the component R_M. [3]

iii) Calculate the power losses in the transformer for the circuit conditions given. [4]

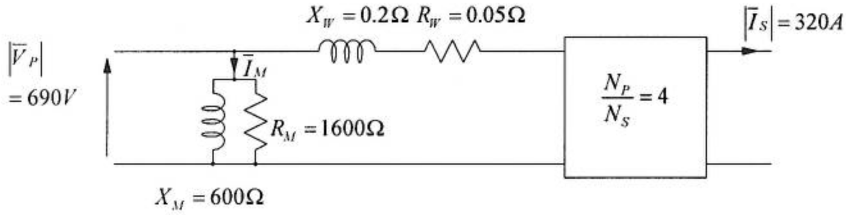


Figure 1.1 Annotated Transformer Equivalent Circuit

c) i) Figure 1.2 shows a buck switch-mode power supply. Mark on a copy of the diagram the current paths during the **on** and **off** states of the transistor and state whether the inductor current is increasing or decreasing in these states. [4]

ii) Explain the difference between continuous and discontinuous conduction. [2]

ii) A buck SMPS operating in continuous mode has an input voltage of 35 V, a switching frequency of 40 kHz and an on-time of 15 μs. Calculate the output voltage. [4]

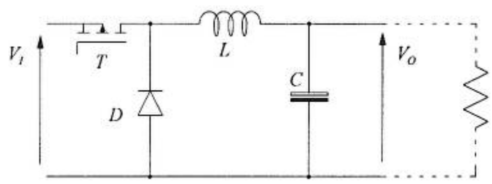


Figure 1.2 A Buck Switch-Mode Power Supply

d) i) Explain why AC rather than DC is used in electricity supply systems. [4]

ii) Explain why 50Hz is a good choice of frequency. [2]

iii) Explain why a system that has a nominal frequency of 50 Hz may suffer small variation of frequency around this value. [4]

2.

a) Sketch the typical torque against speed characteristic of an induction machine. Explain why torque is positive in the sub-synchronous region, zero at synchronous speed and negative in the super-synchronous region. [8]

b) A 2 pole-pair induction machine is rated as follows: 230 V phase; 50 Hz; 3-phase; star-connected. The equivalent circuit parameters, referred to the stator, are:

stator resistance	1.4 Ω,
referred rotor resistance	0.7 Ω,
stator leakage reactance	2.0 Ω,
referred rotor leakage reactance	0.8 Ω,
magnetising reactance	50 Ω,
magnetising resistance	negligible (infinite).

When driving a particular mechanical load, the machine is observed to spin at 1455 rpm.

i) Calculate the slip. [3]

ii) Give an expression for the total input impedance of the machine and calculate its value. [7]

iii) Calculate the stator current. [4]

iv) Calculate the referred rotor current. [4]

v) Calculate the electro-magnetic torque. [4]

3. Figure 3.1 shows the circuit of a boost switch-mode power supply (SMPS). It is to be operated in continuous conduction mode with an input voltage of 9.0 V and output voltage of 20.0 V. It will be required to process a maximum throughput power of 36 W and have an output voltage ripple which is to be no greater than 75 mV. The turn-on and turn-off energy losses of the MOSFET operated at 4 A are 5 μJ and 7 μJ respectively. The on-state resistance of the MOSFET is 0.1 Ω.

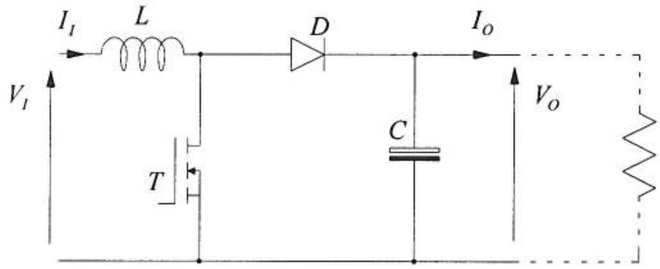


Figure 3.1 A Boost SMPS

a) i) Calculate the duty-cycle assuming continuous conduction and input current when the circuit operates at maximum power. [4]

ii) Sketch graphs of the current through the inductor, the diode and the capacitor. [4]

b) i) The maximum power loss in the MOSFET is to be 2 W. Calculate the maximum switching frequency that can be used. [6]

ii) Calculate a value of inductor to keep the inductor current continuous at an input current of 0.5 A. [6]

ii) Assuming that all of the output voltage ripple is attributed to the effective series resistance, ESR, of the capacitor, specify the ESR to achieve the required voltage ripple. [6]

c) If the switching frequency were doubled, what advantages and disadvantages would arise in the design of this SMPS? [4]

IEE2-3B Power Engineering

1. This question covers several topics and all parts should be attempted.

   a) A current of 120 A is drawn at an angle of 15° lagging with respect to the 11 kV supply voltage.

      i) Calculate the real and reactive power. [5]

      ii) Determine what combination of resistance and capacitance or inductance forms the simplest impedance that would draw this current if the supply frequency is 50 Hz. [5]

[(i) Calculation – 2 marks per correct answer, 1 mark for correct units]

$$P = \overline{V}.\overline{I} = V\,I\cos\left(\angle V - \angle I\right)$$
$$= 11{,}000 \times 120 \times \cos\left(-15°\right) = 1.275\,MW$$
$$Q = V\,I\sin\left(\angle V - \angle I\right)$$
$$= 11{,}000 \times 120 \times \sin\left(-15°\right) = 0.342\,MVAr$$

[(ii) Calculation – 2 marks for R in ohms and 3 marks for L in henries]

$$\overline{Z} = \frac{\overline{V}}{\overline{I}} = \frac{11{,}000}{120}\angle\left(0 - {}^{-}15°\right) = 91.\dot{6}\angle 15°$$
$$R = Z\cos\left(\theta\right) = 88.5\Omega$$
$$X_L = Z\sin\left(\theta\right) = 23.7\Omega$$
$$L = \frac{X_L}{2\pi f} = 75.5mH$$

   b) Figure 1.1 shows a transformer equivalent circuit annotated with various voltage and current measurements.

      i) Describe what is meant by flux leakage and identify the components of the equivalent circuit that model this effect. [3]

[Bookwork]
Leakage flux produced by current in a particular winding (primary or secondary) that does not link with the other winding (typically because it takes a local path through air rather than the core). Leakage flux does not contribute to transformer action but does cause inductive voltage drop in the winding that produces the leakage flux. Thus, leakage is modelled by (small valued) inductors in series with the ideal transformer in each of the primary and secondary connections.

      ii) Explain what physical effects in the transformer are modelled by the component R_M. [3]

[Bookwork]
RM is a dissipative component that models power loss due to magnetisation of the core material. The power loss mechanisms are hysteresis in the B/H characteristic of the core material and eddy currents in the core. (To a reasonable approximation, the total loss is dependent on flux magnitude squared, and therefore voltage squared, so a parallel component is used.)

iii) Calculate the power losses in the transformer given the voltages and currents shown. [4]

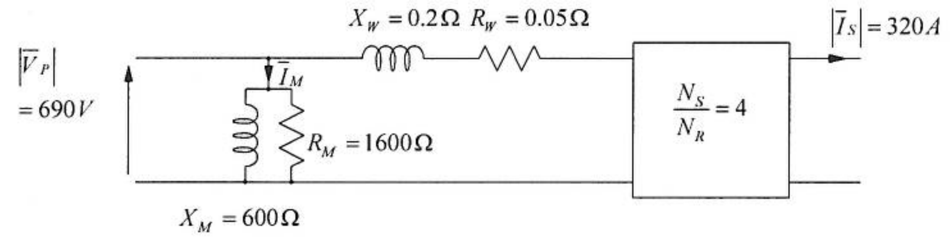


Figure 1.1 Annotated Transformer Equivalent Circuit

[Standard Calculation – 2 marks for each correct answer with no extraneous terms]

$$P_{copper} = I'^2_S\, R_W = (320/4)^2 \times 0.05 = 320W$$

$$P_{core} = \frac{V_S^2}{R_M} = \frac{690^2}{1200} = 397W$$

c) i) Figure 1.2 shows a buck switch-mode power supply. Mark on a copy of the diagram the current paths during the **on** and **off** states of the transistor and state whether the inductor current is increasing or decreasing in these states. [4]

ii) Explain the difference between continuous and discontinuous conduction. [2]

ii) A buck SMPS operating in continuous mode has an input voltage of 35 V, a switching frequency of 40 kHz and a conduction time of 15 μs. Calculate the output voltage. [4]

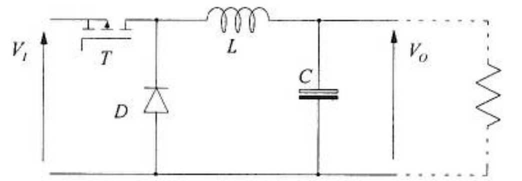
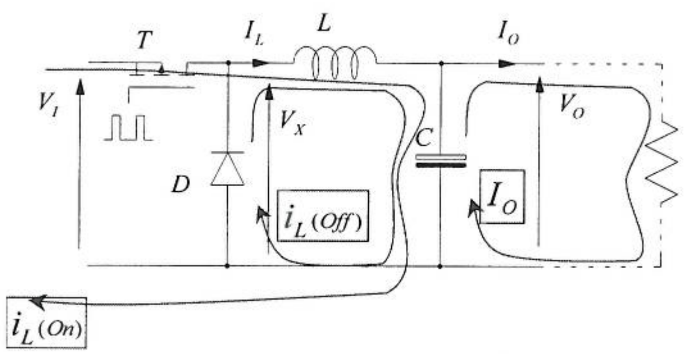


Figure 1.2 A Buck Switch-Mode Power Supply

[(i) Bookwork and interpretation]



When switch is on, input voltage is applied to LHS of inductor (RHS is at output voltage) and current builds up in inductor under the influence of a positive voltage. This current acts to charge the capacitor and meanwhile a load current acts to discharge the capacitor. (Alternatively, the inductor current divides with the average (DC) portion feeding the load and the ripple (AC) portion shunted through the capacitor).

When the switch is off, the diode is pushed into forward bias and carries the inductor current. The LHS of the inductor is slightly negative (by one diode drop) and the RHS is positive at the output voltage. The inductor current falls under the influence of the negative voltage.

[(ii) Bookwork and interpretation]
In continuous conduction mode, the average current in the inductor is large compared with the ripple such that the decrease of current during the off-time does not cause the current to reach zero and the switch turns back on with current still flowing in the inductor.

In discontinuous conduction mode, the average current in the inductor is small compared with the ripple such that the decrease of current during the off-time does causes the current to reach zero (and then hold at zero for the remainder of the off-time) and the switch turns back on with no current flowing in the inductor.

[(iii) Standard calculation]

$$T = \frac{1}{f_{sw}} = \frac{1}{40,000} = 25\mu s$$

$$\delta = \frac{t_{On}}{T} = \frac{15}{25}$$

$$V_O = V_I\,\delta = 35 \times \frac{15}{25} = 21V$$

d) i) Explain why AC rather than DC is used in electricity supply systems. [4]

ii) Explain why 50Hz is a good choice of frequency. [2]

iii) Explain why a system that has a nominal frequency of 50 Hz may suffer small variation of frequency around this value. [4]

[(i) Bookwork and interpretation – logical chain in the answer is important]
Being able to use a variety of voltages in transmission and distribution is key to optimising capital costs and running costs. The most efficient and effective way of converting power between voltages is with a transformer. A transformer requires a rate-of-change of flux which can only be achieved in steady-state with an alternating current (or voltage)

[(ii) Bookwork]
Higher frequencies would pose problems in bearings and shaft stiffness of generators. Higher frequencies would also cause excessive inductive voltage drops in overhead lines and excessive capacitive shunt currents in cables.
Lower frequencies would cause lighting flicker. Lower frequencies would require larger generators and transformers for the same power throughput.

[(iii) Bookwork]

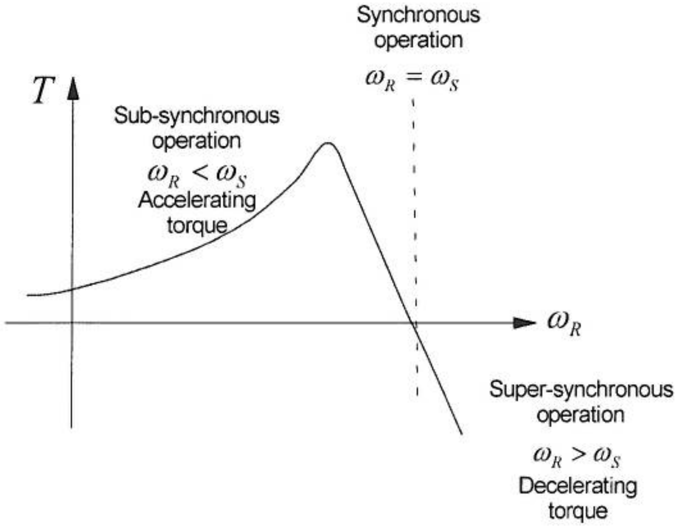
The shafts (of the synchronised) generators will continue to run at a constant speed (constant generated frequency) if there is no net torque on the shaft. This will be true if the electrical load on the generator and the mechanical input of the turbine are matched. Excess of turbine input power causes the frequency to rise; excess of electrical load causes the frequency to fall. Such mismatches occur due to variations of load power, forecasting error in load power. Similar variations apply to intermittent sources such as wind turbines.

2.

a) Sketch the typical torque against speed characteristic of an induction machine. Explain why torque is positive in the sub-synchronous region, zero at synchronous speed and negative in the super-synchronous region. [8]

[Bookwork – 4 for accurate shape of graph; 4 for explanation of regions]

Graph should be approx. straight line through sync speed; should have a clear peak and should have clear starting torque (>0 and zero speed).



Sub-sync: field rotates faster than rotor bars. Speed differences causes rate-of-change of flux linkage which induces voltages. Currents flow in direction that creates torque to cause rotor to accelerate toward sync speed. This is positive torque, i.e., in same direction as rotation.

At sync speed field and rotor bars rotate together and there is no change of flux linkage; no induced voltage; no current and no torque.

Super-sync: field rotates more slowly than rotor bars. Speed differences causes rate-of-change of flux linkage which induces voltages. Currents flow in direction that creates torque to cause rotor to decelerate toward sync speed. This is negative torque, i.e., in opposite direction to rotation.

b) A 2 pole-pair induction machine is rated as follows: 230 V phase; 50 Hz; 3-phase; star-connected. The equivalent circuit parameters, referred to the stator, are:

stator resistance	1.2 Ω,
stator leakage reactance	2.0 Ω,
referred rotor resistance	0.7 Ω,
referred rotor leakage reactance	0.8 Ω,
magnetising reactance	50 Ω,
magnetising resistance	negligible (infinite).

When driving a particular mechanical load, the machine is observed to spin at 1455 rpm.

(i) Calculate the slip. [3]

[Standard calculation]

$$\omega_S = \frac{2\pi f}{P} = \frac{2\pi 50}{2} = 50\pi \quad rad/s$$

$$n_S = \frac{60 \times 50}{2} = 1,500\ rpm$$

$$s = \frac{\omega_S - \omega_R}{\omega_S} = \frac{n_S - n_R}{n_S} = \frac{1500-1455}{1500} = 0.03$$

(ii) Give an expression for the total input impedance of the machine and calculate its value. [7]

[Standard calculation – 3 marks for expression; 4 marks for accurate answer]

$$\overline{Z}_T = R_S + jX_S + jX_M \,//\left({R'_R}/{s} + jX'_R \right)$$

$$\begin{aligned}\overline{Z}_T &= 1.2 + j2.0 + j50 // \left(0.7/0.03 + j0.8\right)\\ &= 1.2 + j2.0 + \frac{j50 \times \left(23.\dot{3} + j0.8\right)}{23.\dot{3} + j50.8}\\ &= 1.2 + j2.0 + \frac{50\angle 90^\circ \times 23.35\angle 1.96^\circ}{55.18\angle 65.3^\circ}\\ &= 1.2 + j2.0 + 21.16\angle 26.66^\circ\\ &= 20.11 + j11.49\Omega\\ &= 23.16\angle 29.7^\circ\Omega\end{aligned}$$

(iii) Calculate the stator current. [4]

[Standard calculation – 2 for method; 2 for answer]

$$\begin{aligned}\overline{I}_S &= \frac{\overline{V}_S}{\overline{Z}_T} = \frac{230\angle 0^\circ}{23.16\angle 29.7^\circ}\\ &= 9.93\angle -29.7^\circ\ A\end{aligned}$$

(iv) Calculate the referred rotor current. [4]

[Standard calculation – 2 for method; 2 for answer]

Apply current divider rule (or calculate voltage across rotor branch and hence current)

$$\begin{aligned}\overline{I}'_R &= \overline{I}_S \frac{jX_M}{jX_M + {R'_R}/{s} + jX'_R}\\ &= 9.93\angle -29.7^\circ \times \frac{50\angle 90^\circ}{55.18\angle 65.3^\circ}\\ &= 9.00\angle -5.0^\circ\end{aligned}$$

(v) Calculate the electro-magnetic torque. [4]

[Standard calculation – 2 for eqn.; 1 for answer; 1 for unit]

$$
\begin{aligned}
T &= 3\left|\overline{I}'_R\right|^2 R'_R \frac{\left(\frac{1}{s}-1\right)}{\omega_R} \\
&= 3\left|\overline{I}'_R\right|^2 R'_R \frac{1}{s\,\omega_S} \\
&= \frac{3\times 9.00^2 \times 0.8}{0.03\times 50\pi} \\
&= 41.25\ Nm
\end{aligned}
$$

3. Figure 3.1 shows the circuit of a boost switch-mode power supply (SMPS). It is to be operated in continuous conduction mode with an input voltage of 9.0 V and output voltage of 20 V. It will be required to process a maximum throughput power of 36 W and have an output voltage ripple (measured peak to peak) which is to be no greater than 75 mV. The turn-on and turn-off energy losses of the MOSFET operated at 4 A are 5 μJ and 7 μJ respectively. The on-state resistance of the MOSFET is 0.1 Ω.

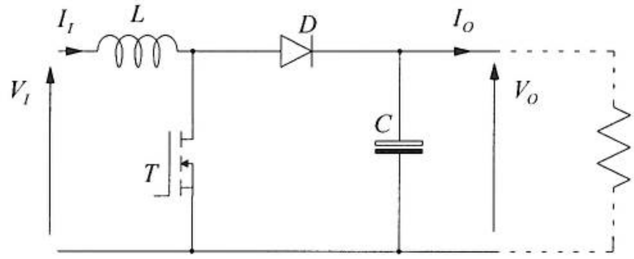


Figure 3.1 A Boost SMPS

a) i) Calculate the values of the duty-cycle assuming continuous conduction and input current when the circuit operates at maximum power. [4]

[Calculation – 2 for each answer]
Duty-cycle found from voltage ratio.

$$V_O = \frac{V_I}{1-\delta}$$

$$-V_O\delta = V_I - V_O$$

$$\delta = \frac{V_O - V_I}{V_O}$$

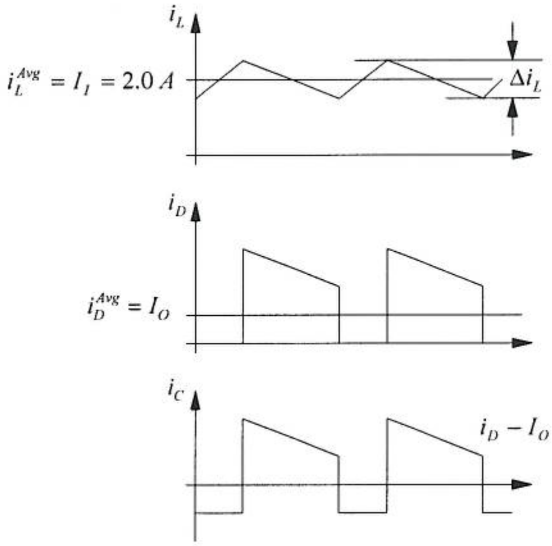
$$= \frac{20-9}{20} = 0.55$$

Input current is found from throughput power.

$$I_I = \frac{P}{V_I} = \frac{36}{9} = 4.0\ A$$

ii) Sketch graphs of the current through the inductor, the diode and the capacitor. [4]

[Bookwork]



b) i) The maximum power loss in the MOSFET is to be 2 W. Calculate the maximum switching frequency that can be used. [6]

[Standard Calculation]

$$P_{loss} = \delta I_I^2 R_{DS(on)} + f_{sw}\left(E_{sw(on)} + E_{sw(off)}\right)$$

$$f_{sw}^{Max} = \frac{P_{loss}^{Max} - \delta I_I^2 R_{DS(on)}}{E_{sw(on)} + E_{sw(off)}}$$

$$= \frac{2.0 - 0.55 \times 4^2 \times 0.1}{12 \times 10^{-6}} = 93.\ddot{3}\ kHz$$

ii) Calculate a value of inductor to keep the inductor current continuous at an input current of 0.5 A. [6]

[Application of know principles]

Inductor is in input connection so magnitude of input current is key. At critical conduction, the inductor current is a triangle wave that just touches zero and has amplitude equal to the ripple current. Input current is the average of this.

$$\Delta I_L = \frac{V_I}{L}\frac{\delta}{f_{sw}}$$

$$I_I = I_L^{Avg} = \frac{1}{T}\int^{T} i_L.dt$$

$$I_I^{crit} = \tfrac{1}{2}\Delta I_L$$

$$L^{\min} = \frac{V_I \delta}{2 I_I^{crit} f_{sw}} = \frac{9 \times 0.55}{2 \times 0.5 \times 93{,}333} = 53\,\mu H$$

ii) Assuming that all of the output voltage ripple is attributed to the effective series resistance, ESR, of the capacitor, specify the ESR to achieve the required voltage ripple. [6]

[Application of know principles]
Capacitor current has a peak to peak amplitude equal to the average inductor current (input current) plus half the inductor current ripple. Voltage ripple is found by multiplying by ESR. This should be evaluated at worst case: full power.

$$\Delta v_{ESR} = R_{ESR}\Delta i_C$$

$$\Delta i_C = I_L^{Avg} + \tfrac{1}{2}\Delta I_L$$

$$R_{ESR}^{Max} = \frac{0.075}{4+0.5} = 16.7\,m\Omega$$

c) If the switching frequency were doubled, what advantages and disadvantages would arise in the design of this SMPS? [4]

[Interpretation]
Advantages:

- Ripple current in the inductor would halve for the same conditions. This is of little impact since the output voltage ripple is largely determined by the average inductor current and only slightly influenced by the inductor ripple current. It would allow operation down to lower currents while maintaining continuous mode or could allow a reduction in the inductor value (and volume and cost).

Disadvantages:

- There is a direct impact on the switching power loss. A consequence would be a lower efficiency and a higher temperature for the MOSFET is nothing else were changed. At present this is 0.88 W of the allowed 2.0 W so losses would rise to 2.88 W. A better heatsink or a better MOSFET would be needed to allow the higher switching frequency.