EE4-47
EE9-CS6-1

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

MODELLING AND CONTROL OF MULTI-BODY MECHANICAL SYSTEMS

Friday, 2 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions

All questions carry equal marks.

This is an OPEN BOOK examination.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	S. Evangelou
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MODELLING AND CONTROL OF MULTIBODY MECHANICAL SYSTEMS

1. A uniform circular disk of mass m and radius a rolls without sliding on a horizontal plane as shown in Figure 1.1 (the disk in the figure has non-zero width for illustration purposes). The plane of the disk remains always vertical, i.e. the spin axis of the disk remains always horizontal. The moment of inertia of the disk about its spin axis is I_{yy} and about a diameter is I_{zz}.

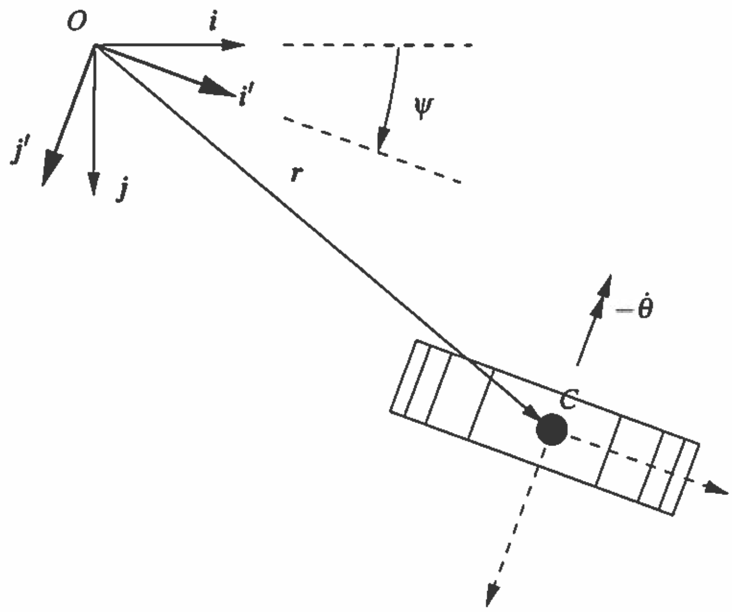

Figure 1.1 A disk rolling on a horizontal plane.

A moving Cartesian coordinate system with unit vectors $\boldsymbol{i}'$, $\boldsymbol{j}'$ and $\boldsymbol{k}'$ ($\boldsymbol{k}'$ not shown) is used to analyse the motion of the object. This coordinate system has a fixed origin O but it rotates by an angle ψ so that it has the same orientation as the axes fixed on the body but not spinning with it (shown with dashed lines on the object). The disk angle of rotation about its spin axis is θ.

a) The coordinates of the centre of mass, C, in the moving reference frame are (x', y'). Write in terms of the four generalised coordinates x', y', ψ, θ the following quantities.

i) The velocity vector of C. [2]

ii) The two equations of the rolling constraint. [4]

iii) The acceleration vector of C. [4]

b) The rolling constraint is maintained by two forces of constraint, $F_{long}\boldsymbol{i}'$ and $F_{lat}\boldsymbol{j}'$. Use the vectorial approach to:

i) compute F_{long} and F_{lat}; [4]

ii) derive the equations of motion of the object. [6]

2. A sphere of radius a has volume $\frac{4}{3}\pi a^3$. A hemisphere is half the sphere and therefore one of its sides is a flat disk with the same radius as that of the sphere.

a) Consider a uniform hemisphere of radius a, mass m and density ρ. Use spherical coordinates to find the moment of inertia of the hemisphere about:

i) its axis of symmetry;
(Hint: $\int \cos^3 x \, dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x$); [6]

ii) an axis perpendicular to the axis of symmetry and passing though the centre of the flat side of the hemisphere. [6]

b) Write the principal moments of inertia (relative to the centre of the flat side) of the hemisphere and state the direction of the principal axes. [4]

c) Compute the moment of inertia of the hemisphere about an axis parallel to its axis of symmetry that touches the outer surface of the hemisphere. [4]

3. A uniform wheel of radius a, mass M and spin inertia I rolls without slipping on a horizontal surface, as shown in Figure 3.1. The wheel centre moves horizontally by a distance x and the wheel rotates by an angle ϕ. A pendulum is attached on the wheel at its centre. The pendulum consists of a mass m connected to the wheel via a massless rod of length l, as shown in Figure 3.1. Assume that the pendulum is free to move in a vertical plane under the influence of gravity and the interaction with the wheel at the wheel centre joint. A control torque T_d is applied on the wheel and reacts on earth.

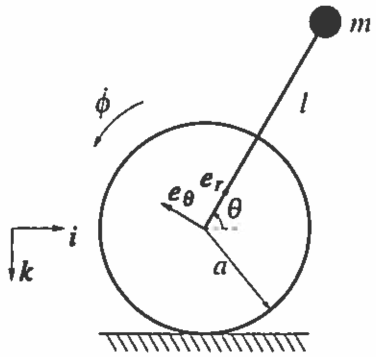

Figure 3.1 Rolling wheel and pendulum.

A fixed Cartesian coordinate system with unit vectors $\boldsymbol{i}$ and $\boldsymbol{k}$, and a moving Cartesian coordinate system with unit vectors $\boldsymbol{e_r}$ and $\boldsymbol{e_\theta}$ (see Figure 3.1) are used to analyse the motion of the two masses. The origin and rotation angle of the moving coordinate system are the wheel centre and θ respectively.

a) Write the position vectors of the wheel centre of mass and of the pendulum mass. [1]

b) Determine the velocity vector of the wheel centre of mass and of the pendulum mass. [2]

c) Compute the kinetic energy of the system. [2]

d) Compute the potential energy of the system. [2]

e) Calculate the Lagrangian function of the system. [2]

f) Derive the equation of the rolling constraint. [1]

g) Use the Lagrangian approach to derive the equations of motion of the system. [10]

4. The body-fixed axes of a rigid body are initially aligned with an earth-fixed set of axes. The rotation of this body is represented by three Euler angles ψ, θ and ϕ in the 3-2-3 convention. In this convention the body is first rotated from its nominal configuration by an angle ψ about the z-axis, then by an angle θ about the intermediate y-axis of the body and finally by an angle ϕ about the new z-axis of the body.

a) By making use of the standard single-axis-rotation transformation matrices compute the complete transformation from:

i) Earth-fixed coordinates to body-fixed coordinates; [8]

ii) body-fixed coordinates to Earth-fixed coordinates. [4]

b) Write the body angular velocity vector, $\boldsymbol{\Omega}$, in terms of the Euler angles in the Earth-fixed coordinate system. [8]