IMPERIAL COLLEGE LONDON

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E3.02
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DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

MSc and EEE PART III/IV: MEng, BEng.and ACGI

INSTRUMENTATION

Tuesday, 27 April 2:30 pm

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : C. Papavassiliou
Second Marker(s) : S. Lucyszyn

Special instructions for students

The following constants are given:

Electron charge:	$e = 1.6 \times 10^{-19}$ C
Speed of light:	$c = 3 \times 10^{8}$ m s^{-1}
Boltzmann constant:	$k = 1.38 \times 10^{-23}$ J K^{-1}
Standard temperature:	$T = 290$ K

The Questions

1. An antenna operating at a frequency of 2 GHz behaves as a voltage source of magnitude 1 μV RMS from an impedance of 75 Ω. The antenna has a bandwidth of 200 MHz, and a noise temperature of 290 K. The antenna will be used with a receiver which consists of a cascade of 6 stages of amplification. Each stage has a gain of 15 dB and a noise figure of 6 dB.

   a) Write a formula for the noise voltage at the terminals of a resistor over a bandwidth B operating at a temperature T. Evaluate the RMS noise voltage at the terminals of the antenna.

   [2]

   b) Define the Signal to Noise ratio of a signal source. Evaluate, in dB, the Signal to Noise ratio for this antenna, assuming it is properly impedance matched. Is there anything unusual about this value?

   [3]

   c) Define the noise measure of an amplifier. Derive an expression for the noise measure in terms of the gain and the noise factor of an amplifier.

   [3]

   d) Calculate the noise measure of a receiver stage.

   [2]

   e) Calculate the Signal to Noise ratio at the output of the receiver.

   HINT: The receiver is effectively a cascade of an infinite number of identical amplifiers.

   [5]

   f) Calculate the maximum data rate that can be supported at the antenna terminals, and also by this receiver

   [5]

2.

a) With the aid of a diagram, describe the Colpitts oscillator. Write an expression for its resonant frequency. Suggest a way to convert the Colpitts oscillator into a voltage-controlled oscillator.

[10]

b) Consider an analogue multiplier operating from a voltage supply $\pm 5\,\mathrm{V}$. Assume the multiplier's voltage transfer function is given by:

$$V_{out} = V_S \tanh\left(\frac{V_1 V_2}{4(kT)^2}\right)$$

Calculate the gain and range of this multiplier when used as a phase detector.

(Hint: approximate the response by a straight line equal to its response for small inputs)

[5]

c) Design an FM demodulator designed around the VCO and phase detector discussed in parts (a) and (b) above. Assume the VCO has been designed for a free-running frequency of 100 MHz and a gain of 10 MHz/V. What should the loop filter be so that the demodulator has zero steady-state phase error for a frequency ramp input?

[5]

3.

a) What is the minimum frequency at which a signal containing energy between DC and $f_{\max}$ can be sampled so that it can be reconstructed without loss of information? Write an expression for the reconstruction formula.

[5]

b) We need to sample a band-pass signal of small bandwidth B, centered around a carrier frequency $f_c \gg B$. What is the minimum sampling frequency $f_{SBP\min}$ that can be used to sample this signal without loss of information? Explain why some frequencies greater than $f_{\min}$ cannot be used to sample such a signal. What is the minimum frequency f_{LS} so that all frequencies above f_{LS} can be used for lossless sampling?

[5]

c) Calculate the minimum sampling frequency for a second order band-pass ΣΔ modulator which can be used as an FM receiver for a remote control system in the 27 MHz band. The signal bandwidth is 20 KHz. The receiver must have 10 bits signal to quantisation noise.

HINT: A band-pass system is obtained by the following mapping function applied on a low pass system:

$$\omega' \to \omega + \frac{1}{\omega}.$$

This means that the usual low pass loop filter maps into a band pass filter for a band pass converter. The resolution of a band-pass converter is that of the corresponding low pass converter with the same oversampling ratio.

[5]

d) Compare the thermal noise power spectral density when the signal in part 3(b) is:

i) low-pass sampled, as described in part 3(a)

ii) band-pass sampled, as described in part 3(b)

HINT: The total noise power must be the same in both cases!

[5]

4.

a) Describe the method of spline interpolation in general, and for polynomials of order N in particular. Assume a measurement has generated data pairs $\{x_i, y_i\}$.Write equations describing what happens at the measured points. Assume for the interpolation you are using a set of functions $\{y = f_n(x)\}$. Assume that the first and second derivatives of the functions at the measured points can be computed.

[8]

b) A set of $\{x, y\}$ experimental data pairs are given, and a quadratic curve must be fitted to them by the method of least squares. Explain why it is preferable to use orthogonal, rather than algebraic, polynomials for this operation. Write the equations to determine the coefficients for least squares fitting the following model to the experimental data:

$$y_i = \sum_{j=0}^{2} a_j P_j(x_i)$$

Where P_j are the first 3 Legendre polynomials:

$$P_0 = 1$$
$$P_1 = x$$
$$P_2 = \frac{1}{2}(3x^2 - 1)$$

[12]

5.

a) Define the sensitivity, resolution and dynamic range for a linear and a non-linear sensor.

[5]

b) Briefly describe 2 ways for performing each of the following measurements. Write an expression or sketch a graph for the response of each technique.

Which of the two methods you describe is expected to have a higher resolution? Explain your answer.

i) Extremely high frequency

ii) Current

iii) Magnetic field

iv) Distance

v) Temperature

[15]

6.

a) Write an expression for the noise factor of an amplifier driven by a source of Thevenin resistance R_S. Explain the meaning of each constant appearing in this equation. Does the noise factor of an amplifier depend on the load impedance?

[5]

b) Show that the noise factor of any amplifier driven by an ideal voltage source or an ideal current source is infinite. Show that in both of these cases the signal power transferred into the amplifier is zero if the source has a finite available power.

[5]

c) Write an equation for the frequency dependence of the input impedance of a cable of length $X = 0.75$ m, inductance $L = 1\ \mu\text{H}/\text{m}$ and capacitance $C = 100\ \text{pF}/\text{m}$, which is terminated with impedance $Z_T = 50\ \Omega$. What are the minimum and maximum real values that the input impedance takes? At which frequency is each of these values obtained?

[5]

d) The cable in part (c) is used to connect a signal source of impedance $Z_T = 50\ \Omega$ to an oscilloscope with real input impedance $Z_{in} = 1\ \text{M}\Omega$. Estimate the bandwidth of this measurement.

[5]

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The Answers 2010

ANSWER 1.

a) (bookwork+computed example)

$$V_n = \sqrt{4kTRB} = \sqrt{4kTRB} = \sqrt{4\times4\times10^{-21}\times75\times2\times10^{8}} = 15.49\mu V$$

[2]

b) (bookwork+computed example)

$SNR = S/N$, S and N are powers. Then,

$$S = V_{RMS}^2 / R = 10^{-12}/75 = 13.3\times10^{-15}W$$

$$N = kTB = 800\times10^{-15}W$$

$$SNR = .0167 = -17.8\,dB$$

[3]

c) (bookwork)

Noise measure is the noise factor of an infinitely long cascade of identical amplifiers. From the Friis formula:

$$M = F + \frac{F-1}{G} + \frac{F-1}{G^2} + \cdots + \frac{F-1}{G^n} =$$

$$= 1 + (F-1)\sum_{n=0}^{\infty}\frac{1}{G^n} = 1 + \frac{(F-1)}{1-\frac{1}{G}} = 1 + \frac{GF-G}{G-1} = \frac{GF-1}{G-1}\cdot$$

[3]

d) (computed example)

$$10\,\log F = 6dB \Rightarrow F = 10^{0.6} = 3.98$$

$$10\,\log G = 15dB \Rightarrow G = 10^{1.5} = 31.6$$

$$M = \frac{FG-1}{G-1} = \frac{124.8}{30.6} = 4.077 = 6.1dB$$

[2]

e) (computed example)

at the output of the receiver,

$$SNR_{out} = SNR_{in}/M = -17.8\text{ dB} - 6.077\text{ dB} = -23.88\text{ dB} = 4.093\times10^{-3}$$

[5]

f) (computed example)

the data rate is $D = B\log_2(SNR+1)$. At the antenna this evaluates to

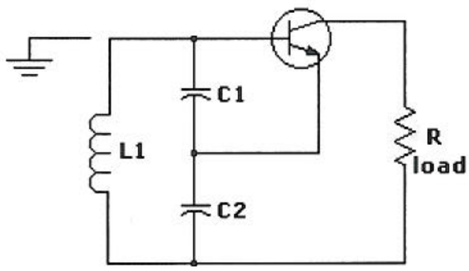
$$D_{ant} = B\log_2(SNR+1) = 2\times10^8\log_2(1.0167) = 4.78MB/s$$

$$D_{out} = B\log_2(SNR+1) = 2\times10^8\log_2\left(1+4.093\times10^{-3}\right) = 1.18MB/s$$

[5]

ANSWER 2.

a) (Bookwork+ interpretation)



The current transfer ratio of the resonant circuit viewed as a current divider between C2 and L is:

$$\frac{i_{out}}{i_{in}} = \frac{j\omega C_2}{j\omega C_2 + \frac{1}{j\omega L}} = \frac{-\omega^2 LC_2}{1-\omega^2 LC_2}$$

At resonance , i.e. at a frequency of $\sqrt{\frac{C_1+C_2}{LC_1C_2}}$ the current gain of the feedback network is :

$$\frac{i_{out}}{i_{in}} = \frac{-\omega^2 LC_2}{1-\omega^2 LC_2} = \frac{C_1+C_2}{C_2} = 1+\frac{C_1}{C_2} > 1$$

This suggests that there is more than unity real positive feedback and the Barkhausen criterion is satisfied.

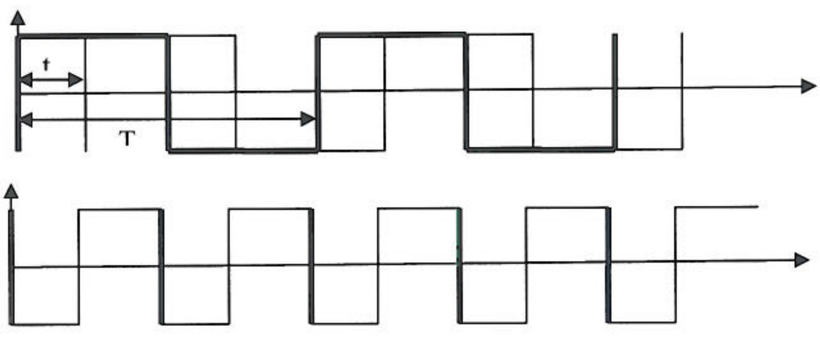
The Colpitts oscillator can be turned into a VCO by using a varactor, voltage controlled capacitor.

[10]

b) (Computed example)

A multiplier with a response given by: $V_{out} = V_S \tanh\left(\frac{V_1V_2}{4(kT)^2}\right)$

As a phase detector for bipolar square signals , we note that $V_S >> kT$



From the diagram, the detector is a XOR detector between the 2 logic levels. The average of the output:

At $\varphi = 0, V_{out} = V_s$, at $\varphi = \pi/2, V_{out} = 0$, and at $\varphi = \pi, V_{out} = -V_S$.

The response is therefore monotonic between $0 \le \varphi \le \pi$ and the gain is:

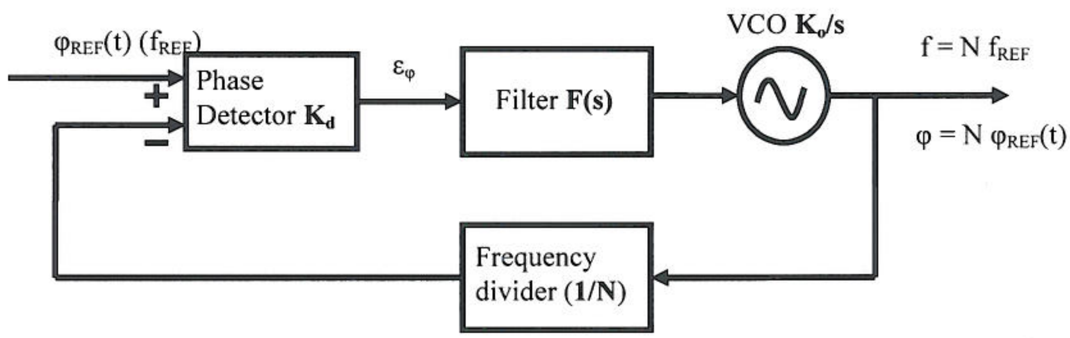
$$\frac{dV_{out}}{d\varphi} = \frac{-2V_S}{\pi}$$

The detector can also show the negative of this response between $\pi \le \varphi \le 2\pi$

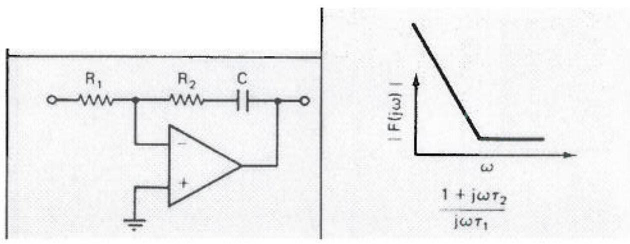
[5]

c) (design)

A simple PLL loop will do. The gain info for the VCO is superfluous.



For zero phase error one needs a pole at zero frequency in the loop filter, so an active filter must be used. A lead-lag filter is best because the input is frequency agile, i.e. a large phase margin is desirable. The most suitable filter would be:



[5]

ANSWER 3.

a) (bookwork)

The Nyquist rate $f_N = 2f_{\max}$. Reconstruction from the samples $\{y_n\}$ obtained at a sampling frequency f_s can be achieved by sinc interpolation:

$$y(t) = \sum y_n \operatorname{sinc}\left(\pi f_s\left(t - t_n\right)\right)$$

[5]

b) (bookwork)

This is bandpass sampling and in principle $f_{SBP\min} = 2B$. However, the sampling process is equivalent to mixing, and only those sampling frequencies that do not result into an overlap of images of the positive and negative frequency signal energy bands are usable. The lowest frequency at which this cannot happen is the low pass Nyquist rate.

[5]

c) (design, and extension of theory)

The answer to this question is the same as obtaining 10 bits of SNQR from a low pass 1st order modulator. Since in a $\Sigma\Delta$ modulator we get 1.5 bits/bit of OSR, it follows that we need 6 bits of OSR, or OSR=$2^6 = 64$.

The minimum sampling frequency then is $f_{\min} = 20kHz \times 64 = 1.28MHz$

[5]

d) (Extension of theory)

In bandpass sampling the noise still occupies the entire range to the low pass Nyquist frequency. Then the noise is "folded " into a bandwidth of twice the actual sampling rate. if N_N the noise floor of the Nyquist low pass sampled signal, the noise floor of a signal of bandwidth B bandpass sampled at 2B will be

$$N_{BP} = N_{LP} f_C / \left(f_B \times OSR\right)$$

[5]

ANSWER 4.

a) (Bookwork+ interpretation)

In spline interpolation curves are passed through pairs of adjacent points. The curves are adjusted to agree by value and the first M derivatives at the experimental points. If polynomials of order N are used, they are required to agree by value and the N-1 derivatives at the points.

[10]

b) (Computed example)

The error in the measurements will introduce large errors in the coefficients if algebraic polynomials are used, because powers of x are not orthogonal in the interval of the measurement.

First the x variable must be scaled to the interval of domain of the orthogonal polynomials:

$$(x_{\min}, x_{\max}) \to (-1,1) \Rightarrow x' = -1 + 2\frac{x - x_{\min}}{x_{\max} - x_{\min}}$$

the equations are derived from minimising

$$\sum_i \left(y_i - \sum_0^2 a_j P_j(x_i)\right)^2$$

i.e. requiring:

$$\frac{\partial}{\partial a_j}\sum_i\left(y_i - \sum_{j=0}^{2} a_j P_j(x_i)\right)^2 = 0 \Rightarrow$$
$$\sum_i \left(y_i P_j(x_i)\right) = \sum_k a_k \sum_i \left(P_k(x_i) P_j(x_i)\right)$$

this is a linear system: (Note that: $\sum P_0 = N, \sum P_1 = \sum x', \sum P_2 = 3/2\sum x'^2 - 1/2N$)

$$\begin{bmatrix} \left(\sum P_2\right)^2 & \sum P_2 \sum P_1 & N\sum P_2 \\ \sum P_2 \sum P_1 & \left(\sum P_1\right)^2 & N\sum P_1 \\ N\sum P_2 & N\sum P_1 & N^2 \end{bmatrix}\begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} \sum yP_2 \\ \sum yx \\ \sum y \end{bmatrix}$$

[10]

ANSWERS 5.

a) (Bookwork)

Sensitivity, or gain, is derived from the implicit linear definition of the sensor:

$y(x) = Ax$, $0 \le x \le x_{max}$, A>0

If the sensor is not linear then

$$A = \frac{\partial y}{\partial x}$$

The smallest change of input detectable at the output is called the **resolution**. So the resolution is related to the noise floor and has little to do with the sensitivity.

The dynamic range is the ratio of the maximum and minimum measurement possible. The maximum is usually defined by clipping and the minimum by noise or offset.

[5]

b) (Bookwork and interpretation)- Many possible answers

i) frequency: transfer oscillator, heterodyne receiver plus counter, josephson junction. The transfer oscillator should have the highest resolution.

ii) current: Mechanical current meter, transimedance amplifier, high impedance amplifier, SQUID, Hall effect. The SQUID is by far the most sensitive.

iii) Magnetic field: Hall effect, rotating magnetometer, Josephson junction SQUID, GMR. Again the squid has the highest resolution.

iv) Distance: Time of flight, interferometry, strain gauges. Interferometry is the most sensitive.

v) Temperature: Temperature dependent resistors (TDR), thermistors, anything else with a temperature coefficient. TDR are highest resolution because of noise considerations.

[15]

ANSWER 6.

a) (bookwork)

$$F = F_{\min} + \frac{R_n}{G_s}\left[\left(G_s - G_{opt}\right)^2 + \left(B_s - B_{opt}\right)^2\right]$$

F: Noise factor

$F_{\min}$ the minimum noise factor.

R_n : Noise resistance, i.e. ho quickly F rises as we deviate from the optimum source impedance

$Y_{opt} = G_{opt} + jB_{opt}$

The noise factor does not depend on the load impedance, provided the amplifier output impedance is at the same temperature as the source!

[5]

b) (interpretation of theory)

The expression in part (a) has

$\lim\limits_{G_s \to \infty} F = \infty$ and $\lim\limits_{G_s \to 0} F = \infty$. I both cases the signal power coupled into the amplifier is zero, while the noise power must be kTB:

Signal power: $P = \mathrm{Re}\left(VI^*\right) = |V|^2 \,\mathrm{Re}\dfrac{Z_{in}}{\left|Z_{in} + Z_s\right|^2}$

However, the source power $P_s = \dfrac{|V_s|^2}{|Z_s|^2}\mathrm{Re}\left(Z_s\right)$ it follows that: $P = \dfrac{\mathrm{Re}\left(Z_{in}\right)}{\mathrm{Re}\left(Z_s\right)} \dfrac{|Z_s|^2}{\left|Z_{in} + Z_s\right|^2} P_s$

since the input impedance does not vanish,

$$\lim_{Z_s \to 0} \frac{|Z_s|^2}{Z_s} = 0, \text{ and } \lim_{Z_s \to \infty} P = \lim_{Z_s \to \infty} \frac{\mathrm{Re}\left(Z_{in}\right)}{\mathrm{Re}\left(Z_s\right)} \frac{|Z_s|^2}{\left|Z_{in} + Z_s\right|^2} P_s = \lim_{Z_s \to \infty} \frac{\mathrm{Re}\left(Z_{in}\right)}{\mathrm{Re}\left(Z_s\right)} = 0$$

[5]

c) (bookwork and computed problem]

$$Z_0 = \sqrt{\frac{L}{C}} = 100\,\Omega, c = \frac{1}{\sqrt{LC}} = 10^8 \, m/s$$

the propagation constant is $k = \dfrac{\omega}{c}$, so that the input impedance equation becomes

$$Z_{in} = Z_0 \frac{Z_T + jZ_0 \tan\left(kX\right)}{Z_0 + jZ_T \tan\left(kX\right)} = 100 \frac{0.5 + j\tan\left(kX\right)}{1 + 0.5j\tan\left(kX\right)} = 100f, \; f = \frac{.5 + jx}{1 + .5jx} \quad x = kX$$

we now have to find when the expression $f = \frac{.5+jx}{1+.5jx}$ becomes real. or when

$$\operatorname{Im} f = 0 \Rightarrow \operatorname{Im}\frac{(.5+jx)(1-.5jx)}{(1+.25x^2)} = 0 \Rightarrow \frac{-.75x}{1+.25x^2} = 0 \Rightarrow$$

$$x = \{0,\infty\} \Rightarrow kX = \left\{\frac{n\pi}{2}, \frac{n\pi}{2}+\frac{\pi}{4}\right\} \Rightarrow \frac{2\pi f}{1\times10^8}0.75 = \frac{\pi}{4} \Rightarrow f = 16.67\text{MHz} \Rightarrow$$

$$f = \{n\times 33.3MHz, n\times 33.3MHz + 16.6MHz\}$$

By direct substitution these values are $50\,\Omega$ and $200\,\Omega$

[5]

d) (Interpretation of theory)

At the first inversion frequency, 16.6MHz, the source will see the oscilloscope input as $Z_L = \frac{Z_0^2}{Z_{in}} = 0.01\Omega$. This frequency is clearly much higher than the bandwidth of the measurement. A better estimate is 8MHz, the lowest frequency at which the magnitude of the apparent scope impedance $Z_0 = 100\Omega$. (I will accept other reasonable estimates, such as

[5]