Series Problems Fun Pack !

I. Do the series below converge or diverge? What test would you use? Find the sum of each series when reasonable.

1. $\sum_{n=1}^{\infty} \dfrac{n}{n^3 + 1}$

2. $\sum_{n=1}^{\infty} \dfrac{(-1)^n \sqrt{n}}{n + 1}$

3. $\sum_{n=1}^{\infty} \dfrac{\cos(3n)}{1 + (1.2)^n}$

4. $\sum_{n=1}^{\infty} \dfrac{2}{n^2 + 4n + 3}$

5. $\sum_{n=0}^{\infty} \dfrac{4^{n+1}}{5^n}$

6. $\sum_{n=2}^{\infty} \dfrac{1}{n^2 - 1}$

7. $\sum_{n=1}^{\infty} \dfrac{1 + \cos n}{n^5}$

8. $\sum_{n=1}^{\infty} \dfrac{\arctan n}{n^4}$

9. $\sum_{n=2}^{\infty} \dfrac{1}{n \ln n}$

10. $\sum_{n=1}^{\infty} \dfrac{1}{n^{1.1}}$

11. $\sum_{n=1}^{\infty} \dfrac{n^2}{5(n + 1)(n + 3)}$

12. $\sum_{n=1}^{\infty} \dfrac{n^n}{n!}$

13. $\sum_{n=1}^{\infty} \dfrac{n \ln n}{(n + 1)^4}$

14. $\sum_{n=1}^{\infty} \dfrac{n}{3n + 1}$

15. $\sum_{n=1}^{\infty} \left(\dfrac{n}{3n + 1} \right)^n$

16. $\sum_{n=1}^{\infty} \ln \left(\dfrac{n}{3n + 1} \right)$

17. $\sum_{n=1}^{\infty} \ln \left(\dfrac{n}{n + 1} \right)$

18. $\sum_{n=1}^{\infty} \dfrac{3n^2 - \sqrt{n}}{\sqrt{2n^5 - n + 8}}$

19. $\sum_{n=1}^{\infty} \dfrac{n}{(n - 1)2^n}$

20. $\sum_{n=1}^{\infty} \dfrac{\sin n}{n\sqrt{n + 2}}$

21. $\sum_{n=1}^{\infty} \dfrac{1}{2n + 3}$

22. $\sum_{n=1}^{\infty} \dfrac{n^2 \arctan n}{n^4 + 1}$

23. $\sum_{n=1}^{\infty} \dfrac{1}{e^{2n}}$

24. $\sum_{n=1}^{\infty} \dfrac{n!}{n^n}$

25. $\sum_{n=1}^{\infty} \dfrac{(n!)^n}{n^{(n^2)}}$

26. $\sum_{n=1}^{\infty} \dfrac{3}{5^n + 2}$

27. $\sum_{n=2}^{\infty} \dfrac{1}{\ln(\ln n)}$

28. $\sum_{n=1}^{\infty} \dfrac{5 + \sin n}{n}$

29. $\sum_{n=1}^{\infty} \dfrac{1}{(\arctan n)^n}$

30. $\sum_{n=1}^{\infty} \dfrac{(-1)^n n}{\sqrt{n^2 + 1}}$

31. $\sum_{n=1}^{\infty} \dfrac{e^n}{n^n}$

32. $\sum_{n=1}^{\infty} (-1)^n \dfrac{2^{5n}}{5^{2n+1}}$

33. $\sum_{n=1}^{\infty} \dfrac{\sqrt{n} + n}{3n^2 + 4}$

34. $\sum_{n=3}^{\infty} \dfrac{\cos(n\pi)}{n - 2}$

35. $\sum_{n=1}^{\infty} \left(\dfrac{n - 2}{n} \right)^{n^2}$

36. $\sum_{n=1}^{\infty} \cos \left(\dfrac{1}{n + 1} \right) - \cos \left(\dfrac{1}{n} \right)$

II. Are the series below absolutely convergent, conditionally convergent, or divergent? What test would you use?

1. $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n}$

2. $\sum_{n=1}^{\infty} \dfrac{(2n + 1)(-1)^n}{6n + 2}$

3. $\sum_{n=1}^{\infty} \dfrac{(-1)^n n^2}{(n + 1)!}$

4. $\sum_{n=1}^{\infty} (-1)^n \dfrac{n^3}{3^n}$

5. $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1} n}{1 + n^2}$

6. $\sum_{n=2}^{\infty} \dfrac{(-1)^n 3^n}{5^{n-1}}$

* Thanks to M. Raven for most of these problems!

Series Problems Fun Pack Answers

Section I

1. c, comp test
2. c, alt series test
3. c
4. c, telescoping, $\frac{5}{6}$
5. c, geometric, 20
6. c, , telescoping, $\frac{3}{4}$
7. c, comp test
8. c, comp test
9. d, integral test
10. c, p-series, int test
11. d, div test
12. d, ratio (lim $=$ e), or div test

13. c, limit comp test
14. d, div test
15. c, ratio or root test
16. d, div test
17. d, telescoping
18. d, limit comp test
19. c, ratio test
20. c, comp test
21. d, int, comp, limit comp tests
22. c, limit comp test
23. c, geometric, $\frac{1}{e^2-1}$
24. c, ratio (lim $=\frac{1}{e}$); comp to $\frac{2}{n^2}$

25. c, root (terms $<\frac{1}{n}$, lim=0)
26. c, comp test
27. d, comp test
28. d, comp test
29. c, root test
30. d, alt series test
31. c, root test
32. d, limit theorem
33. d, lim comp test
34. c, alt series test
35. c, root test
36. c, telescoping, $1-\cos 1$

Section II

1. cc, harmonic, alt series test
2. d, alt series test, limit theorem

3. ac, ratio test
4. ac, ratio test

5. cc, alt series test, comp test
6. ac, geometric, $\frac{9}{8}$