

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2009

EEE/ISE PART II: MEng, BEng and ACGI

Corrected Copy

CONTROL ENGINEERING

Friday, 29 May 2:00 pm

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.

Answer Q1 and any two of questions 2-4.

Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	I.M. Jaimoukha, I.M. Jaimoukha
	Second Marker(s) :	S. Evangelou, S. Evangelou

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1. a) Figure 1.1 illustrates an RLC circuit. The capacitor has capacitance C , the inductor has inductance L and the resistor resistance R . The input is the applied voltage $v_i(t)$ and the output is the voltage across the capacitor $v_o(t)$.

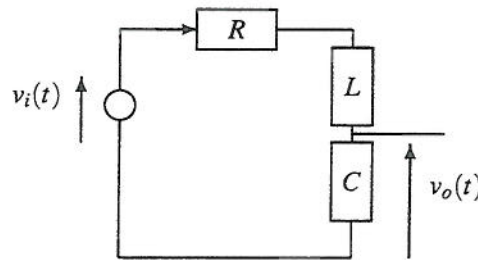


Figure 1.1

- i) Determine the transfer function relating v_o to v_i . [4]
 - ii) Let $v_i(t)$ be a unit step applied at $t = 0$. Use the final value theorem, which should be stated, to find the steady-state value of $v_o(t)$. [5]
 - iii) Set $L = 0.25 \text{ H}$ and $C = 4 \times 10^{-6} \text{ F}$. Derive the value of R so that the response is critically damped. [5]
- b) In Figure 1.2 below, $G(s) = \frac{s-1}{(s+1)(s+2)}$ and K is a variable gain.
- i) Sketch the locus of the closed-loop poles for $0 \leq K < \infty$. [5]
 - ii) Using the gain criterion, find the value of K for which the closed-loop is marginally stable. [4]
 - iii) Hence find the range of $K \geq 0$ for which the closed-loop is stable. [4]
- c) In Figure 1.2 below, $G(s) = \frac{5}{(s+1)^3}$ and K is a variable gain.
- i) Draw the Nyquist diagram of $G(s)$ indicating real-axis intercepts. [5]
 - ii) Take $K = 2$. Use the Nyquist criterion, which should be stated, to determine the number of unstable closed-loop poles. [4]
 - iii) Take $K = 1$. Determine the gain margin. [4]

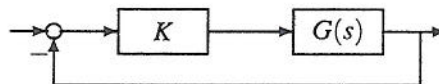


Figure 1.2

2. Consider the feedback system in Figure 2 for voltage regulation. Here, $v_r(t)$ is the reference voltage and $v_o(t)$ is the supplied output voltage. R is a resistance value which is fixed but is unknown.

The op-amp open-loop output voltage E is related to v_e as $E(s) = -G(s)v_e(s)$, where the transfer function $G(s)$:

- is first order,
- has a DC gain A ,
- has a time constant of 1 second.

- a) Derive an expression for $G(s)$ in terms of A . [6]
- b) Derive an expression for $v_e(s)$ in terms of $v_r(s)$ and $v_o(s)$. [6]
- c) Derive an expression for $v_o(s)$ in terms of $v_e(s)$. [6]
- d) Hence, derive and draw a block diagram representation of the feedback loop. Take the reference to be $-v_r(s)$ and the output to be $v_o(s)$. Indicate the signal $v_e(s)$ on the block diagram. [6]
- e) Find the minimum value of the DC gain A such that the closed-loop system has a time constant of 10^{-3} seconds. [6]

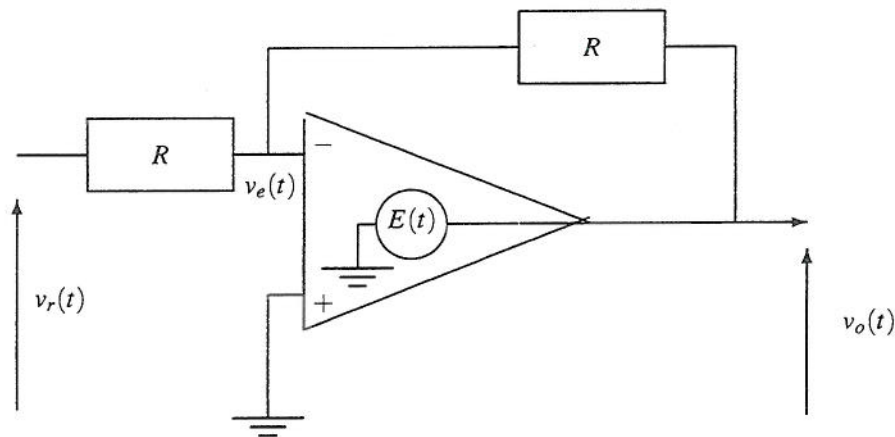


Figure 2

3. Let

$$G(s) = \frac{1}{s(s-1)}$$

and consider the feedback loop shown in Figure 3 below.

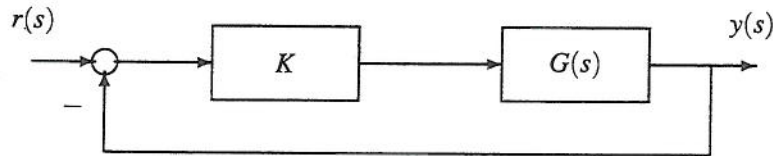


Figure 3

- a) Draw the root locus of $G(s)$ accurately for all $K > 0$. [4]
- b) Draw the root locus of $G(s)$ accurately for all $K < 0$. [3]
- c) Using the answers to Parts (a) and (b), or otherwise, show that there exists no proportional compensator such that the feedback loop is stable. [3]
- d) A feedback compensator utilizing rate feedback is required such that the following design specifications are satisfied:
 - The closed-loop is stable.
 - The closed-loop system has a damping ratio $\zeta = 1/\sqrt{2}$.
 - The closed-loop response has a settling time of 2 seconds.
 - i) Derive the location of the closed-loop poles that satisfy the design specifications. [5]
 - ii) Draw a feedback loop incorporating the rate feedback compensator. The compensator should have two design parameters K and K_v which should be clearly shown on the diagram. [5]
 - iii) Derive the values of the parameters K_v and K that achieve the design specifications. [5]
 - iv) Draw the root locus of the compensated system. [5]

4. Consider the feedback control system in Figure 4 below. Here,

$$G(s) = \frac{2(s+1)}{(s-1)^2}$$

and $K(s)$ is the transfer function of a compensator.

- a) Sketch the Nyquist diagram of $G(s)$, clearly indicating the low and high frequency portions, as well as the real-axis intercepts. [6]
- b) Let $K(s)$ be a constant compensator $K(s) = K$, where $K > 0$. State the Nyquist stability criterion and use the Nyquist diagram to determine the stability of the close-loop system when:
- i) $K > 1$, [4]
- ii) $K < 1$, [4]
- iii) $K = 1$. [4]
- c) Take $K = 1$. Comment on the gain margin for the feedback loop. [6]
- d) Without doing any actual design, briefly describe how a phase-lag compensator,

$$K(s) = \frac{1 + s/\omega_0}{1 + s/\omega_p}, \quad 0 < \omega_p < \omega_0$$

would affect the stability of the feedback loop. [6]

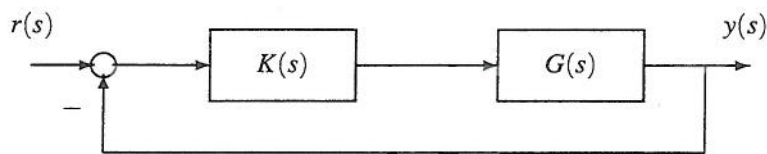


Figure 4

SOLUTIONS

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1. a) i) Using the potential divider rule and the impedances we have

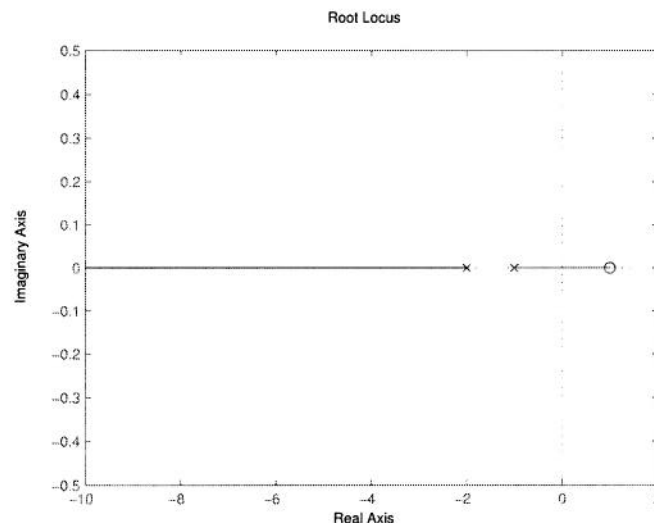
$$G(s) := \frac{v_o(s)}{v_i(s)} = \frac{(LC)^{-1}}{s^2 + sRL^{-1} + (LC)^{-1}}.$$

- ii) Using the final value theorem and the fact that $v_i(s) = 1/s$,

$$\lim_{t \rightarrow \infty} v_o(t) = \lim_{s \rightarrow 0} s v_o(s) = \lim_{s \rightarrow 0} s G(s) v_i(s) = \lim_{s \rightarrow 0} s G(s) \frac{1}{s} = G(0) = 1.$$

- iii) For a critically damped response, the poles are equal and so $R^2 = 4L/C$. Thus $R = 2\sqrt{L/C} = 500 \Omega$.

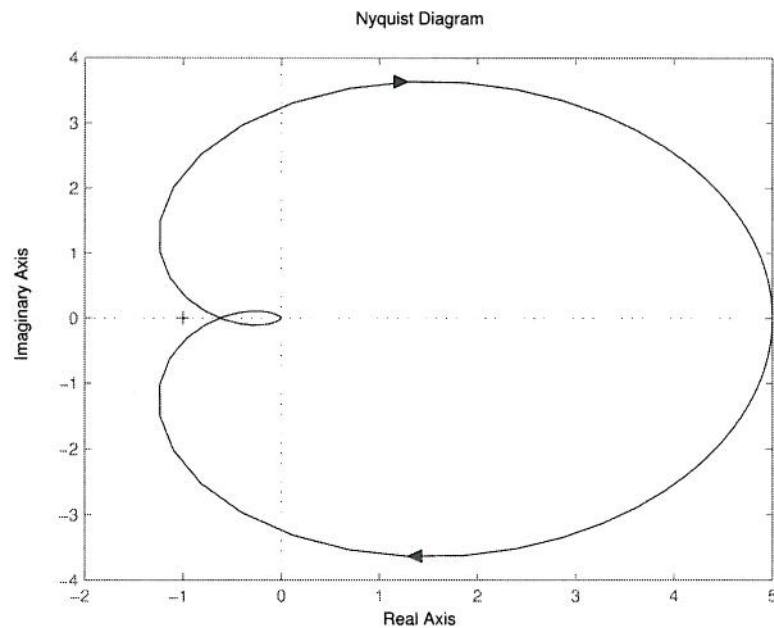
- b) i) The root locus is shown below.



- ii) The closed-loop is marginally stable when at least one pole is on the imaginary axis and all others are in the left half-plane. It follows from the root locus that the marginal pole is at $s = 0$. Using the gain criterion $K = -1/G(0) = 2$.

- iii) It follows from the root locus that the closed-loop is stable for all $0 \leq K < 2$.

- c) i) The Nyquist diagram is shown below. To find the real-axis intercepts,



we consider the extra gain K necessary for marginal stability for the characteristic equation $1 + G(s)K = 0$:

The Routh array :

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 3 & 1+5K \\ s & \frac{8-5K}{3} & \\ 1 & 1+5K & \end{array}$$

Thus $K = 8/5$ and so the intercept is at $-5/8 = -0.625$.

- ii) The Nyquist criterion states that $N = Z - P$ where N is the number of clockwise encirclements by $G(s)$ of the point $-1/K$ as s traverses the Nyquist contour, which in this case is equal to 2 ; P is the number of unstable open-loop poles, which in this case is 0; and Z is the number of closed-loop poles. Thus there are $Z = N + P = 2$ unstable closed-loop poles.
- iii) When $K = 1$, the Nyquist criterion indicates the closed-loop is stable. The gain margin is the amount of gain that the loop can tolerate before becoming marginally stable and in this case is $1/0.625 = 1.6$.

2. a) A first order transfer function $G(s)$ with DC gain A and time constant 1 second has the form

$$G(s) = \frac{A}{s+1}$$

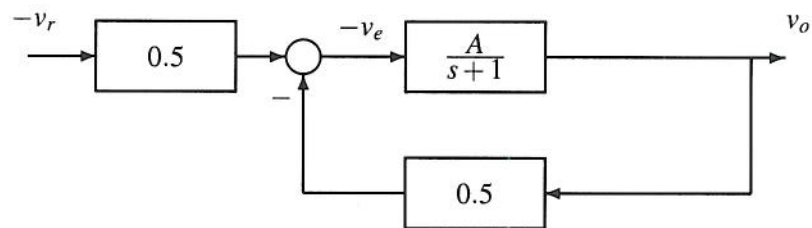
- b) Using a potential divider rule at the op-amp input gives

$$\frac{v_e(s) - v_r(s)}{v_o(s) - v_r(s)} = \frac{R}{R+R} = \frac{1}{2} \Rightarrow -v_e(s) = -0.5v_r(s) - 0.5v_o(s).$$

- c) At the op-amp output we have

$$E(s) = v_o(s) \Rightarrow v_o(s) = -\frac{A}{s+1}v_e(s).$$

- d) Using parts (a) and (b), the block diagram becomes,



- e) Using the block diagram in part (d) and a manipulation gives

$$v_o(s) = -\frac{0.5A}{s+1+0.5A}v_r(s).$$

The time constant is now

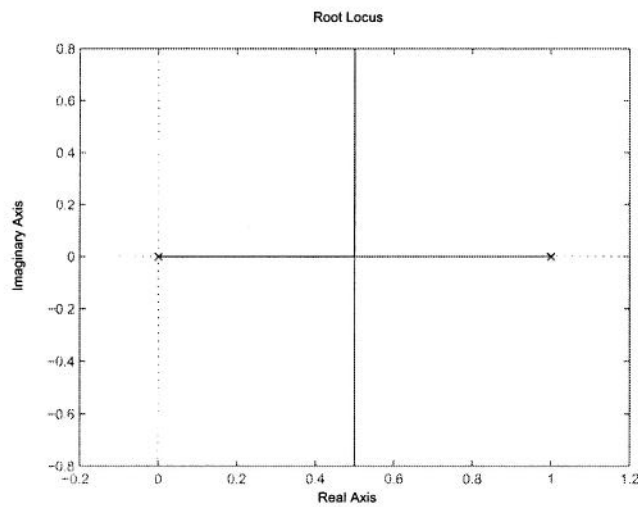
$$T = \frac{1}{1+0.5A}.$$

So, for a time constant $T \leq 10^{-3}$ we need

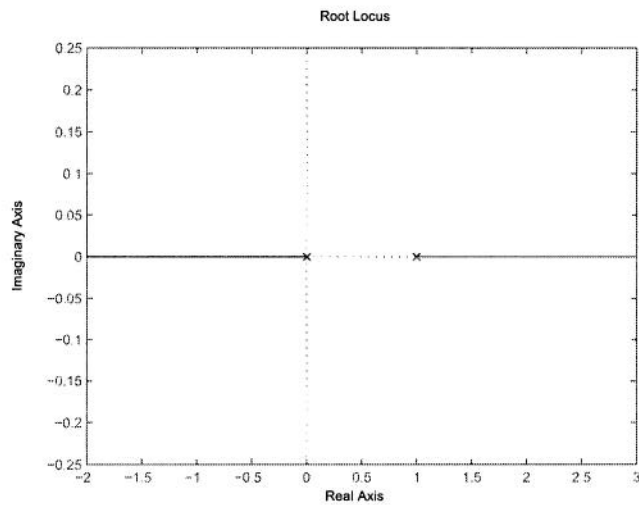
$$\frac{1}{1+0.5A} \leq 10^{-3}$$

or $A \geq 2 \times 10^3$.

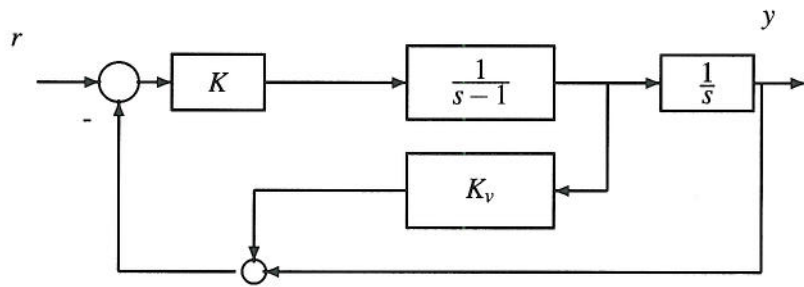
3. a) The root locus is shown below.



- b) The root locus is shown below.



- c) It is clear from the root loci above that there is at least one closed-loop pole in the right half plane and so the closed loop is always unstable.
- d) i) For $\zeta = 1/\sqrt{2}$ the real and imaginary parts of the pole are equal. For a settling time of 2 seconds, the real part must be equal to -2. Thus the closed-loop poles must be placed at $s_1, \bar{s}_1 = -2 \pm j2$.
- ii) The block diagram is shown below.



- iii) The characteristic equation is $1 + KK_v \frac{s+1/K_v}{s(s-1)} = 0$. The location of the zero $z = -1/K_v$ can be determined from the angle criterion:

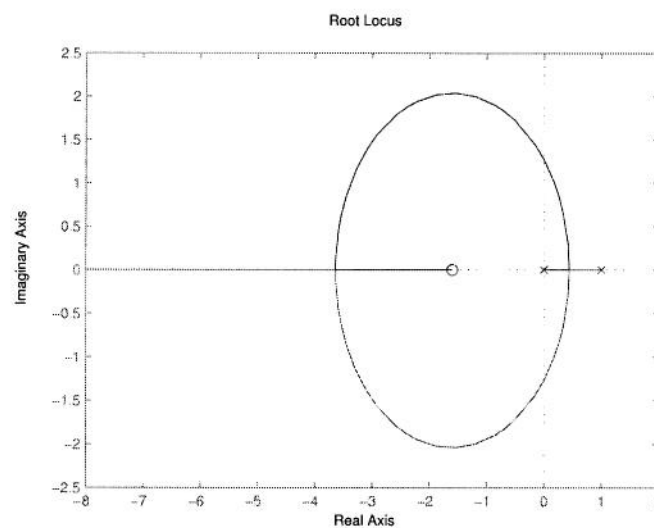
$$\theta = 116^\circ + 146.3^\circ - 180^\circ \sim 101.3^\circ$$

which is satisfied by $z = -1.6$. So, $K_v = 0.625$. Finally, K is obtained from the gain criterion:

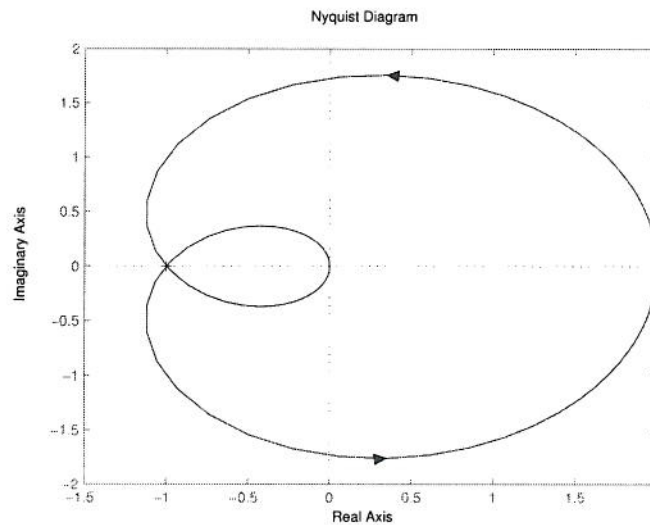
$$KK_v = -s_1(s_1 - 1)/(s_1 + 1.6) = 5 \Rightarrow K = 8$$

where $s_1 = -2 + j2$.

- iv) The root locus of the compensated system is shown below.



4. a) The Nyquist diagram is shown below. The real-axis intercepts can be found by setting the imaginary part of $G(j\omega)$ to zero. This gives $G(j\omega_i) = 2, -1, 0$, respectively.



- b) When $K(s) = K$, we have $N = Z - P$ where N is the number of clockwise encirclement by the Nyquist diagram of the point $-K^{-1}$, P is the number of unstable open-loop poles and Z is the number of unstable closed-loop poles. Here, $P = 2$.
- i) When $K > 1$, $N = -2$ so $Z = 0$ and the closed-loop is stable.
 - ii) When $K < 1$, $N = 0$ so $Z = 2$ and the closed-loop is unstable.
 - iii) It follows that when $K = 1$, the closed-loop is marginally stable.
- c) Since the gain can be increased without bound the system has infinite gain margin for increasing gain. Since any decrease in gain will result in an unstable closed-loop, the system has zero gain margin for decreasing gain.
- d) The phase-lag compensator has gain close to unity for frequencies below ω_p and gain close to $\frac{\omega_p}{\omega_0} < 1$ for frequencies beyond ω_0 . It follows that the compensator will destabilize the feedback loop since the open-loop is marginally stable and any reduction in gain at high frequencies will make the closed-loop system unstable.