Exercise 5.4 - Weight Space Symmetry (1 + 0.5 points)

Consider a neural network with a single hidden layer consisting of M neurons and *tanh* activation function. For any neuron in the hidden layer, simultaneous change of sign of input and output weights from the neuron leads to no change in the output layer, therefore, producing an equivalent transformation. Similarly, the simultaneous interchange of input and output weights between any pair of neurons produces an equivalent transformation.

a) Find the total number of equivalent transformation for the hidden layer.

b) Consider a deep neural network with N hidden layers. Each hidden layer consists of M_i neurons where $i \in 1, 2, ..., N$ and *tanh* activation function. Find the total number of equivalent transformations for the network.

a) For any given hidden layer of size M, there will be M such sign-flips and thus 2^M equivalent transformations.

Furthermore there are $M!$ possible permutations of the weight vector under interchange symmetry.

Thus the total number of equivalent transformations is

$$2^M M!$$

b) For deep neural networks the number of equivalent transformations will be given by the product of the equivalent transformations of each layer:

$$\prod_{i=1}^{N} 2^{M_i} M_i!$$