

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2010

EEE PART I: MEng, BEng and ACGI

Tuesday, 25 May 10:00 am

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.

Answer Q1 and any two of questions 2-4.

Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : W.T. Pike, E.M. Yeatman
Second Marker(s) : T.J. Tate, T.J. Tate

Special instructions for students

Fundamental constants

Permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H/m

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K

Electron charge, $e = 1.6 \times 10^{-19}$ C

Electron mass, $m = 9.1 \times 10^{-31}$ kg

Speed of light, $c = 3.0 \times 10^8$ ms⁻¹

Schrödinger's equation

General form:

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

In one dimension:

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

In spherical coordinates:

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr}$$

Free-electron theory

Density of states (3D):

$$g(E) = \frac{1}{\pi^2 \hbar^3} (m)^{3/2} \sqrt{2E}$$

Fermi energy

$$E_f = \frac{\hbar^2 \pi^2}{2m} \left(\frac{3n}{\pi} \right)^{2/3}$$

Fermi distribution

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{kT}\right)}$$

Electrons in semiconductors

Effective mass:

$$m_e^* = \frac{\hbar^2}{d^2 E(k)/dk^2}$$

Concentration of electrons in a semiconductor of bandgap E_g :

$$n = \frac{1}{\sqrt{2}\hbar^3} \left(\frac{m_e^* kT}{\pi} \right)^{3/2} e^{-\frac{(E_g - E_f)}{kT}}$$

$$= N_c e^{-\frac{(E_g - E_f)}{kT}}$$

Concentration of holes

$$p = \frac{1}{\sqrt{2}\hbar^3} \left(\frac{m_h^* kT}{\pi} \right)^{3/2} e^{-\frac{E_f}{kT}}$$

$$= N_v e^{-\frac{E_f}{kT}}$$

Polarization

Lorentz correction for local field:

$$\mathbf{E}_{loc} = \mathbf{E} + \frac{\mathbf{P}}{3\epsilon_0}$$

Electronic polarization:

$$P_0 = \frac{\epsilon_0 \omega_p^2 E_0}{\omega_m^2 - \omega^2 + j\omega\gamma}$$

where

$$\gamma = \frac{r}{m},$$

$$\omega_m^2 = \omega_0^2 - \frac{\omega_p^2}{3},$$

$$\omega_0^2 = k/m,$$

$$\omega_p^2 = \frac{ne^2}{m\epsilon_0}.$$

Orientational Polarization:

Static:

$$P = n\mu L(\mu E/kT) \text{ where } L(x) = \coth(x) - 1/x$$

Dynamic:

$$P_0 = \frac{P_s}{1 + j\omega\tau},$$

Magnetism

Magnet dipole due to electron angular momentum:

$$\mu_m = -\frac{e\mathbf{L}}{2m}$$

Magnet dipole due to electron spin:

$$\mu_m = -\frac{e\mathbf{S}}{m}$$

Paramagnetism:

$$M = n\mu_m L\left(\frac{\mu_m \mu_0 H}{kT}\right)$$

The Questions

1. [Compulsory]

- a) Explain why the polarity of the Hall voltage is inverted when the sign of the charge of the carriers is reversed for a given current direction. [4]
- b) An electron gun operates at a voltage of 10 kV. What is the wavelength of the electrons emitted by the gun? [4]
- c) Figure 1c shows the probability distributions of an electron in a diatomic molecule, with the two nuclei represented by black dots.

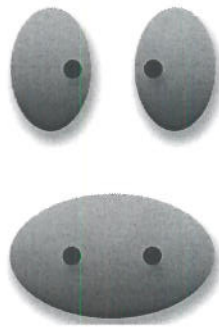


Figure 1c: Probability distributions of two orbitals in a diatomic atom.

- Identify the bonding and antibonding orbitals and explain what might cause any energy difference between them. [4]
- d) By consideration of the occupancy of electron states in a pure metal, explain what would be the expected conductivity at 0 K. [4]
 - e) Silicon has an intrinsic carrier concentration of 10^{10} cm^{-3} at room temperature. What is the minority carrier concentration when it is doped with phosphor at a concentration of 10^{15} cm^{-3} ? [4]
 - f) Draw the free-body diagram for a diving board of length l and negligible mass when a diver of mass m stands still at the unsupported end. [4]
 - g) A charged electrolytic capacitor uses a dielectric exhibiting orientational polarisation. Sketch the variation in the polarisation of the dielectric with time if the capacitor is suddenly discharged. [4]
 - h) An optical fibre has a real refractive index of 1.5 and a loss tangent of 0.001 at the signal wavelength. What is the complex dielectric constant, $\epsilon = \epsilon' - j\epsilon''$. [4]
 - i) Why are diamagnetic materials weakly repelled by a magnetic field? [4]
 - j) By consideration of energy minimisation, explain why domains form in a ferromagnetic material. [4]

2.

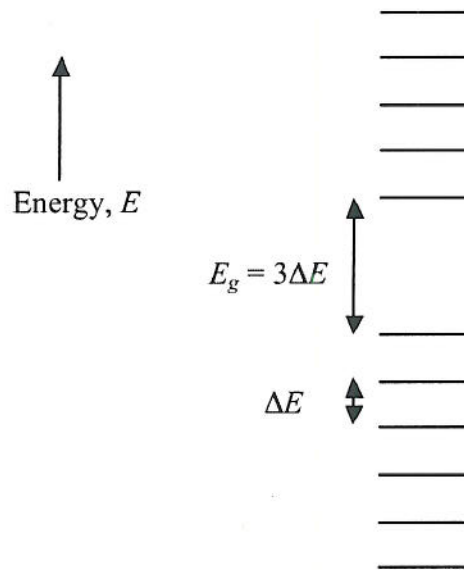


Figure 2: Single electron states of the model system.

Figure 2 shows a simplified model of the single-electron states in a material. The states are uniformly spaced in energy, ΔE except for an energy gap of $3\Delta E$. The spin of the electrons can be ignored.

- a) Given the material is an intrinsic semiconductor, draw the occupancies of these states at 0 K, identifying the valence band, conduction band, and Fermi level. [6]
- b) If the temperature is increased to a value T_1 , where $kT_1 = 6\Delta E$ and k is Boltzmann's constant, draw all the possible electron configurations and hence plot the average values of occupancy for each energy. [12]
- c) An acceptor state is introduced at energy ΔE above the valence band.
 - i) Draw the occupancies for this doped material at 0 K.
 - ii) Sketch the electron configuration at a temperature T_2 given by $kT_2 = \Delta E$ and explain how the conductivity of the material is increased compared to (i). [8]
- d) Which of the previously sketched configurations correspond to freeze-out, extrinsic conduction and intrinsic conduction in this model semiconductor. [4]

3. (a) Figure 3 shows the variation with frequency of the real part of the dielectric constant ϵ_r' , of a material in an oscillating electric field. Describe the mechanism associated with the changes in ϵ_r' in regions 1, 2 and 3.

What is the value of $\epsilon_r'(\infty)$ and why?

Sketch the corresponding plot for the imaginary component, ϵ_r'' .

[10]

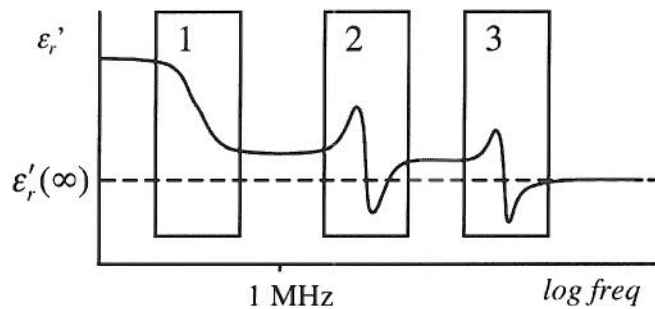


Figure 3: Plot of real part of dielectric constant with frequency

- (b) In region 1 the complex dielectric constant has the general form

$$\epsilon_r(\omega) = \epsilon_{r1} + \frac{\Delta\epsilon_r}{1 + j\omega\tau}$$

An electrolytic capacitor contains a certain dielectric for which $\epsilon_{r1} = 1.5$ and $\Delta\epsilon_r = 2$. The relaxation time, τ , is $1 \mu\text{s}$. What is the fraction of the DC capacitance at the excitation frequency of 1 MHz, whose position is shown on the plot?

[14]

- (c) How will the complex dielectric constant change if the capacitor is cooled sufficiently to freeze the dielectric.

[6]

4. (a) Define the meaning of all the variables and constants in the 1-D Schrödinger equation

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x) \quad [6]$$

(b) An electron is trapped in a infinitely deep one-dimensional potential well:

$$V(x) = \infty, \quad x < 0$$

$$= 0, \quad 0 \leq x \leq L$$

$$= \infty, \quad x > L$$

Show that the wavefunction

$$\psi(x) = 0, \quad x < 0$$

$$= A \sin kx, \quad 0 \leq x \leq L$$

$$= 0, \quad x > L$$

is a solution of the one-dimensional Schrödinger equation and solve for the possible values of k in terms of a quantum number n .

Hence show that the energy difference between the two states with quantum numbers n_1 and n_2 is

$$\Delta E = \frac{\hbar^2 \pi^2}{2mL^2} (n_1^2 - n_2^2) \quad [14]$$

(c) A quantum-well infrared photodetector (QWIP) has been designed to detect infrared radiation of wavelength less than $8 \mu\text{m}$. Assuming the quantum well of the QWIP can be approximated by the potential $V(x)$ in part (b), electrons in the quantum well have an effective mass equal to their free-electron value and the infrared absorption occurs for a transition between the two lowest energy states, what should be the width of the well? [10]

The Answers

2010

1.

a) Positive and negatively charged carriers, or holes and electrons, move in opposite directions to produce the same current [1]. As the Lorentz force on the carriers depends on both the direction and charge sign [1], both holes and electrons feel a force in the same direction perpendicular to motion and the magnetic field [1]. Hence, in both cases a positive or negative charge will accumulate on the same face of the semiconductor, producing fields in opposite directions [1]. [bookwork]

b) From

$$T = eV = \frac{p^2}{2m}, \quad [1]$$

$$p = \sqrt{2meV}$$

and

$$p = h/\lambda,$$

$$\lambda = \frac{h}{\sqrt{2meV}} \quad [2]$$

For 10 kV this gives $1.1 \times 10^{-11} \text{ m}$ [1]. [new calculation]

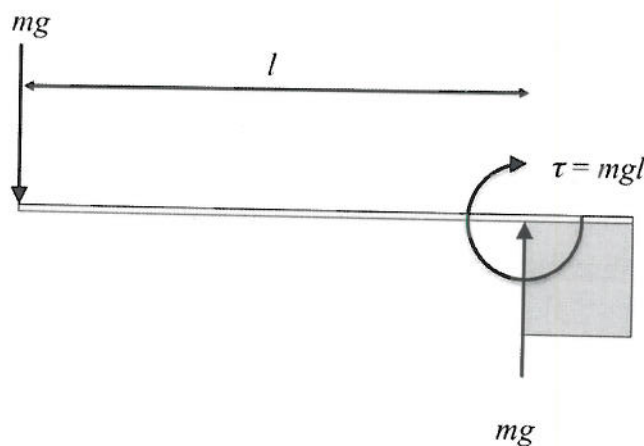
c) Top: antibonding; bottom: bonding. [2]

The bonding orbital has a lower energy as the electron has a greater probability of being between the two nuclei [1] where it will reduce the repulsive forces between the two core states and nuclei [1]. [bookwork]

d) All the states below the Fermi level will be full, all above will be empty [2]. Hence there will be no empty states available for electrons to be transferred into during transport [1]. Hence the conductivity will be zero [1]. [application of theory]

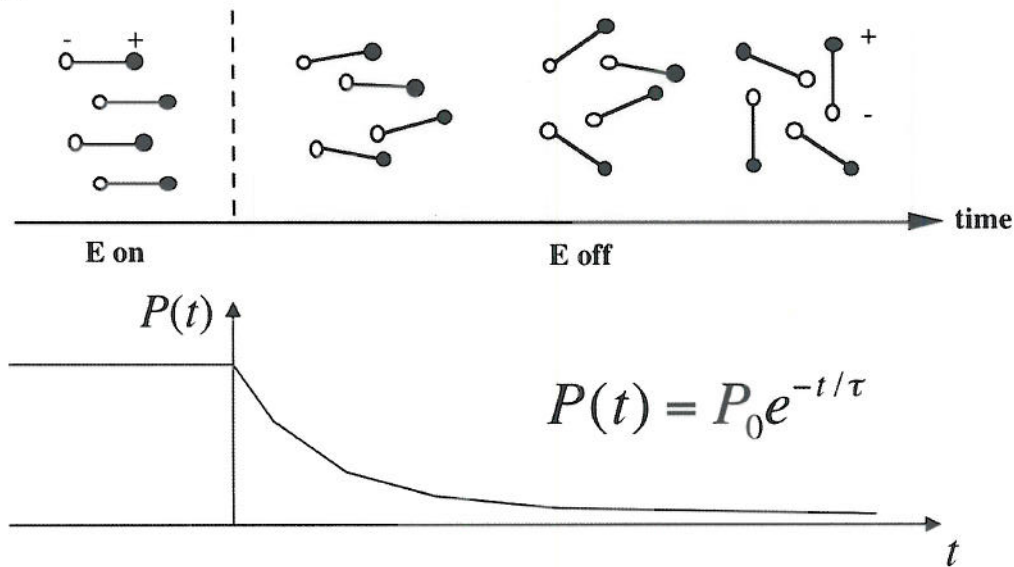
e) $np = n_i^2 = 10^{20} \text{ cm}^{-3}$ [2]. For an n-doped semiconductor, $n = N_D = 10^{15} \text{ cm}^{-3}$ [1]. Hence $p = 10^5 \text{ cm}^{-3}$ [1]. [new calculation]

f)



[4] [new analysis]

g)

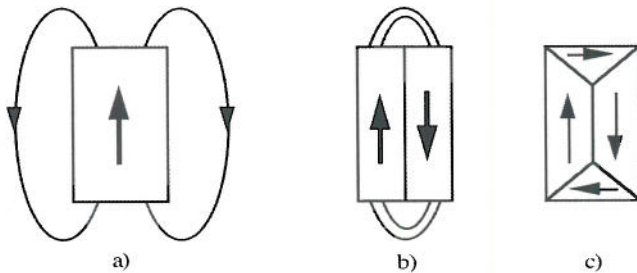


Either sketch will get full marks. [4] [bookwork]

h) This is a weakly absorbing dielectric [1]. Hence the real part on the dielectric index is given by $\epsilon = n^2 = 2.25$ [1]. As the loss tangent is the ratio of the imaginary part to the real part of the dielectric constant, $\epsilon = \epsilon' - j\epsilon'' = 2.25 - j 2.25 \times 10^{-3}$ [2]. [new calculation]

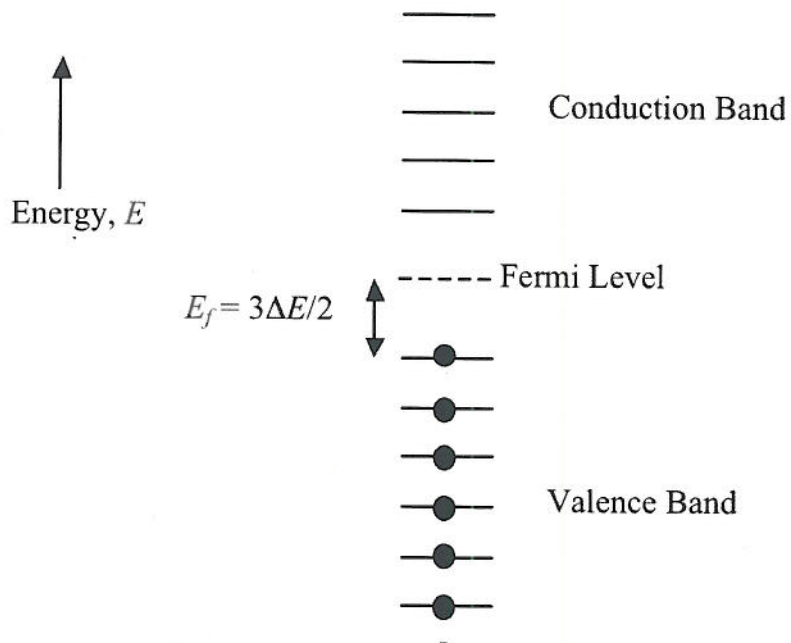
i) The electron orbitals in the atoms of the material can be considered as current loops [1]. Hence if a the loop is introduced into a field a voltage will be developed that opposes the build up of the magnetic field, $V = -dB/dt$, [1] and hence current that will act to resist the increase in the field [1], producing a small resistive force [1]. [bookwork]

j)



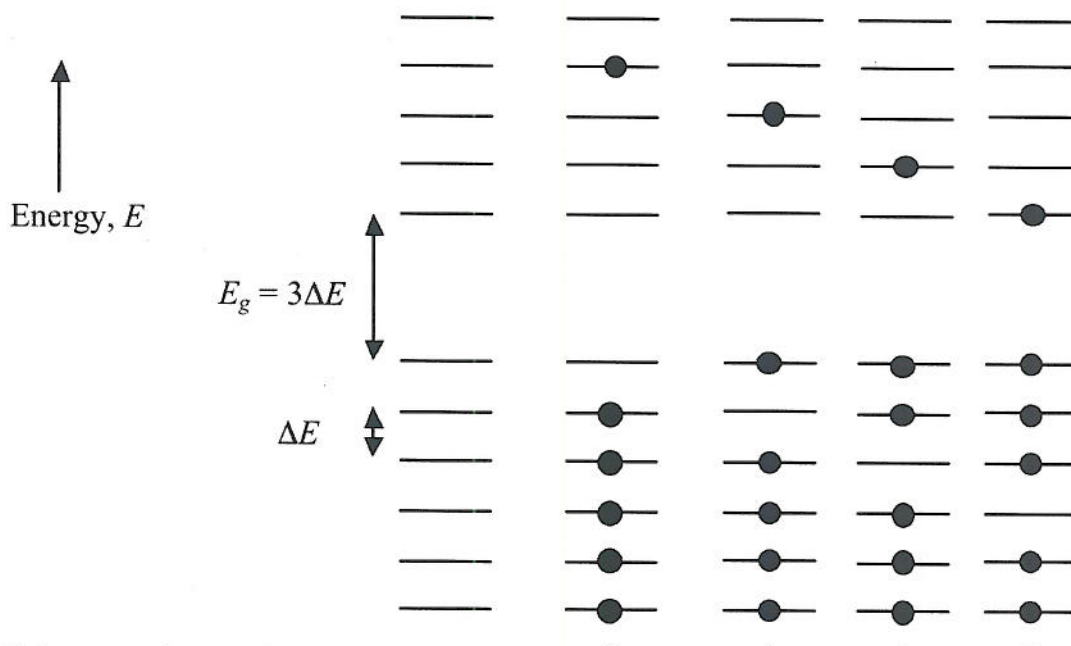
Formation of domains reduces the magnetic field energy [1] but increases the domain-wall energy [1]. Domains be energetically favoured if the reduction in the field energy is larger than the increase in the domain-wall energy [2].

2. a)



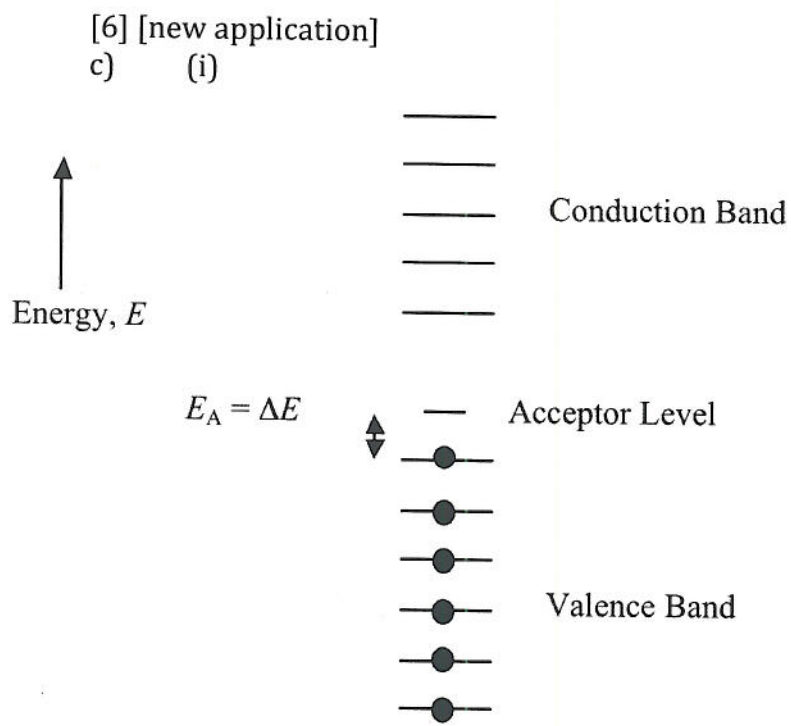
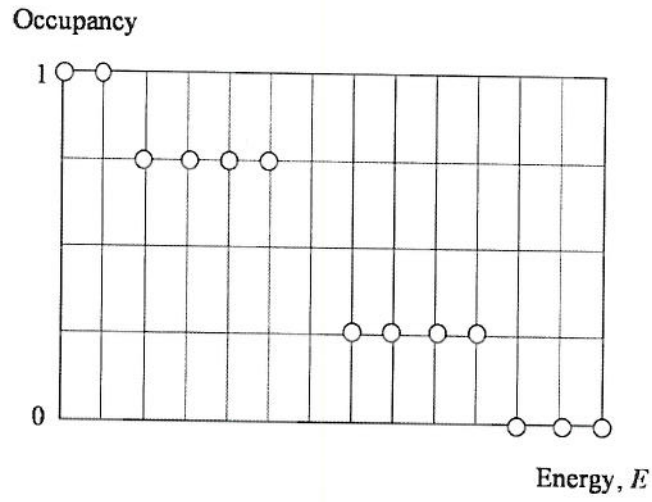
[6] [new application]

b)

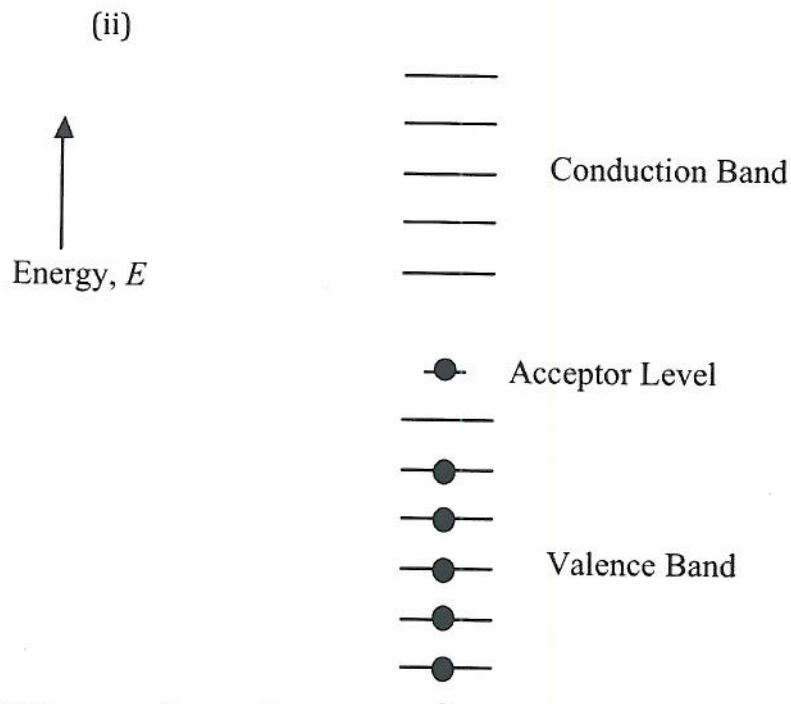


[6] [new application]

The occupancies will be:



[3] [new application]



[3] [new application]

There is now an unoccupied state in the valence band. Hence electrons can produce overall charge movement as there is now the possibility of scattering into an empty state under the influence of an electric field. Hence the conductivity is increased compared to (i). [2] [bookwork]

d) Freeze out is (a) for intrinsic or (c) (i) for doped semiconductors. Extrinsic is (c) (ii). Intrinsic is (b). [4] [new application]

3. a)

Region 1: orientational polarization: dipoles in a fluid will tend to align with the electric field increasing the dielectric constant at low frequencies. Thermal effects will tend to randomize the alignment, with a characteristic relaxation time, corresponding to the reciprocal of frequency of the mechanisms interaction.[2]

Region 2: molecular or ionic polarization. The ions or fractionally charged molecules will respond to the electric field, increasing the dielectric constant at low frequencies.

[2]

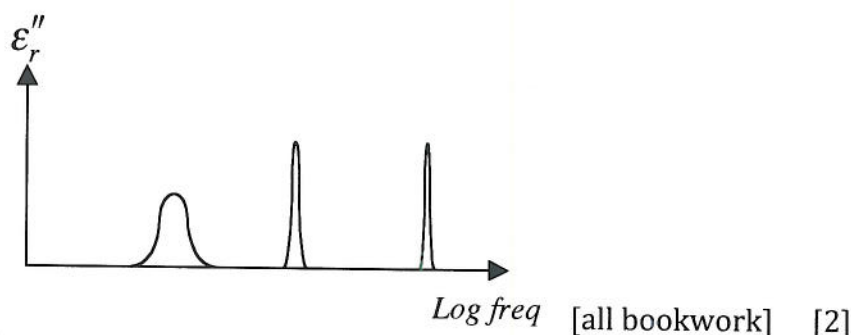
Region 3: electronic polarization. The electrons themselves respond to the electric field, adding to the dielectric constant at low frequencies.

[2]

$\epsilon_r'(\infty)$ will be one, as at very high frequencies there are no mechanisms that can respond fast enough to the electric field, and no polarization interaction is seen.

[2]

The imaginary part:



[all bookwork] [2]

b) Capacitance is given by

$$C = \frac{\epsilon_0(\epsilon_r' - j\epsilon_r'')A}{d} \quad [3]$$

giving an effective capacitance of

$$C' = \frac{\epsilon_0\epsilon_r'A}{d} \quad [2]$$

Separating the dielectric constant into real and imaginary parts:

$$\epsilon_r(\omega) = \epsilon_{r1} + \frac{\Delta\epsilon_r}{1 + j\omega\tau} \quad [2]$$

$$\epsilon_r'(\omega) = \epsilon_{r1} + \frac{\Delta\epsilon_r}{1 + \omega^2\tau^2}$$

For $\tau = 1 \mu s$, $f = 1 \text{ MHz}$, $\omega\tau = 2\pi$ [1]

Hence the fraction of the DC capacitance at 1 MHz

$$\frac{\Delta C}{C} = \frac{\epsilon_r'(f = 1\text{MHz})}{\epsilon_r'(f = 0\text{Hz})} \quad [1]$$

$$\epsilon_r'(f = 0\text{Hz}) = 1.5 + \frac{2.5}{1+0} = 4 \quad [1]$$

$$\epsilon_r'(f = 1\text{MHz}) = 1.5 + \frac{2.5}{1+(2\pi)^2} = 1.56 \quad [2]$$

Therefore the fraction left is $1.56/4 = 0.39$ or 39% [2] [new calculation]

c) If the dielectric is frozen, orientational polarisation will no longer occur. The real part will not rise in region 1 but stay flat [3], and for the imaginary part the peak in region 1 will disappear [3]. [new application]

4. (a) S.E.:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

\hbar : Planck's constant divided by 2π

m : the mass of the particle

ψ : the wavefunction of the particle

x : the spatial dimension

$V(x)$: the potential energy, varying as a function of x

E : the total energy of the particle [6] [bookwork]

b) Inside the quantum well, $V(x)$ is zero, so S.E. becomes:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x) \quad [2]$$

Substituting for $\psi = A \sin k_n x$ gives

$$\frac{\hbar^2 k^2}{2m} A \sin kx = EA \sin kx$$

$$\Rightarrow E = \frac{\hbar^2 k^2}{2m} \quad [2]$$

The boundary conditions automatically match at $x = 0$ through the choice of a sine for the wavefunction. At $x = L$, we have

$$\begin{aligned} \sin kL &= 0 \\ \Rightarrow kL &= n\pi \end{aligned} \quad [2]$$

for integer n . $n = 0$ is not a solution as this would imply ψ is zero everywhere, and hence there is no electron in the state. Hence we

$$\text{obtain: } k = \frac{n\pi}{L}, \quad n = 1, 2, 3, \dots \quad [2]$$

$$\text{and } E = \frac{n^2 \hbar^2 \pi^2}{2mL^2}, \quad n = 1, 2, 3, \dots \quad [2]$$

Hence

$$\Delta E = \frac{\hbar^2 \pi^2}{2mL^2} (n_1^2 - n_2^2) \quad [4] \text{ [bookwork, new application]}$$

c) The lowest energy transitions will be

$$n = 2 \text{ to } 1: \Delta E = 3 \frac{\hbar^2 \pi^2}{2mL^2} \quad [2]$$

And the corresponding wavelengths will be

9
9

$$hf = \frac{hc}{\lambda} = \Delta E,$$

$$\frac{hc}{\lambda} = \frac{3\hbar^2\pi^2}{2mL^2}$$

$$\frac{2\pi\hbar c}{\lambda} = \frac{3\hbar^2\pi^2}{2mL^2}$$

[8] [new calculation]

$$L = \sqrt{\frac{3\hbar\pi\lambda}{4mc}}$$

$$= \sqrt{\frac{3 \times 1.05 \times 10^{-34} \times 3.14 \times 8 \times 10^{-6}}{4 \times 9.1 \times 10^{-31} \times 3 \times 10^8}}$$

$$= 2.7\text{nm}$$