Chapter 03
Problem Solving

Instructor

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Instructor's Information

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Acknowledgment

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- **Prof. Stuart Russell and Peter Norvig**: They are currently from University of California, Berkeley. They are also the author of the book "Artificial Intelligence: A Modern Approach", which is used as the textbook for the course
- **Prof. Tom Lenaerts**, from Université Libre de Bruxelles

Outline

- Problem-solving agents
  - A kind of goal-based agent
- Problem types
  - Single state (fully observable)
  - Search with partial information
- Problem formulation
  - Example problems
- Basic search algorithms
  - Uninformed

Problem-solving agent

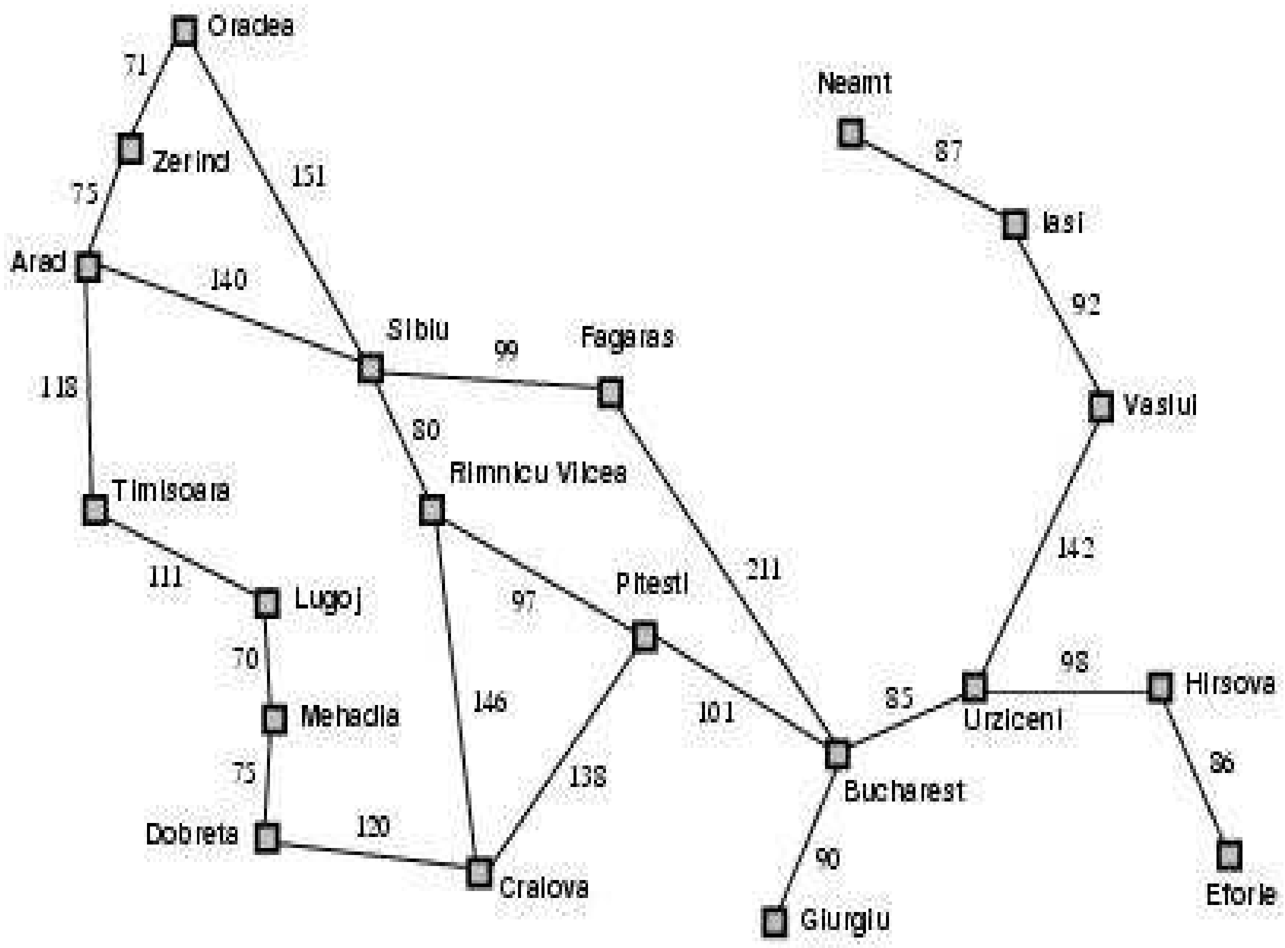
- Four general steps in problem solving:
  - Goal formulation
    - ✓ What are the successful world states
  - Problem formulation
    - ✓ What actions and states to consider to give the goal
  - Search
    - ✓ Determine the possible sequence of actions that lead to the states of known values and then choosing the best sequence.
  - Execute
    - ✓ Give the solution perform the actions.

Problem-solving agent

function SIMPLE-PROBLEM-SOLVING-AGENT(*percept*) **return** an action
    static: *seq*, an action sequence
        state, some description of the current world state
        goal, a goal
        problem, a problem formulation

    state ← UPDATE-STATE(*state*, *percept*)
    if *seq* is empty **then**
        goal ← FORMULATE-GOAL(*state*)
        problem ← FORMULATE-PROBLEM(*state*,*goal*)
        seq ← SEARCH(*problem*)
    action ← FIRST(*seq*)
    seq ← REST(*seq*)
    return *action*

Example: Romania



Example: Romania

- On holiday in Romania; currently in Arad
  - Flight leaves tomorrow from Bucharest
- Formulate goal
  - Be in Bucharest
- Formulate problem
  - States: various cities
  - Actions: drive between cities
- Find solution
  - Sequence of cities; e.g. Arad, Sibiu, Fagaras, Bucharest, …

Problem types

- Deterministic, fully observable $\Rightarrow$ *single state problem*
  - Agent knows exactly which state it will be in; solution is a sequence.
- Partial knowledge of states and actions:
  - Non-observable $\Rightarrow$ *sensorless or conformant problem*
    - ✓ Agent may have no idea where it is; solution (if any) is a sequence.
  - Nondeterministic and/or partially observable $\Rightarrow$ *contingency problem*
    - ✓ Percepts provide *new* information about current state; solution is a tree or policy; often interleave search and execution.
  - Unknown state space $\Rightarrow$ *exploration problem* ("online")
    - ✓ When states and actions of the environment are unknown.

Problem formulation

- A problem is defined by:
  - An initial state, e.g. *Arad*
  - Successor function *S(X)*= set of action-state pairs
    - e.g. *S(Arad)={<Arad → Zerind, Zerind>,…}*

  intial state + successor function = state space

  - Goal test, can be
    - Explicit, e.g. *x='at bucharest'*
    - Implicit, e.g. *checkmate(x)*
  - Path cost (additive)
    - e.g. sum of distances, number of actions executed, …
    - *c(x,a,y)* is the step cost, assumed to be >= 0

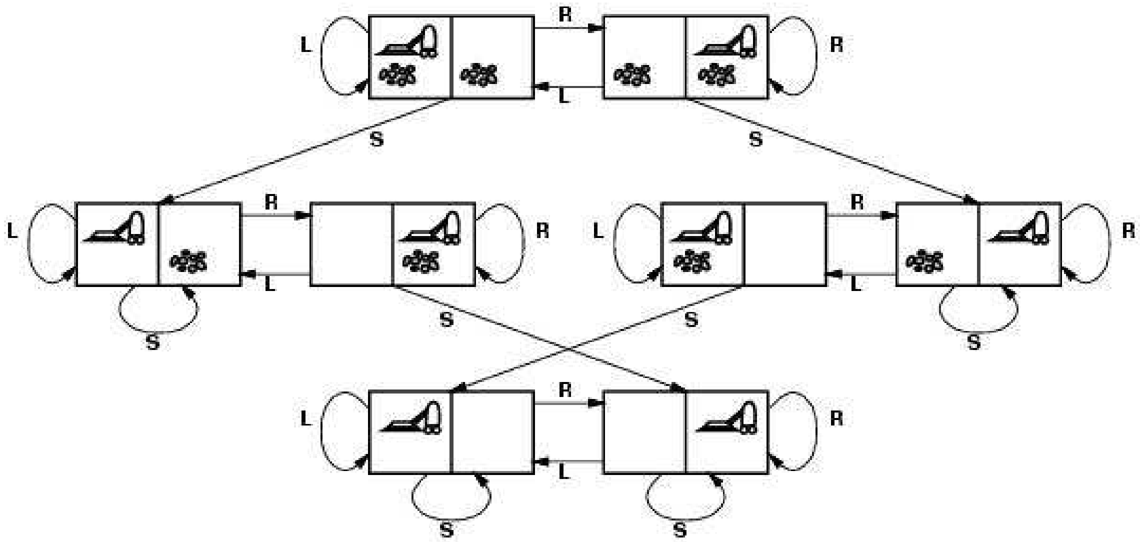
A solution is a sequence of actions from initial to goal state.
Optimal solution has the lowest path cost.

Selecting a state space

- Real world is absurdly complex.

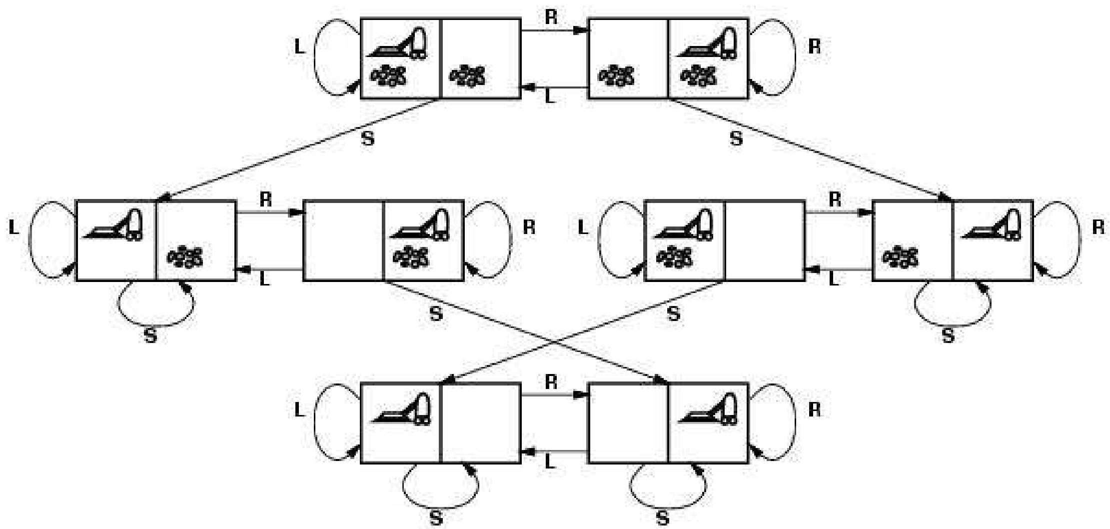
  State space must be *abstracted* for problem solving.
- (Abstract) state = set of real states.
- (Abstract) action = complex combination of real actions.
  - e.g. Arad →Zerind represents a complex set of possible routes, detours, rest stops, etc.
  - The abstraction is valid if the path between two states is reflected in the real world.
- (Abstract) solution = set of real paths that are solutions in the real world.
- Each abstract action should be "easier" than the real problem.

Example: vacuum world



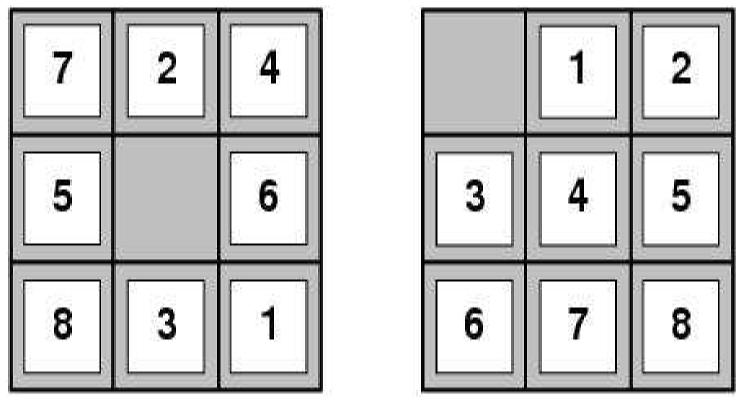
- ❖ States??
- ❖ Initial state??
- ❖ Actions??
- ❖ Goal test??
- ❖ Path cost??

Example: vacuum world



- States?? two locations with or without dirt: 2 x 2^2=8 states.
- Initial state?? Any state can be initial
- Actions?? {*Left*, *Right*, *Suck*}
- Goal test?? Check whether squares are clean.
- Path cost?? Number of actions to reach goal.

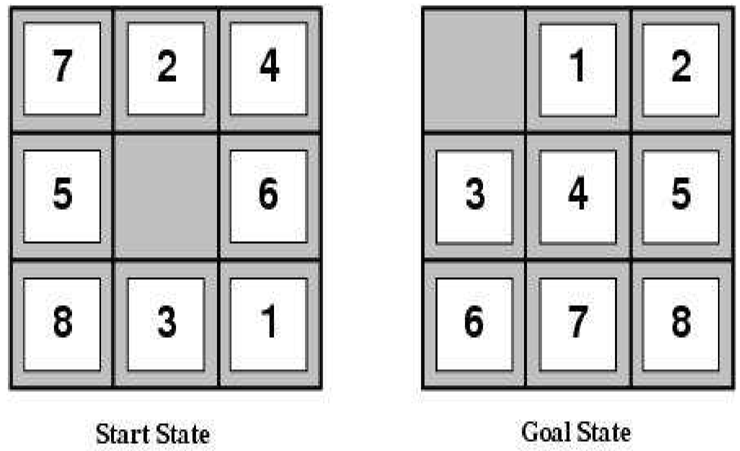
Example: 8-puzzle



Start State Goal State

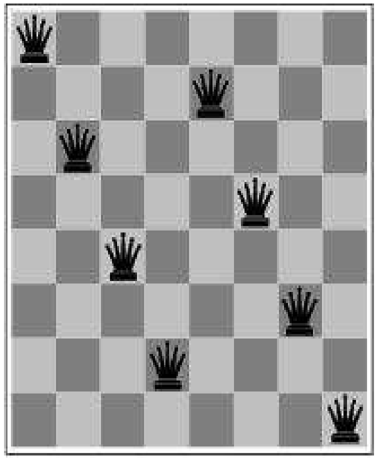
- States??
- Initial state??
- Actions??
- Goal test??
- Path cost??

Example: 8-puzzle



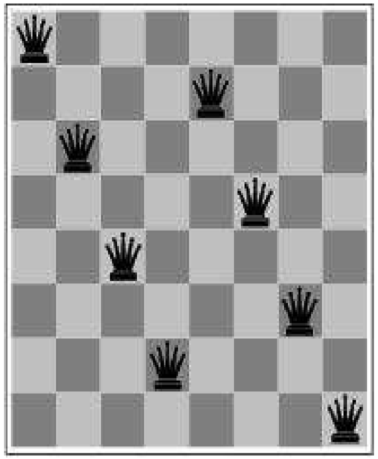
- States?? Integer location of each tile
- Initial state?? Any state can be initial
- Actions?? {*Left*, *Right*, *Up*, *Down*}
- Goal test?? Check whether goal configuration is reached
- Path cost?? Number of actions to reach goal

Example: 8-queens problem



- States??
- Initial state??
- Actions??
- Goal test??
- Path cost??

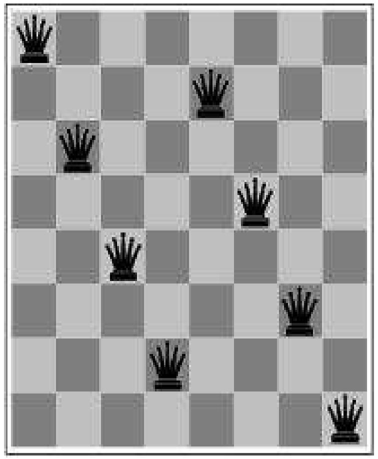
Example: 8-queens problem



Incremental formulation vs. complete-state formulation

- States??
- Initial state??
- Actions??
- Goal test??
- Path cost??

Example: 8-queens problem

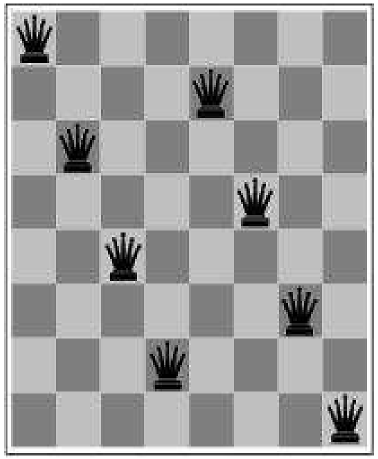


Incremental formulation

- States?? Any arrangement of 0 to 8 queens on the board
- Initial state?? No queens
- Actions?? Add queen in empty square
- Goal test?? 8 queens on board and none attacked
- Path cost?? None

3×10^{14} possible sequences to investigate

Example: 8-queens problem

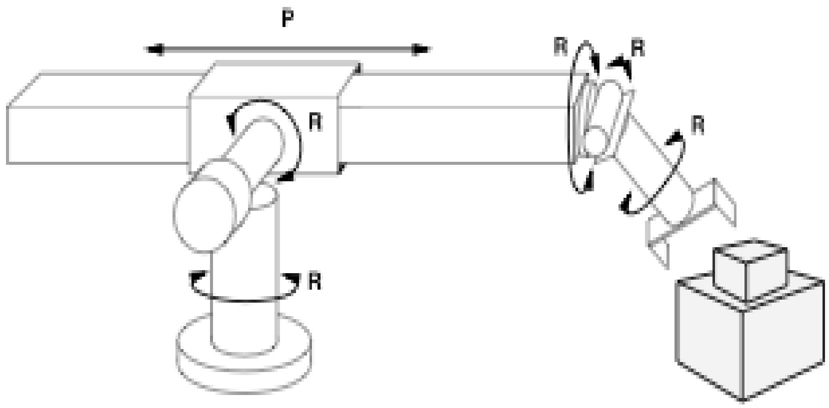


Incremental formulation (alternative)

- States?? n ($0 \leq n \leq 8$) queens on the board, one per column in the n leftmost columns with no queen attacking another.
- Actions?? Add queen in leftmost empty column such that is not attacking other queens

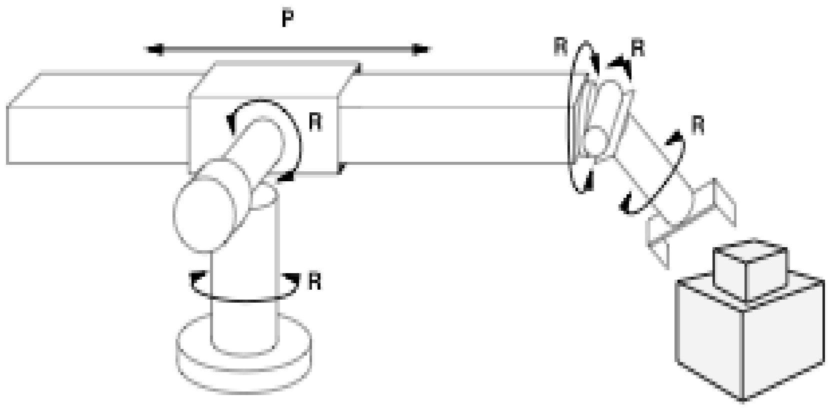
  2057 possible sequences to investigate; Yet makes no difference when n=100

Example: robot assembly



- States??
- Initial state??
- Actions??
- Goal test??
- Path cost??

Example: robot assembly



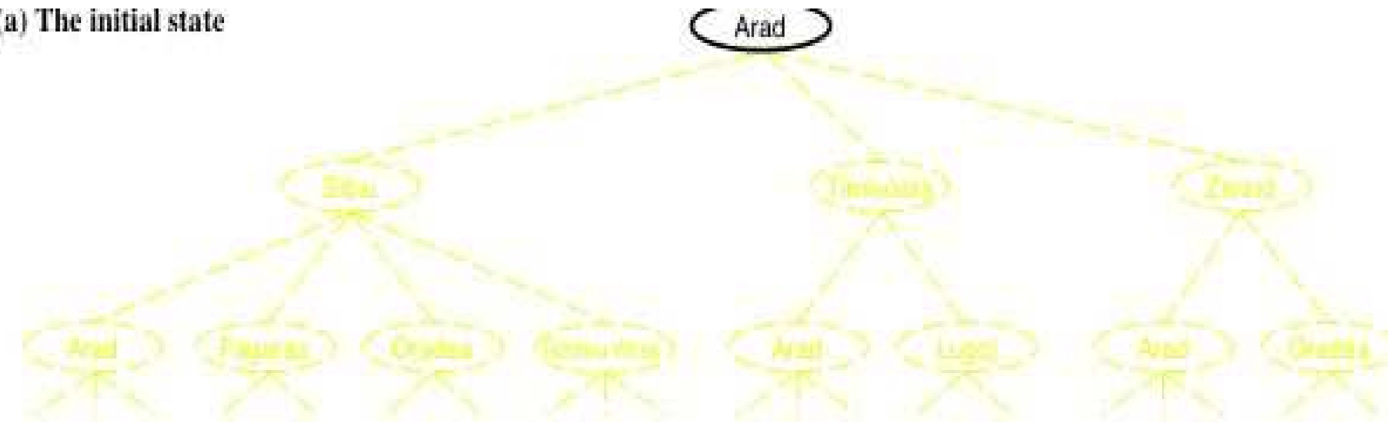
- States?? Real-valued coordinates of robot joint angles; parts of the object to be assembled.
- Initial state?? Any arm position and object configuration.
- Actions?? Continuous motion of robot joints
- Goal test?? Complete assembly (without robot)
- Path cost?? Time to execute

Basic search algorithms

- How do we find the solutions of previous problems?
  - Search the state space (remember complexity of space depends on state representation)
  - Here: search through *explicit tree generation*
    - ✓ ROOT= initial state.
    - ✓ Nodes and leafs generated through successor function.
  - In general search generates a graph (same state through multiple paths)

Simple tree search example

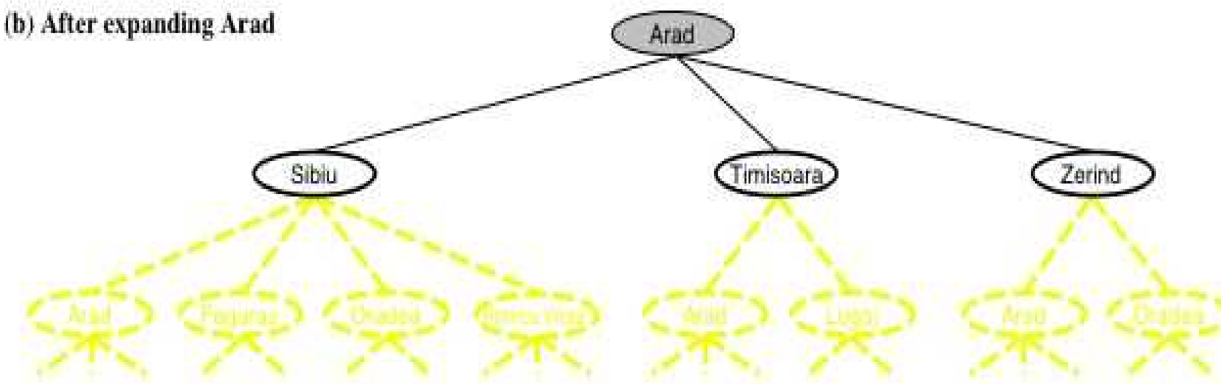
(a) The initial state



function TREE-SEARCH(*problem*, *strategy*) **return** a solution or failure
Initialize search tree to the *initial state* of the *problem*
do
if no candidates for expansion **then return** *failure*
choose leaf node for expansion according to *strategy*
if node contains goal state **then return** *solution*
else expand the node and add resulting nodes to the search tree
enddo

Simple tree search example

(b) After expanding Arad



function TREE-SEARCH(*problem*, *strategy*) **return** a solution or failure

Initialize search tree to the *initial state* of the *problem*

do

if no candidates for expansion **then return** *failure*

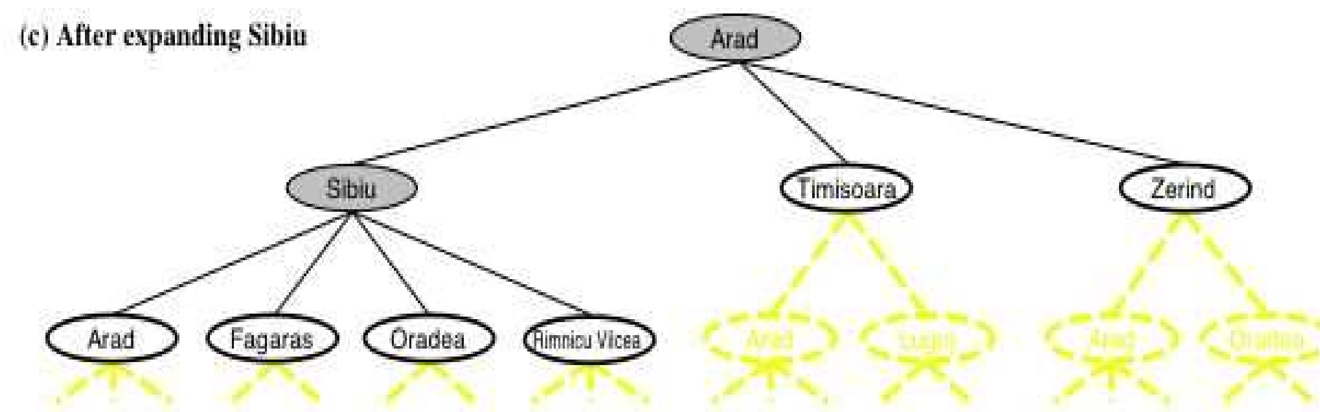
choose leaf node for expansion according to *strategy*

if node contains goal state **then return** *solution*

else expand the node and add resulting nodes to the search tree

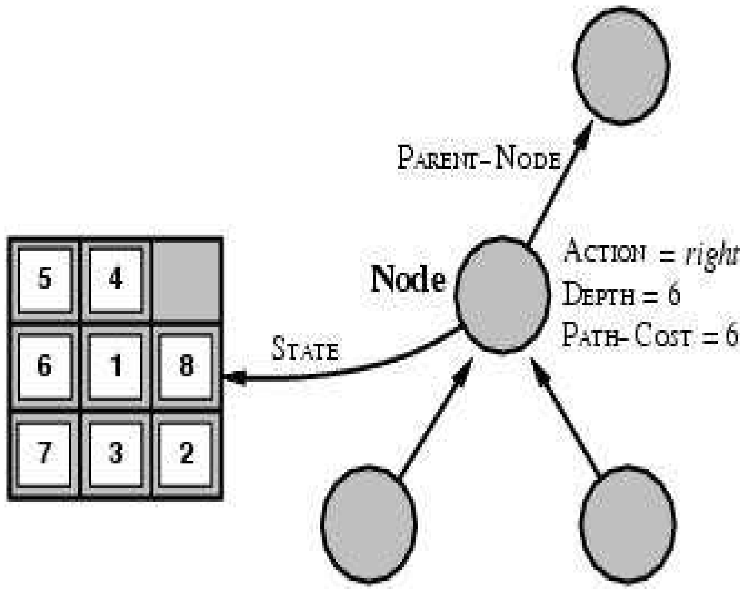
enddo

Simple tree search example



function TREE-SEARCH(*problem*, *strategy*) **return** a solution or failure
Initialize search tree to the *initial state* of the *problem*
do
if no candidates for expansion **then return** *failure*
choose leaf node for expansion according to *strategy* ← **Determines search process!!**
if node contains goal state **then return** *solution*
else expand the node and add resulting nodes to the search tree
enddo

State space vs. search tree



- A *state* is a (representation of) a physical configuration
- A *node* is a data structure belong to a search tree
  - A node has a parent, children, … and ncludes path cost, depth, …
  - Here *node= <state, parent-node, action, path-cost, depth>*
  - *FRINGE*= contains generated nodes which are not yet expanded.
    - ✓ White nodes with black outline

Tree search algorithm

function TREE-SEARCH(*problem*,*fringe*) **return** a solution or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
    loop do
        if EMPTY?(*fringe*) **then return** failure
        node ← REMOVE-FIRST(*fringe*)
        if GOAL-TEST[*problem*] applied to STATE[*node*] succeeds
                then return SOLUTION(*node*)
        fringe ← INSERT-ALL(EXPAND(*node*, *problem*), *fringe*)

Tree search algorithm (2)

function EXPAND(*node*,*problem*) **return** a set of nodes

successors ← the empty set

for each <*action*, *result*> **in** SUCCESSOR-FN**[***problem***]**(STATE[*node*]) **do**

s ← a new NODE

STATE[*s*] ← *result*

PARENT-NODE[*s*] ← *node*

ACTION[*s*] ← *action*

PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node*, *action*,*s*)

DEPTH[*s*] ← DEPTH[*node*]+1

add *s* to *successors*

return *successors*

Search strategies

- A strategy is defined by picking the order of node expansion.
- Problem-solving performance is measured in four ways:
  - Completeness
    - ✓ *Does it always find a solution if one exists?*
  - Optimality
    - ✓ *Does it always find the least-cost solution?*
  - Time Complexity
    - ✓ *Number of nodes generated/expanded?*
  - Space Complexity
    - ✓ *Number of nodes stored in memory during search?*

Search strategies

- Time and space complexity are measured in terms of problem difficulty defined by:
  - *b :maximum branching factor of the search tree*
  - *d :depth of the least-cost solution*
  - *m :maximum depth of the state space (may be ∞)*

Uninformed search strategies

- (a.k.a. blind search) = use only information available in problem definition.
  - When strategies can determine whether one non-goal state is better than another → informed search.
- Categories defined by expansion algorithm:
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search.
  - Bidirectional search

Breadth First Search, an example

- Breadth-First-Search: BF-Search
- Expand *shallowest* unexpanded node
- Implementation: *fringe* is a FIFO queue

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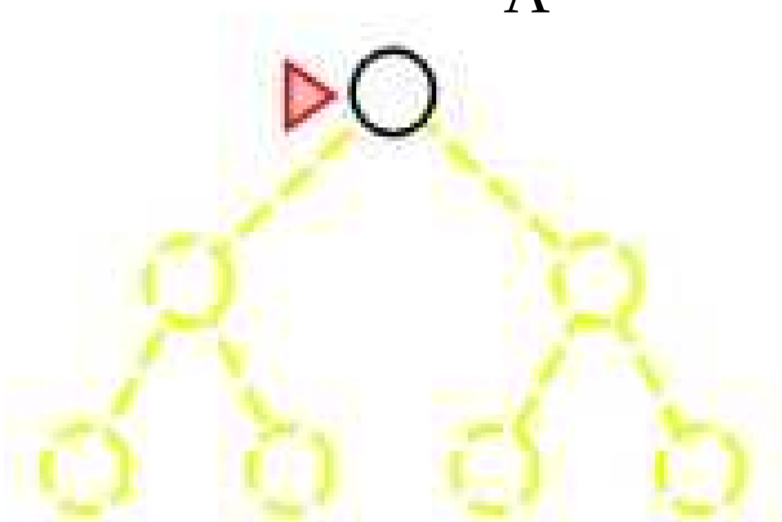

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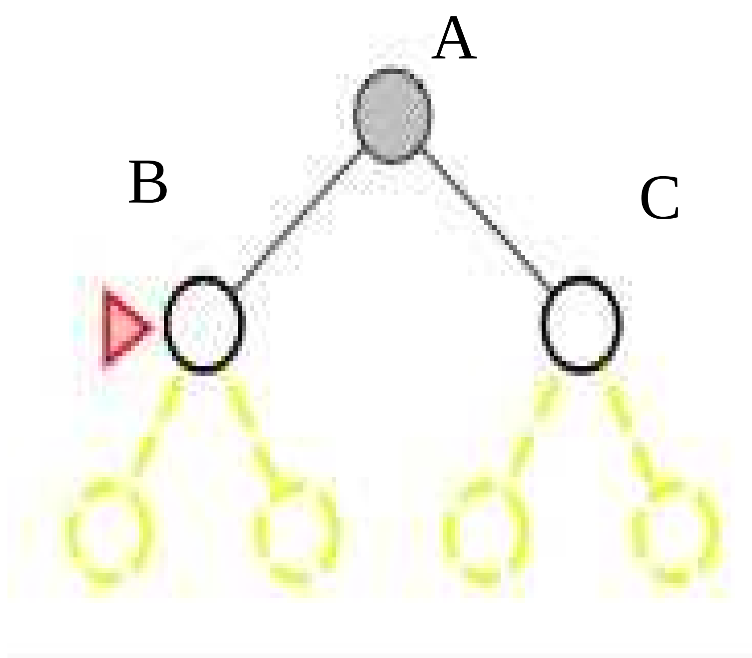

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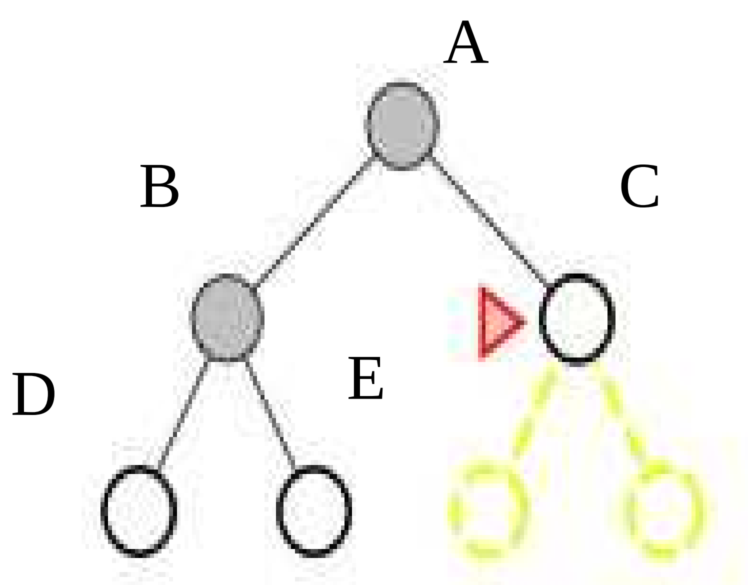

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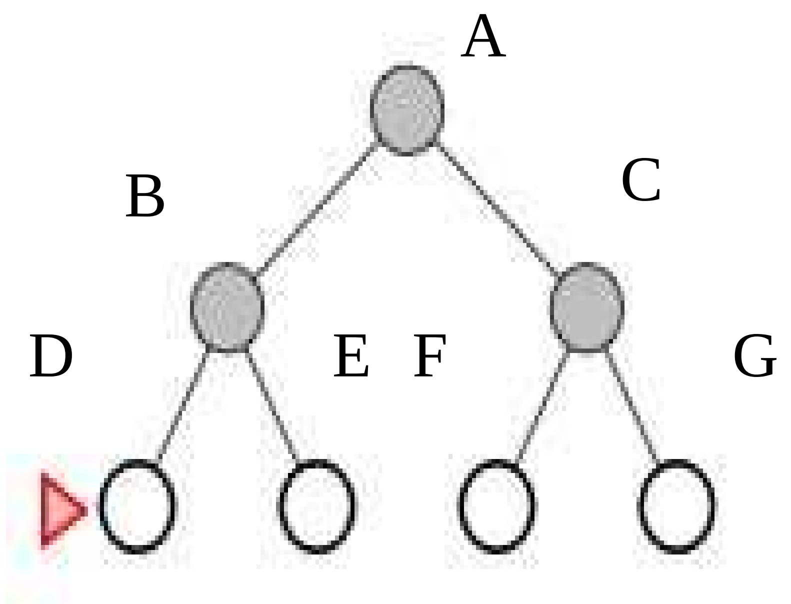


Breadth First Search; evaluation

- Completeness:
  - *Does it always find a solution if one exists?*
  - YES
    - If shallowest goal node is at some finite depth d
    - Condition: If *b* is finite
      - (maximum num. Of succ. nodes is finite)

BF-search; evaluation

- Completeness:
  - YES (if *b* is finite)
- Time complexity:
  - Assume a state space where every state has *b* successors.
    - ✓ root has *b* successors, each node at the next level has again b successors (total b^2), …
    - ✓ Assume solution is at depth *d*
    - ✓ Worst case; expand all but the last node at depth *d*
    - ✓ Total numb. of nodes generated:

$$b + b^2 + b^3 + \ldots + b^d + (b^{d+1} - b) = O(b^{d+1})$$

BF-search; evaluation

- Completeness:
  - YES (if *b* is finite)
- Time complexity:
  - Total numb. of nodes generated:

$$b + b^2 + b^3 + ... + b^d + (b^{d+1} - b) = O(b^{d+1})$$

- Space complexity:
  - Idem if each node is retained in memory

BF-search; evaluation

- Completeness:
  - YES (if *b* is finite)
- Time complexity:
  - Total numb. of nodes generated:

$$b + b^2 + b^3 + ... + b^d + (b^{d+1} - b) = O(b^{d+1})$$

- Space complexity:
  - Idem if each node is retained in memory
- Optimality:
  - *Does it always find the least-cost solution?*
  - In general YES
    - ✓ unless actions have different cost.

BF-search; evaluation

- Two lessons:
  - Memory requirements are a bigger problem than its execution time.
  - Exponential complexity search problems cannot be solved by uninformed search methods for any but the smallest instances.

DEPTH2	NODES	TIME	MEMORY
2	1100	0.11 seconds	1 megabyte
4	111100	11 seconds	106 megabytes
6	10^7	19 minutes	10 gigabytes
8	10^9	31 hours	1 terabyte
10	10^{11}	129 days	101 terabytes
12	10^{13}	35 years	10 petabytes
14	10^{15}	3523 years	1 exabyte

Uniform-cost search

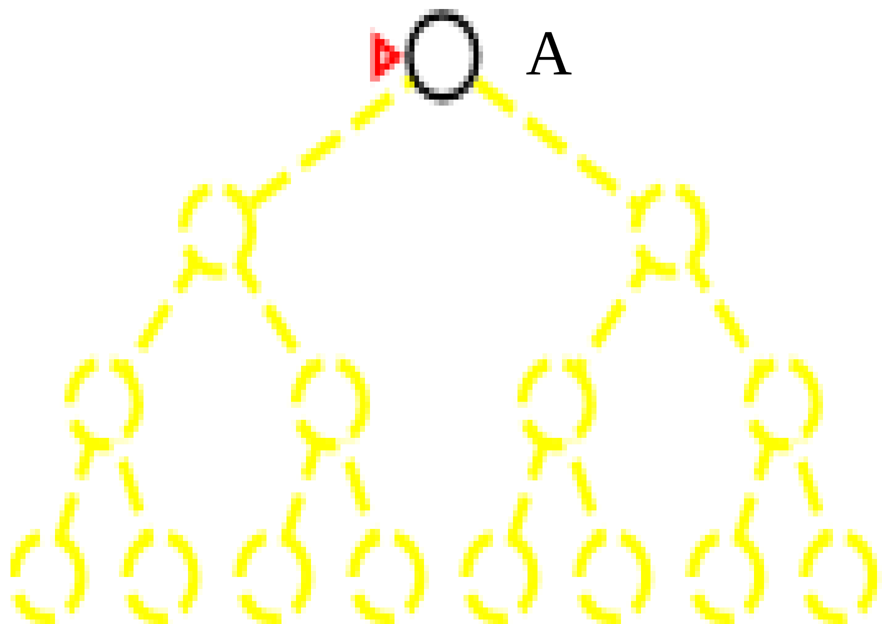
- ❖ Extension of BF-search:
  - Expand node with *lowest path cost*
- ❖ Implementation: *fringe* = queue ordered by path cost.
- ❖ UC-search is the same as BF-search when all step-costs are equal.

Uniform-cost search

- Completeness:
  - YES, if step-cost > ε (small positive constant)
- Time complexity:
  - Assume C* the cost of the optimal solution.
  - Assume that every action costs at least ε
  - Worst-case: $O(b^{C^*/\varepsilon})$
- Space complexity:
  - Idem to time complexity
- Optimality:
  - nodes expanded in order of increasing path cost.
  - YES, if complete.

Depth-First-Search, an example

- ❖ Depth-First-Search: DF-Search
- ❖ Expand *deepest* unexpanded node
- ❖ Implementation: *fringe* is a LIFO queue (=stack)



DF-search, an example

- Expand *deepest* unexpanded node
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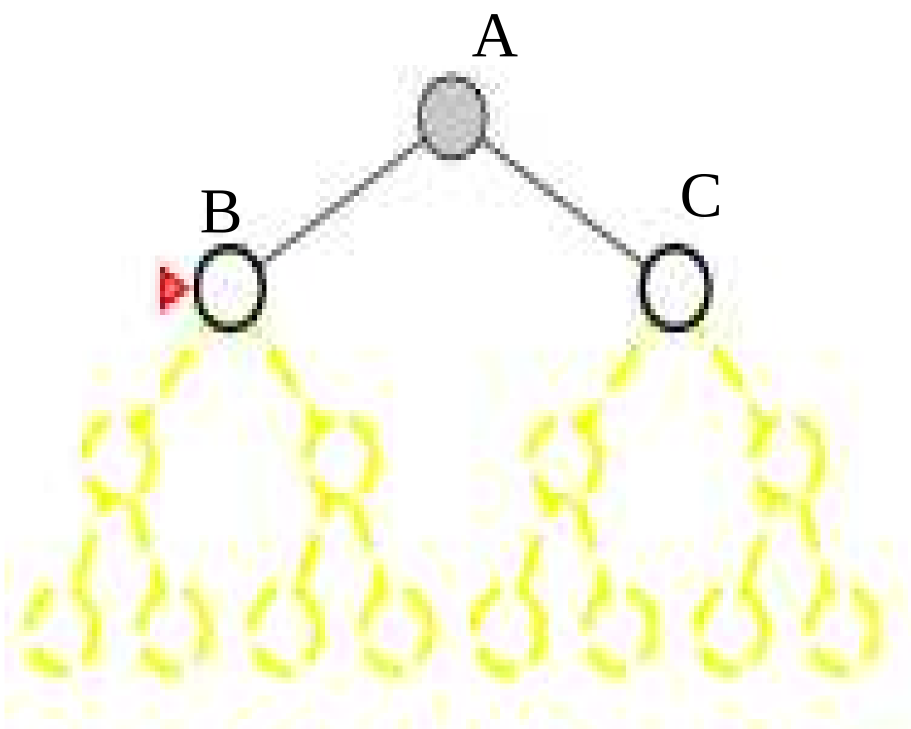

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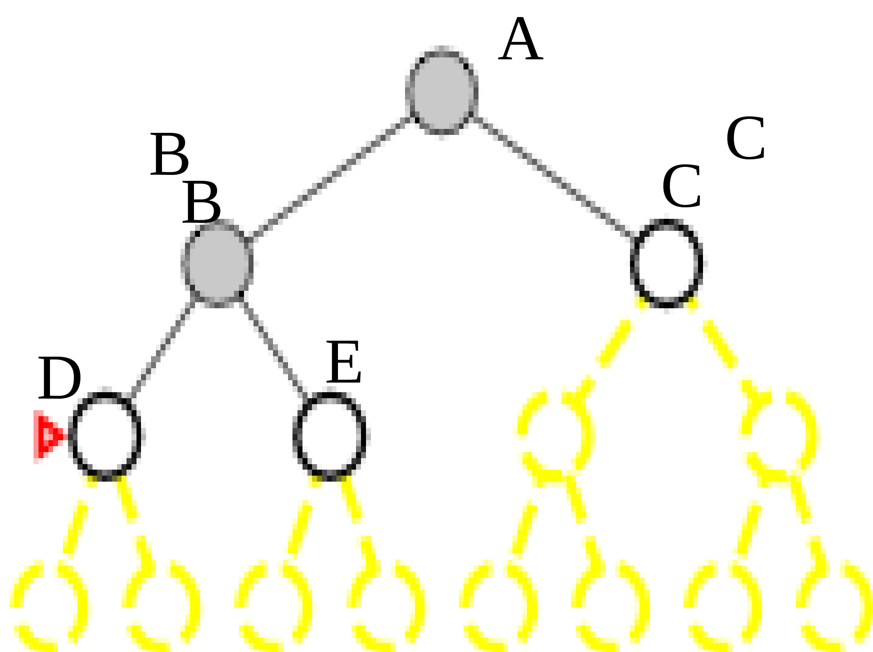

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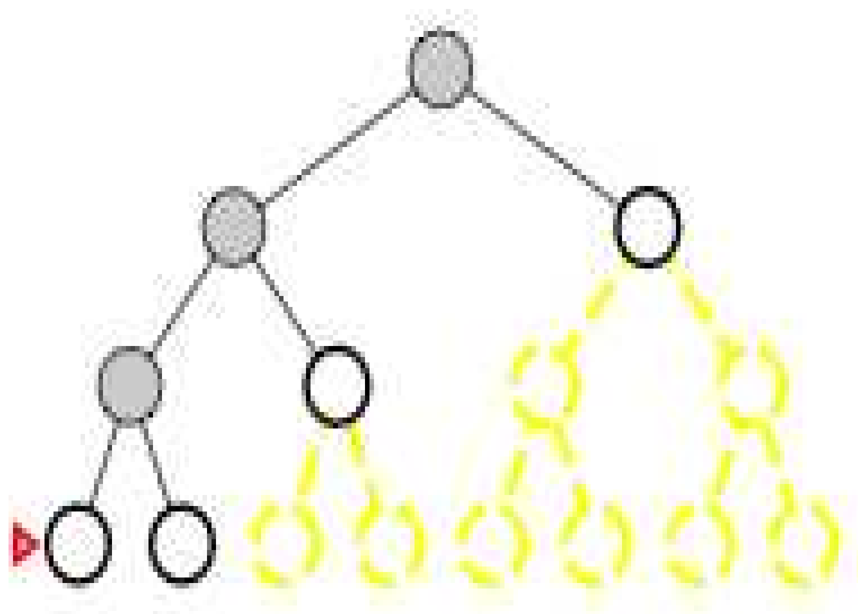

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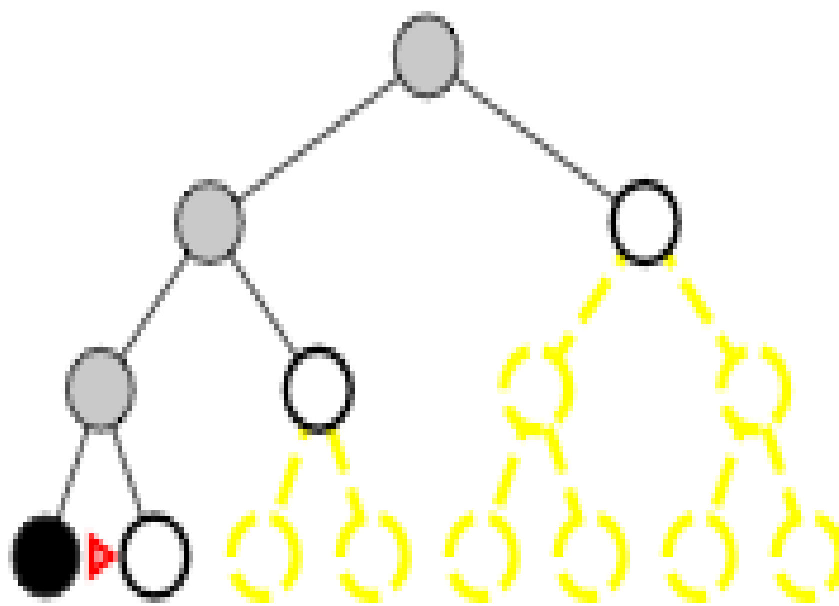

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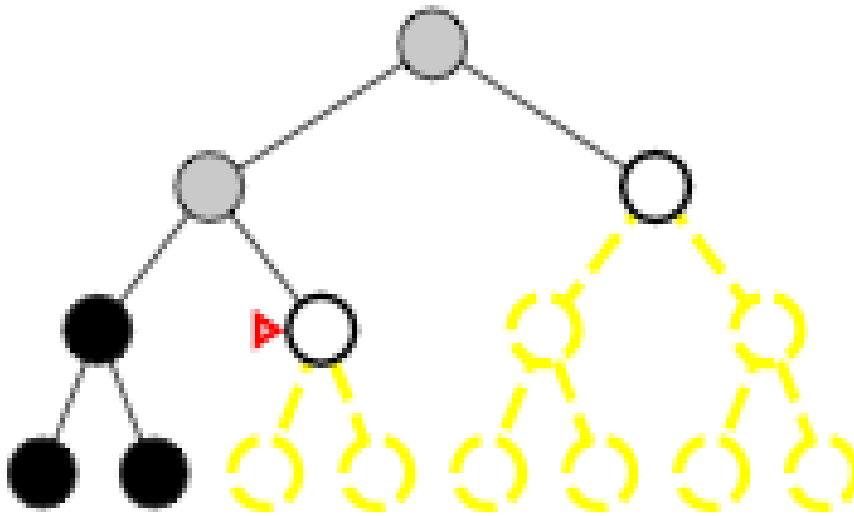

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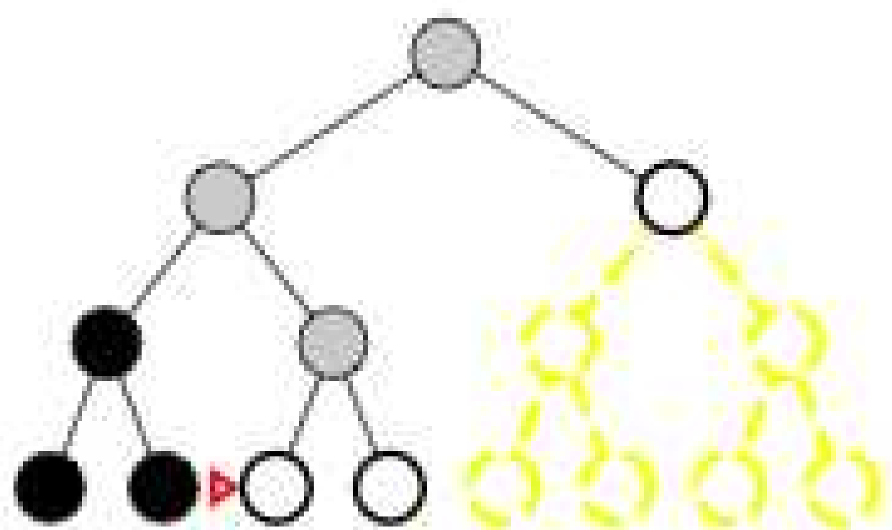

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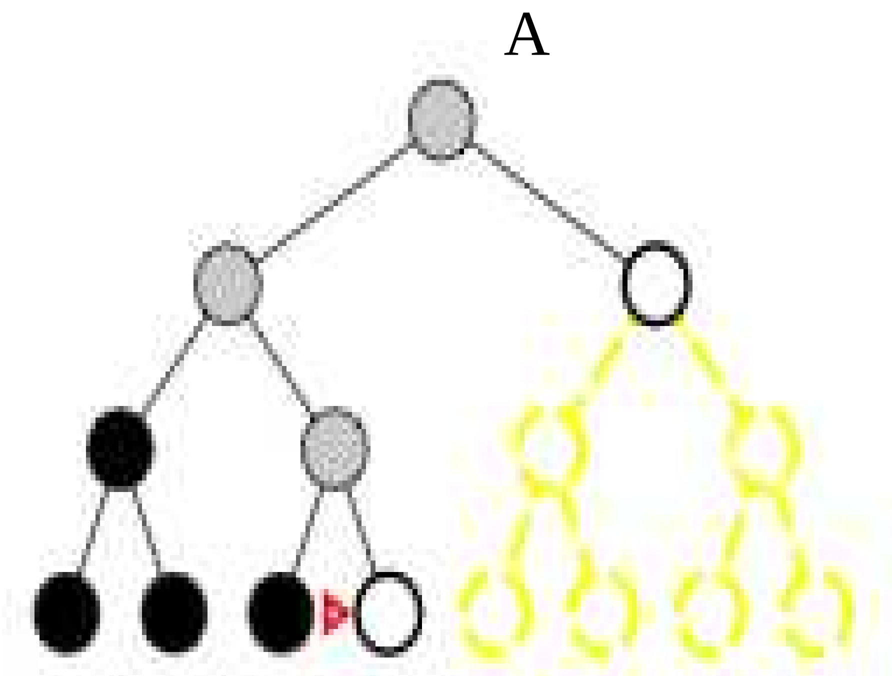

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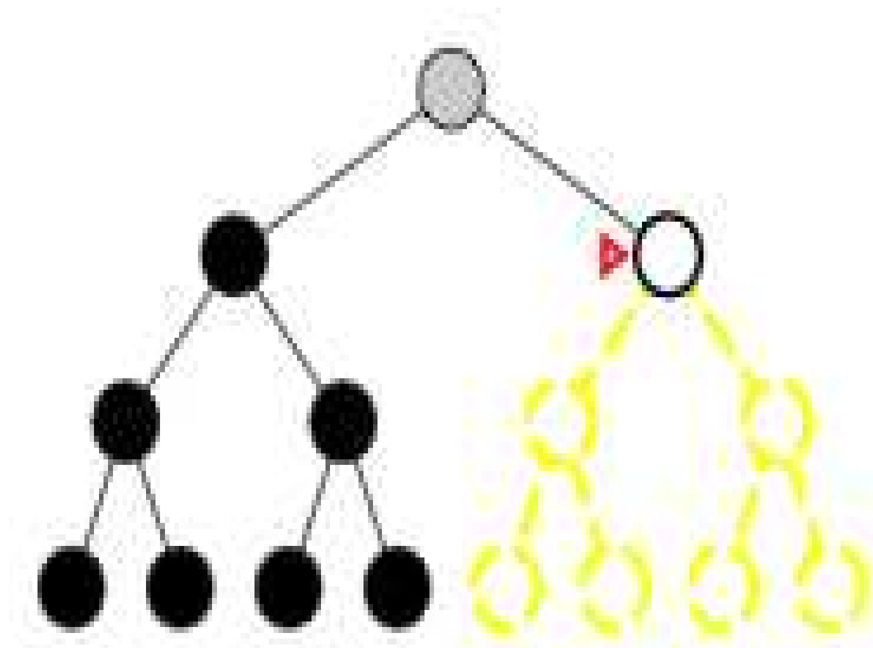

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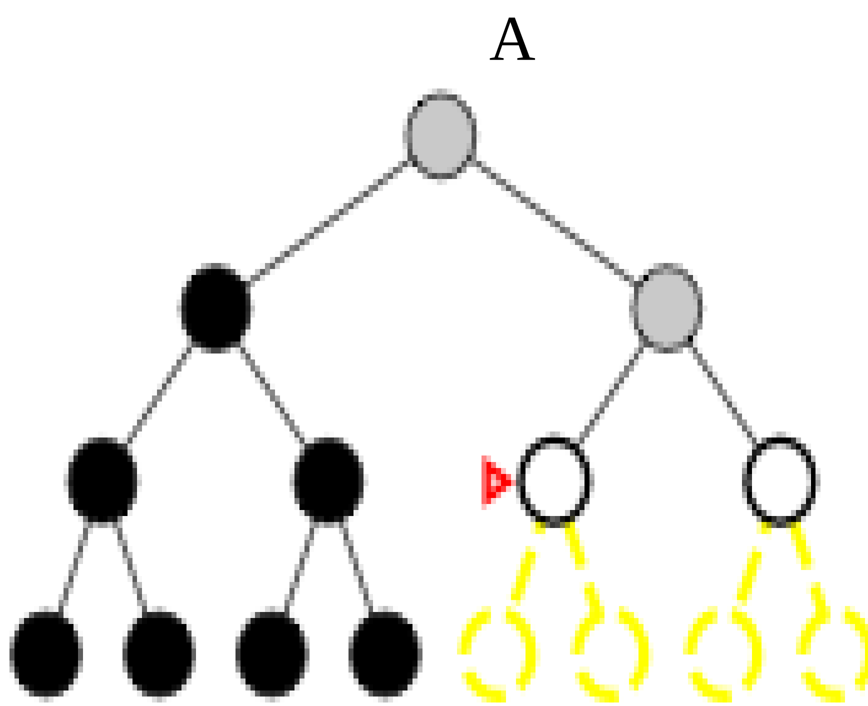

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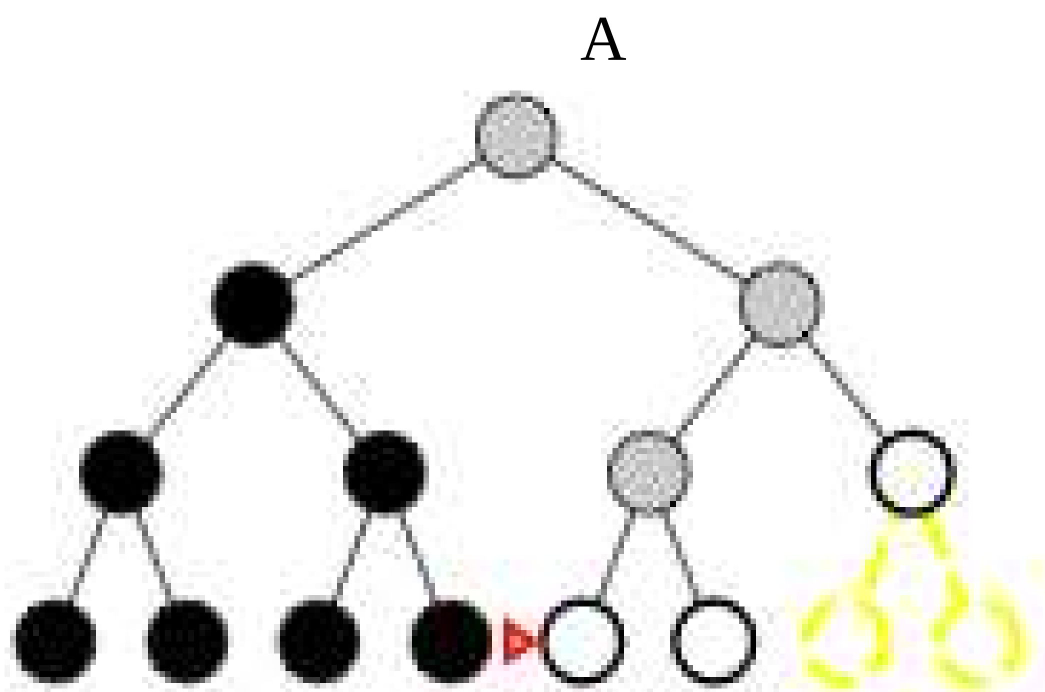

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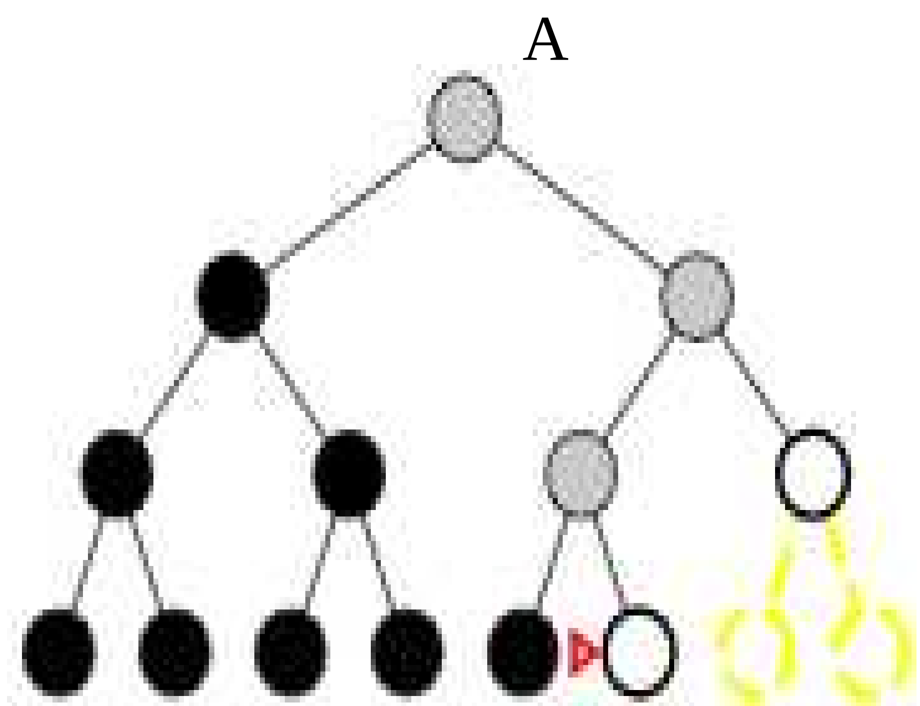


DF-search; evaluation

- Completeness;
  - *Does it always find a solution if one exists?*
  - NO
    - ✓ *unless* search space is finite and no loops are possible.

DF-search; evaluation

- Completeness;
  - NO unless search space is finite.
- Time complexity; $O(b^m)$
  - Terrible if *m* is much larger than *d* (depth of optimal solution)
  - But if many solutions, then faster than BF-search

DF-search; evaluation

- Completeness;
  - NO unless search space is finite.
- Time complexity; $O(b^m)$
- Space complexity; $O(bm+1)$
  - Backtracking search uses even less memory
    - ✓ One successor instead of all *b*.

DF-search; evaluation

- Completeness;
  - NO unless search space is finite.
- Time complexity; $O(b^m)$
- Space complexity; $O(bm+1)$
- Optimality; No
  - Same issues as completeness
  - Assume node J and C contain goal states

Depth-limited search

- Is DF-search with depth limit l.
  - i.e. nodes at depth l have no successors.
  - Problem knowledge can be used
- Solves the infinite-path problem.
- If $l < d$ then incompleteness results.
- If $l > d$ then not optimal.
- Time complexity: $O(b^l)$
- Space complexity: $O(bl)$

Depth-limited algorithm

function DEPTH-LIMITED-SEARCH(*problem*,*limit*) **return** a solution or failure/cutoff

return RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[*problem*]),*problem*,*limit*)

function RECURSIVE-DLS(*node*, *problem*, *limit*) **return** a solution or failure/cutoff

cutoff_occurred? ← false

if GOAL-TEST[*problem*](STATE[*node*]) **then return** SOLUTION(*node*)

else if DEPTH[*node*] == *limit* then return *cutoff*

else for each *successor* **in** EXPAND(*node*, *problem*) **do**

result ← RECURSIVE-DLS(*successor*, *problem*, *limit*)

if *result* == *cutoff* **then** *cutoff_occurred?* ← true

else if *result* ≠ *failure* **then return** *result*

if *cutoff_occurred?* **then return** *cutoff* **else return** failure

Iterative deepening search

- What?
  - A general strategy to find best depth limit *l*.
    - Goals is found at depth *d*, the depth of the shallowest goal-node.
  - Often used in combination with DF-search
- Combines benefits of DF- en BF-search

Iterative deepening search

function ITERATIVE_DEEPENING_SEARCH(*problem*) **return** a solution or failure

inputs: *problem*

for *depth* $\leftarrow$ 0 to ∞ **do**

result $\leftarrow$ DEPTH-LIMITED_SEARCH(*problem*, *depth*)

if *result* $\neq$ *cuttoff* **then return** *result*

ID-search, example

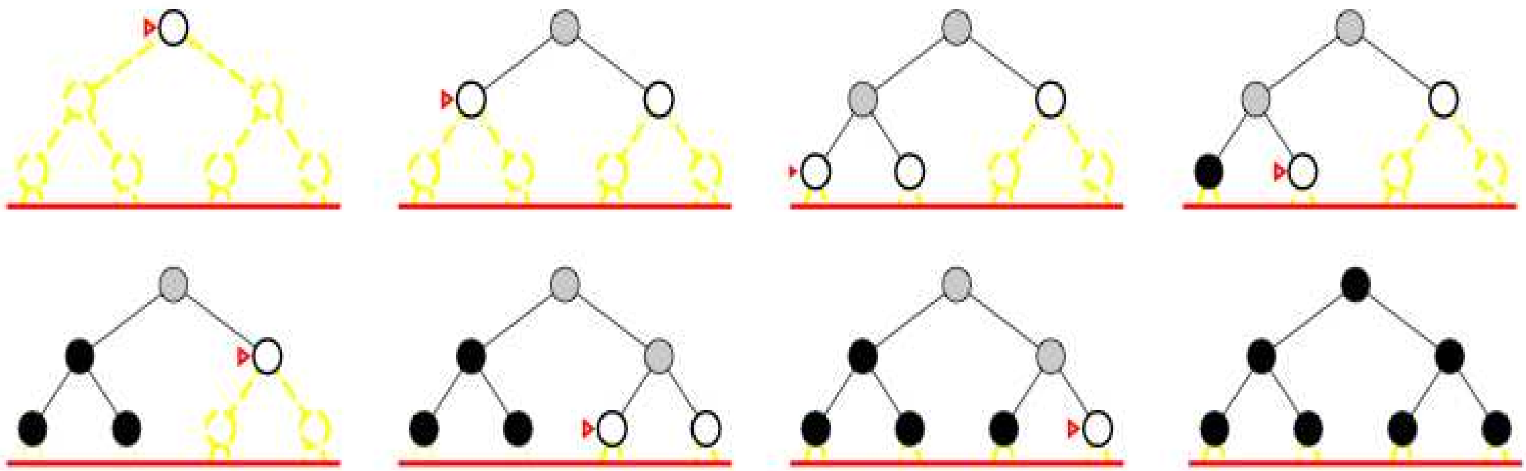
- Limit=0

ID-search, example

- Limit=1

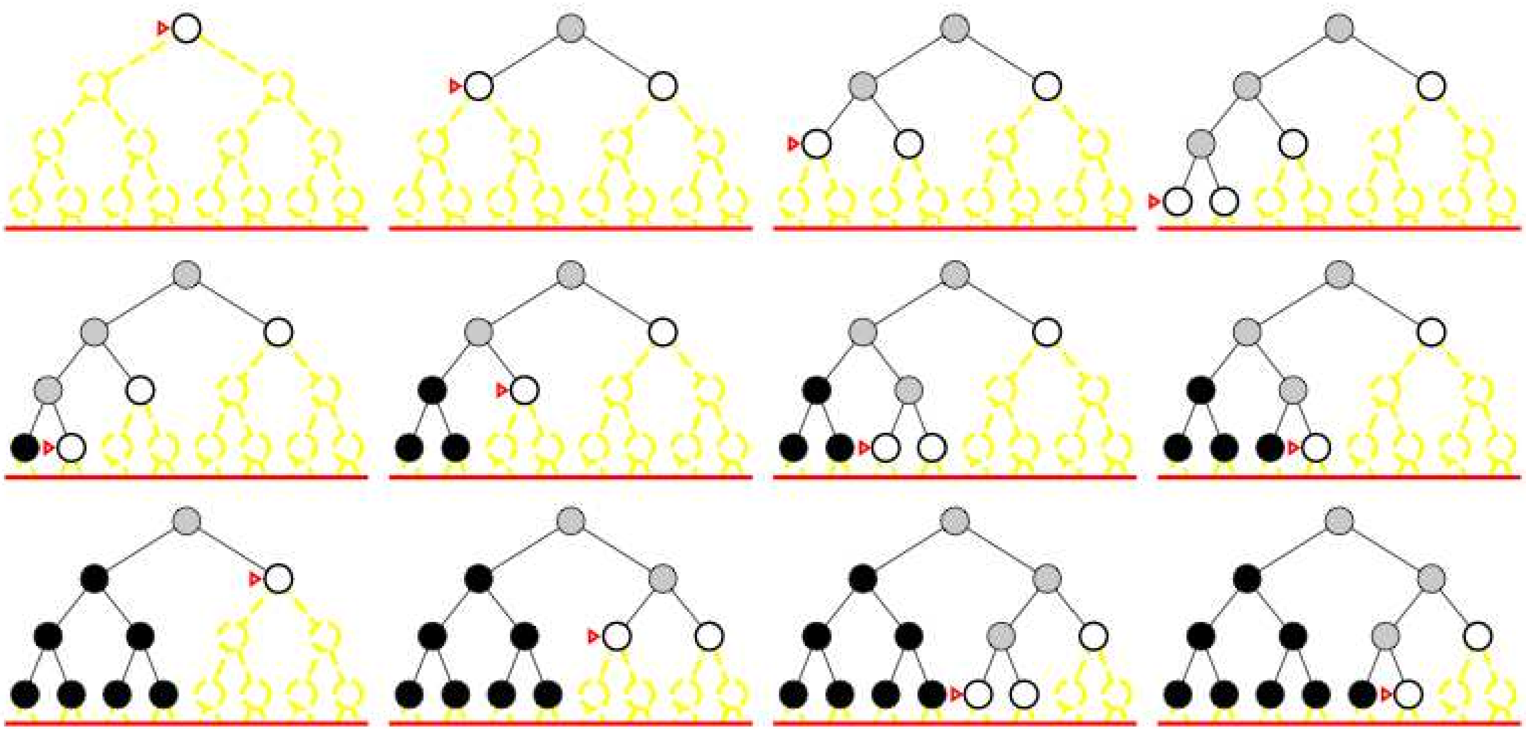
ID-search, example

❖ Limit=2



ID-search, example

❖ Limit=3



ID search, evaluation

- Completeness:
  - YES (no infinite paths)

ID search, evaluation

- Completeness:
  - YES (no infinite paths)
- Time complexity:
  - Algorithm seems costly due to repeated generation of certain states.
  - Node generation:
    - level d: once
    - level d-1: 2
    - level d-2: 3
    - …
    - level 2: d-1
    - level 1: d

$$O(b^d)$$

$$N(IDS) = (d)b + (d-1)b^2 + \ldots + (1)b^d$$

$$N(BFS) = b + b^2 + \ldots + b^d + (b^{d+1} - b)$$

Num. Comparison for b=10 and d=5 solution at far right

$$N(IDS) = 50 + 400 + 3000 + 20000 + 100000 = 123450$$

$$N(BFS) = 10 + 100 + 1000 + 10000 + 100000 + 999990 = 1111100$$

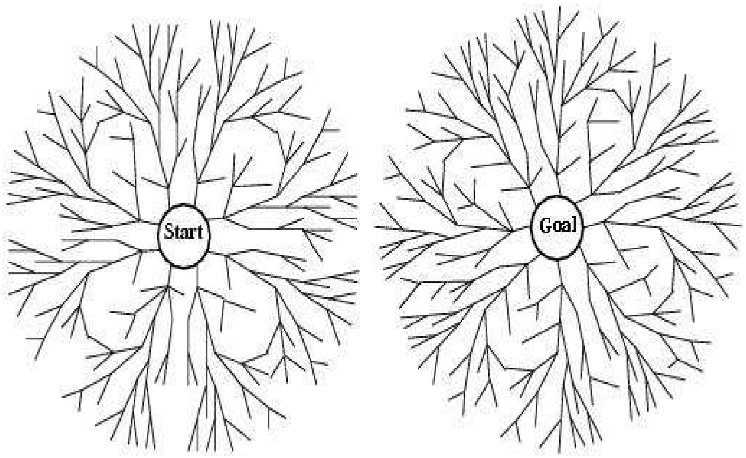
ID search, evaluation

- Completeness:
  - YES (no infinite paths)
- Time complexity: $O(b^d)$
- Space complexity: $O(bd)$
  - Cfr. depth-first search

ID search, evaluation

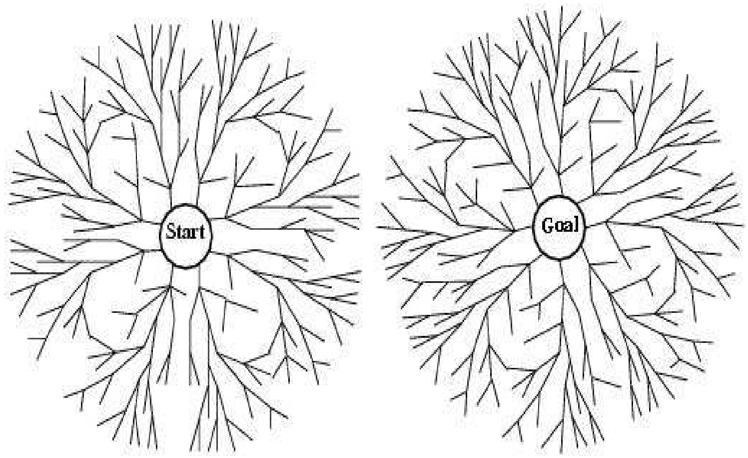
- Completeness:
  - YES (no infinite paths)
- Time complexity: $O(b^d)$
- Space complexity: $O(bd)$
- Optimality:
  - YES if step cost is 1.
  - Can be extended to iterative lengthening search
    - ✓ Same idea as uniform-cost search
    - ✓ Increases overhead.

Bidirectional search



- Two simultaneous searches from start an goal.
  - Motivation: $b^{d/2} + b^{d/2} \neq b^{d}$
- Check whether the node belongs to the other fringe before expansion.
- Space complexity is the most significant weakness.
- Complete and optimal if both searches are BF.

How to search backwards?



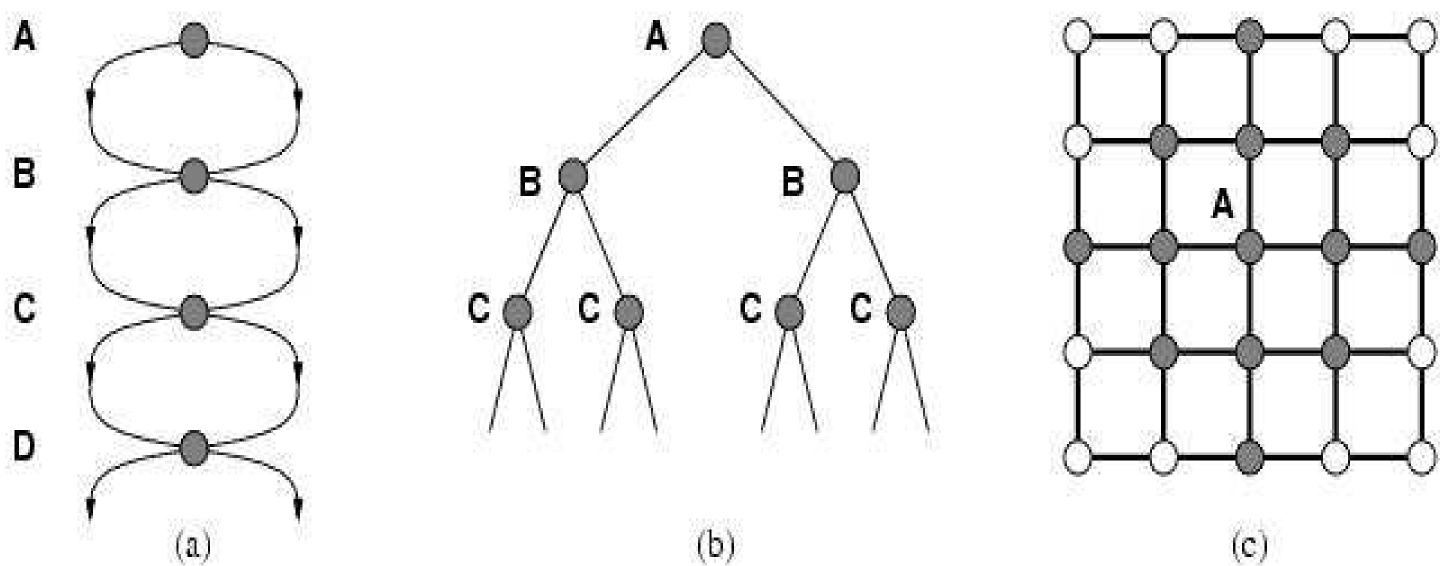
- The predecessor of each node should be efficiently computable.
  - When actions are easily reversible.

Summary of algorithms

Criterion	Breadth-First	Uniform-cost	Depth-First	Depth-limited	Iterative deepening	Bidirectional search
Complete?	YES*	YES*	NO	YES, if $l \geq d$	YES	YES*
Time	b^{d+1}	$b^{C^*/e}$	b^m	b^l	b^d	$b^{d/2}$
Space	b^{d+1}	$b^{C^*/e}$	bm	bl	bd	$b^{d/2}$
Optimal?	YES*	YES*	NO	NO	YES	YES

Repeated states

❖ Failure to detect repeated states can turn a solvable problems into unsolvable ones.



Graph search algorithm

❖ Closed list stores all expanded nodes

function GRAPH-SEARCH(*problem*,*fringe*) **return** a solution or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
    loop do
        if EMPTY?(*fringe*) **then return** failure
        node ← REMOVE-FIRST(*fringe*)
        if GOAL-TEST[*problem*] applied to STATE[*node*] succeeds
            then return SOLUTION(*node*)
        if STATE[*node*] is not in *closed* **then**
            add STATE[*node*] to *closed*
            fringe ← INSERT-ALL(EXPAND(*node*, *problem*), *fringe*)

Graph algorithm (2)

function EXPAND(*node*,*problem*) **return** a set of nodes

successors ← the empty set

for each <*action*, *result*> **in** SUCCESSOR-FN**[***problem***]**(STATE[*node*]) **do**

s ← a new NODE

STATE[*s*] ← *result*

PARENT-NODE[*s*] ← *node*

ACTION[*s*] ← *action*

PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node*, *action*,*s*)

DEPTH[*s*] ← DEPTH[*node*]+1

add *s* to *successors*

return *successors*

Modify for graph search

Graph search, evaluation

- Optimality:
  - GRAPH-SEARCH discard newly discovered paths.
    - ✓ This may result in a sub-optimal solution
    - ✓ YET: when uniform-cost search or BF-search with constant step cost
- Time and space complexity,
  - proportional to the size of the state space

    (may be much smaller than $O(b^d)$).
  - DF- and ID-search with closed list no longer has linear space requirements since all nodes are stored in closed list!!

Search with partial information

❖ Previous assumption:
- Environment is fully observable
- Environment is deterministic
- Agent knows the effects of its actions

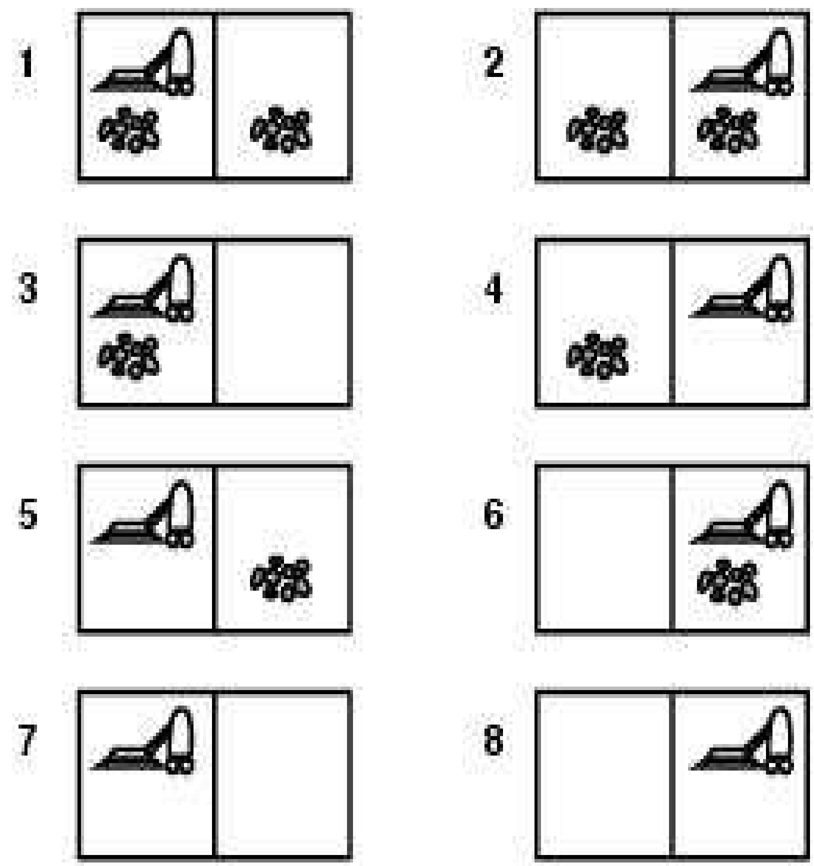
What if knowledge of states or actions is incomplete?

Search with partial information

- (SLIDE 7) Partial knowledge of states and actions:
  - *sensorless or conformant problem*
    - ✓ Agent may have no idea where it is; solution (if any) is a sequence.
  - *contingency problem*
    - ✓ Percepts provide *new* information about current state; solution is a tree or policy; often interleave search and execution.
    - ✓ If uncertainty is caused by actions of another agent: *adversarial problem*
  - *exploration problem*
    - ✓ When states and actions of the environment are unknown.

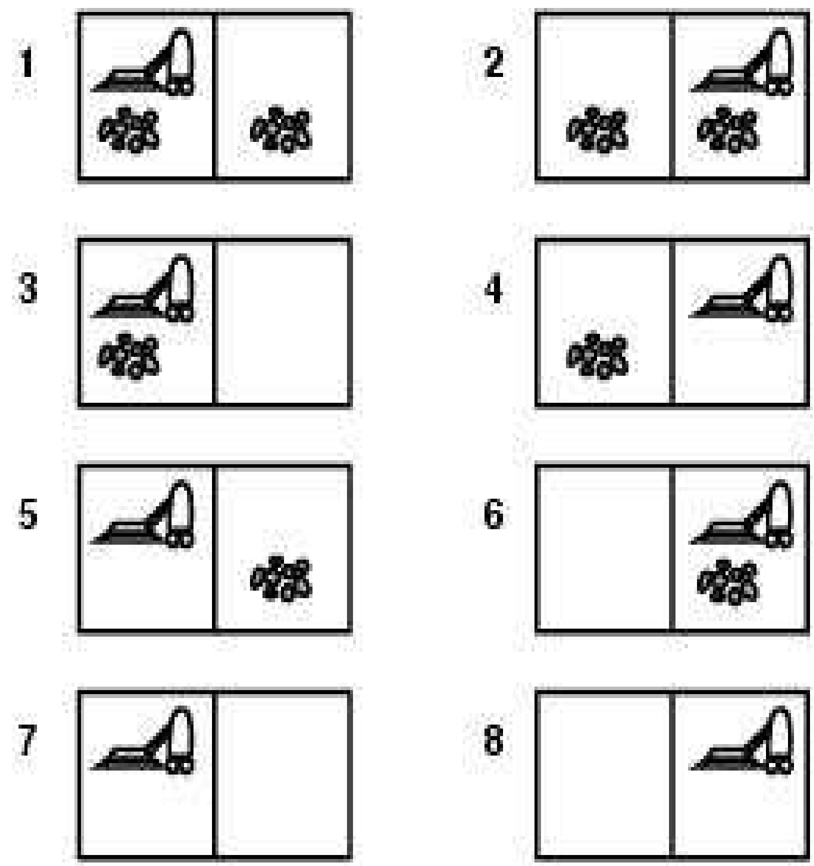
Conformant problems

- start in {1,2,3,4,5,6,7,8} e.g Right goes to {2,4,6,8}. Solution??
  - *[Right, Suck, Left,Suck]*
- *When the world is not fully observable: reason about a set of states that migth be reached*
  - =belief state

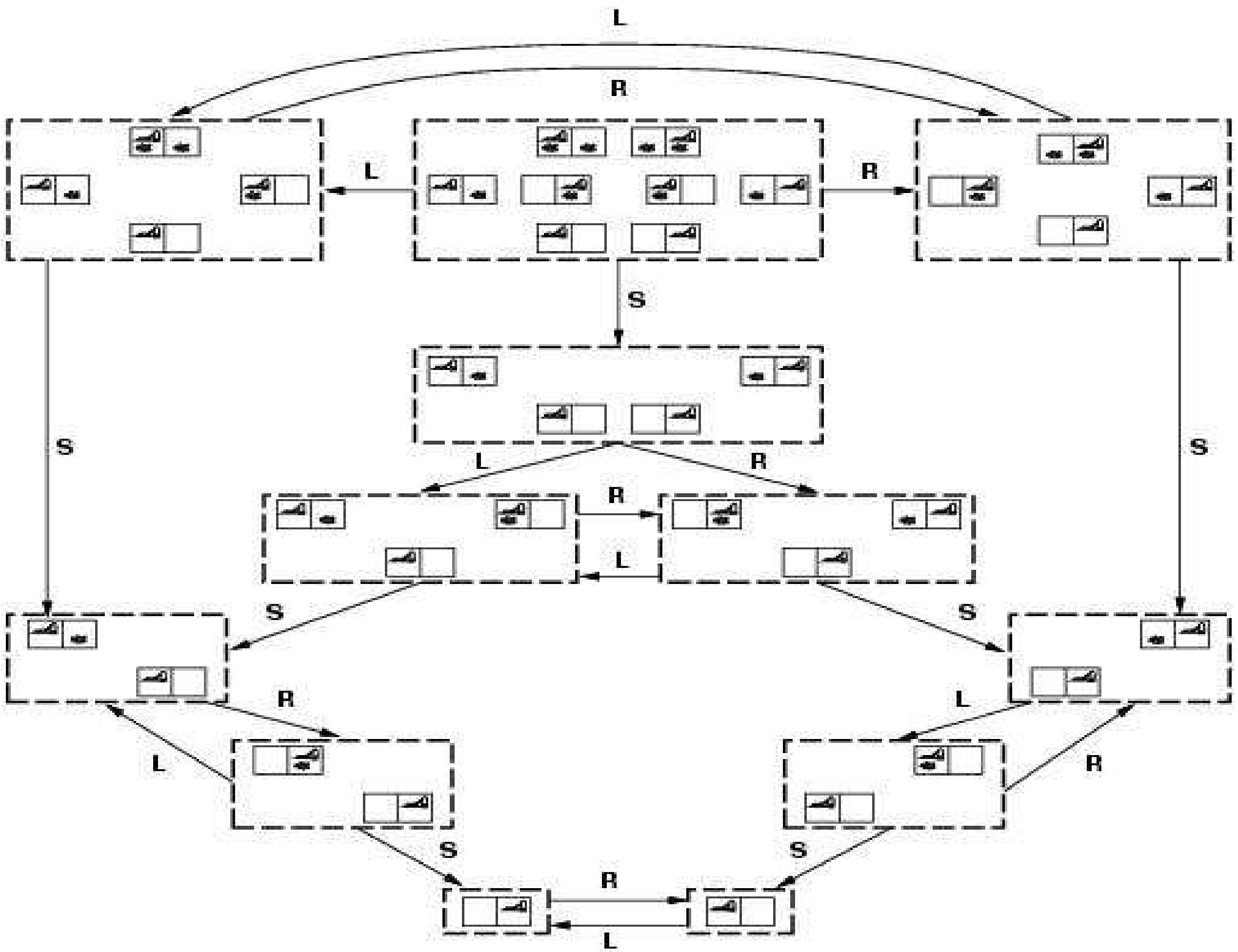


Conformant problems

- Search space of belief states
- Solution = belief state with all members goal states.
- If S states then 2^S belief states.
- Murphy's law:
  - *Suck can dirty a clear square.*



Belief state of vacuum-world



Contingency problems

- Contingency, start in {1,3}.
- Murphy's law, Suck *can* dirty a clean carpet.
- Local sensing: dirt, location only.
  - Percept = [L,Dirty] ={1,3}
  - [Suck] = {5,7}
  - [Right] ={6,8}
  - [Suck] in {6}={8} (Success)
  - BUT [Suck] in {8} = failure
- Solution??
  - Belief-state: no fixed action sequence guarantees solution
- Relax requirement:
  - [*Suck*, *Right*, if *[R,dirty]* then *Suck*]
  - Select actions based on contingencies arising during execution.