Linear Algebra

(Recap session at Math Club, GDG Yangon)

Presenter: Thura Aung

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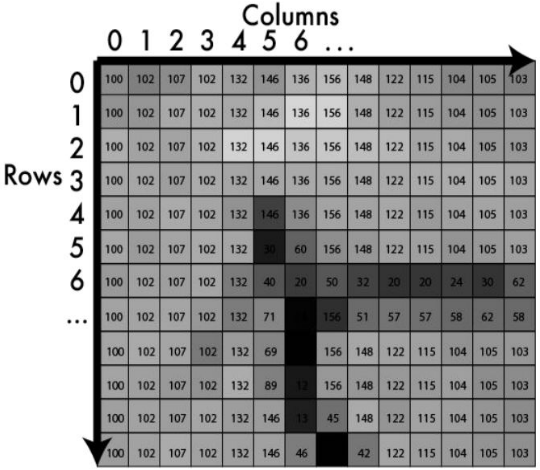
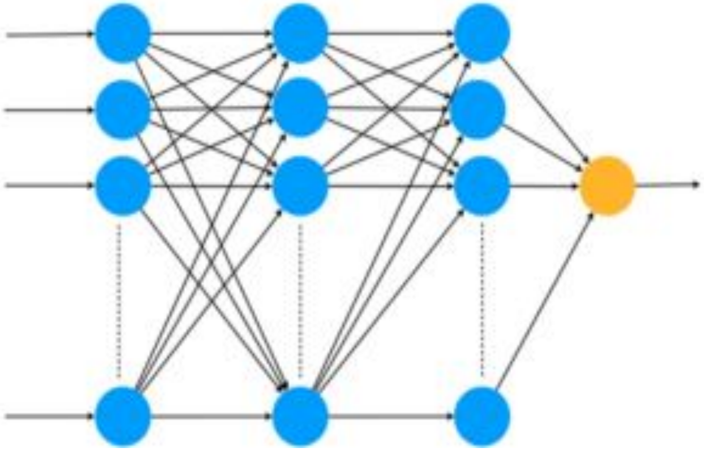
Motivation





System of Information

System 1	System 2	System 3	System 4
The dog is **black**	The dog is **black**	The dog is **black**	The dog is **black**
The cat is **orange**	The dog is **black**	The dog is **black**	The dog is **white**
The bird is **red**	The bird is **red**	The dog is **black**	The bird is **red**
Complete	Redundant	Redundant	Contradictory
Non-singular	**Singular**	**Singular**	**Singular**

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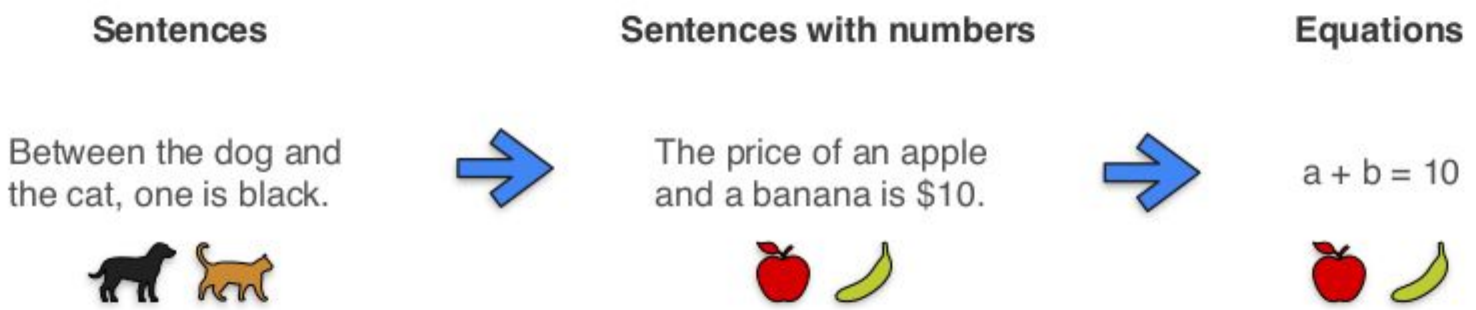
Cont’d Systems of Information



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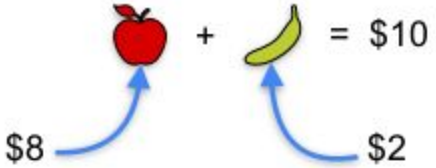
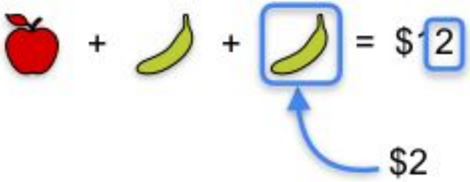
Cont’d Systems of Information

- **Day 1:** You bought an apple and a banana and they cost $10.



- **Day 2:** You bought an apple and two bananas and they cost $12.



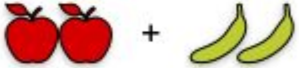
- **Solution:** An apple costs $8, a banana costs $2.

Cont’d Systems of Information

- **Day 1:** You bought an apple and a banana and they cost \$10.

 = \$10

- **Day 2:** You bought two apples and two bananas and they cost \$20.

 = \$20

Same thing!!!

8	2
5	5
8.3	1.7
0	10

Infinitely many solutions!

Cont’d Systems of Information

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $24.

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Cont’d Systems of Information

System 1

- a + b = 10
- a + 2b = 12

Unique solution:

a = 8
b = 2

Complete

Non-singular

System 2

- a + b = 10
- 2a + 2b = 20

Infinite solutions

a = 8, 7, 6 ...
b = 2, 3, 4

Redundant

Singular

System 3

- a + b = 10
- 2a + 2b = 24

No solution

Contradictory

Singular

Calculation

System

- $5a + b = 17$
- $4a - 3b = 6$

Eliminate 'a' from this equation

Divide by coefficient of a

- $a + 0.2b = 3.4$
- $a - 0.75b = 1.5$

Subtract equation 1 from equation 2

$$
\begin{array}{rl}
 & a - 0.75b = 1.5 \\
- & a + 0.2b = 3.4 \\
\hline
 & 0a - 0.95b = -1.9 \\
 & -0.95b = -1.9 \\
 & b = 2
\end{array}
$$

Solved system

- $a = 3$
- $b = 2$

$a + 0.2(2) = 3.4$

$a + 0.4 = 3.4$

$a = 3$

Cont'd Calculation

What if the system is singular (redundant)?

System

- $a + b = 10$
- $2a + 2b = 20$

Eliminate 'a' from this equation

Divide by coefficient of a

- $a + b = 10$
- $a + b = 10$

Subtract equation 1 from equation 2

$$
\begin{array}{rl}
 & a + b = 10 \\
- & a + b = 10 \\
\hline
 & 0 = 0
\end{array}
$$

Degree of freedom x

Solved system

- $a = x$
- $b = 10 - x$

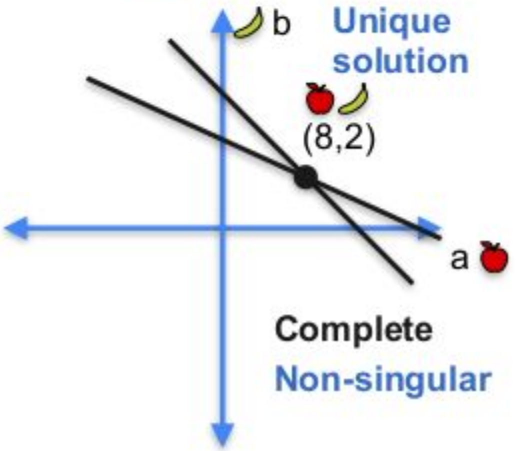
Solved system

- $a + b = 10$
- no other equation

Cont’d Systems of Information

System 1

- $a + b = 10$
- $a + 2b = 12$

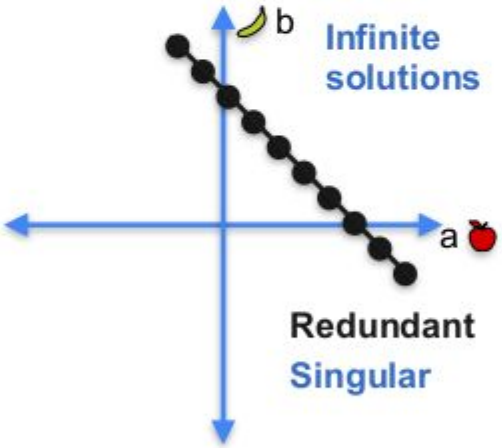


Complete

Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

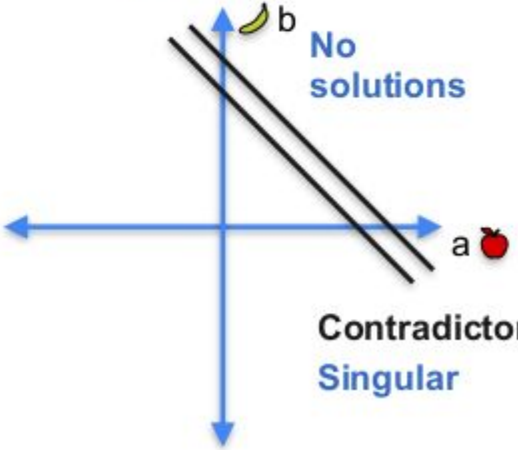


Redundant

Singular

System 3

- $a + b = 10$
- $2a + 2b = 24$



Contradictory

Singular

In the context of Matrix

System 1

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

Non-singular system

Non-singular matrix

(Unique solution)

System 2

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Singular system

Singular matrix

(Infinitely many solutions)

Cont’d In the context of Matrix

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

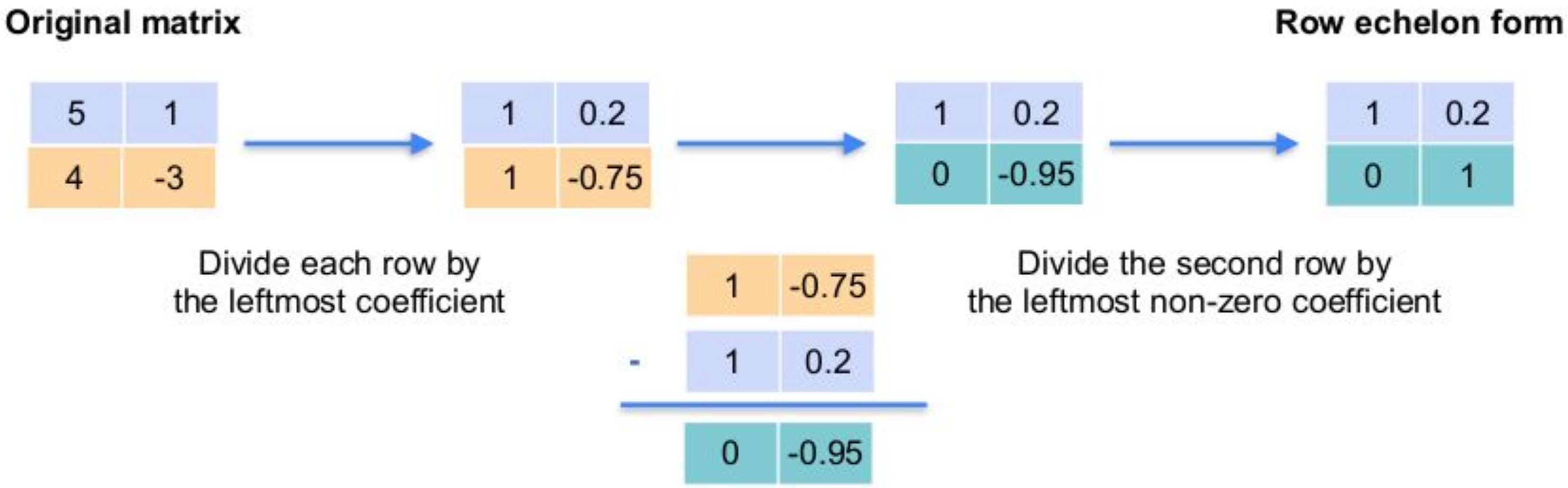
Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Cont’d In the context of Matrix



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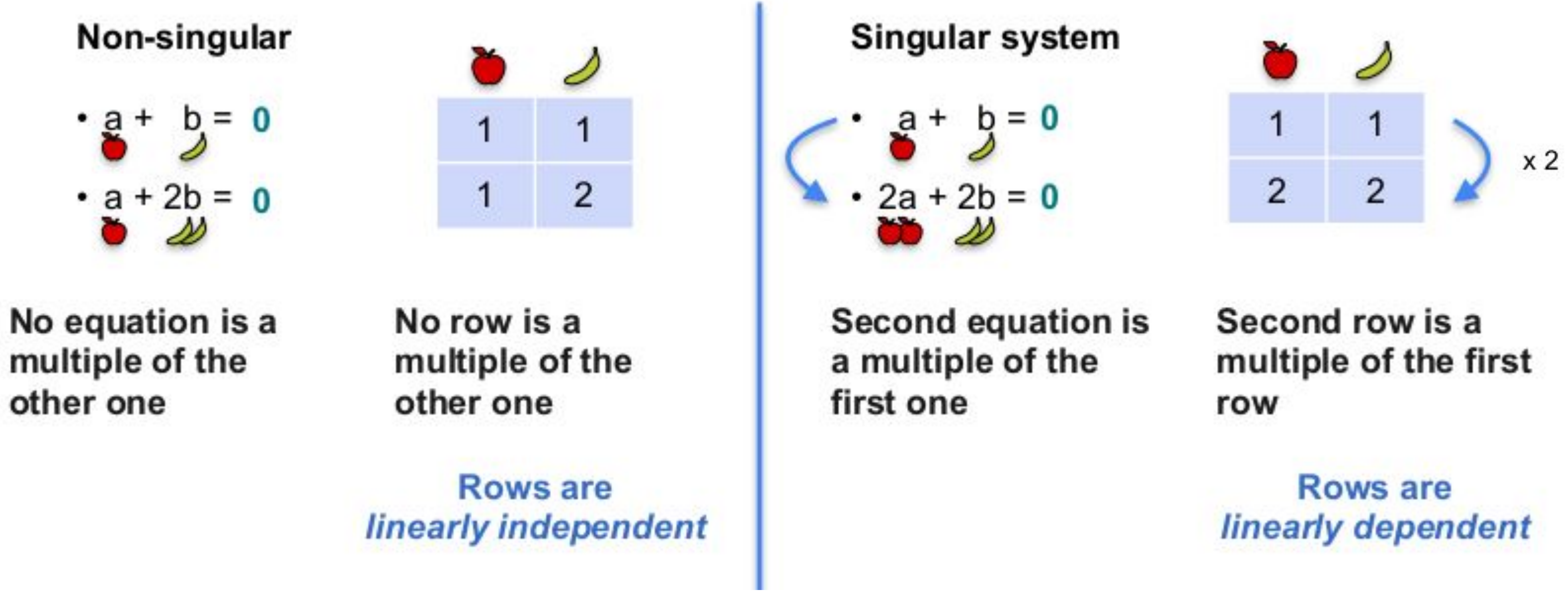
Linear Dependent/Independent



Determinant

a	b
c	d

Determinant $= ad - bc$

$$\begin{matrix} a & \\ & d \end{matrix} \quad - \quad \begin{matrix} & b \\ c & \end{matrix}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{bmatrix} a & b \end{bmatrix} * k = \begin{bmatrix} c & d \end{bmatrix}$$

$$ad = bc$$

Determinant

$$\boxed{ad - bc} = 0$$

Cont’d Determinant

Non-singular matrix

1	1
1	2

Determinant

$$1 \cdot 2 - 1 \cdot 1 = \boxed{1}$$

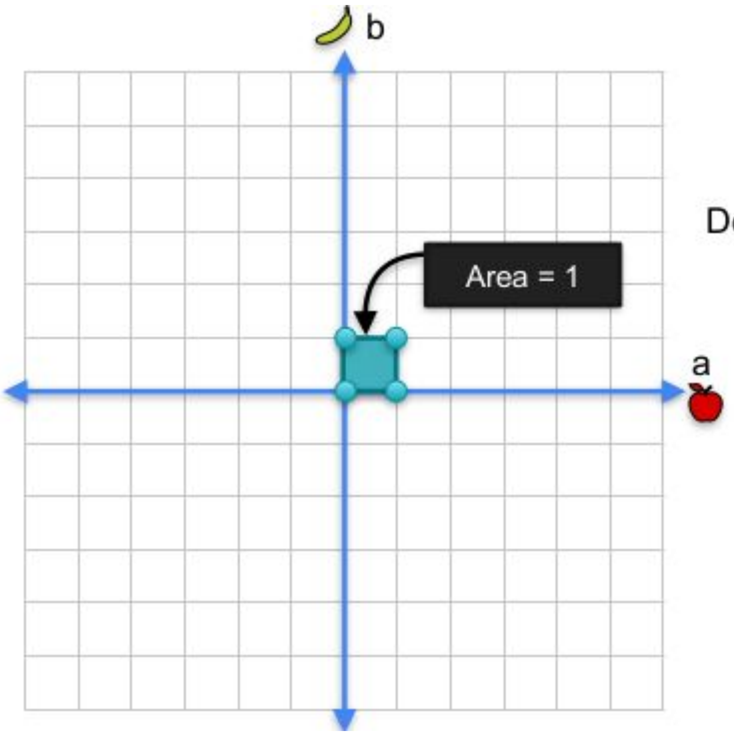
Singular matrix

1	1
2	2

Determinant

$$1 \cdot 2 - 2 \cdot 1 = \boxed{0}$$

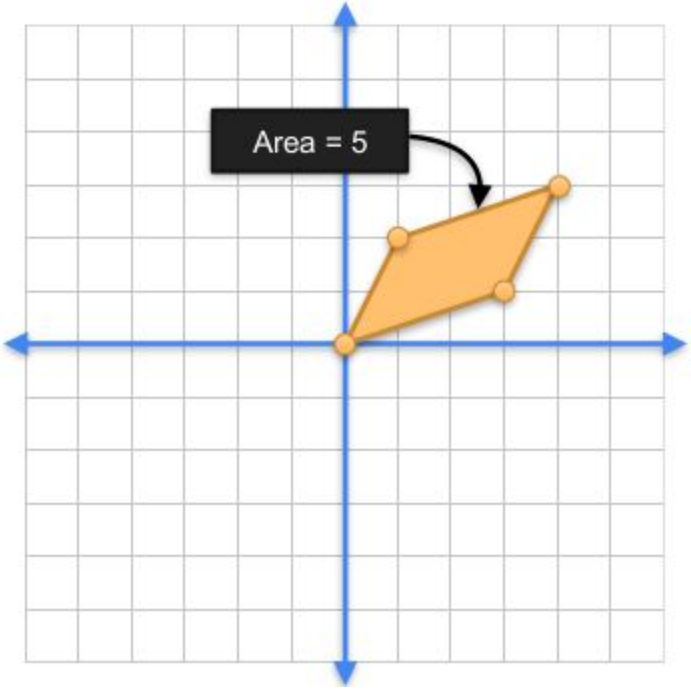
Determinant as area



3	1
1	2

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$



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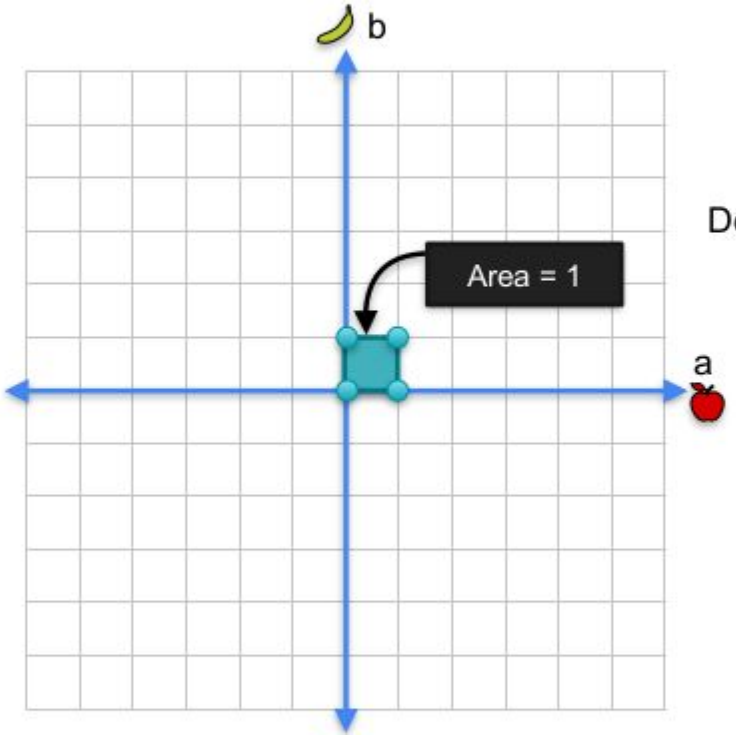
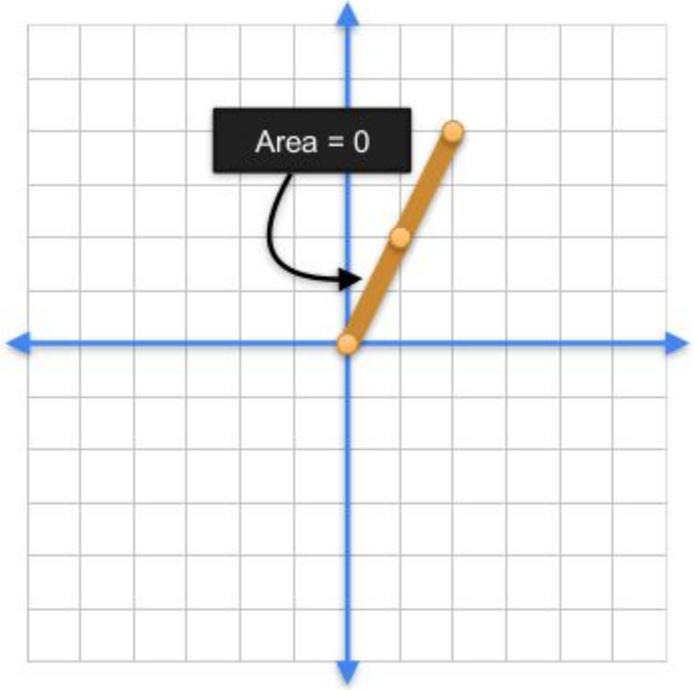
Cont'd Determinant as area



1	1
2	2

$$\text{Det} = 1 \cdot 2 - 1 \cdot 2$$

$$\text{Det} = 0$$



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Variables

System of equations 1

$$a + b + c = 10$$
$$a + 2b + c = 15$$
$$a + b + 2c = 12$$

Solution

$a = 3$
$b = 5$
$c = 2$

Rank of Matrix

System 1

The dog is **black**
The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

The dog is **black**
The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3

The dog
The dog

Two sentences

Zero pieces of information

Rank = 0

Cont'd Rank of Matrix

System 1

$a + b = 0$

$a + 2b = 0$

🍎	🍌
1	1
1	2

Rank = 2

System 2

$a + b = 0$

$2a + 2b = 0$

🍎	🍌
1	1
2	2

Rank = 1

System 3

$0a + 0b = 0$

$0a + 0b = 0$

🍎	🍌
0	0
0	0

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

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Cont’d Rank of Matrix

1	1
1	2

Rank = 2

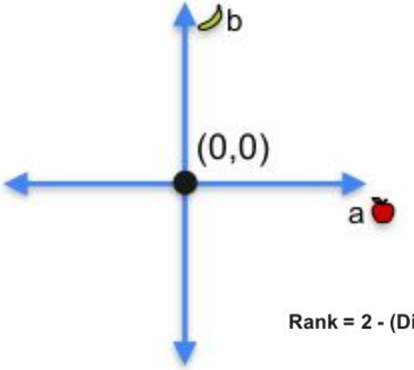
1	1
2	2

Rank = 1

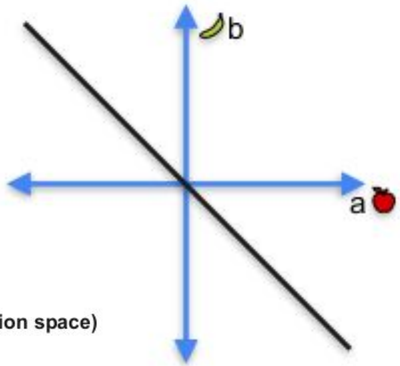
0	0
0	0

Rank = 0

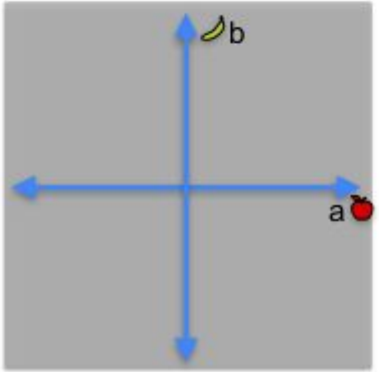
Dimension of solution space = 0



Dimension of solution space = 1

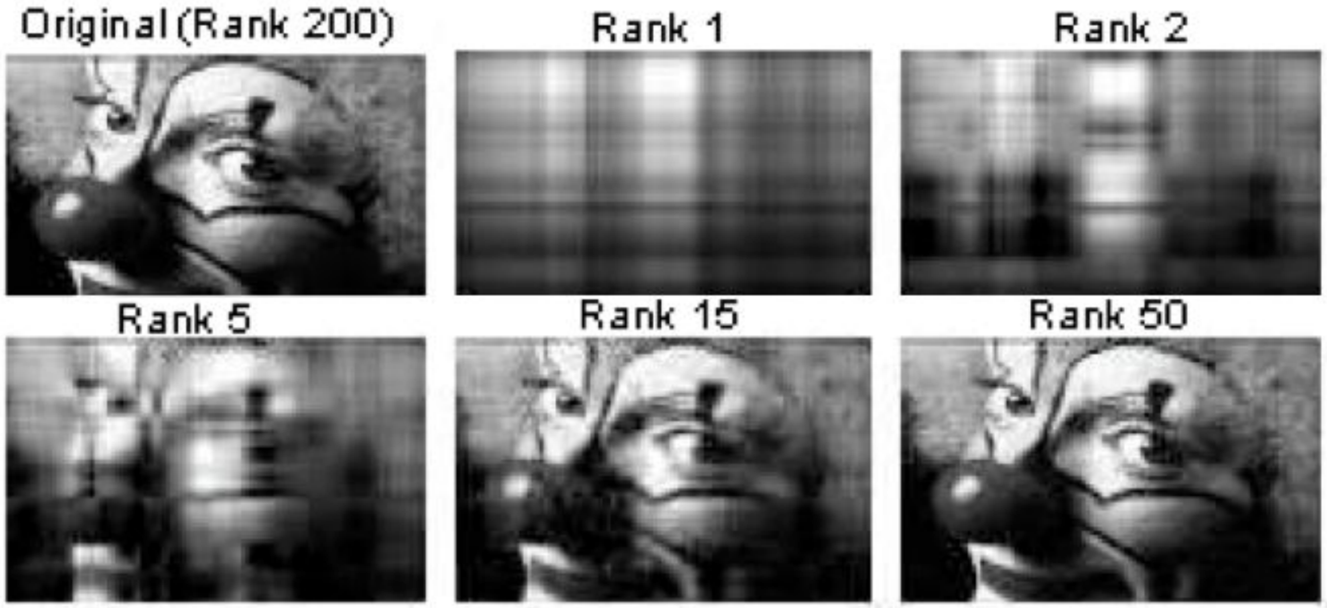


Dimension of solution space = 2



Rank = 2 - (Dimension of solution space)

Cont'd Rank of Matrix



Cont'd Rank of Matrix

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Matrix

1	1	2
3	-3	-1
2	-1	6

System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

Row echelon form matrix

1	1	2
0	-6	7
0	0	6

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

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Cont’d Rank of Matrix

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Rank = 0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

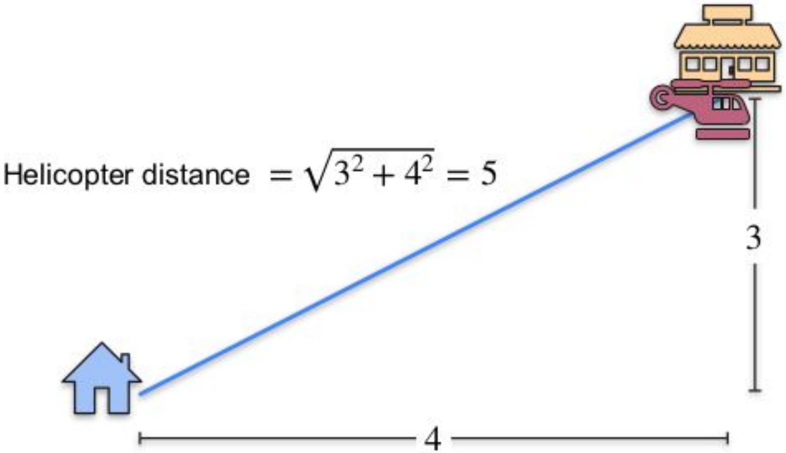
Number of pivots = 1

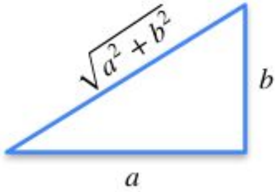
0	0	0
0	0	0
0	0	0

Number of pivots = 0

Vector Spaces

How to get from point A to point B?

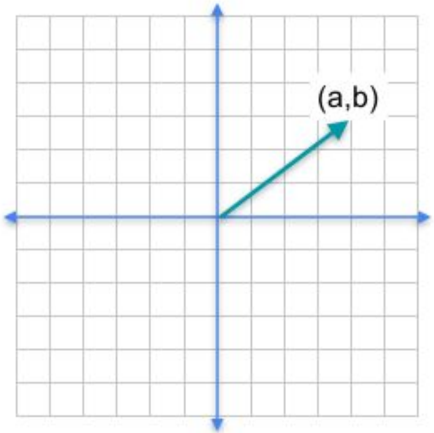




Pythagorean Theorem

Cont'd Vector Spaces

Norms

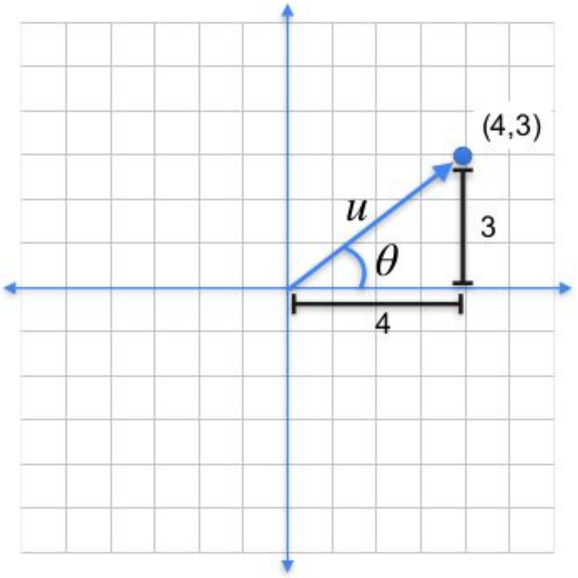


L1-norm $= |(a,b)|_1 = |a| + |b|$

L2-norm $= |(a,b)|_2 = \sqrt{a^2 + b^2}$

Cont’d Vector Spaces

Direction of a vector

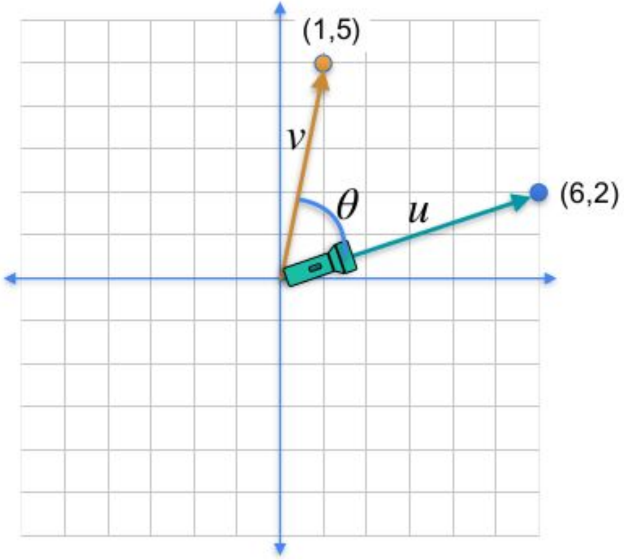


$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 \ = 36.87°$$

Cont'd Vector Space

Distances



L1-distance $|u-v|_1 = |5| + |-3| = 8$

L2-distance $|u-v|_2 = \sqrt{5^2+3^2} = 5.83$

Cosine distance $\cos(\theta)$

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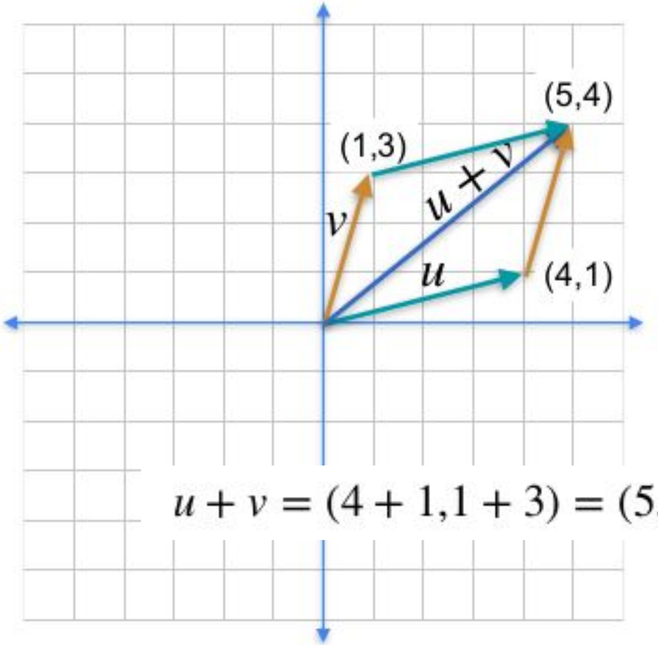
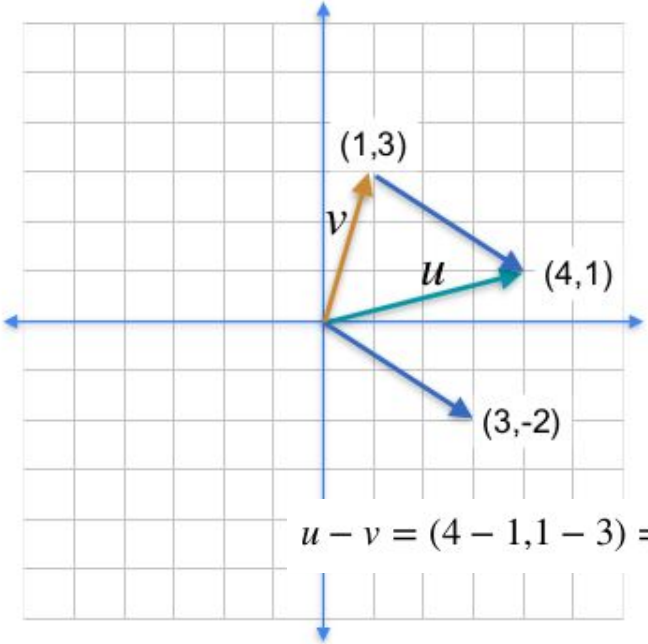
Vector Arithmetic

Sum of vectors



$$u + v = (4 + 1, 1 + 3) = (5,4)$$

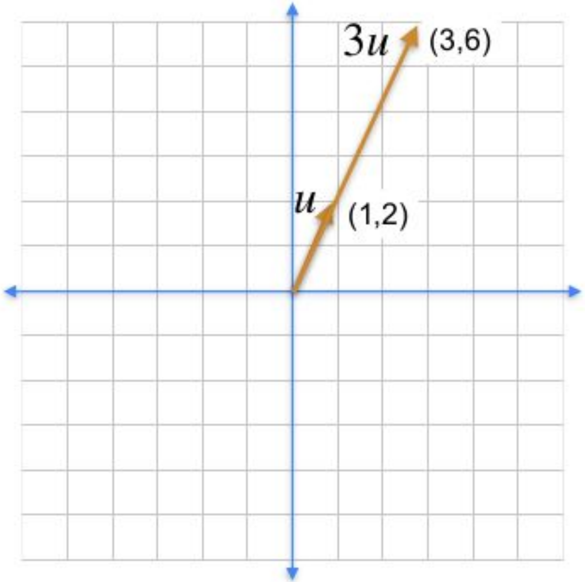
Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Linear Transformation

Multiplying a vector by a scalar

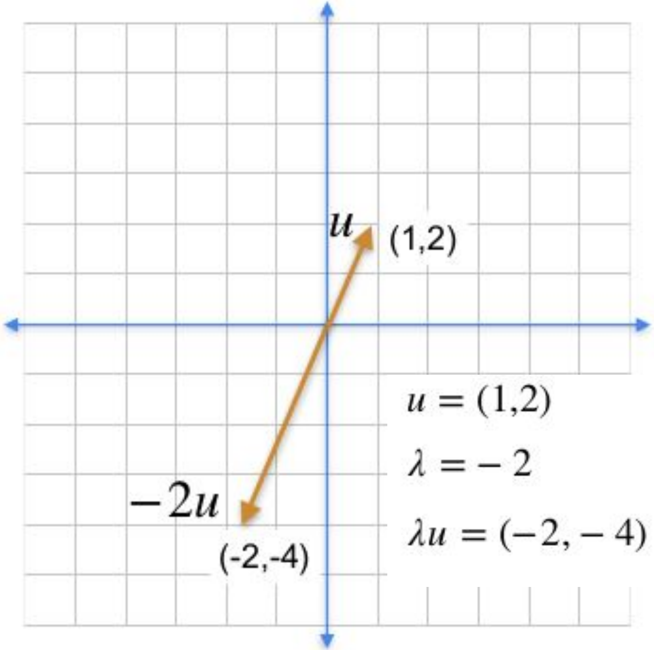


$u = (1,2)$

$\lambda = 3$

$\lambda u = (3,6)$

If the scalar is negative

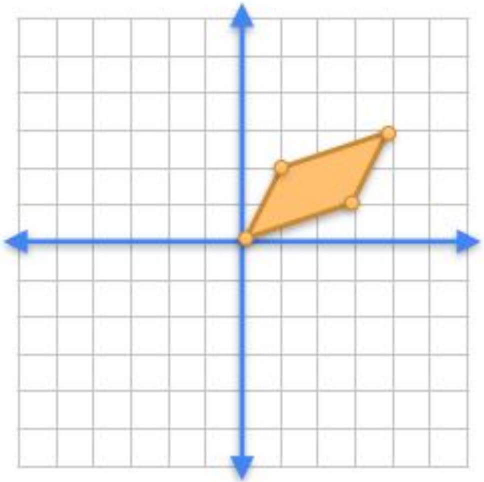


$u = (1,2)$

$\lambda = -2$

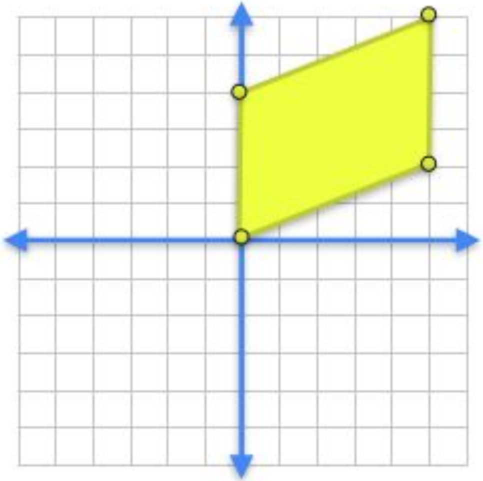
$\lambda u = (-2, -4)$

Cont'd Linear Transformation



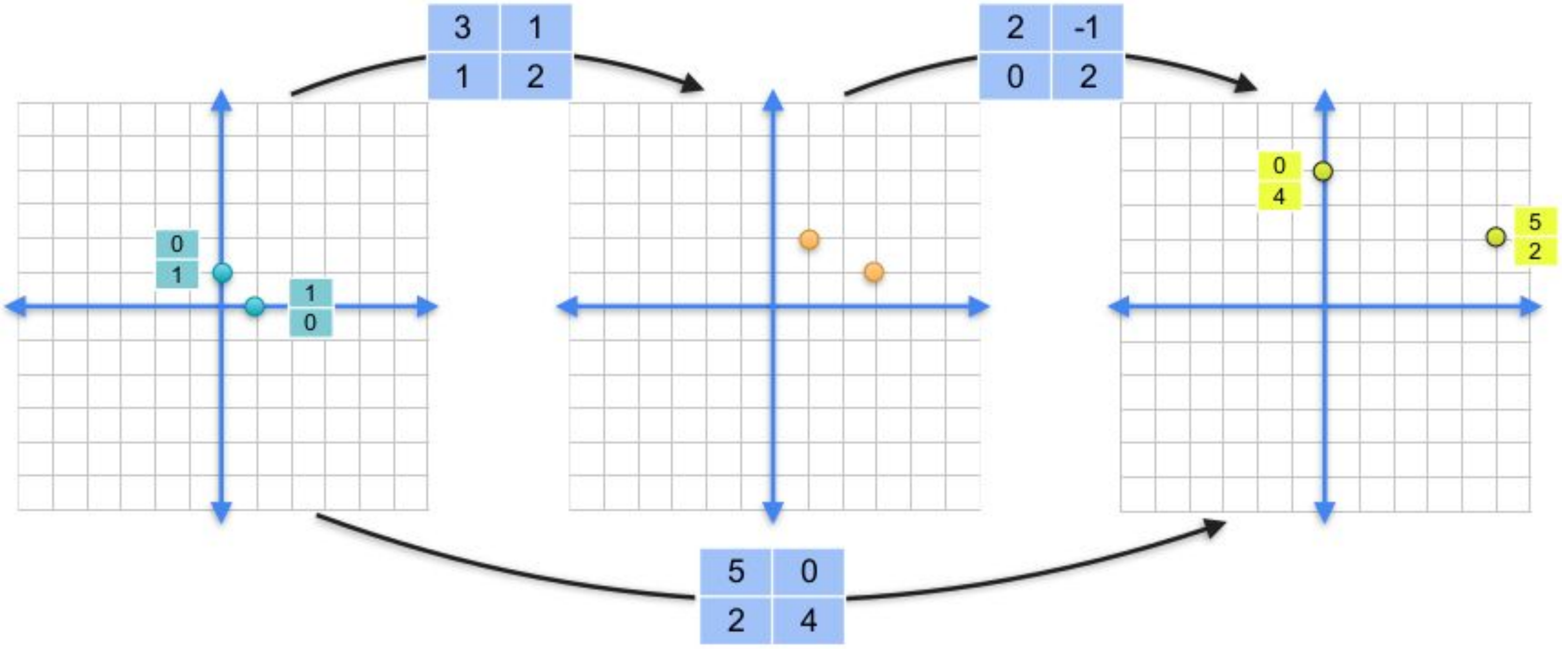
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$



Cont'd Linear Transformation

Combining linear transformations



Cont’d Linear Transformation

Singular and non-singular transformations

Non-singular

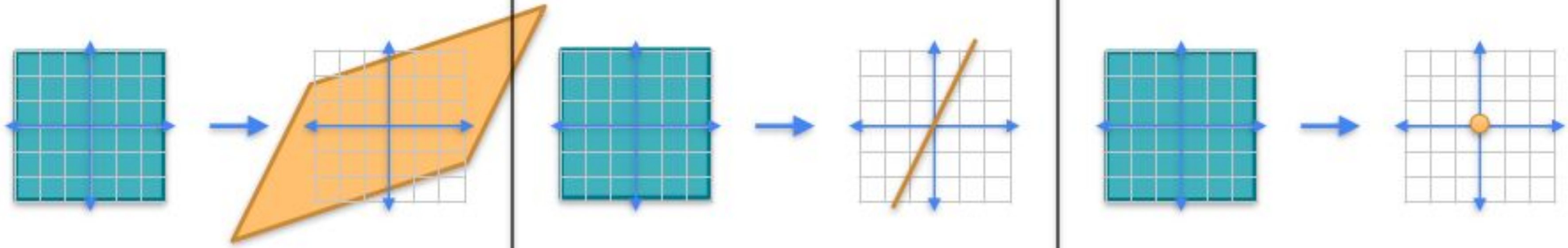
🍎	🍌
3	1
1	2

Singular

🍎	🍌
1	1
2	2

Singular

🍎	🍌
0	0
0	0



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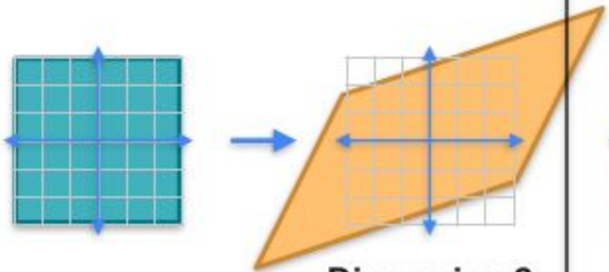
Cont’d Linear Transformation

Rank of linear transformations

Rank 2

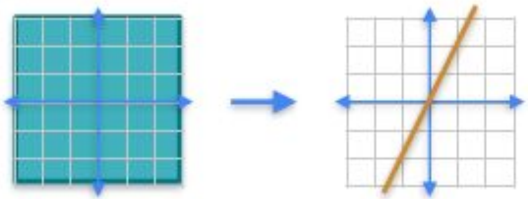
3	1
1	2



Dimension 2

Rank 1

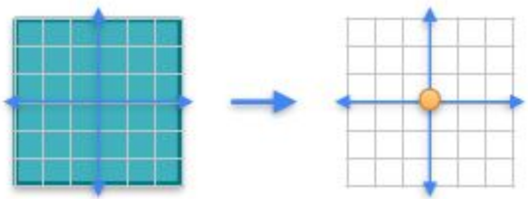
1	1
2	2



Dimension 1

Rank 0

0	0
0	0



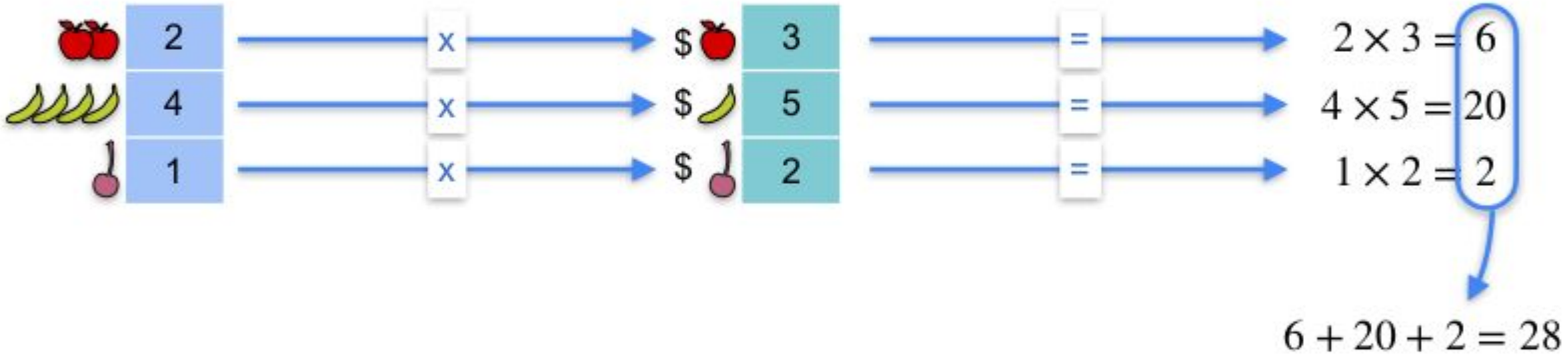
Dimension 0

Dot Product

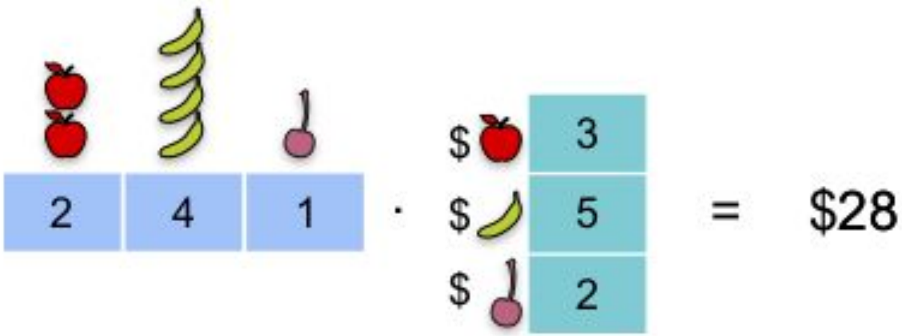
Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price
$28



Cont’d Dot Product



$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

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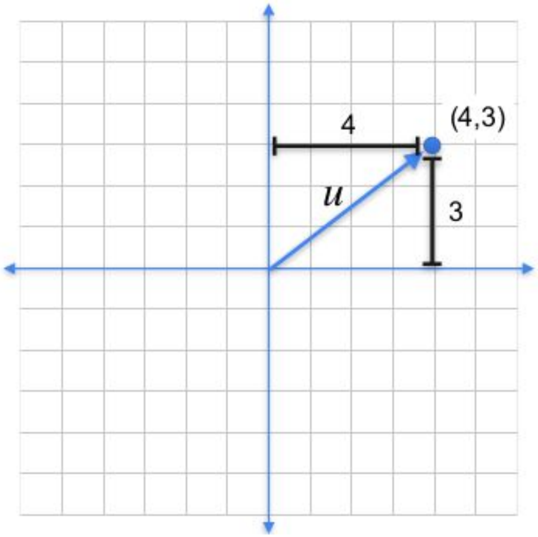
Cont'd Dot Product

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

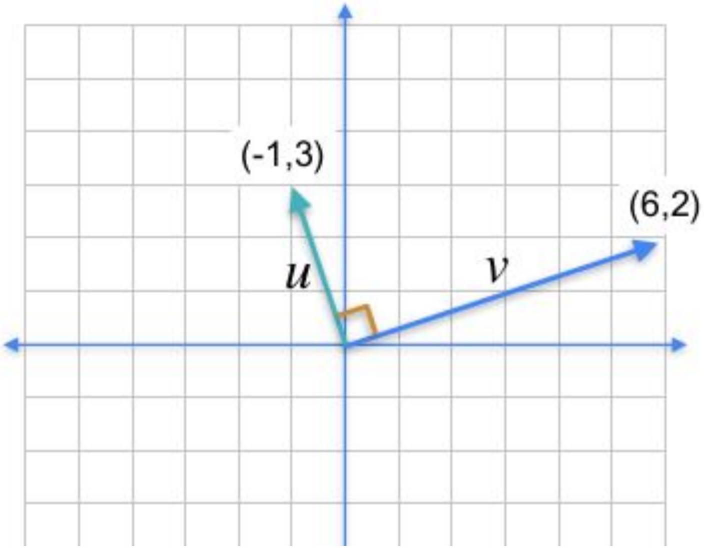
$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2-norm = \sqrt{dot\ \ product(u, u)}$$

$$|u|_2 = \sqrt{\langle u, u \rangle}$$

Cont’d Dot Product

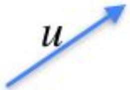
Orthogonal vectors have dot product 0



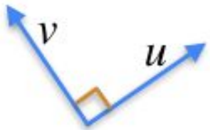
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\langle u, v \rangle = 0$$

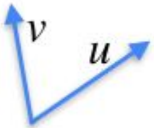
Cont’d Dot Product



$$\langle u, u \rangle = |u|^2$$

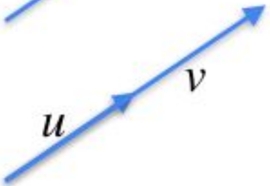


$$\langle u, v \rangle = 0$$

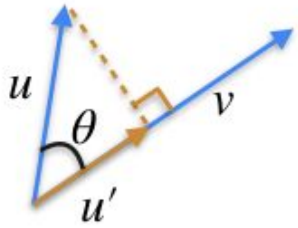


$$\langle u, v \rangle = ?$$

$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



$$\langle u, v \rangle = |u| \cdot |v|$$



$$\begin{aligned}\langle u, v \rangle &= |u'| \cdot |v| \\ &= |u||v|\cos(\theta)\end{aligned}$$

Cont’d Dot Product

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{bmatrix}1&1&1\\1&2&1\\1&1&2\end{bmatrix} \cdot \begin{array}{c}\$\\\$\\\$\end{array}\begin{bmatrix}a\\b\\c\end{bmatrix} = \$\begin{bmatrix}10\\15\\12\end{bmatrix}$$

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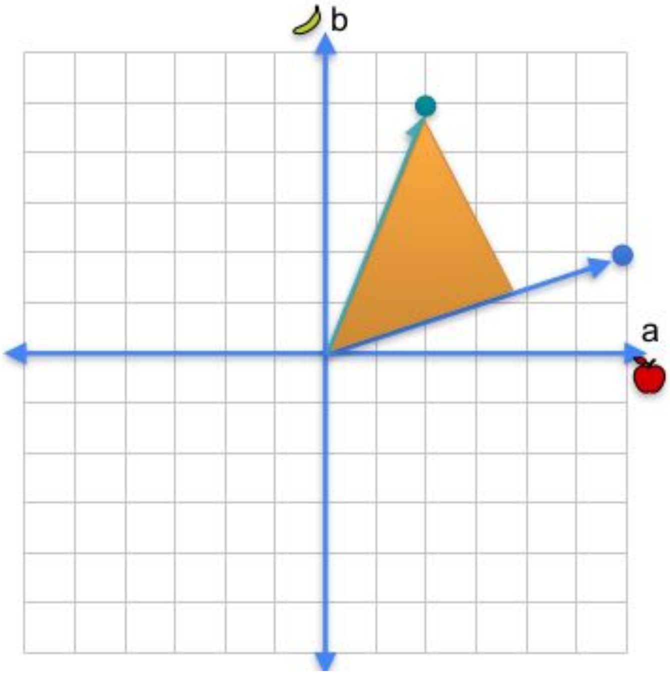
Dot Product as an area

Dot product as an area



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Matrix Multiplication

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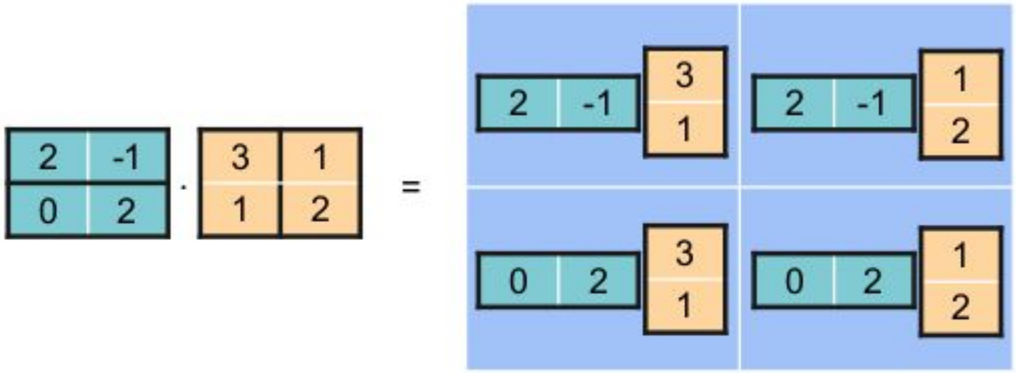
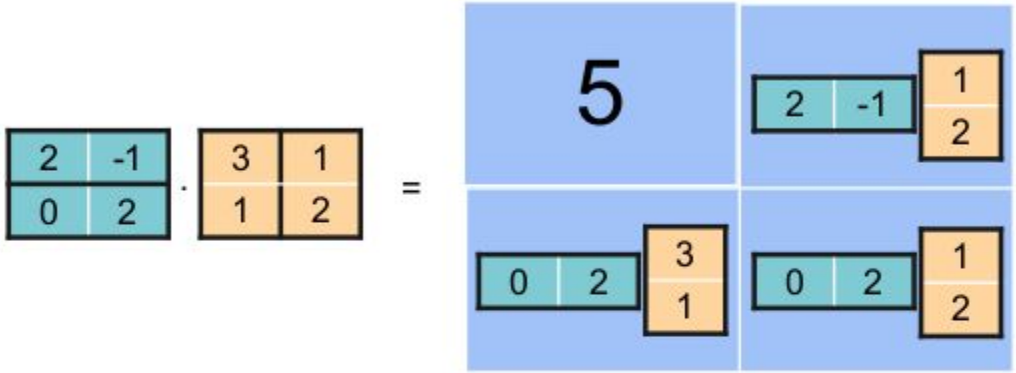
Cont’d Matrix Multiplication

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} \, & \, \\ \, & \, \end{bmatrix}$$

Cont’d Matrix Multiplication



Cont’d Matrix Multiplication



Cont’d Matrix Multiplication

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

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Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} a \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

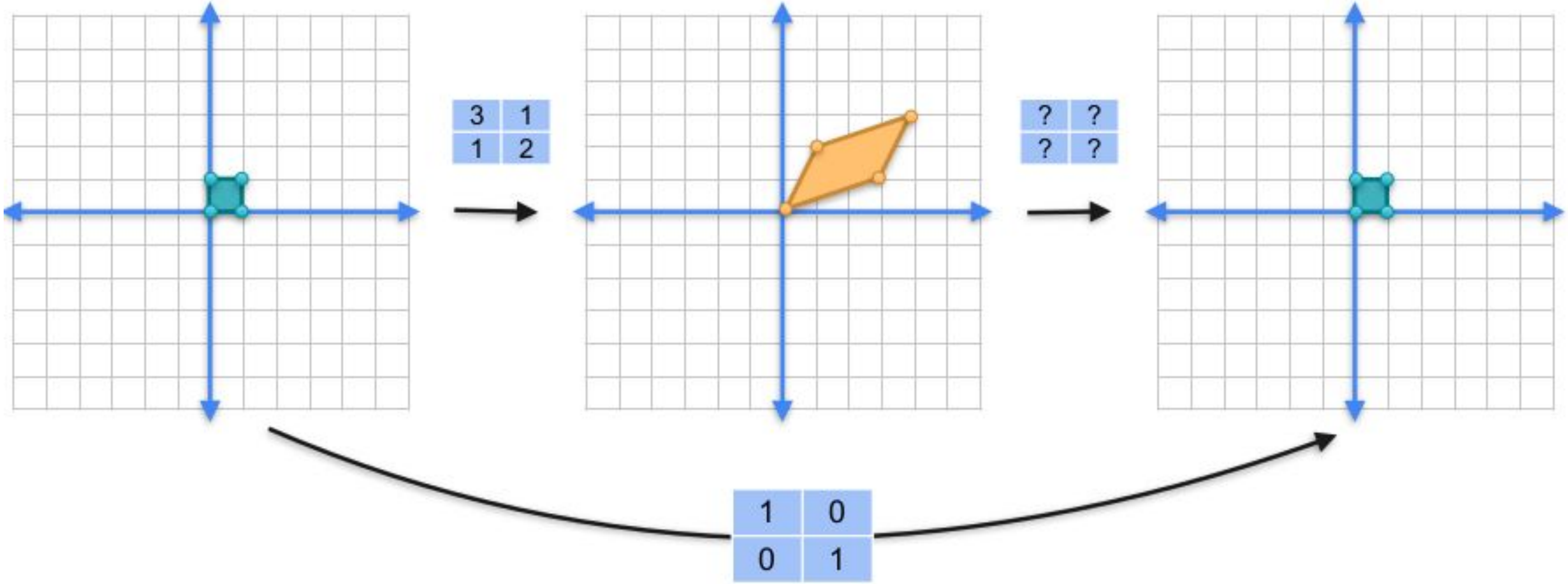
Cont'd Identity Matrix

Determinant of the identity matrix

$$\det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \cdot 1 - 0 \cdot 0 = 1$$

$$\det(I) = 1$$

Inverse Matrix



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Cont’d Inverse Matrix

Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

be the 2 x 2 matrix. The inverse matrix of A is given by the formula,

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

be the 3 x 3 matrix. The inverse matrix is:

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}$$

Cont'd Inverse Matrix

Determinant of an inverse

$$\det(AB) = \det(A)\det(B)$$

Why is this?

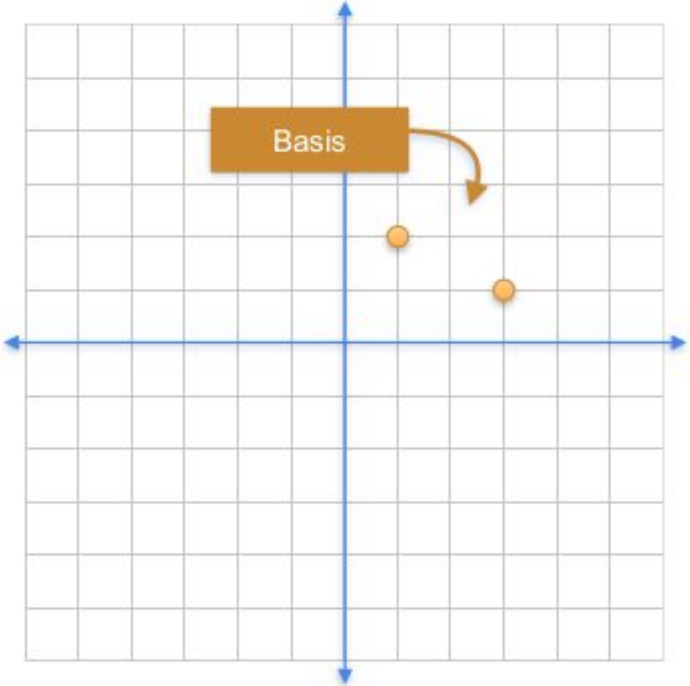
$$\det(A^{-1}) = \frac{1}{\det(A)}$$

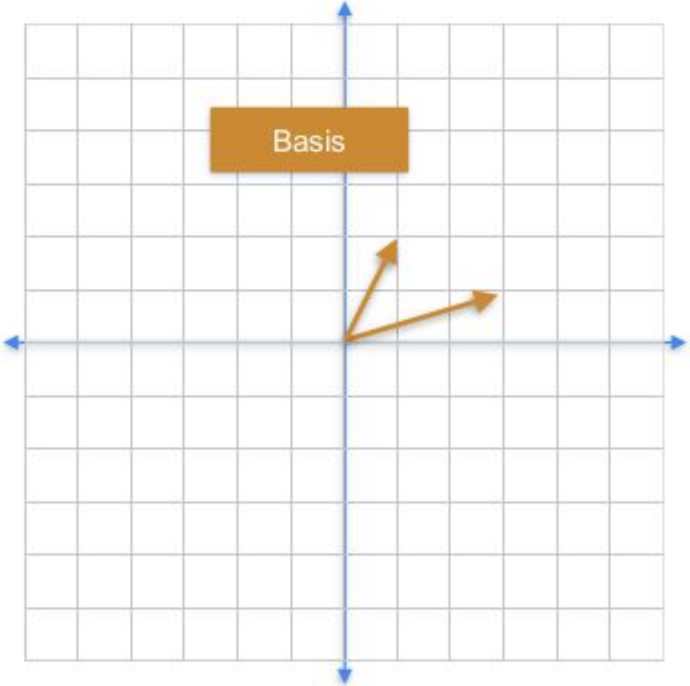
$$\det(AA^{-1}) = \det(A)\det(A^{-1})$$

$$\underbrace{\det(I)}_{1} = \det(A)\underbrace{\det(A^{-1})}_{\frac{1}{\det(A)}}$$

Bases

3	1
1	2





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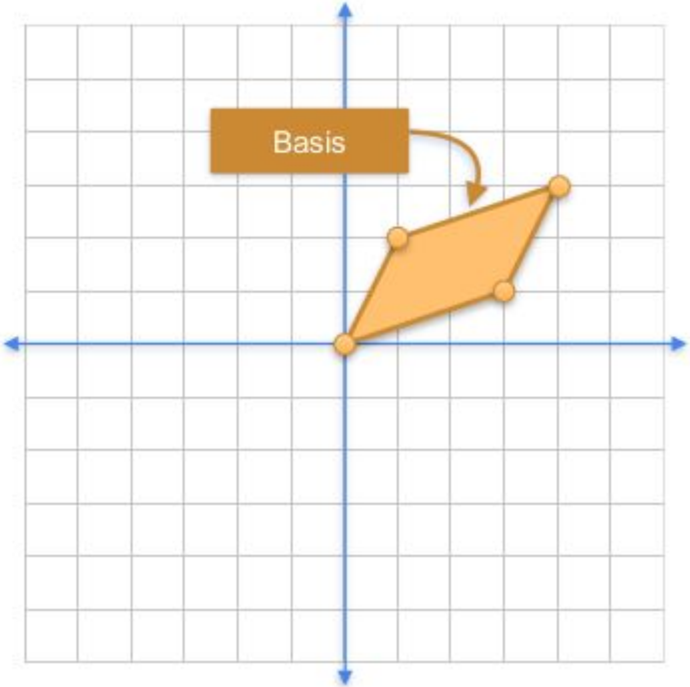
Cont'd Bases

3	1
1	2



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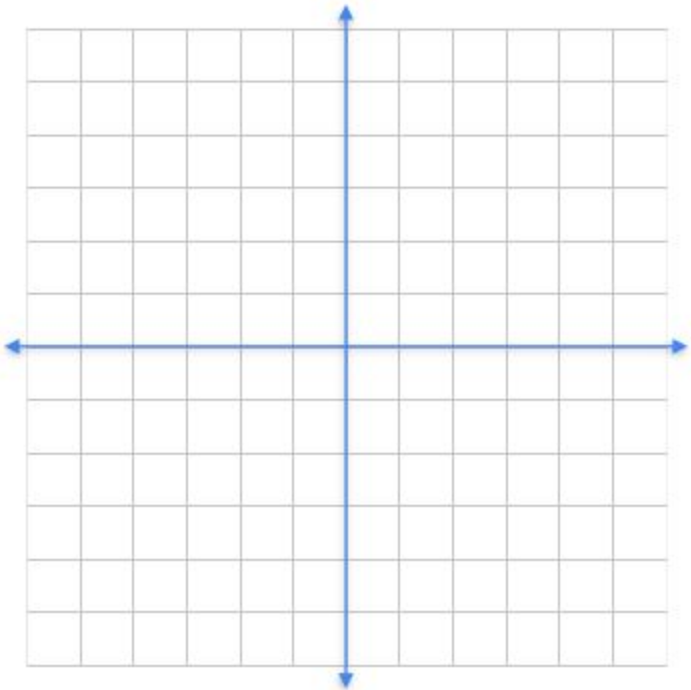
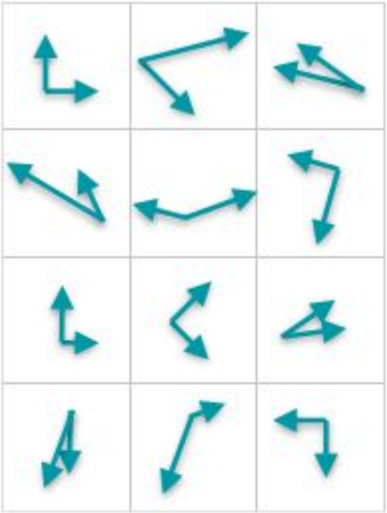
Cont’d Bases

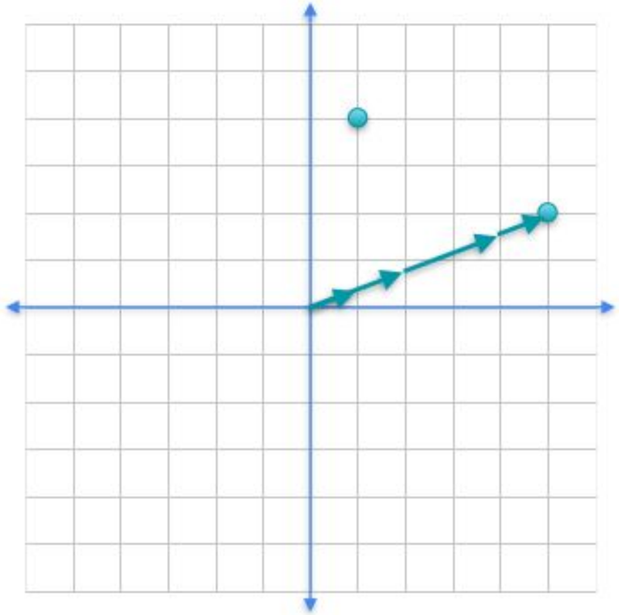


Bases

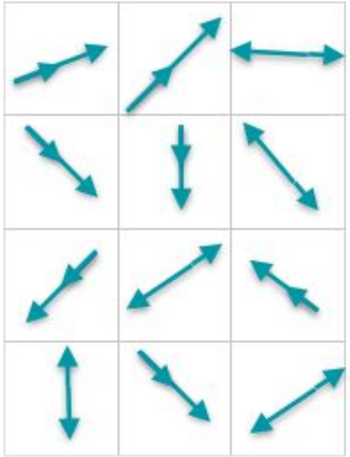


Cont'd Bases

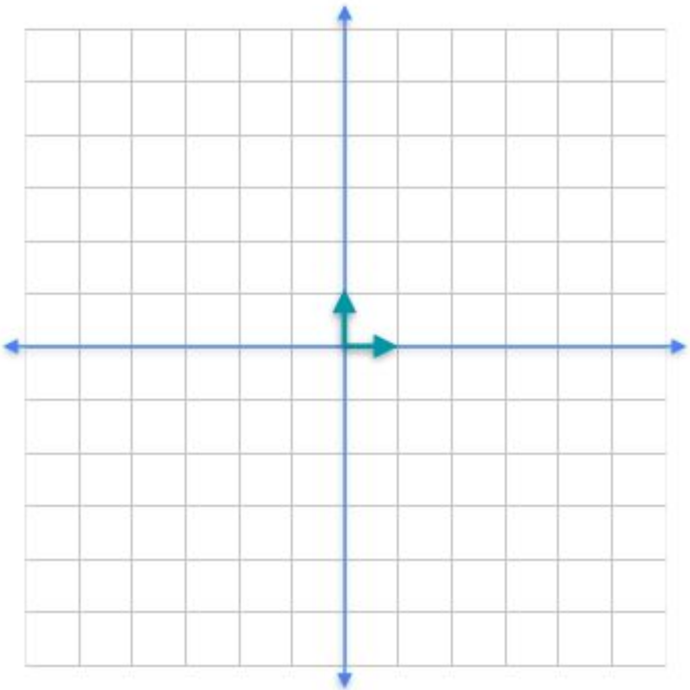
What is not a basis?

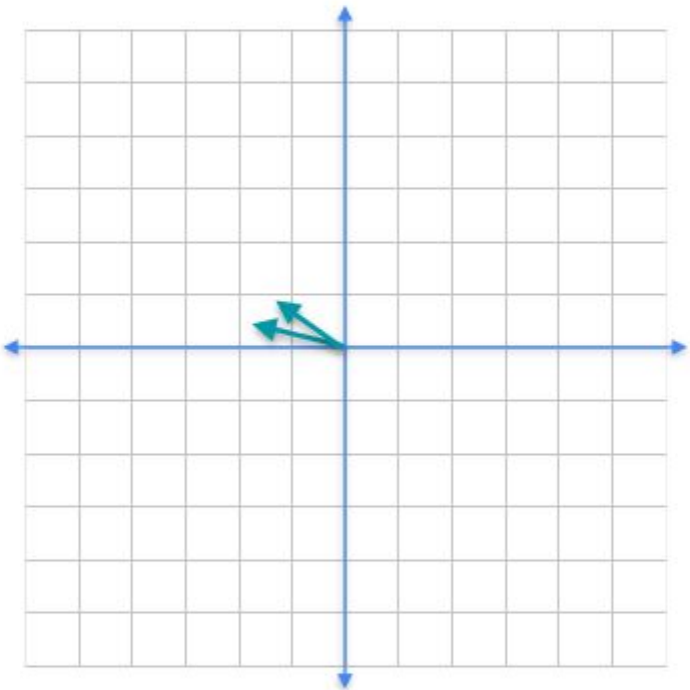


Not bases

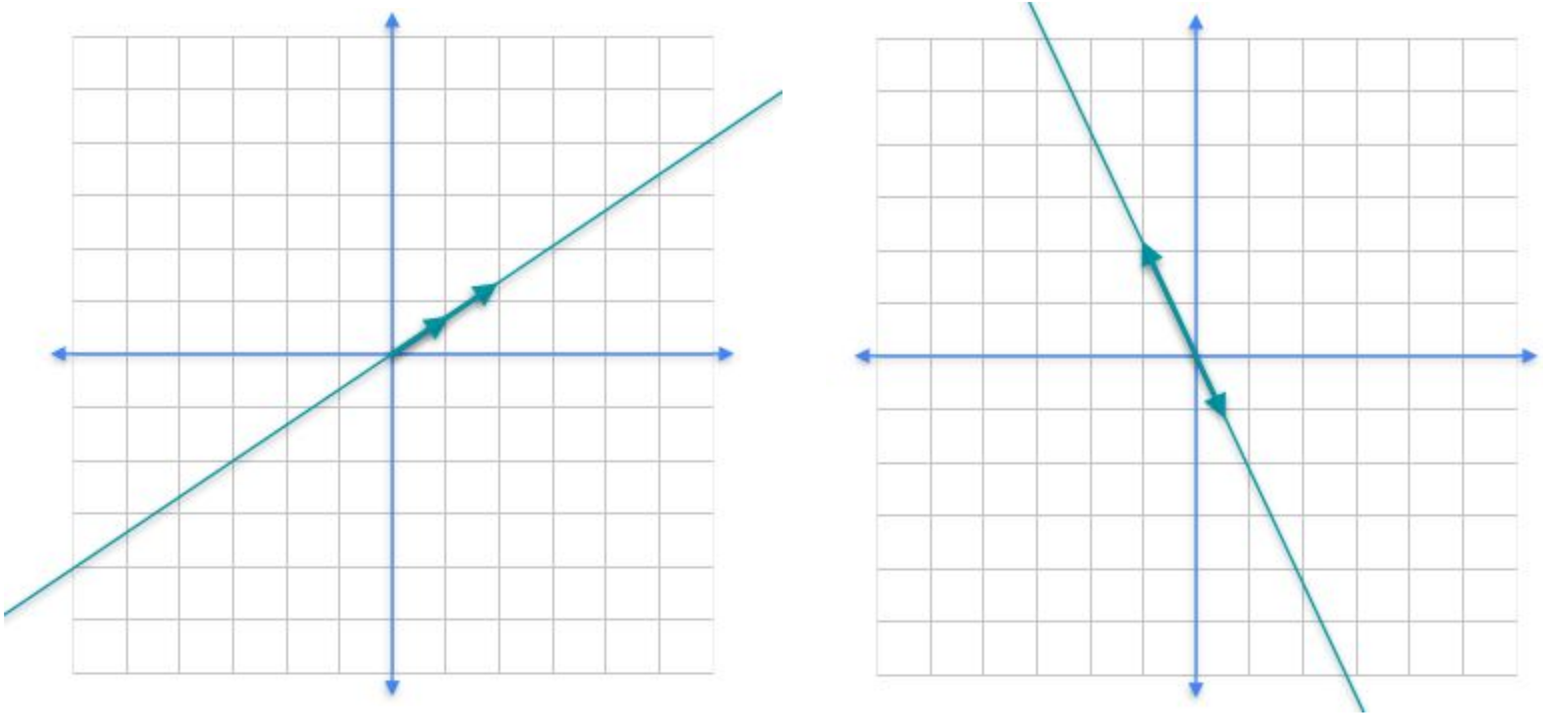


Span



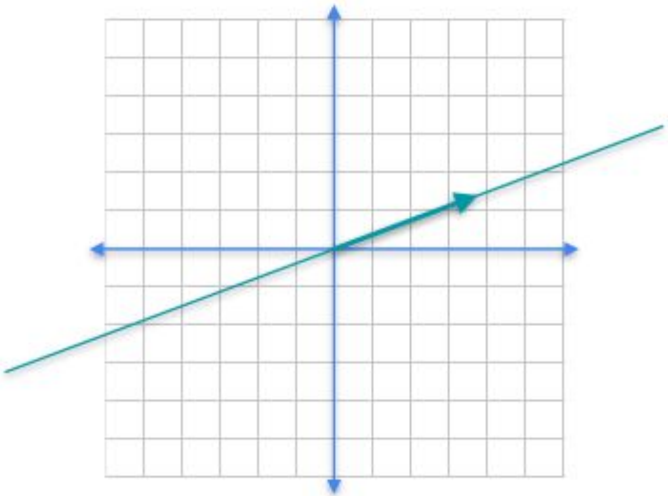


Cont’d Span

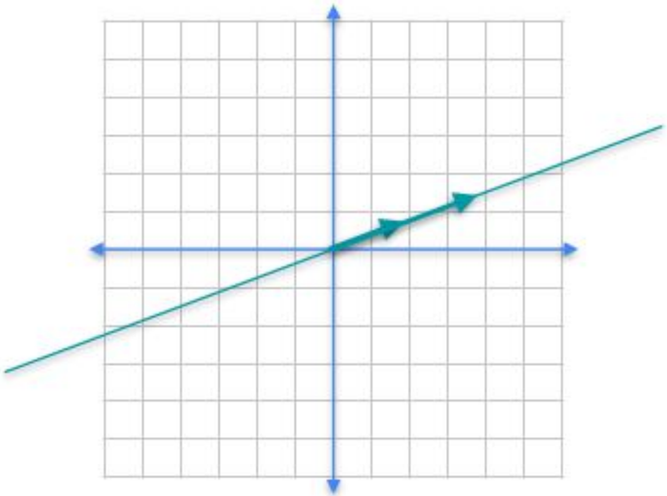


Cont'd Span

A basis is a minimal spanning set



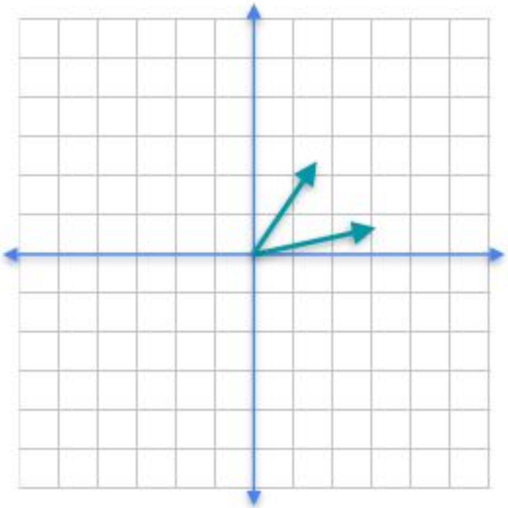
Basis



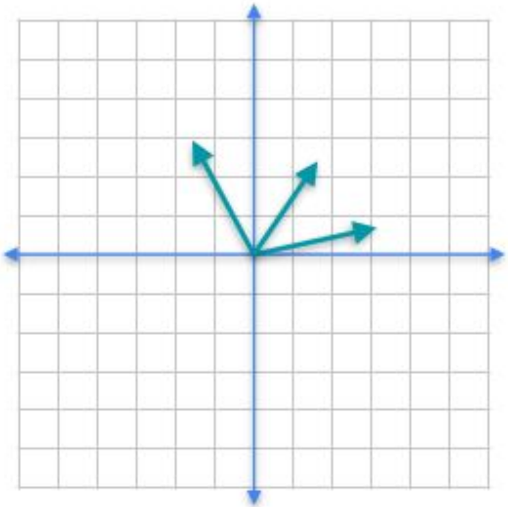
Not a basis

Cont'd Span

A basis is a minimal spanning set



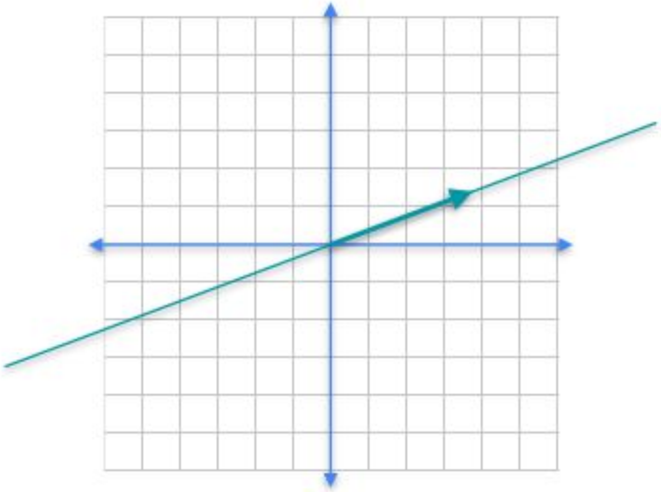
Basis



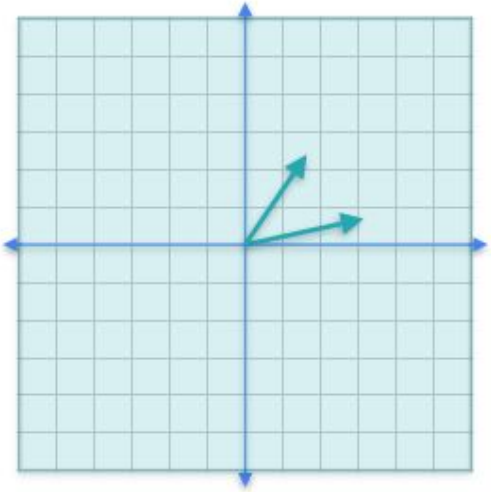
Not a basis

Cont'd Span

Number of elements in the basis is the dimension



1 element
Dimension = 1



2 elements in the basis
Dimension = 2

Row Space

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 11 & 22 & 36 \\ 1 & 5 & 8 \end{bmatrix}$$

$$\begin{aligned} a1 &= \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \\ a2 &= \begin{bmatrix} 11 & 22 & 36 \end{bmatrix} \\ a3 &= \begin{bmatrix} 1 & 5 & 8 \end{bmatrix} \end{aligned}$$

$$R(A) = span(a1, a2, a3)$$

All the linear combinations of row vectors : a1, a2 and a3

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Column Space

$$a1=\begin{bmatrix}1\\11\\1\end{bmatrix},\quad a2=\begin{bmatrix}2\\22\\5\end{bmatrix},\quad a3=\begin{bmatrix}3\\36\\8\end{bmatrix}$$

$$C(A) = span(a1, a2, a3)$$

All the linear combinations of column vectors : a1, a2 and a3

Null Space

A matrix and its corresponding system of equations

System 1

- $a + b = 0$
- $a + 2b = 0$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

The only two numbers a, b, such that
- a+b = 0

and
- a+2b = 0

are:
a=0 and b=0

System 2

- $a + b = 0$
- $2a + 2b = 0$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Any pair (x, -x) satisfies that
- a+b = 0

and
- a+2b = 0

For example:
(1,-1), (2,-2), (-8,8), etc.

System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Any pair of numbers satisfies that
- 0a+0b = 0

and
- 0a+0b = 0

For example:
(1,2), (3,-9), (-90,8.34), etc.

Cont'd Null Space

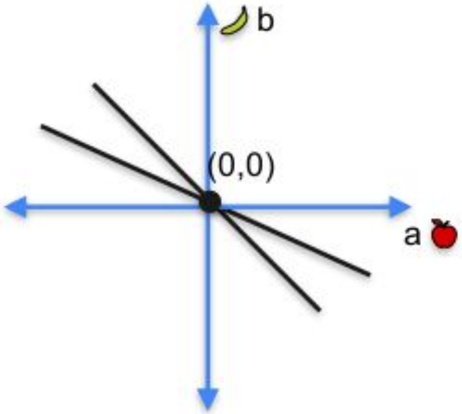
The set of solutions of a system of equations

System 1

- a + b = **0**
- a + 2b = **0**

Solution

- $a = 0$
- $b = 0$

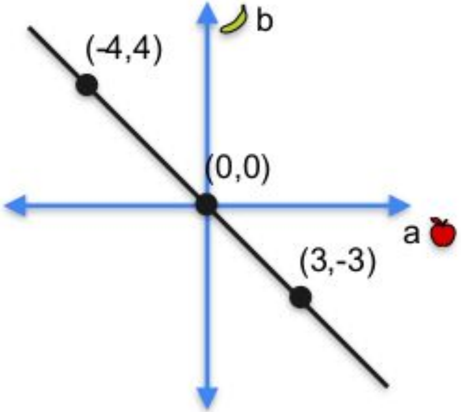


System 2

- a + b = **0**
- 2a + 2b = **0**

Solutions

- any a
- $b = -a$

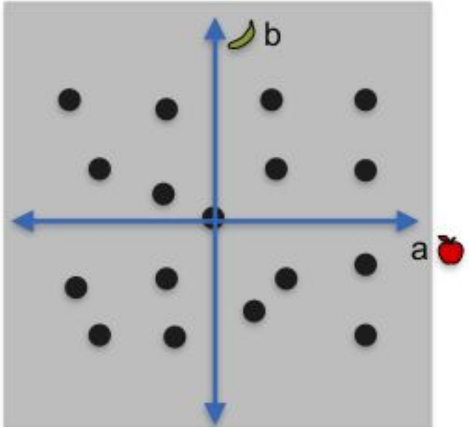


System 3

- 0a + 0b = **0**
- 0a + 0b = **0**

Solutions

- any a
- any b



Cont’d Null Space

The null space of a matrix

1	1
1	2

Null space
- $a = 0$
- $b = 0$

Dimension = 0

Non-singular

1	1
2	2

Null space
- any a
- $b = -a$

Dimension = 1

Singular

0	0
0	0

Null space
- any a
- any b

Dimension = 2

Singular

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Cont'd Null Space

Null space for systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

Solution space

Dimension = 0

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

Solution space

Dimension = 1

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Solution space

Dimension = 2

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Solution space

Dimension = 3

Cont'd Null Space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- a + c = 0
- b = 0
- 3a + 2b + 3c = 0

All points of the form
$(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- a + b + c = 0
- a + b + 2c = 0
- c = 0

All points of the form
$(x, -x, 0)$
Dimension = 1

1	1	1
0	2	2
0	0	3

- a + b + c = 0
- 2b + 2c = 0
- 3c = 0

The point
$(0, 0, 0)$
Dimension = 0

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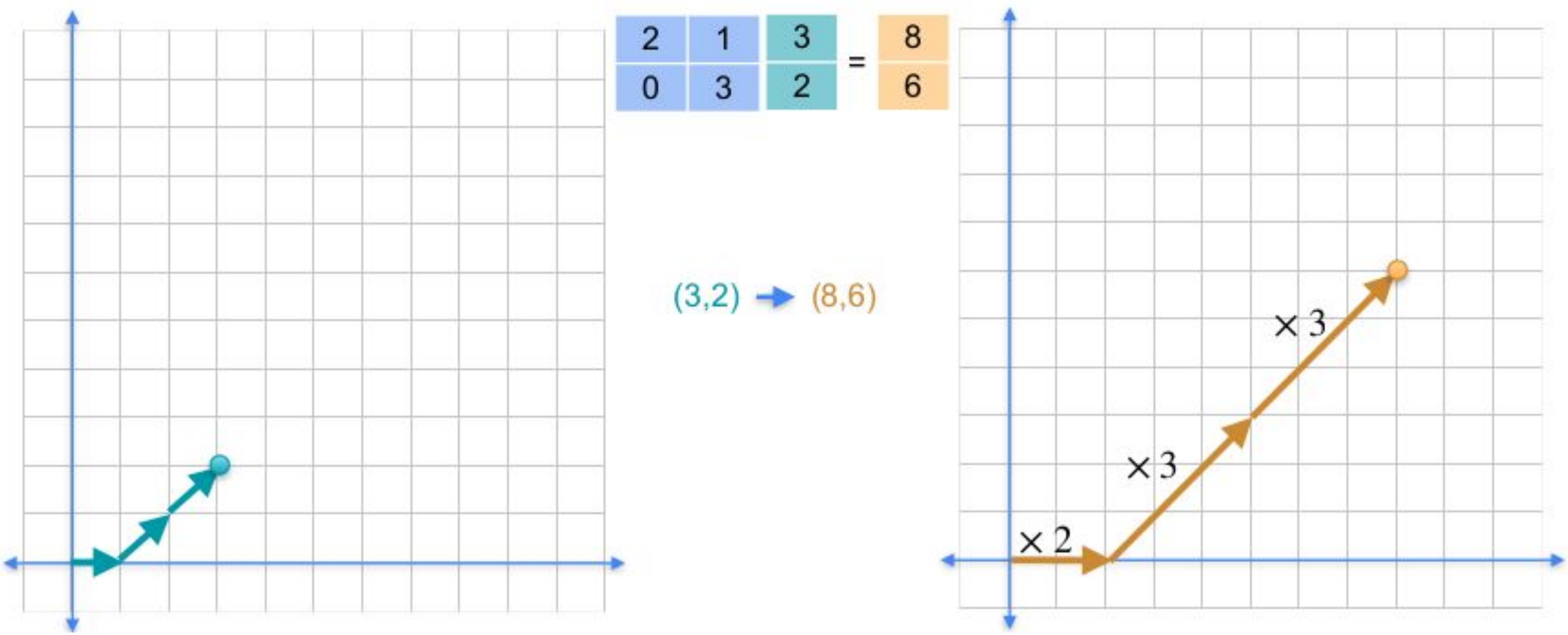
Eigen

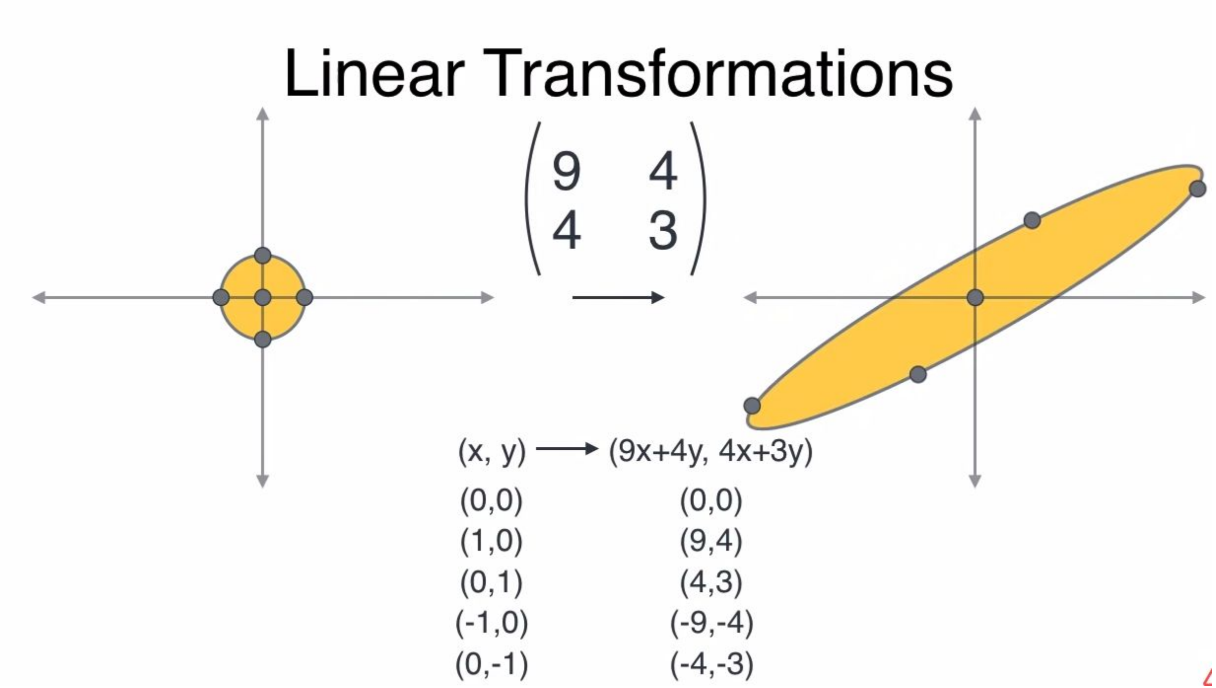
Eigenbasis

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

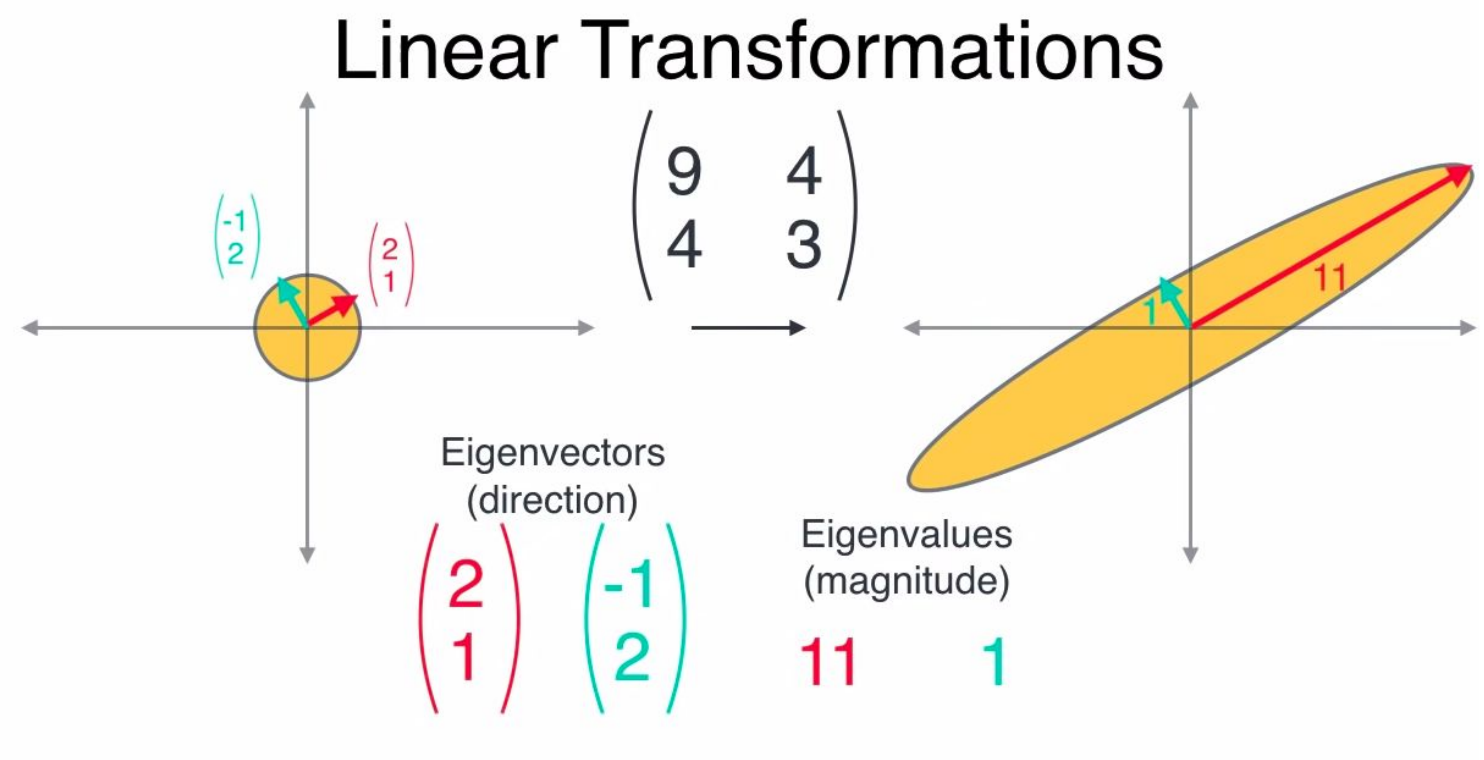
(3,2) → (8,6)



Cont’d Eigen


Linear Transformations
9 4
4 3
(x, y) → (9x+4y, 4x+3y)
(0,0) (0,0)
(1,0) (9,4)
(0,1) (4,3)
(-1,0) (-9,-4)
(0,-1) (-4,-3)

Cont’d Eigen


Linear Transformations
9 4
4 3
-1
2
2
1
11
1
Eigenvectors
(direction)
2
1
-1
2
Eigenvalues
(magnitude)
11
1

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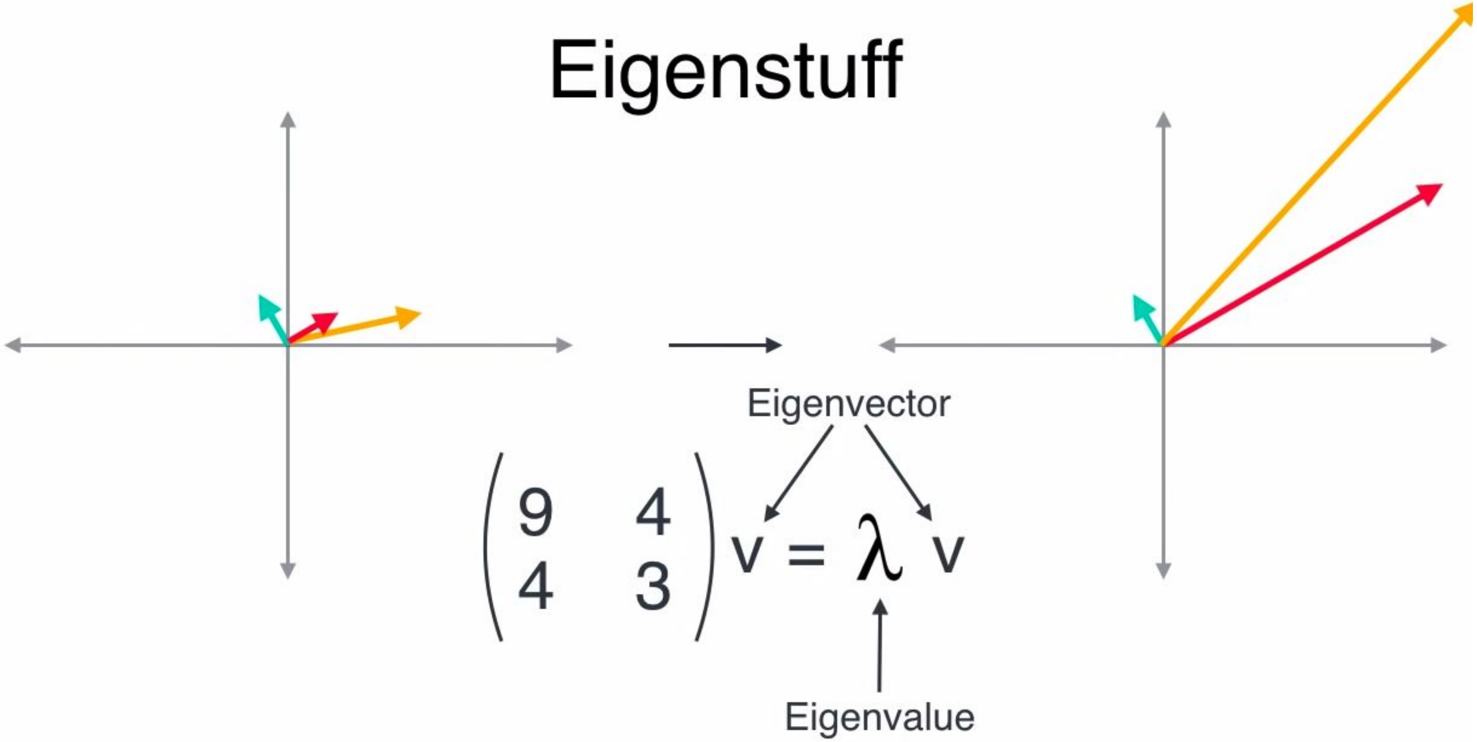
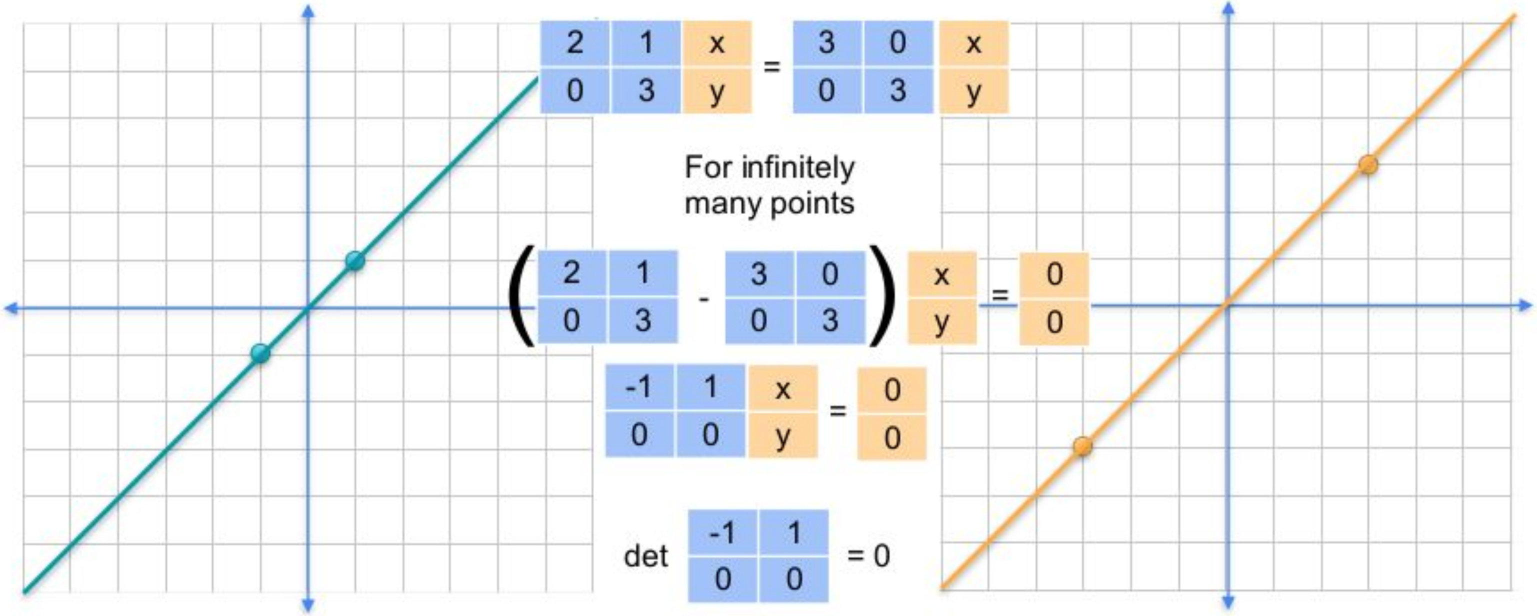
Cont’d Eigen



Cont’d Eigen

Finding eigenvalues



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = 0$$

Cont'd Eigen

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Has infinitely many solutions

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$

$\lambda = 2$

$\lambda = 3$

Cont’d Eigen

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ Has infinitely many solutions

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$ $\quad \lambda = 2$, $\lambda = 3$

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Cont'd Eigen

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 2x \\ 0x + 3y &= 2y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 0 \end{aligned} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 3x \\ 0x + 3y &= 3y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 1 \end{aligned} \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For further experiences …

Essence of Linear Algebra

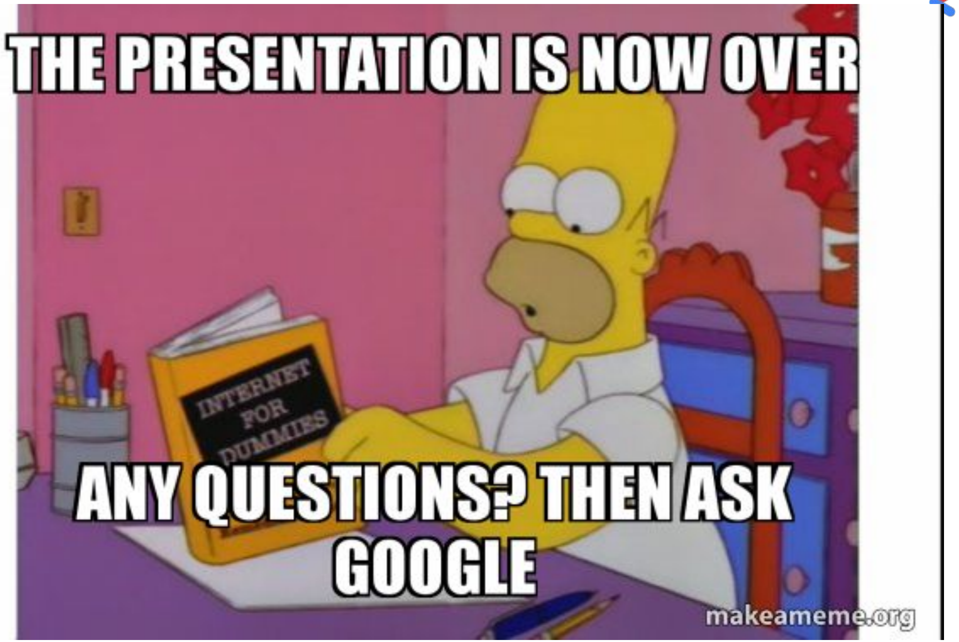
https://youtu.be/fNk_zzaMoSs

Principal Component Analysis (PCA)

https://youtu.be/g-Hb26agBFg

MIT 18.06SC Linear Algebra, Fall 2011

https://www.youtube.com/playlist?list=PL221E2BBF13BECF6C



Thank You.

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