Model answer to Q 1(a): Bookwork

Define the general frequency ranges for the following and briefly give an example of the kind of hardware technologies used to implement integrated circuits at these frequencies and state at least one well-known commercial application.

i) Far- and Mid-infrared.

Far-infrared is from 300 GHz to 30 THz.

Mid-infrared is from 30 THz to 120 THz.

Within these parts of the frequency spectrum, a combination of thermodynamics (e.g. LEDs and pyro-electric infrared detectors) and optics techniques are used to implement circuits used in wireless remote controls and motion sensor applications.

[2]

ii) Millimetre-wave.

30 GHz to 300 GHz

Transmission line techniques are used to implement circuits used for autonomous cruise control (76.5 GHz) and radiometric imaging (35 to 250 GHz) applications.

[2]

iii) Microwave.

1 GHz to 30 GHz.

A mixture of all-transistor, lumped element and transmission line techniques are used to make circuits for mobile phone and WiFi applications.

[2]

iv) Terahertz.

The THz band ranges from 0.3 to 10 THz (although some may say it ranges from 0.1 to 3 THz). Within these bands, a mixture of transmission line and quasi-optical techniques are used to implement circuits for imaging and spectroscopy applications.

[2]

Model answer to Q 1(b): Bookwork

b) From 1(a), which part of the electromagnetic spectrum is capable of detecting metals, plastic, liquids, gels, ceramics and narcotics concealed beneath a person's clothing? State the reasons that restrict both the lower and upper frequency limits for this application.

The millimetre-wave part of the frequency spectrum is ideally suited for these applications. The lower frequency defines the spatial resolution of the image and the upper frequency limits the depth of penetration. Ideally, the larger the bandwidth the faster the integration time for the image pixels to be generated.

[3]

Model answer to Q 1(c): Bookwork

c) From 1(a), which part of the electromagnetic spectrum can be used for wireless energy transfer from space to ground, via a satellite link? State the reasons why this technology has a greater sun-to-earth energy transfer efficiency.

The microwave part of the frequency spectrum is ideally suitable for this application. There are proposals for a Microwave-based Space Solar Power System (M-SSPS) for wireless energy transmission, whereby a satellite having large solar panel arrays generates dc power and most of this energy is transmitted to earth via a microwave link at 5.8 GHz. With conventional solar arrays on earth a significant part of the solar energy is lost on its way through the atmosphere by the effects of reflection and absorption, which is not found at microwave frequencies.

[3]

Model answer to Q 1(d): Bookwork

From 1(a), which part of the electromagnetic spectrum is used in the Active Denial System (also known as the Heat Ray) technology? State the reasons for using this part of the electromagnetic spectrum, in terms of effectiveness, safety and range.

The millimetre-wave part of the frequency spectrum is ideally suited for this application. This is a non-lethal, directed-energy weapon developed by the U.S. military and police. It is a high power millimetre-wave transmitter, primarily used for crowd control, which can operate up to a distance of 1 km. The millimetre waves excite the water and fat molecules on the skin, instantly heating it and causing intense pain. If the frequency is too low then the penetration depth is too large, which could lead to biological harm to humans. If the frequency s too high then it becomes too expensive to generate the very high power levels needed to inflict pain at long distances.

[3]

Model answer to Q 1(e): Computed Example

e) In general, given a metal-based guided-wave structure that is uniformly surrounded by a dielectric media. Calculate the appropriate dielectric property that would allow it to operate in its dominant mode at 10 GHz, 100 GHz, 1 THz and 10 THz. *{Hint, the value calculated is different for each frequency}.*

The dielectric constant of the dielectric medium would have to change. Assuming that the dominant mode is directly scalable with wavelength, which is inversely proportional to the square root of the dielectric constant and that at 10 THz the dielectric is air (for lowest loss operation and feature sizes that are easiest to manufacture): The dielectric constants εr are:

$\varepsilon r(10\text{ THz}) = 1$; $\varepsilon r(1\text{ THz}) = \text{sqrt}(10) \sim 3$; $\varepsilon r(100\text{ GHz}) = 10$; $\varepsilon r(10\text{ GHz}) \sim 32$

[3]

Model answer to Q 2(a): Computed Example

2. At 2.45 GHz, human skin has a dielectric constant of 44 and a conductivity of 1.85 [S/m]. At the same frequency and with the help of equation 2.1 (all variables having their usual meaning), calculate the following for inside the skin:

i) Wavelength.

$$\varepsilon_r' = 44 \; and \; \lambda = \frac{\lambda_o}{\sqrt{\varepsilon_r'}} = 18.46 \; mm$$

[2]

ii) Imaginary part of the relative permittivity.

$$\varepsilon_r'' = \frac{\sigma'}{\omega \varepsilon_o} = 13.57$$

[3]

iii) Loss tangent.

$$tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} = 0.308$$

[2]

iv) Quality factor.

$$Qm = \frac{1}{tan\delta} = 3.247$$

[2]

v) Attenuation constant.

$\alpha = Re\{\gamma\} = Re\{j\beta_o\sqrt{\varepsilon_r'(1 - jtan\delta)}\}$=51.82 [Np/m]

[2]

vi) Phase constant.

$\beta = Im\{\gamma\} = Im\{j\beta_o\sqrt{\varepsilon_r'(1 - jtan\delta)}\}$=344.29 [rad/m]

[2]

vii) Power attenuation in dB per wavelength.

$$Power \;\; Attenuatio\,n = e^{-2(\alpha l)}$$
$$\alpha\lambda = 0.9566[Np/\lambda]$$
$$Power \;\; Attenuatio\,n = \alpha\lambda(20\log_{10} e) = 8.686\alpha\lambda = 8.31\,[dB/\lambda]$$

[3]

viii) Intrinsic impedance.

$$\eta = \frac{j\omega\mu}{\gamma} = 8.3 - j54.9\Omega$$

[4]

Model answer to Q 3(a): Computed Example

3. A straight piece of gold bond wire has a circular cross section, with a diameter of d = 25 microns and resistivity of 22.14 nΩ/mm. The manufacturer of this bond wire quotes the inductance as L ~ 600 pH/mm and resistances of R(0.1 GHz) ~ 45 mΩ/mm and R(1 GHz) ~ 130 mΩ/mm. No other information is given by the manufacturer.

a) Based on this limited amount of information from the manufacturer.

i) Give a crude assumption about the frequency scaling of both the inductance and resistance.

The inductance does not scale very much with, but there is an 8.3:1 scaling in frequency for the resistance. This is close to the 10:1 scaling found with the classical skin-effect model.

[2]

ii) Estimate the resistance at 3.6 GHz, based on classical skin-effect scaling laws in terms of frequency.

Based on the findings in 3(a)(i), it can be assumed that the classical skin-effect scaling laws apply and that R(3.6 GHz) ~ 247 mΩ/mm

[2]

b) If the classical skin depth is much less than the diameter of the bond wire, simple analytical expressions for L and R can be derived in terms of the respective constitutive terms μ and σ (having their usual meaning), skin depth δ and geometric variables of length l and diameter d.

i) Give the values of the constitutive terms and use these to calculate the classical skin-effect skin depth and associated surface impedance.

$$\mu = \mu_o \; and \; \sigma = \frac{1}{\rho} = 4.517 \; x \; 10^7 [S/m]$$

$$\delta = \sqrt{\frac{2}{\omega \mu_o \sigma}} = 1.248 \; microns \; and \; Zs = \frac{(1+j)}{\sigma\delta} = 0.0177\;(1+j)[\Omega]$$

[3]

ii) Write down the simplified expression for the bond wire inductance and calculate its value at 3.6 GHz. How does this value compare to the manufacturers quoted value?

$L = \frac{\mu_o \delta}{2}\left(\frac{l}{2\pi d/2}\right) = 10\; pH/mm$ This is about a factor of 60 too low!

[3]

iii) Write down the simplified expression for the bond wire resistance and calculate its value at 3.6 GHz. How does this value compare to the value estimated in 3(a)(ii)?

$R = \frac{1}{\sigma\delta}\left(\frac{l}{2\pi d/2}\right) = 226\; m\Omega/mm$ This is about right!

[3]

iv) Define the relationship between the bond wire resistance and inductance and how these relate to the surface impedance.

$$R = \omega L = Rs\left(\frac{l}{2\pi d/2}\right)$$

[2]

c) A more accurate expression for the free space inductance is given by equation 3.1, which takes into account the curvature of the bond wire geometry with skin depth.

i) Using equation 3.1, calculate the bond wire inductance and comment on how this relates to the manufacturers quoted value.

$$L = \frac{\mu_o l}{2\pi}\left(5.075 + 0.0125 - 1 + \frac{\tanh(0.2)}{4}\right) = 827\ pH/mm$$

This is about right!

[3]

ii) In terms of physical dimensions, give the two conditions required to reduce equation 3.1 to the simplified expression given in 3(b)(ii).

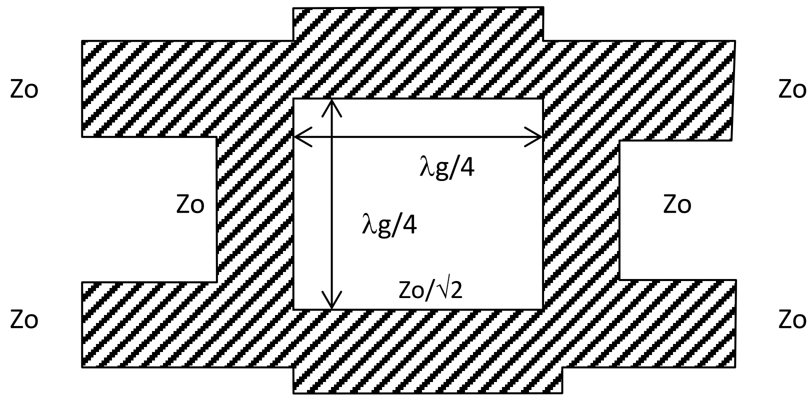
It can be seen from equation 3.1 that when the diameter of the bond wire is much greater than the length the square brackets reduces to the value of C, which also reduces to the value of skin depth divided by the diameter. This in turn gives the expression used in 3(b)(ii). Therefore the two conditions are that $d >> l$ and $\delta << d$.

[2]

Model answer to Q 4(a): Bookwork

a) Describe, with the aid of a diagram, a conventional branch-line coupler having a distributed-element implementation, indicating lengths and impedances. List some of the main characteristics for this coupler.

90° 3dB Directional Coupler (Branch-line Coupler)

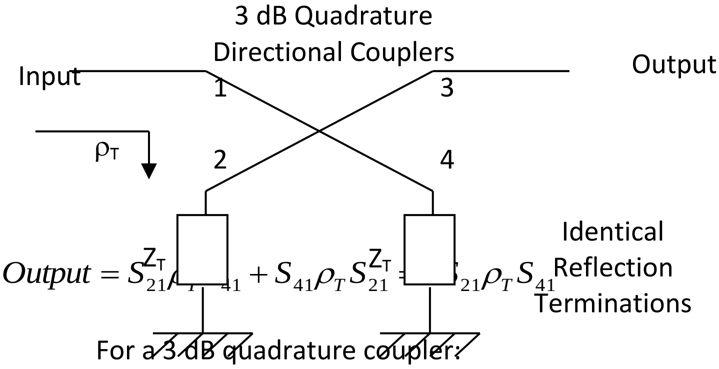


- Works on the interference principle, therefore, narrow fractional bandwidth (15% maximum)
- No bond-wires or isolation resistors required
- Wider tracks make it easier to fabricate and is, therefore, good for lower loss and higher power applications
- Simple design but large
- Meandered lines are possible for lower frequency applications

[5]

Model answer to Q 4(b): New Derivation

b) With the aid of a diagram and simple analysis, explain how a lossless single-stage reflection-topology works with the coupler given in 4(a). *{Hint, assume the input is unity and determine the output}.*



$$Output = S_{21}\rho_T S_{41} + S_{41}\rho_T S_{21} = 2S_{21}\rho_T S_{41}$$

For a 3 dB quadrature coupler:

$$S_{41} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{2}} \quad and \quad S_{21} = \frac{-1}{\sqrt{2}} e^{-j0}$$

$$\therefore S_{21}|_{OVERALL} = 2\frac{1}{\sqrt{2}} e^{-j\frac{\pi}{2}} \rho_T \frac{-1}{\sqrt{2}} = -\rho_T e^{-j\frac{\pi}{2}} = j\rho_T$$

The reflection topology transforms a reflection coefficient into a transmission coefficient with a 90 degrees phase offset.

[5]

Model answer to Q 4(c) New Derivation

c) Explain how an ideal reflection-type attenuator can be implemented and extend the analysis given in 4(b) for this application. *{Hint, first determine the voltage-wave reflection coefficient for the reflection termination}.*

Here, the reflection coefficient is implemented with a PIN diode or cold-FET, to realise a variable resistance, R_T.

$$\rho_T(V) = \frac{R_T(V) - Zo}{R_T(V) + Zo} \equiv |\rho_T(V)|$$

$$\therefore S_{21}|_{OVERALL} = |\rho_T(V)| e^{-j\frac{\pi}{2}}$$

Therefore, the relative attenuation, $\Delta|S_{21}|_{OVERALL} = \Delta|\rho_T(V)|$

[5]

Model answer to Q 4(d) New Derivation

d) Explain how an ideal reflection-type phase shifter can be implemented and extend the analysis given in 4(b) for this application. *{Hint, first determine the voltage-wave reflection coefficient for the reflection termination}.*

Here, the reflection coefficient is implemented with a varactor diode, to realise a variable capacitance:

$$C_T = \frac{-1}{\omega X_T}.$$

$$\rho_T(V) = \frac{X_T(V) - Zo}{X_T(V) + Zo} \equiv 1.e^{j\angle\rho_T(V)}$$

$$\therefore S_{21}|_{OVERALL} = 1.e^{j\angle\rho_T(V) - j\frac{\pi}{2}}$$

Therefore, the relative phase shift, $\Delta\angle S_{21}|_{OVERALL} = \Delta\angle\rho_T(V)$

[5]

Model answer to Q 5(a): bookwork

a) State the boundary conditions for electric and magnetic fields at the interface between air and a perfect electrical conductor.

E-fields can only meet a perfect conductor at normal incidence, while the H-field must be perfectly parallel.

[2]

Model answer to Q 5(b): bookwork

b) Explain why a metal-pipe rectangular waveguide of internal cross-sectional dimensions *a* and *b* has a lowest frequency at which electromagnetic waves will not propagate. Derive an expression for the lowest mode cut-off frequency.

There must be *m*, *n* half wavelengths of fields across the *a*, *b* dimensions, respectively, so that the E-field components vanish at the guide walls.

The cut-off wavelength is given by:

$$k_c = \sqrt{k_x^2 + k_y^2} = \sqrt{\left(m\frac{\pi}{a}\right)^2 + \left(n\frac{\pi}{b}\right)^2} \equiv \frac{2\pi}{\lambda_c}$$

$$\therefore \lambda_c = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}}$$

For example, with the TE_{10} mode, *m = 1* and *n = 0*:

[4]

Model answer to Q 5(c): computed example

c) For a metal-pipe rectangular waveguide of internal cross-sectional dimensions 2.0 mm x 1.0 mm, determine the cut-off frequencies of the following modes: TE_{10}, TE_{01}, TE_{11}, TM_{11}, TE_{20}.

For $a = 0.002$ m and $b = 0.001$ m:

$$f_c\big|_{TE10} = \frac{c}{2a} = 5.9\ GHz$$

$$f_c\big|_{TE01} = \frac{c}{2b} = 11.8\ GHz$$

$$f_c\big|_{TE11} = f_c\big|_{TM11} = \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 13.2\ GHz$$

$$f_c\big|_{TE20} = 2 f_c\big|_{TE10} = 11.8\ GHz$$

[4]

Model answer to Q 5(d): bookwork

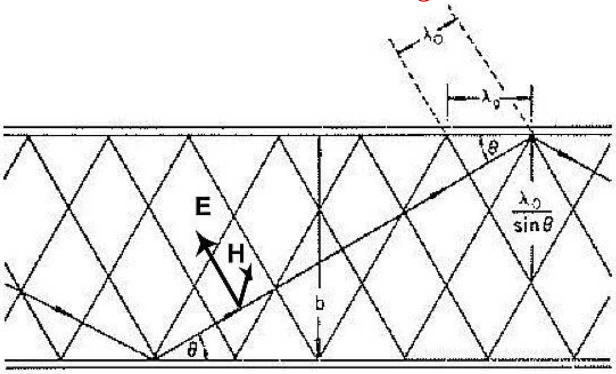
d) List which of these modes are degenerate and discuss mode conversion between degenerate modes.

TE_{11} and TM_{11} are the degenerate modes as they have different field patterns but share the same guided wavelength at any frequency (i.e. they must, therefore, share the same cut-off frequency). They are 'phase matched' over long distances and so mode conversion happens easily at discontinuities.

[2]

Model answer to Q 5(e): bookwork

e) Define the terms phase velocity and group velocity. Also derive formulas for them in terms of the guide mode cut-off frequency and signal frequency.



The above figure shows the side-view of a generic parallel-plate waveguide, supporting two plane-waves propagating between and reflecting off the wall plates. Propagation within the waveguide can only be maintained as long as $\theta < \pi/2$:

$\sin\theta = \frac{n\lambda_o/2}{b} = \frac{\lambda_o}{\lambda_c} < 1 \quad since \quad \lambda_c = \frac{2b}{n}$ i.e. the n^{th} mode cut-off wavelength (e.g. $n = 3$ above)

with λ_0 = free space wavelength and n = the mode index.

The guided wavelength is shown in the above figure, and is given by:

$$\cos\theta = \frac{n\lambda_o/2}{n\lambda_g/2} = \frac{\lambda_o}{\lambda_g} < 1$$

$$\therefore \lambda_g = \frac{\lambda_o}{\cos\theta}$$

$$but, \left(\sin^2\theta + \cos^2\theta\right) \equiv 1$$

$$\therefore \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{\lambda_o}{2b/n}\right)^2} = \sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}$$

$$\therefore \lambda_g = \frac{\lambda_o}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}} > \lambda_o$$

The guided-wave pattern travels at the phase velocity, $v_p = \frac{\omega}{\beta} = \frac{c}{\cos\theta} > c$, while the energy travels at the group velocity $v_g = \frac{\partial\omega}{\partial\beta} = c\cos\theta < c$. The product of these two velocities is $v_p v_g = c^2$.

[4]

<u>Model answer to Q 5(f): computed example</u>

f) Identify the centre of the band of frequencies in the waveguide where only one mode will propagate and calculate the group and phase velocity of the propagating mode at this band centre. Show that the product of your group and phase velocities is 9 x 10^{16} in SI units.

$$f_c\big|_{TE10} = 5.9\ GHz \quad and \quad f_c\big|_{TE20} = 11.8\ GHz$$

$$\therefore\ f\big|_{midband} = \frac{f_c\big|_{TE10} + f_c\big|_{TE20}}{2} = 8.85\ GHz$$

$$\cos\theta = 0.745$$

$$v_p\left(f\big|_{midband}\right) = 4.03x10^8\ \ m/s$$

$$v_g\left(f\big|_{midband}\right) = 2.24x10^8\ \ m/s$$

$$\therefore v_p\left(f\big|_{midband}\right)v_g\left(f\big|_{midband}\right) = 9x10^{16}\ \ m^2/s^2$$

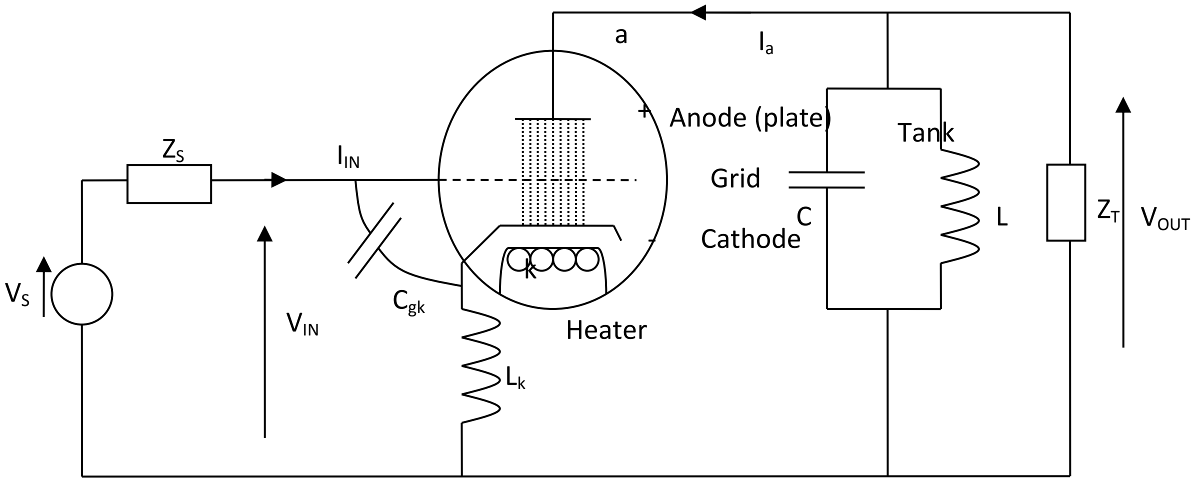
This is the speed of light in a vacuum squared.

[4]

Model answer to Q6(a): Bookwork

a) Explain the operation of an RF thermionic valve amplifier and draw its simplified equivalent circuit model. Compare and contrast the main features of this device with the modern day solid-state MESFET.

Triode Thermionic Valves (or Gridded Tubes)



- With the triode valve, a heater is required to heat the cathode electrode (which is negatively biased, with respect to ground). Within the vacuum or inert gas environment, electron that have absorbed enough thermal energy to overcome the work function of the cathode metal are emitted from the cathode and attracted to the positively biased plate electrode. The resulting flow of electrons can be modulated by controlling the grid-cathode potential. The GaAs MESFET is the modern day analog of the triode valve. With the MESFET, the gate-source potential controls the flow of electrons flowing from the source to drain electrodes. The triode value consumes a great deal of power for the heaters and required very large positive and negative potentials on the plate and cathode electrodes, respectively. The transconductance is generally low and, therefore, the GAIN-Bandwidth Product (GBP) will be low. In contrast, the MESFET has a higher transconductance and, therefore, higher GBP. Also, this device does not need to be heater or have large bias voltages.

[6]

Model answer to Q6(b): Bookwork

b) To increase the frequency of operation, explain how the valve needs to be modified. Explain why the performance of triode valves deteriorates as frequency increases into the microwave spectrum.

- As frequency increases, the distance between the cathode, grid and plate must decrease. This is because the cathode-plate electron transit time must be kept well below the period of the RF cycle, otherwise:
  - The grid and plate signals may not be 180° out-of-phase, which can lead to problems with feedback oscillators.
  - The grid can take power from the driving source, since the grid voltage has time to change during the flow of a particular group of electrons from the cathode to the plate.

As a result, at microwave frequencies, the internal dimensions are so small that:

  - The large parasitic capacitances could resonate with the lead inductances, within the band of operation.
  - Power levels are significantly reduced, since dielectric breakdown of dry air at one atmosphere is only ~ 3×10^6 V/m, and even much less in a vacuum.
  - Input impedance and, therefore, voltage gain are reduced.

[4]

Model answer to Q6(c): Bookwork and Computed Example

c) If the unwanted parasitics of a triode value consist of a grid-cathode capacitance of 20 pF and a cathode inductance of 10 nH, derive a general expression for the input resistance and calculate its value at 2 GHz, given a transconductance value of 2 mA/V. Comment on the significance of the input resistance in relation to the voltage gain of the amplifier. For the unwanted parasitic values given, what will the effect be on the voltage gain of the amplifier?

If only C_{gk} and L_k are considered,

$I_{OUT} = g_m V_{gk}$ and, therefore, $V_{IN} \cong V_{gk} + j\omega L_k I_{OUT} = V_{gk}\,(1 + j\omega L_k g_m)$

But, $V_{gk} = \dfrac{I_{IN}}{j\omega C_{gk}}$

Therefore, $Z_{IN} = \dfrac{V_{IN}}{I_{IN}} = \dfrac{1 + j\omega L_k gm}{j\omega C_{gk}}$

Therefore, $Y_{IN} = \dfrac{1}{Z_{IN}} = \dfrac{j\omega C_{gk}}{1 + j\omega L_k gm} \approx \omega^2 L_k C_{gk} g_m + j\omega C_{gk}$

Since, $(1 \pm jA)^{-1} \approx (1 \mp jA)$ if $|A| << 1$

$\therefore R_{IN} \cong \dfrac{1}{\omega^2 L_k C_{gk} g_m}$

As the voltage gain of the valve increase, due to an increased transconductance, the input resistance decreases.

* With the tank at resonance, voltage gain = $\dfrac{V_{OUT}}{V_{IN}} = -g_m R_T$

But, $V_{IN} = \dfrac{V_S Z_{IN}}{Z_S + Z_{IN}}$ and, therefore, $\dfrac{V_{OUT}}{V_S} = \dfrac{-g_m R_T}{Z_S / Z_{IN} + 1}$

When fo = 1 GHz, L_k = 15 nH, C_{gk} = 30 pF and g_m = 1 mA/V
Therefore, R_{IN} = 56.3 Ω, i.e. a near the driving source impedance of Z_S = 50 Ω and, therefore, the overall voltage gain may not have to be low.

[6]

Model answer to Q6(d): Bookwork and Computed example

c) If the unwanted parasitics of a triode value consist of a grid-cathode capacitance of 20 pF and a cathode inductance of 10 nH, derive a general expression for the input resistance and calculate its value at 2 GHz, given a transconductance value of 2 mA/V. Comment on the significance of the input resistance in relation to the voltage gain of the amplifier. For the unwanted parasitic values given, what will the effect be on the voltage gain of the amplifier?

Output Loaded-Quality factor, Q_{OUT} = $\dfrac{R_T}{|X_c|} = R_T \omega_0 C$

Therefore, output bandwidth, $\Delta\omega = \dfrac{\omega_0}{Q_{OUT}} = \dfrac{1}{R_T C}$

Therefore, for a matched amplifier, gain-bandwidth product, GBP $\cong g_m R_T \dfrac{\Delta\omega}{2\pi} = \dfrac{g_m}{2\pi C}$

For a bandwidth = 100 MHz it is impossible to meet a power gain = 30 dB with the valve amplifier because each stage would have a GBP = 31.8×10^6 and, therefore, this represents a 10 dB power attenuation.

[4]