Math 250 Optimization Problem Fun Pack !!

1. Find the absolute max and min values of f subject to the given constraint:

   (a) $f(x,y) = 8x + 3y$; $(x-1)^2 + (y+2)^2 = 9$
   . [min is $2 - 3\sqrt{73}$ at $(1 - 24/\sqrt{73}, -2 - 9/\sqrt{73})$,
   . max is $2 + 3\sqrt{73}$ at $(1 + 24/\sqrt{73}, -2 + 9/\sqrt{73})$]

   (b) $f(x,y) = xy$; $\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1$ [max $5\sqrt{2}$, min $-5\sqrt{2}$]

   (c) $f(x,y) = x^2 + y^3$; $x^2 + y^2 = 49$ [min -343 at (0,-7), max 343 at (0,7)]

   (d) $f(x,y,z) = z - x^2 - y^2$; $\dfrac{x^2}{4} + \dfrac{y^2}{9} + \dfrac{z^2}{16} = 1$ [max 4 at (0,0,4), min -$\frac{85}{9}$ at $(0, \sqrt{3}, -\frac{8}{9})$]

   (e) $f(x,y) = \cos(x^2 - y^2)$; $x^2 + y^2 = 1$ [max 1, min $\cos 1$]

   (f) $f(x,y) = \cos^2 x + \cos^2 y$; $x + y = \frac{\pi}{4}$.

   (g) $f(x,y,z) = x^2 + y^2 + z^2$; on the set $S = \{(x, \cos x) : x \in R\}$.

   (h) $f(x,y) = e^{2xy}$; $x^3 + y^3 = 16$ [max e^8 at $(2,2)$]

   (i) $f(x,y) = (xy)^{1/2} + y^2$; $x + y = 1$ with $|x| < 1$ and $|y| < 1$.

2. Find the points on the surface $z^2 - xy = 1$ nearest the origin. [(0,0,1), (0,0,-1)]

3. A triangular field is to be enclosed by p feet of fencing so as to maximize the area of the field. Find the lengths of the sides of this triangle. [Hint: Heron's formula for the area of a triangle with side lengths x, y, and z is $A = \sqrt{s(s-x)(s-y)(s-z)}$ where $s = \frac{1}{2}(x+y+z)$.] [$x = y = z = p/3$]

4. Find the extreme values of $f(x,y) = x^2 y$ on the line $x + y = 3$.

5. The temperature at a point (x,y) on a metal plate is $T(x,y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant? [low=0°, high=125°]

6. Find the dimensions of the closed right circular cylindrical can of smallest surface area whose volume is 16π cm^3. [$r = 2$cm, $h = 4$cm]

7. A space probe in the shape of the ellipsoid $4x^2 + y^2 + z^2 = 16$ enters Earth's atmosphere and its surface begins to heat. After 1 hour, the temperature at the point (x,y,z) on the probe's surface is $T(x,y,z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface.
   . [$(0, -2. - \sqrt{12})$]

8. A firm uses wool and cotton fiber to produce cloth. The amount of cloth produced is given by $Q(x,y) = xy - x - y + 1$, where x is the number of pounds of wool, and y is the number of pounds of cotton, $x > 1$ and $y > 1$. If wool costs p dollars per pound, and cotton q dollars per pound, and the firm can spend B dollars on material, what should the ratio of cotton and wool be to produce the most cloth?

9. A rectangular mirror with area A square feet is to have trim along the edges. If the trim along the horizontal edges costs p cents per foot and that for the vertical edges costs q cents per foot, find the dimensions which will minimize the total cost. [horiz = $\sqrt{qA/p}$, vert = $\sqrt{pA/q}$]

10. Find the max and min of $f(x,y) = xy - y + x - 1$ on the set $x^2 + y^2 \le 2$.

11. Find the max and min of $f(x,y) = x^2 + xy + y^2$ on the unit disk. [$\frac{3}{2}$ abs max, 0 abs min]