

IMPERIAL COLLEGE LONDON

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DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2009

MSc and EEE/ISE PART IV: MEng and ACGI

Corrected Copy

OPTICAL COMMUNICATION

Monday, 27 April 2:30 pm

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	E.M. Yeatman
	Second Marker(s) :	A.S. Holmes

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge : $e = 1.6 \times 10^{-19} \text{ C}$

permittivity of free space : $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

relative permittivity of silicon : $\epsilon_r = 12$

Planck's constant : $h = 6.63 \times 10^{-34} \text{ J s}$

Boltzmann's constant : $k = 1.38 \times 10^{-23} \text{ J/K}$

speed of light : $c = 3.0 \times 10^8 \text{ m/s}$

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x} \tan(k_{1x}d/2) \text{ and } \kappa = -k_{1x} \cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, state any assumptions or approximations made, and give a brief (one or two lines) explanation where appropriate. All parts have equal value.

[20]

- a) A slab waveguide supports 3 TE modes, $m = 0, 1$ and 2 . Which of these has the highest fraction of energy propagating in the cladding?
- b) A silica optical fibre has a numerical aperture of 0.18 . Estimate the index difference Δn .
- c) Two single mode optical fibres have different index differences. Which fibre would you expect to suffer more from bend losses, the one with the higher or lower index difference? Briefly explain why.
- d) An optical fibre link is running at 2.5 Gbit/s. Estimate the number of bits propagating in a 2 km long fibre at any one moment.
- e) Give a practical disadvantage of polarisation maintaining fibre in comparison to conventional single mode fibre.
- f) Calculate the optimal thickness and refractive index of an anti-reflection coating acting between a glass of index 1.52 and air, for a free space wavelength of $0.85 \mu\text{m}$.
- g) Why can photodiodes for use in high performance fibre optic communication systems not be based on silicon?
- h) A laser diode with quantum efficiency 0.80 has a slope efficiency of 1.16 W/A. Calculate the output (free space) wavelength.
- i) Briefly explain why an avalanche photodiode usually has the avalanche region confined to the base of the depleted region.
- j) Silica fibres generally have higher attenuation at 1.3 than at $1.5 \mu\text{m}$ wavelength. Which attenuation mechanism is primarily responsible for this higher attenuation?

- 2.
- A plane wave propagating in a medium of refractive index $n_1 = 1.51$ is incident on a planar interface with a medium of refractive index n_2 at an incident angle θ . Sketch a plot of the critical angle θ_c as a function of n_2 for the range of n_2 between 1.3 and 1.50. You should calculate a few specific values in order to obtain a quantitative plot. [6]
 - For $n_2 = 1.40$, find the incident angle θ_i for which the $1/e$ decay distance of the electric field in the second medium is $1.5 \mu\text{m}$. Assume the incident wave has TE polarisation and a free space wavelength of $1.3 \mu\text{m}$. [4]
 - What is the phase change on reflection in the case described in (b)? Note that the TE reflection coefficient for the electric field is given by the difference over the sum of the perpendicular components of the two wavevectors. [2]
 - If the incident wave has TE polarisation, the interface between the media lies on the y - z plane, and the incident wavevector is in the x - z plane, what is the direction of the electric field of the incident wave? [2]
 - For $n_2 = 1.40$ and TE polarisation, find the incident angle θ_i for which the modulus of the reflection coefficient for the electric field is 0.5. [6]
3. A symmetric slab waveguide as shown in Fig. 3.1 has a core thickness $d = 12 \mu\text{m}$, and core and cladding indices of $n_1 = 1.46$ and $n_2 = 1.45$ respectively. For a free-space wavelength of $1.50 \mu\text{m}$, a certain mode of the waveguide has an effective index of $n' = 1.45264$.
- State the boundary conditions that must be satisfied at the core-cladding interfaces for TE modes, with respect to the electric field distribution $E(x)$. Show that the effective index given above is consistent with these conditions. [10]
 - Sketch the field profile $E(x)$ for the mode having this value of n' , and state its mode number m . [5]
 - Determine the total number of TE modes supported by the guide at this wavelength. [5]

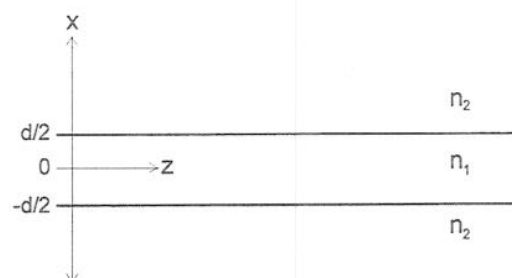


Figure 3.1

4.

- a) Briefly define material dispersion and waveguide dispersion as they relate to light propagation in dielectric waveguides.

[6]

- b) The refractive index n of silica in the near infra-red region can be closely approximated by :

$$n^2 - 1 = \frac{a_0}{1 - (b_0/\lambda)^2} + \frac{a_1}{1 - (b_1/\lambda)^2} + \frac{a_2}{1 - (b_2/\lambda)^2}$$

where λ is the free space wavelength, and the constants a and b are given by :

$$\begin{array}{lll} a_0 = 0.696 & a_1 = 0.408 & a_2 = 0.897 \\ b_0 = 0.068 \mu\text{m} & b_1 = 0.116 \mu\text{m} & b_2 = 9.90 \mu\text{m} \end{array}$$

Using this equation and the approximations $b_0, b_1 \ll \lambda \ll b_2$, show that

$$\frac{d^2n}{d\lambda^2} = \frac{3(a_0b_0^2 + a_1b_1^2)}{n\lambda^4} - \frac{a_2}{nb_2^2}$$

Hence, find the wavelength at which there is no material dispersion. How does your approximation compare with the known value for silica?

[8]

- c) For a strongly confining single-moded slab waveguide, the effective index n' of the TE mode can be approximated by :

$$n' \cong n_2 [1 + 2 \Delta n^2 (\pi d/\lambda)^2]$$

where d is the guide thickness, n_2 the cladding index and Δn the index difference. Using this approximation, find an expression for the waveguide dispersion as a function of λ .

Using your results above, find the value of $\Delta n \cdot d$ at which the dispersion minimum is shifted to $\lambda = 1.55 \mu\text{m}$.

[6]

5. a) Describe and discuss the important attenuation mechanisms in optical fibres, and the influence these have in choice of operating wavelengths. Use diagrams and equations where appropriate. [6]
- b) An optical receiver can be implemented as a photodiode in series with a resistor, followed by a voltage amplifier. State the principal noise sources in this case, and draw a noise equivalent circuit for the receiver. Describe an alternative receiver configuration, and give its main advantage in terms of noise. [6]
- c) A receiver with an effective input resistance of $10\text{ k}\Omega$ is at the end of 20 km of step index, multi-mode fibre with index difference 0.003 and attenuation of 0.8 dB/km at the operating wavelength of 1510 nm . The transmitted power is $1\text{ }\mu\text{W}$. Making an appropriate assumption for the required bandwidth, find the maximum bit-rate B for this link in the two cases
- if the link is limited by the SNR requirement of 12 , assuming perfect quantum efficiency in the receiver, and that thermal noise is the dominant noise source;
 - if the link is limited by dispersion.

Which is the more restricting factor for the parameters given? [8]

6. a) A silicon p-i-n photodiode has intrinsic layer thickness $w_i = 9\text{ }\mu\text{m}$, and p and n doping levels respectively of $N_A^+ = 1.5 \times 10^{21}\text{ m}^{-3}$ and $N_D^+ = 10^{21}\text{ m}^{-3}$. The intrinsic layer doping level is N_D^- . The electron and hole velocities can be approximated as linearly proportional to applied field, with mobilities (drift velocity per unit electric field) of $0.125\text{ m}^2/\text{Vs}$ and $0.05\text{ m}^2/\text{Vs}$ respectively, up to a saturation velocity of 10^5 m/s (for both electrons and holes). Find the applied electric field amplitude at which the electrons reach their saturation velocity. Hence, find the value of N_D^- such that the electric field strength reaches this saturation value at the i-n junction, and is 10% above this value at the p-i junction. Calculate also the corresponding applied voltage needed to reach this condition. [10]
- b) For the structure and conditions of part (a), find the maximum time for a carrier pair to be swept out of the depletion region for a photon absorbed in the intrinsic region (you may neglect the propagation in the depleted parts of the n^+ and p^+ regions). You may find the following integral useful:

$$\int \frac{dx}{C + ax} = \frac{1}{a} \ln(C + ax)$$

[6]

- c) Explain why the p-i-n structure is preferred to an n-i-p structure for photodiodes. [4]

SOLUTIONS

1. a) A slab waveguide supports 3 TE modes, $m = 0, 1$ and 2. Which of these has the highest fraction of energy propagating in the cladding?

$m=2$. Higher order modes have higher k_x values (as can be seen in eigen-equation plots), thus lower β values, thus lower effective indices, indicating that more light is in the cladding.

$\frac{1}{8}$

- b) A silica optical fibre has a numerical aperture of 0.18. Estimate the index difference Δn .

$NA = \sqrt{2n \cdot \Delta n}$. We need to assume $n = 1.5$, giving $0.18^2 = 3 \cdot \Delta n$, $\Delta n = 0.011$

- c) Two single mode optical fibres have different index differences. Which fibre would you expect to suffer more from bend losses, the one with the higher or lower index difference? Briefly explain why.

The one with lower index difference has a higher critical angle, and so could be expected to have a higher mode angle, and thus be more affected by bend loss.

- d) An optical fibre link is running at 2.5 Gbit/s. Estimate the number of bits propagating in a 2 km long fibre at any one moment.

Approximating the speed as $c/1.5 = 2 \times 10^8$, no of bits = $L/(c/nB) = 25,000$ bits.

- e) Give a practical disadvantage of polarisation maintaining fibre in comparison to conventional single mode fibre.

Alignment of two fibres is more difficult because their rotation as well as position need to be matched.

- f) Calculate the optimal thickness and refractive index of an anti-reflection coating acting between a glass of index 1.52 and air, for a free space wavelength of $0.85 \mu\text{m}$.

$n = \sqrt{1.52 \times 1} = 1.233$, $t = \lambda/4n = 0.85/(4 \times 1.233) = 0.172 \mu\text{m}$.

- g) Why can photodiodes for use in high performance fibre optic communication systems not be based on silicon?

High performance systems work at 1.3 or $1.5 \mu\text{m}$, at which photon energies are less than the Si bandgap so Si is transparent and can't be used to detect.

- h) A laser diode with quantum efficiency 0.80 has a slope efficiency of 1.16 W/A . Calculate the output (free space) wavelength.

$S = \eta hc/e\lambda$ so $\lambda = \eta hc/eS$; using $hc/e = 1.24 \times 10^{-6}$, $\lambda = 0.8 \times 1.24/1.16 = 0.855 \mu\text{m}$.

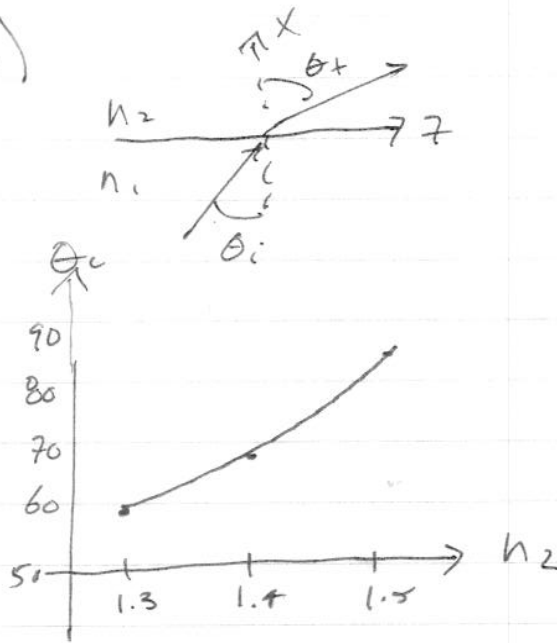
- i) Briefly explain why an avalanche photodiode usually has the avalanche region confined to the base of the depleted region.

In this way very few photons are absorbed in the avalanche region, so the region receives all the photo-generated electrons, and none of the holes. This makes the avalanche factor more predictable and controllable.

- j) Silica fibres generally have higher attenuation at 1.3 than at $1.5 \mu\text{m}$ wavelength. Which attenuation mechanism is primarily responsible for this higher attenuation?

Rayleigh scattering.

2. a)



$$\theta_c = \sin^{-1}\left(\frac{n_2}{1.51}\right)$$

n_2	θ_c
1.3	59.4°
1.4	68.0°
1.5	83.4°

b) For $\theta > \theta_c$, $k_{2x} = jK$

$$k_{1z}^2 = k_{2z}^2 \therefore n_1^2 k_0^2 \sin^2 \theta_i = n_2^2 k_0^2 + K^2$$

$$\sin^2 \theta_i = \frac{n_2^2 + (K/k_0)^2}{n_1^2}$$

$\frac{1}{e}$ decay distance (amplitude) = $\frac{1}{K} = 1.5 \mu\text{m}$

$$k_0 = \frac{2\pi}{\lambda_0} = 2\pi / 1.3 \mu\text{m} \therefore \frac{K}{k_0} = \frac{1.3}{2\pi(1.5)} = 0.138$$

$$\sin^2 \theta_i = \frac{1.4^2 + 0.138^2}{1.51^2} \quad \theta_i = 68.7^\circ$$

c) $\frac{E_r}{E_i} = \frac{k_{1x} - k_{2x}}{k_{1x} + k_{2x}} = \frac{k_{1x} - jK}{k_{1x} + jK} \therefore \tan \frac{\phi}{2} = -\frac{K}{k_{1x}}$

$$\phi = 2 \tan^{-1} \left(\frac{0.138 k_0}{1.51 \cos(68.7^\circ) k_0} \right) = 28.2^\circ$$

d) TE: \hat{E} is parallel to interface and $\perp \underline{k}$,
so lies in $\pm \hat{y}$ direction.

e) $\left| \frac{E_r}{E_i} \right| = \left| \frac{k_{1x} - k_{2x}}{k_{1x} + k_{2x}} \right| = \frac{1}{2}$, k_{1x} & k_{2x} are real ($\theta < \theta_c$)

Since $n_1 > n_2$, $k_{1x} > k_{2x} \therefore k_{1x} - k_{2x} = \frac{1}{2}(k_{1x} + k_{2x})$
 $k_{2x} = k_{1x} / 3$

$$k_{1z} = k_{2z} \therefore n_1^2 k_0^2 - k_{1x}^2 = n_2^2 k_0^2 - (k_{1x}/3)^2$$

$$1 - (n_2^2/n_1^2) = \frac{8}{9} \cos^2 \theta_i \quad \theta_i = 66.6^\circ$$

3 a) If $E_1(x)$ and $E_2(x)$ are the field in the core ($-d/2 < x < d/2$) and the cladding ($x > d/2$) respectively, then need:

$$E_1(d/2) = E_2(d/2), \quad \frac{dE_1(d/2)}{dx} = \frac{dE_2(d/2)}{dx}$$

For even modes $E_1(x) = A \cos(k_x x)$ (i)

$E_2(x) = B e^{-Kx}$ (ii)

$\frac{dE_1(x)}{dx} = -k_x A \sin(k_x x)$ (iii) $\frac{dE_2(x)}{dx} = -KB e^{-Kx}$ (iv)

Dividing (iii) by (ii) = (iv) by (i):

$$k_x \tan(k_x d/2) = K$$

For odd modes $E_1(x) = A \sin(k_x x)$

This gives: $K = -\cot(k_x d/2)$

Also, need $k_{1z} = k_{2z} (= \beta)$

$$\beta = n' k_0 = 1.45264 (2\pi/1.5) = 6.0848 \mu\text{m}^{-1}$$

$$K^2 = \beta^2 - n_2^2 k_0^2 \quad \therefore K = 0.3666 \mu\text{m}^{-1}$$

$$k_x^2 = (n_1 k_0)^2 - \beta^2 \quad \therefore k_x = 0.61333 \mu\text{m}^{-1}$$

Assuming even m:

$$K = k_x \tan(k_x d/2) = 0.61333 \tan\left(\frac{0.61333 \cdot 12}{2}\right)$$

$$= 0.3663 \mu\text{m}^{-1}$$

→ agrees. (close enough)

b) $E_1(x) = A \cos(k_x x)$

$k_x d/2 = 3.68$, This is between π and $3\pi/2$, so

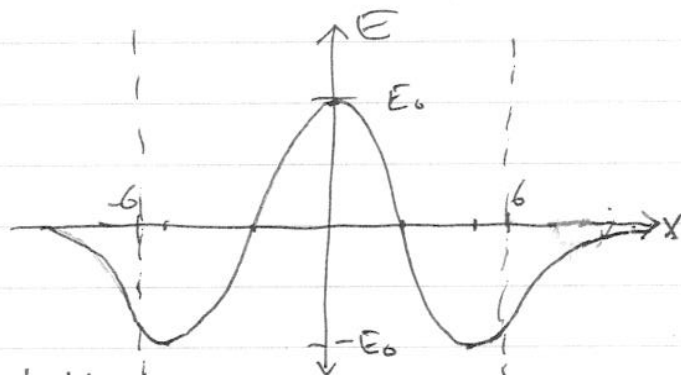
$m=2$ (3rd mode).

$E=0$ for $k_x x = \pm\pi/2$

$x = \pm 2.56 \mu\text{m}$

$E=A$ for $k_x x = 0, \pm\pi$

$x = 0, \pm 5.1 \mu\text{m}$



c) Cutoff: need $m\pi/2 < NA \cdot k_0 d/2$

$$m < NA \cdot (2d/\lambda_0) = 2.73$$

highest m is 2, 3 modes supported.

④ Material Dispersion: variation in group velocity, caused by the wavelength dependence of the refractive index of glass, giving pulse spreading proportional to $\Delta\lambda \frac{d^2n}{d\lambda^2}$

Waveguide Dispersion: variation in group velocity over the signal spectrum resulting from the dependence of mode shape, and therefore effective index n' , on wavelength. (prop. to $\Delta\lambda \frac{d^2n'}{d\lambda^2}$)

$$\tau_g = L/v_g = L / (d\omega/d\beta) = L \frac{d\beta}{d\omega} \frac{d\omega}{d\beta} \quad \beta = n k_0 \quad k_0 = 2\pi/\lambda$$

$$\tau_g = \frac{L}{c} \left(n + \frac{dn}{dk_0} k_0 \right) = \frac{L}{c} \left(n - \lambda \frac{dn}{d\lambda} \right)$$

$$\Delta\tau_g = \Delta\lambda \frac{d\tau_g}{d\lambda} = \frac{\Delta\lambda L}{c} \left(\frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda \frac{d^2n}{d\lambda^2} \right) = -\lambda \frac{\Delta\lambda L}{c} \frac{d^2n}{d\lambda^2}$$

L = path length, v_g = group veloc, τ_g = group delay

$$b_0, b_1 \ll \lambda \ll b_2 :$$

$$n^2 - 1 \approx q_0 \left(1 + \frac{b_0^2}{\lambda^2} \right) + q_1 \left(1 + \frac{b_1^2}{\lambda^2} \right) - q_2 \lambda^2 / b_2^2$$

$$\lambda n \frac{dn}{d\lambda} = -\lambda \frac{q_0 b_0^2}{\lambda^3} - \lambda \frac{q_1 b_1^2}{\lambda^3} - \frac{2q_2 \lambda}{b_2^2}$$

$$n \frac{d^2n}{d\lambda^2} + \left(\frac{dn}{d\lambda} \right)^2 = \frac{3(q_0 b_0^2 + q_1 b_1^2)}{\lambda^4} - \frac{q_2}{b_2^2}$$

$$\frac{d^2n}{d\lambda^2} = \frac{1}{n} \left[\frac{3(q_0 b_0^2 + q_1 b_1^2)}{\lambda^4} - \frac{q_2}{b_2^2} - \frac{1}{n^2} \left(\frac{q_0 b_0^2 + q_1 b_1^2}{\lambda^3} + \frac{q_2 \lambda}{b_2^2} \right)^2 \right]$$

Using $b_0, b_1 \ll \lambda \ll b_2$ we can show that all terms from expanding the third term are \ll the first two, so

$$\frac{d^2n}{d\lambda^2} = \frac{3(q_0 b_0^2 + q_1 b_1^2)}{n \lambda^4} - \frac{q_2}{n b_2^2}$$

= 0 for $\lambda = 1.30 \mu\text{m}$. Agrees well with known value.

4) (continued)

$$n' = n_2 \left[1 + 2\Delta n^2 \left(\pi d / \lambda \right)^2 \right]$$

$$\frac{dn'}{d\lambda} = 2n_2 \Delta n^2 (\pi d)^2 \left(-\frac{2}{\lambda^3} \right) + \underbrace{2n_2 \left(\frac{\pi d}{\lambda} \right)^2 \cdot 2\Delta n \frac{d\Delta n}{d\lambda}}_{\text{can be neglected } \frac{5}{8}}$$

$$\frac{d^2 n'}{d\lambda^2} = \frac{12 n_2 (\Delta n \pi d)^2}{\lambda^4} \quad \Delta \phi = -\frac{\Delta \lambda L}{c} \left(\frac{12 n_2 (\Delta n \pi d)^2}{\lambda^4} \right)$$

Combined dispersion:

$$\frac{d^2 n'}{d\lambda^2} = \frac{12 n_2 (\Delta n \pi d)^2}{\lambda^4} + \frac{3(a_2 b_0^2 + a_1 b_1^2)}{n \lambda^4} - \frac{a_2}{n b_2^2}$$

taking $n = n_2 = 1.5$, $d^2 n' / d\lambda^2 = 0$ for:

$$\frac{18 \pi^2 (\Delta n \cdot d)^2}{(1.55)^4} + \frac{3(-0.0087)}{1.5(1.55)^4} = 0.0061$$

$$\Delta n \cdot d = 0.01 \text{ } \underline{\mu\text{m}}$$

⑤ a) bookmark, see notes B pp 22-23

b) bookmark, see notes C pp. 36-37

$$c) i) SNR = \frac{I_{ph}}{\left[\frac{4kT}{R} \right]^{1/2} \Delta f^{1/2}} = 12$$

$$\text{take } \Delta f = B/2$$

$$I_{ph} = \Phi_R \times \frac{e \lambda}{hc} \quad (\text{for } \eta = 1)$$

$$\Phi_R = -30 \text{ dBm} - 20 \times 0.8 \text{ dB} = -46 \text{ dBm} \\ = 25 \text{ nW}$$

$$I_{ph} = \frac{25 \times 10^{-9} \times 1.6 \times 10^{-19} \times 1.51 \times 10^{-6}}{6.63 \times 10^{-34} \times 3 \times 10^8} = 30.4 \text{ nA}$$

$$\left(\frac{4kT}{R} \right) (B/2) = \left(\frac{30.4 \times 10^{-9}}{12} \right)^2 = 6.4 \times 10^{-18} \\ B = \frac{6.4 \times 10^{-18} \cdot 10^4}{150 \times 10^{-3} \times 1.6 \times 10^{-19}} = 2 \times 10^6 \text{ Bits/sec}$$

$$ii) \text{Frachal spread in velocity} \approx \text{frachal} \\ \text{spread in effective index} = .003/1.5 = .002 \\ \Delta \tau = \tau \frac{\Delta v}{v} = \frac{nL}{c} (.002) = \frac{1.5 \cdot 20 \times 10^3}{3 \times 10^8} \times .002 = 2 \times 10^{-7}$$

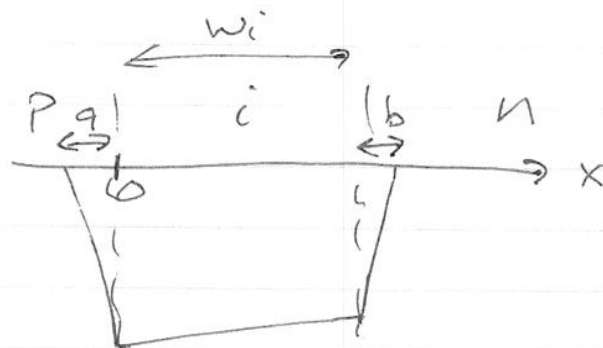
If we allow $\frac{1}{4}$ bit of $\Delta \tau$ then

$$0.25/B = \Delta \tau \quad B = 0.25/2 \times 10^{-7} = 1.25 \times 10^6 \text{ bit/s}$$

\therefore Dispersion is the more limiting factor.

(6a)

7/8



$$E_{sat} = \frac{V_{dsat}}{\mu} = \frac{10^{-5}}{0.125} = 8 \times 10^5 \text{ V/m (electrons)}$$

$$= \frac{10^5}{0.05} = 2 \times 10^6 \text{ V/m (holes)}$$

$$E(x=w_i) = 8 \times 10^5$$

$$E(x=0) = 1.1 \times 8 \times 10^5 = 8.8 \times 10^5 \text{ V/m}$$

$$\frac{dE}{dx} = \frac{\rho}{\epsilon} \Rightarrow \rho = \frac{\epsilon_0 \epsilon_r \Delta E}{w_i} = \frac{8.85 \times 10^{-12} \times 12 \times 8 \times 10^4}{9 \times 10^{-6}}$$

$$= 0.944 \text{ C/m}^3$$

$$\rho = e N_0^- \quad N_0^- = .944 / 1.6 \times 10^{-19} = 5.9 \times 10^{18} \text{ m}^{-3}$$

To find voltage, need to calculate a and b

$$\frac{1.1 E_{sat}}{a} = \frac{e N_A^+}{\epsilon_r \epsilon_0} \quad a = \frac{8.8 \times 10^5 \times 12 \times 8.85 \times 10^{-12}}{1.6 \times 10^{-19} \times 1.5 \times 10^{21}} = 0.39 \mu\text{m}$$

$$\frac{E_{sat}}{b} = \frac{e N_D^+}{\epsilon_r \epsilon_0} \Rightarrow b = \frac{8 \times 10^5 \times 12 \times 8.85 \times 10^{-12}}{1.6 \times 10^{-19} \times 10^{21}} = 0.53 \mu\text{m}$$

$$V = -\int E dx = \frac{1}{2} a (1.1 E_{sat}) + 1.05 E_{sat} \times w_i + E_{sat} \times b \times \frac{1}{2}$$

$$= 8 \times 10^5 \left[\frac{1.1 \times .39}{2} + 1.05 \times 9 + \frac{.53}{2} \right] \times 10^{-6}$$

$$= 7.94 \text{ V}$$

(6) b)

longest time is for holes to travel from bottom to top.

We can write $E(x)$ in the intrinsic region as $E(x) = E_{sat} (1.1 - .1x/9)$ for x in μm .

This does not reach the saturation field for hole velocity, so $V_d = E(x) \mu_{ph}$

$$dt = \frac{dx}{V(x)} = \frac{dx}{E_{sat} (1.1 - .011x) \mu_{ph}}$$

$$\begin{aligned} \Delta t &= \int dt = \frac{1}{8 \times 10^5 \times .05} \int_0^9 \frac{-dx}{1.1 - .011x} = +2.5 \times 10^{-5} \int_0^9 \frac{dx}{1.1 - .011x} \\ &= \frac{-2.5 \times 10^{-5}}{.011 \times 10^6} \ln \left(\frac{1}{1.1} \right) \quad \underbrace{9}_{x \text{ in } \mu m} \\ &= 0.22 \text{ ns} \end{aligned}$$

Check: if E_{sat} throughout, $V_h = 8 \times 10^5 \times .05 = 4 \times 10^4$

$$\Delta t = \frac{9 \times 10^{-6}}{4 \times 10^4} = .225 \text{ ns (slightly more).}$$

c) most photons are absorbed closer to the top, so we want to give the faster electrons the longest distance to travel to maximize the speed, this means the top layer should be biased negative, i.e. p-in, not n-i-p.