Traffic Theory & Queueing Systems

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Q1 a) i) Each of the channels can be viewed as a Bernoulli trial with success probability

$$P(\text{Busy} \rightarrow \text{Free}) = \mu \Delta t$$

ii) Since the channels are acting independently the probability that exactly k channels will become idle in $(t, t+\Delta t)$ is binomial

$$P(k \text{ channels} \rightarrow \text{idle}) = \binom{i}{k} (\mu \Delta t)^k (1-\mu\Delta t)^{i-k}$$

iii) Therefore

$$P(1 \text{ channel} \rightarrow \text{idle}) = \binom{i}{1} \mu \Delta t (1-\mu\Delta t)^{i-1}$$

$$= i\mu\Delta t + o(\Delta t)$$

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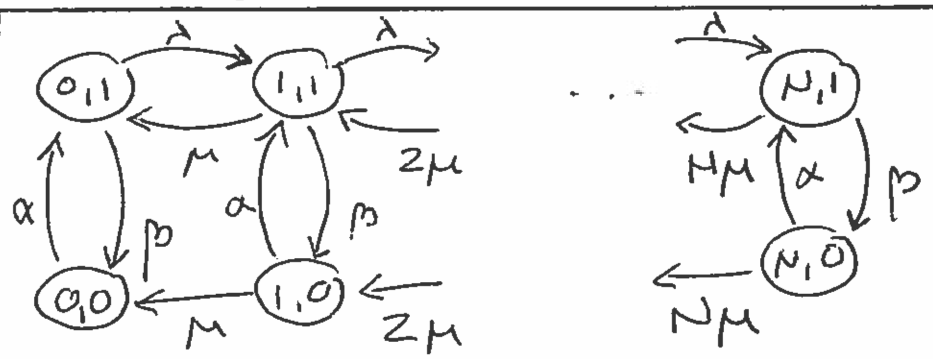
Q1 a)



offered traffic, $\rho = \frac{\lambda}{\mu} = 9.4$ Erlangs

$\frac{1}{\mu} = 12.5$ s — $\mu = 0.08$

$P[\text{link 1 in saturation}] = E_{12}(9.4) = 0.1$

$P(\text{link in saturation}) = \frac{T_{on}}{T_{on} + T_{off}} = 0.1$

$\Rightarrow T_{on}/T_{off} = 1/9$

$E[T_{on}] = E[\text{sojourn time in saturation}]$

$= \frac{1}{N\mu} = 2.08$ s

$E[T_{off}] = 18.76$ s

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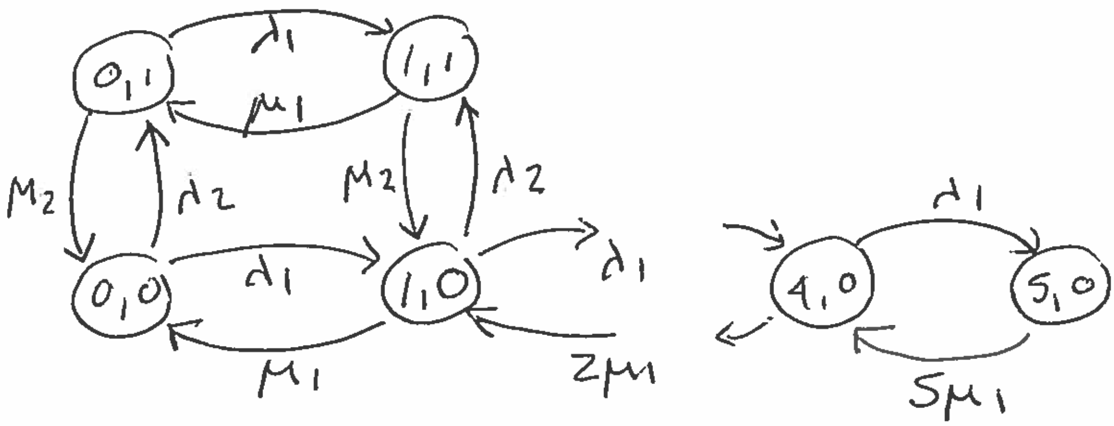
Q2 a)



$(N_t, B_t) =$ N_t = Nr of Type 1 calls in prog.
B_t = Nr of Type 2 calls in prog.

If the Nr of channel were infinite the processes $\{N_t\}$ and $\{B_t\}$ would not interfere i.e. would be independent B/D processes

Truncation of the state space of a reversible process does not destroy reversibility

$\Rightarrow$ Equilibrium distribution is of product form with Poisson-like factors:

iii) $$\pi_{ij} = K\left(\frac{\rho_1^i}{i!}\right)\left(\frac{\rho_2^j}{j!}\right) \qquad i+4j \le 5$$

$$K = \left[\sum_{i=0}^{5}\frac{\rho_1^i}{i!} + \rho_2\sum_{i=0}^{1}\frac{\rho_1^i}{i!}\right]^{-1} = 0.269$$

iv) B Type 1 $= \pi_{5,0} + \pi_{1,1} = 0.137$

B Type 2 $= 1 - (\pi_{0,0} + \pi_{1,0}) = 0.462$

Q 2 (b)

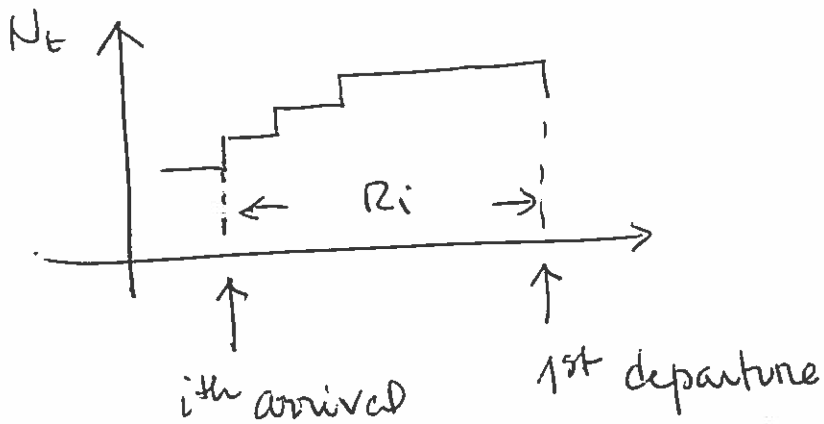
i) The effect of re-attempts is important under heavy-traffic conditions. If re-attempts occur soon after the initial blocking the resulting arrival stream will not be a Poisson stream.

ii) S_i = service time

W_i = waiting time

Q_i = queue length found on arrival

R_i = Residual service time = time until first departure seen by ith arrival



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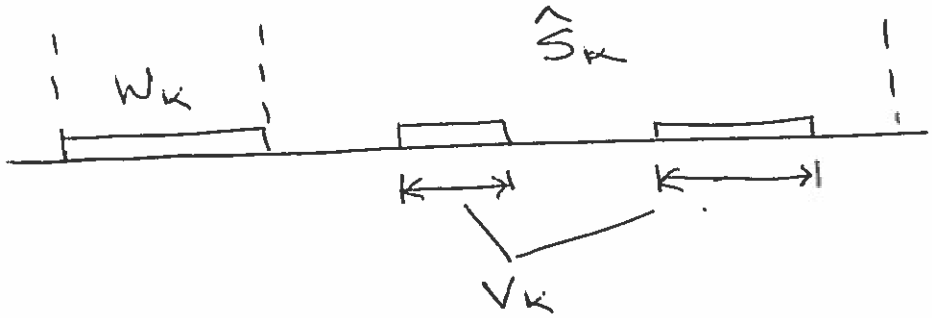
Q3 a)



V_K = work brought into the system, during $\hat{S}_K$ by higher-priority arrivals

$$E[V_K] = \sum_{i=1}^{k-1} (\lambda_i E[\hat{S}_K]) E(s_i)$$

$$= \left(\sum_{i=1}^{k-1} \rho_i\right) E[\hat{S}_K] \quad , \rho_i = \lambda_i E[s_i]$$

$$= \sigma_{K-1} E[\hat{S}_K]$$

Then

$$E[\hat{S}_K] = E[\text{true service time}] + E[\text{Interrupt time}]$$

$$= E[S_K] + E[V_K]$$

$$= E[S_K] + \sigma_{K-1} E[\hat{S}_K]$$

$$E[\hat{S}_K] = \frac{E[S_K]}{1 - \sigma_{K-1}}$$

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Q3
a)
i) system capacity VC calls/s.

x incremented by $\frac{V}{\alpha}$ cells during a talk spurt

Equivalent capacity $\frac{VC}{\frac{V}{\alpha}} = \alpha C$ "unit of information"

ii) If i sources ON the iV cells/s

$$\frac{iV}{\frac{V}{\alpha}} = \alpha i \text{ "unit of information"}$$

iii)
$$F_i(t+\Delta t, x) = [N-(i-1)]\lambda \Delta t\, F_{i-1}(t, x)$$
$$+(i+1)\alpha \Delta t\, F_{i+1}(t, x)$$
$$+\{1-[(N-i)\lambda + i\alpha]\Delta t\} F_i[t, x-\underbrace{(i-c)\alpha\Delta t}_{\Delta t}]$$
$$+ o(\Delta t)$$

explanation and discussion

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Q4 a) i) M|M|K|N system

$N = K + B \qquad B = N - K$

N = system capacity , K = NR of servers

ii)

$P(\text{delay}) = P(\text{all } K \text{ servers busy} \mid \text{buffer not full})$

$= P(K \leq N_t < K+B)$

$= \Pi_K \left(\dfrac{1-\rho^B}{1-\rho} \right)$ .. (derivations)

iii) $P(\text{loss}) = P(\text{buffer full})$

$= P(N_t = K+B)$

$= \Pi_K \rho^B$

where $\Pi_K = \left(\dfrac{A^K}{K!} \right) \Pi_0$

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Q4 i) The state space of the system

$S = \{0, S, L, 1, 2, 3\}$

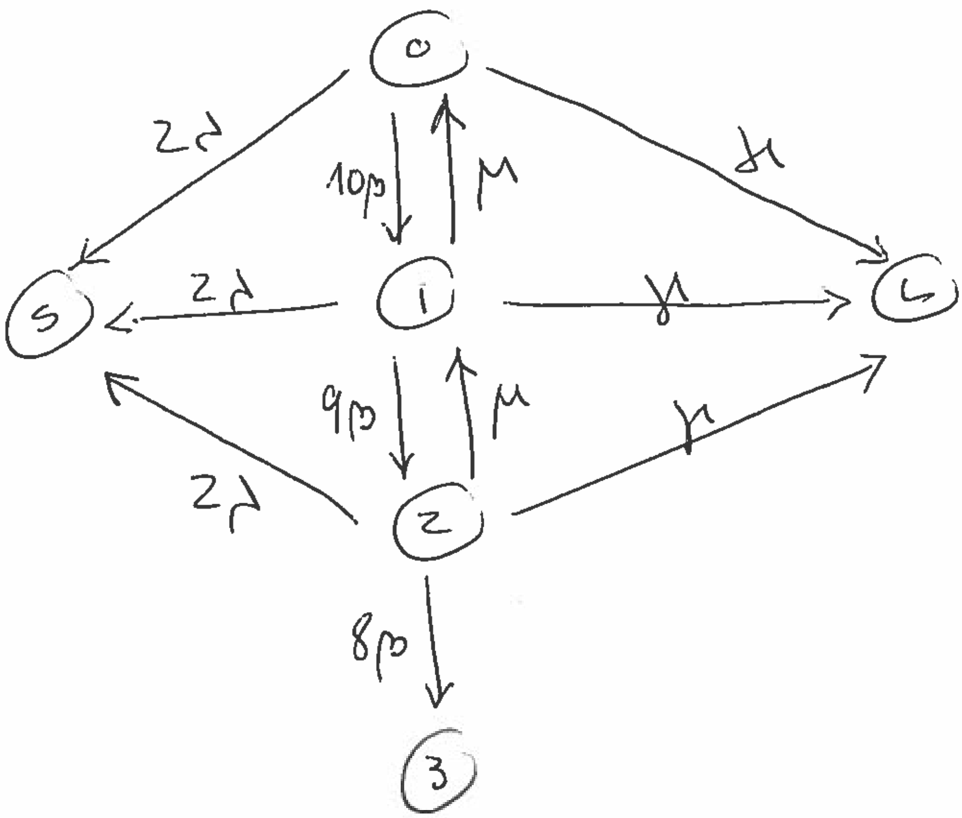
0 = fully operational state

S = switch failure

L = link failure

1, 2, 3 = Nr of access node in failure (1, 2, 3)

ii)



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$$
Q = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}
\begin{array}{cccccc}
0 & 1 & 2 & 3 & S & L \\
\end{array}
$$

$$
Q = \left[\begin{array}{cccccc}
-(10\mu+2\lambda+\gamma) & 10\mu & 0 & 0 & 2\lambda & \gamma \\
\mu & -(9\mu+2\lambda+\mu+\gamma) & 9\mu & 0 & 2\lambda & \gamma \\
0 & \mu & -(8\mu+2\lambda+\mu+\gamma) & 8\mu & 2\lambda & \gamma \\
0 & \cdots & & & \cdots & 0 \\
0 & \cdots & & & \cdots & 0 \\
0 & \cdots & & & \cdots & 0
\end{array}\right]
$$