Electronic Circuits
Chapter 5: Frequency Response

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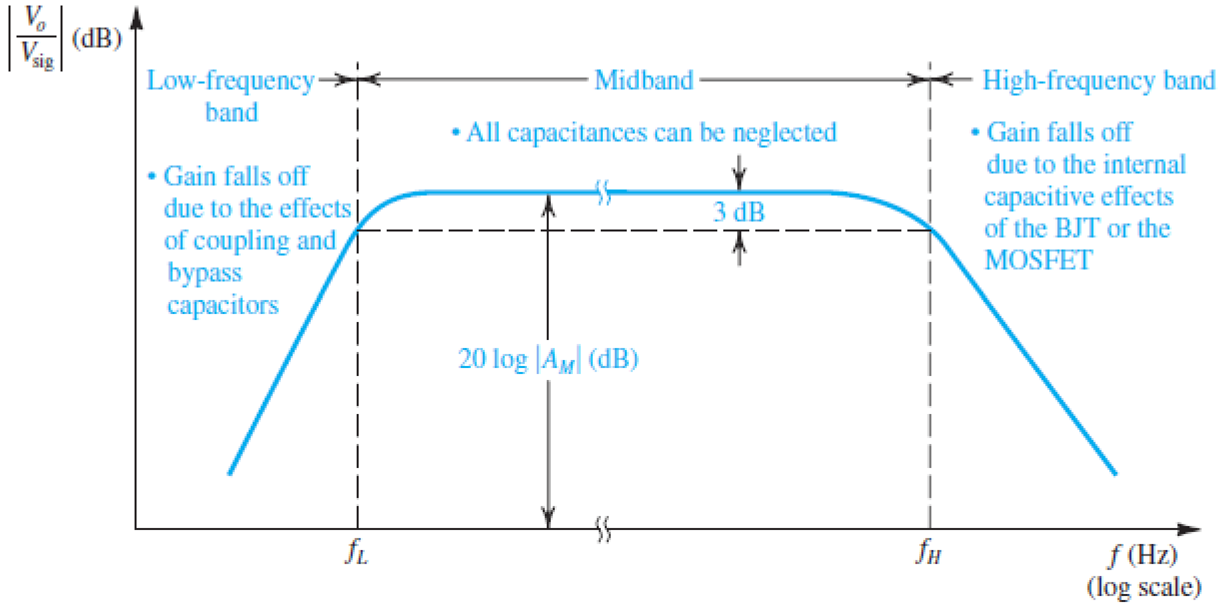
Outline

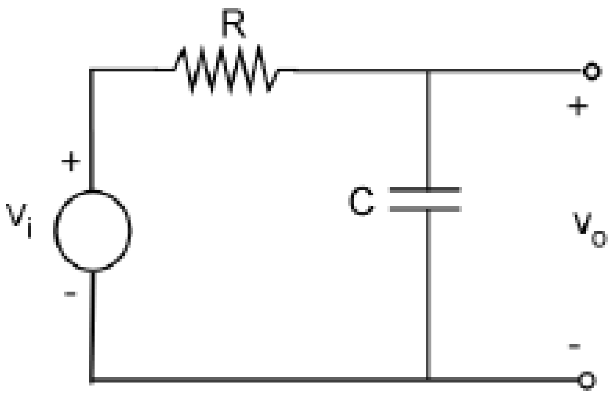
Introduction

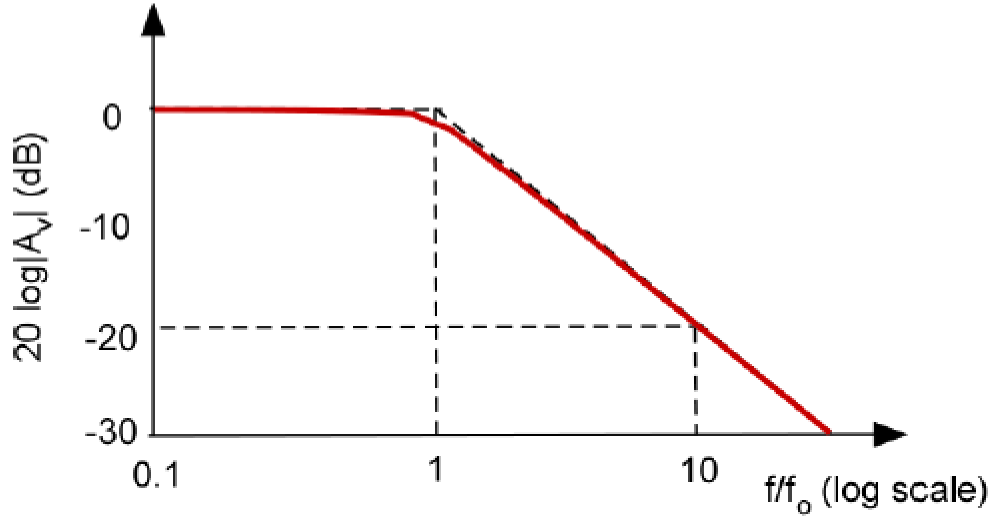


At lower frequencies, the magnitude of the amplifier gain falls off. This occurs because the *coupling and bypass capacitors* no longer have low impedances.

The gain of the amplifier falls off at *the high-frequency* end. This is due to internal capacitive effects in the BJT and in the MOSFET.

Introduction: Low-pass Circuit



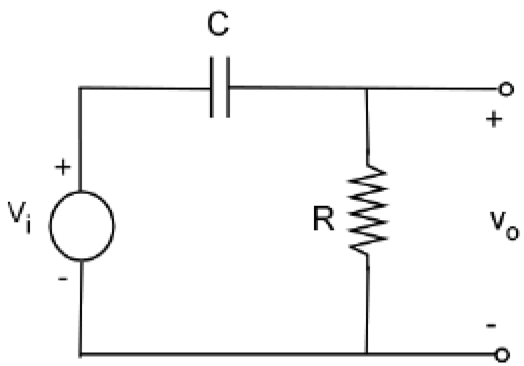


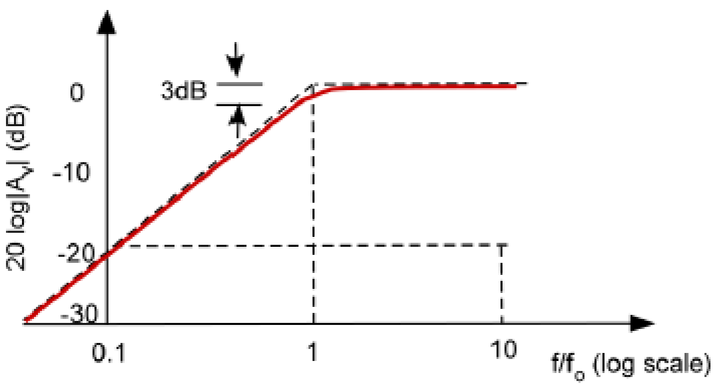
In frequency domain: $V_o = \dfrac{V_i}{R + \dfrac{1}{j\omega C}} \cdot \dfrac{1}{j\omega C} = \dfrac{V_i}{1 + j\omega RC}$

$$\rightarrow A_v = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\, {}^{f}\!/_{f_0}}$$

$$f_0 = \frac{1}{2\pi RC} = \frac{1}{\tau} \qquad \tau = 2\pi RC$$

Introduction: High-pass Circuit





In frequency domain: $$V_o = \frac{V_i R}{R + \frac{1}{j\omega C}} = \frac{V_i}{1 + \frac{1}{j\omega RC}}$$

$$\rightarrow A_v = \frac{V_o}{V_i} = \frac{1}{1 - j\frac{1}{\omega RC}} = \frac{1}{1 - j\, {}^{f_o}\!/_{f}}$$

$$\boldsymbol{f_0 = \frac{1}{2\pi RC} = \frac{1}{\tau}} \qquad \boldsymbol{\tau = 2\pi RC}$$

Introduction: Octave vs Decade

If $f_2 = 2f_1$, then f_2 is one octave above f_1.

If $f_2 = 10f_1$, then f_2 is one decade above f_1

$$\#\ of\ octave = log_2\frac{f_2}{f_1} = 3.32log_{10}\frac{f_2}{f_1}$$

$$\#\ of\ decade = log_{10}\frac{f_2}{f_1}$$

Example:

2 GHz is one octave above 1 GHz

10 GHz is one decade above 1 GHz

Introduction: 3dB definition

3dB points are points at which the magnitude is $\frac{1}{\sqrt{2}}$ *that at mid-band frequency.*

Power is halved. Voltage is scaled as:

$$\frac{V_o}{|1+j|} = \frac{V_o}{\sqrt{2}}$$

From which:

$$A_{dB} = 20log_{10}\left(\sqrt{2}\right) = 3dB$$

Introduction: Gain

Amplifier has intrinsic gain: A_0

Low-pass characteristics: $\frac{1}{1+j^{f}/_{f_{hi}}}$

High-pass characteristics: $\frac{j^{f}/_{f_{lo}}}{1+j^{f}/_{f_{lo}}}$

Overall gain: $A(f) = A_0 \frac{1}{1+j^{f}/_{f_{hi}}} \frac{j^{f}/_{f_{lo}}}{1+j^{f}/_{f_{lo}}}$

At very high frequency, the gain becomes:

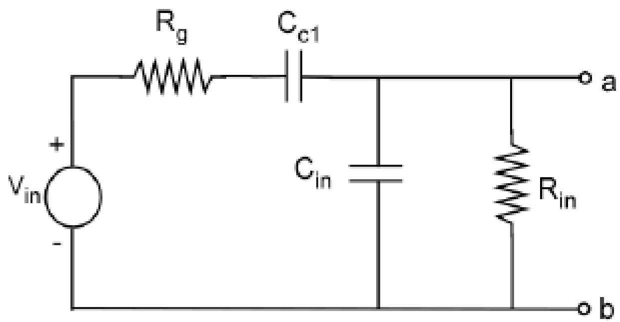
$$G = -20log_{10}\sqrt{1+\left({}^{\omega}/_{\omega_0}\right)^2} \approx -20log_{10}\left({}^{\omega}/_{\omega_0}\right)$$

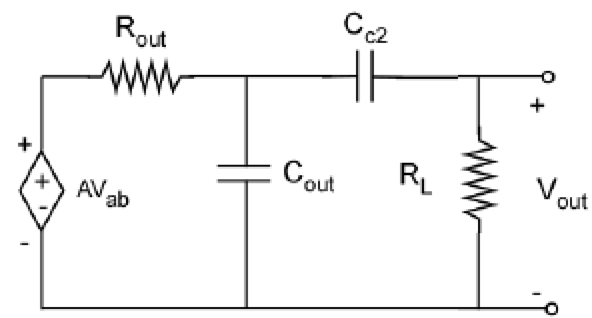
Slope of curve is **-20db/decade**

Model for general amplifying element

C_{C1} and C_{C2} are coupling capacitors (large): μF.

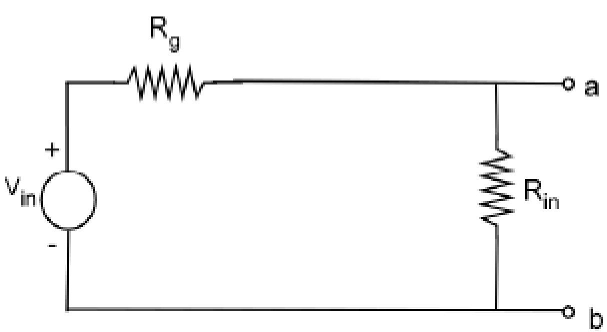
C_{in} and C_{out} are parasitic capacitors (small): pF.

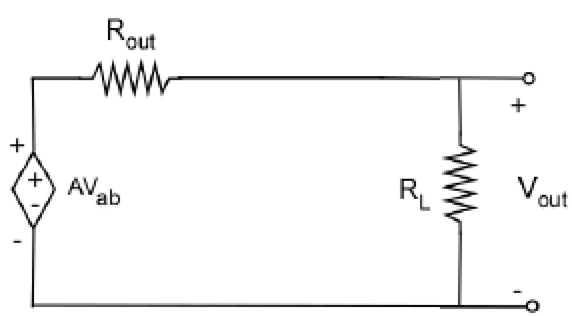




Mid-band frequency: - Coupling capacitors are short circuits

- Parasitic capacitors are open circuits





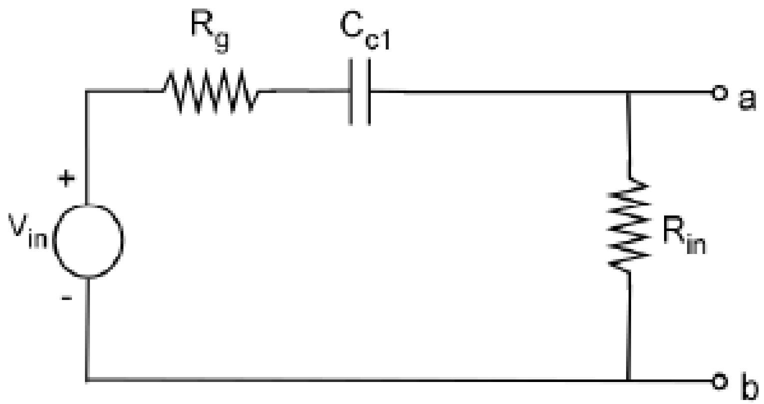
$$A_{MB} = \frac{v_{out}}{v_{in}}$$

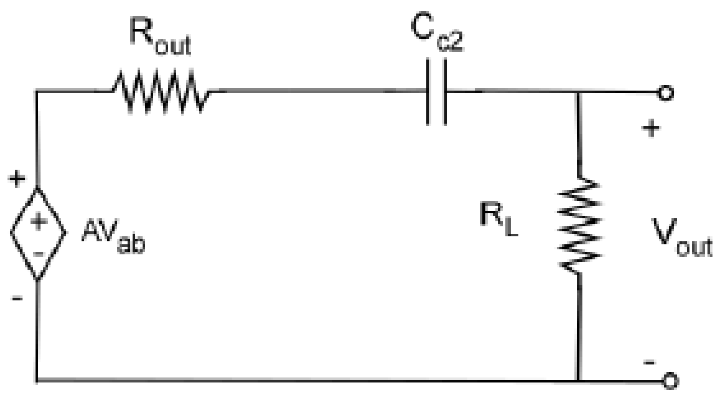
$$= \frac{R_{in}}{R_{in} + R_g} A \frac{R_L}{R_L + R_{out}}$$

Low frequency model

Low frequency model: *- Coupling capacitors are present.*

- Parasitic capacitors are open circuits.

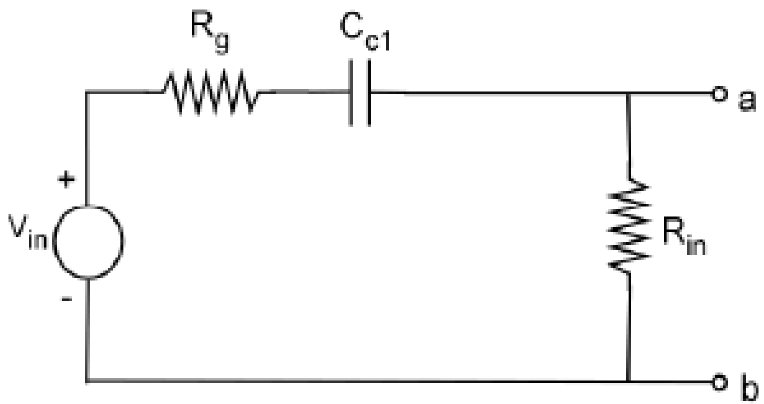


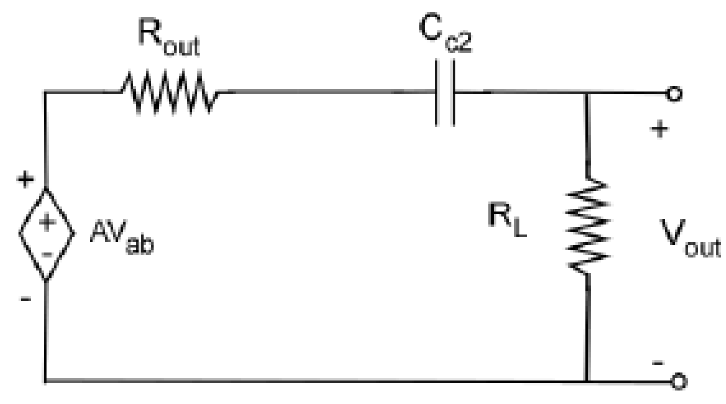


$$v_{ab} = v_{in}\frac{R_{in}}{R_{in}+R_g+\frac{1}{j\omega C_{c1}}} = v_{in}\frac{j\omega R_{in}C_{c1}}{1+j\omega C_{c1}\left(R_{in}+R_g\right)}$$

$$= v_{in}\frac{R_{in}}{R_{in}+R_g}\frac{j\omega C_{c1}\left(R_{in}+R_g\right)}{1+j\omega C_{c1}\left(R_{in}+R_g\right)}$$

Low frequency model



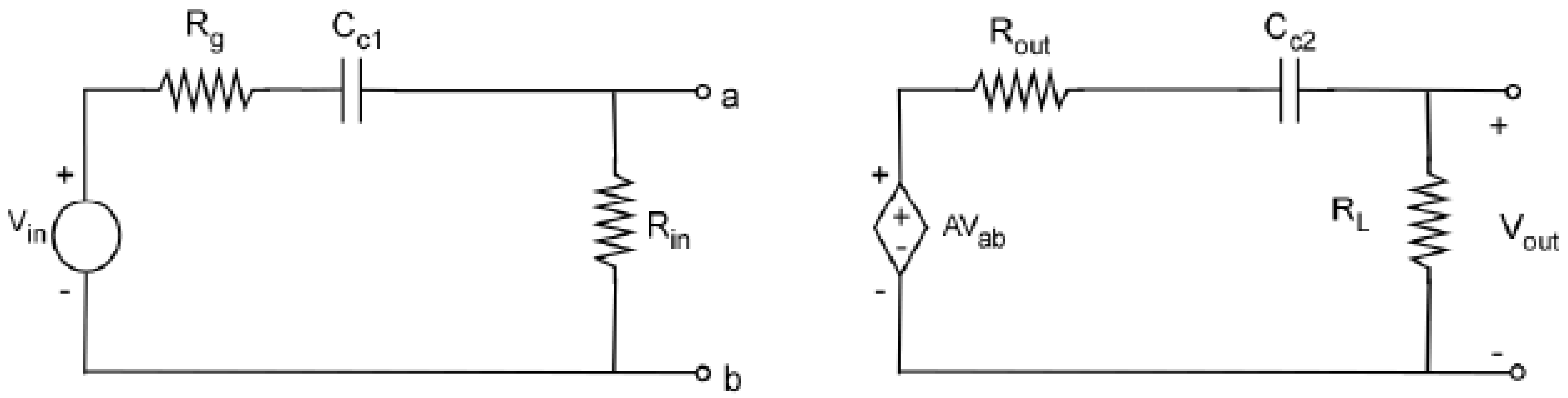


Define: $f_{l1} = \dfrac{1}{2\pi(R_{in} + R_g)C_{c1}}$ $\quad f_{l2} = \dfrac{1}{2\pi(R_{out} + R_L)C_{c2}}$

$$v_{ab} = \boldsymbol{v_{in}} \frac{R_{in}}{R_{in} + R_g} \frac{j\omega C_{c1}(R_{in} + R_g)}{1 + j\omega C_{c1}(R_{in} + R_g)} = \boldsymbol{v_{in}} \frac{R_{in}}{R_{in} + R_g} \frac{j\, {}^{f}\!/_{f_{l1}}}{1 + j\, {}^{f}\!/_{f_{l1}}}$$

$$v_{out} = \boldsymbol{Av_{ab}} \frac{R_L}{R_L + R_{out}} \frac{j\omega C_{c2}(R_L + R_{out})}{1 + j\omega C_{c2}(R_L + out)} = \boldsymbol{Av_{ab}} \frac{R_L}{R_L + R_{out}} \frac{j\, {}^{f}\!/_{f_{l2}}}{1 + j\, {}^{f}\!/_{f_{l2}}}$$

Low frequency model



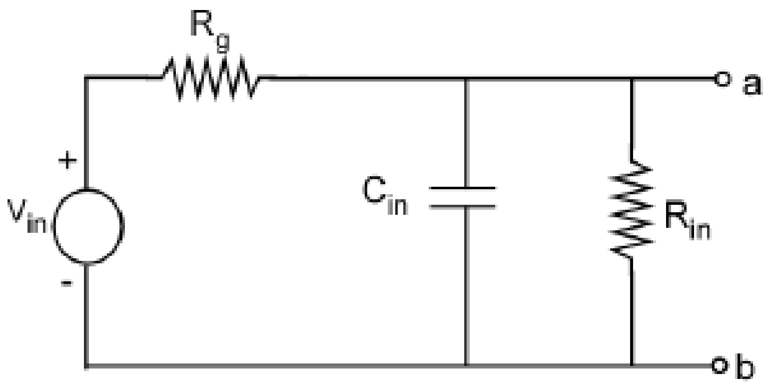
Overall gain:
$$\frac{v_{out}}{v_{in}} = A\frac{R_{in}}{R_{in}+R_g}\frac{R_L}{R_L+R_{out}}\frac{j\,{}^{f}/_{f_{l1}}}{1+j\,{}^{f}/_{f_{l1}}}\frac{j\,{}^{f}/_{f_{l2}}}{1+j\,{}^{f}/_{f_{l2}}}$$

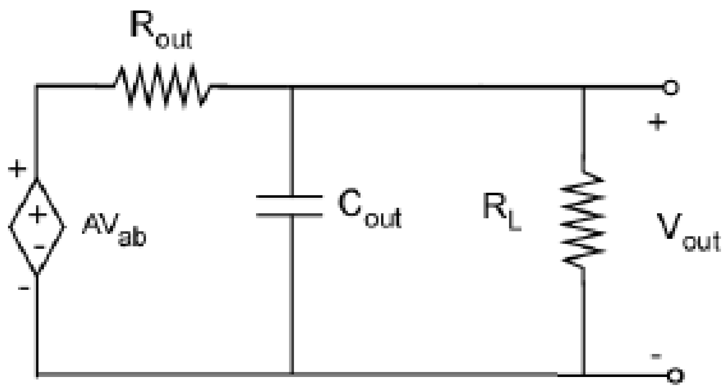
$$\frac{v_{out}}{v_{in}} = \boldsymbol{A_{MB}}\frac{j\,{}^{f}/_{f_{l1}}}{1+j\,{}^{f}/_{f_{l1}}}\frac{j\,{}^{f}/_{f_{l2}}}{1+j\,{}^{f}/_{f_{l2}}}$$

High frequency model

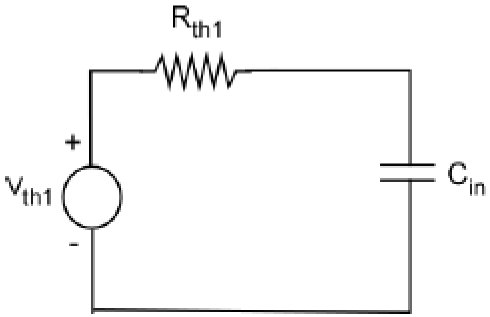
Low frequency model: *- Coupling capacitors are short.*

- Parasitic capacitors are present.





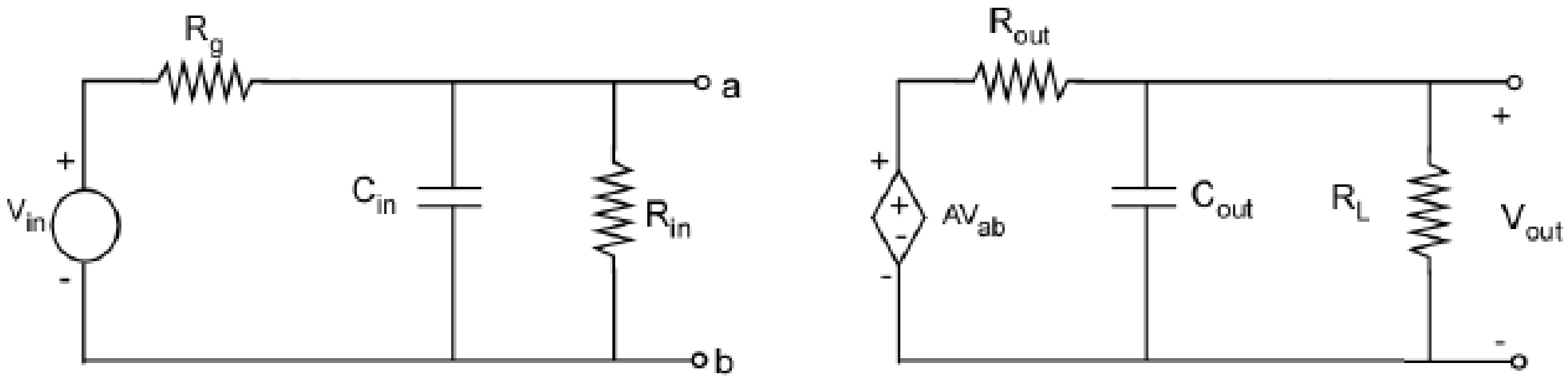
Equivalent Thevenin Circuit



$$V_{th1} = v_{in}\frac{R_{in}}{R_{in} + R_g}$$

$$R_{th1} = R_{in} \parallel R_g$$

High frequency model



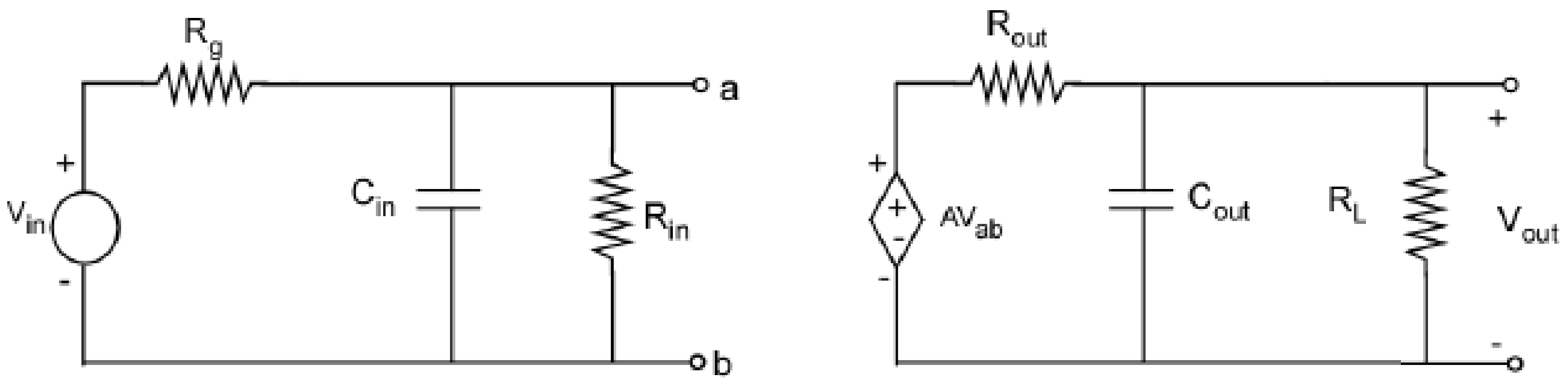
$$v_{ab} = v_{in}\frac{R_{in}}{R_{in}+R_g}\frac{1}{1+j\omega(R_{in}\parallel R_g)C_{in}}$$

$$= v_{in}\frac{R_{in}}{R_{in}+R_g}\frac{1}{1+j\,{}^{f}/_{f_{h1}}}\quad \text{where:}\quad f_{h1}=\frac{1}{2\pi(R_{in}\parallel R_g)C_{in}}$$

$$v_{out} = Av_{ab}\frac{R_L}{R_L+R_{out}}\frac{1}{1+j\omega C_{out}(R_L\parallel R_{out})} = Av_{ab}\frac{R_L}{R_L+R_{out}}\frac{1}{1+j\,{}^{f}/_{f_{h2}}}$$

$$\text{where:}\quad f_{h2}=\frac{1}{2\pi(R_{out}\parallel R_L)C_{out}}$$

High frequency model



Overall gain:

$$\frac{v_{out}}{v_{in}} = Av_{ab}\frac{R_{in}}{R_{in}+R_g}\frac{R_L}{R_L+R_{out}}\frac{1}{1+j\,{}^{f}\!/\!_{f_{h1}}}\frac{1}{1+j\,{}^{f}\!/\!_{f_{h2}}}$$

$$\frac{v_{out}}{v_{in}} = A_{MB}\frac{1}{1+j\,{}^{f}\!/\!_{f_{h1}}}\frac{1}{1+j\,{}^{f}\!/\!_{f_{h2}}}$$

High frequency model

Example 1: Given $R_{out} = 3k\Omega$, $R_g = 200\Omega$, $R_{in} = 12k\Omega$, $R_L = 10k\Omega$, $C_{c1} = 5\mu F$, $C_{c2} = 1\mu F$, $C_{in} = 200pF$, $C_{out} = 40pF$. Compute f_{l1}, f_{l2}, f_{h1}, f_{h2}.

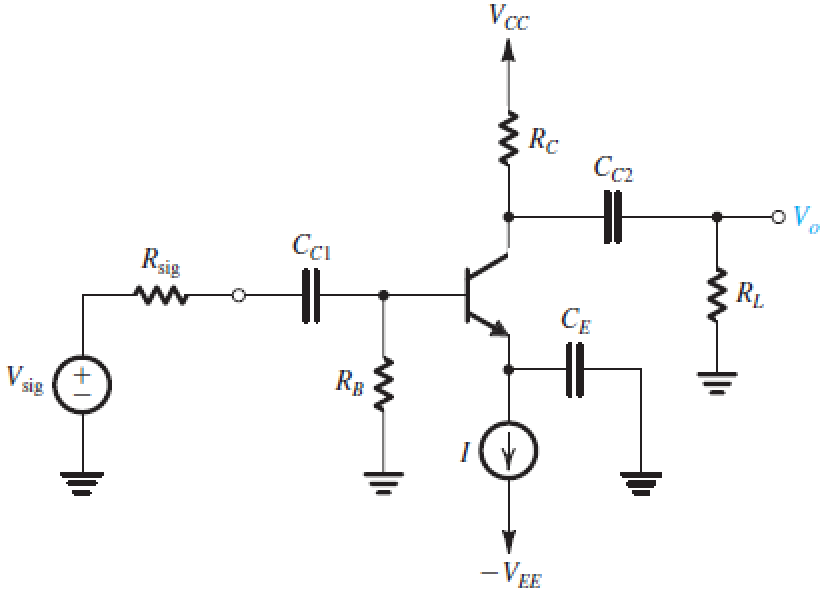
$$f_{l1} = \frac{1}{2\pi \times 12200 \times 5 \times 10^{-6}} = 2.61\ Hz$$

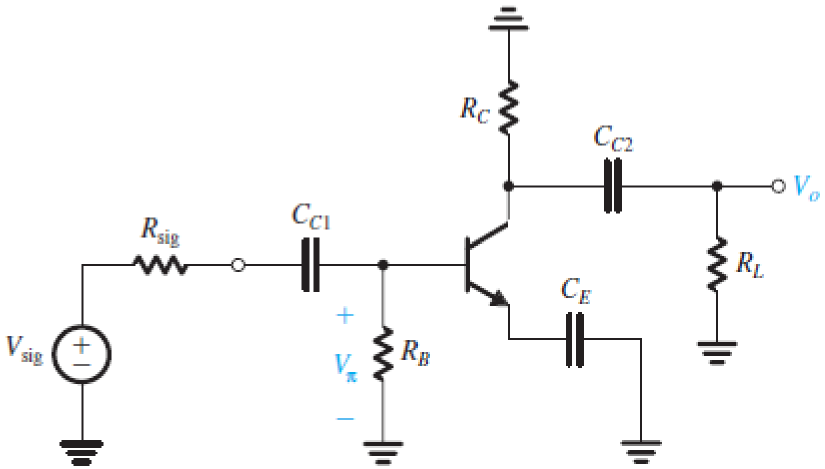
$$f_{l2} = \frac{1}{2\pi \times 13000 \times 1 \times 10^{-6}} = 12.2\ Hz$$

$$f_{h1} = \frac{1}{2\pi \times (12000 \parallel 200) \times 2 \times 10^{-10}} = 4.05\ MHz$$

$$f_{h2} = \frac{1}{2\pi \times (10000 \parallel 3000) \times 4 \times 10^{-11}} = 1.72\ MHz$$

LF Response of CE Amplifier



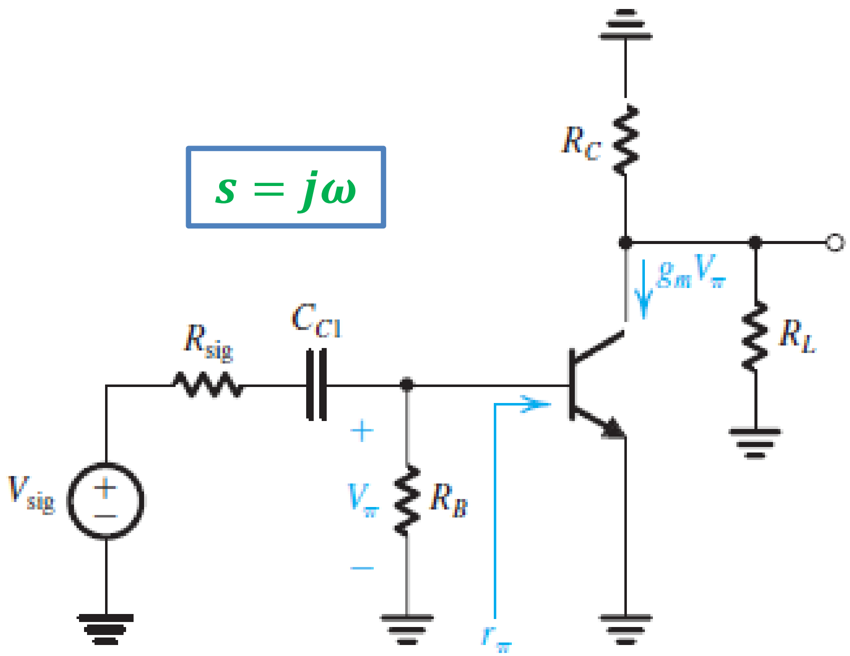


LF Response of CE Amplifier

Considering the Effect of Each of the Three Capacitors Separately

Case 1: Consider C_{C1}, short circuit C_E and C_{C2}

$\boxed{s = j\omega}$



$$V_\pi = V_{sig} \frac{R_B \parallel r_\pi}{(R_B \parallel r_\pi) + R_{sig} + \frac{1}{sC_{c1}}}$$

$$V_o = -g_m V_\pi (R_C \parallel R_L)$$

$$\frac{V_o}{V_{sig}} = -\frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_{sig}} g_m (R_C \parallel R_L)$$

$$\times \frac{s}{s + \frac{1}{C_{c1}[R_B \parallel r_\pi + R_{sig}]}}$$

$$\boxed{f_{P1} = \frac{1}{2\pi C_{c1}[R_B \parallel r_\pi + R_{sig}]}}$$

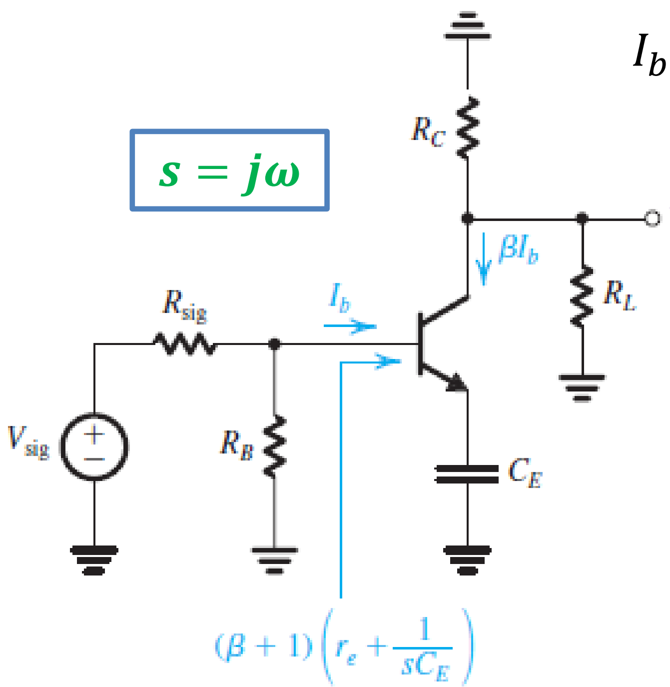
$$\boxed{A_{MB} = -\frac{R_B \parallel r_\pi}{[R_B \parallel r_\pi + R_{sig}]} g_m (R_C \parallel R_L)}$$

LF Response of CE Amplifier

Considering the Effect of Each of the Three Capacitors Separately

Case 2: Consider C_E, short circuit C_{C1} and C_{C2}

$$\boldsymbol{s = j\omega}$$



$$I_b = V_{sig} \frac{R_B}{R_B + R_{sig}} \frac{1}{(R_B \parallel R_{sig}) + (\beta + 1)\left(r_e + \frac{1}{sC_E}\right)}$$

$$V_o = -\beta I_b (R_C \parallel R_L)$$

$$\frac{V_o}{V_{sig}} = -\frac{R_B}{R_B + R_{sig}} \frac{\beta (R_C \parallel R_L)}{(R_B \parallel R_{sig}) + (\beta + 1) r_e}$$

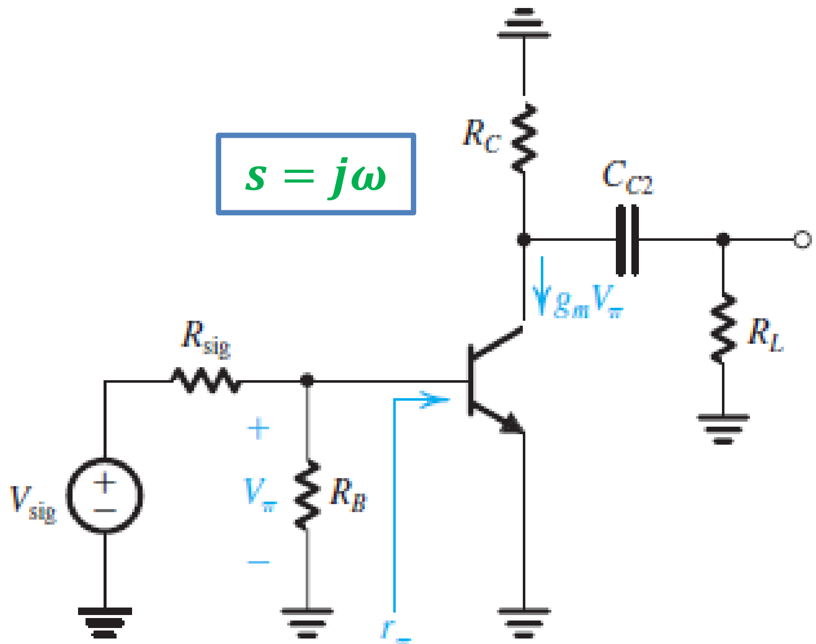
$$\times \frac{s}{s + \dfrac{1}{C_E\left[r_e + \dfrac{R_B \parallel R_{sig}}{\beta + 1}\right]}}$$

$$\boxed{f_{P2} = \frac{1}{2\pi C_E \left[r_e + \dfrac{R_B \parallel R_{sig}}{\beta + 1}\right]}}$$

LF Response of CE Amplifier

Considering the Effect of Each of the Three Capacitors Separately

Case 3: Consider C_{C2}, short circuit C_E and C_{C1}



$$\boxed{s = j\omega}$$

$$V_\pi = V_{sig}\frac{R_B \parallel r_\pi}{(R_B \parallel r_\pi) + R_{sig}}$$

$$V_o = -g_m V_\pi \frac{R_C}{R_C + \dfrac{1}{sC_2} + R_L} R_L$$

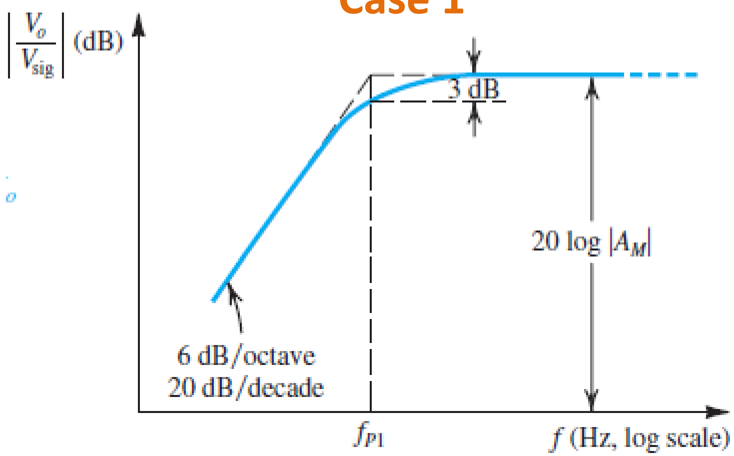
$$\frac{V_o}{V_{sig}} = -\frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_{sig}} g_m (R_C \parallel R_L)$$

$$\times \frac{s}{s + \dfrac{1}{C_{c2}(R_C + R_L)}}$$

$$\boxed{f_{P3} = \frac{1}{2\pi C_{c2}(R_C + R_L)}}$$

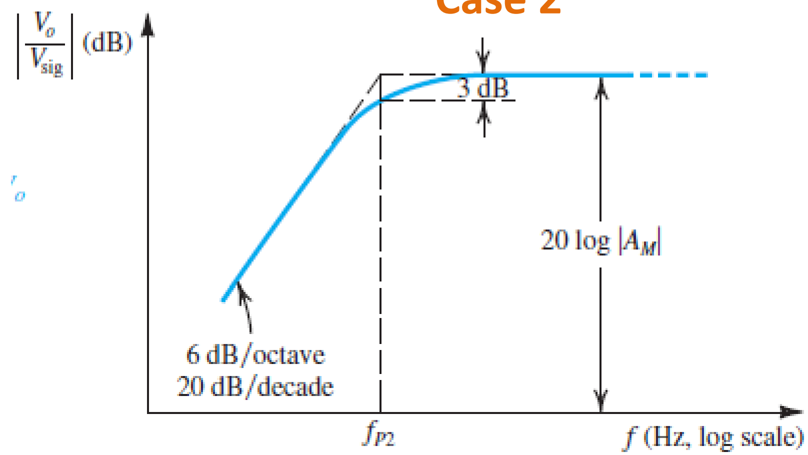
LF Response of CE Amplifier

Case 1



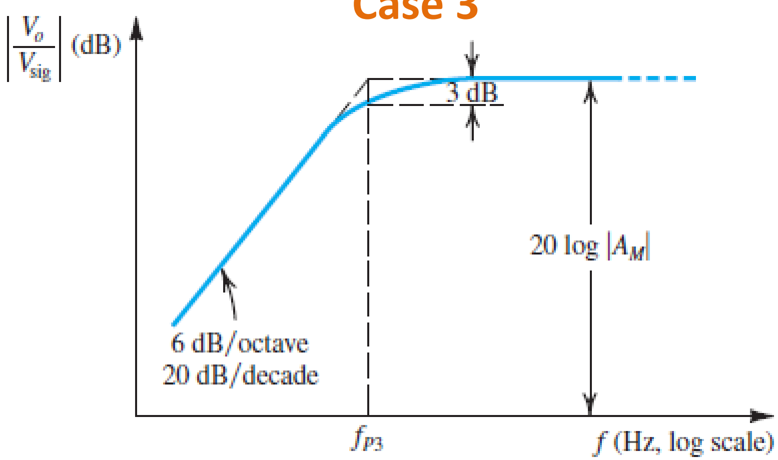
$f_{P1} = 1/2\pi\, C_{C1}\, [(R_B \| r_\pi) + R_{sig}]$

Case 2

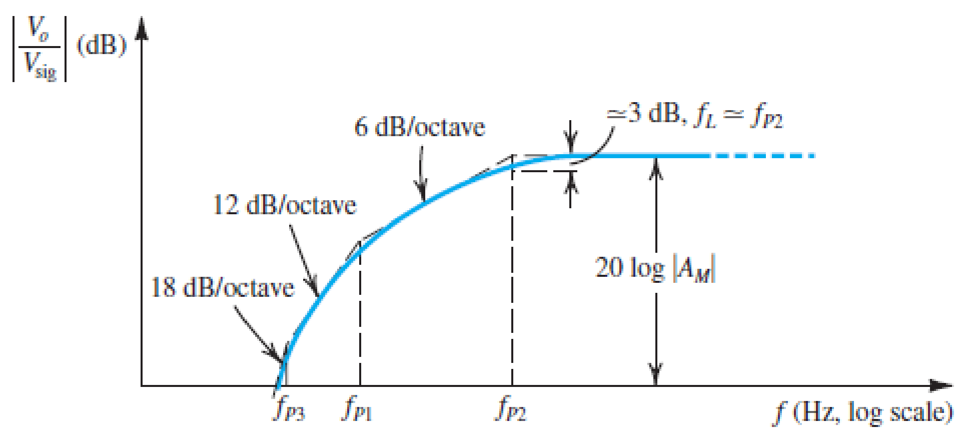


$f_{P2} = 1/2\pi C_E \left[r_e + \frac{R_B \| R_{sig}}{\beta + 1} \right]$

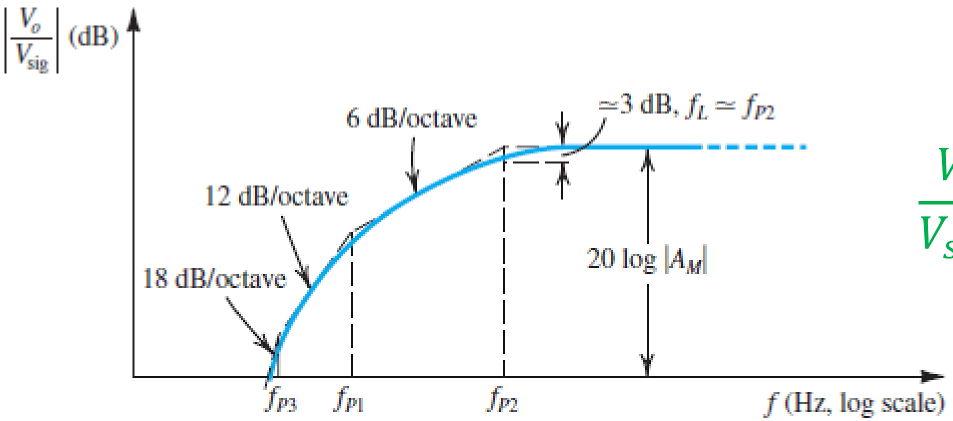
Case 3



$f_{P3} = 1/2\pi\, C_{C2}\, (R_C + R_L)$



LF Response of CE Amplifier



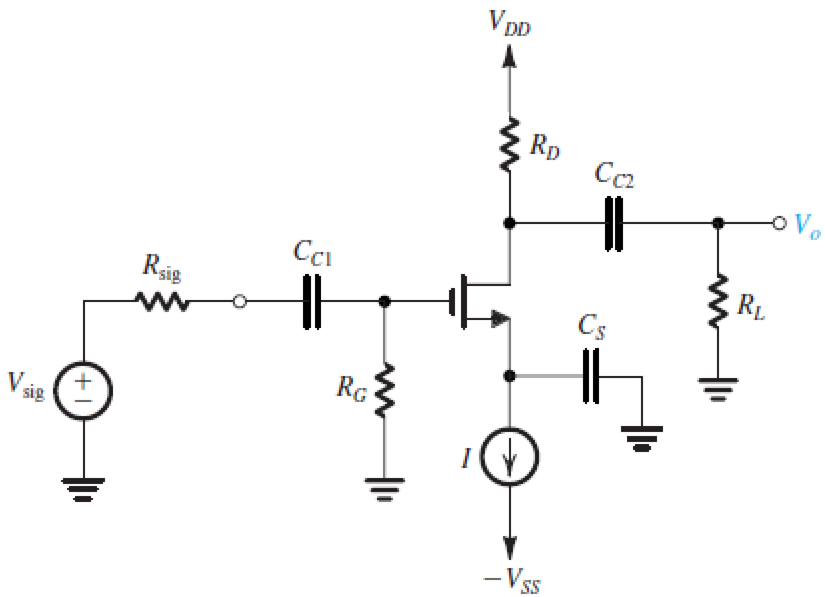
$$\frac{V_o}{V_{sig}} = -A_M\left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

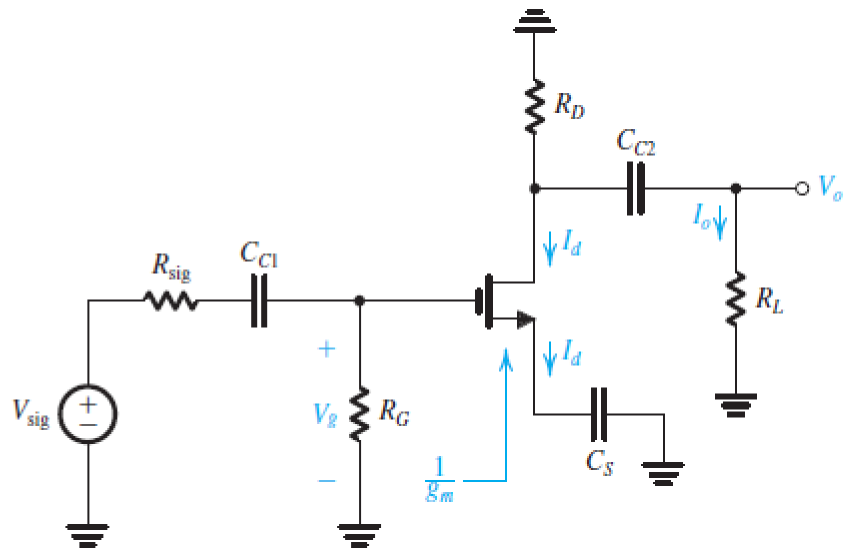
If f_{P1}, f_{P2} and f_{P3} are widely separated: $f_L = \max(f_{P1}, f_{P2}, f_{P3})$

If f_{P1}, f_{P2} and f_{P3} are close together: $f_L \approx \frac{1}{2\pi}\left[\frac{1}{R_{c1}C_{c1}} + \frac{1}{R_E C_E} + \frac{1}{R_{c3}C_{c3}}\right] = f_{P1} + f_{P2} + f_{P3}$

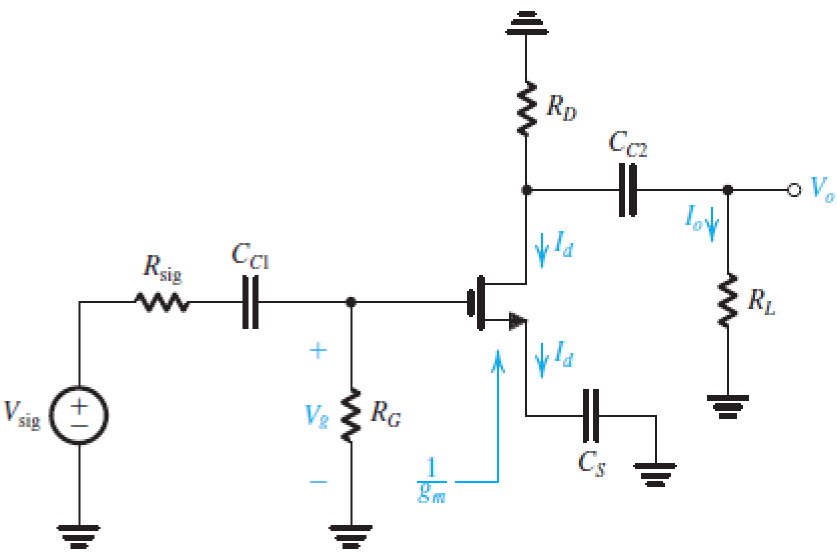
Example 2: Select appropriate values for C_{c1}, C_{c2} and C_E for the CE amplifier which has $R_B = 100k\Omega$, $R_C = 8k\Omega, R_L = 5k\Omega$, $R_{sig} = 5k\Omega$, $\beta = 100$, $g_m = 4mA/V$ and $r_\pi = 2.5k\Omega$. It is required $f_L = 100Hz$.

LF Response of CS Amplifier





LF Response of CS Amplifier



$$V_g = V_{sig} \frac{R_G}{R_G + R_{sig} + \frac{1}{sC_{c1}}}$$

$$V_g = V_{sig} \frac{R_G}{R_G + R_{sig}} \frac{s}{s + \frac{1}{C_{c1}(R_G + R_{sig})}}$$

$$I_d = \frac{V_g}{\frac{1}{g_m} + \frac{1}{sC_s}} = g_m V_g \frac{s}{s + \frac{g}{C_s}}$$

$$I_o = -I_d \frac{R_D}{R_D + R_L + \frac{1}{sC_{c2}}}$$

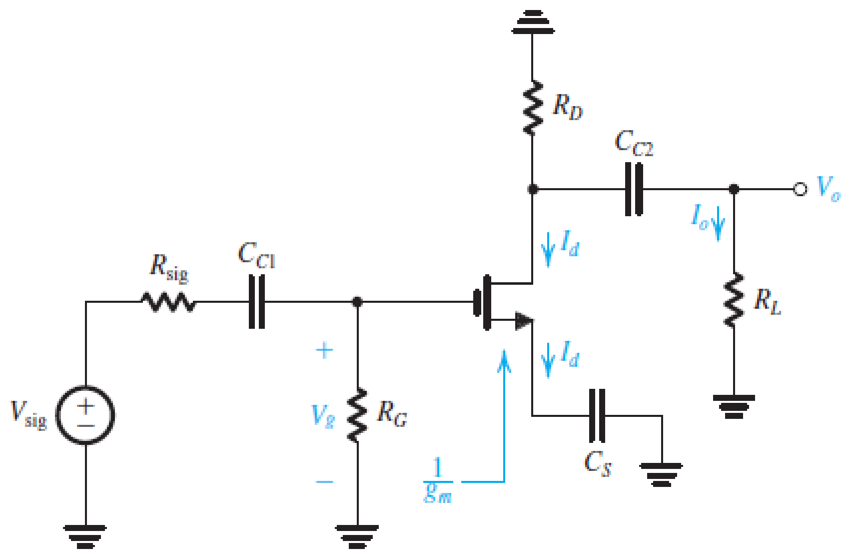
$$V_o = -I_d \frac{R_D R_L}{R_D + R_L} \frac{s}{s + \frac{1}{C_{c2}(R_D + R_L)}}$$

$$f_{P1} = \frac{1}{2\pi C_{c1}[R_G + R_{sig}]}$$

$$f_{P2} = \frac{g_m}{2\pi C_s}$$

$$f_{P3} = \frac{1}{2\pi C_{c2}[R_D + R_L]}$$

LF Response of CS Amplifier



$$\frac{V_o}{V_{sig}} = -\frac{R_G}{R_G + R_{sig}} g_m (R_D \parallel R_L) \left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

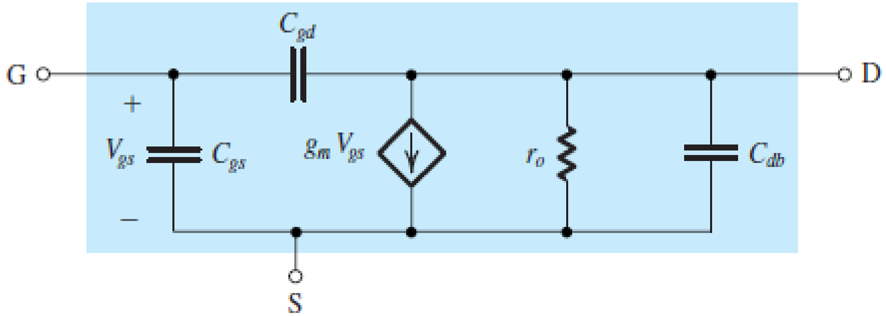
$$\frac{V_o}{V_{sig}} = A_{MB} \left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

where: $A_{MB} = -\frac{R_G}{R_G + R_{sig}} g_m (R_D \parallel R_L)$

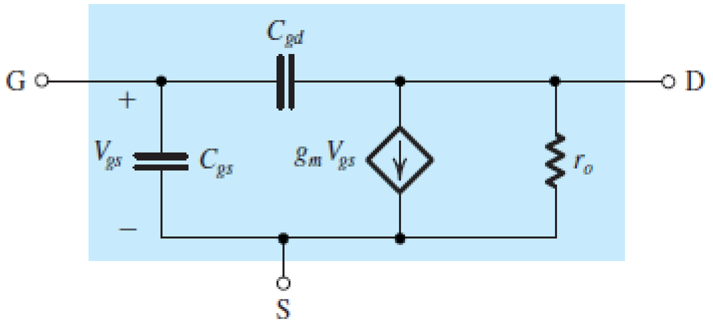
Gate Capacitive Effect

The gate capacitive effect can be modeled by the capacitances C_{gs}, C_{gd}.

- Triode region: $C_{gs} = C_{gd} = \frac{1}{2} WLC_{ox}$
- Saturation region: $C_{gs} = \frac{2}{3} WLC_{ox}$ $\quad C_{gd} = 0$
- Cutoff region: $C_{gs} = 0$ $\quad C_{gd} = 0$

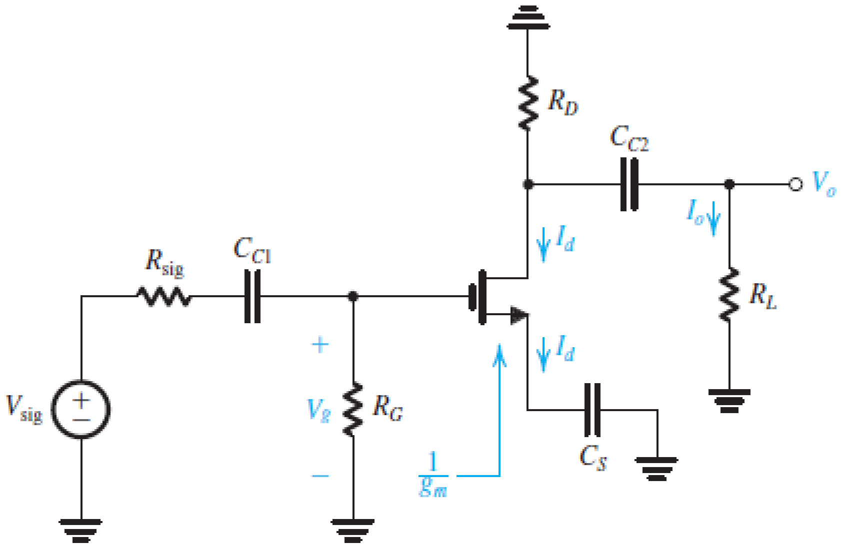


Equivalent circuit for the case in which the source is connected to the substrate

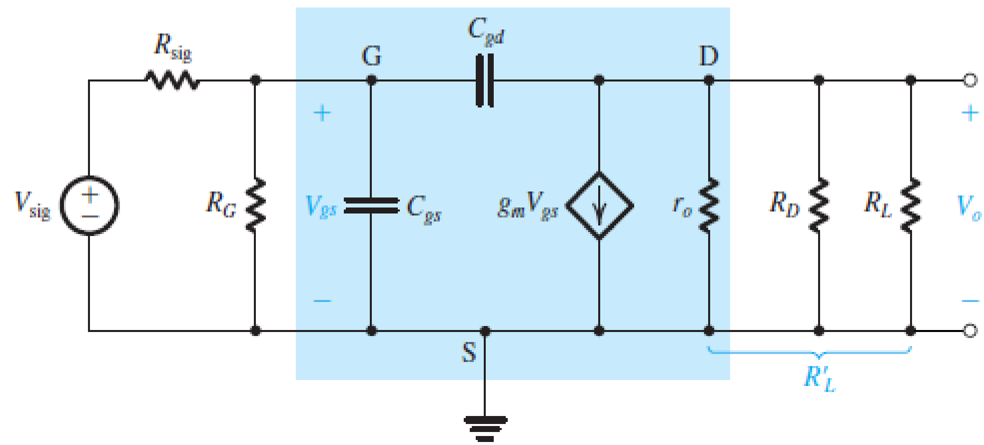


Equivalent circuit with C_{db} neglected (to simplify analysis)

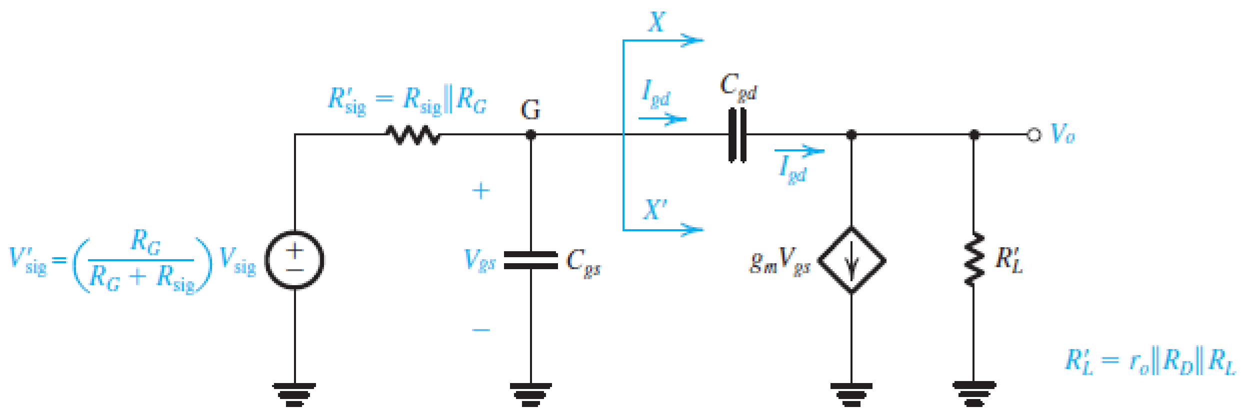
HF Response of CS Amplifier



CS Amplifier

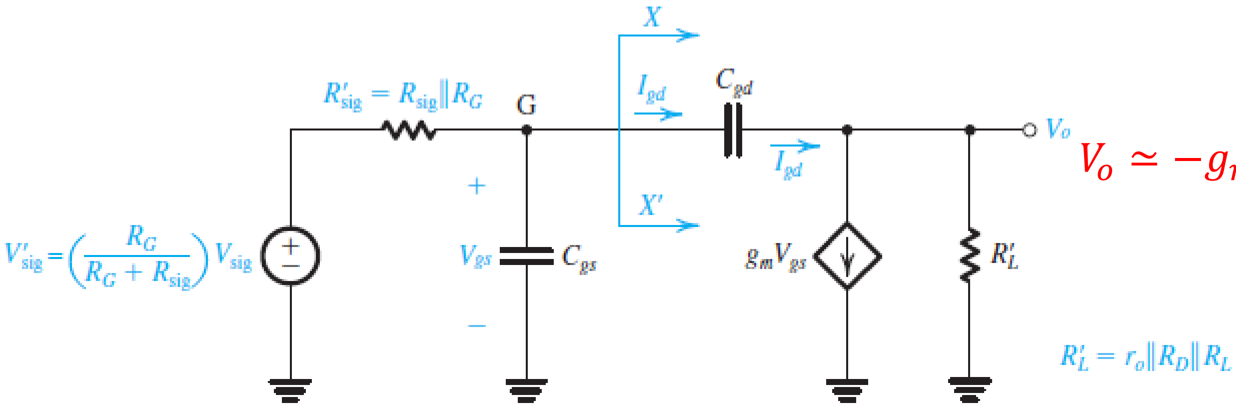


HF Equivalent circuit



Simplified HF Equivalent circuit

HF Response of CS Amplifier



$V_o \simeq -g_m R'_L V_{gs}$

Simplified HF Equivalent circuit

Midband gain: $A_{MB} = \frac{V_o}{V_{sig}} = -\frac{R_G}{R_G + R_{sig}}(g_m R'_L)$

Load current: $I'_L = g_m V_{gs} - I_{gd}$

At frequencies in the vicinity of f_H: $I'_L \approx g_m V_{gs}$ $\rightarrow V_o \simeq -g_m R'_L V_{gs}$

The current I_{gd} can now be found as: $I_{gd} = sC_{gd}(V_{gs} - V_o) = sC_{gd}(V_{gs} + g_m R'_L V_{gs})$

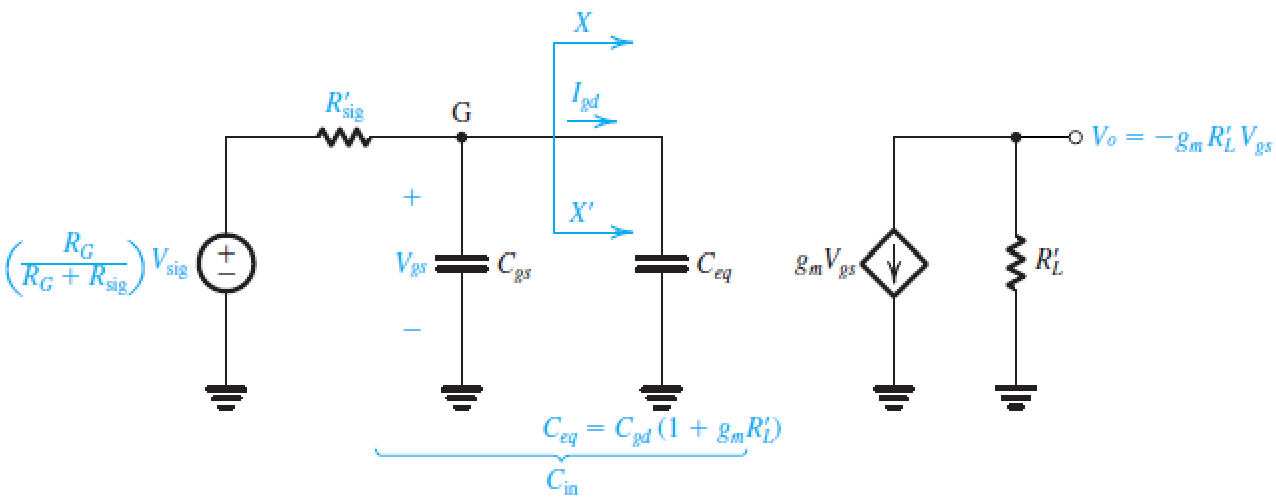
$$= sC_{gd}(1 + g_m R'_L)V_{gs}$$

HF Response of CS Amplifier

❖ Therefore, the left hand side of *XX'* could be replaced by C_{eq}, where

$$sC_{eq}V_{gs} = sC_{gd}(1 + g_m R_L')V_{gs} \quad \text{or} \quad \mathbf{C_{eq} = C_{gd}(1 + g_m R_L')}$$

Miller effect



$$C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}(1 + g_m R_L')$$

$$R_{sig}' = R_{sig} \parallel R_G$$

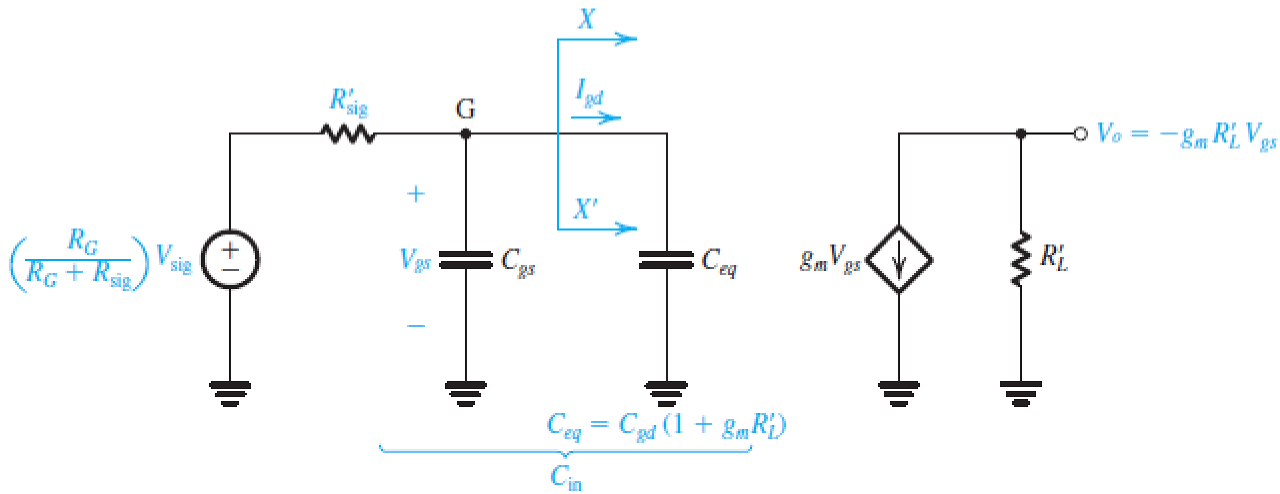
HF Response of CS Amplifier

V_{gs} can be written as:

$$V_{gs} = \left(\frac{R_G}{R_G + R_{sig}} V_{sig}\right) \frac{1}{1 + \frac{jf}{f_0}} = \left(\frac{R_G}{R_G + R_{sig}} V_{sig}\right) \frac{1}{1 + \frac{s}{\omega_0}}$$

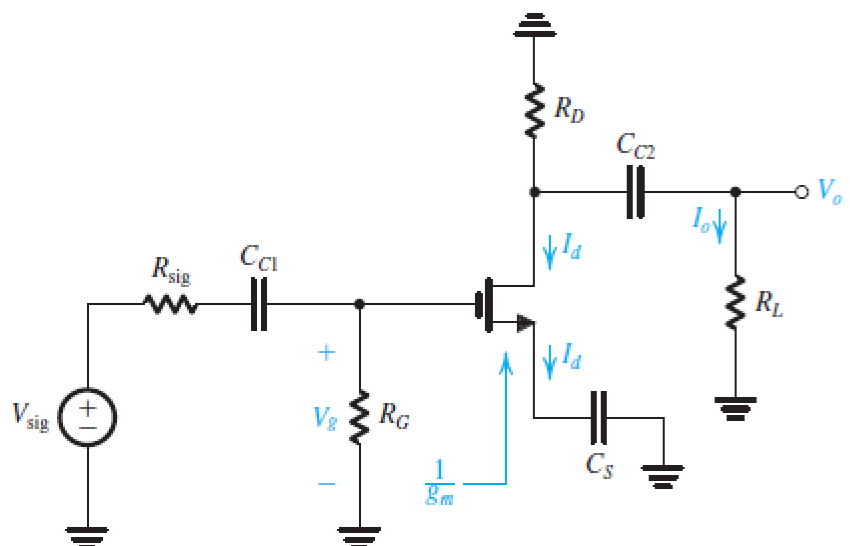
where f_0 is the 3dB frequency: $\boldsymbol{f_0 = \frac{1}{2\pi C_{in} R'_{sig}}}$

$$C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}(1 + g_m R'_L) \qquad R'_{sig} = R_{sig} \parallel R_G$$

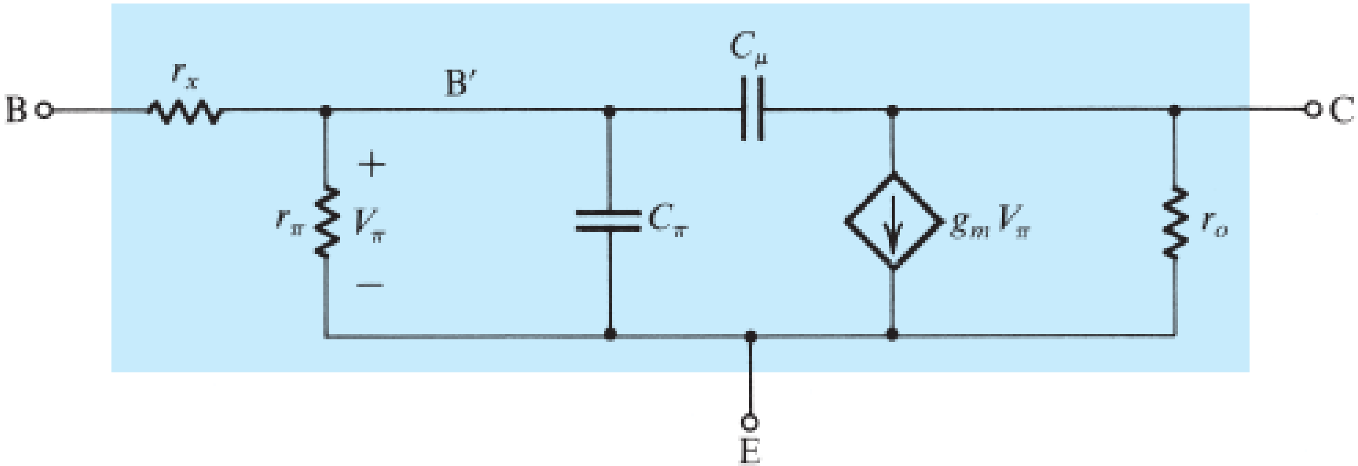


HF Response of CS Amplifier

Example 3: Find the mid-band gain A_{MB} and the upper 3-dB frequency f_H of a CS amplifier fed with a signal source having an internal resistance $R_{sig} = 100k\Omega$. The amplifier has $R_G = 4.7M\Omega$, $R_D = R_L = 15k\Omega$, $g_m = 1mA/V$, $r_o = 150k\Omega$, $C_{gs} = 1pF$, $C_{gd} = 0.4pF$.



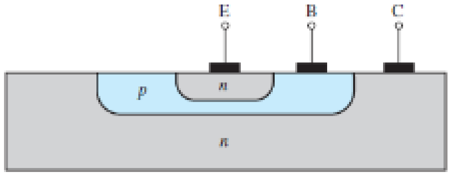
BJT High Frequency model



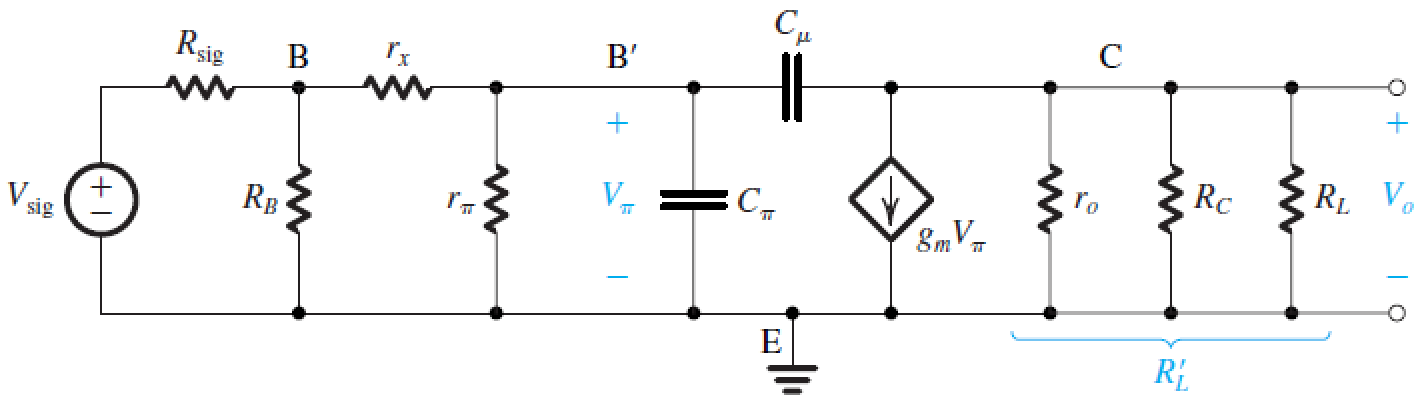
Emitter-base capacitance C_π is in the range of a few picofarads.

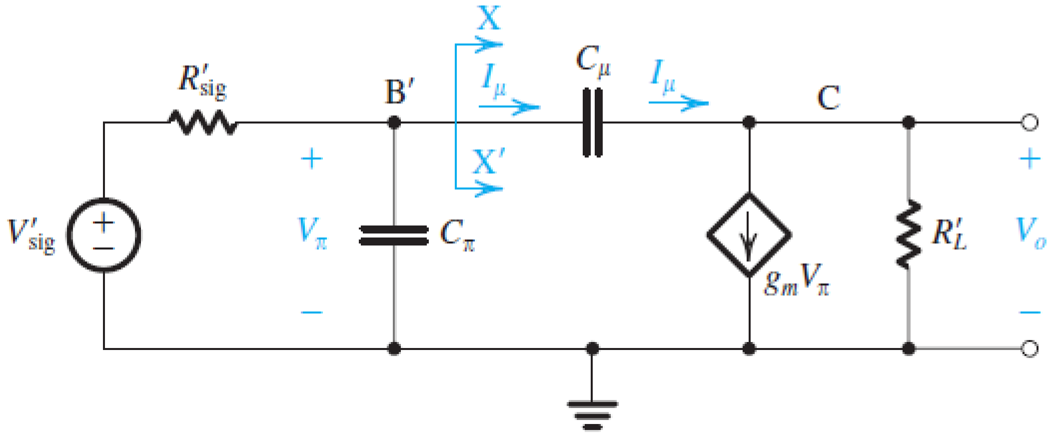
Collector-base capacitance C_μ is in the range of a fraction of pF to a few pF.

r_x is added to model the resistance of the silicon material of the base region between the base terminal B and a fictitious internal, or intrinsic, base terminal that is right under the emitter region.



HF Response of CE Amplifier



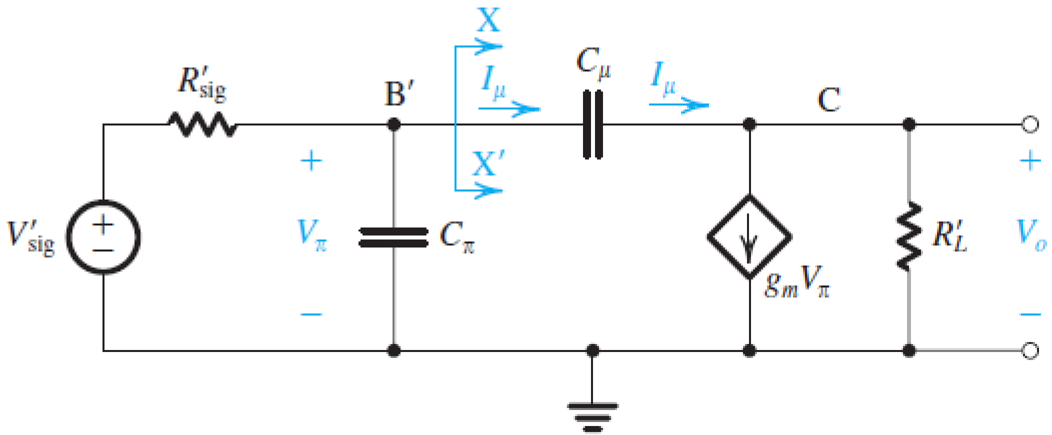


$$V'_{sig} = V_{sig} \frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \| R_B)}$$

$$R'_L = r_o \| R_C \| R_L$$

$$R'_{sig} = r_\pi \| [r_x + (R_B \| R_{sig})]$$

HF Response of CE Amplifier



$$V'_{sig} = V_{sig}\frac{R_B}{R_B + R_{sig}}\frac{r_\pi}{r_\pi + r_x + (R_{sig}\|R_B)}$$

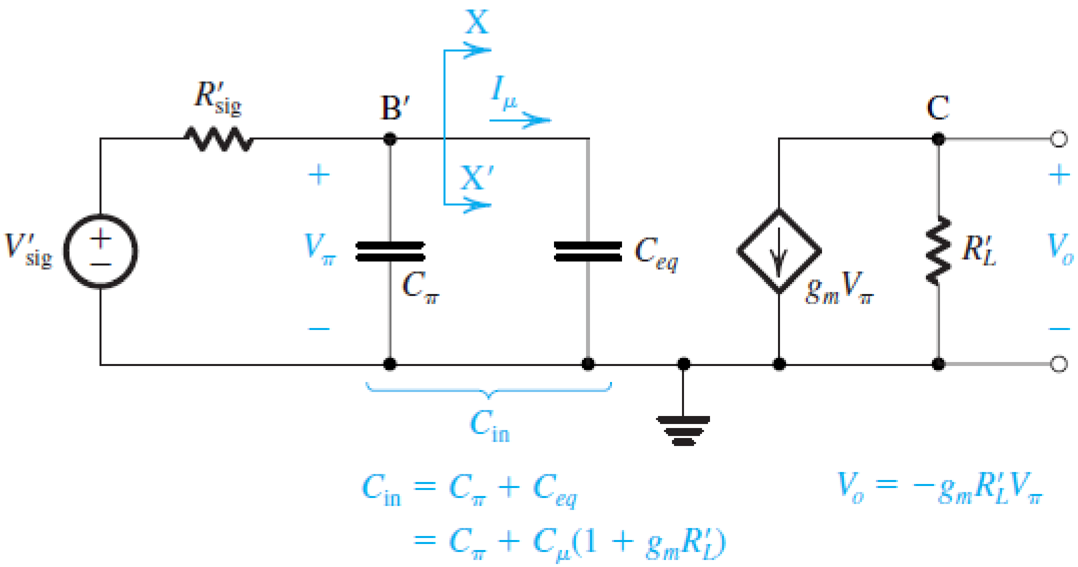
$$R'_L = r_o\|R_C\|R_L$$

$$R'_{sig} = r_\pi\|[r_x + (R_B\|R_{sig})]$$

Midband gain: $$A_{MB} = \frac{V_o}{V_{sig}} = -\frac{R_B}{R_B + R_{sig}}\frac{r_\pi}{r_\pi + r_x + R_B \parallel R_{sig}}(g_m R'_L)$$

And: $$\frac{V_o}{V_{sig}} = A_{MB}\frac{1}{1 + \frac{jf}{f_0}}$$

HF Response of CE Amplifier



the 3dB frequency: $f_0 = \dfrac{1}{2\pi C_{in} R'_{sig}}$

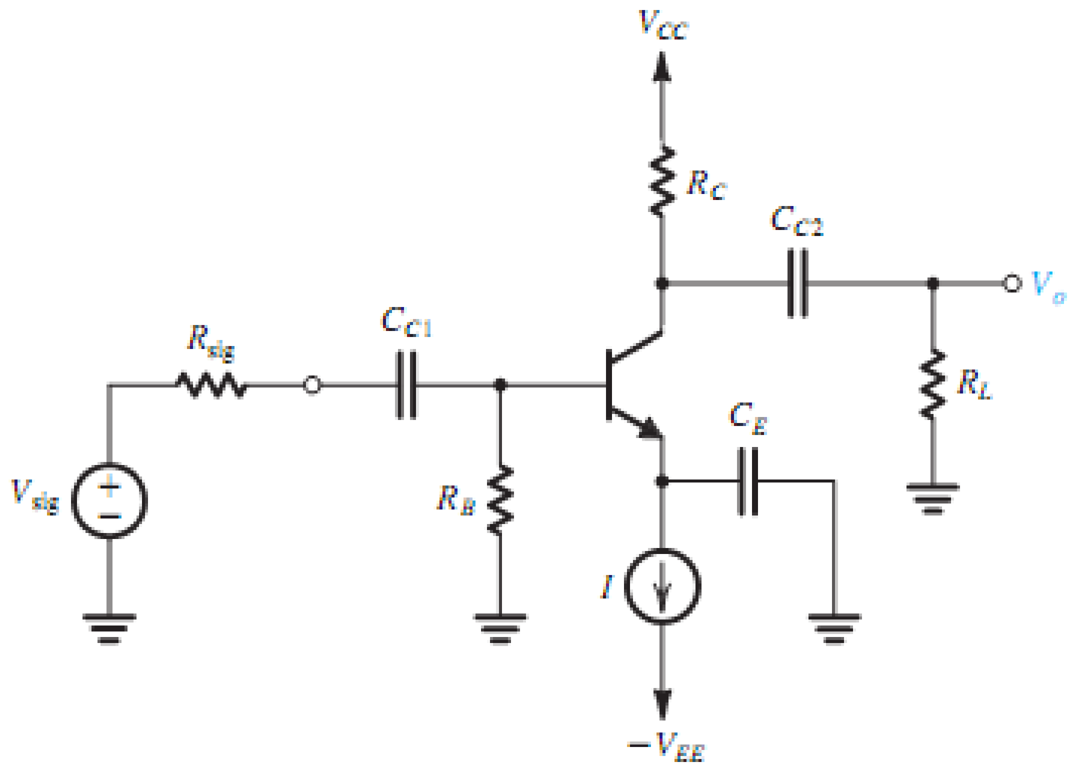
Miller effect

$$C_{in} = C_\pi + C_{eq} = C_\pi + C_\mu(1 + g_m R'_L)$$

$$R'_{sig} = R_{sig} \parallel R_G$$

HF Response of CE Amplifier

Example 4: It is required to find the mid-band gain and the upper 3-dB frequency of the common-emitter amplifier. Given: $V_{CC} = V_{EE} = 10V$, $I = 1mA$, $R_B = 100k\Omega$, $R_{sig} = 5k\Omega$, $R_L = 5k\Omega$, $\beta_0 = 100$, $V_A = 100V$, $C_\mu = 1pF$, $C_\pi = 7pF$ and $r_x = 50\Omega$.



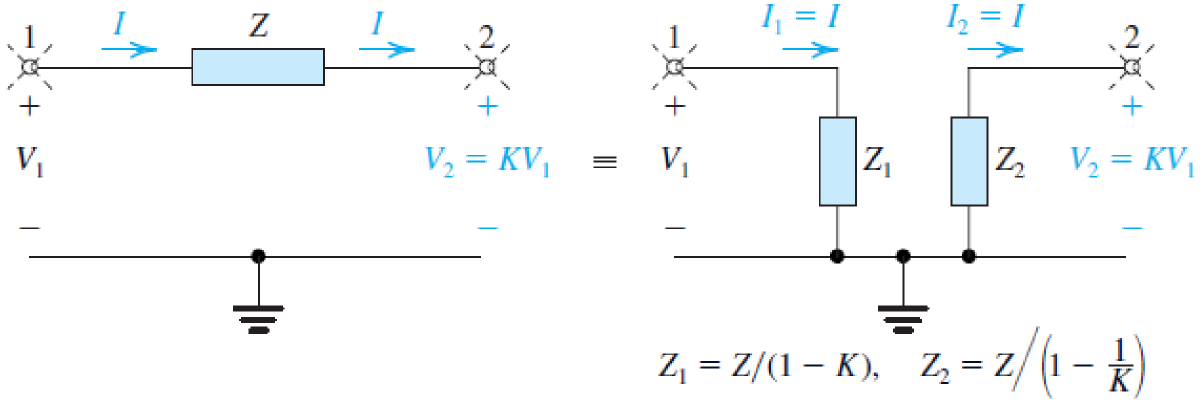
Miller's Theorem

In the analysis of HF response of CE and CS amplifiers, a technique for replacing the bridging capacitance by an equivalent input capacitance.

This technique is based on a general theorem known as **Miller's theorem**.

Assume that $V_2 = KV_1$. Miller's theorem states that impedance Z can be replaced by two impedances:

$$Z_1 = \frac{Z}{1-K} \qquad Z_2 = \frac{Z}{1 - {}^{1}/_{K}}$$

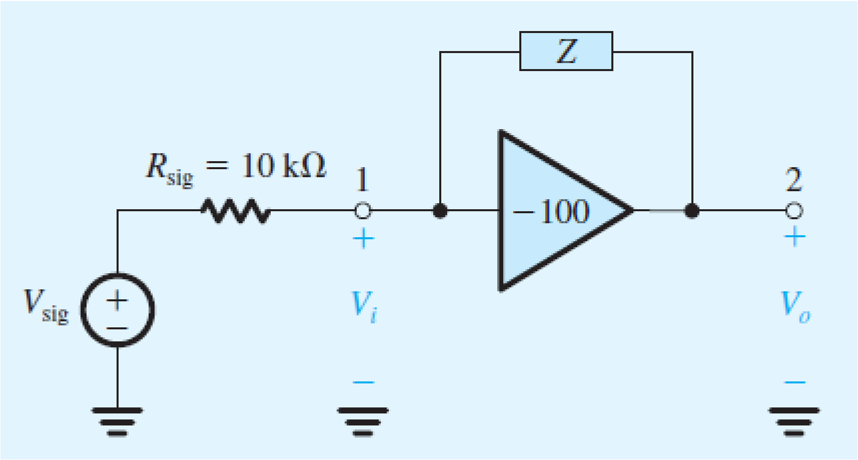


Miller's Theorem

Example 5: An ideal voltage amplifier having a gain of $-100V/V$ with an impedance Z connected between its output and input terminals. Find the Miller equivalent circuit when Z is

a. A $1M\Omega$ resistance.

b. a 1pF capacitance. In each case, use the equivalent circuit to determine $^{V_o}/_{V_{sig}}$.



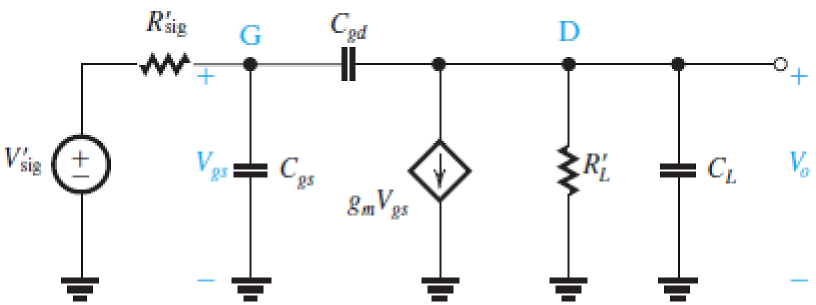
Analyzing using Miller's theorem

The value of and can be determined using Miller's theorem:

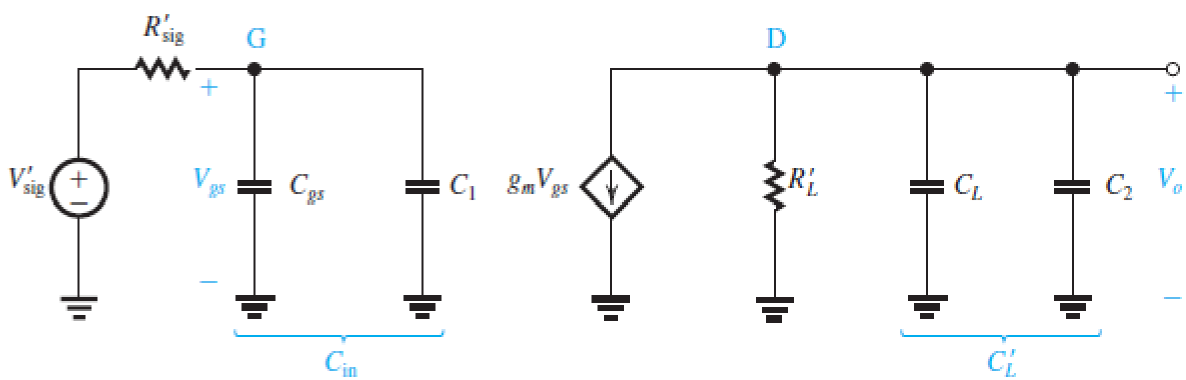
$$C_1 = C_{gd}(1 - K)$$

$$C_2 = C_{gd}\left(1 - {}^{1}\!/_{K}\right)$$

where: $K = {}^{V_o}\!/_{V_{gs}} = -g_m R'_L$



Generalized HF equivalent circuit for the CS amplifier



HF equivalent circuit model of the CS amplifier after the application of Miller's theorem

Analyzing using Miller's theorem

$\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ will be used to determine the overall transfer function.

$$C_1 = C_{gd}(1-K) = C_{gd}(1+g_mR'_L) \qquad C_2 = C_{gd}\left(1-{}^1\!/_K\right) = C_{gd}\left(1+{}^1\!/_{g_mR'_L}\right)$$

At the input side:

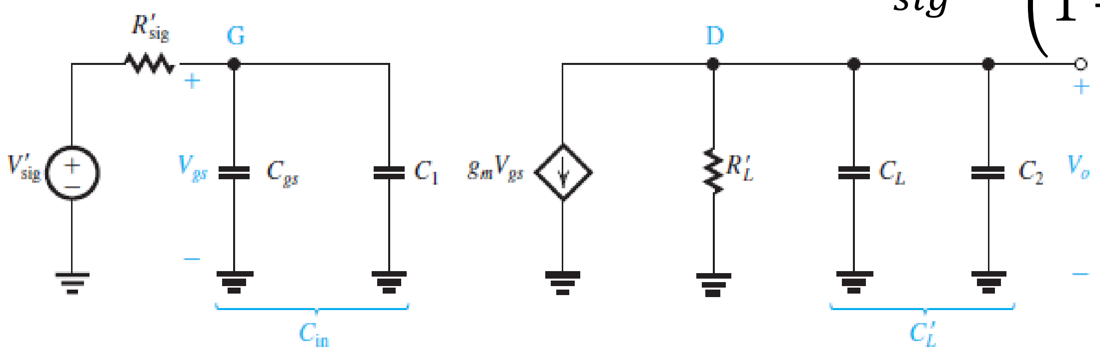
$$f_{hi} = \frac{1}{2\pi(C_{gs}+C_1)R'_{sig}}$$

At the output side:

$$f_{ho} = \frac{1}{2\pi(C_{gs}+C_2)R'_L}$$

The approximated transfer function:

$$\frac{V_o}{V'_{sig}} = \frac{-g_mR'_L}{\left(1+\frac{s}{\omega_{hi}}\right)\left(1+\frac{s}{\omega_{ho}}\right)}$$



HF equivalent circuit model of the CS amplifier after the application of Miller's theorem

$$\boxed{f_H = \frac{1}{\sqrt{\frac{1}{f_{hi}^2}+\frac{1}{f_{ho}^2}}}}$$

The HF Gain Function

The amplifier gain can be expressed in the general form: $A(s) = A_M F_H(s)$

where: $$F_H(s) = \frac{(1 + s/\omega_{z1})(1 + s/\omega_{z2}) \dots (1 + s/\omega_{zn})}{(1 + s/\omega_{P1})(1 + s/\omega_{P2}) \dots (1 + s/\omega_{Pn})}$$

The designer needs to estimate the value of the upper 3-dB frequency f_H → particularly interested in the part of the HF band close to the midband.

If the dominant pole exists: $$F_H(s) \approx \frac{1}{(1 + s/\omega_{P1})}$$

If the dominant pole does not exist: For simplicity, consider the following case:

$$F_H(s) = \frac{(1 + s/\omega_{z1})(1 + s/\omega_{z2})}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})}$$

The HF Gain Function

The magnitude of f_H can be written as:

$$|F_H(j\omega)|^2 = \frac{\left(1+\omega^2/\omega_{Z1}^2\right)\left(1+\omega^2/\omega_{Z2}^2\right)}{\left(1+\omega^2/\omega_{P1}^2\right)\left(1+\omega^2/\omega_{P1}^2\right)}$$

By definition $\omega = \omega_H$. $|F_H|^2 = \frac{1}{2}$, thus:

$$\frac{1}{2} = \frac{\left(1+\omega^2/\omega_{Z1}^2\right)\left(1+\omega^2/\omega_{Z2}^2\right)}{\left(1+\omega^2/\omega_{P1}^2\right)\left(1+\omega^2/\omega_{P1}^2\right)}$$

Since $\omega_H < \omega_P, \omega_Z$, we can neglect ω_H^4: $\omega_H \approx 1 \Big/ \sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} - \frac{2}{\omega_{Z1}^2} - \frac{2}{\omega_{Z2}^2}}$

This relationship can be extended to any number of poles and zeros:

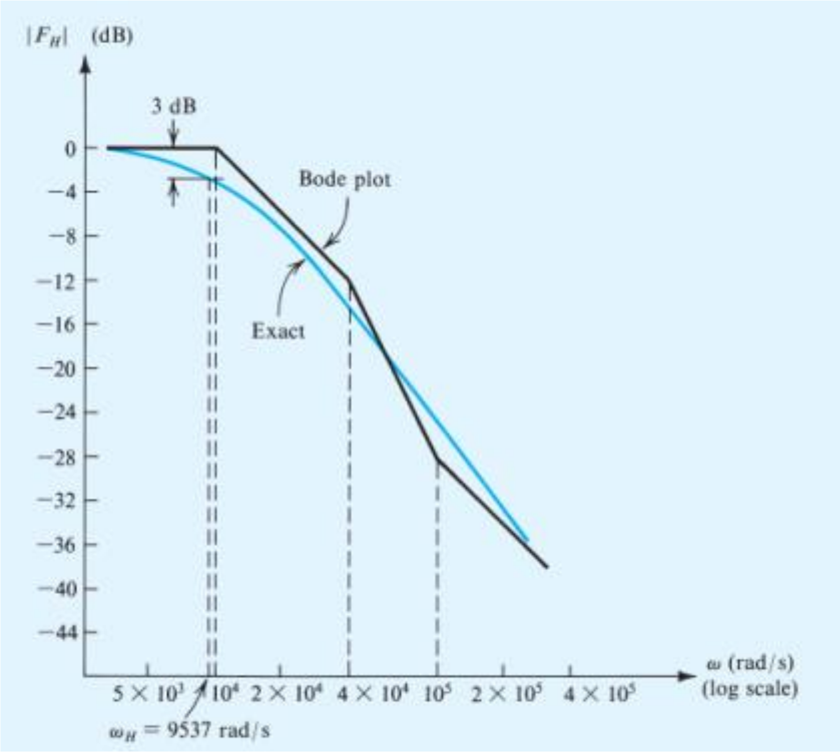
$$\omega_H \approx 1 \Big/ \sqrt{\left(\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} + \cdots\right) - \left(\frac{2}{\omega_{Z1}^2} + \frac{2}{\omega_{Z2}^2} + \cdots\right)}$$

The HF Gain Function

Example 6: The high-frequency response of an amplifier is characterized by the transfer function:

$$F_H(s) = \frac{1 - s/10^5}{(1 + s/10^4)(1 + s/4 \times 10^5)}$$

Determine the 3-dB frequency approximately and exactly.

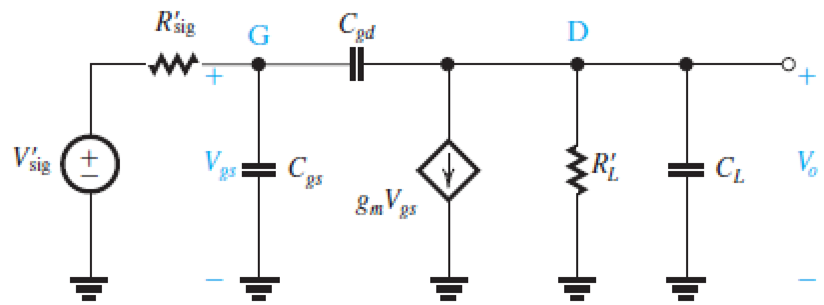


Analyzing using Exact Method

Node equation at the drain provides:

$$sC_{gd}\left(V_{gs}-V_o\right)=g_m V_{gs}+\frac{V_o}{R'_L}+sC_L V_o$$

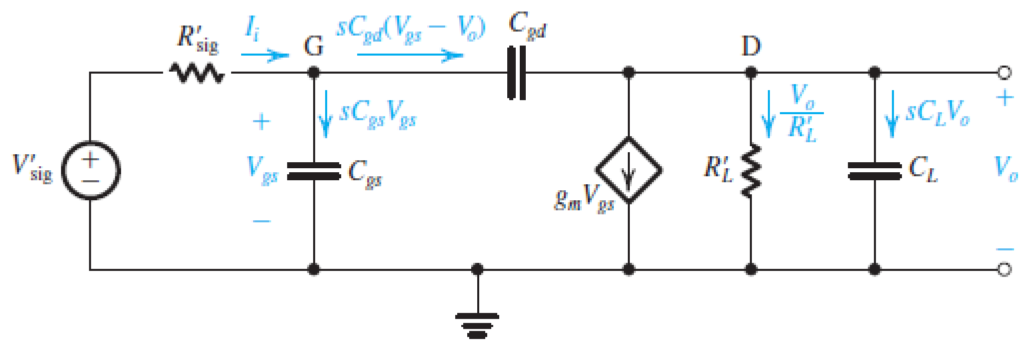
$$\rightarrow V_{gs}=\frac{-V_o}{g_m R'_L}\,\frac{1+s\left(C_L+C_{gd}\right)R'_L}{1-s\,C_{gd}/g_m}$$



Generalized HF equivalent circuit for the CS amplifier

Node equation at G provides:

$$I_i=sC_{gs}V_{gs}+sC_{gd}\left(V_{gs}-V_o\right)$$



Then:

$$V'_{sig}=V_{gs}\left[1+s\left(C_{gs}+C_{gd}\right)R'_{sig}\right]-sC_{gs}R'_{sig}V_o$$

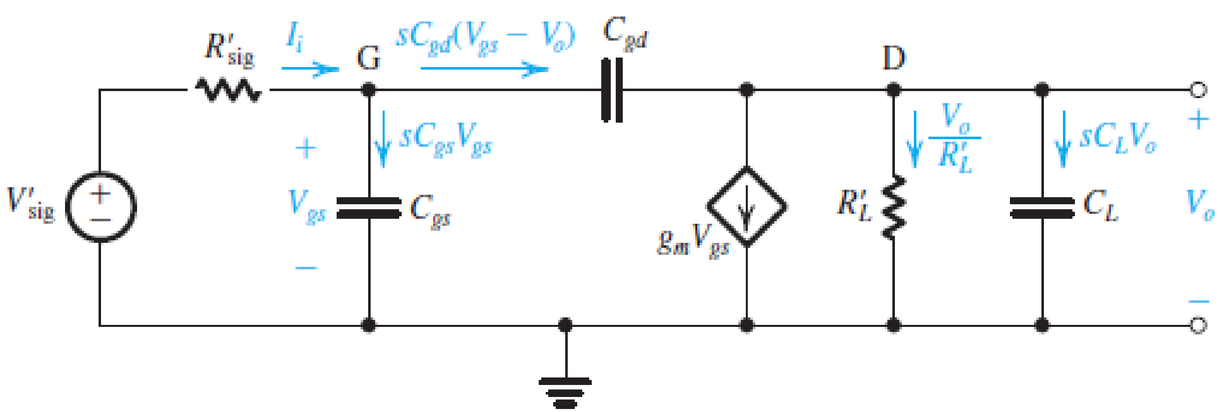
Analyzing using Exact Method

The amplifier gain is: $$\frac{V_o}{V'_{sig}} = \frac{-g_m R'_L\left[1 - s\,C_{gd}/g_m\right]}{1 + s\boldsymbol{A} + s^2\boldsymbol{B}}$$

where: $$\boldsymbol{A} = \left[C_{gs} + C_{gd}(1 + g_m R'_L)\right]R'_{sig} + \left(C_L + C_{gd}\right)R'_L$$

$$\boldsymbol{B} = \left[\left(C_L + C_{gd}\right)C_{gs} + C_L C_{gd}\right]R'_{sig}R'_L$$

The transfer function has a second-order denominator, and thus the amplifier has two poles. Also the numerator is of the first order.



Analyzing using Exact Method

The amplifier gain is: $$\frac{V_o}{V'_{sig}} = \frac{-g_m R'_L\left[1 - s\,C_{gd}/g_m\right]}{1 + s\boldsymbol{A} + s^2\boldsymbol{B}}$$

Zeros: $\omega_{P1} = \infty \qquad \omega_{P2} = g_m/C_{gd}$

Poles: the denominator polynomial $D(s)$ can be expressed as:

$$D(s) = \left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right) = 1 + s\left(\frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}}\right) + \frac{s^2}{\omega_{P1}\omega_{P2}}$$

$$\approx 1 + s\frac{1}{\omega_{P1}} + s^2\frac{1}{\omega_{P1}\omega_{P2}}$$

This gives: $$\omega_{P1} \approx \frac{1}{A} = \frac{1}{\left[C_{gs} + C_{gd}(1 + g_m R'_L)\right]R'_{sig} + (C_L + C_{gd})R'_L}$$

$$\omega_{P2} = \frac{1}{B} = \frac{\left[C_{gs} + C_{gd}(1 + g_m R'_L)\right]R'_{sig} + (C_L + C_{gd})R'_L}{\left[(C_L + C_{gd})C_{gs} + C_L C_{gd}\right]R'_{sig}R'_L}$$

Analyzing using Exact Method

Example 7: Consider an IC CS amplifier for which $g_m = 1.25mA/V^2$, $C_{gs} = 20fF$, $C_{gd} = 5fF$, $C_L = 25fF$, $R'_{sig} = 10k\Omega$, $R'_L = 10k\Omega$. Determine f_H using

a. the Miller approximation.

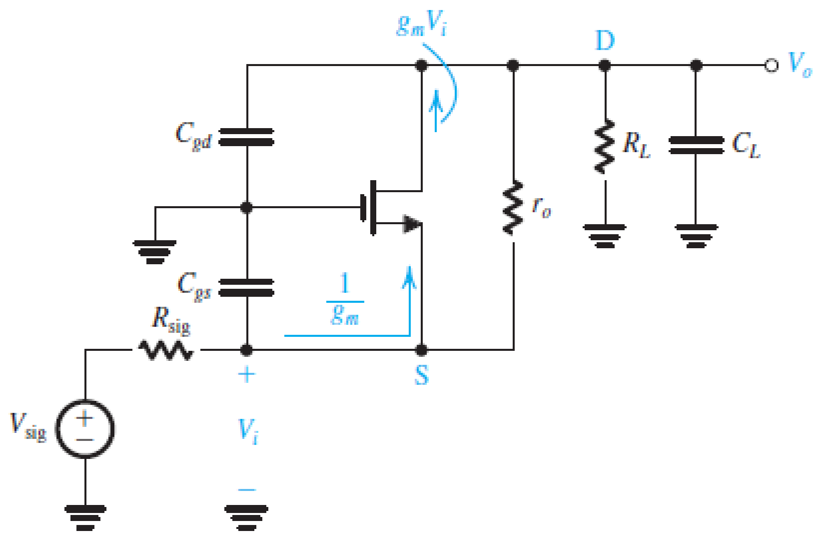
b. Miller's theorem.

c. Determine the frequencies of the two poles and the zero and hence the 3-dB frequency.

HF Response of the CG Amplifiers

- CS and CE amplifier:
  - Substantial gain at mid-band frequencies.
  - Low f_H due to the large input capacitance C_{in} (Miller effect).

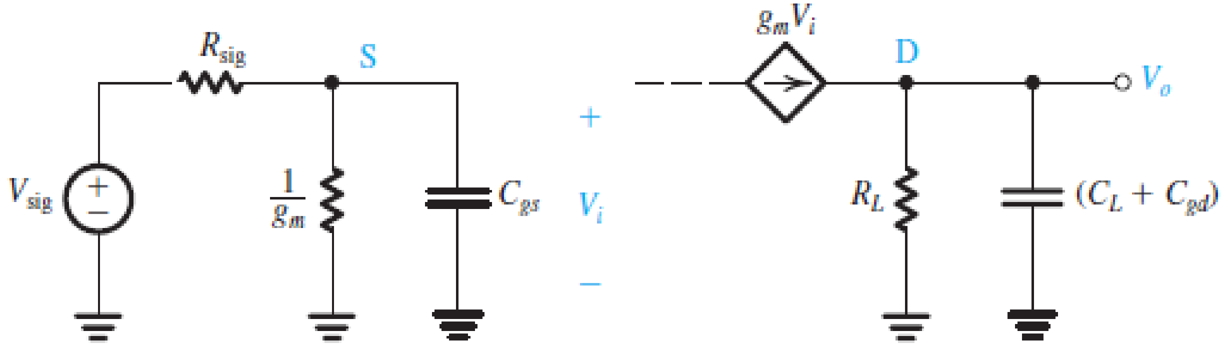
→ In order to obtain wide bandwidth: need circuit configurations that do not suffer from the Miller effect: Common Gate (CG) circuit.



HF Response of the CG Amplifiers

If r_0 is neglected: the circuit is greatly simplified.

Two poles: $$f_{P1} = \frac{1}{2\pi C_{gs}\left(R_{sig} \parallel \frac{1}{g_m}\right)} \qquad f_{P2} = \frac{1}{2\pi R_L(C_{gs} + C_L)}$$

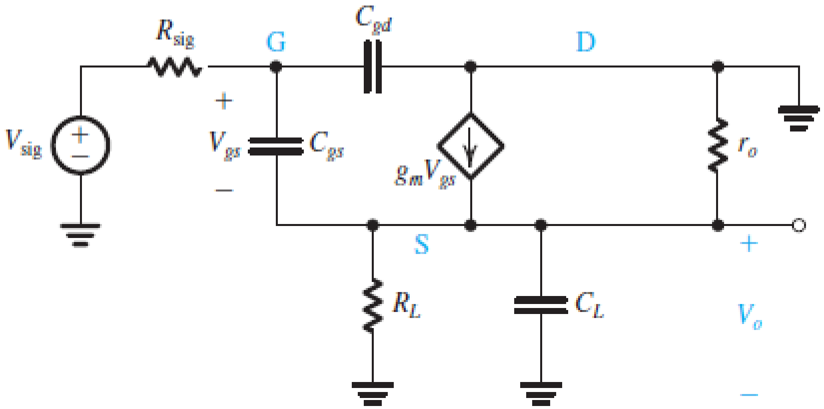


If r_0 is not neglected: reading Ref. page 746-750.

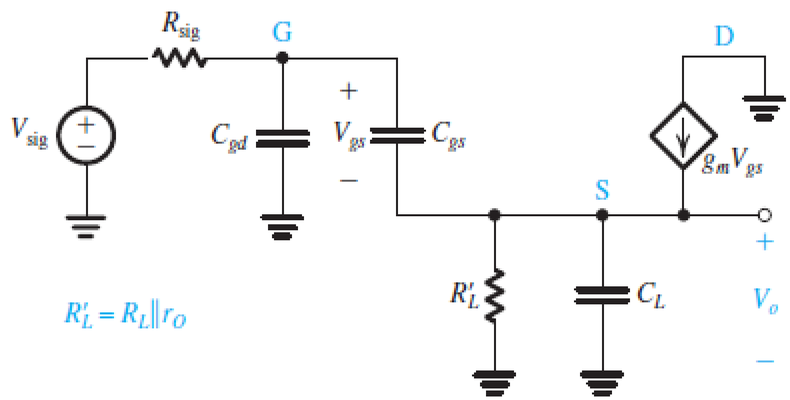
HF Response of the Source and Emitter Amplifiers

Major advantage of the source follower: its excellent high-frequency response.

Midband gain: $$A_M = \frac{(R_L \parallel r_o)}{(R_L \parallel r_o) + \frac{1}{g_m}}$$ $$R_o = \frac{1}{g_m} \parallel r_o$$



Equivalent circuit of Source Follower Amplifier



Simplified equivalent circuit of Source Follower Amplifier

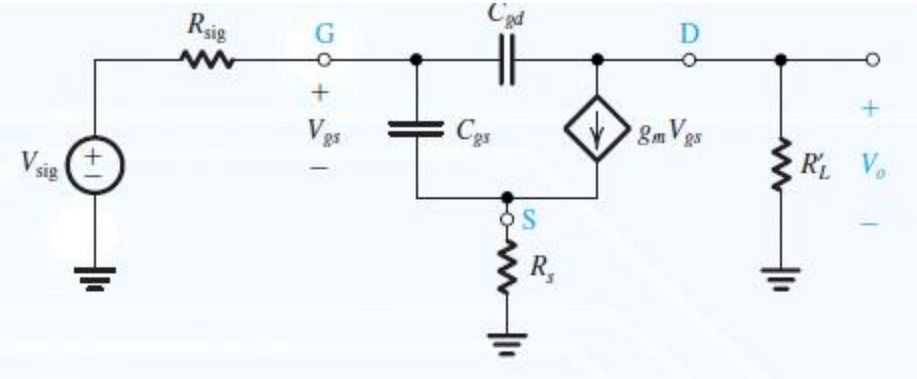
Exercises

Exercise 4: The following figure shows the high-frequency equivalent circuit of a CS amplifier with a resistance R_s connected in the source lead. The purpose of this problem is to show that the value of R_s can be used to control the gain and bandwidth of the amplifier, specifically to allow the designer to trade gain for increased bandwidth.

a. Derive an expression for the low-frequency voltage gain.

b. Derive R_{gs} and R_{gd}.

c. Let $R_{sig} = 100k\Omega$, $g_m = 4mA/V$, $R_L' = 5k\Omega$, $C_{gs} = C_{gd} = 1pF$. Determine the low frequency gain and 3dB frequency f_H for 3 cases: $R_s = 0$, $R_s = 100\Omega$ and, $R_s = 250\Omega$. Comment.

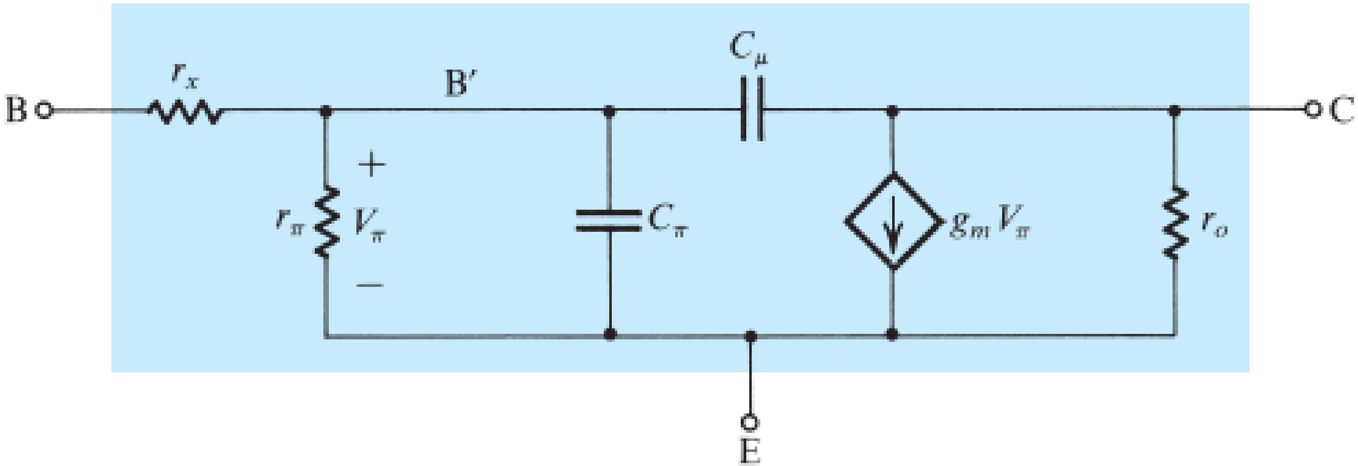


$R_s = 0$: $A_v = -20$, $f_H = 72kHz$

$R_s = 100$: $A_v = -14.3$, $f_H = 99kHz$

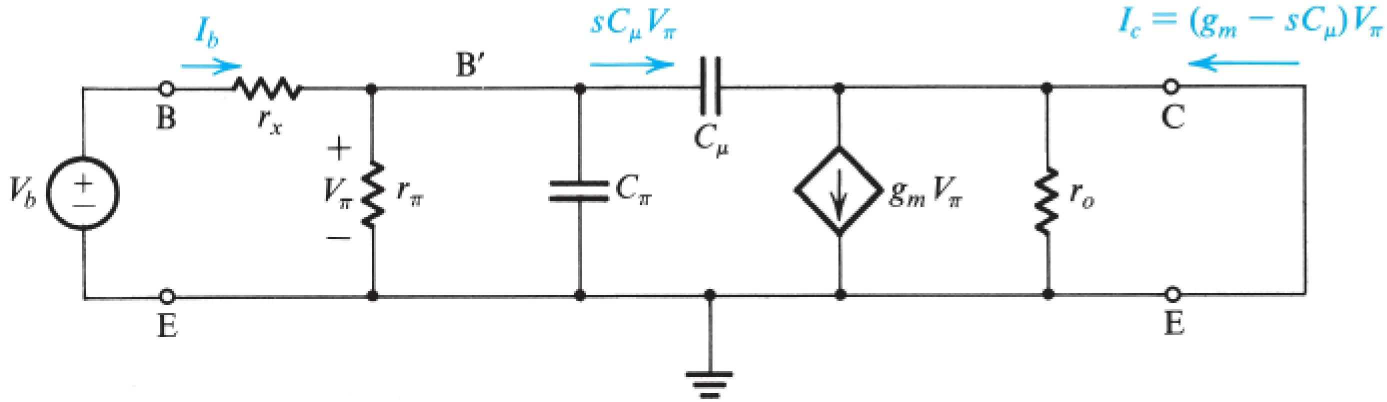
$R_s = 250$: $A_v = -10$, $f_H = 137kHz$

A1 - BJT High Frequency model



- The transistor data sheets do not usually specify the value of C_π .
- Rather, the behavior of β (or h_{fe}) versus frequency is normally given.
- Need to derive an expression for h_{fe} to determine C_π and C_μ

A1 - BJT High Frequency model



The short-circuit collector current I_c can be written as: $I_c = (g_m - sC_\mu)V_\pi$

And $V_\pi = I_b(r_\pi \parallel C_\pi \parallel C_\mu) = \dfrac{I_b}{1/r_\pi + sC_\pi + sC_\mu}$

Then $h_{fe} \equiv \dfrac{I_c}{I_b} = \dfrac{(g_m - sC_\mu)}{1/r_\pi + sC_\pi + sC_\mu} \simeq \dfrac{g_m r_\pi}{1 + s(C_\pi + C_\mu)r_\pi}$

Q&A