EJ 02 /AC4

The Answers 2008

Q1 ANSWER: [all computed example]

(a) $V = V_R \dfrac{SR R_L / (SR + R_L)}{(10-S)R + SR R_L / (SR + R_L)} = \dfrac{V_R S\rho}{10S - S^2 + 10\rho}$

[3]

(b) $G = \dfrac{dV}{dS} = \dfrac{d}{dS} \dfrac{\rho V_R S}{10S - S^2 + 10\rho} = V_R \dfrac{S^2\rho + 10\rho^2}{\left(10S - S^2 + 10\rho\right)^2}$

The mean gain is $\bar{G} = \dfrac{V(9)}{9} = \dfrac{\rho}{9+10\rho} V_R$

[3]

(c) It is always monotonic since the gain is always positive.

[2]

(d) $V = V_R \dfrac{S\rho}{10S - S^2 + 10\rho} \simeq V_R \dfrac{S}{10}\left(1 - \dfrac{10S - S^2}{10\rho}\right)$

[2]

(e) Absolute non-linearity: . $\max \delta_A \simeq \max \dfrac{10S - S^2}{10\rho} = \dfrac{2.5}{\rho}$

[4]

(f) differential non-linearity: $\max \delta_A \simeq \max \dfrac{\dfrac{d}{dS}\dfrac{10S^2 - S^3}{100\rho}}{\dfrac{1}{10}} = \max \dfrac{20S - 3S^2}{10\rho}$

the maximum of the last expression is at $S = 20/6 \simeq 3$, $\delta_A \simeq \dfrac{60-27}{10\rho} \simeq 3\rho$

[4]

(g) We require $\dfrac{10S - S^2}{10\rho} < .01 \ \forall S \in [0,9] \Rightarrow 2.5/\rho < 0.01 \Rightarrow \rho > 250$

[2]

Q2 ANSWER [bookwork, examples and an extension]

a) Define, very briefly, each of the following quantities for a sensor:

i. Sensitivity: is the derivative of the output with respect to the input: $S = \frac{\partial y}{\partial x}$

[1]

ii. Threshold is the minimum detectable input

[1]

iii. Zero Offset is the input which results into a zero output

[1]

iv. Absolute non-linearity:

Express the response as: $y = Ax + g(x)$. Then $\delta_A = \max \frac{g(x)}{Ax}$

[1]

v. Differential Non-linearity

Express the response as: $y = Ax + g(x)$. Then $\delta_D = \frac{1}{A} \max \frac{dg(x)}{dx}$

[1]

b) A sensor is monotonic if the gain does not change sign over the sensor's input range. A Phase detector is not monotonic, since its response is periodic in the input.
No phase detector is monotonic because of the periodicity of the input. A D/A converter IS monotonic.

[6]

c) A linear sensor's response function $y = f(x)$ does not depend on past input. As a result the output is the convolution of the Fourier transform of the input and the fourier transform of the impulse response of the sensor.

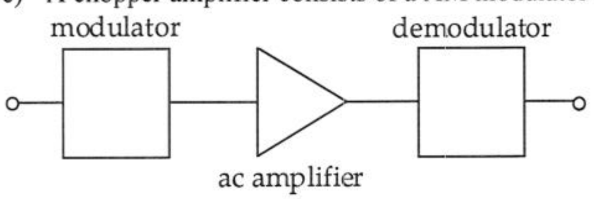
A non-linear sensor's response function depends on its past input. As a result the output is a multiple integral over past inputs.

[2]

d) A sensor's response y is often dependant on other parameters z_i beside the intended stimulus x. The partial derivatives $\frac{\partial y}{\partial z_i}$ are called the cross sensitivities. By applying a signal on one of the z_i's the measurement can be AM modulated. Subsequent averaging (Low pass filtering) can increase the SNR of the measurement at the expense of a long observation time.

[3]

e) A chopper amplifier consists of a AM modulator-AM demodulator pair:



The modulator needs to be preceded by an anti-aliasing filter. This diagram is functionally identical to that of a communication link, suggesting that any type of modulation (FM, PM, QAM,…) can in principle be used for chopper amplifiers. [4]

Q3. ANSWERS [computed example]

a) Johnson Noise $V_{J1}^2 = 4kTRB = 320\times10^{-15}V^2$. Spectrally white, but we are told there is also pink noise (uncorrelated to this source) of magnitude $V_{P1}^2 = 4kTR\frac{f_0}{f}$

The total contribution of the source V_{n1} is $V_{n1}^2 = 4kTR\left(1+\frac{f0}{f}\right)$ since these two sources are uncorrelated. Over the entire frequency band we need to integrate V_{n1} to get

$V_{n1\_total}^2 = 4kTR\left(B + f_0\ln\left(f_{up}/f_{low}\right)\right)$

[5]

b) $V_{n2} = g_m R_L V_{n1} = g_m R_L\sqrt{4kTR\left(1+\frac{f0}{f}\right)}$. $V_{n2\_total}^2 = 4kTRg_m^2R_L^2\left(B+\ln\left(f_{up}/f_{low}\right)\right)$

[3]

c) $V_{n3}^2 = 2eI_{DC}R^2$, spectrally white, so that $V_{n3\_total}^2 = 2eI_{DC}R^2B$

[3]

d) $V_{n\_total}^2 = \left(V_{n1}+V_{n2}\right)^2 + V_{n3}^2 = 4kTR\left(B+f_0\ln\left(1000\right)\right)\left(1+g_mR_L\right)^2 + 2eI_{DC}R^2B$

The numbers given are $4kTR = 1.6\times10^{-17}$, $B + f_0\ln 1000 = 20000 + 1000\times6.9 = 26900$

$\left(1+g_mR_L\right)^2 = 9$

$2eI_{DC}R^2B = 6.4\times10^{-13}$ so that

$V_{n\_total}^2 = 3.87\times10^{-12} + 6.4\times10^{-13} = 4.51\times10^{-12} \Rightarrow V_{nRMS} = 2.12\mu V$

[5]

e) $$F = 20\log\frac{SNR_{in}}{SNR_{out}} = 20\log\frac{\frac{S}{N}}{\frac{GS}{GN+N_A}} = 20\log\frac{GN+N_A}{GN} = 20\log\left(1+\frac{NA}{GN}\right)$$

The source noise voltage is $V_{NS}^2 = 4kTR_SB = 192\times10^{-15}V^2$

And the noise figure is $F = 20\log\left(1+\frac{4.51\times10^{-12}}{100\times192\times10^{-15}}\right) = 1.83dB$

[4]

Q4 ANSWER [Mostly bookwork, presented as computed example]

a) Gains:

Phase detector: K_d.

Filter $F(s)$.

VCO: K_o / s.

Multiplier: N.

$$B(s) = \frac{\varphi_{out}}{\varphi_{in}} = \frac{K_d K_o F(s)}{s + K_d K_o F(s) N}$$

The steady state response is :

$$B(0) = \frac{K_d K_o F(s)}{K_d K_o F(s) N} = \frac{1}{N}$$

This circuit divides the frequency of the input signal by a factor of N.

[4]

b) The loop gain for this circuit is $G_L = K_d K_o F(s) N / s$. If N>>1 and $F(s) = \frac{1}{1+s\tau}$,

$G_L = K_d K_o F(s) N / s = \frac{1}{1+s\tau} K_d K_o N / s$. If $N \to \infty$, we can estimate G_L at high frequencies as $G_L = K_d K_o N / s^2 \tau$ So that the phase margin is zero. Any delay in the loop will make this circuit unstable.

[4]

c) A device that emits one or two very narrow impulses in every period of an input signal. Consequently, its output spectrum contains power in high order harmonics (>100) of the input signal. Used as high order frequency multiplier. If a comb generator is used in fig. 4 then the input can be a very high frequency and the phase detector can be a diode.

[4]

d) The circuit just described is called the transfer oscillator. It is a PLL with a multiplier in the feedback path. It functions as a frequency divider, and is used to generate sub-harmonics of extremely high frequencies. for precise measurements.

[4]

e) A transfer oscillator can be used to divide an input frequency by a high number (eg 100). A couple such sub-harmonic generation stages can bring the signal in a range where direct counters can measure it t any required precision.

[4]

Q5 ANSWER

a) [BOOKWORK] Linearity, offset, missing codes, monotonicity

[4]

b) [Computed example]

Input RMS power is $P = 0.5mW$. since $SQNR = 10\log\left(6A^2/q^2\right) = 6$, for 1 bit system, the quantisation noise power is $P_{NQ} = P_{sig} / SNQR = 0.5mW / 6 = 84\mu W$ RMS. The power spectral density is $\frac{dP_{QN}}{df} = \frac{P_{QN}}{2f_S} = \frac{84\mu W}{10^7 s^{-1}} = 8.33pJ$. This is spectrally white.

c) [Computed example]

The thermal noise power over a bandwidth of $B = 2f_s$ is simply $P_J = kT = 4\times10^{-21}J$ so the ratio is 2×10^9 . This is a voltage ratio of 44×10^3, or 15.45 bits.

[4]

d) [Computed example]

By simple oversampling plus averaging we gain 0.5 bits/bit. An oversampling ratio of $OSR = 2^{24} = 16.78\times10^6$ is required. This allows a maximum of 0.3 measurements / second.

[4]

e) [Computed example]

With a 1st order $\Delta\Sigma$ modulator, we get 1.5 bits / bit of oversampling. So we need $OSR = 2^8 = 256$ for a maximum sampling rate of $f_s = 5\times10^6 / 256 = 19500$

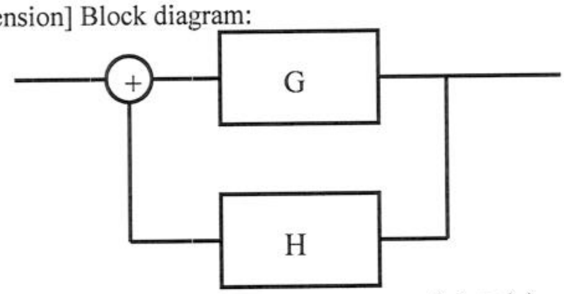
[4]

Q6 ANSWER

a) [bookwork] Frequency and phase can be defined for periodic signals. The period of a periodic signal is the smallest T such that $f\left(t+T\right) = f\left(t\right)$, $\forall t$. The phase of a periodic signal is the fractional part of time normalised to the period. So we can write that for times between: $nT \le t \le (n+1)T \Rightarrow t = \left(n + \phi/2\pi\right)T$. Phase is ambiguous and as such not a good measurement of time. The (fundamental) frequency is the inverse of the period.

[4]

b) [bookwork + extension] Block diagram:



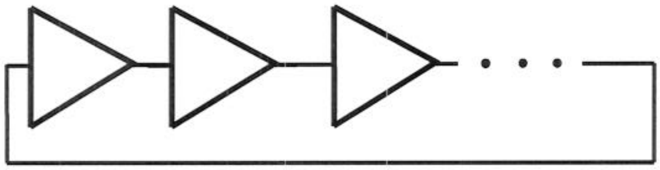
Necessary condition: satisfy the Barkhausen criteria: $G\left(s\right)H\left(s\right) = 1$, or $\left|G\left(s\right)H\left(s\right)\right| = 1$, $\arg\left(G\left(s\right)H\left(s\right)\right) = 1$.

Sufficient condition: $\ni f$: $\left|G\left(s\right)H\left(s\right)\right| > 1$, $\mathrm{Re}\left(G\left(s\right)H\left(s\right)\right) > 0$.

Example oscillator: Phase delay oscillator, G is an inverting amp, H is a number of 1st order low pass sections.

[4]

c) [new theory]



i. Gain magnitude must exceed unity to start up.

ii. With an odd number of inverters the condition of oscillation is: $(2n+1)\tau = \frac{T}{2}$

iii. With an even number of inverters the oscillation condition is satisfied at DC: no oscillation

iv. $(2n+1)\tau = \frac{T}{2} \Rightarrow (2n+1)\omega_0\tau = \pi$ . . The odd harmonics of f satisfy:

$$\omega_k = (2k+1)\omega_0 \Rightarrow (2n+1)\omega_k\tau = (2n+1)(2k+1)\omega_0\tau = (2k+1)\pi$$

Since phase is understood $\bmod \pi$, the odd harmonics are supported (and as a result the oscillator supports square and triangle wave modes)

[6]

f) [Computed example]

The uncertainty in a counting experiment where an unknown signal at f_x is divided by N and used to gate a counter running at f_{ref}, is:

$$N = \text{int}\left(\frac{\Delta T}{T}\right) = \text{int}\left(\frac{Df_x}{f_{ref}}\right) \Rightarrow \frac{N}{D} f_{ref} < f_x < \frac{N+1}{D} f_{ref}$$

If we exchange the roles of f_{ref} and f_x we can similarly write

$$\frac{M}{D} f_x < f_{ref} < \frac{M+1}{D} f_x$$

$$\left.\begin{array}{l} \frac{N}{D} f_{ref} < f_x < \frac{N+1}{D} f_{ref} \\ \frac{M}{D} f_x < f_{ref} < \frac{M+1}{D} f_x \Rightarrow \frac{D}{M+1} f_{ref} < f_x < \frac{D}{M} f_{ref} \end{array}\right\} \Rightarrow$$

$$\max\left(\frac{N}{D}, \frac{D}{M+1}\right) < \frac{f_x}{f_{ref}} < \min\left(\frac{N+1}{D}, \frac{D}{M}\right)$$

[6]