

B.ENG. AND M.ENG. EXAMINATIONS 2013

PART II Paper 3 : MATHEMATICS (ELECTRICAL ENGINEERING)

Date Thursday 30th May 2013 2.00 - 4.00 pm

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO.

Answer FOUR questions.

A mathematical formulae sheet is provided.

[Before starting, please make sure that the paper is complete; there should be 6 pages, with a total of SIX questions. Ask the invigilator for a replacement if your copy is faulty.]

1. (i) Find where the function below is stationary, characterize such points and sketch it.

$$F(x, y) = x^2 + 4x + y^2$$

- (ii) Define the gradient vector of a function $f(x, y)$ and explain its direction and magnitude.

A bead is a circular ellipsoid with volume $V = \frac{4}{3}\pi ab^2$ and $a = 10$, $b = 20$. I make a small error in fabricating the shape. I first measure a and find $\Delta a = 0.01$ (a small error in a). I then measure the volume of the ellipsoid and find it is unchanged. What can I conclude about my error in b , Δb ?

I construct a chain of beads out of alternating ellipsoidal and spherical beads (the chain has an even number of beads, $2N$). Each spherical bead has radius $r = 10$. Write out an expression for the volume of the chain in terms of N, a, b, r .

I make a small change $\Delta a, \Delta b, \Delta r$ which maximizes the change in volume of my chain (which still has $2N$ beads).

What ratios $\frac{\Delta b}{\Delta a}$ and $\frac{\Delta r}{\Delta a}$ did I use?

2. (i) Write out the Cauchy-Riemann equations. Provide a one or two line explanation of their meaning.

- (ii) Consider the map

$$w = \frac{1}{z - (1 + i)}$$

Show that it is analytic everywhere except at $z = 1 + i$.

Is this conformal?

Two straight lines intersect in the z -plane at $z = 10 + 10i$. What two things can be said, briefly, about this intersection under the map w ?

- (iii) What do the lines $y = 0$ and $x = 0$ in the z -plane become under the map w above? (It might help to find expressions for u, v and $u^2 + v^2$ which connect points $x + iy$ in the z -plane to points $u + iv$ in the w -plane.)

Provide a sketch showing the transformed versions of the locuses $x = 0$ and $y = 0$ in the w -plane.

Identify any points of intersection of these two locuses in the w -plane. What is the x, y co-ordinate in the z -plane in the limit as these intersection points are approached in the w -plane?

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3. (i) Evaluate

$$\oint_C \frac{z^3}{(z-2)^3} dz ,$$

where the closed contour C is a counter clockwise unit circle in the z -plane with centre at $z = 2$.

What is this integral when the contour C is instead specified as the boundary of a square with the same centre?

- (ii) By considering a unit circle contour in the z -plane, and a suitable substitution for $\sin \theta$, evaluate the integral below:

$$\int_0^{2\pi} \frac{1}{\sin \theta + i} d\theta .$$

- (iii) Write down the residue theorem for

- (a) a contour containing a single simple pole,
- (b) a contour containing two simple poles.

Explain very briefly, through a diagram of an appropriate contour or otherwise, how result (b) can be obtained from result (a).

Note: The residue of a complex function $f(z)$ at a pole $z = a$ of multiplicity m is given by

$$\lim_{z \rightarrow a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{ (z-a)^m f(z) \} .$$

4. (i) Provide a simple sketch of

$$\frac{1}{(a^2 + t^2)^2}$$

as a function of t with a being a positive constant.

Write out an expression for its Fourier Transform (using the frequency space variable ω).

Write out Jordan's Lemma.

Explain how this can be used to express integrals of the form

$$\int_{-\infty}^{\infty} e^{imx} F(x) dx \quad (m \geq 0)$$

in terms of a contour integral.

- (ii) Using the above and assuming $\omega < 0$ find the Fourier Transform of $\frac{1}{(a^2 + t^2)^2}$, $a > 0$.

Briefly explain how you would solve this for $\omega > 0$.

Note: The residue of a complex function $f(z)$ at a pole $z = a$ of multiplicity m is given by

$$\lim_{z \rightarrow a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{(z-a)^m f(z)\} .$$

PLEASE TURN OVER

5. (i) If $\bar{f}(\omega)$ is the Fourier Transform of $f(t)$ prove Parseval's equality

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\bar{f}(\omega)|^2 d\omega$$

You might need the identity

$$\int_{-\infty}^{\infty} e^{\pm i\Omega t} dt = 2\pi\delta(\Omega)$$

- (ii) Find the Fourier transform of $e^{-|t|}$
- (iii) Find the Fourier Transform of $H(t)e^{-bt}$ where $H(t)$ is the Heaviside step function $H(t) = 1$ for $t > 0$ and $H(t) = 0$ for $t < 0$ and where $b > 0$.
- (iv) Using the Fourier Convolution theorem write down the Fourier Transform of

$$\int_{-\infty}^{\infty} e^{-|t'|} H(t-t') e^{-b(t-t')} dt' \quad (1)$$

where $b > 0$.

- (v) By performing the integral (1) directly, while carefully considering the cases $t > 0$ and $t < 0$, show that

$$\int_{-\infty}^{\infty} e^{-|t'|} H(t-t') e^{-b(t-t')} dt' = \begin{cases} \frac{(b+1)e^{-t} - 2e^{-bt}}{b^2 - 1} & t > 0 \\ \frac{e^t}{1+b} & t < 0 \end{cases} \quad (2)$$

- (vi) Find the Fourier Transform of the right hand side of equation (2). (You might like to use this to check your answer to (iv)).

PLEASE TURN OVER

6. (i) Given that $\bar{f}(s) = \mathcal{L}\{f(t)\}$ is the Laplace transform of $f(t)$, prove that when a is a real constant

$$\mathcal{L}\{e^{at}f(t)\} = \bar{f}(s-a) \quad \text{Re}(s) > a.$$

- (ii) Find the inverse Laplace Transform of

$$\bar{f}(s) = \frac{4}{s(s-4)}$$

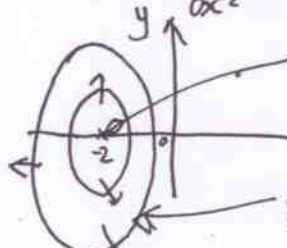
- (iii) Show that the Laplace Transforms of $\sin \omega t$ and $\cos \omega t$ are $\frac{\omega}{s^2 + \omega^2}$, $s > 0$ and $\frac{s}{s^2 + \omega^2}$, $s > 0$ respectively.

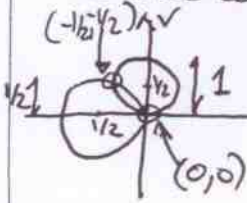

Use the Laplace Convolution theorem to solve the equation below for $x(t)$:

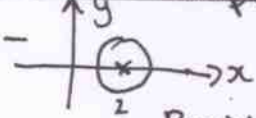

$$\frac{d^2x}{dt^2} + 4x = \cos 2t \quad \text{when } x(t=0) = 0 \quad \text{and} \quad \frac{dx}{dt}(t=0) = 0$$

Recall the identity $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

END OF PAPER

	EXAMINATION QUESTIONS/SOLUTIONS 2012-13	Course <u>EE3</u>
Question 1	TOPIC Functions of multiple variables	Marks & seen/unseen
Parts	<p> $F(x, y) = x^2 + 4x + y^2$ $\frac{\partial f}{\partial x} = 2x + 4$ $\frac{\partial^2 f}{\partial x^2} = 2$ $\frac{\partial^2 f}{\partial x \partial y} = 0$ stationary at $x = -2$ $y = 0$ $\frac{\partial f}{\partial y} = 2y$ $\frac{\partial^2 f}{\partial y^2} = 2$ at $(-2, 0)$ $\frac{\partial^2 f}{\partial x^2} > 0$; $(\frac{\partial^2 f}{\partial x \partial y})^2 - \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} = 0 - 4 < 0 \Rightarrow$ minimum. </p> 	<p>4.5</p> <p>4</p> <p>seen</p> <p>4</p> <p>1</p> <p>6</p>
	<p> ii) - $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})$. It points in the direction in the (x, y)-plane (which yields the greatest positive change in f). [or accept variants]. - its magnitude is the rate of change of f in the direction specified above. </p> <p> $V = \frac{4}{3}ab^2\pi \Rightarrow \Delta V = 4b^2\Delta a + 8ab\Delta b$ $a=10, b=20$ if $\Delta V=0, \Delta a=0.01 \Rightarrow 0 = [4 \times 20^2 \times 0.01 + 8 \times 10 \times 20 \times \Delta b] \frac{\pi}{3}$ $\Delta b = -0.005$ </p> <p> $V_{\text{chan}} = N (\frac{4}{3}ab^2\pi + \frac{4}{3}\pi r^3)$ $\Delta V = N [\frac{4}{3}b^2\Delta a + 8ab\Delta b + 4\pi r^2\Delta r]$ $\nabla V = \begin{pmatrix} N4b^2\pi/3 \\ N8ab\pi/3 \\ N4\pi r^2 \end{pmatrix}$ so $\frac{\Delta b}{\Delta a} = \frac{2a}{b} = 2$ $\frac{\Delta r}{\Delta a} = \frac{3r^2}{b^2} = \frac{3 \times 100}{400} = \frac{3}{4}$ </p>	
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	EXAMINATION QUESTIONS/SOLUTIONS 2012-13	Course EE 3
Question 2	TOPIC Complex Variables I	Marks & seen/unseen
Parts	<p>$u_x = u_y$; $u_y = -v_x$</p> <p>- They represent the constraint that $\lim_{\Delta z \rightarrow 0} \frac{\Delta f(z)}{\Delta z}$ should be independent of the direction in which $\Delta z \rightarrow 0$. Accept $\frac{\partial f(z)}{\partial \bar{z}} = 0$ or any reasonable alternative.</p> <p>- Check satisfies CR-equations. $w = \frac{1}{(x-1) + i(y-1)} = \frac{(x-1) - i(y-1)}{(x-1)^2 + (y-1)^2} = u + iv$</p> <p>$u_x = \frac{1}{(x-1)^2 + (y-1)^2} + \frac{-2(x-1)}{((x-1)^2 + (y-1)^2)^2}$; $u_y = \frac{-2(y-1)(x-1)}{((x-1)^2 + (y-1)^2)^2}$</p> <p>$v_y = \frac{-1}{(x-1)^2 + (y-1)^2} + \frac{2(y-1)}{((x-1)^2 + (y-1)^2)^2}$; $v_x = \frac{(y-1) \cdot 2 \cdot (x-1)}{((x-1)^2 + (y-1)^2)^2}$</p> <p>$u_y = -v_x$ clearly.</p> <p>$u_x = [(x-1)^2 + (y-1)^2 - 2(x-1)] / g(x,y)$; $v_y = [-x-1^2 - (y-1)^2 + 2(y-1)] / g(x,y)$ $g(x,y) = ((x-1)^2 + (y-1)^2)^2$</p> <p>So analytic everywhere save at $z = 1+i$. \Rightarrow conformal where it is analytic. we note $f'(z) = 0$ occurs nowhere.</p> <p>- The angle of intersection is preserved and so too is the ordering of the lines (except equivalent ordering remarks). $\frac{1}{2} \rightarrow \frac{1}{2}$</p> <p>This works because the map is conformal at this point.</p> <p>- $u^2 + v^2 = \frac{1}{(x-1)^2 + (y-1)^2}$. When $y=0$ $v = \frac{1}{(x-1)^2 + 1}$ $u^2 + v^2 = v$ $\Rightarrow u^2 + (v - v/2)^2 = 1/4 \Rightarrow$ circle centre $(0, 1/2)$ radius $1/2$.</p> <p>when $x=0$ $u = \frac{-1}{(y-1)^2 + 1}$; $u^2 + v^2 = -u \Rightarrow (u + 1/2)^2 + v^2 = 1/4$ \Rightarrow circle centre $(-1/2, 0)$ radius $1/2$</p> <p>$(-1/2, 1/2)$ meet at right angles at both points. $(-1/2, -1/2) \rightarrow (0,0)$ in z-plane. $(0,0) \rightarrow$ any mention of infinity scores.</p> 	<p>2</p> <p>2</p> <p>3 4</p> <p>2</p> <p>5</p> <p>5</p>
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	EXAMINATION QUESTIONS/SOLUTIONS 2012-13	Course EE 3
Question 3	TOPIC Complex Variables II	Marks & seen/unseen
Parts	<p>i)  Pole of multiplicity 3 at $z=2$. Pole is contained in contour. Residue at pole is $\lim_{z \rightarrow 2} \frac{1}{2!} \frac{d^2}{dz^2} \frac{(z-2)^3 z^3}{(z-2)^3}$ $= \frac{1}{2} \times 3 \times 2 \times 2 = 6$.</p> <p>By the residue thm the value of integral is $12\pi i$. Pole still enclosed and no other poles \Rightarrow answer unchanged</p> <p>ii) Consider contour $z=e^{i\theta}$ $dz=ie^{i\theta} d\theta = iz d\theta$ noting that $\sinh\theta = (z-z')/2i$ consider integral $\oint \frac{dz}{((\frac{z-z'}{2i})+i)iz} = \oint \frac{2}{z^2-2z+1} dz$ $\frac{z}{(z-1+i\sqrt{2})(z-1-i\sqrt{2})}$ $= \oint \frac{2}{(z-(1+i\sqrt{2}))(z-(1-i\sqrt{2}))} dz$ <p>Note that only pole at $z=1-i\sqrt{2}$ is contained in contour and is a simple pole.</p> <p>\Rightarrow residue at $z=1-i\sqrt{2}$ $\lim_{z \rightarrow 1-i\sqrt{2}} \left[\frac{2}{z-(1+i\sqrt{2})} \right] = \frac{2}{1-i\sqrt{2}-1-i\sqrt{2}} = -\frac{1}{\sqrt{2}}$ <p>By the residue theorem, which is $2\pi i \times (\text{sum of residues enclosed})$ integral is $-\sqrt{2}\pi i$. Because of the equivalence of the contour integral and the original we this is the desired answer.</p> <p>iii) a) $2\pi i \times$ value of residue at the simple pole b) $2\pi i \times$ the sum of the two residues at each of the simple poles.</p> <p>- Accept any contour/schematic or argument using cuts e.g.</p>  </p></p>	<p>5</p> <p>2</p> <p>10</p> <p>1</p> <p>2</p>
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EXAMINATION QUESTIONS/SOLUTIONS 2012-13

Course

EE 3

Question

4

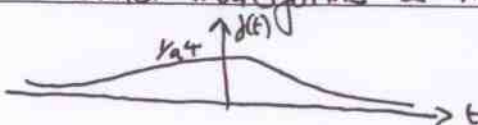
TOPIC

Fourier Transforms 1-A

Marks &

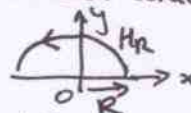
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Parts

-  symmetrical and decaying away at $\pm\infty$.

$$- \int_{-\infty}^{\infty} e^{-i\omega t} \frac{1}{(a^2 + t^2)^2} dt (*)$$

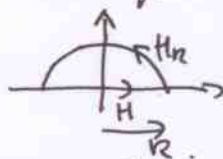
- If the only singularities of $F(z)$ are poles then on a semicircular contour of radius R as drawn below



$$\text{then } \lim_{R \rightarrow \infty} \int_{H_R} e^{imz} F(z) dz = 0$$

provided $m > 0$ and $|F(z)| \rightarrow 0$ as $R \rightarrow \infty$. If $m = 0$ faster convergence to zero is required for $F(z)$. Or equivalent.

- consider the contour C_R



$$\text{then } \lim_{R \rightarrow \infty} \oint_{C_R} e^{imz} F(z) dz = 2\pi i \times (\text{sum of residues enclosed by above contour})$$

$$= \lim_{R \rightarrow \infty} \left[\int_H e^{imz} F(z) dz + \int_{H_R} e^{imz} F(z) dz \right]$$

where if J's Lemma applies $\textcircled{2} \rightarrow 0$ and we note that $\textcircled{1}$ becomes $\int_{-\infty}^{\infty} e^{imx} F(x) dx$ since $z = x$ along H .

$$\Rightarrow \int_{-\infty}^{\infty} e^{imx} F(x) dx = 2\pi i \times (\text{sum of residues enclosed}).$$

- Suppose $F(z) = \frac{1}{(a^2 + z^2)^2}$. we note that $m > 0$ since $-i\omega > 0$ for $(*)$ above and that $F(z)$ decays away sufficiently fast. So can use J's Lemma.


$$\text{so } \lim_{R \rightarrow \infty} \oint_{C_R} e^{-i\omega z} \frac{1}{(a^2 + z^2)^2} dz \quad \text{using the contour above}$$

$$= \int_{-\infty}^{\infty} e^{-i\omega t} \frac{1}{(a^2 + t^2)^2} dt + \lim_{R \rightarrow \infty} \int_{H_R} = 2\pi i \times \text{sum of residues enclosed}$$

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	EXAMINATION QUESTIONS/SOLUTIONS 2012-13	Course EE 3
Question 4	TOPIC Fourier Transforms 1-B	Marks & seen/unseen
Parts	<p>Residues $z^2 = -a^2$ $z = \pm ai$</p> <p>Two double poles at ai and $-ai$ ($a > 0$) \Rightarrow one pole at ai in upper half plane so</p> $\lim_{z \rightarrow ai} \frac{1}{1!} \frac{d}{dz} \left[\frac{e^{-i\omega z} (z - ai)^2}{(z - ai)^2 (z + ai)^2} \right]$ $= \lim_{z \rightarrow ai} \left[\frac{-i\omega e^{-i\omega z}}{(z + ai)^2} + \frac{e^{-i\omega z} \cdot -2}{(z + ai)^3} \right]$ $= \frac{-2a i \omega e^{+a\omega} (a\omega - 1)}{4a^3}$ <p>so $\int_{-\infty}^{\infty} e^{-i\omega t} \frac{1}{(a^2 + t^2)^2} dt = \frac{-\pi e^{+a\omega} (a\omega - 1)}{2a^3} \quad \omega < 0$</p> <p>- For $\omega > 0$ consider a corresponding semicircular contour in the lower half plane: but no</p> <div style="display: flex; align-items: center; margin-top: 20px;">  <p>some indication of clockwise direction.</p> </div>	
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	EXAMINATION QUESTIONS/SOLUTIONS 2012-13	Course EE 3
Question 5	TOPIC Fourier Transforms 2	Marks & seen/unseen
Parts	<p>i) $\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \bar{f}(w') e^{i w' t} dw' \right] \left[\int_{-\infty}^{\infty} \bar{f}^*(w'') e^{-i w'' t} dw'' \right] dt = \int_{-\infty}^{\infty} f(t) ^2 dt$ 3</p> <p>$= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \bar{f}(w') \int_{-\infty}^{\infty} \bar{f}^*(w'') \int_{-\infty}^{\infty} e^{i(w'-w'')t} dt dw'' dw'$</p> <p>$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{f}(w') ^2 dw'$ $\frac{1}{2\pi} \delta(w'-w'')$</p> <p>ii) $\int_{-\infty}^{\infty} e^{-(i\omega+1)t} dt + \int_{-\infty}^0 e^{-(i\omega-1)t} dt = \left[\frac{e^{-(i\omega+1)t}}{-(i\omega+1)} \right]_0^{\infty} + \left[\frac{e^{-(i\omega-1)t}}{-(i\omega-1)} \right]_0^0$ 3</p> <p>$= \frac{1}{i\omega+1} + \frac{1}{1-i\omega} = \frac{2}{4\omega^2}$</p> <p>iii) $\int_0^{\infty} e^{-bt} e^{-i\omega t} dt = \left[\frac{e^{-(i\omega+b)t}}{-(i\omega+b)} \right]_0^{\infty} = \frac{1}{b+i\omega}$ 3</p> <p>iv) $FT[f(t) * g(t)] = \bar{f}(w) \bar{g}(w)$ so Eq(1) becomes $\frac{2}{(1+\omega^2)(b+i\omega)}$ under transformation. 2</p> <p>v) consider first $t > 0$ $H(t-t') = 0$ $t-t' < 0$, $t < t'$ Integral becomes $\int_{-\infty}^0 e^{- t' } e^{-b(t-t')} dt' + \int_0^t e^{- t' } e^{-b(t-t')} dt'$ $= e^{-bt} \left(\left[\frac{e^{(1+b)t'}}{1+b} \right]_{-\infty}^0 + \left[\frac{e^{(b-1)t'}}{(b-1)} \right]_0^t \right)$ $= \frac{1}{b^2-1} ((1+b)e^{-t} - 2e^{-bt})$ now consider $t < 0$. $\int_{-\infty}^t e^{t'} e^{-b(t-t')} dt' = e^{-bt} \left[\frac{e^{t'(1+b)}}{1+b} \right]_{-\infty}^t = \frac{e^t}{1+b}$</p> <p>vi) consider FT of two parts. $t < 0$. $\int_{-\infty}^0 e^{-i\omega t} \frac{e^t}{1+b} dt = \frac{1}{(1+b)(1-i\omega)}$ ①</p> <p>$t > 0$. $\int_0^{\infty} e^{-i\omega t} [(b+1)e^{-t} - 2e^{-bt}] dt = \frac{1}{(b^2-1)} \left[\frac{e^{-(i\omega+1)t}}{-(i\omega+1)} \right]_0^{\infty} + \left[\frac{-2e^{-(i\omega+b)t}}{-(i\omega+b)} \right]_0^{\infty}$ 3</p> <p>$= \frac{1}{b^2-1} \left[\frac{b+1}{i\omega+1} + \frac{2}{i\omega+b} \right]$ ②</p> <p>FT of (2) is ①+②. Accept this as an answer</p>	
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	EXAMINATION QUESTIONS/SOLUTIONS 2012-13		Course
Question	TOPIC		Marks & seen/unseen
Parts	<p>check step (not required)</p> $\textcircled{1} + \textcircled{2} = \frac{2b - 2iw}{(b^2 - 1)(w^2 + 1)} \times \frac{[(b-1)(1+iw) + (b+1)(1-iw)]}{(iw+b)(b^2-1)} = \frac{-2}{(iw+b)(b^2-1)}$ $= \frac{(2b-2iw)(b+iw) - 2(w^2+1)}{(iw+b)(b^2-1)(w^2+1)} = \frac{2(b^2-1)}{(iw+b)(w^2+1)(b^2-1)}$ <p>This checking step is not required for full marks.</p>		
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EXAMINATION QUESTIONS/SOLUTIONS 2012-13

Course

EE 3

Question

6

TOPIC

Laplace Transforms

Marks &

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Parts

ii)

$$\bar{f}(s) = \frac{A}{s} + \frac{B}{s-4} \quad A=-B, A=-1.$$

$$= \frac{1}{s-4} - \frac{1}{s}$$

5

By shift theorem $e^{at} f(t) \Rightarrow F(s-a)$ and also

$$1 \Rightarrow 1/s$$

$$\text{so } f(t) = e^{4t} - 1.$$

$$\text{iii)} - \int_0^{\infty} e^{-st} e^{i\omega t} dt = \left[\frac{e^{(i\omega-s)t}}{i\omega-s} \right]_0^{\infty} = \frac{1}{s-i\omega} = \frac{s+i\omega}{s^2+\omega^2}$$

5

$$\text{since } e^{i\omega t} = \cos \omega t + i \sin \omega t.$$

$$\text{it follows that } \cos \omega t \Rightarrow \frac{s}{s^2+\omega^2}, s>0$$

$$\sin \omega t \Rightarrow \frac{\omega}{s^2+\omega^2}, s>0$$

$$- s^2 \bar{x}(s) - 1 + 4 \bar{x}(s) = \frac{s}{s^2+4}$$

$$\bar{x}(s) = \frac{1}{s^2+4} + \frac{s}{(s^2+4)^2}$$

to 8

L convolution theorem $f * g \Rightarrow F(s)G(s)$

$$x(t) = \frac{1}{2} \sin 2\omega t + \int_0^t \frac{1}{2} \sin 2\omega t' \cos 2\omega(t-t') dt'$$

using double angle formula

$$\sin 2\omega t' \cos 2\omega(t-t') = \frac{1}{2} [\sin 2\omega t + \sin (4\omega t' - 2\omega t)]$$

$$x(t) = \frac{1}{2} \sin 2\omega t + \frac{t}{2} \sin 2\omega t + 0.$$

$$\text{ii)} \mathcal{L}[e^{at} f(t)] = \int_0^{\infty} e^{-(s-a)t} f(t) dt = \bar{f}(s-a)$$

only valid if $\text{Re}(s) > a$

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