

# QUESTION 1

SOLUTIONS -  
E1.6 Communications 1

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2008

$$a) i. E_{x_1} = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_0^1 \sin^2(2\pi t) dt =$$

$$= \frac{1}{2} - \frac{1}{2} \int_0^1 \cos(4\pi t) dt = \frac{1}{2}$$

ii.  $E_{x_2} = E_{x_1} = \frac{1}{2}$  (ENERGY IS NOT INFLUENCED  
BY A CHANGE OF SIGN OR  
A TIME-SHIFT)

$$\begin{aligned} iii. E_{(x_1+x_2)} &= \int_{-\infty}^{\infty} (x_1(t) + x_2(t))^2 dt = \int_{-\infty}^{\infty} x_1^2(t) dt + \\ &+ \int_{-\infty}^{\infty} x_2^2(t) dt + 2 \int_{-\infty}^{\infty} x_1(t) x_2(t) dt = \\ &= \frac{1}{2} + \frac{1}{2} + 2 \int_{-\infty}^{\infty} x_1(t) x_2(t) dt = 1 + 2 \int_{0.5}^1 \sin^2(2\pi t) dt, \\ &= 1 + 2 \left( \frac{1}{4} - \frac{1}{2} \int_{0.5}^1 \sin(4\pi t) dt \right) = 1 + \frac{1}{2} = \frac{3}{2} \end{aligned}$$

$$b) \quad \omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$$

i.  $x(t)$  IS EVEN  $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt = \frac{1}{4} \int_{-2}^2 x(t) dt = \frac{1}{2}$$

$$a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cos n\omega_0 t dt = \frac{1}{2} \int_{-2}^2 x(t) \cos n\omega_0 t dt =$$

$$= \int_0^2 \cos \frac{n\pi}{2} t dt = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

$$x(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos \frac{n\pi}{2} t$$

ii. THE FILTER REMOVES ALL THE HARMONIC WITH THE EXCEPTION OF THE FIRST ONE:

$$y(t) = a_0 + a_1 \cos \pi t/2 = \frac{1}{2} + \frac{2}{\pi} \cos \frac{\pi t}{2}$$

$$c) X(\omega) = \frac{1}{\omega^2} (e^{j\omega} - j\omega e^{j\omega} - 1)$$

$$Y(t) = x(t-1) + x(-t-1)$$

$$x(t-1) \Leftrightarrow X(\omega) e^{-j\omega}$$

$$x(-t) \Leftrightarrow X(-\omega)$$

THEREFORE

$$Y(\omega) = [X(\omega) + X(-\omega)] e^{-j\omega}$$

$$= \frac{2e^{-j\omega}}{\omega^2} (\cos \omega + \omega \sin \omega - 1)$$

d) i.

$$R_x(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \cos \omega_0 t \cos(\omega_0 t + \omega_0 \tau) dt =$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \cos^2 \omega_0 t \cos \omega_0 \tau dt =$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sin \omega_0 \tau \cos \omega_0 t \sin \omega_0 t dt =$$

$$= \cos \frac{\omega_0 \tau}{2}$$

$$R_x(\tau) = \cos \frac{\omega_0 \tau}{2} = \frac{\cos 100\tau}{2}$$

d) i.i.

$$R_x(\tau) \quad (=) \quad S_x(\omega)$$

WHERE  $S_x(\omega)$  IS THE POWER SPECTRAL DENSITY OF  $x(t)$ .

THUS

$$S_x(\omega) = \frac{\pi}{2} [\delta(\omega - 100) + \delta(\omega + 100)]$$

$$\begin{aligned} 2) \quad \varphi_{FM}(t) &= A \cos\left(\omega_c t + K_f \int_{-\infty}^t m_1(d) dd\right) \\ &= A \cos\left(\omega_c t + \int_0^t (a_1 d + a_0) dd\right) \\ &= A \cos\left(\omega_c t + \frac{a_1 t^2}{2} + a_0 t\right). \end{aligned}$$

$$\varphi_{PM} = A \cos(\omega_c t + K_p m_2(t)) = A \cos(\omega_c t + b_2 t^2 + b_1 t + b_0).$$

THE TWO OUTPUTS ARE THE SAME WHEN

$$b_2 = \frac{a_1}{2}, \quad b_1 = a_0, \quad b_0 = 0.$$

$$1) \quad Z_{in} = j 50 \tan(KL)$$

$Z_{in}$  is zero when  $\tan KL = 0$

But  $KL = \frac{\omega}{u} \cdot L \Rightarrow \frac{\omega}{u} \cdot L = \pi$

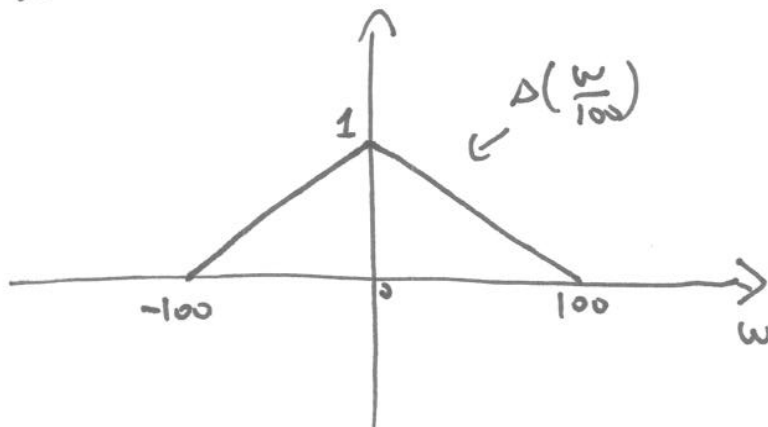
$$\frac{2\pi f_0}{u} \cdot L = \pi \quad f_0 = \frac{u}{2L} = 1 \text{ MHz}$$

## QUESTION 2

a) i.  $\frac{d^2}{2\pi} \text{sinc}^2\left(\frac{dt}{2}\right) \Leftrightarrow \Delta\left(\frac{\omega}{2}\right)$

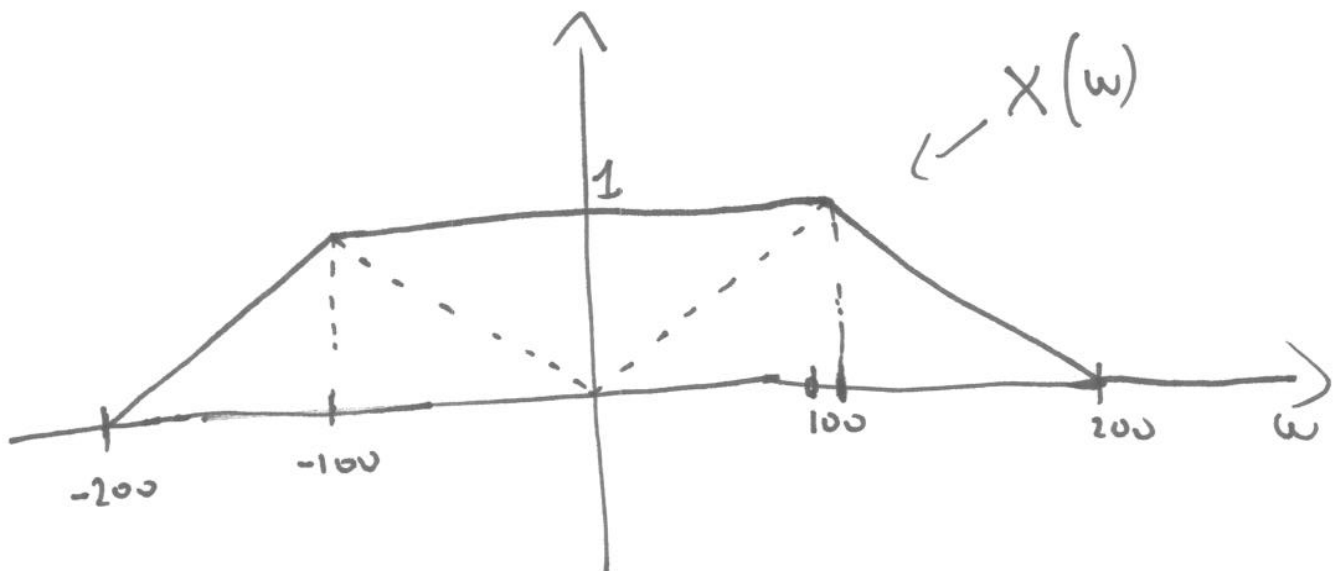
IN OUR CASE  $d=100$

$$\frac{5000}{\pi} \text{sinc}^2(50t) \Leftrightarrow \Delta\left(\frac{\omega}{100}\right)$$



ii.  $x(t) = \frac{5000}{\pi} \text{sinc}^2(50t) + \frac{10000}{\pi} \text{sinc}^2(50t) \cos 100t$

$$X(\omega) = \Delta\left(\frac{\omega}{100}\right) + \Delta\left(\frac{\omega-100}{100}\right) + \Delta\left(\frac{\omega+100}{100}\right)$$



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$$B_{FH} = 2 \left( \Delta f + B \right) = 2 \left( \frac{K_f \cancel{A_m}}{2\pi} + B \right)$$

$$B = \frac{200}{2\pi} \text{ Hz}$$

$$\cancel{A_m} = x(0) = \frac{5000}{\pi} + \frac{10000}{\pi} = \frac{15000}{\pi}$$

$$B_{FH} = 2 \left( \cancel{A} \cdot \frac{15000}{\cancel{\pi}} + \frac{200}{2\pi} \right) = \frac{34000}{2\pi} \text{ Hz}$$

b) i.  $\beta = \frac{\Delta f}{B} = \frac{K_f \cdot A_m}{2\pi \cdot f_m} = \frac{K_f \cdot 2}{2\pi \cdot 1000}$

FIRST ROOT AT  $\beta = 2.405 \Rightarrow$

$$\frac{K_f \cdot 2}{2\pi \cdot 1000} = 2.405$$

$$K_f = 2405\pi$$

a) b) ii.  $S.S = \frac{K_f A_m}{2\pi \cdot 1000} \Rightarrow$

$$A_m = \frac{11 \cdot \cancel{\pi} \cdot 1000}{2405 \cancel{\pi}} = 4.57$$

## QUESTION 3

$$(a) i(t) = I_0 \left( e^{-\frac{V(t)}{V_T}} - 1 \right).$$

$$e^x = 1 + x + \frac{x^2}{2} + \dots$$

$$i(t) = I_0 \left( 1 - \frac{V(t)}{V_T} + \frac{V^2(t)}{2V_T^2} - 1 \right) = I_0 \left( \frac{V^2(t)}{2V_T^2} - \frac{V(t)}{V_T} \right).$$

$$V(t) = m(t) - C(t)$$

THEREFORE

$$i(t) = I_0 \left( \frac{m^2(t)}{2V_T^2} + \frac{C^2(t)}{2V_T^2} - \frac{m(t)C(t)}{V_T^2} - \frac{m(t)}{V_T} + \frac{C(t)}{V_T} \right)$$

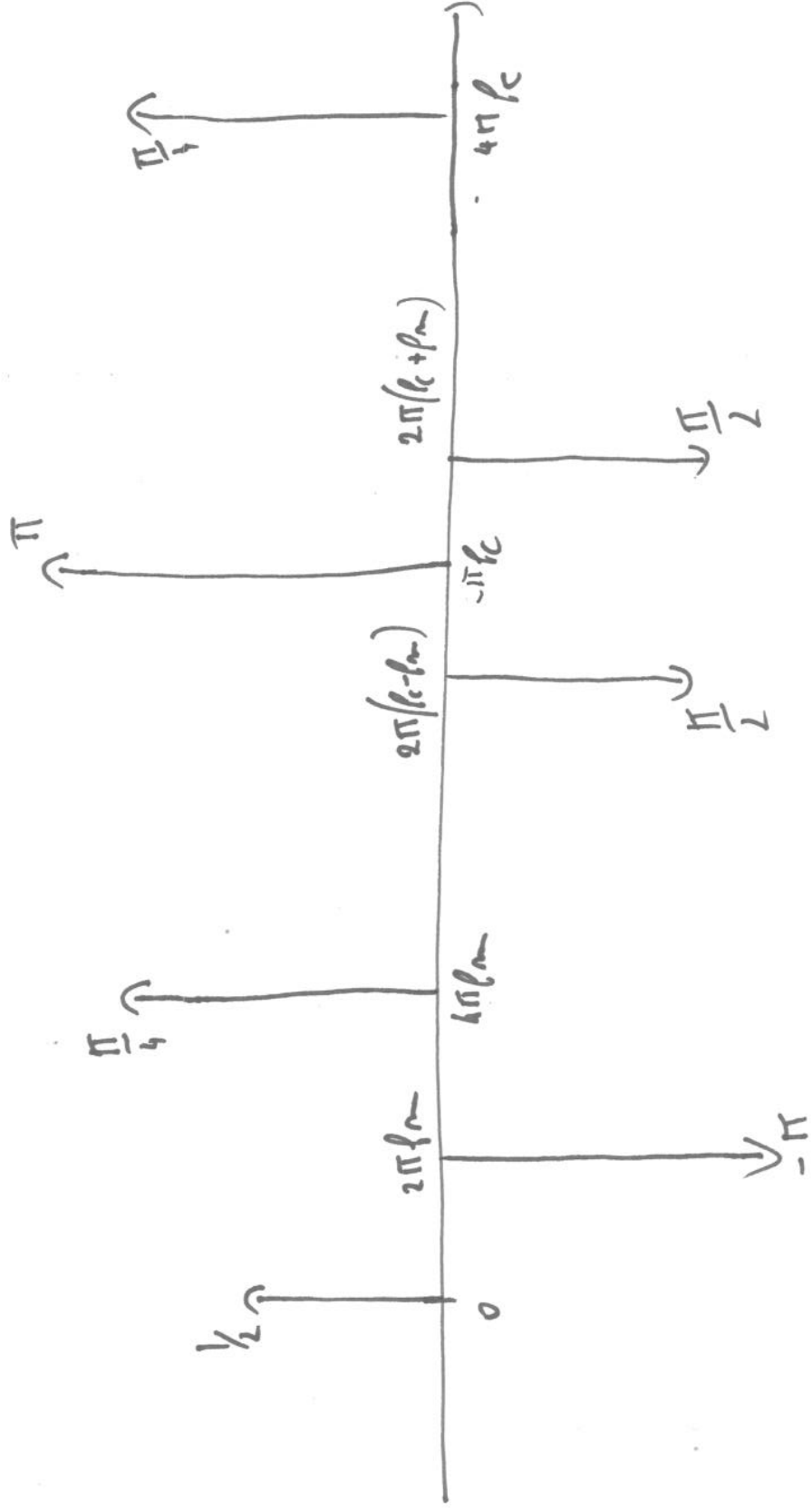
$$= I_0 \left( \frac{\cos^2 2\pi f_m t}{2} + \frac{\cos^2 2\pi f_c t}{2} - \cos 2\pi f_m t \cos 2\pi f_c t + \right. \\ \left. - \cos 2\pi f_m t + \cos 2\pi f_c t \right)$$

$$= I_0 \left( \frac{1}{4} + \frac{1}{4} \cos 4\pi f_m t + \frac{1}{4} + \frac{1}{4} \cos 4\pi f_c t + \right. \\ \left. - \frac{1}{2} \cos 2\pi (f_c - f_m) t - \frac{1}{2} \cos 2\pi (f_c + f_m) t + \right. \\ \left. - \cos 2\pi f_m t + \cos 2\pi f_c t \right)$$

$$= I_0 \left( \frac{1}{2} + \frac{1}{4} \cos 4\pi f_m t + \frac{1}{4} \cos 4\pi f_c t - \frac{1}{2} \cos 2\pi (f_c - f_m) t \right. \\ \left. - \frac{1}{2} \cos 2\pi (f_c + f_m) t - \cos 2\pi f_m t + \cos 2\pi f_c t \right).$$



SPECTRUM FOR  $\omega \geq 0$  (THE SPECTRUM IS EVEN)



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$$c) \quad 2\pi(f_c + f_m) < W < 4\pi f_c$$

THE OUTPUT IS

$$y(t) = I_0 (1 - \cos 2\pi f_m t) \cos 2\pi f_c t =$$

$$= \frac{I_0}{V_T} (V_T - m(t)) \cos 2\pi f_c t$$

d) SINCE  $\varphi_{AM}(t) = \frac{I_0}{V_T} (V_T - m(t)) \cos 2\pi f_c t$

THEN

$$\mu = \frac{m_p}{A} = \frac{V_T}{V_T} = 1$$

e)  $\eta = \frac{P_s}{P_c + P_s} = \frac{1/4}{1/2 + 1/4} = \frac{1}{3}$

# QUESTION 4

a)  $K_p = |K_v|^2$  AND WE WANT  $K_p = 0.01$

THE LOAD AT THE JUNCTION IS  $z_0 // z_1$

SO  $K_v$  IS GIVEN BY

$$K_v = \frac{z_0 // z_1 - z_0}{z_0 // z_1 + z_0} = \frac{\frac{z_0 \cdot z_1}{z_0 + z_1} - z_0}{\frac{z_0 \cdot z_1}{z_0 + z_1} + z_0} = - \frac{z_0}{2z_1 + z_0} = - \frac{1}{1+2\lambda}$$

WHERE  $\lambda = \frac{z_1}{z_0}$

$$K_p = |K_v|^2 = \frac{1}{(1+2\lambda)^2} = \frac{1}{100} \Rightarrow 1+2\lambda = 10$$

$$\lambda = 4.5 \Rightarrow z_1 = 4.5 z_0 = 225 \Omega$$

b)  $V_2 = K_v V_+ = - \frac{1}{1+2\lambda} \cdot 1$   $\lambda = 2 \Rightarrow V_2 = -0.2 \text{ Volts}$

$$I_2 = K_I \cdot I_+ = -K_v \frac{V_+}{z_0} = + \frac{1}{5} \cdot \frac{1}{50} = 4 \text{ mA}$$

$$V_T = V_2 + V_+ = -0.2 + 1 = 0.8 \text{ Volts}$$

$$I_T = I_2 + I_+ = 16 \text{ mA}$$

c)  $P_2 = \frac{1}{2} |V_2 \cdot I_2| = \frac{1}{2} (0.2 \cdot 4 \cdot 10^{-3}) \text{ W} = 0.4 \text{ mW}$

$$P_T = \frac{1}{2} |V_T \cdot I_T| = \frac{1}{2} (0.8 \cdot 16 \cdot 10^{-3}) \text{ W} = 6.4 \text{ mW}$$