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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2011

MSc and EEE PART IV: MEng and ACGI

HIGH PERFORMANCE ANALOGUE ELECTRONICS

Friday, 27 May 10:00 am

Time allowed: 3:00 hours

There are FOUR questions on this paper.

Answer ALL questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	C. Papavassiliou
	Second Marker(s) :	P. Georgiou

The Questions

1. (a) Explain briefly the role of the oscillator in a superheterodyne receiver.

[3]

(b) Define the noise factor for an electronic system and give an equation for it.

[2]

(c) The input referred noise of an electronic system integrated within a 1KHz band is $100uV^2$. This system is connected to a source which has a total input noise of $2uV^2$ within a 100Hz band. Assuming that all the noise sources are white, what is the noise factor of the system?

[3]

(d) A double superheterodyne receiver is designed using two local oscillators, one of them providing a single tone harmonic at 1MHz, and the second one providing a single tone harmonic at 100KHz. However, in the fabricated circuit the measured first oscillator frequency is 990KHz and the measured second oscillator frequency is 90KHz.

i) Calculate the value of the first and second IF frequencies (i.e. the IF frequencies after mixing with the first and second oscillators) for an RF signal of 500KHz.
ii) How does the value of the second IF frequency differ from the originally intended one?
iii) Based on your answer, mention one advantage double superheterodyne receivers have over single superheterodyne receivers.

[6]

(e) A single superheterodyne receiver is designed to receive RF signals centered at 4MHz with a 500KHz bandwidth. The frequency of the local oscillator is 12MHz. A first order low pass filter is used to attenuate the image signal before mixing with the local oscillator. The cutoff frequency of the filter is 10MHz and its passband gain 0dB.

i) Where in the frequency spectrum is the image signal?
ii) What is the attenuation of the image signal in the middle of its frequency range?
iii) What would you modify in the filter to improve image rejection?

[6]

2. For the circuit in Figure 2.1, where all the transistors are equally sized and operating in the strong inversion saturation region:

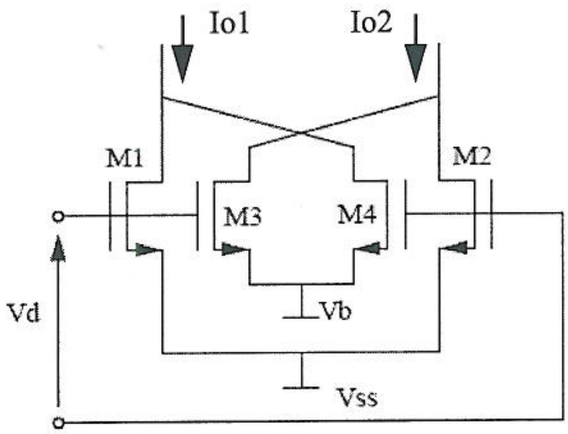

Figure 2.1

(a) Find an expression for the output current Iout=(Io1-Io2) as a function of transistor voltages Vb, Vss and Vd.

[4]

(b) Draw a schematic to indicate how this circuit could be used as a mixer with differential inputs Vin1 and Vin2. Assume that there are no special requirements on the input impedances for both inputs.

[3]

(c) If Vb is set to 0.5V and Vss to 0V, find the β (transconductance parameter) value of the transistors necessary to implement a transconductor with a transconductance of 100mS.

[4]

(d) Assuming that the value of β calculated in (c) is fixed, calculate the voltage that needs to be applied to Vss in order to reduce the transconductance by a factor of 2.

[3]

(e) Explain how you would build an integrator using a fully differential transconductor and two identical capacitors in order to minimize the effects caused by parasitic capacitances. Draw a schematic of such an integrator.

[3]

(f) Explain briefly how transconductors and capacitors can be used to design LC ladder filters.

[3]

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3.

(a) Draw the schematic diagram of a fully linearised differential transconductor using four MOS transistors operating in the strong inversion triode region. Derive an expression for the transconductance of this circuit.

[4]

(b) Is the transconductor in Fig. 3.1 a fully linear transconductor? (Hint: M1 and M2 are operating in the strong inversion saturation region and M3 in the strong inversion ohmic region. Vin=(Vin1-Vin2) and Iout= (Io1-Io2)).

Justify your answer. If your answer is "YES" give an expression for the transconductance. If you answer is "NO" explain which circuit technique you would use to linearise it.

[4]

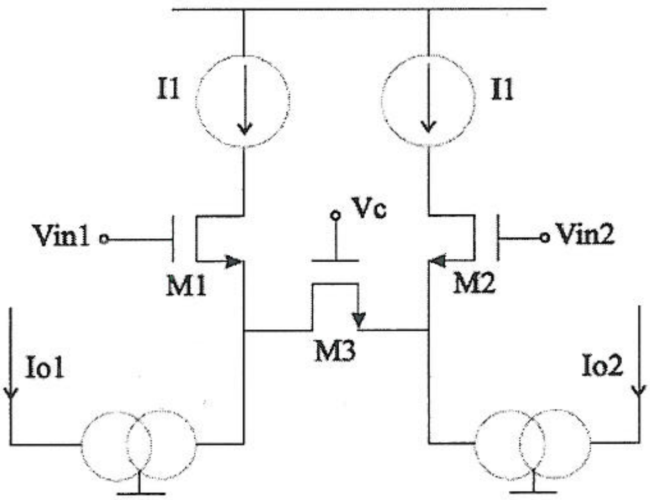

Figure 3.1

Figure 3.2 shows the schematic of a fully differential transconductor where: M1 is operating in the strong inversion ohmic region; M2 and M3 are both equally sized and operate in the strong inversion saturation region; M4, Ma1 and Ma3 operate in the strong inversion saturation region; M3 and Ma3 are equally sized; M4 and Ma1r are equally sized; and N2 is an ideal voltage amplifier. Based on this:

(c) Give an expression for the current flowing through M1, as a function of the transistor's parameters, V_{1p} and V_b.

[3]

(d) Give expressions for the currents flowing through M2, M3 and M_{a3}, M4 and M_{a1r} as a function of transistor voltages V_{1p} and V_b.

[3]

(e) Give an expression for I_{out} as a function of V_{1p}, V_{1n} and V_b.

[3]

(f) Give an expression for the transconductance as a function of voltage V_b.

[3]

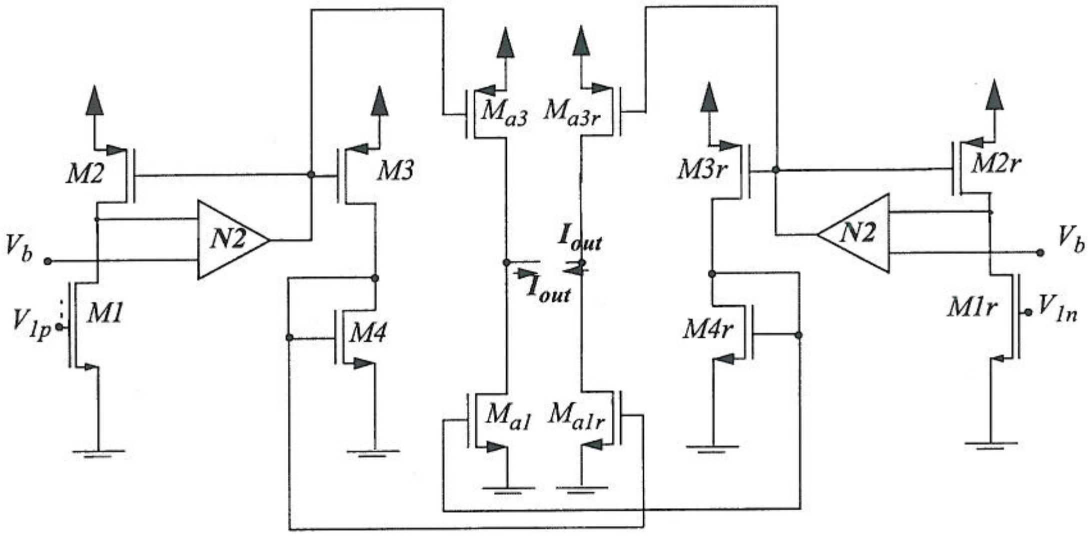

Figure 3.2

4. For the circuit in Figure 4.1:

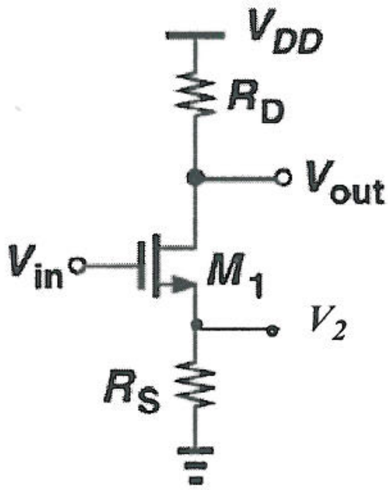

Figure 4.1

(a) Draw the small signal equivalent circuit. You can assume that the output resistance of the transistor is much larger than any other resistance in the circuit.

[3]

(b) Analyse the equivalent circuit from (a) and find an expression for the gain V_{out}/V_{in}.

[4]

(c) Draw the small signal equivalent circuit including all possible noise sources, and give expressions for them.

[4]

(d) Find the equivalent noise power spectral density at the input of the circuit. Assume that the circuit operates in a frequency range where for the transistor flicker noise dominates. Assume also that the noise contribution due to the passive resistors is at least the same order of magnitude as the flicker noise of the transistor.

[3]

(e) Find the equivalent noise power spectral density in node V_2.

[3]

(f) A low pass filter is connected at the output (V_{out}) of the circuit with transfer function

$$H(s) = \frac{1}{1+\left(\frac{s}{10^6}\right)}$$

Give an expression for the total noise power at the input of the filter in a 5Hz to 6HZ bandwidth.

[3]

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1.

(a) (Theory)

If we are trying to select one particular frequency channel from the complete RF spectrum, then we need a bandpass filter to reject any unwanted frequencies. Generally this filter has to be narrowband, and high Q filters are difficult to design at high frequencies. This problem is compounded if the input signal frequency is variable (i.e. the signal is transmitted in one of a number of possible channels, each with the same bandwidth). A tuneable, high Q bandpass filter with constant bandwidth is now required.

The solution is to use a superhet receiver (supersonic heterodyne). This system downconverts the input signal to an intermediate frequency (IF), and a bandpass IF filter is then used to select the wanted signal. The design of the bandpass IF filter is eased since it doesn't have to be tuneable, and the IF centre frequency is much lower than the input RF signal.

The downconversion is performed by 'mixing' (multiplying) the RF input signal (fRF) with a local oscillator signal (fLO), such that the resulting output is at the required IF frequency (fIF).

The oscillator frequency fLO is often tuneable to ensure that a range of input RF frequencies can be selected.

(b) (Theory)

Noise factor is defined as the total output (or input) noise power of the system divided by the output (or input) noise power due to the source alone (Pni).

Noise factor F = (Psys + Pni)/Pni

(c) (Application of theory)

The noise factor must be calculated using the same bandwidth for the system and the source. If all the sources are white, it means that the power spectral densities are, 100nV^2/Hz and 20nV^2/Hz. Therefore, using the formula in (b) the noise factor is 6.

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(d) (Application of theory)

Fist IF is at (990kHz-500kHz)=490kHz
Second IF is at (490kHz-90kHz)= 400kHz

There is no difference with the originally intended one. One of the advantages of double superheterodyne receivers is that if the second local oscillator can be designed such that it tracks any frequency offsets or drift of the first LO, then the effects of these frequency offsets can be minimised.

(e) (Application of theory)

The image signal would be centered at 20MHz, with a frequency range from 19.75MHz to 20.25MHz. The attenuation would be 23dB. To improve the attenuation of the image the filter could be designed with a lower cut-off frequency.

2.

(a) (Theory)

$$Id1 - Id2 = \beta\, Vd(Vc1 - Vth) \qquad Id3 - Id4 = \beta Vd\,(Vc2 - Vth)$$

$$\text{where}\quad Vc1 = \frac{Vgs1 + Vgs2}{2} \quad \text{and} \quad Vc2 = \frac{Vgs3 + Vgs4}{2}$$

$$\text{Thus}\quad Iout = (Id1 - Id2) - (Id3 - Id4)$$

$$= \beta\, Vd\,(Vc1 - Vc2) = \beta\, Vd\,(Vb - Vss)$$

(b) (Application of theory)

Vin1 can be connected as Vd and Vin2 between Vd and Vss, or viceversa.

(c) (Computed example)

The transconductance is given by:

$G_m = \beta(Vb - Vss)$. From there, β should be 200mA/V.

(d) (Computed example)

Once β is fixed, in order to reduce the transconductance, the difference between Vb and Vss has to be also reduced. In this case Vss=0.25V.

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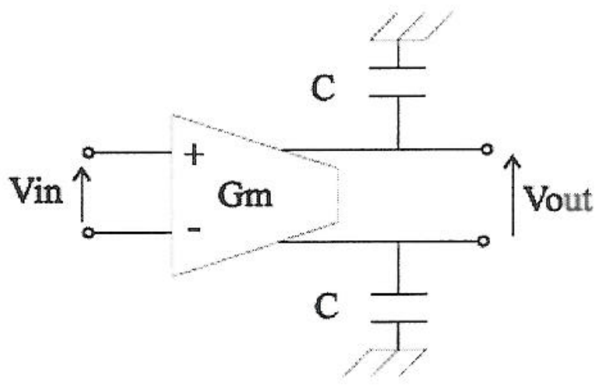
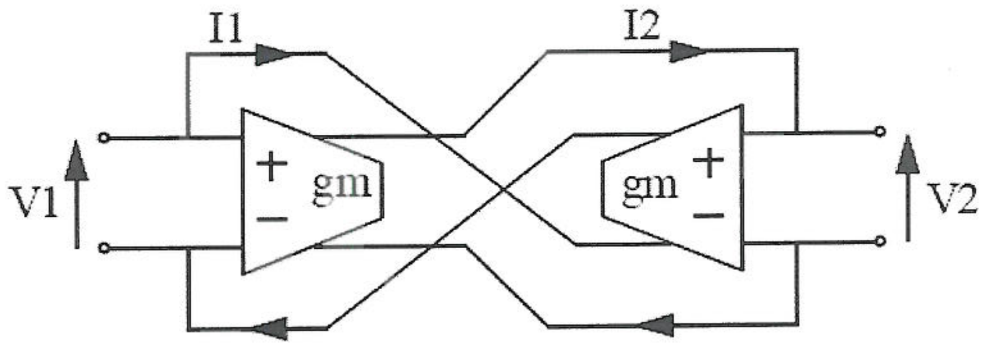
(e) (Theory)



Alternatively the two capacitors could be connected in parallel across Vout, but with their top and bottom plates inverted.

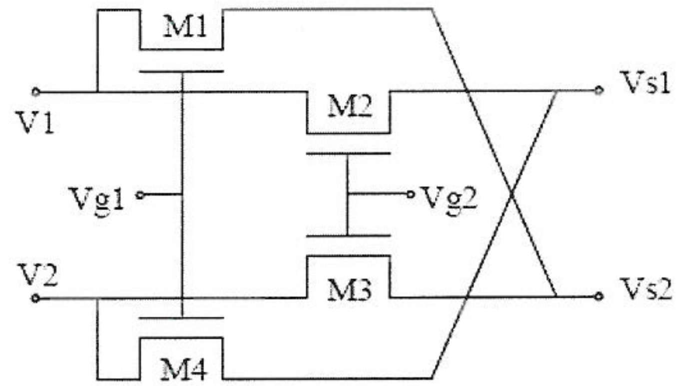
(f) (Theory)

Substituting the inductors in the ladder by gyrators:



3.

(a) (Theory)



If Vs1=Vs2=Vs

$$Id1 = G1(V1\text{-}Vs) - \beta\,(V1\text{-}Vs)^2 \qquad Id2 = G2\,(V1\text{-}Vs) - \beta\,(V1\text{-}Vs)^2$$
$$Id3 = G2\,(V2\text{-}Vs) - \beta\,(V2\text{-}Vs)^2 \qquad Id4 = G1\,(V2\text{-}Vs) - \beta\,(V2\text{-}Vs)^2$$

where $G1 = 2\,\beta(Vg1 - Vth)$ $G2 = 2\,\beta\,(Vg2 - Vth)$

$$I1 - I2 = (Id1 + Id3) - (Id2 + Id4)$$
$$= (G1 - G2)\,(V1 - V2)$$

And the transconductance is:

$$G_m = 2\beta(Vg1 - Vg2)$$

(b) (Application of theory)

No,

$$Iout = 2\ Id3 = 4\beta(Vgs3\text{-}Vth)Vin - 2\beta Vin^2$$

It can be linearised using a cross-couple configuration instead.

(c) (New theory)

$$I_1 = \beta_1(V_{1p} - V_T)V_b$$

(d) (New theory)

The same as in (c) for all of them.

(e) (New theory)

$$I_{out} = (I_{a3} - I_{a1})$$

Since it is a fully differential configuration, the current in Ma1r is the same as in Ma1, but with V1n in the expression instead. Hence:

$$I_{out} = \beta_1(V_{1p} - V_{1n})V_b$$

(f) $G_m = \beta_1 V_b$

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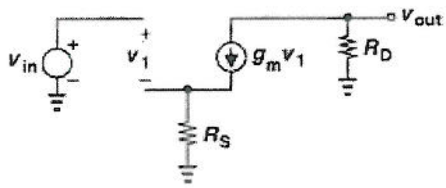
4.

(a) (New theory)



(b) (New theory)

$$A_v = \frac{v_{out}}{v_{in}} = -\frac{g_m R_D}{1+g_m R_S}$$

(c) (New theory)

The new schematic would be as in (a) but with a current (or voltage) noise source from v_{out} to v_2 that accounts for the thermal noise in the channel plus a voltage noise source at the input that accounts for the flicker noise of the transistor. Another voltage noise source in v_2 (or negative terminal of v_1) that represents the thermal noise of R_s, and another voltage noise source in vout which represents the thermal noise of R_D.

The expressions are;

Thermal between drain and source, i.e. between vout and v2: $ind^2 = \frac{8kTgm}{3}\left(\frac{A^2}{Hz}\right)$

Flicker in series with the gate: $vng^2 = \frac{k_f}{C_{ox}WLf}\left(\frac{V^2}{Hz}\right)$

Thermal due to R_D: $v_{RD}^2 = 4kTR_D\left(\frac{V^2}{Hz}\right)$

Thermal due to Rs: $i_{Rs}^2 = 4kTR_s\left(\frac{V^2}{Hz}\right)$

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(d) (New theory)

Since the Flicker noise dominates in the transistor, thermal noise does not need to be considered. The noise contribution of the Flicker noise, is already at the input of the circuit, so it is just a matter of transfering the other two sources of noise (i.e. the ones generated by RD and Rs) back to the input. The noise generated by RD is alredy at the output so, the equivalent at the input can be found dividing by the power gain between input and output, i.e. the square value of the expression calculated in (b):

$$v^2_{RD(input)} = \frac{4kTR_D}{A_v^2}\left(\frac{V^2}{Hz}\right)$$

And for Rs, first the power gain from V2 to Vin has to be found from the circuit in (a):

$g_m(v_{in} - v_2) = \frac{v_2}{R_s}$, and from here:

$$v^2_{Rs(input)} = \frac{4kTR_s\left(g_m + \frac{1}{R_s}\right)^2}{g_m^2}\left(\frac{V^2}{Hz}\right)$$

The total power spectral density of noise at the input is:

$$P_{input} = \left[\frac{k_f}{C_{ox}WLf} + \frac{4kTR_D}{A_v^2} + \frac{4kTR_s\left(g_m + \frac{1}{R_s}\right)^2}{g_m^2}\right]\left(\frac{V^2}{Hz}\right)$$

(e) (New theory)

Using the result in (d), and the power gain from V_{in} to V_2:

$$P_{V_2} = \left[\frac{k_f}{C_{ox}WLf} + \frac{4kTR_D}{A_v^2} + \frac{4kTR_s\left(g_m + \frac{1}{R_s}\right)^2}{g_m^2}\right]\left(\frac{g_m}{g_m + \frac{1}{R_s}}\right)^2\left(\frac{V^2}{Hz}\right)$$

(f) (New theory and computed example)

The filter has no influence in the power spectral density within that band, since the cutoff frequency is at a much larger frequency. Based on that it is a matter of integrating the expression in (d) using as integration limits 6Hz and 5Hz:

$$\text{Noise}_{\text{input}} = \left[\frac{k_f}{C_{ox}WLf}\ln\left(\frac{6}{5}\right) + \frac{4kTR_D}{A_v^2} + \frac{4kTR_s\left(g_m + \frac{1}{R_s}\right)^2}{g_m^2}\right](V^2)$$