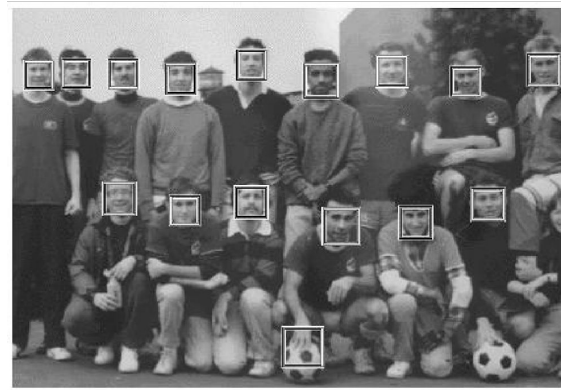


Intro to Machine Learning Algorithms

Outline

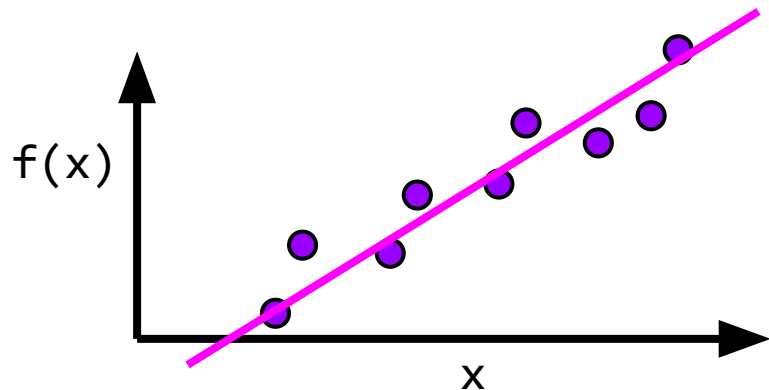
- Linear Regression
- K-nearest neighbour
- Decision Tree
- Logistic Regression



Acknowledgement : Most of slide credits go to CSE455, University of Washington and CS131, Stanford University

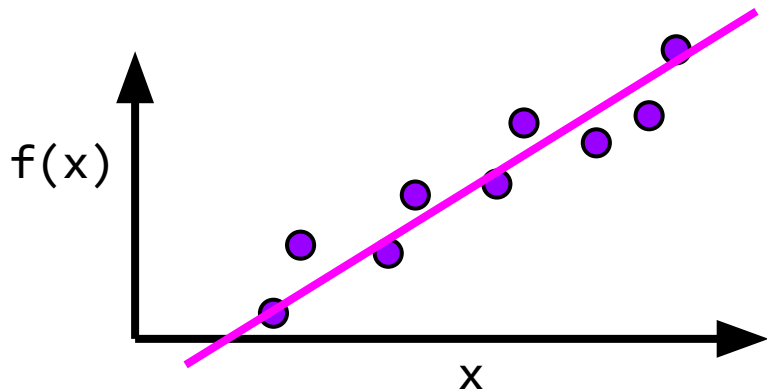
Linear regression

- $f^*(x) = ax + b$
- Learn a and b from data (how?)



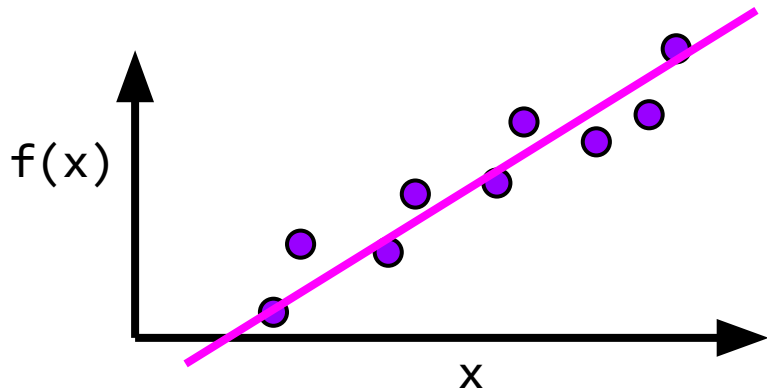
Linear regression

- $f^*(x) = ax + b$
- Learn a and b from data (how?)
 - Minimize squared error!
 - Loss function $L(f^*) = \sum_i ||f(x_i) - f^*(x_i)||^2$



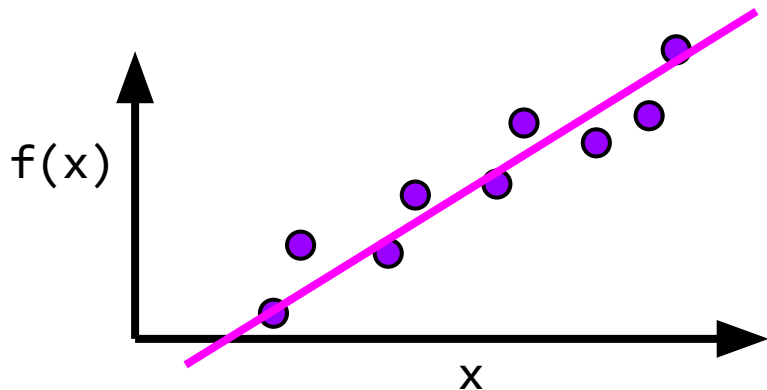
Linear regression

- $f^*(x) = ax + b$
- Learn a and b from data (how?)
 - Minimize squared error!
 - Loss function $L(f^*) = \sum_i ||f(x_i) - f^*(x_i)||^2$
 - Want $\operatorname{argmin}_{a,b}[L(f^*)]$
 - Extrema when derivative = 0



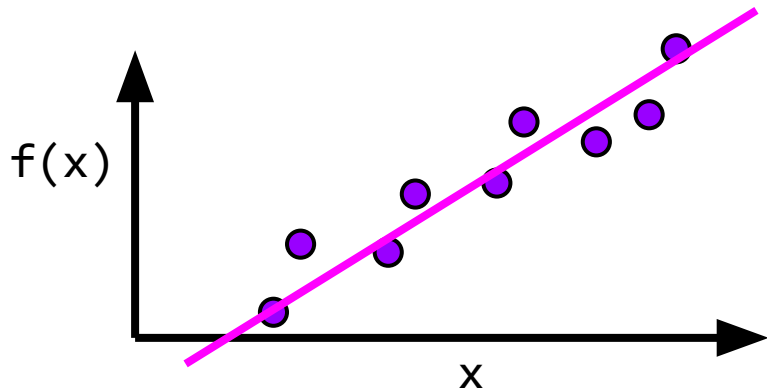
Linear regression

- $f^*(x) = ax + b \cdot 1$
- Learn a and b from data (how?)
 - Minimize squared error!
 - Loss function $L(f^*) = \sum_i ||f(x_i) - f^*(x_i)||^2$
 - Want $\operatorname{argmin}_{a,b}[L(f^*)]$
 - Extrema when derivative = 0
 - Solve linear system of equations
 - $Ma = b$
 - Already did this!



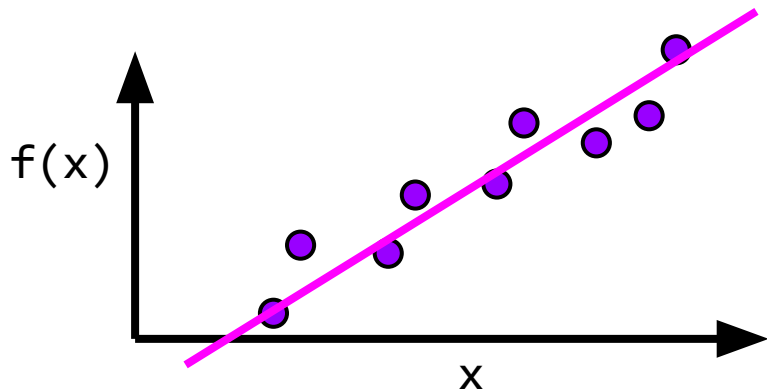
Linear regression

- $f^*(x) = ax + b$
- Learn a and b from data (how?)
- High bias: linear assumption
- Low variance
- Benefits:
 - Closed form solution
 - Fast to compute for new data



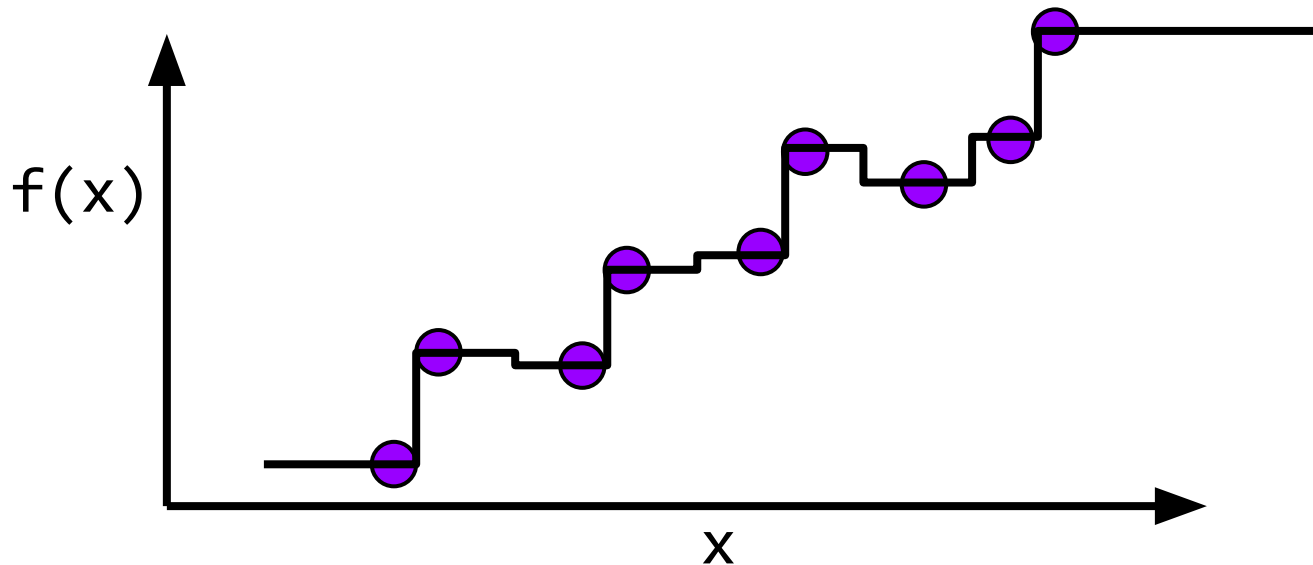
Linear regression

- $f^*(x) = ax + b$
- Learn a and b from data (how?)
- High bias: linear assumption
- Low variance
- Benefits:
 - Closed form solution
 - Fast to compute for new data
- Weaknesses:
 - Not very powerful, **assumes linear**
 - **Underfit** more interesting data



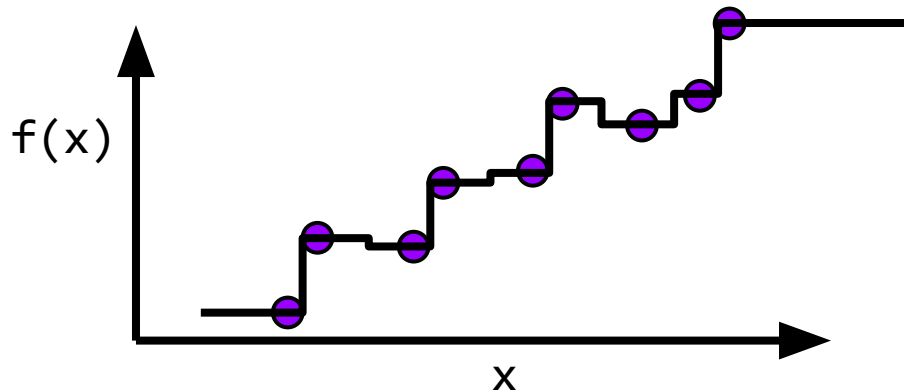
Nearest neighbor

- $f^*(x) = f(x')$ for nearest x' in training set



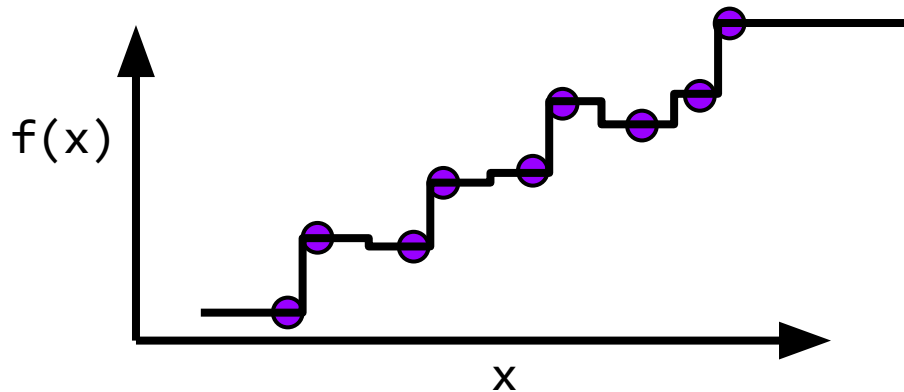
Nearest neighbor

- $f^*(x) = f(x')$ for nearest x' in training set
- Low bias: no assumptions about data
- High variance: very sensitive to training set



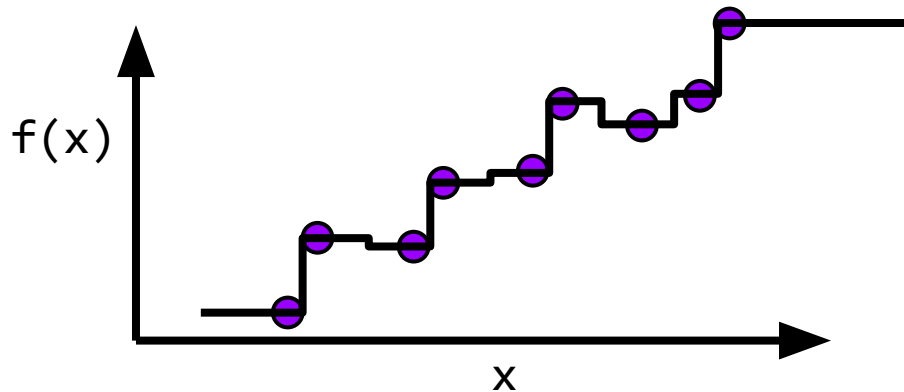
Nearest neighbor

- $f^*(x) = f(x')$ for nearest x' in training set
- Low bias: no assumptions about data
- High variance: very sensitive to training set
- Benefits:
 - Super easy to implement
 - Easy to understand
 - Arbitrarily powerful, esp with lots of data



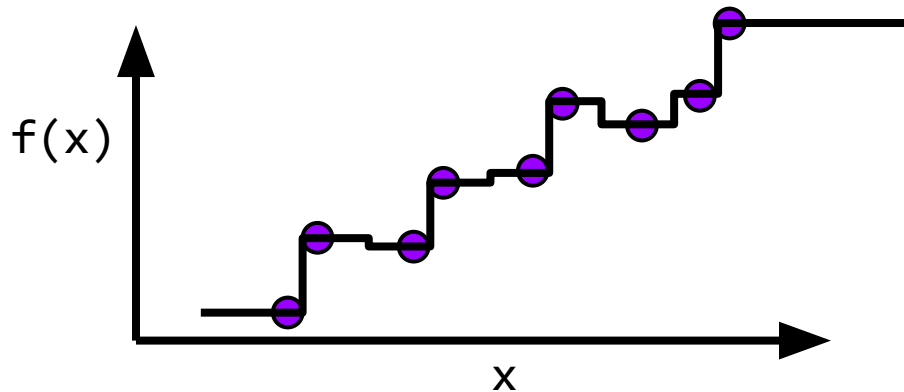
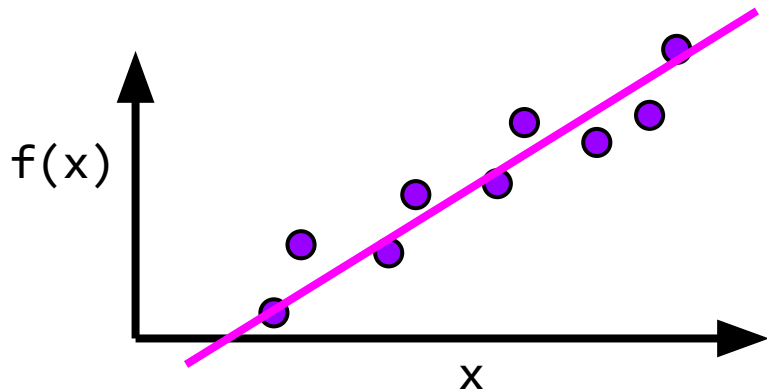
Nearest neighbor

- $f^*(x) = f(x')$ for nearest x' in training set
- Low bias: no assumptions about data
- **High variance**: very sensitive to training set
- Benefits:
 - Super easy to implement
 - Easy to understand
 - Arbitrarily powerful, esp with lots of data
- Weaknesses:
 - Hard to scale
 - Prone to **overfitting to noise**



These are examples of *regression*

- Given training data
 - input variables X , output variables Y
- And new data point x'
- Predict corresponding output variables y'



A different task: *Classification*

- Training data: points associated with a class
 - Also other data about that point
- Example: Does patient have the flu?
 - Binary classification (yes or no)
 - Different types of variables (continuous, discrete)

sore throat	runny nose	nausea	temp	chills	pain	age	days	diagnosis
no	yes	yes	101.3	yes	7	15	5	flu
yes	yes	no	98.8	no	3	74	3	not flu
yes	yes	no	100.1	yes	4	46	4	flu
yes	yes	yes	99.8	yes	6	27	1	flu
yes	no	no	98.4	yes	5	35	2	not flu
yes	yes	yes	99.0	no	3	42	4	not flu

One approach: partitions

- Find best split or splits to data along one variable
- One possibility, $\text{Pr}(\text{flu}) = (\text{temp} > 99.5)$
 - Pretty accurate on our training data

sore throat	runny nose	nausea	temp	chills	pain	age	days	diagnosis
no	yes	yes	101.3	yes	7	15	5	flu
yes	yes	no	98.8	no	3	74	3	not flu
yes	yes	no	100.1	yes	4	46	4	flu
yes	yes	yes	99.8	yes	6	27	1	flu
yes	no	no	98.4	yes	5	35	2	not flu
yes	yes	yes	99.9	no	3	42	4	not flu

Trees: layers of partitions

Very simple models

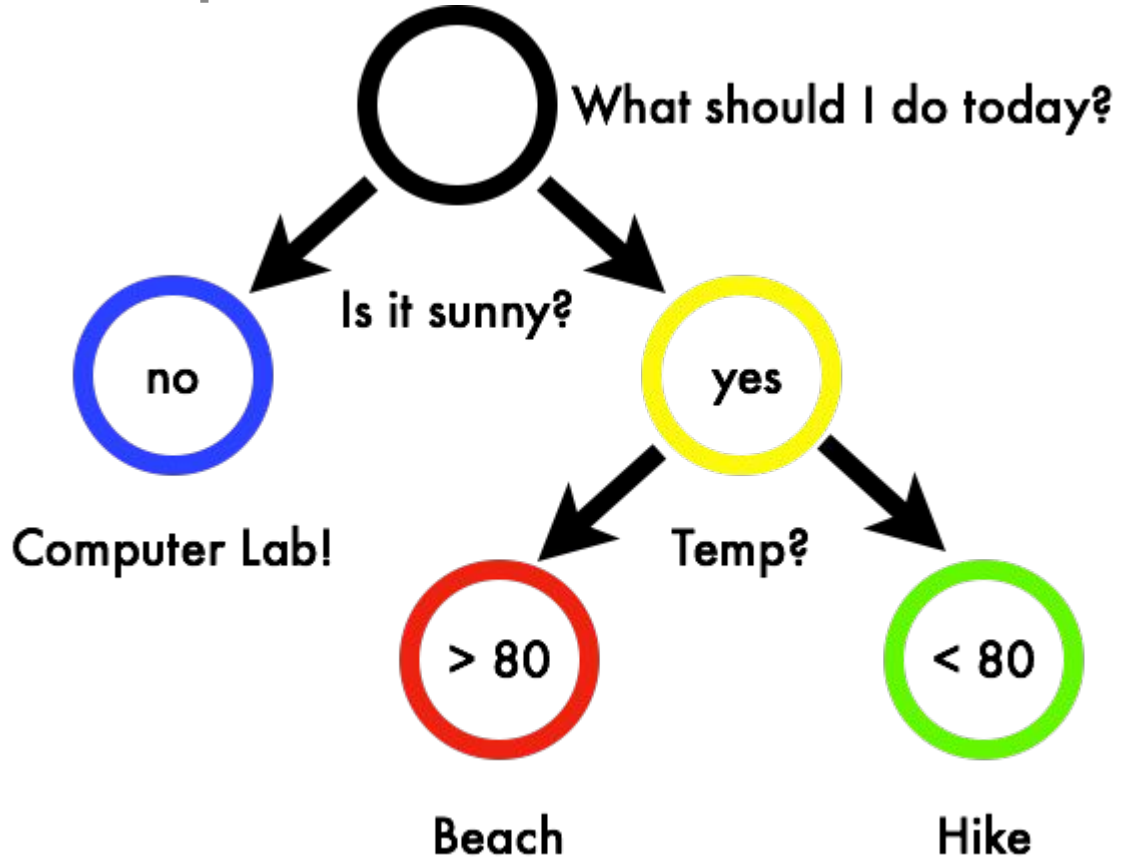
Benefits:

Interpretable

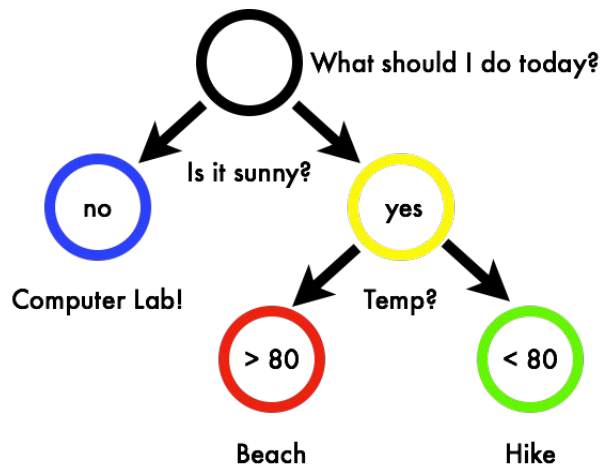
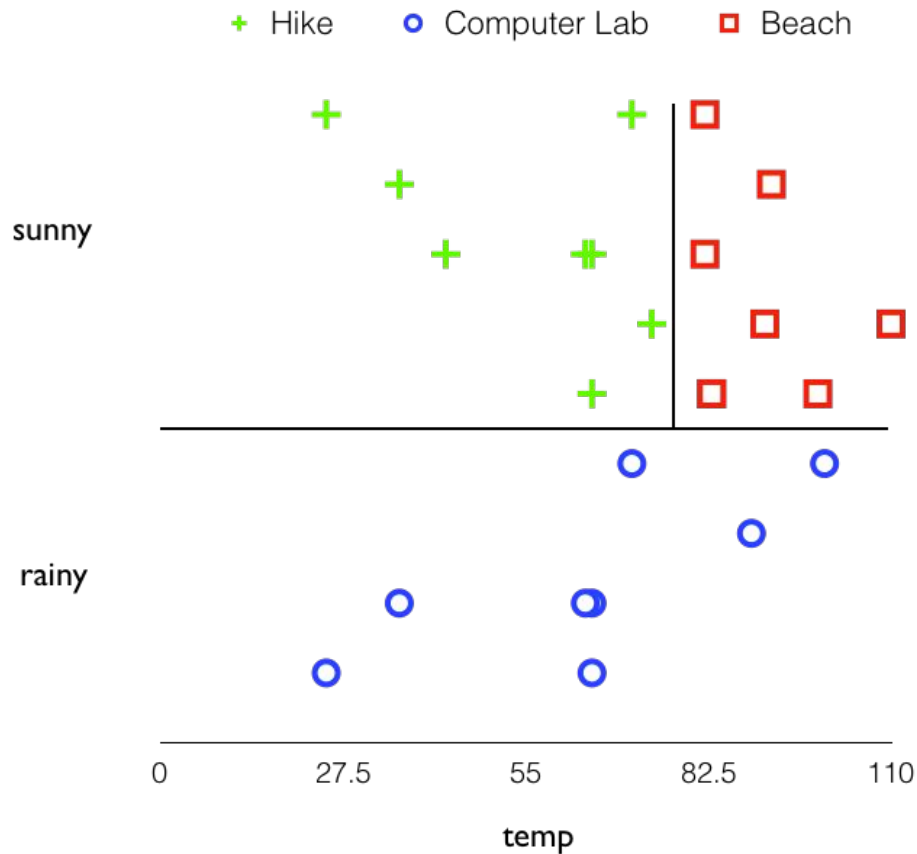
Easy to use

Good for applications

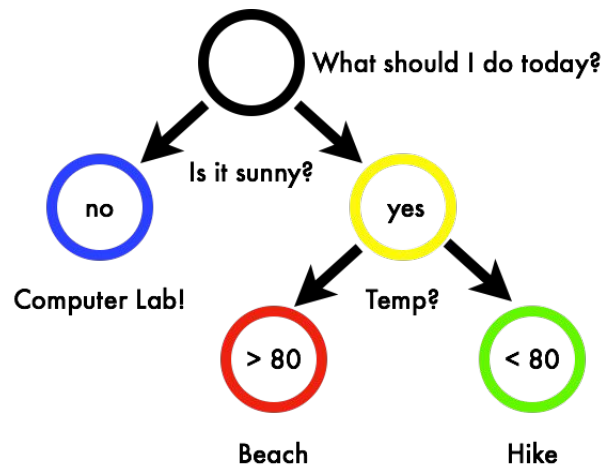
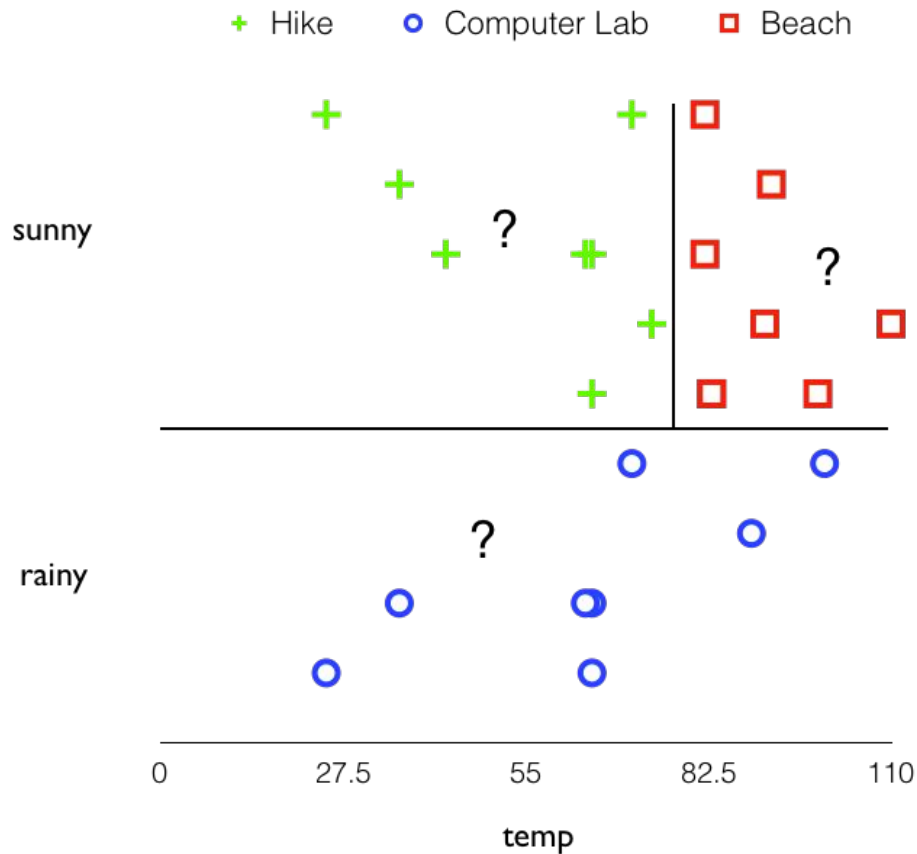
E.g. medicine



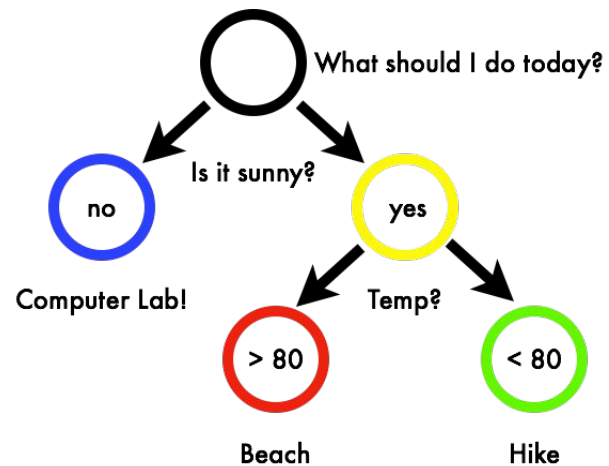
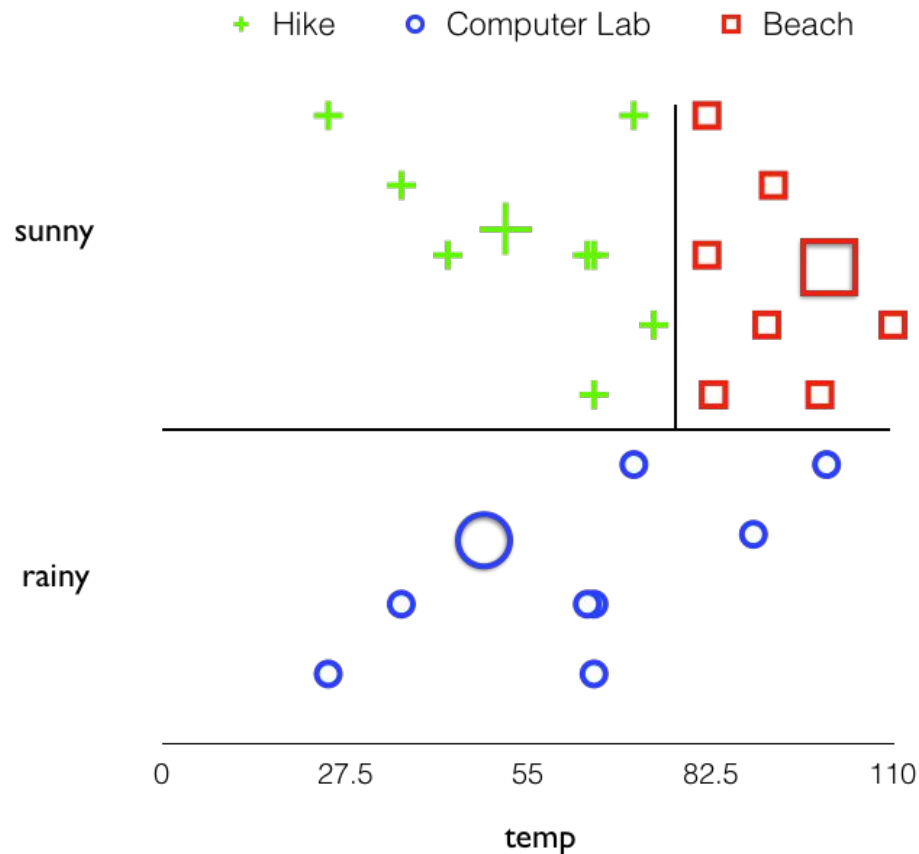
Trees are partitions of data



Predict new data based on what region it falls into

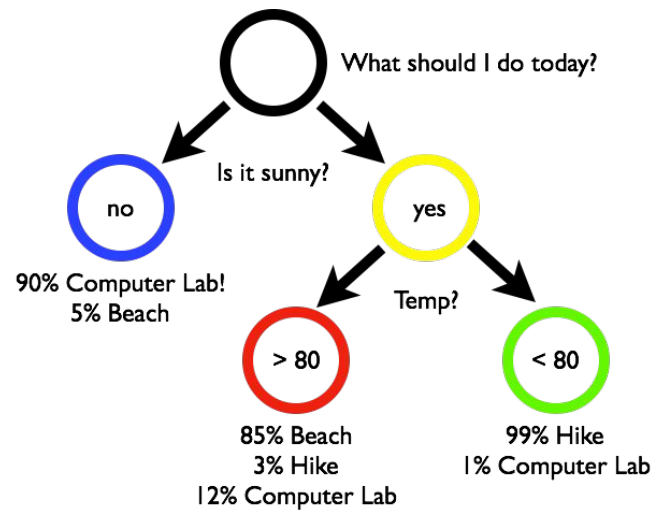
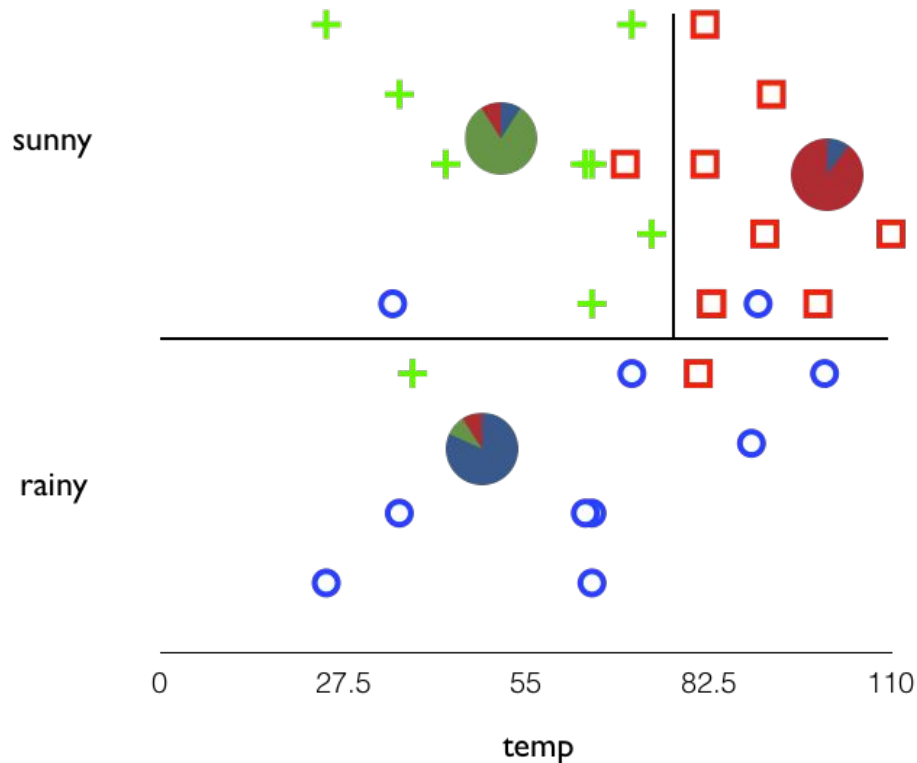


Predict new data based on what region it falls into



Data might be noisy, use soft assignments

+ Hike ○ Computer Lab □ Beach



Case study: Viola-Jones Face detection

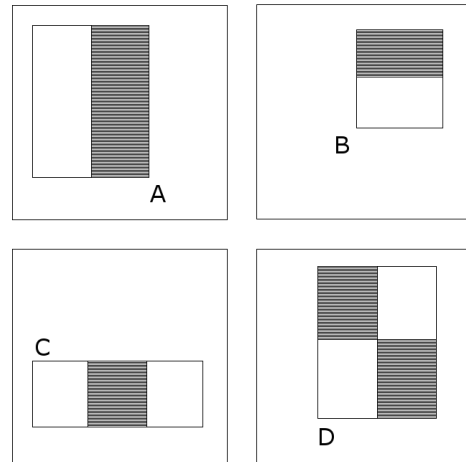
Want it to be very fast and accurate

Run on a camera or cell phone, low cost

Use simple features and simple classifiers

Haar features:

Response = $\sum \text{pix in black region} - \sum \text{pix in white region}$



Case study: Viola-Jones Face detection

Why do Haar features work?

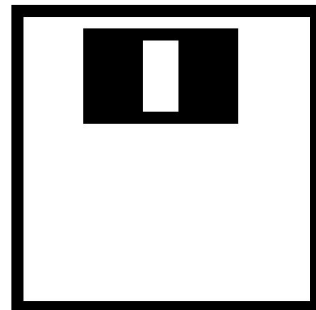
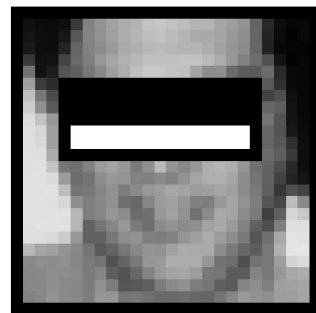
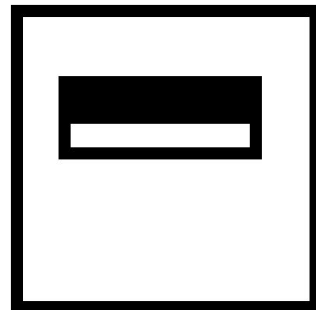
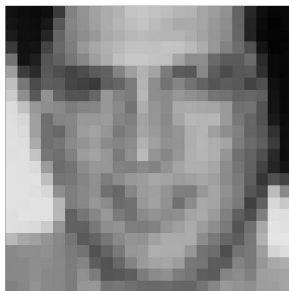
Eyes are generally darker than cheeks

Bridge of nose lighter than eyes

Etc.

Also, fast to compute!

Integral images - fast sums
over regions.



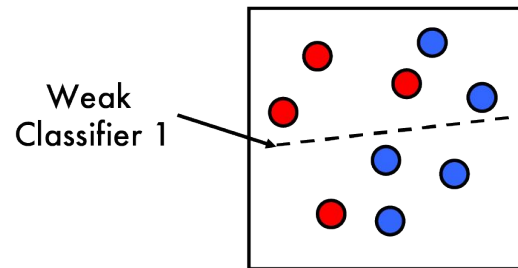
Case study: Viola-Jones Face detection

Classifier: *boosted* partitions

Boosting

Way to make weak classifiers better

Train a weak classifier



Case study: Viola-Jones Face detection

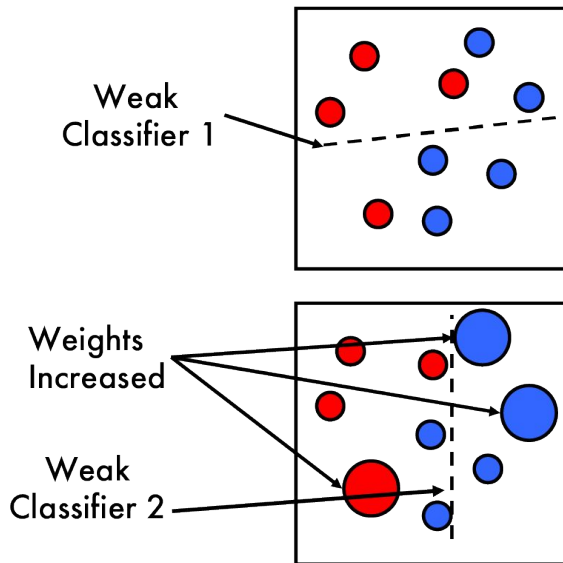
Classifier: *boosted* partitions

Boosting

Way to make weak classifiers better

Train a weak classifier

Reweight data we got wrong, train again



Classifier: *boosted* partitions

Boosting

Way to make weak classifiers better

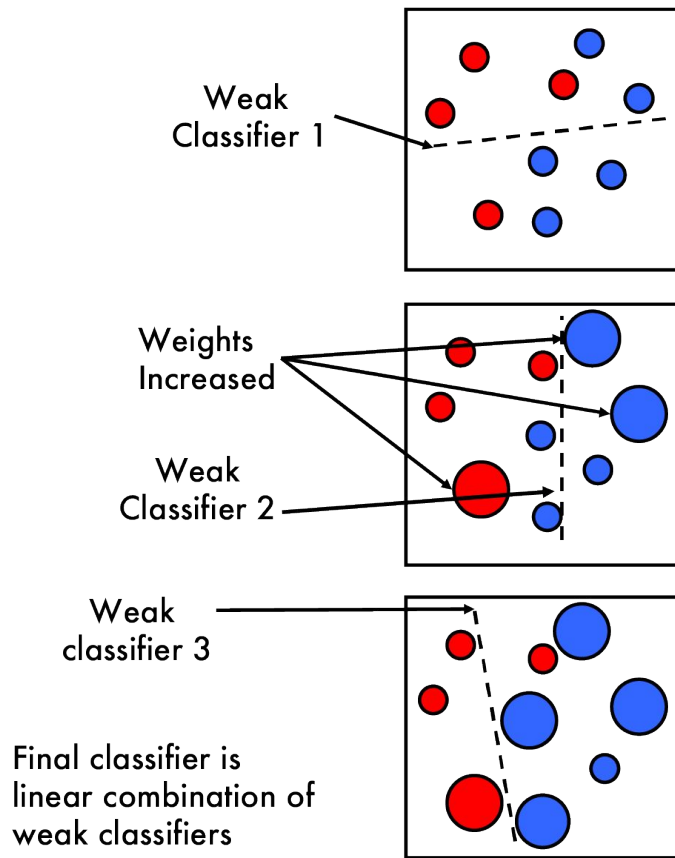
Train a weak classifier

Reweight data we got wrong, train again

...and again

Until you feel like stopping

Final classifier is combination of all



Case study: Viola-Jones Face detection

Finally, use a **cascade of classifiers**

1st classifier

Very fast, throws out easy negatives

2nd classifier

Fast, throws out harder negatives

3rd classifier

Slower, throws out hard negatives

Only run slow, good classifiers on hard examples

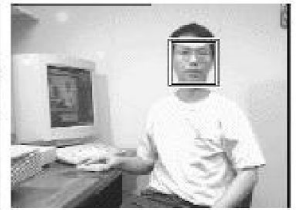
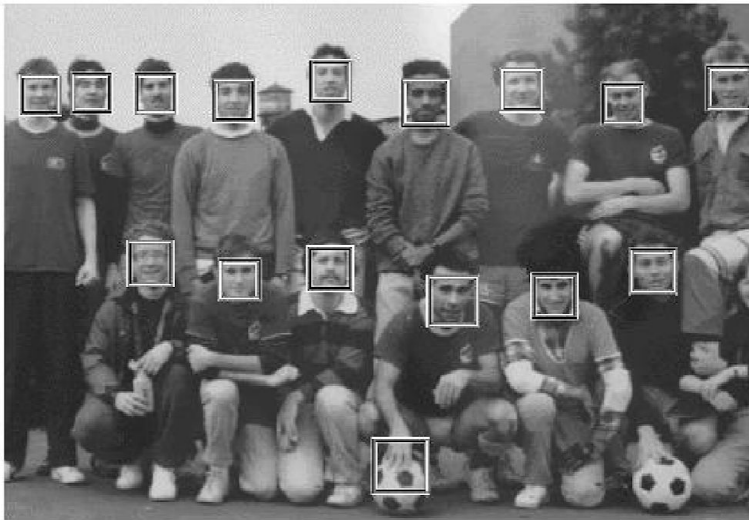
Fast classifier that is still very accurate

Case study: Viola-Jones Face detection

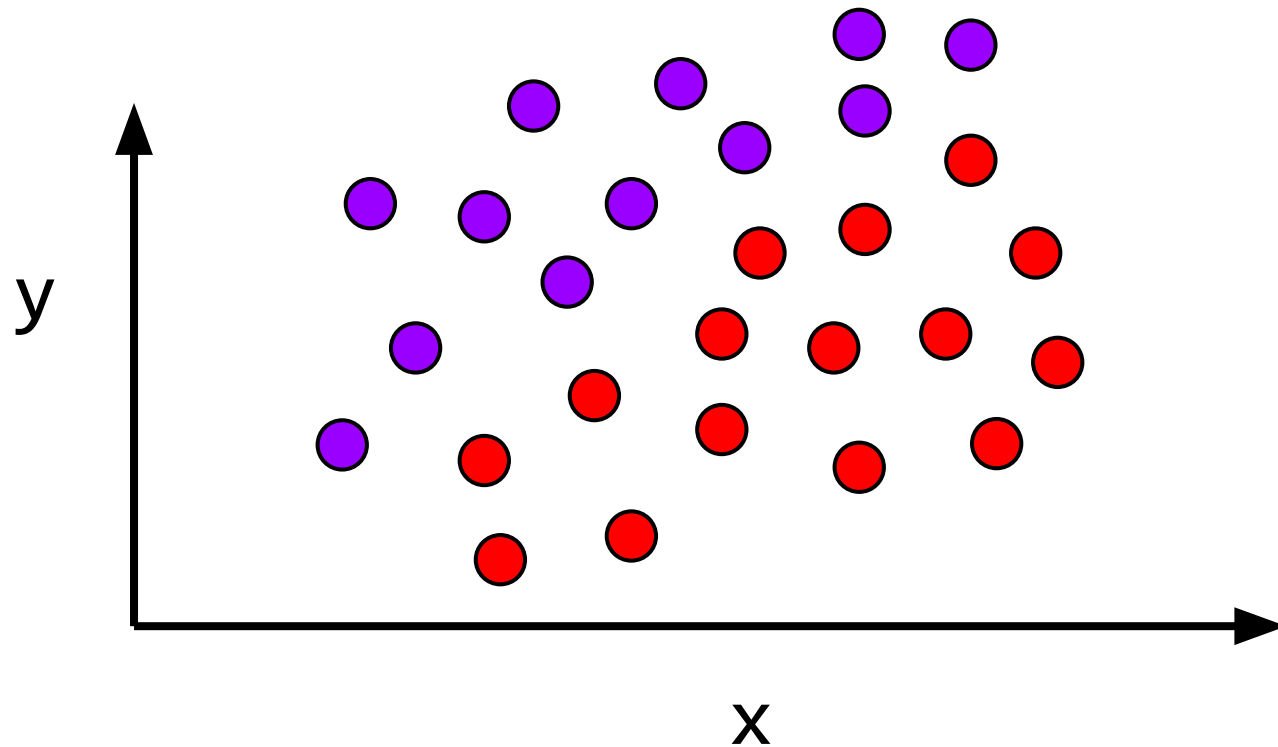
Haar features

Cascade

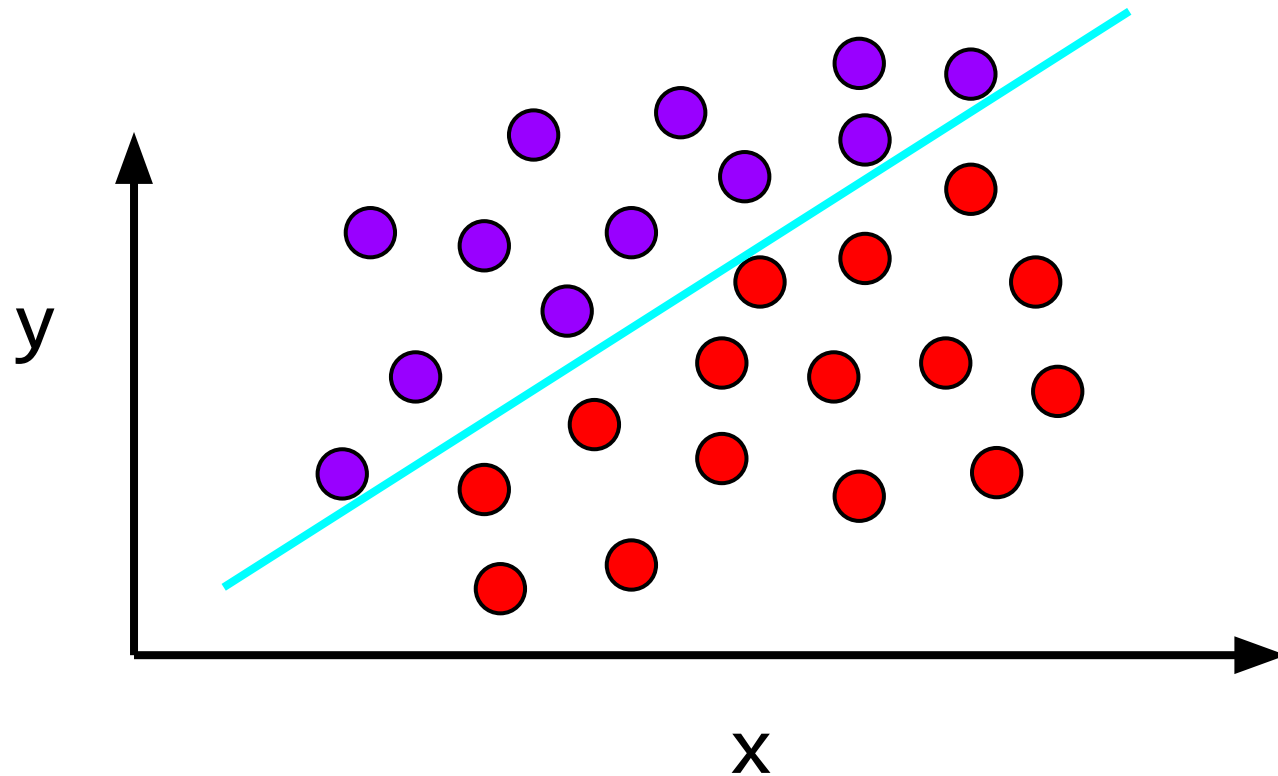
Of boosted classifiers



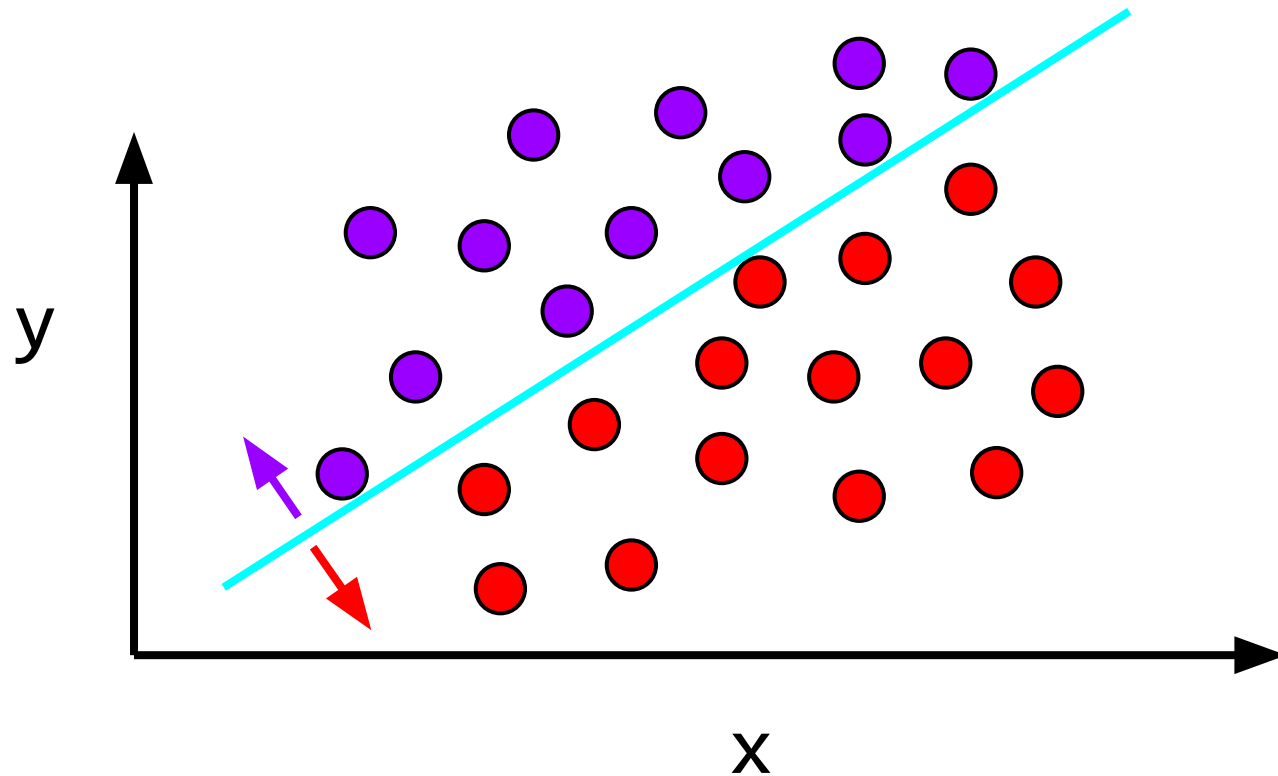
Classification in two dimensions



Classification in two dimensions

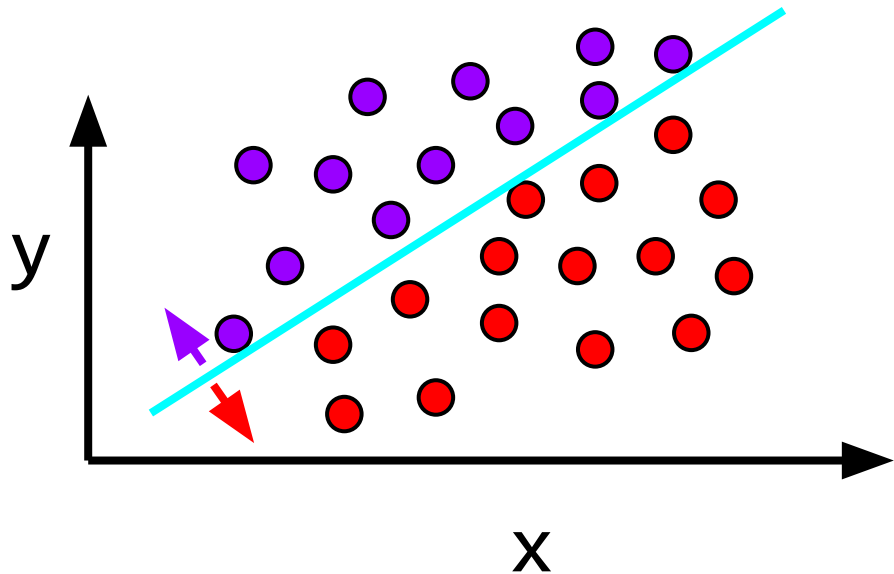


Classification in two dimensions



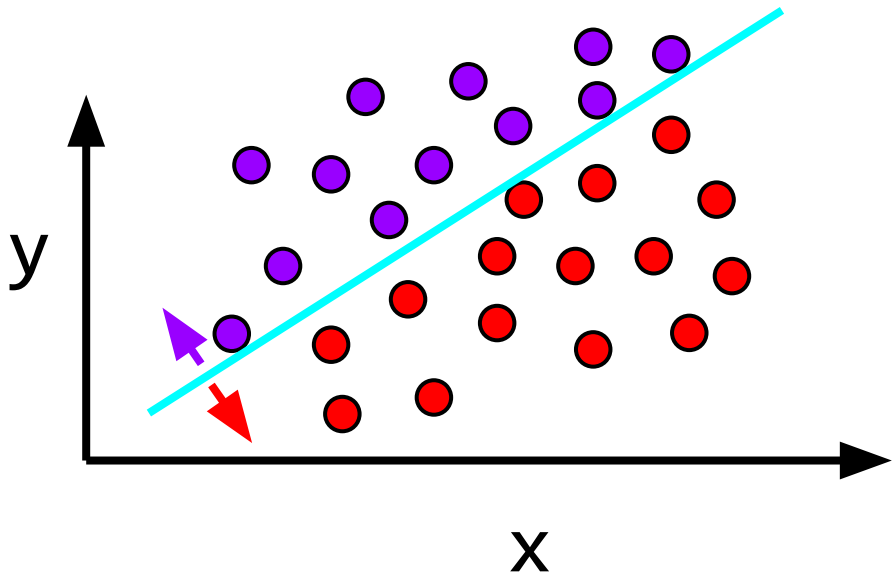
Classification in two dimensions

- Linear classifier
 - Given dataset, learn weights w
 - Output of model is weighted sum of inputs
 - $P(\text{purple} \mid \mathbf{x}) = f(\mathbf{w} \cdot \mathbf{x}) = f(\sum_i (w_i x_i))$
 - Where f is some function (a few options)



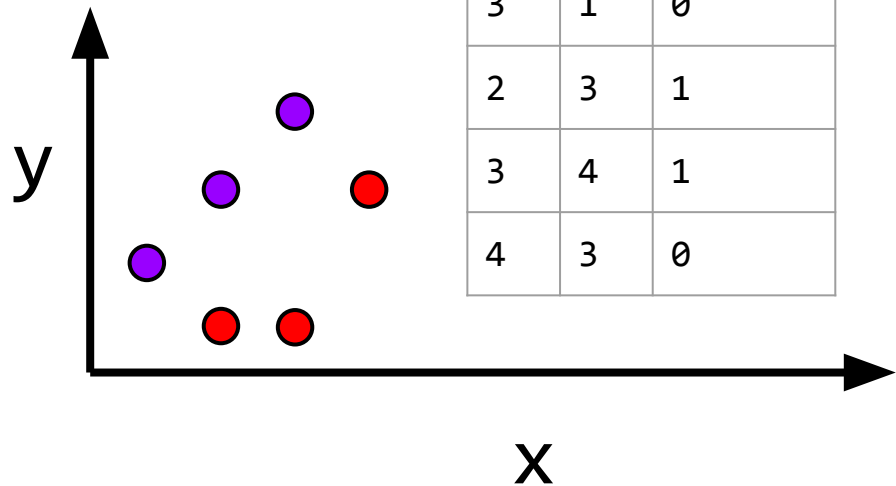
Classification in two dimensions

- Linear classifier
 - Given dataset, learn weights w
 - Output of model is weighted sum of inputs
 - $P(\text{purple} \mid \mathbf{x}) = f(\mathbf{w} \cdot \mathbf{x}) = f(\sum_i (w_i x_i))$
 - Where f is some function (a few options)
 - Typically a bias term:
 - $f(\sum_i (w_i x_i) + w_{\text{bias}})$



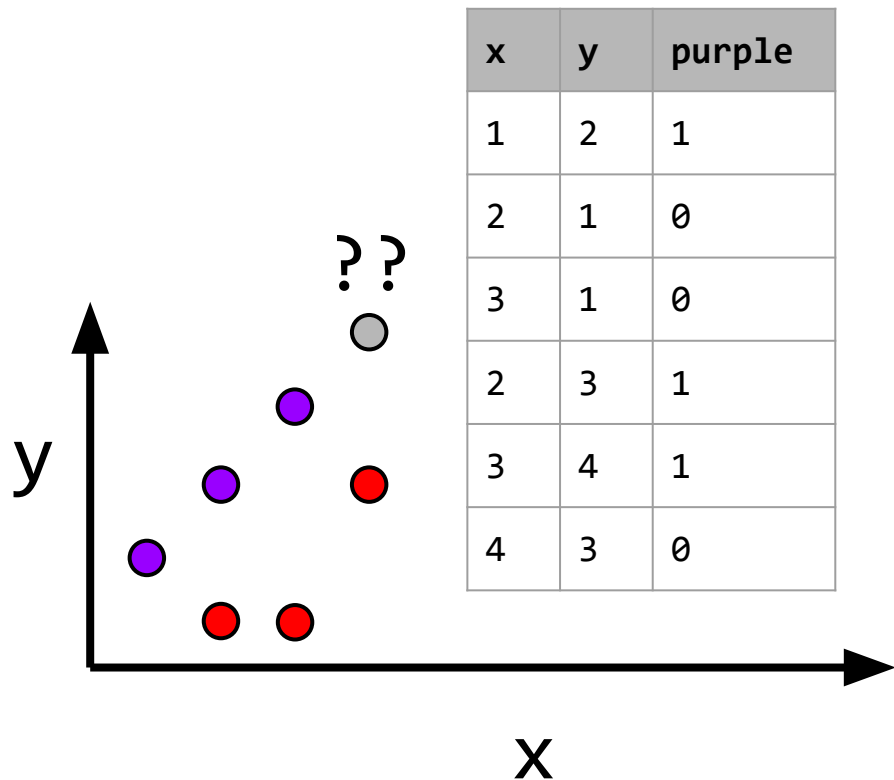
Classification in two dimensions

- Simple example:
 - Learned weights: $[-1, 1]$
 - f is threshold at 0
 -



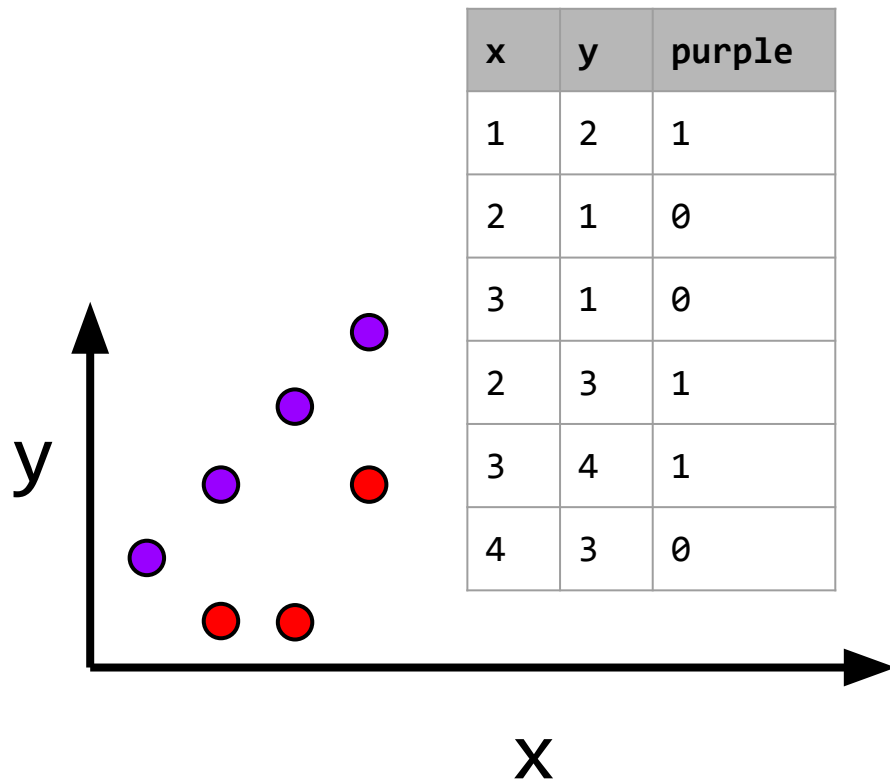
Classification in two dimensions

- Simple example:
 - Learned weights: $[-1, 1]$
 - f is threshold at 0
- New data point $(4, 5)$
 - $(w \cdot x) = (4, 5) \cdot (-1, 1)$
 $= 4 \cdot -1 + 5 \cdot 1 = 1$



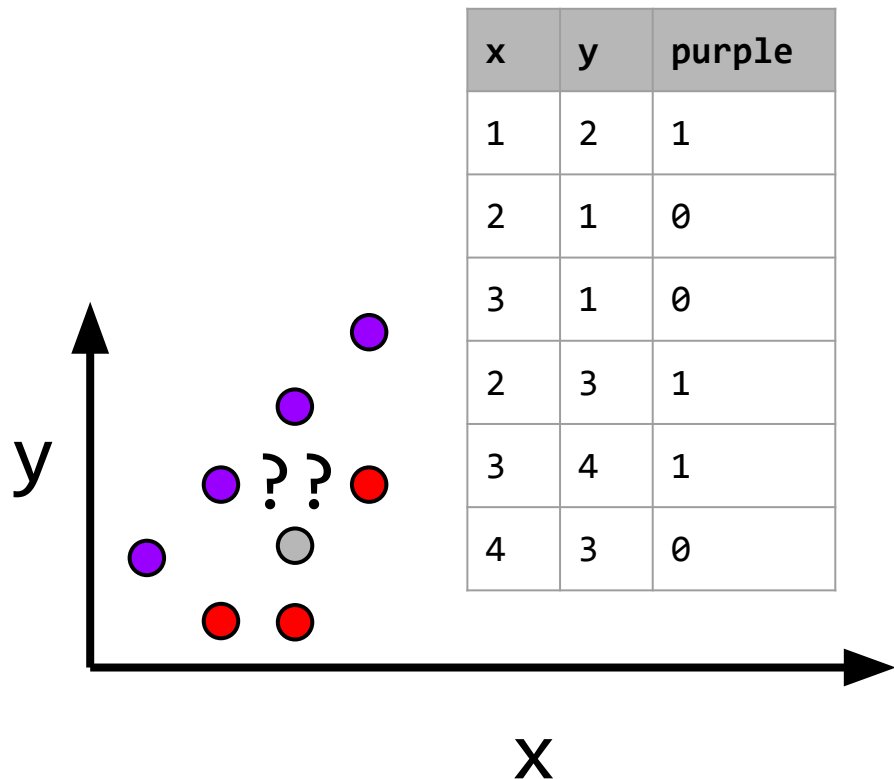
Classification in two dimensions

- Simple example:
 - Learned weights: $[-1, 1]$
 - f is threshold at 0
- New data point $(4, 5)$
 - $(\mathbf{w} \cdot \mathbf{x}) = (4, 5) \cdot (-1, 1)$
 $= 4 \cdot -1 + 5 \cdot 1 = 1$
 - $f(\mathbf{w} \cdot \mathbf{x}) = f(1) = 1$



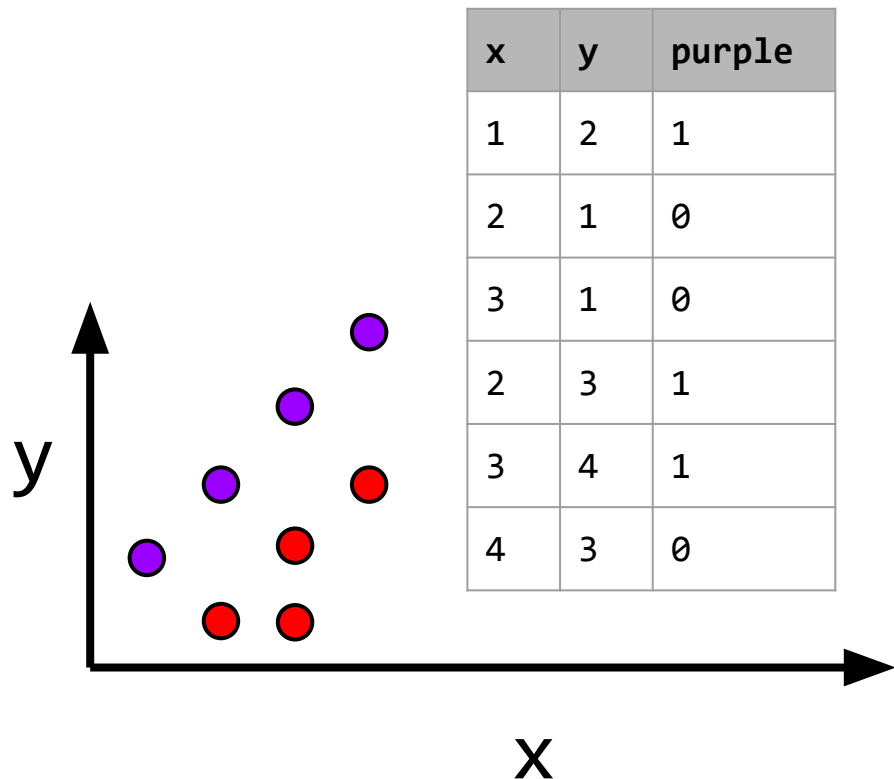
Classification in two dimensions

- Simple example:
 - Learned weights: $[-1, 1]$
 - f is threshold at 0
- New data point $(4, 5)$
 - $f(\mathbf{w} \cdot \mathbf{x}) = 1$
- New data point $(3, 2)$
 - $(\mathbf{w} \cdot \mathbf{x}) = (3, 2) \cdot (-1, 1)$
 $= 3 \cdot -1 + 2 \cdot 1 = -1$
 - $f(\mathbf{w} \cdot \mathbf{x}) = f(-1) = 0$
 -



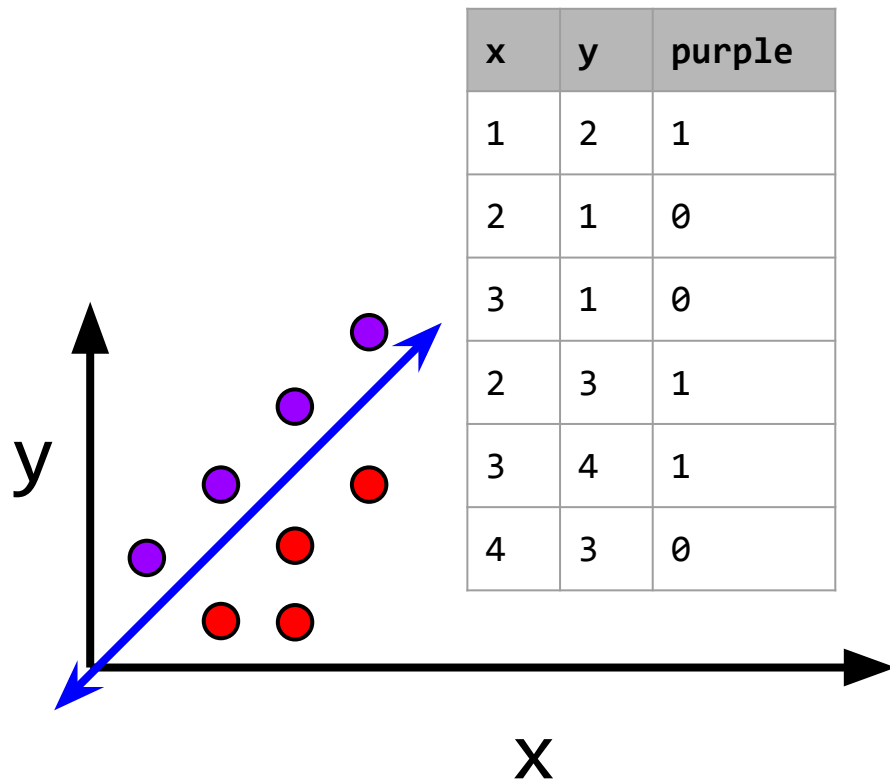
Classification in two dimensions

- Simple example:
 - Learned weights: $[-1, 1]$
 - f is threshold at 0
- New data point (4, 5)
 - $f(\mathbf{w} \cdot \mathbf{x}) = 1$
- New data point (3, 2)
 - $f(\mathbf{w} \cdot \mathbf{x}) = 0$



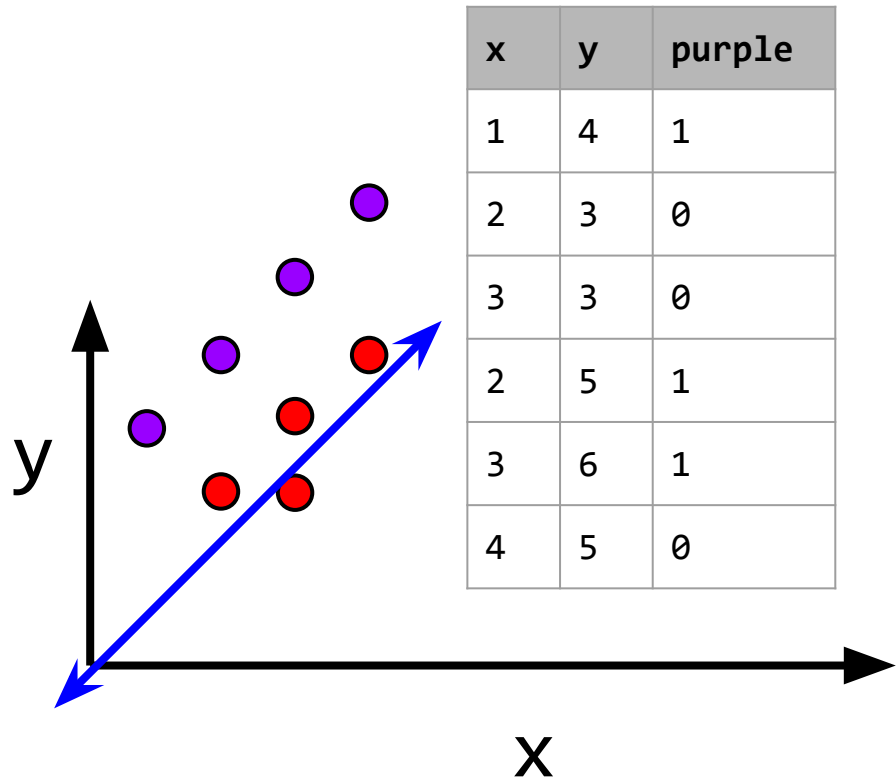
Classification in two dimensions

- Simple example:
 - Learned weights: $[-1, 1]$
 - f is threshold at 0
- New data point (4, 5)
 - $f(\mathbf{w} \cdot \mathbf{x}) = 1$
- New data point (3, 2)
 - $f(\mathbf{w} \cdot \mathbf{x}) = 0$
- Decision boundary: $x=y$



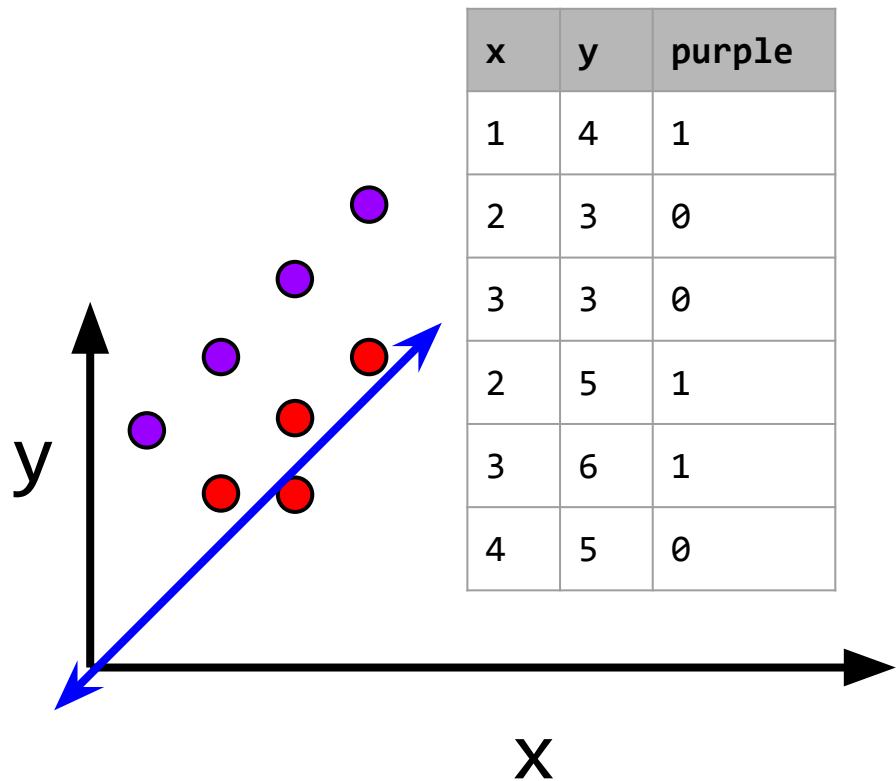
What if data is shifted up by two?

- Need a bias!:
 - Learned weights: $[-1, 1]$
 - f is threshold at 0



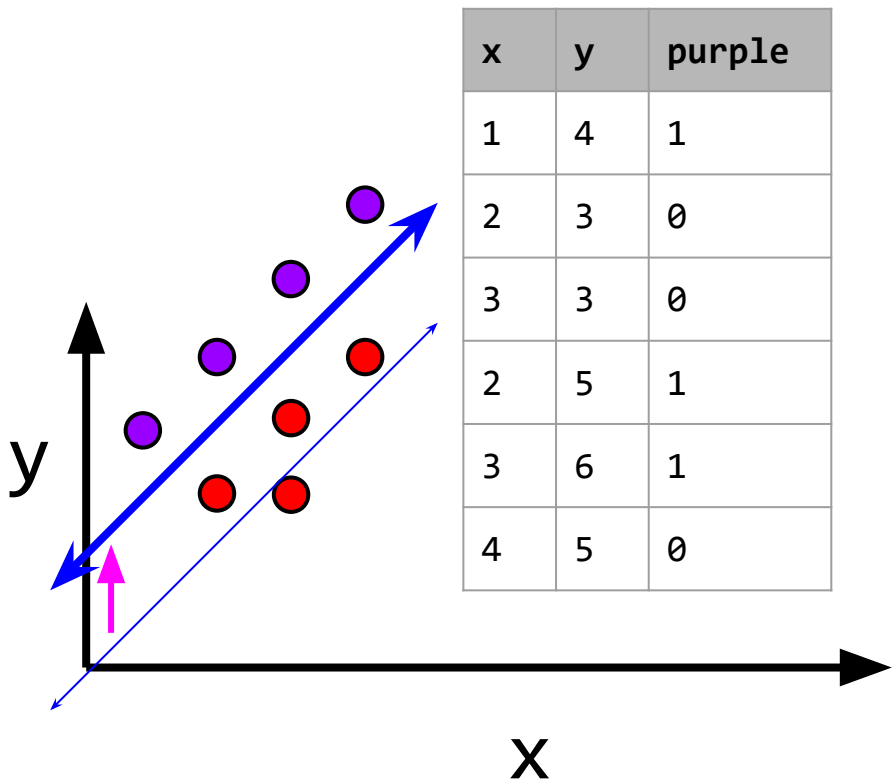
What if data is shifted up by two?

- Need a bias!:
 - Learned weights: $[-1, 1, -2]$
 - f is threshold at 0



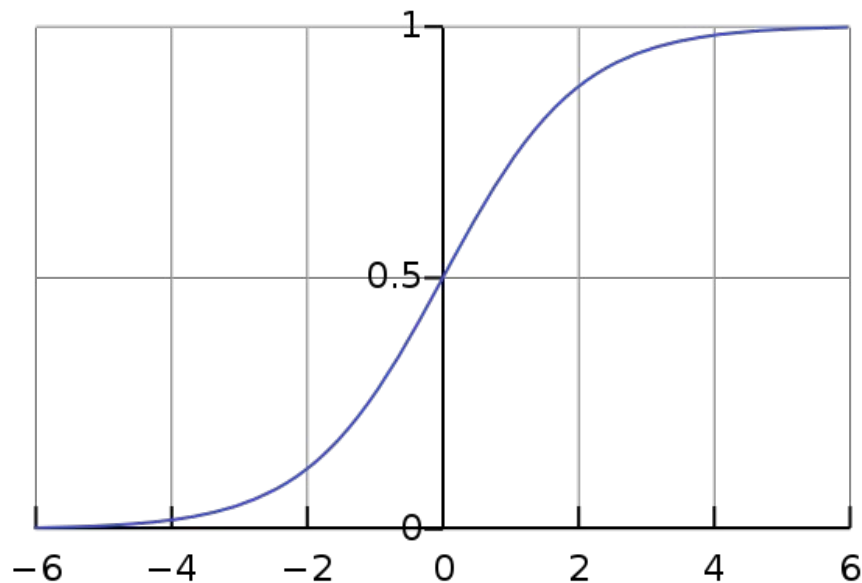
What if data is shifted up by two?

- Need a bias!:
 - Learned weights: $[-1, 1, -2]$
 - f is threshold at 0
- New data point $(4, 7, 1)$
 - $(\mathbf{w} \cdot \mathbf{x}) = (4, 7, 1) \cdot (-1, 1, -2)$
 $= 4 \cdot -1 + 7 \cdot 1 + 1 \cdot -2 = 1$
 - $f(\mathbf{w} \cdot \mathbf{x}) = f(1) = 1$



Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
 - Maps all reals $\rightarrow [0,1]$, **probabilities!**



Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - Good choice: how well our model fits the data, likelihood

Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - Good choice: how well our model fits the data, likelihood
 - **X**: training input variables, **Y**: training dependant variables
 - Want to learn w that models training data well

Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - Good choice: how well our model fits the data, likelihood
 - \mathbf{X} : training input variables, \mathbf{Y} : training dependant variables
 - Want to learn \mathbf{w} that models training data well
 - $L(\mathbf{w} \mid \mathbf{X}, \mathbf{Y}) = \Pr(\mathbf{Y} \mid \mathbf{X}, \mathbf{w}) = \prod_i \Pr(Y_i \mid \mathbf{X}_i, \mathbf{w})$

Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - Good choice: how well our model fits the data, likelihood
 - \mathbf{X} : training input variables, \mathbf{Y} : training dependant variables
 - Want to learn \mathbf{w} that models training data well
 - $L(\mathbf{w} \mid \mathbf{X}, \mathbf{Y}) = \Pr(\mathbf{Y} \mid \mathbf{X}, \mathbf{w}) = \prod_i \Pr(Y_i \mid X_i, \mathbf{w})$
 - $\Pr(Y_i \mid X_i, \mathbf{w}) =$
 - If $Y_i = 1$, $\sigma(\mathbf{w} \cdot \mathbf{X}_i)$

Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - Good choice: how well our model fits the data, likelihood
 - \mathbf{X} : training input variables, \mathbf{Y} : training dependant variables
 - Want to learn \mathbf{w} that models training data well
 - $L(\mathbf{w} \mid \mathbf{X}, \mathbf{Y}) = \Pr(\mathbf{Y} \mid \mathbf{X}, \mathbf{w}) = \prod_i \Pr(Y_i \mid X_i, \mathbf{w})$
 - $\Pr(Y_i \mid X_i, \mathbf{w}) =$
 - If $Y_i = 1$, $\sigma(\mathbf{w} \cdot \mathbf{X}_i)$
 - If $Y_i = 0$, $1 - \sigma(\mathbf{w} \cdot \mathbf{X}_i)$

Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - Good choice: how well our model fits the data, likelihood
 - \mathbf{X} : training input variables, \mathbf{Y} : training dependant variables
 - Want to learn \mathbf{w} that models training data well
 - $L(\mathbf{w} \mid \mathbf{X}, \mathbf{Y}) = \Pr(\mathbf{Y} \mid \mathbf{X}, \mathbf{w}) = \prod_i \Pr(Y_i \mid X_i, \mathbf{w})$
 - $\Pr(Y_i \mid X_i, \mathbf{w}) =$
 - If $Y_i = 1$, $\sigma(\mathbf{w} \cdot \mathbf{X}_i)$
 - If $Y_i = 0$, $1 - \sigma(\mathbf{w} \cdot \mathbf{X}_i)$
 - $L(\mathbf{w} \mid \mathbf{X}, \mathbf{Y}) = \prod_i [(\sigma(\mathbf{w} \cdot \mathbf{X}_i))^{Y_i} * (1 - \sigma(\mathbf{w} \cdot \mathbf{X}_i))^{(1-Y_i)}]$

Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - Good choice: how well our model fits the data, likelihood
 - \mathbf{X} : training input variables, \mathbf{Y} : training dependant variables
 - Want to learn \mathbf{w} that models training data well
 - $L(\mathbf{w} \mid \mathbf{X}, \mathbf{Y}) = \Pr(\mathbf{Y} \mid \mathbf{X}, \mathbf{w}) = \prod_i \Pr(Y_i \mid X_i, \mathbf{w})$
 - $\Pr(Y_i \mid X_i, \mathbf{w}) =$
 - If $Y_i = 1$, $\sigma(\mathbf{w} \cdot \mathbf{X}_i)$
 - If $Y_i = 0$, $1 - \sigma(\mathbf{w} \cdot \mathbf{X}_i)$
 - $L(\mathbf{w} \mid \mathbf{X}, \mathbf{Y}) = \prod_i [(\sigma(\mathbf{w} \cdot \mathbf{X}_i))^{Y_i} * (1 - \sigma(\mathbf{w} \cdot \mathbf{X}_i))^{(1-Y_i)}]$
 - In practice we use log likelihood, it's simpler later!!

Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - $\log L(w \mid X, Y) = \log \prod_i [(\sigma(w \cdot X_i))^{Y_i} * (1 - \sigma(w \cdot X_i))^{(1-Y_i)}]$
 - $= \sum_i \log[(\sigma(w \cdot X_i))^{Y_i} * (1 - \sigma(w \cdot X_i))^{(1-Y_i)}]$
 - $= \sum_i [Y_i \log(\sigma(w \cdot X_i)) + (1-Y_i) \log(1 - \sigma(w \cdot X_i))]$

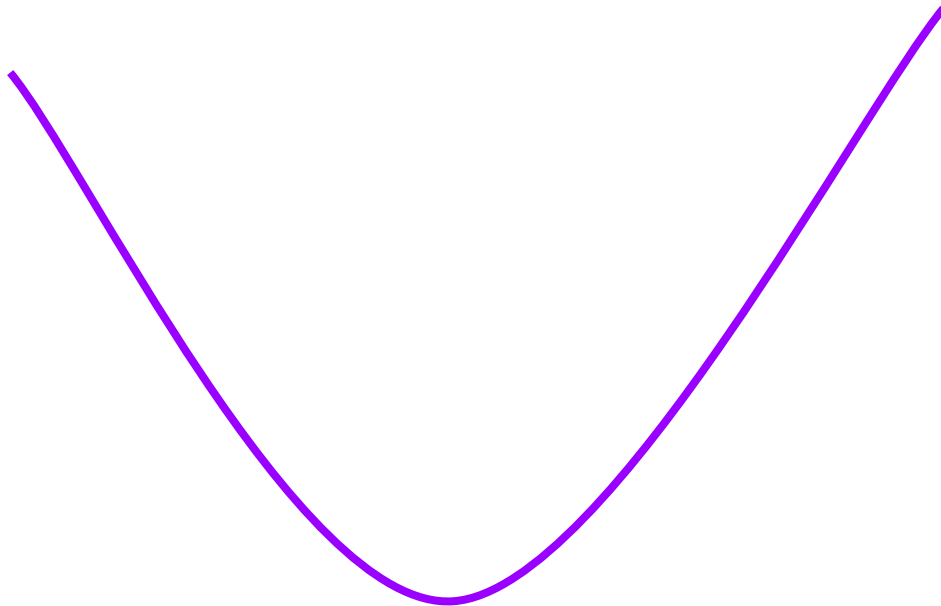
Logistic regression

- Linear classifier, f is logistic function
 - $\sigma(x) = 1/(1 + e^{-x}) = e^x/(1 + e^x)$
- Want something to optimize!
 - $\log L(w \mid X, Y) = \log \prod_i [(\sigma(w \cdot X_i))^{Y_i} * (1 - \sigma(w \cdot X_i))^{(1-Y_i)}]$
 - $= \sum_i \log[(\sigma(w \cdot X_i))^{Y_i} * (1 - \sigma(w \cdot X_i))^{(1-Y_i)}]$
 - $= \sum_i [Y_i \log(\sigma(w \cdot X_i)) + (1-Y_i) \log(1 - \sigma(w \cdot X_i))]$
- Can we take derivative and set to 0?
 - No! :-(no closed form solution
 - BUT! We can still optimize

Gradient descent

For some loss function $L(w)$, gradient $\nabla L(w)$ points towards in direction of steepest ascent.

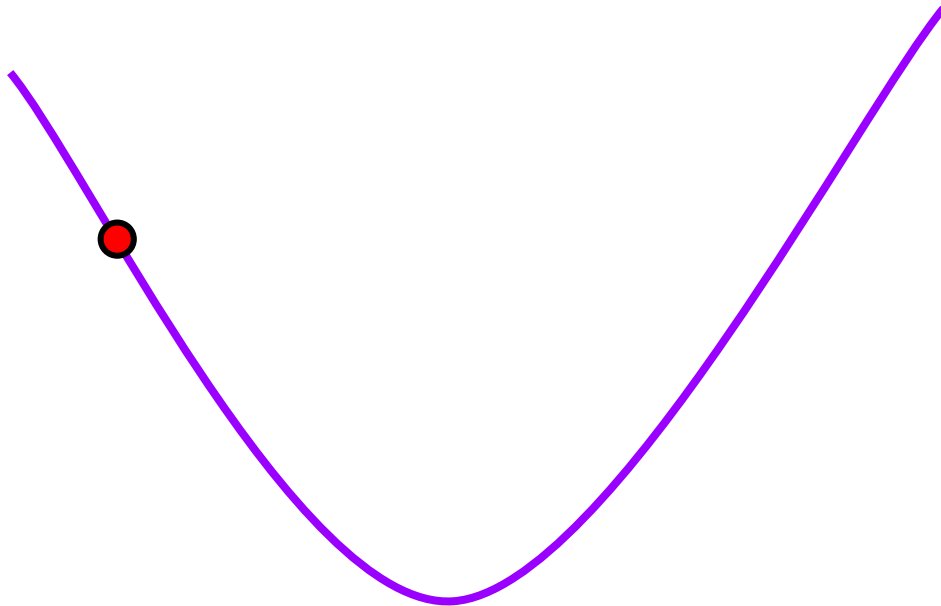
In 1d, either points left or right



Gradient descent

For some loss function $L(w)$, gradient $\nabla L(w)$ points towards in direction of steepest ascent.

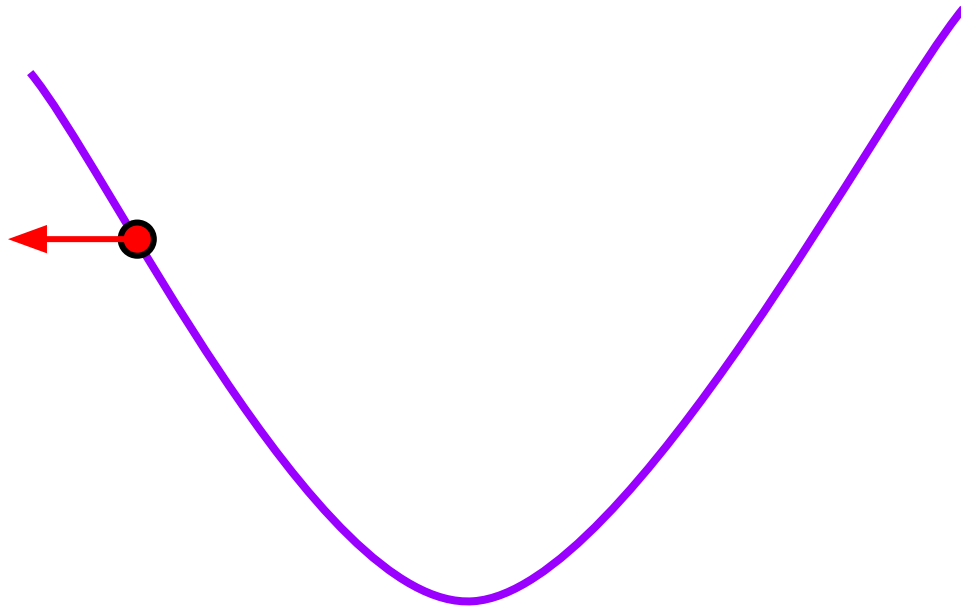
In 1d, either points left or right



Gradient descent

For some loss function $L(w)$, gradient $\nabla L(w)$ points towards in direction of steepest ascent.

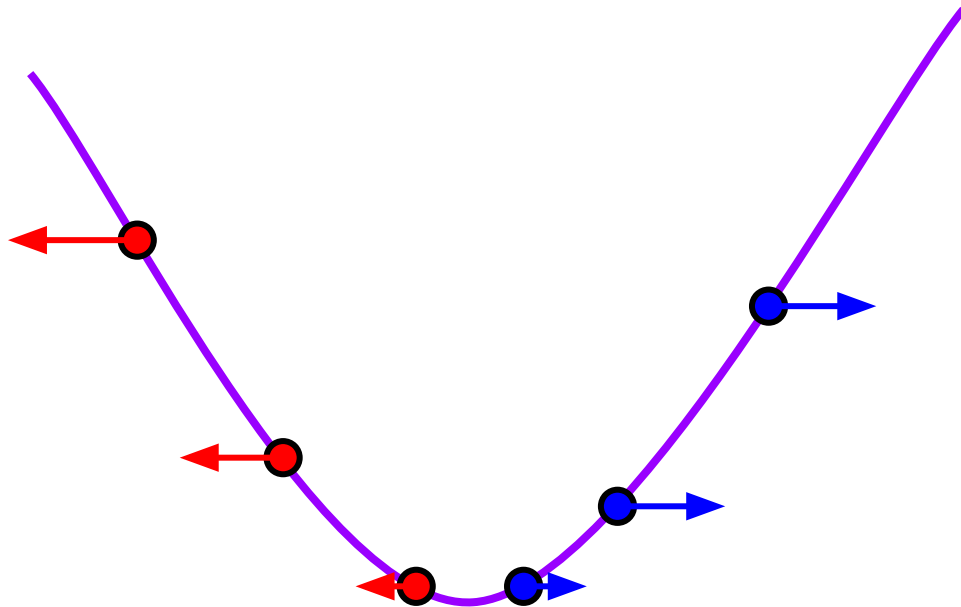
In 1d, either points left or right



Gradient descent

For some loss function $L(w)$, gradient $\nabla L(w)$ points towards in direction of steepest ascent.

In 1d, either points left or right



Gradient descent

For some loss function $L(w)$, gradient $\nabla L(w)$ points towards in direction of steepest ascent.

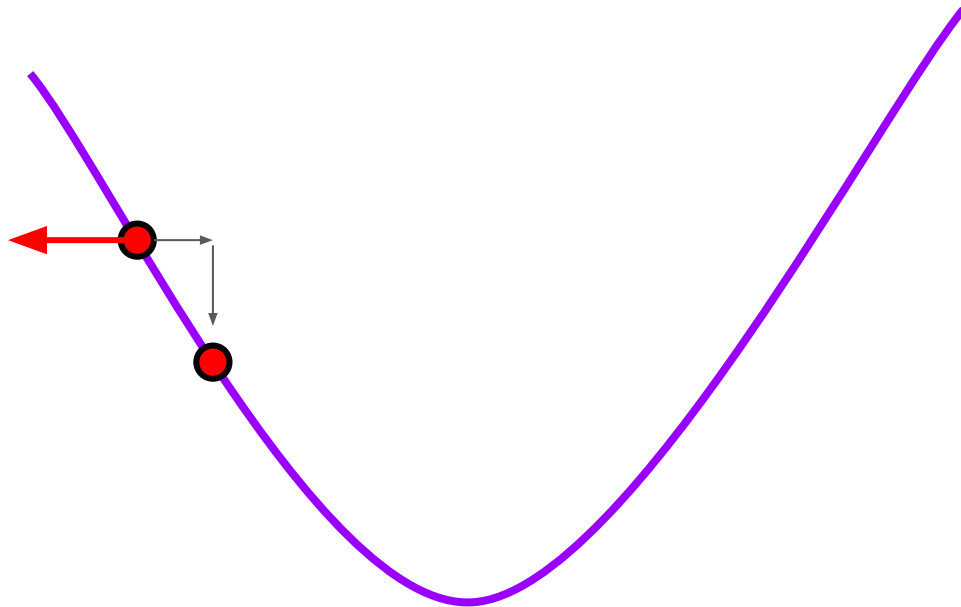
In 1d, either points left or right

Algorithm:

Take derivative

Move slightly in other
direction

Repeat



Gradient descent

For some loss function $L(w)$, gradient $\nabla L(w)$ points towards in direction of steepest ascent.

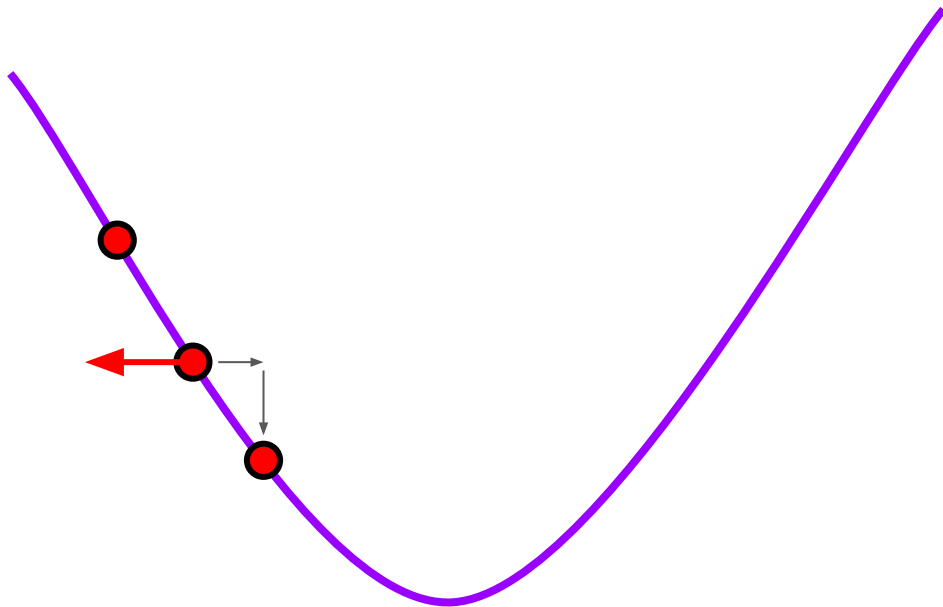
In 1d, either points left or right

Algorithm:

Take derivative

Move slightly in other
direction

Repeat



Gradient descent

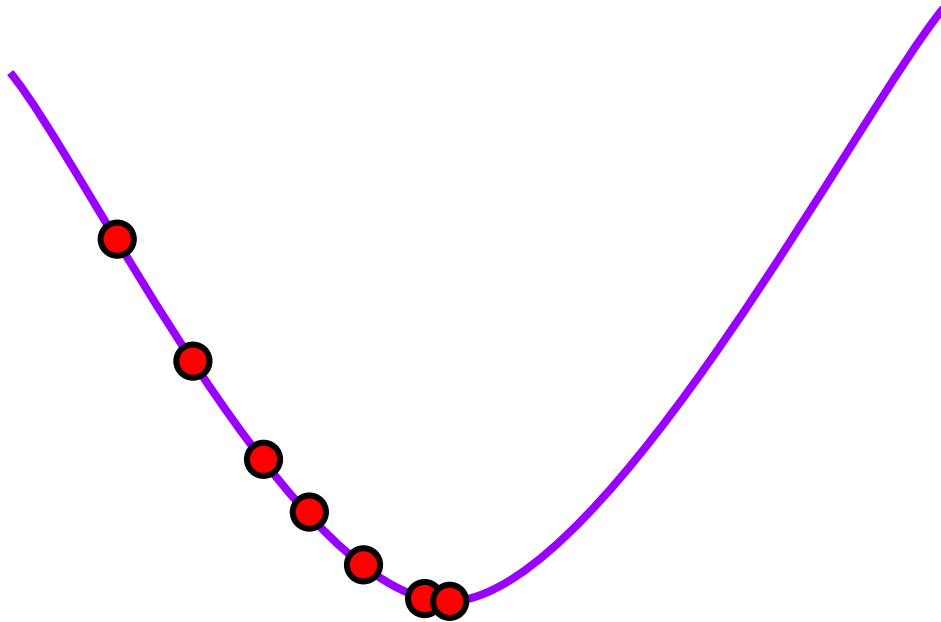
Algorithm:

Take derivative

Move slightly in other
direction

Repeat

End up at local optima

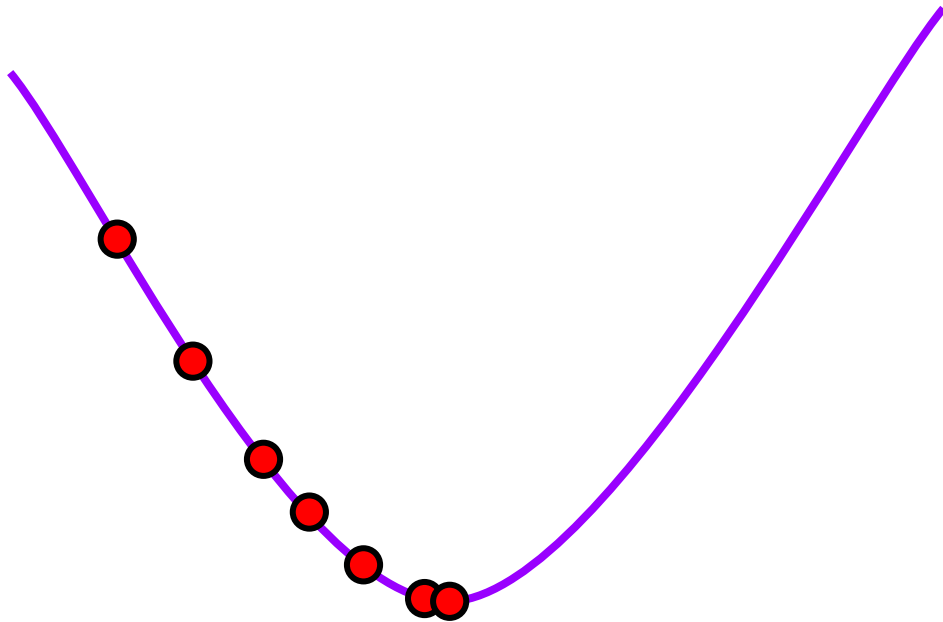


Gradient descent

Formally:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta \nabla L(\mathbf{w})$$

Where η is *step size*, how far to step relative to the gradient



Gradient descent

Calculating $\nabla L(w)$ can be hard, especially for big data, $|data|$ very large.

What if we estimate it instead?

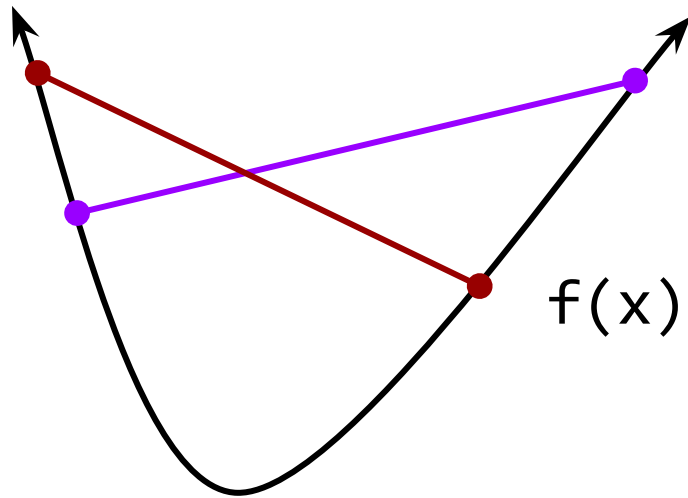
How do we estimate things?

Convex vs Non-convex

Convex function: connect any two points on graph with a line, that line lies above function everywhere

Why is it important? Any local extrema is global extrema!

If our loss function is convex, can set
derivative = 0, solve for parameters
(sometimes still no closed-form)

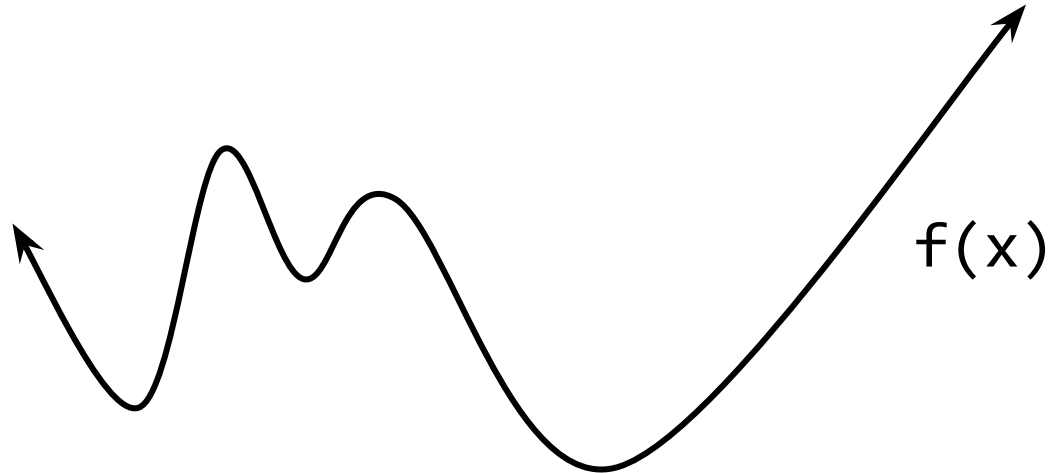


Convex vs Non-convex

Non-convex function: no rules!

Local optima are not global optima

Usually no easy way to find global or local optima, harder to optimize



Loss

Measure of how wrong our model is

Want a smaller loss

Try to find local or global minima of our loss function

Derivative = 0, gradient descent

Likelihood

How probable our model thinks our training data is

Want high likelihood, model that explains the data well

Find local or global maxima of likelihood function

Derivative = 0, gradient ascent

What if we have multiple classes?

Use an extension of logistic regression to multiple classes

For each class k we have weights w_k

Want to predict probability distribution over classes, what's wrong with:

$$\Pr(Y_i=1) = \sigma(w_1 \cdot X_i), \Pr(Y_i=2) = \sigma(w_2 \cdot X_i), \dots \text{etc}$$

What if we have multiple classes?

Use an extension of logistic regression to multiple classes

For each class k we have weights w_k

Want to predict probability distribution over classes, what's wrong with:

$$\Pr(Y_i=1) = \sigma(w_1 \cdot X_i), \Pr(Y_i=2) = \sigma(w_2 \cdot X_i), \dots \text{etc}$$

No normalization! Might sum to $\neq 1$.

What if we have multiple classes?

What if we normalized logistic regression across classes?

Softmax!

$$\sigma(\mathbf{z})_j = \frac{e^{z_j}}{\sum_{k=1}^K e^{z_k}}$$

If we have 2 classes and we assume $z_0 = 1$, $z_1 = \mathbf{w} \cdot \mathbf{X}$ then this is normal logistic regression.

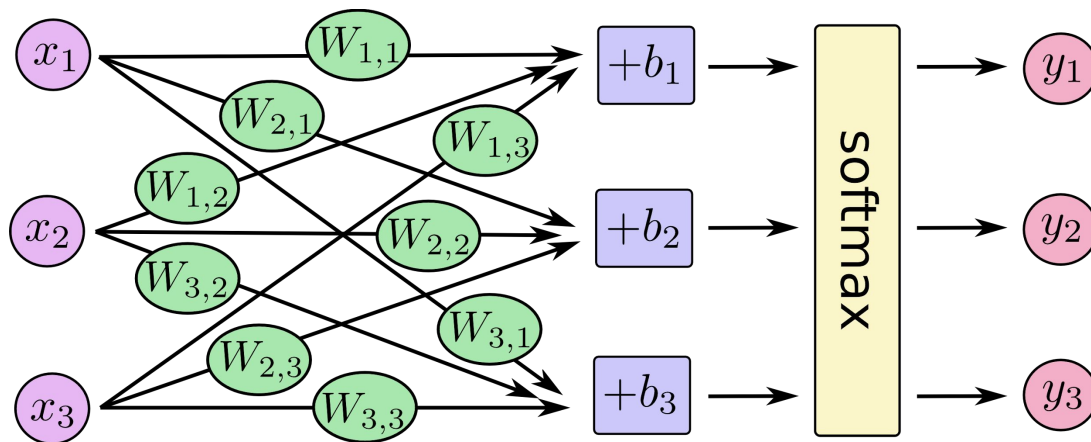
Multinomial logistic regression

Probability of that a data point belongs to a class is the normalized, weighted sum of the input variables with the learned weights.

$$P(y = j \mid \mathbf{x}) = \frac{e^{\mathbf{x}^\top \mathbf{w}_j}}{\sum_{k=1}^K e^{\mathbf{x}^\top \mathbf{w}_k}}$$

Multinomial logistic regression

Probability of that a data point belongs to a class is the normalized, weighted sum of the input variables with the learned weights.



Multinomial logistic regression

Probability of that a data point belongs to a class is the normalized, weighted sum of the input variables with the learned weights.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \text{softmax} \left(\begin{bmatrix} W_{1,1} & W_{1,2} & W_{1,3} \\ W_{2,1} & W_{2,2} & W_{2,3} \\ W_{3,1} & W_{3,2} & W_{3,3} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \right)$$

MNIST: Handwriting recognition

50,000 images of handwriting

28 x 28 x 1 (grayscale)

Numbers 0-9

10 class softmax regression

Input is 784 pixel values

Train with SGD

> 95% accuracy





Q & A

External Readings :



- Viola Jones Face Detection with OpenCV

<https://towardsdatascience.com/viola-jones-algorithm-and-haar-cascade-classifier-ee3bfb19f7d8>

- Introduction to Linear Regression

<https://thuraaung-1601.medium.com/introduction-to-linear-regression-with-normal-equation-98e6c1f839f8>

- Mnist Logistic Regression

<https://aigeekprogrammer.com/binary-classification-using-logistic-regression-and-keras/>

- Scikit-learn Image Classification

<https://youtu.be/bwZ30iuj3i8>

- Deep Learning Introduction (Optional)

<https://youtu.be/h11Xt02jRWM>

Thank You !

