IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2014

EEE/EIE PART I: MEng, Beng and ACGI

Corrected Copy

ANALYSIS OF CIRCUITS

Friday, 6 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer ALL questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : D.M. Brookes
Second Marker(s) : P. Georgiou

Analysis of Circuits

Information for Candidates:

The following notation is used in this paper:

1. The voltage waveform at node X in a circuit is denoted by $x(t)$, the phasor voltage by X and the root-mean-square (or RMS) phasor voltage by $\widetilde{X} = \frac{X}{\sqrt{2}}$. The complex conjugate of X is X^*.
2. Component and source values in a circuit are normally given in Ohms, Farads, Henrys, Volts or Amps with the unit symbol omitted. Where an imaginary number is specified, it represents the complex impedance or phasor value.
3. Times are given in seconds unless otherwise stated.
4. Unless otherwise indicated, frequency response graphs should use a linear axis for phase and logarithmic axes for frequency and magnitude.

1. a) Using nodal analysis, calculate the voltages at nodes X and Y of Figure 1.1. [5]

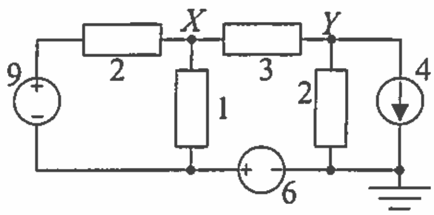

Figure 1.1

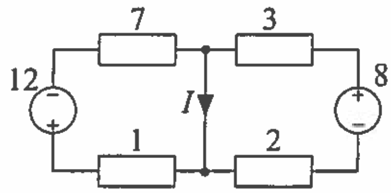

Figure 1.2

b) Use the principle of superposition to find the current I in Figure 1.2. [5]

c) Draw the Thévenin equivalent circuit of the network in Figure 1.3 and find the value of its components. [5]

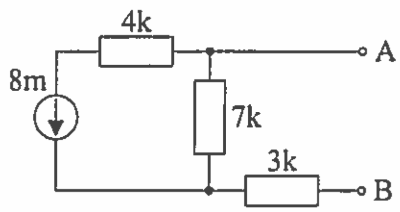

Figure 1.3

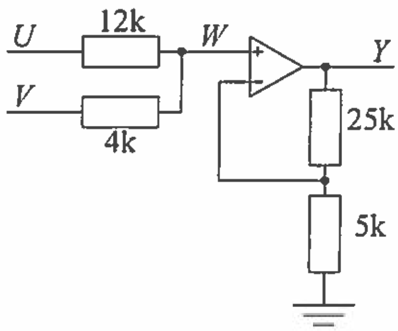

Figure 1.4

d) Assuming the opamp in the circuit of Figure 1.4 is ideal, give an expression for Y in terms of U and V. [5]

e) The graph of Figure 1.5 plots the output voltage, Y, against the input voltage, X, for the circuit shown in Figure 1.6. The graph consists of two straight lines that intersect at the point (10, 10) and that pass through the origin and the point (20, 12) respectively. Assuming that the forward voltage drop of the diode is 0.7 V, determine the values of the resistor, R, and the voltage source, V. [5]

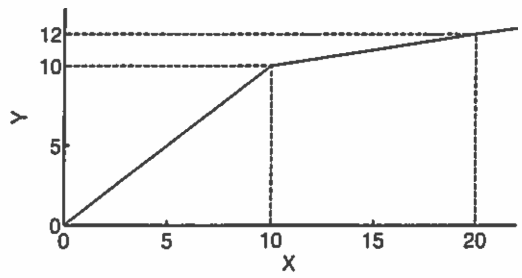

Figure 1.5

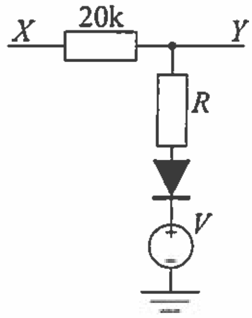

Figure 1.6

f) Determine the gain, $\frac{Y}{X}$, for the block diagram shown in Figure 1.7. The rectangular blocks are drawn with inputs at the left and outputs at the right and have gains of F and G respectively. The open circles represent adder/subtractors; their inputs have the signs indicated on the diagram and their outputs are W and Y respectively. [5]

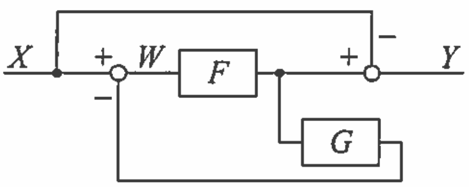

Figure 1.7

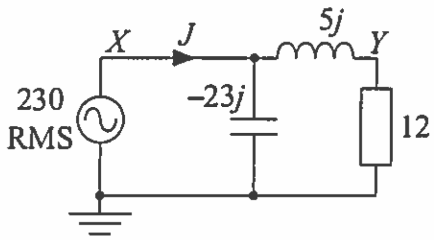

Figure 1.8

g) In the circuit of Figure 1.8, the RMS phasor $\widetilde{X} = 230$ and the component values shown indicate complex impedances. Determine the value of the RMS current $\widetilde{J}$ and of the complex power, $\widetilde{V} \times \widetilde{I}^*$, absorbed by each of the four components. [5]

h) Figure 1.10 shows a transmission line of length 100 m that is terminated in a resistive load, R, with reflection coefficient $\rho = +0.6$. The line has a propagation velocity of $u = 2 \times 10^8$ m/s. At time $t = 0$, a forward-travelling (i.e. left-to-right) pulse arrives at X with amplitude 4 V and duration 1.5 μs, as shown in Figure 1.9.

Draw a dimensioned sketch of the waveform at Y, a point 60 m from the end of the line, for $0 \leq t \leq 3\,\mu$s. Assume that no reflections occur at point X. [5]

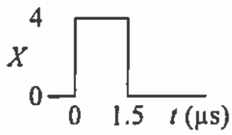

Figure 1.9

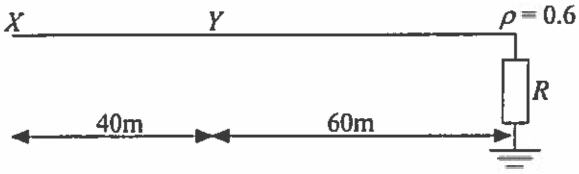

Figure 1.10

2. a) Show that the transfer function of the circuit of Figure 2.1 can be written in the form

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\frac{j\omega}{\omega_0} + 1}$$

and express the values of ω_0 and ζ in terms of the component values L, C and R. [5]

b) Give expressions for the low and high frequency asymptotes of $H(j\omega)$ and the frequency at which they have the same magnitude. [3]

c) Determine the magnitude and phase of $H(j\omega)$ at $\omega = \omega_0$. [2]

d) Show that $|H(j\omega)|^{-2}$ may be written as a polynomial with real coefficients in x where $x = \left(\frac{\omega}{\omega_0}\right)^2$. By differentiating this polynomial, or otherwise, show that the maximum value of $|H(j\omega)|$ occurs at $\omega = \omega_0\sqrt{1-2\zeta^2}$. [6]

e) Determine values of C and R so that $\omega_0 = 5000\,\text{rad/s}$ and $\zeta = 0.1$ given that $L = 100\,\text{mH}$. [2]

i) Sketch a dimensioned graph of $|H(j\omega)|$ in decibels using a logarithmic frequency axis. Your graph should include both the high and low frequency asymptotes in addition to a sketch of the true magnitude response. [3]

ii) If $x(t) = 3\cos\omega_0 t$, determine the average power dissipation of the circuit and the peak value of the energy, $\frac{1}{2}Cy^2(t)$, stored in the capacitor.

[3]

iii) Determine the values of ω for which $\angle H(j\omega) = -45°$ and $-135°$.

Hence sketch a dimensioned graph of $\angle H(j\omega)$ using a straight-line approximation with three segments. Your graph should use a logarithmic frequency axis and a linear phase axis. [6]

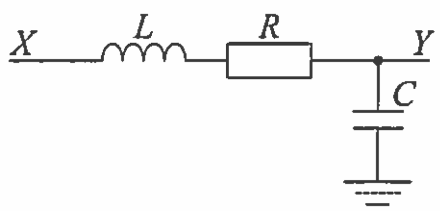

Figure 2.1

3. In the circuit of Fig. 3.1, the input, X, is driven by a voltage source as shown.

a) Derive an expression for the transfer function, $\frac{Y(j\omega)}{X(j\omega)}$ and determine the corner frequencies in its magnitude response. [4]

b) Determine the Thévenin equivalent voltage and resistance of the remainder of the circuit at the terminals of the capacitor. [4]

c) Derive the time constant of the circuit, τ, in two ways: (i) from the Thévenin resistance found in part b) and (ii) from the denominator corner frequency found in part a). [2]

d) If the input voltage, $x(t)$, is given by

$$x(t) = \begin{cases} -2 & \text{for } t < 0 \\ +3 & \text{for } t \geq 0 \end{cases},$$

determine an expression for the output waveform, $y(t)$. Sketch its waveform over approximately the range $-\tau \leq t \leq 4\tau$. [7]

e) Assuming that the opamp in Fig. 3.2 is ideal, determine the transfer function, $\frac{V(j\omega)}{U(j\omega)}$. [4]

f) By considering the voltage across the capacitor, explain why an input voltage discontinuity of Δu will result in an output voltage discontinuity of the same amplitude. [2]

g) If $R = 20\,\text{k}\Omega$, $C = 20\,\text{nF}$ and the input voltage, $u(t)$, is given by

$$u(t) = \begin{cases} \sin 1000t & \text{for } t < 0 \\ 2\cos 2000t & \text{for } t \geq 0 \end{cases},$$

determine expressions for the output $v(t)$ for both positive and negative t. [7]

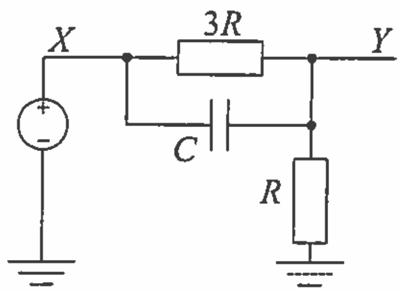

Figure 3.1

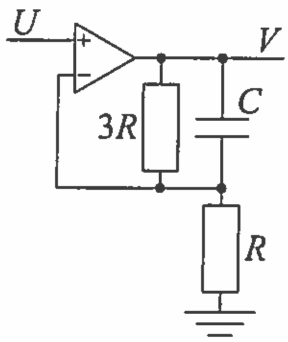

Figure 3.2