

Paper Number(s): **EE1-5**

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING  
EXAMINATIONS 2011

EEE Part I: MEng, BEng and ACGI

## **ENERGY CONVERSION**

Monday, 6 June 10:00 am

Time allowed: 2:00 hours

**There are THREE questions on this paper.**

**Answer ALL questions.**

**Question 1 carries 40% of the marks. Questions 2 and 3 carry equal marks (30% each).**

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible:

First Marker(s): W.T. Pike, E.M. Yeatman

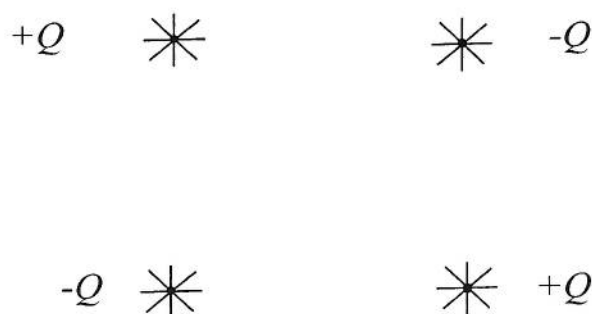
Second Marker(s): B.C. Pal

## Constants and Formulae

permittivity of free space:  $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$   
permeability of free space:  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$   
charge of an electron:  $e = 1.6 \times 10^{-19} \text{ C}$

1.

- a) Give an example of how a smart grid better allows a transmission system to handle unpredictability in electricity transmission. [2]
- b) Using Ampere's law, derive the magnitude of the magnetic flux density one metre away from a straight conductor in air, carrying a 1 A current. [4]
- c) Why does the introduction of a dielectric between the plates of a capacitor cause an increase in the capacitance? [2]
- d) List three major options to reduce the carbon emissions of electricity production. [3]
- e) What are the modes of energy transfer that take place between a unit of coal at a generating station to a unit of electrical power supplied to a domestic customer? [4]
- f) Why are diamagnetic materials repelled by a magnetic field? [2]
- g) Two positive and two negative charges are arranged at the corners of a square as below in Fig. 1g. Copy the figure to your answer book, and complete the lines of electric field, indicating the direction of the field lines. [3]



*Fig. 1g: Arrangement of four charges showing electric field lines in the vicinity of the charges*

- h) An oscillating magnetic flux of peak magnitude 10 mWb and frequency 50 Hz passes through a single turn, open circuit coil. What is the magnitude of the induced voltage? [4]
- i) An ideal transformer of secondary-to-primary turns ratio 20:1 is connected to a load of 100  $\Omega$ . What is the input-referred load? [4]
- j) A simple DC motor with an applied voltage of 10 V and an armature resistance of 5  $\Omega$  has an armature current of 400 mA. What is the mechanical power? [4]
- k) Briefly explain how laminating the core of a transformer can improve its efficiency. [4]
- l) Give two advantages of the use of permanent magnet excitation in a DC machine. [4]

2.

- a) By consideration of the power expended by a constant current source charging an initially uncharged capacitor to a voltage  $V$  with a charge  $Q$ , show that the energy stored in a capacitor is given by  $\frac{1}{2} QV$ . Hence show that the energy per unit volume between the plates of a parallel plate capacitor with a dielectric of relative permittivity  $\epsilon_r$  is given by  $\frac{1}{2} \epsilon_0 \epsilon_r E^2$ , where  $E$  is the electric field between the plates. [12]
- b) A power transmission system comprises a series resistive and inductive load of impedance  $23+j7\Omega$  connected to a 50-Hz AC supply via a resistive cable.
- i) Describe qualitatively how introducing a compensating capacitance in parallel with the load can maximise the efficiency of transmission. [4]
- ii) Calculate the capacitance of the optimal compensating capacitor in this case. [14]

3.

- a) An inductor is constructed by winding  $N$  turns of a coil around an iron toroidal core as shown in Figure 3.1(a). The toroid has mean radius  $r_o$  and cross-sectional area  $A$ , and relative permeability  $\mu_r$ . Using Ampere's Law, derive an expression for the inductance  $L$ . Clearly state any assumptions or approximations used. [6]
- b) The core in part (a) is now replaced with one having a narrow air gap of width  $g$  as shown in Figure 3.1(b). Derive an expression for the inductance  $L$  in this case, again clearly stating any assumptions or approximations used. [8]

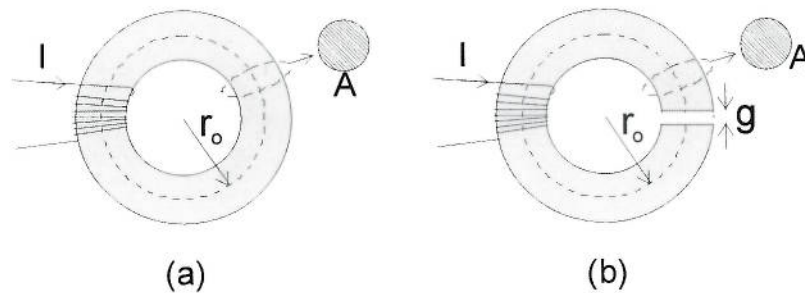


Figure 3.1

- c) Figure 3.2 shows an equivalent circuit for a transformer. Briefly discuss the physical effects corresponding to each of the four components  $R_b$ ,  $X_m$ ,  $X_{tt}$  and  $R_{tt}$ . [4]
- d) Open and short circuit tests are carried out on the transformer of Figure 3.2, with results as below. Calculate the value of each of the components  $R_b$ ,  $X_m$ ,  $X_{tt}$  and  $R_{tt}$ . Clearly state any assumptions or approximations used. [12]

Open Circuit Test:  $V_I = 110 \text{ V}$ ,  $I_{in} = 23 \text{ mA}$ ,  $I_{in}$  lags  $V_I$  by  $31^\circ$ .  
 Short Circuit Test:  $V_I = 4 \text{ V}$ ,  $I_{in} = 0.62 \text{ A}$ ,  $P_{in} = 1.10 \text{ W}$ .

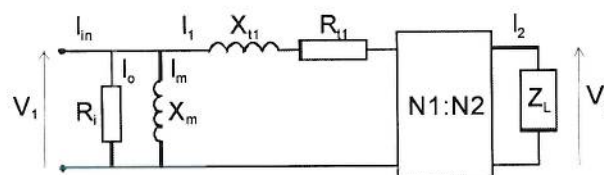


Figure 3.2

## ANSWERS

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1a) A dishwasher providing scheduling information to the supplier so that it is provided with power to complete its cycle by a certain time, for instance return from work, but allows the supplier to schedule when according to energy availability.

b) From

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I,$$

$$2\pi r B = \mu_0 I,$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Hence 1 m away from a current of 1 A,  $B = 2 \times 10^{-7}$  T.

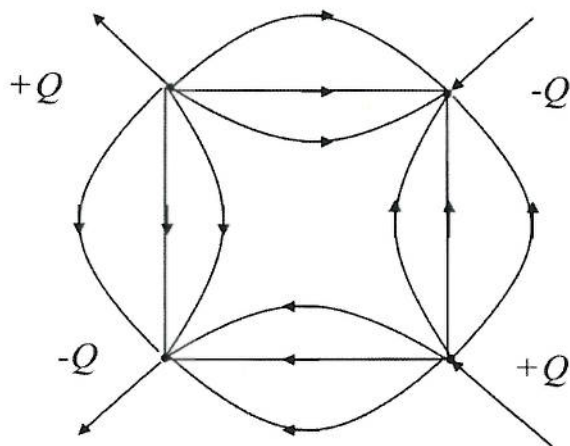
c) The dielectric will be polarised by the field between the plates, producing surface charges on the faces of the dielectric opposite to the charges on the capacitors. This additional surface charge attracts more charge on to the plates, hence increasing the capacitance.

d) Renewables, carbon capture with fossil fuel, nuclear.

e) Chemical energy in carbon-carbon bonds of coal converted on combustion to produce thermal energy in heated steam. Thermal energy of steam converted to kinetic energy in turbine. Kinetic energy of turbine converted to electrical energy in generator. Electrical energy transmitted through a series of transformers to consumer as a unit of electrical energy.

f) In diamagnets, the orbiting electron, which can be regarded in this case as small current loops, adjust their effective currents to produce an internal field opposing any change in field as per Lenz's law. Hence diamagnetic materials will be repelled by the introduction of a magnetic field.

g)



h)  $V = d(NF)/dt$ ,  $F = F \sin(\omega t)$ ,  $N=1$ ,  $V = \omega F \cos(\omega t)$ ,  $V_0 = 2\pi f F_0 = 3.14$  V

i)  $ZL' = (N_1/N_2)^2 ZL = 202(100) = 40$  kW.

j)  $e_a = V_a - I_a R_a = 8$  V;  $P_{\text{mech}} = E_a I_a = 8(0.4) = 3.2$  W



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k) The oscillating magnetic flux induces current loops in the conducting core, which wastes power. Laminating the core as iron slices separated by insulating oil layers breaks the current paths, greatly reducing the size of the eddy current loops and thus their power.

l) Possible answers include:

- Avoid wasted  $I^2R$  power in the field winding
- Possibility of brushless design (magnets in rotor)
- Fewer parts in assembly

2a) The current source produces a constant increase in the charge, and hence voltage on the capacitor until the voltage reaches  $V$ .

If the time to charge the capacitor is  $T$ ,  $v(t) = Vt/T$  and the power expended by the current source will be given by  $P(t) = Iv(t)$ . [3]

The work done by the current source will be

$$\begin{aligned} W &= \int_0^T Iv(t) dt \\ &= \frac{IV}{T} \int_0^T t dt \\ &= \frac{IVT}{2} \end{aligned} \quad [4]$$

As  $IT = Q$ , the final charge on the capacitor,  $W = 1/2 QV$ . Hence as there are no other energy losses, the energy stored on the capacitor  $E = W = 1/2 QV$ . [2]

The capacitance is given by  $C = Q/V$ , and hence  $E = W = 1/2 CV^2$ . [1]

For a parallel-plate capacitor of area  $A$ , spacing  $d$  and a dielectric of permittivity,  $\epsilon_r$ ,

$$\begin{aligned} C &= \frac{\epsilon_r \epsilon_0 A}{d}, \\ E &= \frac{\epsilon_r \epsilon_0 AV^2}{2d} \end{aligned} \quad [1]$$

The volume between the plates is  $Ad$ , giving an energy density,  $\xi$ ,

$$\xi = \frac{\epsilon_r \epsilon_0 V^2}{2d^2} \quad [1]$$

As the field between the plates of a parallel plate capacitor  $E = V/d$ , this gives the energy density as  $\frac{1}{2} \epsilon_0 \epsilon_r E^2$  as requested. [1]

b) i) With the correct compensating capacitor, the reactive power for the inductor element of the load is supplied by the capacitor and not by the supply. Hence the current to supply this power does not need to be transmitted along the resistive transmission line, avoiding the power losses here. [4]

ii) The impedance of the load with the parallel capacitance  $C$  is given by

$$\begin{aligned} Z_L &= \frac{1}{j\omega C} \parallel (25 + j7) \\ &= \frac{25 + j7}{j\omega 25C + \omega 7C + 1} \end{aligned} \quad [3]$$

For maximum efficiency, the power transmitted should not have any reactive element, which implies the load must be real. [3]

Hence the phase of the numerator should equal the phase of the denominator: [2]



$$\begin{aligned}\frac{7}{25} &= \frac{25\omega C}{1+7\omega C} \\ \omega C(25^2 - 7^2) &= 7 \\ \omega C &= \frac{7}{24} \\ C &= \frac{7}{2\pi \times 50 \times 24} \quad [4] \\ &= 0.93mF\end{aligned}$$

3a) Taking a path around the centre of the core. Ampere's Law:

$$\oint_P \underline{H} \cdot d\underline{\ell} = NI$$

We assume no flux leakage from the coil, and that  $\underline{H} \parallel d\underline{\ell}$ . By symmetry  $H$  is constant on path  $P$  so  $H(2\pi r_0) = NI$

$$B = \mu_r \mu_0 H = \frac{\mu_r \mu_0 NI}{2\pi r_0}$$

Assume  $B$  along the core centre equals the average  $B$  in the core, so that  $\Phi = BA$ , then using  $L = N\Phi/I$  we get  $L = \frac{\mu_r \mu_0 N^2 A}{2\pi r_0}$

b) Now we have  $\oint \underline{H} \cdot d\underline{\ell} = H_i l_i + H_g g = NI$   
Assume  $l_i \approx 2\pi r_0$ . We know  $B_g = \mu_0 H_g$  and  $B_i = \mu_r \mu_0 H_i$ . Approximate  $B_i = B_g = B$ .  
Then  $B\left(\frac{l_i}{\mu_r} + g\right) = \mu_0 NI$

$$L \approx NBA/I = \frac{\mu_r \mu_0 N^2 A}{2\pi r_0 + \mu_r g}$$

c)  $R_{ti}$ : coil resistance  $X_{ti}$ : flux leakage  
 $X_m$ : magnetisation reactance, ~~and~~  $R_i$ : iron losses

$$\begin{aligned} \text{d) Open ckt test: admittance} &= \frac{1}{R_i} - \frac{j}{X_m} = \frac{I_{in}}{V_{in}} \\ &= \frac{0.023 \cos 31^\circ}{110} - j \frac{0.023 \sin 31^\circ}{110} = (1.79 - j 1.08) \times 10^{-4} \end{aligned}$$

$$R_i = 5587 \Omega \quad X_m = 9259 \Omega$$

Short ckt: assume current through  $R_i$  &  $X_m$  negligible

$$P_{in} = I_{in}^2 R_{ti} \quad R_{ti} = 2.86 \Omega$$

$$V_{in}^2 = I_{in}^2 (R_{ti}^2 + X_{ti}^2) \quad X_{ti} = 5.78 \Omega$$