

MSc and EEE PART IV: MEng and ACGI

Corrected Copy

Monday, 12 May 10:00 am

Time allowed: 3:00 hours

There are THREE questions on this paper.

Answer THREE questions.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible

First Marker(s) :	B. Clerckx
Second Marker(s) :	K.K. Leung

Important information for students

Notations:

- (a) A $n_r \times n_t$ MIMO channel consists in n_r receive antennas and n_t transmit antennas.
- (b) a , \mathbf{a} , \mathbf{A} denote a scalar, vector and matrix respectively.
- (c) \mathbf{A}^H denotes conjugate transpose (Hermitian).
- (d) \mathbf{A}^* denotes conjugate.
- (e) \mathbf{A}^T denotes transpose.
- (f) $|a|$ denotes the absolute value of scalar a .
- (g) $\|\mathbf{a}\|$ denotes the (Euclidean) norm of vector \mathbf{a} .
- (h) “i.i.d.” means “independent and identically distributed”.
- (i) “CSI” means “Channel State Information”.
- (j) “CSIT” means “Channel State Information at the Transmitter”.
- (k) “CDIT” means “Channel Distribution Information at the Transmitter”.
- (l) $\mathcal{E}\{.\}$ denotes Expectation.
- (m) $\text{Tr}\{.\}$ denotes the Trace of a matrix.

Assumptions:

- (a) The CSI is assumed to be always perfectly known to the receiver.
- (b) The receiver noise is a $n_r \times 1$ vector with i.i.d. entries modeled as zero mean complex additive white Gaussian noise with variance σ_n^2 .

Some useful relationships:

- (a) $\|\mathbf{A}\|_F^2 = \text{Tr}\{\mathbf{A}\mathbf{A}^H\} = \text{Tr}\{\mathbf{A}^H\mathbf{A}\}$
- (b) $\text{Tr}\{\mathbf{A}\mathbf{B}\} = \text{Tr}\{\mathbf{B}\mathbf{A}\}$
- (c) $\det(\mathbf{I} + \mathbf{A}\mathbf{B}) = \det(\mathbf{I} + \mathbf{B}\mathbf{A})$

THE QUESTIONS

1. [40]

- a) Consider the Multiuser Multiple-Input Single-Output (MU-MISO) transmission of two independent streams to two independent users. The transmitter is equipped with 4 antennas while each user is equipped with a single receive antenna. The Channel State Information (CSI) is perfectly known to the transmitter. Denoting the vector of transmitted symbols as $\mathbf{c} = [c_1, c_2]^T$ with c_1 and c_2 intended for user 1 and 2 respectively, the received signals are written as $y_1 = \mathbf{h}_1 \mathbf{P} \mathbf{c} + n_1$ and $y_2 = \mathbf{h}_2 \mathbf{P} \mathbf{c} + n_2$ at user 1 and 2, respectively. The precoder \mathbf{P} is made of two columns \mathbf{p}_1 and \mathbf{p}_2 , each column subject to a power constraint $\|\mathbf{p}_i\|^2 = 1, i = 1, 2$. The channel vector of user 1 is given by

$$\mathbf{h}_1 = [1 \ 2 \ -1 \ 1]$$

while that of user 2 is given by

$$\mathbf{h}_2 = [3 \ 1 \ 4 \ -1]$$

At the transmitter we would like to apply a precoder \mathbf{P} with the lowest possible complexity such that the multi-user interference is completely nulled out.

- i) Derive the expression of the precoder \mathbf{P} . Provide your reasoning. [3]
 - ii) What kind of precoder is this? Explain your result. [3]
- b) Figure 1.1 displays the average Error Probability of one scheme (i.e., one transmission and reception strategy) vs. SNR for point-to-point channels with i.i.d. Rayleigh slow fading and four different antenna configurations (a) to (d). The CSI is unknown to the transmitter.
- i) What is the diversity gain (at high SNR) achieved by that scheme in each antenna configuration? Provide your reasoning. [4]
 - ii) For each scenario (a) to (d), identify an antenna configuration (i.e., n_t and n_r) and the corresponding transmission/reception strategy that can achieve such diversity gain. Provide your reasoning. [4]
- c) Figure 1.2 displays the ergodic capacity of point-to-point i.i.d. Rayleigh fast fading channels with Channel Distribution Information at the Transmitter (CDIT) for five antenna ($n_r \times n_t$) configurations (denoted as (a) to (e)) with $n_t + n_r = 8$.
- i) What is the achievable (spatial) multiplexing gain (at high SNR) for cases (a), (b), (c), (d) and (e)? Provide your reasoning. [5]
 - ii) For (a), (b), (c), (d) and (e), identify an antenna configuration, i.e. n_t and n_r , satisfying $n_t + n_r = 8$ that achieves such multiplexing gain.

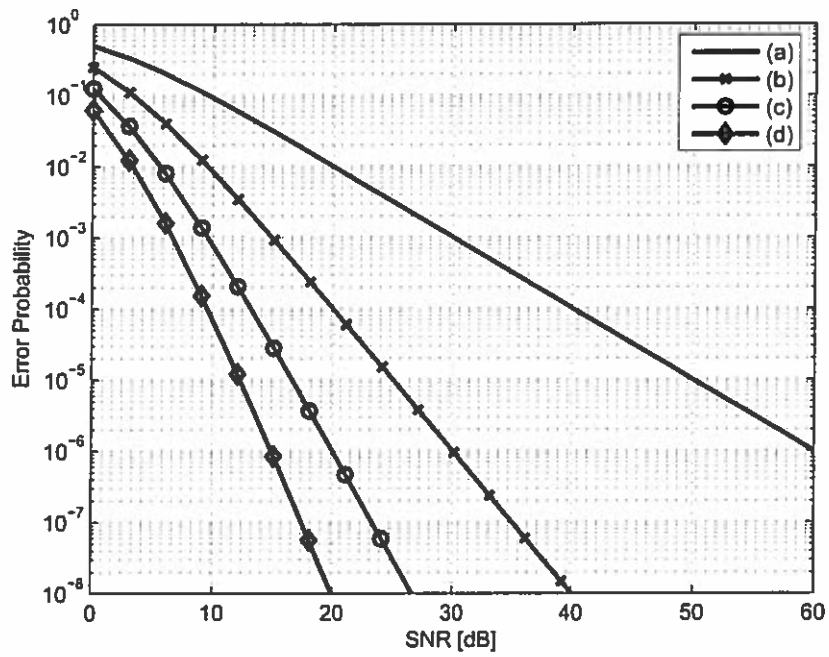


Figure 1.1 Average Error Probability vs. SNR.

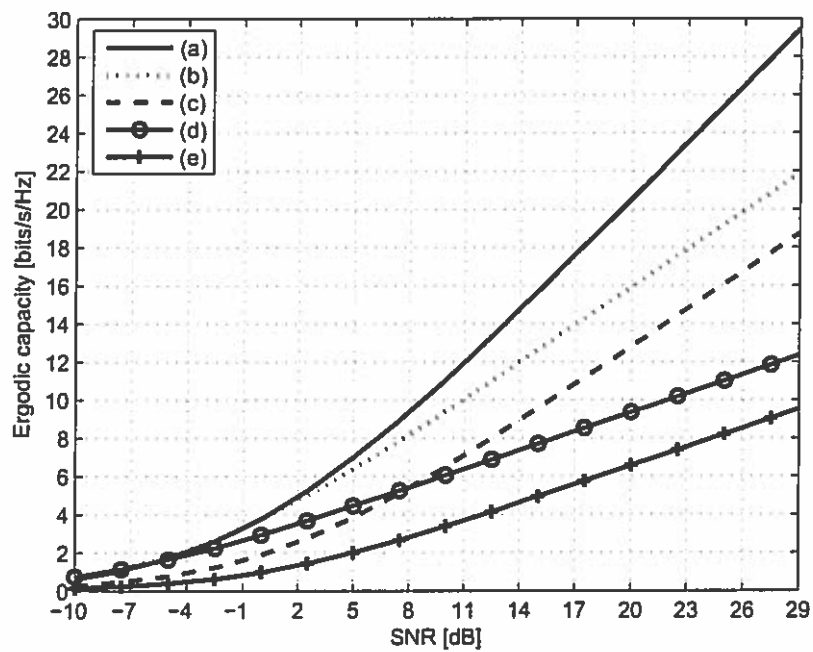


Figure 1.2 Ergodic capacity vs. SNR.

Provide your reasoning.

[5]

- d) Consider a MISO channel $h = [h_1 \ h_2]$ with two transmit antennas and one receive antenna, constant over two consecutive symbol durations. Symbols c_1 and c_2 are transmitted over the channel during two symbol durations using an Alamouti space-time code.
- i) Write the received signals at the receiver. [2]
 - ii) Provide a simple receiver strategy to detect c_1 and c_2 . [3]
 - iii) What happens if the same transmission strategy is applied to a channel that is not constant over two consecutive symbol durations? Provide your reasoning. [3]
- e) Consider the very strong interference regime of a two-user SISO interference channel.
- i) Explain why a per-user multiplexing gain of 1 is achievable and which receiver strategy allows to achieve such multiplexing gain? [4]
 - ii) Is there any condition on the Signal to Noise Ratio (SNR) and Interference to Noise Ratio (INR) of each link for such strategy to be valid? [4]

Discuss the validity of the following statements. Detail your argument.

- a) In a two-user SISO Broadcast Channel, for any ordering of the channels, the capacity region can be achieved with superposition coding with Successive Interference Cancellation (SIC).

[6]

- b) Consider a point-to-point i.i.d. MISO Rayleigh slow fading channel with 4 transmit antennas and 1 receive antenna and a transmission strategy based on partial transmit channel knowledge consisting of transmitting a single stream using quantized precoding. The codebook of precoders is given by

$$\mathcal{W} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

This transmission strategy achieves a diversity gain of 4.

[6]

- c) In point-to-point MISO and in multi-user MISO, the multiplexing gain (at high SNR) is not affected by the quality of Channel State Information at the Transmitter (CSIT).

[6]

- d) In a cellular system where users are uniformly located in a cell and have the same Quality of Service (QoS), the proportional fair (PF) scheduler is a fair scheduler that benefits from multi-user diversity in the limit of very small scheduling time scale.

[6]

- e) A narrowband transmission using a transmission strategy characterized by the following set of codewords

$$\mathbf{a} = [a \ a \ b \ c],$$

$$\mathbf{b} = [a \ d \ a \ b],$$

$$\mathbf{c} = [b \ a \ a \ b],$$

with $a = \frac{1}{\sqrt{2}}(1+j)$, $b = \frac{1}{\sqrt{2}}(-1+j)$, $c = \frac{1}{\sqrt{2}}(-1-j)$ and $d = \frac{1}{\sqrt{2}}(1-j)$ being the four constellation symbols taken from a unit average energy QPSK constellation, and a Maximum Likelihood (ML) receiver, achieves a diversity gain of 2 in i.i.d. fast Rayleigh fading channels with a single receive antenna and a single transmit antenna.

[6]

Assume a downlink narrowband transmission in a cellular network consisting of 2 cells, each equipped with one transmitter, denoted as transmitter 1 in cell 1 and transmitter 2 in cell 2. In cell 1, there are two terminals (denoted as terminal 1 and 2) and in cell 2, there is a single terminal (denoted as terminal 3). The two transmitters are equipped with 2 antennas and the terminals equipped with 1 receive antenna. In cell 1, each receiver perfectly estimates and reports its channel state information to transmitter 1 and transmitter 1 schedules the two terminals at a time and sends using MU-MISO linear precoding a total of 2 streams, one stream per terminal. In cell 2, transmitter 2 sends a single stream to terminal 3. Each transmitter is subject to a total transmit power P .

- a) Write an expression for the received signal of terminal 1 in cell 1 in terms of channel parameters, precoders and transmit symbol vectors. Clearly define each variable and identify the terms responsible for the intra-cell interference (also called multi-user interference) and inter-cell interference in your expression.

[4]

- b) Derive the expression of the rate achievable by terminal 1 in cell 1.

[4]

- c) Assume that terminal 1 in cell 1 has the ability to recommend one precoder to transmitter 2 so as to maximize its rate. In order to decrease the feedback overhead, the precoder recommended by terminal 1 has to be selected in a codebook (with codewords normalized to have unit norm). Two codebooks, each containing four precoders, are available

$$\mathcal{W}_1 = \left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix} \right\}$$

and

$$\mathcal{W}_2 = \left\{ \begin{bmatrix} -0.1612 - 0.7348i \\ -0.5135 - 0.4128i \end{bmatrix}, \begin{bmatrix} -0.0787 - 0.3192i \\ -0.2506 + 0.9106i \end{bmatrix}, \right. \\ \left. \begin{bmatrix} -0.2399 + 0.5985i \\ -0.7641 - 0.0212i \end{bmatrix}, \begin{bmatrix} -0.9541 \\ 0.2996 \end{bmatrix} \right\}.$$

- i) Assuming that the channel vector between transmitter 2 and terminal 1 is given by $\begin{bmatrix} 1 & e^{\pi/3} \end{bmatrix}$ and that codebook \mathcal{W}_1 is used, which precoder will terminal 1 recommend to transmitter 2? Provide your reasoning.
- ii) Assuming that the channel vector between transmitter 2 and terminal 1 is random and has entries modeled as i.i.d. circularly symmetric complex Gaussian variables, which codebook (among \mathcal{W}_1 and \mathcal{W}_2) would you recommend to use?
- d) Assume the interference from transmitter has been completely mitigated (i.e. there is no interference from transmitter 2 to terminal 1 and 2) and focus exclusively on cell 1, i.e. transmitter 1 and its two terminals.
- i) Derive the expression of the transmit precoder such that the two terminals do not experience any multi-user interference. Provide your reasoning.

[5]

- ii) Derive the total multiplexing gain that is achieved at high SNR with such a transmit precoder. Provide your reasoning. [5]
- iii) Let us improve the design of part i) such that the sum-rate is maximized under the constraint that the two terminals do not experience any multi-user interference. In particular, derive the expressions of the transmit precoding and power allocation strategy (assuming a fixed total power constraint) for transmitter 1 such that the sum-rate is maximized under the constraint that the two terminals do not experience any multi-user interference. Provide your reasoning. [5]