IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2013

EEE PART I: MEng, BEng and ACGI

ENERGY CONVERSION

Monday, 10 June 10:00 am

Time allowed: 2:00 hours

There are THREE questions on this paper.

Answer All questions.
Q1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : W.T. Pike, E.M. Yeatman

Second Marker(s) : B.C. Pal, B.C. Pal

Special instructions for students

Physical Constants

permittivity of free space:	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
permeability of free space:	$\mu_o = 4\pi \times 10^{-7}$ H/m
charge of an electron:	$e = 1.6 \times 10^{-19}$ C

The Questions

1.

a) What is the major difference between lines of electric flux and lines of magnetic flux? [4]

b) Draw a labelled plot of the B-H curve for a magnet, indicating on it the coercive field, remnant flux density and saturation flux density. Draw a line on the plot giving a measure of the relative permeability of the material of the magnet. [4]

c) In terms of energy transfer, how can the introduction of a power-factor-compensation capacitor increase the efficiency of driving an inductive load through a resistive transmission line. [4]

d) Why has 50/ 60 Hz been selected as the frequency of mains voltage worldwide? [4]

e) Suggest what measurement of the mains voltage at any socket would indicate how well the National Grid is matching supply to demand. [4]

f) An open circuit, 10 turn coil of area 5 cm^2, has a uniform magnetic flux passing through if which oscillates sinusoidally at 100 Hz. If the flux density magnitude is 0.50 T, what will be the peak induced voltage? [4]

g) Each coil in a DC generator has N turns. Why is the output voltage of each coil less than N times the induced voltage amplitude for each turn? [4]

h) Briefly describe two mechanisms by which energy is lost in an iron core ("iron losses") when it is magnetised by an alternating field. [4]

i) Sketch and label an equivalent circuit for a simple DC motor. [4]

j) A certain DC motor has motor constant $\boldsymbol{K} = 30$, an armature resistance $\boldsymbol{R_A}$ = 5.0 Ω, a total flux $\boldsymbol{\Phi}$= 50 mWb, and an applied voltage V_A = 20V. Calculate the maximum torque this motor can generate. At what motor speed will this be achieved? [4]

2.

a) For a conducting sphere of radius R in air with a charge Q, using Gauss's law derive the variation of the electric field, E, with r, the distance from the centre of the sphere. Determine the electric potential of the sphere, taking the reference potential as zero far from the sphere.

Hence show the capacitance of the sphere is $4\pi\varepsilon_0 R$. [8]

b) Two well-separated, identical spheres in air are given a charge of Q_1 and Q_2 respectively. Show that when the spheres are connected via a long conducting wire, the electrostatic energy will be reduced, after current has stopped flowing in the wire, by

$$\Delta U = \frac{(Q_1 - Q_2)^2}{16\pi\varepsilon_o R}$$ [14]

c) How is the energy is transferred, and lost, over time, and what will be the effect of changing the resistance of the wire? [8]

3.

a) An inductor is constructed by winding N_l turns of a coil around an iron toroidal core as shown in Figure 3.1. The toroid has mean radius r_o and cross-sectional area A, and relative permeability μ_r. Using Ampere's Law, derive expressions for the magnetic field strength $\boldsymbol{H}$ and the magnetic flux density $\boldsymbol{B}$ in the core at the mean radius, for an applied current in the coil I. Clearly state any assumptions or approximations used. [6]

b) Using your results from (a), derive an expression for the average energy density stored in the magnetic field. [6]

c) Give an expression for the energy stored in an inductor of inductance L through which a current I is flowing (no derivation is required). Using this expression and your result from (b), derive an expression for the inductance of the inductor in Figure 3.1. [6]

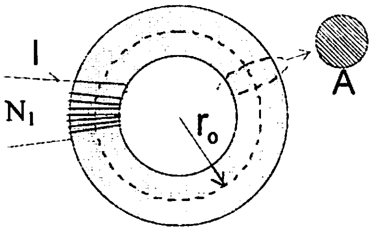


Figure 3.1

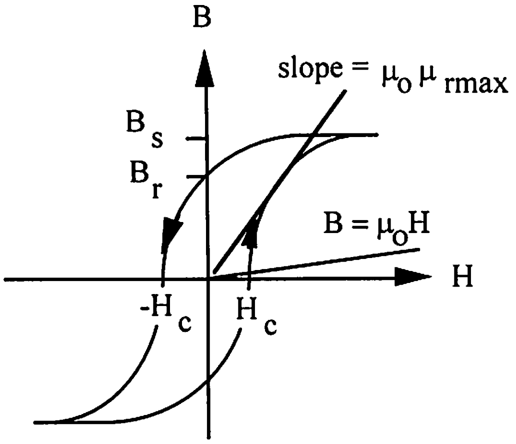
d) A transformer has $N_1 = 20$ and $N_2 = 50$ turns on the primary and secondary coils respectively. These two coils have resistance of 10 Ω and 50 Ω respectively. A real load of Z_L is connected to the secondary terminals. Draw and label an equivalent circuit for the transformer, assuming it is ideal apart from the coil resistances. Calculate the efficiency, and the regulation, of the transformer for $Z_L = 500\ \Omega$. [12]

Answers

1.

a) Lines of electric flux start on positive charges and end on negative charges in a static field. Lines of magnetic flux are loops as there is no source for magnetic flux lines.

b)



either of the two straight lines is acceptable.

c) A capacitor acts as a temporary store of energy for an inductive load during part of the cycle. If this capacitor put in the power circuit near the load it will ensure that this energy does not have to be supplied along the resistive path for the generator.

d) This frequency is fast enough to avoid flicker of electric lights due to the persistence of vision; it is also compatible with the rotation speed of generators, which in turn is set by the strength of the materials used to fabricate the moving parts.

e) The frequency of supply indicates how supply is matching demand. If the demand is higher than supply, the generators will slow down and the frequency will drop, releasing rotational kinetic energy, and vice versa.

f) An open circuit, 10 turn coil of area 5 cm^2, has a uniform magnetic flux passing through if which oscillates sinusoidally at 100 Hz. If the flux density magnitude is 0.50 T, what will be the peak induced voltage? [4]

$V = -Nd\Phi/dt$, $\Phi = BA = B_o\sin(\omega t)A$, $V = -N\omega B_o\cos(\omega t)A$, $V_o = N2\pi fB_oA = 1.57$ V

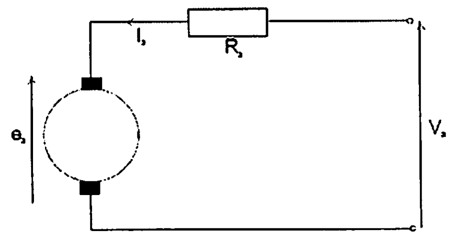
g) Each coil in a DC generator has N turns. Why is the output voltage of each coil less than N times the induced voltage amplitude for each turn? [4]

Because each turn is at a different orientation around the stator, it's induced voltage has a different phase. Adding N sinusoids of different phases gives less than N times the voltage.

h) Briefly describe two mechanisms by which energy is lost in an iron core ("iron losses") when it is magnetised by an alternating field. [4]

The two mechanisms are (i) hysteresis, whereby the energy put in to magnetise the core is more than the energy that flows out as it is de-energised, as can be seen by the different paths taken by the B-H curve in the two directions; and (ii) eddy current losses, whereby the oscillating magnetic flux induces loops of current in the conducting core, which result in ohmic losses.

i) Sketch and label an equivalent circuit for a simple DC motor. [4]



j) A certain DC motor has motor constant $K = 30$, an armature resistance $R_A = 5.0\ \Omega$, a total flux $\Phi = 50$ mWb, and an applied voltage $V_A = 20$V. Calculate the maximum torque this motor can generate. At what motor speed will this be achieved? [4]

This happens at a speed of zero, at which point there is no back-EMF so $I_A = V_A/R_A$, and since $T = K\Phi I$ then $T_{max} = K\Phi(V_A/R_A) = 30 \times 50 \times 10^{-3} \times 20/5 = \underline{6.0\ \text{Nm}}$

2.

a) Consider Gauss's Law applied to a sphere of radius $r > R$ centered on the charged sphere:

$$Q = \oint_S \mathbf{D} \cdot \mathbf{dS}$$
$$= \oint_S D \cdot dS$$
$$= D \oint_S dS$$
$$= DS$$
$$= D\left(4\pi r^2\right)$$
$$E = D / \varepsilon_o$$
$$= \frac{Q}{4\pi\varepsilon_o\varepsilon_r r^2}$$

Calculating the voltage along a path from an infinite distance away from the sphere, taken as zero potential, to the surface of the sphere at $r = R$

$$V(R) = -\int_\infty^R \mathbf{E} \cdot \mathbf{dl}$$
$$= -\frac{Q}{4\pi\varepsilon_o} \int_\infty^R \frac{dr}{r^2}$$
$$= \frac{Q}{4\pi\varepsilon_o R}$$

The capacitance will be given by

$$C = \frac{Q}{V}$$
$$= 4\pi\varepsilon_o R$$

b) Energy for a capacitance with a charge C is given by

$$E = \frac{1}{2} C V^2$$
$$= \frac{Q2}{2C}$$

The initial energy is therefore

$$E_i = \frac{Q_1^2}{2C} + \frac{Q_2^2}{2C}$$

When the spheres are connected current will flow until the two voltages are equal (and thus the 2 charges are equal, since the capacitances are the same). By conservation of charge, this final charge will be given by

$$Q_f = \frac{Q_1 + Q_2}{2}$$

The final energy will be given by

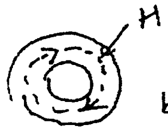
$$E_f = 2\frac{Q_f^2}{2C}$$
$$= \frac{1}{C}\left(\frac{Q_1 + Q_2}{2}\right)^2$$

Hence the reduction of energy,

$$\Delta E = E_f - E_i$$
$$= \frac{Q_1^2}{2C} + \frac{Q_2^2}{2C} - \frac{1}{C}\left(\frac{Q_1 + Q_2}{2}\right)^2$$
$$= \frac{1}{C}\left(\frac{Q_1^2}{4} + \frac{Q_2^2}{4} - \frac{Q_1 Q_2}{2}\right)$$
$$= \frac{1}{4C}\left(Q_1 - Q_2\right)^2$$
$$= \frac{1}{16\pi\varepsilon_o R}\left(Q_1 - Q_2\right)^2$$

c) The energy is lost due to resistive heating of the wire as the current flows through it to equalise the charge. Increasing the resistance of the wire will mean the charge will take longer to flow between the two spheres, though the energy lost will be the same.

3 a) An inductor is constructed by winding N_1 turns of a coil around an iron toroidal core as shown in Figure 3.1. The toroid has mean radius r_0 and cross-sectional area A, and relative permeability μ_r. Using Ampere's Law, derive expressions for the magnetic field strength $\boldsymbol{H}$ and the magnetic flux density $\boldsymbol{B}$ in the core at the mean radius, for an applied current in the coil $\boldsymbol{I}$. Clearly state any assumptions or approximations used. [6]



$\oint \underline{H} \cdot d\underline{l} = NI$

by symmetry $\underline{H}$ is uniform, and $\underline{H} \parallel d\underline{l}$ around loop so

$H \oint dl = H(2\pi r_0) = NI$

$H = \frac{NI}{2\pi r_0}$ $\quad B = \mu H = \mu_r \mu_0 \frac{NI}{2\pi r_0}$

assume no flux leakage.

b) Using your results from (a), derive an expression for the average energy density stored in the magnetic field. [6]

energy density $u' = \frac{1}{2} BH = \frac{1}{2}\mu_r\mu_0 H^2$

$= \frac{\mu_r\mu_0 (NI)^2}{2(2\pi r_0)^2}$

c) Give an expression for the energy stored in an inductor of inductance $\boldsymbol{L}$ through which a current $\boldsymbol{I}$ is flowing (no derivation is required). Using this expression and your result from (b), derive an expression for the inductance of the inductor in Figure 3.1. [6]

$U_L = \frac{1}{2} L I^2$

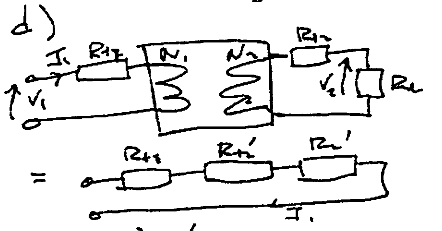
$U_L = u' \cdot (Vol) \quad Vol \approx 2\pi r_0 A$

$U_L = \frac{1}{2} \frac{\mu_r\mu_0 (NI)^2}{(2\pi r_0)^2} (2\pi r_0) A = \frac{1}{2} \frac{\mu_r\mu_0 (NI)^2 A}{\vdots}$

$\frac{1}{2} L I^2 = U_L \therefore L = \frac{U_L}{\frac{1}{2} I^2} = \frac{\mu_r\mu_0 \,\cdot}{2\pi r_0}$

d) A transformer has $\boldsymbol{N_1}$ = 20 and $\boldsymbol{N_2}$ = 50 turns on the primary and secondary coils respectively. These two coils have resistance of 10 Ω and 50 Ω respectively. A real load of $\boldsymbol{Z_L}$ is connected to the secondary terminals. Draw and label an equivalent circuit for the transformer, assuming it is ideal apart from the coil resistances. Calculate the efficiency, and the regulation, of the transformer for $\boldsymbol{Z_L}$ = 500 Ω. [12]

d)



$R_{t1} = 10\,\Omega$

$R_{t2}' = \left(\frac{N_1}{N_2}\right)^2 R_{t2}$

$= \left(\frac{2}{5}\right)^2 \cdot 50 = 8\,\Omega$

$R_L' = \left(\frac{N_1}{N_2}\right)^2 R_L = \left(\frac{2}{5}\right)^2 \cdot 500$

$= 80\,\Omega$

$\eta = \frac{I_1^2 R_L'}{I_1^2 (R_L' + R_{t}' + R_{t1})} = \frac{80}{98} = \underline{81.6\%}$

$Reg = \frac{V_{2oc} - V_2}{V_{2oc}} \times 100\% \qquad V_{2oc}' = V_1$

$= \frac{V_{2oc}' - V_2'}{V_{2oc}'} \times 100\%$ (referred to primary)

$V_2' = V_1 \left(\frac{R_L'}{R_{t1} + R_{t2}' + R_L'}\right) = .816 V_1$

$Reg = \frac{1 - .816}{1} \times 100\% = +18.4\%$