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## Exercise Sheet 2

Linear Algebra and PCA

**Deadline: 23.09.2022 08:00**

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### Exercise 2.1 - Linear Algebra

(0.5+1+2 points)

- Compute the eigenvalues and eigenvectors of the matrix  $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$
- Given a matrix  $A$  for which an inverse exists, find the relationship between the eigenvalues of  $A$  and  $A^{-1}$ .
- Show that if  $\lambda$  is an eigenvalue of  $AB$ , then it is also an eigenvalue of  $BA$  where  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times n}$ .

### Exercise 2.2 - Matrix Calculus

(0.5+1+1+1 points)

In this lecture we will often compute the derivatives of multivariate functions and matrix valued functions. Let  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ ;  $w, x, c \in \mathbb{R}^n$ ,  $A \in \mathbb{R}^{n \times n}$  and  $B \in \mathbb{R}^{n \times n}$ . Prove that the following rules hold.

- $f(x) = w^T x$ , then  $\nabla_x f(x) = w$
- $f(x) = x^T A x$ , then  $\nabla_x f(x) = Ax + A^T x$
- $f(x) = \|Bx\|_2^2$ , then  $\nabla_x f(x) = 2B^T Bx$
- $f(x) = \|Bx - c\|_2^2$ , then  $\nabla_x f(x) = 2B^T (Bx - c)$

### Exercise 2.3 - PCA

(3 points)

See assignment2.ipynb

## Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

- Please submit the assignment as a **team of two to three** students.

- Write the Microsoft Teams user name, student id and the name of each member of your team on your submission.
- Hand in zip file containing a **single** PDF with your solutions and the completed ipython notebook. Do not include any data or cache files (e.g. `--pycache--`).
- Important: please name the submitted zip folder and files inside using the format: **Name1\_id1\_Name2\_id2**.
- Your assignment solution must be uploaded by only **one** of your team members to the 'Assignments' tab of the tutorial team (in **Microsoft Teams**). Please remember to press the **Hand In** button after uploading your work.
- If you have any trouble with the submission, contact your tutor **before** the deadline.