

[E1.10 (Maths 1) 2012]

B.ENG. and M.ENG. EXAMINATIONS 2012

**PART I : MATHEMATICS 1 (ELECTRICAL AND INFORMATION
SYSTEMS ENGINEERING)**

Date Thursday 7th June 2012 10.00 - 12.00

DO NOT OPEN THIS PAPER UNTIL THE INVIGILATOR TELLS YOU TO.

Answer Question 1 and THREE of the remaining five

Answer Section A and Section B in different answerbooks.

Question 1 carries twice the marks of each of the other questions.

CALCULATORS MAY NOT BE USED.

A mathematical formulae sheet is provided.

[Before starting, please make sure that the paper is complete; there should be 6 pages, with a total of SIX questions. Ask the invigilator for a replacement if your copy is faulty.]

SECTION A

[E1.10 (Maths 1) 2012]

1. (i) Write

$$z^2 + z^{*2} = -8$$

in Cartesian form.

Given that $z = x + iy$, solve the above equation for all possible values of y in terms of x .

- (ii) Find all possible values of

$$(2 - 2i)^{2/3}$$

in polar form.

- (iii) Find q so that the limit

$$\lim_{x \rightarrow \infty} e^{qx} \left(e^{2x/3} - (2 + e^x)^{2/3} \right)$$

is finite and non-zero.

Do not use L'Hôpital's rule.

- (iv) Find the limit

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2}$$

Do not use L'Hôpital's rule.

You can use $\lim_{x \rightarrow 0} \sin(x)/x = 1$.

- (v) Differentiate

$$(\sin(x))^{\sin(x)}$$

- (vi) Integrate

$$\int \frac{\sin(x) + 1}{\sin(x) - 1} \, dx$$

- (vii) Integrate

$$\int_0^1 \frac{x}{(1 - x^2)^{1/3}} \, dx$$

Q1 CONTINUES ON THE NEXT PAGE

(viii) Find the Taylor expansion of

$$\frac{\ln(x-1)}{x}$$

about $x = 2$ to first order (up to and including the term linear in x) and state the remainder term $R_2(x)$.

(ix) Find the general solution of the following first order ODE:

$$y'(x) = \frac{y(x)}{x} + 1$$

(x) Find the general solutions of the following second order ODE:

$$y''(x) + 4y'(x) + 4y(x) = \exp(x)$$

SECTION B

[E1.10 (Maths 1) 2012]

2. Find $\frac{dy}{dx}$ as a function of x in each of the following cases :

(i) $y = \frac{\exp(x)}{\exp(x^2)} ;$

(ii) $y = \exp(\sin^{-1}(x));$

Note: $\sin^{-1}(x)$ denotes the inverse sin function

(iii) $y = \ln(\cos(x^{\sin(x)})) ;$

(iv) $\cos(y) = \sin(x) ;$

Find the following n th derivative

(v) $\frac{d^n}{dx^n} (x^2 \exp(x/2)).$

3. Evaluate the following limits *without using L'Hôpital's rule unless specified:*

(i) $\lim_{x \rightarrow -1} \frac{(x-2)(x+2)}{(x-3)(x-1)} ;$

Do not use L'Hôpital's rule.

(ii) $\lim_{x \rightarrow -1} \frac{(x-2)(2x^2-2)}{(x-3)(x+1)} ;$

Do not use L'Hôpital's rule.

(iii) $\lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{1+x} - \sqrt{x}) ;$

Do not use L'Hôpital's rule.

(iv) $\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1} \text{ for integer } n > 0;$

Do not use L'Hôpital's rule.

(v) $\lim_{x \rightarrow \pi} \frac{\sin(x) + (x - \pi)}{(x - \pi)^3} ;$

Use L'Hôpital's rule here.

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4. (i) State whether the improper integral $\int_0^1 x \ln(x) \, dx$ is finite and calculate its value if it is.

Note: $\lim_{x \rightarrow 0} x \ln(x) = 0$.

- (ii) Show that $\int_0^\infty x^n \exp(-x) \, dx = n \int_0^\infty x^{n-1} \exp(-x) \, dx$ for any positive integer n , i.e. $0 < n \in \mathbb{N}$

- (iii) Integrate $\int \cos^3(x) dx$.

- (iv) Integrate $\int \frac{x^2 - 1}{x^3 + 2x^2 - 3x} dx$.

5. (i) Write the following equation in Cartesian form and then solve it for all possible values of y in terms of x :

$$|z + 1| = z^* + z$$

- (ii) Express in polar form

$$1 + 2i \quad \text{and} \quad (\sqrt{3} + i)^{1/3}$$

- (iii) Express $\sin(4\theta)$ in terms of $\sin(\theta)$ and $\cos(\theta)$.

- (iv) Integrate $\int \cos(x) \exp(2x) dx$.

Note: You can use $\int \exp(ax) \, dx = \frac{1}{a} \exp(ax) + C$ for complex $a \in \mathbb{C}$.

6. (i) Find the solution $y(x)$ of the differential equation

$$\frac{dy}{dx} = \frac{y^2 + x^2 + 2(x + y) + 2}{2x^2 + 4x + 2}$$

Note: It might help to solve the differential equation for $Y=y+1$ as a function of $X=x+1$.

- (ii) Find the solution $y(x)$ of the differential equation

$$\frac{1}{x} \frac{dy}{dx} + \frac{y}{x^2} + \exp(x) = 0$$

- (iii) Find the solution $y(x)$ of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 2\sin(3x),$$

that satisfies $y(0) = 1$, $\frac{dy}{dx}(0) = 0$.

END OF PAPER

	EXAMINATION QUESTIONS/ <u>SOLUTIONS</u> 2011-2012	Course EE1.1
Question Q1	TOPIC General (long question)	Marks & seen/unseen
Parts	<p>(i) $z^2 + z^{*2} = -8$ find all roots. $2x^2 - 2y^2 = -8 \Rightarrow y = \pm \sqrt{x^2 + 4}$</p> <p>(ii) $(2-2i)^{2/3}$ in polar form. $(2-2i)^{2/3} = 8^{1/3} e^{-i(\frac{\pi}{4} + 2\pi k)\frac{2}{3}}$ $= 2 e^{-i(\frac{\pi}{6} + \frac{4}{3}\pi k)}$ $k=0,1,2$ so: $2e^{-i\frac{\pi}{6}}, 2e^{-i\frac{3\pi}{2}} = 2e^{i\frac{\pi}{2}}$ and $2e^{-i\frac{17}{6}\pi} = 2e^{-i\frac{5}{6}\pi} = 2e^{i\frac{7}{6}\pi}$</p> <p>$(2+2i)^{1/3}$ in polar form: $(2+2i)^{1/3} = 8^{1/6} e^{i(\frac{\pi}{4} + 2\pi k)\frac{1}{3}}$ $= \sqrt{2} e^{i(\frac{\pi}{12} + \frac{2k}{3}\pi)}$ $k=0,1,2$ so: $\sqrt{2}e^{i\frac{\pi}{12}}, \sqrt{2}e^{i\frac{5}{4}\pi} = \sqrt{2}e^{-i\frac{3}{4}\pi}$ and $\sqrt{2}e^{i\frac{17}{12}\pi} = \sqrt{2}e^{-i\frac{7}{12}\pi}$</p>	<p>4 seen similar</p> <p>4 seen similar</p>
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	EXAMINATION QUESTIONS /SOLUTIONS 2011-2012		Course EEL1
Question Q1	TOPIC		Marks & seen/unseen
Parts (iii)	<p>Find q so that $\lim_{x \rightarrow \infty} e^{qx} \left(e^{\frac{2x}{3}} - (2+e^x)^{\frac{2}{3}} \right)$ is finite.</p> <p>$A = e^{\frac{2x}{3}}$, $B = (2+e^x)^{\frac{2}{3}}$ and</p> $A-B = \frac{A^3 - B^3}{A^2 + AB + B^2} = \frac{e^{2x} - (2+e^x)^2}{A^2 + AB + B^2}$ $= -\frac{4e^x + 4}{A^2 + AB + B^2} = -\frac{e^x}{e^{\frac{4x}{3}}} \frac{4 + 4e^{-x}}{1 + (2e^{-x}+1) + (2e^{-x}+1)^2}$ $\Rightarrow e^{qx} (A-B) = -e^{(q-\frac{1}{3})x} \frac{4 + 4e^{-x}}{1 + (2e^{-x}+1) + (2e^{-x}+1)^2}$ <p>For $q = \frac{1}{3}$ this converges to $-\frac{4}{3}$ as $x \rightarrow \infty$.</p>		4 unseen
(iv)	$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2} = \lim_{x \rightarrow 0} \frac{-\sin^2 x}{x^2(1+\cos x)} = -\frac{1}{2}$		seen 4
(v)	$(\sin x)^{\sin x} = \exp(\sin x \ln \sin x)$ $\frac{d}{dx} (\sin x)^{\sin x} = (\cos x \ln \sin x + \cos x) (\sin x)^{\sin x}$ $= \cos x (1 + \ln(\sin x)) (\sin x)^{\sin x}$		unseen 4
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	EXAMINATION QUESTIONS /SOLUTIONS 2011-2012		Course EE1.1
Question Q1	TOPIC		Marks & seen/unseen
Parts (vi)	$\int \frac{\sin x + 1}{\sin x - 1} dx = \int \frac{(\sin x + 1)^2}{-\cos^2 x} dx$ $= \int -\frac{1}{\cos^2 x} - 2 \frac{\sin x}{\cos^2 x} - \underbrace{\frac{\tan^2 x}{\frac{1}{\cos^2 x} - 1}} dx$ $= x - 2 \tan x - 2 \frac{1}{\cos x} + C' \quad \textcircled{A}$ <p>Alternatively</p> $t = \tan\left(\frac{x}{2}\right), \quad \sin(x) = \frac{2t}{1+t^2}, \quad dx = \frac{2}{1+t^2} dt$ $\int \frac{\sin x + 1}{\sin x - 1} dx = x + 2 \int \frac{1}{\sin x - 1} dx$ $= x + 2 \int \frac{1}{\frac{2t}{1+t^2} - 1} \cdot \frac{2}{1+t^2} dt = x - 4 \int \frac{dt}{(1-t)^2}$ $= x - 4 \frac{1}{1 - \tan \frac{x}{2}} + C' \quad \textcircled{B} \quad (\text{differs by a const from above})$ $\textcircled{A} - \textcircled{B} = 2$		4 seen similar
(vii)	$\int_0^1 \frac{x}{(1-x^2)^{1/3}} dx = \frac{1}{2} \int_0^1 \frac{du}{(1-u)^{1/3}} = \frac{1}{2} \lim_{\epsilon \rightarrow 0^-} \left[-\frac{3}{2} (1-u)^{2/3} \right]_0^{1+\epsilon}$ $= \frac{1}{2} \left(-\frac{3}{2} (-1) \right) = \frac{3}{4}$		4 unseen
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	EXAMINATION QUESTIONS /SOLUTIONS 2011-2012		Course EE1.1
Question Q1	TOPIC		Marks & seen/unseen
Parts (viii)	$f(x) = \frac{\ln(x-1)}{x} \quad \text{Taylor expansion:}$ $f'(x) = -\frac{\ln(x-1)}{x^2} + \frac{1}{x(x-1)}$ $f''(x) = 2\frac{\ln(x-1)}{x^3} - \frac{1}{x^2(x-1)} - \frac{2x-1}{x^2(x-1)^2}$ $f(2) = 0$ $f'(2) = \frac{1}{2}$ $\Rightarrow f(x) = \frac{1}{2}(x-2) + R_2(x)$ $\text{with } R_2(x) = \frac{(x-2)^2}{2} f''(\xi), \xi \in [2, x]$		4 unseen
(ix)	$y' = \underbrace{\frac{y}{x} + 1}_{f(v)} \quad \text{homogeneous, } v = \frac{y}{x}, f(v) = v+1$ $\ln x = \int \frac{1}{f(v)-v} dv = v + C' \Rightarrow y = x(\ln x - C')$		4 seen similar
(x)	$y'' + 4y' + 4y = \exp(x)$ $a=1, b=4, c=4 \quad b^2 - 4ac = 0 \quad \text{critical case}$ $\lambda = -\frac{b}{2a} = -2$ $\Rightarrow y_{CF} = Ae^{-2x} + Bxe^{-2x} \quad y(x) = Ae^{-2x} + Bxe^{-2x} + \frac{1}{5}e^x$ $y_{PI} = \frac{1}{5}e^x$		4 seen similar
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	EXAMINATION QUESTIONS/ <u>SOLUTIONS</u> 2011-2012		Course EE1.1
Question Q2	TOPIC Differentiation		Marks & seen/unseen
Parts			
(i)	$y = \frac{e^x}{e^{x^2}}$ $y' = \frac{e^x}{e^{x^2}} - 2x e^{x^2} \frac{e^x}{e^{2x^2}} = (1-2x) e^{x-x^2}$		3 seen similar
(ii)	$y = \exp(\sin^{-1}(x))$ $y' = \frac{1}{\sqrt{1-x^2}} \exp(\sin^{-1}(x))$		3 unseen
(iii)	$y = \ln(\cos(x^{\sin x}))$ $= \ln(\cos(\exp(\sin(x) \ln(x))))$ $y' = -\left(\frac{\sin x}{x} + \cos(x) \ln(x)\right) \exp(\sin(x) \ln(x))$ $\tan(\exp(\sin(x) \ln(x)))$ $= -\left(\frac{\sin x}{x} + \cos(x) \ln(x)\right) x^{\sin x} \tan(x^{\sin x})$		4 unseen
	Setter's initials GP	Checker's initials HM	Page number 55

	EXAMINATION QUESTIONS /SOLUTIONS 2011-2012		Course EE1.1
Question 3	TOPIC Limits		Marks & seen/unseen
Parts			
(i)	$\lim_{x \rightarrow -1} \frac{(x-2)(x+2)}{(x-3)(x-1)} = \frac{-3}{8}$		3 seen similar
(ii)	$\lim_{x \rightarrow -1} \frac{(x-2)(x^2-2)}{(x-3)(x+1)} = \lim_{x \rightarrow -1} 2 \frac{(x-2)(x-1)}{(x-3)}$ $= 2 \frac{6}{-4} = -3$		3 seen similar
(iii)	$\sqrt{1+x} - \sqrt{x} = \frac{1}{\sqrt{1+x} + \sqrt{x}}$ $\Rightarrow \lim_{x \rightarrow \infty} \sqrt{x} (\sqrt{1+x} - \sqrt{x}) = \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{1+x} + \sqrt{x}} = \frac{1}{2}$		4 seen similar
(iv)	$\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1} = \lim_{x \rightarrow 1} \underbrace{x^{n-1} + x^{n-2} + \dots + 1}_{n \text{ terms}} = n$		5 seen similar
(v)	$\lim_{x \rightarrow \pi} \frac{\sin(x) + (x-\pi)}{(x-\pi)^2}$ <p>To use L'Hôpital, show that π is a root in numerator and denominator. 1st obvious for 0th, 1st, 2nd but not third derivative. Former obvious for 0th derivative, and then</p> <p>1st $\cos(x) + 1 = 0$ at $x = \pi$ 2nd $-\sin(x) = 0$ at $x = \pi$ 3rd $-\cos(x) = 1$ at $x = \pi$</p>		5 unseen
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	EXAMINATION <u>QUESTIONS/SOLUTIONS</u> 2011-2012		Course EE1.1
Question 3	TOPIC		Marks & seen/unseen
Parts	<p>Third derivative of denominator: 6</p> $\Rightarrow \lim_{x \rightarrow \pi} \frac{\sin(x) + (x-\pi)}{(x-\pi)^3} = \frac{1}{6}$		
	Setter's initials GP	Checker's initials MB	Page number 58

	EXAMINATION QUESTIONS / <u>SOLUTIONS</u> 2011-2012		Course EE1.1
Question 4	TOPIC Integration		Marks & seen/unseen
Parts			
(i)	$\int_0^1 x \ln x \, dx$ <p>integrand singular at $x=0$.</p> $\int x \ln x \, dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x \, dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$ $\Rightarrow \int_0^1 x \ln x \, dx = \lim_{\epsilon \rightarrow 0^+} \left[\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 \right]_{\epsilon}^1$ $= -\frac{1}{4} \quad \text{using} \quad \lim_{x \rightarrow 0} x^2 \ln x = 0$ <p>Since $\lim_{x \rightarrow 0} x \ln x = 0$.</p>		2 unseen 3
(ii)	$\int_0^{\infty} x^n e^{-x} \, dx = \left[-x^n e^{-x} \right]_0^{\infty} + \int_0^{\infty} n x^{n-1} e^{-x} \, dx$ <p style="text-align: center;">" 0</p> $= n \int_0^{\infty} x^{n-1} e^{-x} \, dx$		5 unseen
(iii)	$\int \cos^3(x) \, dx = \int (1 - \sin^2(x)) \cos(x) \, dx$ $= \sin(x) - \frac{1}{3} \sin^3(x) + C$		4 seen similar
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	EXAMINATION QUESTIONS /SOLUTIONS 2011-2012	Course EE1.1
Question 4	TOPIC	Marks & seen/unseen
Parts (iv)	$\int \frac{x^2-1}{x^3+2x^2-3x} dx = I$ <p>Denominator: $x^3+2x^2-3x = x(x^2+2x-3) = x(x+3)(x-1)$ Numerator: $(x+1)(x-1)$</p> $\Rightarrow I = \int \frac{x+1}{x(x+3)} dx$ $\frac{x+1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3} \Rightarrow A = \frac{1}{3} \quad B = \frac{2}{3}$ $\Rightarrow I = \frac{1}{3} \int \frac{1}{x} + \frac{2}{x+3} dx = \frac{1}{3} \ln x + \frac{2}{3} \ln x+3 + C'$	3 seen similar 3
(v)	$\int \frac{dx}{\sqrt{8+2x-x^2}} = I$ $8+2x-x^2 = -(x-4)(x+2)$ $y = x-1$ $\Rightarrow 8+2x-x^2 = -(y-3)(y+3) = 9-y^2$ $I = \int \frac{1}{3} \frac{dy}{\sqrt{1-\frac{y^2}{9}}} = \sin^{-1}\left(\frac{y}{3}\right) + C'$ $= \sin^{-1}\left(\frac{x-1}{3}\right) + C'$	seen similar -
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	EXAMINATION QUESTIONS/SOLUTIONS 2011-2012	Course EE1.1
Question 5	TOPIC Complex numbers + Taylor series	Marks & seen/unseen
Parts	<p>(i) $z+1 = 2^x + z \Rightarrow (x+1)^2 + y^2 = 4x^2$ $z = x + iy$</p> <p>$\Rightarrow y^2 = 3x^2 - 2x - 1$</p> <p>$y = \pm \sqrt{3x^2 - 2x - 1}$ for $3x^2 - 2x - 1 \geq 0, x \in \mathbb{R}$</p> <p>Roots of $3x^2 - 2x - 1$ by inspection: $x=1$ $\Rightarrow 3x^2 - 2x - 1 = 3(x-1)(x+\frac{1}{3})$ other root $x = -\frac{1}{3}$</p> <p>$\Rightarrow x \geq 1$ and $x \leq -\frac{1}{3}, x \in (-\infty, -\frac{1}{3}] \cup [1, \infty)$</p> <p>$\arg(z(z-2)) = \pi \Rightarrow$ $z(z-2) = -r$ with $0 \leq r \in \mathbb{R}$ Where is $z(z-2)$ negative? $z \in (0, 2) \subset \mathbb{R}, z = 1 \pm 2\sqrt{\frac{5}{2}-1}$ for $r > 2$</p> <p>(ii) $1+2i = \sqrt{5} e^{i \tan^{-1}(2)}$</p> <p>$(\sqrt{3}+i)^{\frac{1}{3}} = 2^{\frac{1}{3}} e^{i \tan^{-1}(\frac{1}{\sqrt{3}}), \frac{1}{3} + i 2\pi \frac{k}{3}}$ $\frac{\pi}{6}$ $k=0,1,2$</p> <p>$2^{\frac{1}{3}} e^{i \frac{\pi}{18}}, 2^{\frac{1}{3}} e^{i \frac{13\pi}{18}}, 2^{\frac{1}{3}} e^{i \frac{25\pi}{18}} = 2^{\frac{1}{3}} e^{-i \frac{11\pi}{18}}$</p>	<p>5 seen similar</p> <p>2 seen similar</p> <p>3 seen similar</p>
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	EXAMINATION QUESTIONS/SOLUTIONS 2011-2012	Course EE1.1
Question 5	TOPIC	Marks & seen/unseen
Parts (iii)	$\sin(4\theta) = \operatorname{Im}(e^{i4\theta}) = \operatorname{Im}((\cos\theta + i\sin\theta)^4)$ $= 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$	4 seen similar
(iv)	$\int \cos(x) e^{2x} dx = \int \frac{1}{2}(e^{2x} + e^{-2x})e^{2x} dx$ $= \frac{1}{2} \frac{1}{2+i} e^{(2+i)x} + \frac{1}{2} \frac{1}{2-i} e^{(2-i)x} + C$ $= \frac{1}{2} e^{2x} \frac{1}{5} \left\{ (2-i)e^{ix} + (2+i)e^{-ix} \right\} + C$ $= \frac{1}{5} e^{2x} \{ 2\cos(x) + \sin(x) \} + C$ <p>Alternatively integration by parts twice:</p> $\int \cos(x) e^{2x} dx = \sin x e^{2x} - 2 \int \sin(x) e^{2x} dx$ $= \sin x e^{2x} + 2\cos x e^{2x} - 4 \int \cos(x) e^{2x} dx$ $\Rightarrow \int \cos(x) e^{2x} dx = \frac{1}{5} e^{2x} \{ 2\cos(x) + \sin(x) \} + C$	3 unseen 3
(v)	$\left. \begin{aligned} f(x) &= x \ln(x) \\ f'(x) &= \ln(x) + 1 \\ f''(x) &= \frac{1}{x} \\ f'''(x) &= -\frac{1}{x^2} \end{aligned} \right\} \begin{aligned} f(x) &= (x-1) + \frac{1}{2}(x-1)^2 + R_3(x) \\ R_3(x) &= \frac{(x-1)^3}{6} \left(-\frac{1}{x^2}\right) \\ &\quad \xi \in [1, x] \end{aligned}$	unseen
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		Page number S12

EXAMINATION QUESTIONS /SOLUTIONS 2011-2012		Course EE1.1
Question 6	TOPIC ODEs	Marks & seen/unseen
Parts (i)	$y' = \frac{y^2 + x^2 + 2(x+y) + 2}{2x^2 + 4x + 2} = \frac{(y+1)^2 + (x+1)^2}{2(x+1)^2}$ $X = x+1 \quad Y = y+1 \quad Y' = \frac{Y^2 + X^2}{2X^2} = \frac{1}{2} \underbrace{(v^2 + 1)}_{f(v)}$ $\text{With } v = \frac{Y}{X}$ $\Rightarrow \ln X = \int \frac{dv}{f(v) - v} = \int \frac{2}{(1-v)^2} dv = \frac{2}{1-v} + C'$ $\frac{1}{2}(\ln X - C') = \frac{1}{1-v}$ $v = 1 - \frac{2}{\ln X - C'}$ $\Rightarrow y = (x+1) \left\{ 1 - \frac{2}{\ln(x+1) - C'} \right\} - 1$	<p>3 seen similar</p> <p>4</p>
(ii)	$\frac{1}{x} y' + \frac{y}{x^2} + \exp(x) = 0 \Rightarrow$ $y' + \frac{y}{x} = -x e^x \quad \text{linear} \quad R = e^{\int \frac{1}{x} dx} = x$ $\Rightarrow y(x) = -\frac{1}{x} \int x^2 e^x dx$ $\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2(x e^x - e^x) + C'$ $\Rightarrow y(x) = -e^x \frac{x^2 - 2x + 2}{x} + \frac{C'}{x}$	<p>6 seen similar</p>
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	EXAMINATION QUESTIONS/SOLUTIONS 2011-2012	Course EE1.1
Question 6	TOPIC	Marks & seen/unseen
Parts (iii)	$y'' + 2y' - 3y = 2 \sin(3x)$ $a=1, b=2, c=-3 \quad b^2 - 4ac = 16 > 0$ $\lambda_1 = \frac{-2 + \sqrt{16}}{2} = 1 \quad \lambda_2 = \frac{-2 - \sqrt{16}}{2} = -3$ $y_{cf} = Ae^x + Be^{-3x}$ $y_{p1} = \alpha \sin(3x) + \beta \cos(3x)$ $y_{p1}' = 3\alpha \cos(3x) - 3\beta \sin(3x)$ $y_{p1}'' = -9\alpha \sin(3x) - 9\beta \cos(3x)$ $y_{p1}'' + 2y_{p1}' - 3y_{p1} = -9\alpha \sin(3x) - 9\beta \cos(3x) - 6\beta \sin(3x) + 6\alpha \cos(3x) - 3\alpha \sin(3x) - 3\beta \cos(3x)$ $-9\alpha - 6\beta - 3\alpha = 2 \Rightarrow -3\beta - 1 = 6\alpha$ $-9\beta + 6\alpha - 3\beta = 0 \Rightarrow -9\beta - 3\alpha - 1 - 3\alpha = 0$ $\beta = -\frac{1}{15}$ $\alpha = -\frac{2}{15}$ $\Rightarrow y_{p1}(x) = -\frac{2}{15} \sin(3x) - \frac{1}{15} \cos(3x)$ $y(x) = Ae^x + Be^{-3x} - \frac{2}{15} \sin(3x) - \frac{1}{15} \cos(3x)$ $y(0) = A + B - \frac{1}{15} = 1 \Rightarrow A + B = \frac{16}{15}$ $y'(x) = Ae^x - 3Be^{-3x} - \frac{2}{5} \cos(3x) + \frac{1}{5} \sin(3x)$ $y'(0) = A - 3B - \frac{2}{5} = 0 \Rightarrow \frac{16}{15} - 4B - \frac{2}{5} = 0 \Rightarrow B = \frac{1}{6}$ $A = \frac{16}{15} - \frac{1}{6} = \frac{22}{30} = \frac{11}{15}$	<p>2 seen sin/cos</p> <p>2</p> <p>3</p>
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