

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2014

MSc and EEE/EIE PART IV: MEng and ACGI

Corrected Copy

**DISCRETE-EVENT SYSTEMS**

Thursday, 22 May 10:00 am

Time allowed: 3:00 hours

**There are FOUR questions on this paper.**

**Answer ALL questions.**

*All questions carry equal marks*

**Any special instructions for invigilators and information for candidates are on page 1.**

Examiners responsible      First Marker(s) :      D. Angeli  
Second Marker(s) :      E.C. Kerrigan

1. An office is organized with 3 queues of people waiting to be served. Arrivals at the  $i$ -th queue ( $i = 1, 2, 3$ ) are denoted by  $a_i$ , departures from the  $i$ -th queue are denoted by  $d_i$ .
  - a) Build a finite deterministic automaton  $G$  that models the queues, arrivals and departures, assuming each queue can at most contain 1 person waiting. [ 4 ]
  - b) Assume next that when a new customer needing service from queue 1 arrives and queue 1 is already full while queue 2 or queue 3 is not, then an exception  $o_1$  is generated (meaning 'overflow' 1), and the customer is physically sent to the next available empty queue. Similarly, if a new customer needing service from queue 2 arrives and queue 2 is already full while queue 3 or queue 1 is not, then an exception  $o_2$  is generated (meaning 'overflow' 2), and the customer is physically sent to the next available empty queue (queue 3 or 1, respectively). Finally, if a new customer needing service from queue 3 arrives and queue 3 is already full while queue 1 or queue 2 is not, then an exception  $o_3$  is generated (meaning 'overflow' 3), and the customer is physically sent to the next available empty queue (queue 1 or 2, respectively). Modify the previous automaton by including exceptions  $o_1$ ,  $o_2$  and  $o_3$  as possible transitions. Denote the new automaton by  $G_O$ . [ 2 ]
  - c) Design an automaton  $G_L$  that is meant to act as a labeling device to discriminate between the situation in which  $o_1$  has not occurred and the one in which  $o_1$  has occurred. [ 2 ]
  - d) Compute the parallel composition  $G_O || G_L$ . [ 3 ]
  - e) Assume that events  $o_i$  ( $i = 1, 2, 3$ ) are partially observable, namely only a generic event  $o$ , "overflow" would be generated each time any of  $o_1$ ,  $o_2$  or  $o_3$  occurs. Replace in  $G_O || G_L$  events  $o_i$  with a generic event  $o$  and denote by  $G_N$  the resulting non-deterministic automaton and by  $f_N$  the associated transition map; identify the state(s)  $x$  of  $G_N$  for which  $f_N(x, o)$  is a set of cardinality bigger than 1. [ 2 ]
  - f) Build a diagnoser  $G_D$  that can decide if events  $o_1$  have occurred (meaning YES / NO / MAYBE) by processing events of type  $a_1, a_2, a_3, d_1, d_2, d_3$  and  $o$  only. [ 7 ]

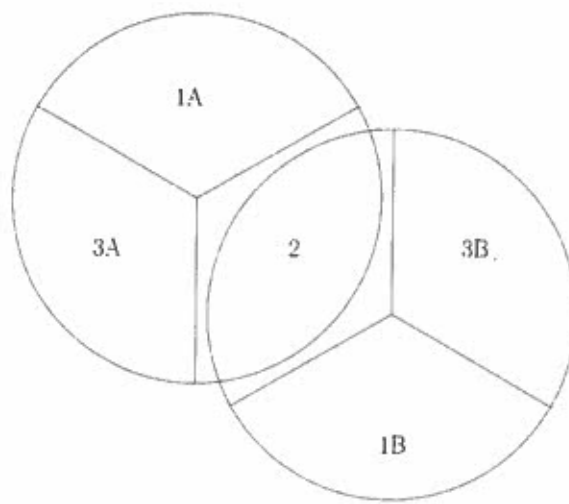


Figure 2.1 Robots A and B operating areas

2. Two robots operate in overlapping regions of space, as shown in Fig. 2.1. Robot A is a high priority robot that rotates clockwise between 3 regions of space (regions 1A, 2 and 3A, respectively). At each tick of a clock it either stays in the same position (event  $t_s$ ) or it moves to the next position in a clockwise direction, event  $t_c$ . Robot B is a low priority robot that can move asynchronously between positions 3B and 1B, and 1B and 3B, following events  $c$  or  $a$ , respectively. When in position 1B an event  $c$  can trigger transition to a synchronized mode of operation in which, following events  $t_s$  or  $t_c$ , the robot enters position 2, and then leaves it at the following tick of the clock in favour of position 3B (again event  $t_c$  or  $t_s$ ). Back in position 3B or 1B, the robot is free to operate asynchronously, and events  $t_s$  or  $t_c$  leave its position unaffected.
- Build a finite deterministic automaton  $G_A$  modeling Robot A and its occupation of positions 1, 2, 3, respectively, assuming position 1A as initial state. [ 4 ]
  - Build a finite deterministic automaton  $G_B$  modeling Robot B, its occupation of positions 1, 2, 3 and the transition to the synchronous mode of operation, assuming position 1B as initial state. [ 4 ]
  - Compute the parallel composition  $G_A || G_B$ . [ 4 ]
  - Design an automaton  $H$  to implement the specification that robots should not both operate simultaneously in position 2. [ 2 ]
  - Is this specification controllable, assuming the set of uncontrollable events to be  $E_{uc} = \{t_c, t_s\}$ ? (justify your answer). [ 2 ]
  - Realize a supervisor that implements the supremal controllable sublanguage  $\mathcal{L}(H)^C$ . [ 4 ]

3. A machine works according to the following set of rules: pieces to be processed are dispatched to the machine, event  $a$ , and accumulated in a stack; in order to process the pieces, event  $p$ , the machine needs to first pick up two distinct tools, Tool A and Tool B, (events  $pA$  and  $pB$  respectively); after processing the pieces the machine may decide to release the tools (event  $rA$  and  $rB$  respectively). Processed pieces can then exit the factory, event  $x$ .
- a) Build a model of the machine, the arrival of pieces and their departure from the factory as well as the tools  $A$  and  $B$  employed by means of a marked Petri Net  $\langle N, M_0 \rangle$ . [ 5 ]
  - b) Assume that two machines are sharing the same tools, how can you modify the previous model to take into account this situation? [ 3 ]
  - c) How can you modify the previous network in order to take into account that only a finite number of processed pieces can be hosted inside the factory? [ 2 ]
  - d) For the Petri Net in item a), compute the P-invariant vectors; what is their physical meaning? [ 3 ]
  - e) For the Petri Net in item a), sketch the coverability graph. [ 4 ]
  - f) Which places of network  $N$  are structurally bounded ? Do they differ from the places which are behaviourally bounded for  $N$ , with initial marking  $M_0$  as specified in item a)? [ 3 ]

4.

In a car park it has been observed that cars arrive with a normal probability distribution of arrival times, at a rate  $\lambda_a$ . On the other hand, cars parked inside leave with a normal probability distribution of departure times with a rate  $\lambda_d$ .

- a) Assume next that the car park has a total of 3 available parking spaces and that arrivals of cars when the parking is full do not occur. Build a continuous time Markov Chain to model the time behaviour of the total number of parked cars. (*Hint: pay attention to the rate at which cars leave the car park when occupancy is equal to n*). [ 6 ]
- b) What is the asymptotic average number of cars in the car park? [ 4 ]
- c) Assume next that parking lots are numbered 1,2,3. Assume that when a car comes it always takes the first available slot (viz. the one with the smallest number). Sketch the transition diagram of a Markov chain modelling the occupancy of all 3 parking slots. (*Hint: 8 states are needed*). [ 5 ]
- d) Consider now the occupancy of parking slot 1 only. Consider the stochastic process that takes the value TAKEN when parking 1 is occupied and AVAILABLE otherwise. Show, by using the Markov chain in the previous item, that this can be modeled as a Markov chain with 2 states. Find the average occupancy of parking slot 1. [ 3 ]
- e) Explain why a similar argument cannot be repeated for parking slot 2. [ 2 ]