ADVANCED DATA COMMUNICATION

SOLUTIONS - 2008

E4.04 | ISE 4.9 | SC6

$\frac{1}{14}$

Problem 1.a.

(i) As an orthonormal set of basis functions we consider the set

$$\psi_1(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{o.w} \end{cases} \qquad \psi_2(t) = \begin{cases} 1 & 1 \le t < 2 \\ 0 & \text{o.w} \end{cases}$$

$$\psi_3(t) = \begin{cases} 1 & 2 \le t < 3 \\ 0 & \text{o.w} \end{cases} \qquad \psi_4(t) = \begin{cases} 1 & 3 \le t < 4 \\ 0 & \text{o.w} \end{cases}$$

In matrix notation, the four waveforms can be represented as

$$\begin{pmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \\ s_4(t) \end{pmatrix} = \begin{pmatrix} 2 & -1 & -1 & -1 \\ -2 & 1 & 1 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & -2 & -2 & 2 \end{pmatrix} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \\ \psi_4(t) \end{pmatrix}$$

Note that the rank of the transformation matrix is 4 and therefore, the dimensionality of the waveforms is 4

(ii) The representation vectors are

$$\begin{aligned} \mathbf{s}_1 &= \begin{bmatrix} 2 & -1 & -1 & -1 \end{bmatrix} \\ \mathbf{s}_2 &= \begin{bmatrix} -2 & 1 & 1 & 0 \end{bmatrix} \\ \mathbf{s}_3 &= \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix} \\ \mathbf{s}_4 &= \begin{bmatrix} 1 & -2 & -2 & 2 \end{bmatrix} \end{aligned}$$

(iii) The distance between the first and the second vector is

$$d_{1,2} = \sqrt{|\mathbf{s}_1 - \mathbf{s}_2|^2} = \sqrt{\left|\begin{bmatrix} 4 & -2 & -2 & -1 \end{bmatrix}\right|^2} = \sqrt{25}$$

Similarly we find that

$$\begin{aligned} d_{1,3} &= \sqrt{|\mathbf{s}_1 - \mathbf{s}_3|^2} = \sqrt{\left|\begin{bmatrix} 1 & 0 & -2 & 0 \end{bmatrix}\right|^2} = \sqrt{5} \\ d_{1,4} &= \sqrt{|\mathbf{s}_1 - \mathbf{s}_4|^2} = \sqrt{\left|\begin{bmatrix} 1 & 1 & 1 & -3 \end{bmatrix}\right|^2} = \sqrt{12} \\ d_{2,3} &= \sqrt{|\mathbf{s}_2 - \mathbf{s}_3|^2} = \sqrt{\left|\begin{bmatrix} -3 & 2 & 0 & 1 \end{bmatrix}\right|^2} = \sqrt{14} \\ d_{2,4} &= \sqrt{|\mathbf{s}_2 - \mathbf{s}_4|^2} = \sqrt{\left|\begin{bmatrix} -3 & 3 & 3 & -2 \end{bmatrix}\right|^2} = \sqrt{31} \\ d_{3,4} &= \sqrt{|\mathbf{s}_3 - \mathbf{s}_4|^2} = \sqrt{\left|\begin{bmatrix} 0 & 1 & 3 & -3 \end{bmatrix}\right|^2} = \sqrt{19} \end{aligned}$$

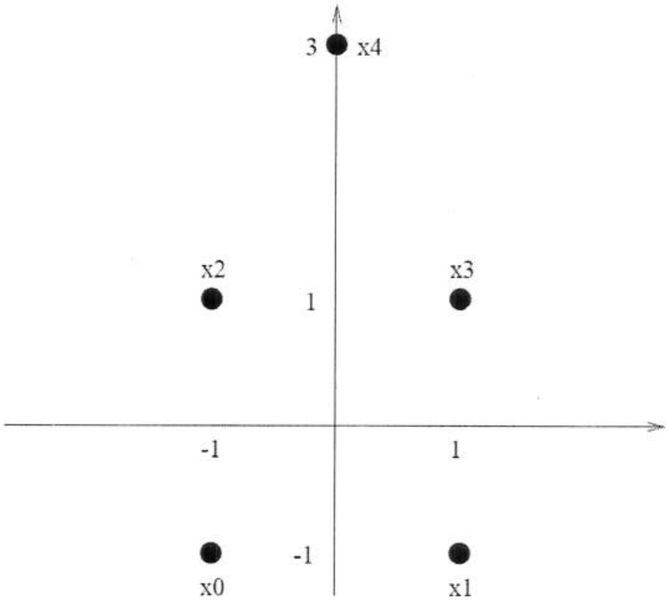
Thus, the minimum distance between any pair of vectors is $d_{\min} = \sqrt{5}$.

Problem 1.b

i) The signal constellation is shown in the figure below.

From the signal constellation, we get $d_{min} = 2$. Since we have 5 signals, the Union Bound is given by,

$$\begin{aligned} P_e &\le 4\,Q\left(\frac{d_{min}}{2\sigma}\right) \\ &= 4\,Q\left(\frac{1}{\sigma}\right). \end{aligned}$$



ii) The number of Nearest Neighborhood $N_e = 1/5(2+2+3+3+2) = 2.4$. Therefore, the Nearest Neighborhood Union Bound is given by,

$$P_e = 2.4\,Q\left(\frac{1}{\sigma}\right).$$

iii) Since $\mathcal{E}_x = \frac{1}{5}(2 \times 4 + 3^2) = 3.4$,

$$\sigma = \sqrt{\frac{\bar{\mathcal{E}}_x}{SNR}} = \sqrt{\frac{\frac{1}{2}\mathcal{E}_x}{SNR}} = \sqrt{\frac{1.7}{10^{1.4}}}$$

Therefore, we get

$$P_e(\text{NNUB}) = 2.4 \times Q\left(\sqrt{\frac{10^{1.4}}{1.7}}\right) = 1.45 \times 10^{-4}.$$

Problem 1.c

(i) The impulse response of the matched filter is

$$s(t) = u(T-t) = \begin{cases} \frac{A}{T}(T-t)\cos(2\pi f_c(T-t)) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

(ii) The output of the matched filter at $t = T$ is

$$\begin{aligned}
g(T) &= u(t) \star s(t)|_{t=T} = \int_0^T u(T-\tau)s(\tau)d\tau \\
&= \frac{A^2}{T^2}\int_0^T (T-\tau)^2 \cos^2(2\pi f_c(T-\tau))d\tau \\
&\overset{v=T-\tau}{=} \frac{A^2}{T^2}\int_0^T v^2 \cos^2(2\pi f_c v)dv \\
&= \frac{A^2}{T^2}\left[\frac{v^3}{6} + \left(\frac{v^2}{4\times 2\pi f_c} - \frac{1}{8\times(2\pi f_c)^3}\right)\sin(4\pi f_c v) + \frac{v\cos(4\pi f_c v)}{4(2\pi f_c)^2}\right]\Bigg|_0^T \\
&= \frac{A^2}{T^2}\left[\frac{T^3}{6} + \left(\frac{T^2}{4\times 2\pi f_c} - \frac{1}{8\times(2\pi f_c)^3}\right)\sin(4\pi f_c T) + \frac{T\cos(4\pi f_c T)}{4(2\pi f_c)^2}\right]
\end{aligned}$$

Problem 1.d

The maximum likelihood criterion selects the maximum of $f(\mathbf{r}|\mathbf{s}_m)$ over the M possible transmitted signals. When $M = 2$, the ML criterion takes the form

$$\frac{f(\mathbf{r}|\mathbf{s}_1)}{f(\mathbf{r}|\mathbf{s}_2)} \underset{s_2}{\overset{s_1}{\gtrless}} 1$$

or, since

$$\begin{aligned} f(\mathbf{r}|\mathbf{s}_1) &= \frac{1}{\sqrt{\pi N_0}} e^{-(r-\sqrt{\mathcal{E}_b})^2/N_0} \\ f(\mathbf{r}|\mathbf{s}_2) &= \frac{1}{\sqrt{\pi N_0}} e^{-(r+\sqrt{\mathcal{E}_b})^2/N_0} \end{aligned}$$

the optimum maximum-likelihood decision rule is

$$r \underset{s_2}{\overset{s_1}{\gtrless}} 0$$

The average probability of error is given by

$$\begin{aligned} P(e) &= p\int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-(r+\sqrt{\mathcal{E}_b})^2/N_0} dr + (1-p)\int_{-\infty}^0 \frac{1}{\sqrt{\pi N_0}} e^{-(r-\sqrt{\mathcal{E}_b})^2/N_0} dr \\ &= p\int_{\sqrt{2\mathcal{E}_b/N_0}}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx + (1-p)\int_{-\infty}^{-\sqrt{2\mathcal{E}_b/N_0}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= pQ\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] + (1-p)Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] \\ &= Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] \end{aligned}$$

5

Problem 2.a

i The energy of the 64 QAM is given by,

$$\mathcal{E}_x(64) = \frac{d^2}{6}(M-1) = 42$$

The energy of the hybrid 32 QAM constellation is found as follows. The 64 QAM can be decomposed into two 32 hybrid QAM constellations: The first one represented by dots, and the other one represented by $\times$. Since these two constellations are rotations of one another, then their energies must be equal. The energy of the 64 QAM constellation is then,

$$\mathcal{E}_x(64) = \frac{1}{2}\mathcal{E}_x(32) + \frac{1}{2}\mathcal{E}_x(32)$$

(The factor of $1/2$ is present because we have 32 points in each hybrid constellation, and 64 points in the square QAM constellation). Therefore, $\mathcal{E}_x(32) = 42$. The energies are the same.

ii The NNUB probability of error for the 64 QAM constellation is,

$$\begin{aligned} P_e &\le 4(1-\frac{1}{\sqrt{M}})Q(\frac{d}{2\sigma}) \\ \Leftrightarrow P_e &\le 3.5Q(\frac{1}{\sigma}) \end{aligned}$$

To find the NNUB for the 32 hybrid QAM the number of nearest neighbors N_e has to be computed. The constellation points are divided as follows,

- Inner Points (26) have 4 neighbors.
- Side Points (6) have 3 neighbors.

Therefore, $N_e = \frac{1}{32}[26 \times 4 + 6 \times 3] = 3\frac{13}{16}$. Since $d_{min} = 2\sqrt{2}$, then,

$$P_e \le 3.8125Q(\frac{\sqrt{2}}{\sigma})$$

iii The distance d^2 is given by,

$$\begin{aligned} d^2 &= \frac{6\mathcal{E}_x}{\frac{31}{32}M - 1} = 8.4 \\ \Leftrightarrow d &= \sqrt{8.4} \simeq 2.898 \end{aligned}$$

iv The probability of error for the 32 Cross QAM is,

$$P_e \le 3.5Q(\frac{\sqrt{2.1}}{\sigma})$$

For the 32 hybrid QAM constellation, $P_e \le 3.8125Q(\frac{\sqrt{2}}{\sigma})$. Since the argument of the Q function for the 32 Cross QAM is bigger than the one for the 32 hybrid contellation - equivalently $d(cross) > d(hybrid)$, 32 Cross performs better.

v The constellation figure of merit for both constellations is,

$$\begin{aligned} CFM(hybrid) &= \frac{d^2/4}{\mathcal{E}_x} = \frac{2}{21} = 0.0952, \\ CFM(cross) &= \frac{\frac{8.4}{4}}{21} = 0.1 \end{aligned}$$

Obviously, $CFM(cross) > CFM(hybrid)$, a result that is consistent with (d).

Problem 2.b

The three symbols A, 0 and $-A$ are used with equal probability. Hence, the optimal detector uses two thresholds, which are $\frac{A}{2}$ and $-\frac{A}{2}$, and it bases its decisions on the criterion

$$
\begin{array}{ll}
A: & r > \dfrac{A}{2} \\
0: & -\dfrac{A}{2} < r < \dfrac{A}{2} \\
-A: & r < -\dfrac{A}{2}
\end{array}
$$

If the variance of the AWG noise is σ_n^2, then the average probability of error is

$$
\begin{aligned}
P(e) &= \frac{1}{3}\int_{-\infty}^{\frac{A}{2}} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(r-A)^2}{2\sigma_n^2}} dr + \frac{1}{3}\left(1 - \int_{-\frac{A}{2}}^{\frac{A}{2}} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{r^2}{2\sigma_n^2}} dr\right) \\
&\quad + \frac{1}{3}\int_{-\frac{A}{2}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(r+A)^2}{2\sigma_n^2}} dr \\
&= \frac{1}{3}Q\left[\frac{A}{2\sigma_n}\right] + \frac{1}{3}2Q\left[\frac{A}{2\sigma_n}\right] + \frac{1}{3}Q\left[\frac{A}{2\sigma_n}\right] \\
&= \frac{4}{3}Q\left[\frac{A}{2\sigma_n}\right]
\end{aligned}
$$

7

Problem 2.c

i Since QAM transmission is used,

$$\phi 1 = -\sqrt{\frac{2}{T}} sinc(\frac{t}{T}) \sin \omega_c t$$
$$\phi 2 = \sqrt{\frac{2}{T}} sinc(\frac{t}{T}) \cos \omega_c t$$

where $T = 2 \cdot 10^{-7}$.

ii For QAM and $P_e = 10^{-6}$ the gap is $\Gamma = 8.8\ dB$. Hence,

$$\bar{b} = \frac{1}{2}\log_2(1 + \frac{SNR}{\Gamma}) \simeq 3$$

iii Then, $R = \frac{b}{T} = \frac{2 \cdot 3}{2 \cdot 10^{-7}} = 30\ Mbps$.

Since $\bar{b} = 3, b = 6$, we have $M = 2^b = 2^6 = 64$. Therefore this is a 64-square QAM signal constellation.

iv In order to achieve $R = 35\ Mbps$, we should have $b = 7$, thus employing 128-cross QAM. We require the same probability of error, so:

$$\left(\frac{d_{min}}{2\sigma}\right)^2 = \frac{3SNR}{\frac{31}{32}M - 1} = 13.5\ dB \Rightarrow$$
$$SNR = 917.9 = 29.6\ dB$$

8

Problem 3.a

(i) We will proceed using the gap approximation: i.e. $E_n(b_n) = \frac{\Gamma}{g_n}(2^{2\bar{b}_n} - 1) * k$, where k=1 if PAM and k=2 if QAM.

So, we first need to find $g_n = \frac{|H_n|^2}{\sigma_n^2}$. From the system parameters $\sigma_n^2 = .125$. So, we have the following table (the center 3 are QAM channels)

subchannel	0	1	2	3	4
g_n	18	15.6569	10	4.3431	2

Now, using the above formula, we get:

subchannel	0	1	2	3	4
$e_n(1)$	1.2463	.9690	1.5172	3.4932	11.37
$e_n(2)$	5.07	1.938	3.043	6.9860	45.5233
$e_n(3)$	19.2287	3.8760	6.0686	13.97	182.0567
$e_n(4)$	81.9150	7.7521	12.1372	27.94	728.2345

Note for the QAM channels we could have used $e_n(b_n) = \frac{\Gamma}{g_n} 2_n^b$

(ii) With the above table, it is obvious that the bit allocations are as follows:

subchannel	0	1	2	3	4
b_n	2	3	2	1	0
$E_n(b_n)$	6.3215	6.7830	4.5515	3.4932	0

Bits were chosen in the following order: 1,0,2,1,2,3,1,0

(iii) $N * \bar{E_x} = 8$, so we are way over budget. Working backwards, we get

subchannel	0	1	2	3	4
b_n	1	2	1	0	0
$E_n(b_n)$	1.2463	2.9070	1.5172	0	0

(iv) Again, we just work backwards

subchannel	0	1	2	3	4
b_n	1	1	0	0	0
$E_n(b_n)$	1.2463	.9690	0	0	0

The margin in this case is $10 * log_{10}(\frac{8}{1.2463+.9690}) = 5.577dB$

i

Problem 3.b

```
g =

    4.3431   10.0000   15.6569   18.0000   15.6569   10.0000    4.3431    2.000

e =

    1.1124    1.1041    1.0680    0.9377    0.6680

b =

    2.1970    2.0964    1.7730    1.1714    0.6120

N =

     8

b_bar =

    1.6113
```

ii gap $= 10^{.9}/3 = 2.6478 = 4.23\text{dB}$

iii Check the following results
$R = 1.0622$

```
g =

    4.3431   10.0000   15.6569   18.0000   15.6569   10.0000    4.3431    2.00

e =

    1.2977    1.2757    1.1800    0.8351    0.1206

b =

    1.6478    1.5472    1.2239    0.6223    0.0629

N =

     8

b_bar =

    1.0622
```

3.c

i From Problem 3.6, a rate of 1.06 is achieved with a gap of 4.23 dB. Since $\Gamma = 9.8dB$ for $P_e = 10^{-7}$, about 4.5 dB more energy is needed. So, the P_e goal cannot be achieved. We can reach the same conclusion by doing MA, which will give us a negative margin.

ii Since we get $R = 1.0622$ for Γ=4.23dB from 3.6, an easy way to solve this one would be to try a slightly larger gap than 4.23dB. To get the exact gap value, we need to solve MA again. The same approach as in problem 3. will give us Γ = 4.79dB, and the corresponding $P_e = 1.3 \cdot 10^{-3}$.

iii Part ii gives us Γ = 4.79dB with no margin. So, with $\Gamma = 0$, margin = 4.79dB.

Problem 4.a

Substituting the expression of $X_{rc}(f)$ in the desired integral, we obtain

$$\begin{aligned}
\int_{-\infty}^{\infty} X_{rc}(f)df &= \int_{-\frac{1+\alpha}{2T}}^{-\frac{1-\alpha}{2T}} \frac{T}{2}\left[1+\cos\frac{\pi T}{\alpha}(-f-\frac{1-\alpha}{2T})\right]df + \int_{-\frac{1-\alpha}{2T}}^{\frac{1-\alpha}{2T}} T df \\
&+ \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \frac{T}{2}\left[1+\cos\frac{\pi T}{\alpha}(f-\frac{1-\alpha}{2T})\right]df \\
&= \int_{-\frac{1+\alpha}{2T}}^{-\frac{1-\alpha}{2T}} \frac{T}{2}df + T\left(\frac{1-\alpha}{T}\right) + \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \frac{T}{2}df \\
&+ \int_{-\frac{1+\alpha}{2T}}^{-\frac{1-\alpha}{2T}} \cos\frac{\pi T}{\alpha}(f+\frac{1-\alpha}{2T})df + \int_{\frac{1-\alpha}{2T}}^{\frac{1+\alpha}{2T}} \cos\frac{\pi T}{\alpha}(f-\frac{1-\alpha}{2T})df \\
&= 1+\int_{-\frac{\alpha}{T}}^{0} \cos\frac{\pi T}{\alpha}x dx + \int_{0}^{\frac{\alpha}{T}} \cos\frac{\pi T}{\alpha}x dx \\
&= 1+\int_{-\frac{\alpha}{T}}^{\frac{\alpha}{T}} \cos\frac{\pi T}{\alpha}x dx = 1+0 = 1
\end{aligned}$$

Problem 4.b

i The pulse response is $p(t) = \phi(t) * h(t)$. In frequency domain,

$$\begin{aligned} P(f) &= \Phi(f)H(f) \\ &= (\sqrt{T} \sqcap (Tf))(\frac{1}{1+ae^{j2\pi f}} \sqcap (f)) \\ &= \frac{1}{1+ae^{j2\pi f}} \sqcap (f) \qquad \text{(since T=1)} \end{aligned}$$

In terms of ω,

$$P(\omega) = \begin{cases} \frac{1}{1+ae^{j\omega}} & |\omega| \le \pi \\ 0 & |\omega| > \pi \end{cases}$$

ii First, let's find $P(e^{-j\omega T})$.

$$P(e^{-j\omega T}) = \frac{1}{T}\sum_{n=-\infty}^{\infty} P(\omega + \frac{2\pi n}{T})$$

Since T=1 and $P(\omega) = 0$ for $|\omega| > \pi$, $P(e^{-j\omega T}) = \frac{1}{1+ae^{j\omega}}$. Then, by the inverse Fourier transform,

$$p_k = (-a)^{-k} u[-k].$$

Therefore,

$$\begin{aligned} \|p\|^2 &= T \sum_{k=-\infty}^{\infty} |p_k|^2 \\ &= \sum_{k=-\infty}^{0} (-a)^{2k} \\ &= \sum_{k=0}^{\infty} (-a)^{2k} \\ &= \frac{1}{1-a^2}. \end{aligned}$$

iii By substituting $e^{-j\omega T} = D$ into $P(e^{-j\omega T})$, we get $P(D) = \frac{1}{1+aD^{-1}}$. Therefore,

$$Q(D) = \frac{T}{\|p\|^2} P(D)P^*(D^{-*}) = \frac{1-a^2}{(1+aD)(1+aD^{-1})}.$$

iv For the zero-forcing equalizer,

$$W_{ZFE}(D) = \frac{1}{\|p\|Q(D)} = \frac{(1+aD)(1+aD^{-1})}{\sqrt{1-a^2}}.$$

To calculate $W_{MMSE-LE}(D)$, we need SNR_{MFB}:

$$SNR_{MFB} = \frac{\|p\|^2\bar{\mathcal{E}_x}}{\sigma^2} = \frac{10^{1.5}}{1-a^2}.$$

Then,

$$\begin{aligned} W_{MMSE-LE}(D) &= \frac{1}{\|p\|(Q(D) + 1/SNR_{MFB})} \\ &= \frac{\sqrt{1-a^2}}{\frac{1-a^2}{(1+aD)(1+aD^{-1})} + \frac{1+a^2}{10^{1.5}}} \\ &= \frac{(1+aD)(1+aD^{-1})}{\sqrt{1-a^2}[1 + (1+aD)(1+aD^{-1})10^{-1.5}]}. \end{aligned}$$

v When $a = 0$, $Q(D) = 1$ and $\|p\|^2 = 1$. Since $SNR = 15$dB and $\Gamma = 8.8$dB at $P_e = 10^{-6}$,

$$\bar{b} = \frac{1}{2}\log_2(1 + \frac{10^{1.5}}{10^{0.88}}) = 1.18$$

Then, the maximum data rate achievable is

$$R = \frac{b}{T} = \frac{1.18}{1} = 1.18 \text{ bits/sec}$$

Problem 4.c

The bandwidth of the bandpass channel is $W = 4$ KHz. Hence, the rate of transmission should be less or equal to 4000 symbols/sec. If a 8-QAM constellation is employed, then the required symbol rate is $R = 9600/3 = 3200$. If a signal pulse with raised cosine spectrum is used for shaping, the maximum allowable roll-off factor is determined by

$$1600(1+\alpha) = 2000$$

which yields $\alpha = 0.25$. Since α is less than 50%, we consider a larger constellation. With a 16-QAM constellation we obtain

$$R = \frac{9600}{4} = 2400$$

and

$$1200(1+\alpha) = 2000$$

Or $\alpha = 2/3$, which satisfies the required conditions. The probability of error for an M-QAM constellation is given by

$$P_M = 1 - (1 - P_{\sqrt{M}})^2$$

where

$$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left[\sqrt{\frac{3\mathcal{E}_{av}}{(M-1)N_0}}\right]$$

With $P_M = 10^{-6}$ we obtain $P_{\sqrt{M}} = 5 \times 10^{-7}$ and therefore

$$2 \times (1 - \frac{1}{4})Q\left[\sqrt{\frac{3\mathcal{E}_{av}}{15 \times 2 \times 10^{-10}}}\right] = 5 \times 10^{-7}$$

Using the last equation and the tabulation of the $Q[\cdot]$ function, we find that the average transmitted energy is

$$\mathcal{E}_{av} = 24.70 \times 10^{-9}$$

Note that if the desired spectral characteristic $X_{rc}(f)$ is split evenly between the transmitting and receiving filter, then the energy of the transmitting pulse is

$$\int_{-\infty}^{\infty} g_T^2(t)dt = \int_{-\infty}^{\infty} |G_T(f)|^2 df = \int_{-\infty}^{\infty} X_{rc}(f)df = 1$$

Hence, the energy $\mathcal{E}_{av} = P_{av}T$ depends only on the amplitude of the transmitted points and the symbol interval T. Since $T = \frac{1}{2400}$, the average transmitted power is

$$P_{av} = \frac{\mathcal{E}_{av}}{T} = 24.70 \times 10^{-9} \times 2400 = 592.8 \times 10^{-7}$$

If the points of the 16-QAM constellation are evenly spaced with minimum distance between them equal to d, then there are four points with coordinates $(\pm\frac{d}{2}, \pm\frac{d}{2})$, four points with coordinates $(\pm\frac{3d}{2}, \pm\frac{3d}{2})$, four points with coordinates $(\pm\frac{3d}{2}, \pm\frac{d}{2})$, and four points with coordinates $(\pm\frac{d}{2}, \pm\frac{3d}{2})$. Thus, the average transmitted power is

$$P_{av} = \frac{1}{2 \times 16}\sum_{i=1}^{16}(A_{mc}^2 + A_{ms}^2) = \frac{1}{2}\left[4 \times \frac{d^2}{2} + 4 \times \frac{9d^2}{2} + 8 \times \frac{10d^2}{4}\right] = 20d^2$$

Since $P_{av} = 592.8 \times 10^{-7}$, we obtain

$$d = \sqrt{\frac{P_{av}}{20}} = 0.00172$$