Rational Metareasoning in Problem-Solving Search

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Outline

Rational Metareasoning

- A problem-solving agent can perform *base-level* actions from a known set $\{A_i\}$.
- Before committing to an action, the agent may perform a sequence of *meta-level* deliberation actions from a set $\{S_j\}$.
- At any given time there is a base-level action A_α that maximizes the agent's *expected utility.*

The **net VOI** $V(S_j)$ of action S_j is the **intrinsic VOI** Λ_j less the cost of S_j:

$$V(S_j) = \Lambda(S_j) - C(S_j)$$

$$\Lambda(S_j) = \mathbb{E}\left(\mathbb{E}(U(A^j_\alpha)) - \mathbb{E}(U(A_\alpha))\right)$$

- $S_{j_{\max}}$ that maximizes the net VOI is performed: $j_{\max} = \arg\max_j V(S_j)$, if $V(S_{j_{\max}}) > 0$.
- Otherwise, A_α is performed.

Outline

Constraint Satisfaction

- CSP backtracking search algorithms typically employ variable-ordering and value-ordering heuristics.
- Many value ordering heuristics are computationally heavy, e.g. heuristics *based on solution count estimates.*
- Principles of rational metareasoning can be applied to decide when to deploy the heuristics.

Constraint Satisfaction

A constraint satisfaction problem (CSP) is defined by:

variables $\mathscr{X} = \{X_1, X_2, ...\}$,

constraints $\mathscr{C} = \{C_1, C_2, ...\}$.

- Each *variable* X_i has a non-empty domain D_i of possible values.
- Each *constraint* C_i involves some subset of the variables—the *scope* of the constraint—and specifies the allowable combinations of values for that subset.
- An *assignment* that does not violate any constraints is called *consistent* (or solution).

Value Ordering Model

Value ordering heuristics provide information about:

- T_i—the expected time to find a solution containing an assignment $X_k = y_{ki}$;
- p_i—the probability that there is no solution consistent with $X_k = y_{ki}$.

The expected remaining search time in the subtree under X_k for ordering ω is $T^{s|\omega} = T_{\omega(1)} + \sum_{i=2}^{|D_k|} T_{\omega(i)} \prod_{j=1}^{i-1} p_{\omega(j)}$

- The current optimal base-level action is picking the ω which optimizes $T^{s|\omega}$. $T^{s|\omega}$ is minimal if the values are sorted by increasing order of $\frac{T_i}{1-p_i}$.
- The intrinsic VOI Λ_i of estimating T_i, p_i for the ith assignment is the expected decrease in the expected search time: $\Lambda_i = \mathbb{E}\left[T^{s|\omega_-} - T^{s|\omega_{+i}}\right]$.

Main Results

Rational Value Ordering

The intrinsic VOI Λ_i of invoking the heuristic can be approximated as:

$$\Lambda_i \approx \mathbb{E}\left[(T_1 - T_i)|D_k|\,\middle|\, T_i < T_1\right]$$

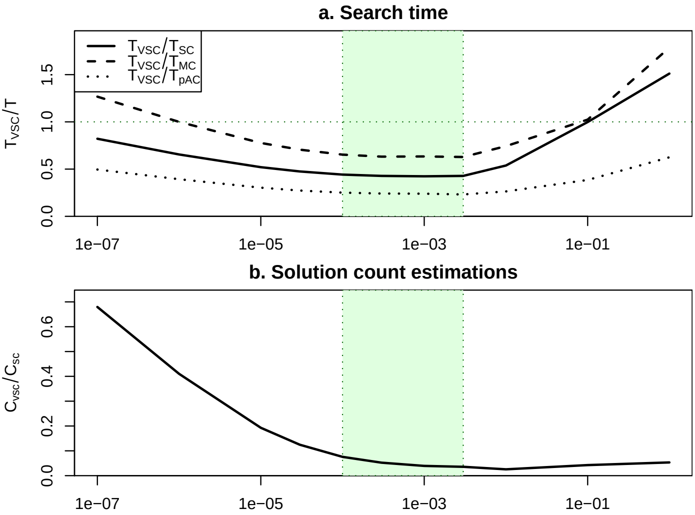
VOI of Solution Count Estimates

The net VOI V of estimating a solution count can be approximated as:

$$V \propto |D_k| e^{-\nu} \sum_{n=n_{\max}}^{\infty} \left(\frac{1}{n_{\max}} - \frac{1}{n}\right) \frac{\nu^n}{n!} - \gamma$$

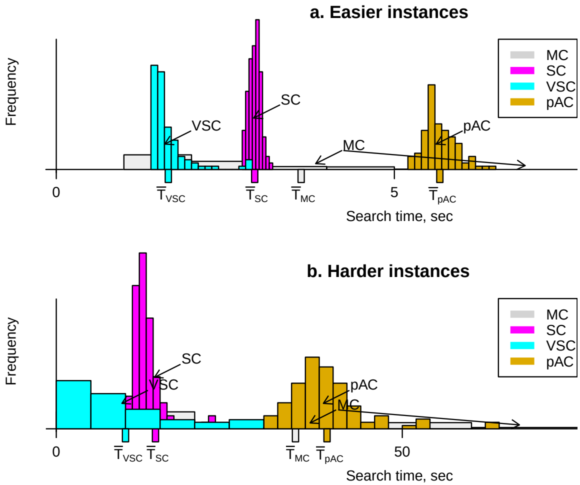
where the constant γ depends on the search algorithm and the heuristic, rather than on the CSP instance, and can be learned offline.

Benchmarks



14 CSP Solver Competion 2005 Benchmarks
solved for $\gamma \in \{0, 10^{-7}, 10^{-6}, \ldots, 1\}$

Random Instances



100 Model RB Random Instances

Generalized Sudoku

- Real-world problem instances often have much more structure than random instances generated according to Model RB.
- We repeated the experiments on randomly generated Generalized Sudoku instances— a highly structured domain.
- Relative performance on Generalized Sudoku was similar to Model RB.

Outline

MCTS

Monte **C**arlo **T**ree **S**earch helps in large search spaces.At each node:

- Repeats:
  1. **Selection:** select an action to explore.
  2. **Simulation:** simulates a rollout until a goal is reached.
  3. **Backpropagation:** updates the action value.
- Selects the best action.

Adaptive Generally, MCTS samples ‘good’ moves more frequently, but sometimes **explores** new directions.

Multi-armed Bandit Problem and UCB

Multi-armed Bandit Problem:

- We are given a set of K arms.
- Each arm can be pulled multiple times.
- The reward is drawn from an **unknown** (but normally *stationary* and *bounded*) distribution.
- The **total reward** must be maximized.

UCB is near-optimal for MAB — solves *exploration/exploitation* tradeoff.

- pulls an arm that maximizes **U**pper **C**onfidence **B**ound: $b_i = \overline{X}_i + \sqrt{\frac{c \log(n)}{n_i}}$
- the cumulative regret is $O(\log n)$.

UCT

UCT (**U**pper **C**onfidence Bounds applied to **T**rees) is based on UCB.

- Adaptive MCTS.
- Applies the UCB selection scheme at each step of the rollout.
- Demonstrated good performance in Computer Go (MoGo, CrazyStone, Fuego, Pachi, ...) as well as in other domains.

However, the first step of a rollout is different:

- The purpose of MCTS is to choose an action with the greatest utility.
- Therefore, the **simple regret** must be minimized.

Upper Bounds on Value of Information

Assuming that:

1. Samples are i.i.d. given the value of the arm.
2. The expectation of a selection in a belief state is equal to the sample mean.

Upper bounds on intrinsic VOI Λ_i^b of testing the ith arm N times are (based on Hoeffding inequality):

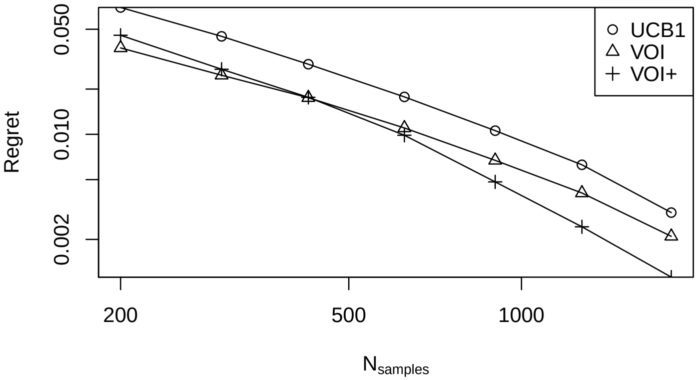
$$\Lambda_\alpha^b < \frac{N\overline{X}_\beta^{n_\beta}}{n_\alpha + 1} \cdot 2\exp\left(-1.37(\overline{X}_\alpha^{n_\alpha} - \overline{X}_\beta^{n_\beta})^2 n_\alpha\right)$$

$$\Lambda_{i|i\neq\alpha}^b < \frac{N(1 - \overline{X}_\alpha^{n_\alpha})}{n_i + 1} \cdot 2\exp\left(-1.37(\overline{X}_\alpha^{n_\alpha} - \overline{X}_i^{n_i})^2 n_i\right)$$

Tighter bounds can be obtained (see the paper).

VOI-based Sampling in Bernoulli Selection Problem

25 arms, 10000 trials:



UCB1 is always worse than VOI-aware policies (VOI, VOI+).

Sampling in Trees

- Hybrid sampling scheme:
  1. At the *root node*: sample based on the VOI estimate.
  2. At *non-root nodes*: sample using UCT.
- Stopping criterion: Assuming sample cost c is known, stop sampling when intrinsic VOI is less than $C = cN$:

$$\frac{1}{N}\Lambda_\alpha^b \leq \frac{\overline{X}_\beta^{n_\beta}}{n_\alpha + 1} \Pr(\overline{X}_\alpha^{n_\alpha+N} \leq \overline{X}_\beta^{n_\alpha}) \leq c$$

$$\frac{1}{N}\max_i \Lambda_i^b \leq \max_i \frac{(1-\overline{X}_\alpha^{n_\alpha})}{n_i + 1} \Pr(\overline{X}_i^{n_i+N} \geq \overline{X}_\alpha^{n_\alpha}) \leq c$$

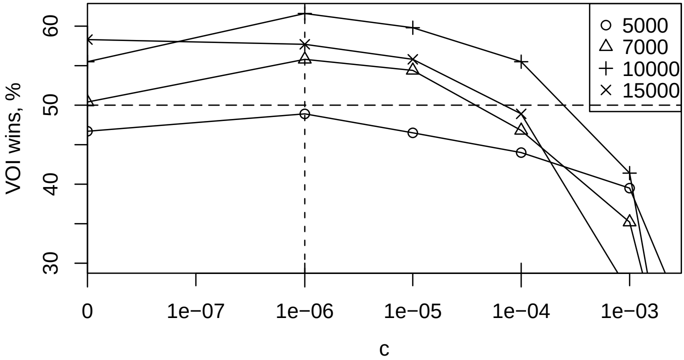
$$\forall i : i \neq \alpha$$

Sample Redistribution

- The VOI estimate assumes that the information is **discarded** between states.
- MCTS **re-uses rollouts** generated at earlier search states.

- Either incorporate ‘future’ influence into the VOI estimate (*non-trivial!*).
- Or behave myopically w.r.t. search tree depth:
  1. Estimate VOI as though the information is discarded.
  2. Stop early if the VOI is below a certain threshold.
  3. Save the unused sample budget for search in future states.
- The cost c of a sample is
  the VOI of increasing a future budget by one sample.

Playing Go Against UCT: Tuning the Sample Cost

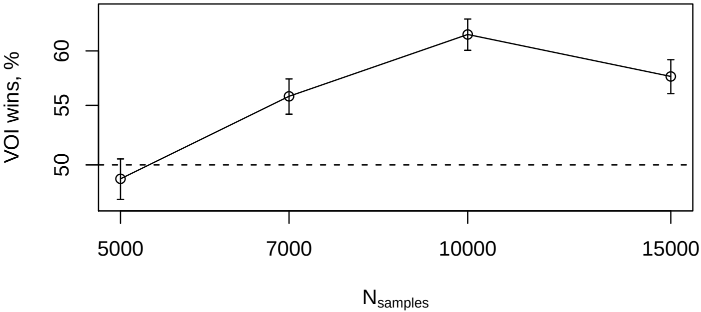


Best results for sample cost $c \approx 10^{-6}$:

winning rate of **64%** for 10000 samples per ply.

Playing Go Against UCT: Winning Rate vs. Number of Samples per Ply

Sample cost c fixed at 10^{-6}:



Best results for *intermediate* $N_{samples}$:

- When $N_{samples}$ is too low, poor moves are selected.
- When $N_{samples}$ is too high, the VOI of further sampling is low.

Outline

A^*

Apply all heuristics to initial state s_0
Insert s_0 into OPEN
while OPEN *not empty* **do**
    $n \leftarrow$ best node from OPEN
    if *Goal(n)* **then**
        return trace(n)
    foreach *child c of n* **do**
        Apply h_1 to c
        insert c into OPEN
    Insert n into CLOSED
return FAILURE

Lazy A^*

Apply all heuristics to initial state s_0
Insert s_0 into OPEN
while OPEN *not empty* **do**
    $n \leftarrow$ best node from OPEN
    if *Goal(n)* **then**
        return trace(n)
    if h_2 *was not applied to n* **then**
        Apply h_2 to n
        re-insert n into OPEN
        continue //next node in OPEN
    foreach *child c of n* **do**
        Apply h_1 to c
        insert c into OPEN
    Insert n into CLOSED
return FAILURE

Rational Lazy A^*

```
Apply all heuristics to initial state s0
Insert s0 into OPEN
while OPEN not empty do
    n ← best node from OPEN
    if Goal(n) then
        return trace(n)
    if h2 was not applied to n  and h2 is likely to pay off then
        Apply h2 to n
        re-insert n into OPEN
        continue        //next node in OPEN
    foreach child c of n do
        Apply h1 to c
        insert c into OPEN
    Insert n into CLOSED
return FAILURE
```

Rational Decision

- When does computing h_2 pay off?
- Suppose h_2 was computed for state s. Then either:
  1. s will be expanded later on anyway
  2. an optimal goal is found before s is expanded
- Computing h_2 pays off only in outcome 2 — call this "h_2 is helpful"

"It is difficult to make predictions, especially about the future"

— Yogi Berra / Neils Bohr

Towards a Rational Decision

- Myopic assumption: this is the *last* meta-level decision to be made, and henceforth the algorithm will act like lazy A^*.
- When a node re-emerges from the open list, compare the regret of computing h_2 as in lazy A^*, vs. just expanding the node.
- Note: if rational lazy A^* is indeed better than lazy A^*, the myopic assumption results in an upper bound on the regret.

	Compute h_2	Bypass h_2
h_2 helpful	0	$\sim b(s)t_1 + (b(s) - 1)t_2$
h_2 not helpful	$\sim t_2$	0

$b(s)$ denotes the number of successors of s

Disclaimer: for the exact analysis, see the paper

From Regret to Rational Decision

	Compute h_2	Bypass h_2
h_2 helpful	0	$\sim b(s)t_1 + (b(s) - 1)t_2$
h_2 not helpful	$\sim t_2$	0

- Suppose that the probability of h_2 being helpful is p_h
- Then the rational decision is to compute h_2 iff:

$$\frac{t_2}{t_1} < \frac{p_h b(s)}{1 - p_h b(s)}$$

Approximating p_h

$$\frac{t_2}{t_1} < \frac{p_h b(s)}{1 - p_h b(s)}$$

- We can directly measure t_1, t_2 and $b(s)$, but need to approximate p_h
- If s is a state at which h_2 was helpful, then we computed h_2 for s, but did not expand s. Denote the number of such states by B.
- Denote by A the number of states for which we computed h_2.
- We can use $\frac{A}{B}$ as an estimate for p_h
- To get an estimate which is more stable, we use a weighted average with k fictitious examples giving an estimate of p_{init}:

$$\frac{(A + p_{init} \cdot k)}{B + k}$$

- We use $p_{init} = 0.5$ and $k = 1000$

Empirical Evaluation: Weighted 15 Puzzle

- h_1 — weighted manhattan distance
- h_2 — lookahead to depth l with h_1

	Generated			Time		
l	A^*	LA^*	RLA^*	A^*	LA^*	RLA^*
2	1,206,535	1,206,535	1,309,574	0.707	0.820	0.842
4	1,066,851	1,066,851	1,169,020	0.634	0.667	0.650
6	889,847	889,847	944,750	**0.588**	0.533	0.464
8	740,464	740,464	793,126	0.648	**0.527**	0.377
10	611,975	611,975	889,220	0.843	0.671	**0.371**
12	**454,130**	**454,130**	807,846	0.927	0.769	0.429

Empirical Evaluation: Planning Domains

- h_{LA} — admissible landmarks
- $h_{\text{LM-CUT}}$ — landmark cut

		623 Commonly Solved		
Alg	Solved	Time (GM)	Expanded	Generated
h_{LA}	698	1.18	183,320,267	1,184,443,684
$h_{\text{LM-CUT}}$	697	0.98	23,797,219	114,315,382
max	722	0.98	**22,774,804**	**108,132,460**
selmax	747	0.89	54,557,689	193,980,693
LA^*	747	0.79	**22,790,804**	**108,201,244**
RLA^*	**750**	**0.77**	25,742,262	110,935,698

- RLA^* solves the most problems, and is fastest on average

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