Chapter 06
Game Playing

Instructor

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Instructor's Information

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Acknowledgment

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- **Prof. Stuart Russell and Peter Norvig**: They are currently from University of California, Berkeley. They are also the author of the book "Artificial Intelligence: A Modern Approach", which is used as the textbook for the course

- **Prof. Tom Lenaerts**, from Université Libre de Bruxelles

Outline

- What are games?
- Optimal decisions in games
  - Which strategy leads to success?
- α-β pruning
- Games of imperfect information

What are and why study games?

- Games are a form of *multi-agent environment*
  - What do other agents do and how do they affect our success?
  - Cooperative vs. competitive multi-agent environments.
  - Competitive multi-agent environments give rise to adversarial problems a.k.a. *games*
- Why study games?
  - Fun; historically entertaining
  - Interesting subject of study because they are hard
    - ✓ Chess game:
      average branch factor: 35, each player: 50 moves → Search tree: 35^{100} nodes ☹

Relation of Games to Search

- Search – no adversary
  - Solution is (heuristic) method for finding goal
  - Heuristics and CSP techniques can find *optimal* solution
  - Evaluation function: estimate of cost from start to goal through given node
  - Examples: path planning, scheduling activities
- Games – adversary
  - Solution is strategy (strategy specifies move for every possible opponent reply).
  - Time limits force an *approximate* solution
  - Evaluation function: evaluate "goodness" of game position
  - Examples: chess, checkers, Othello, backgammon

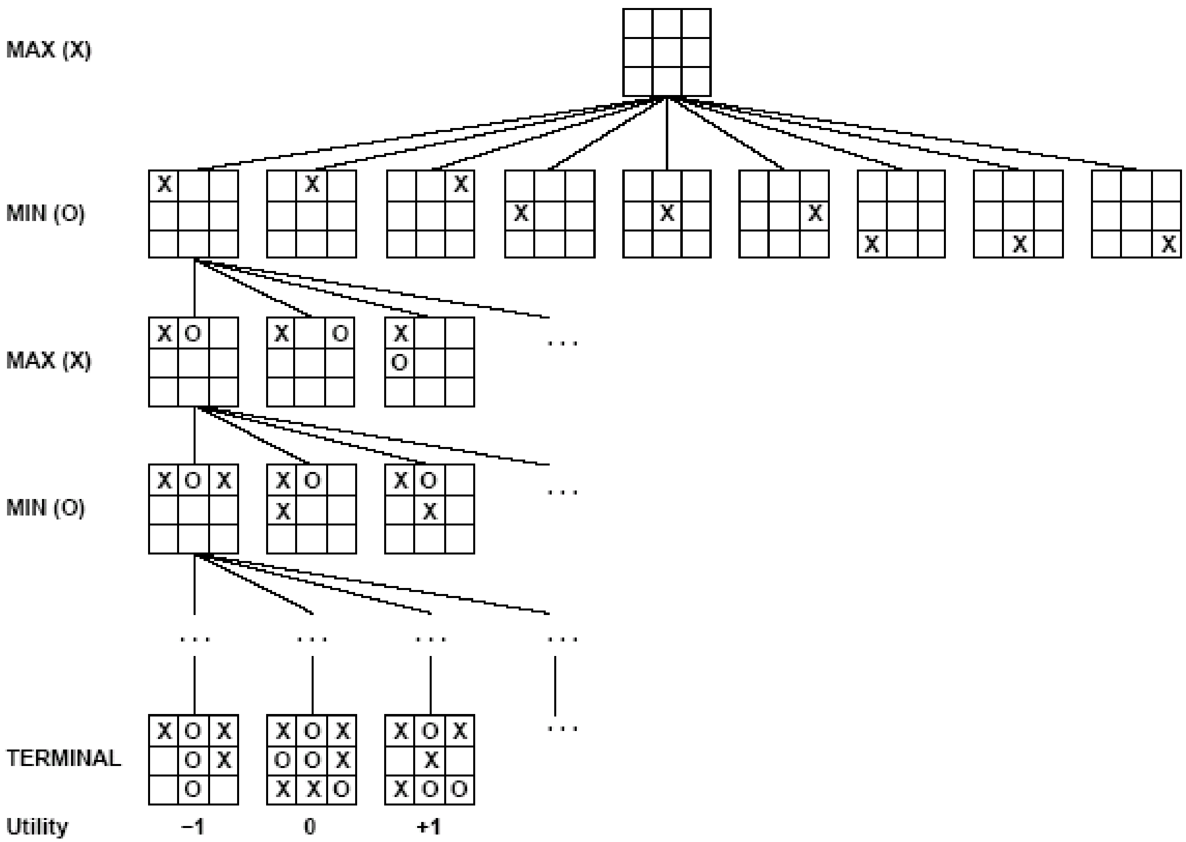
Types of Games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information		bridge, poker, scrabble nuclear war

Game setup

- Two players: MAX and MIN
- MAX moves first and they take turns until the game is over. Winner gets award, looser gets penalty.
- Games as search:
  - Initial state: e.g. board configuration of chess
  - Successor function: list of (move,state) pairs specifying legal moves.
  - Terminal test: Is the game finished?
  - Utility function: Gives numerical value of terminal states. E.g. win (+1), loose (-1) and draw (0) in tic-tac-toe (next)
- MAX uses search tree to determine next move.

Partial Game Tree for Tic-Tac-Toe



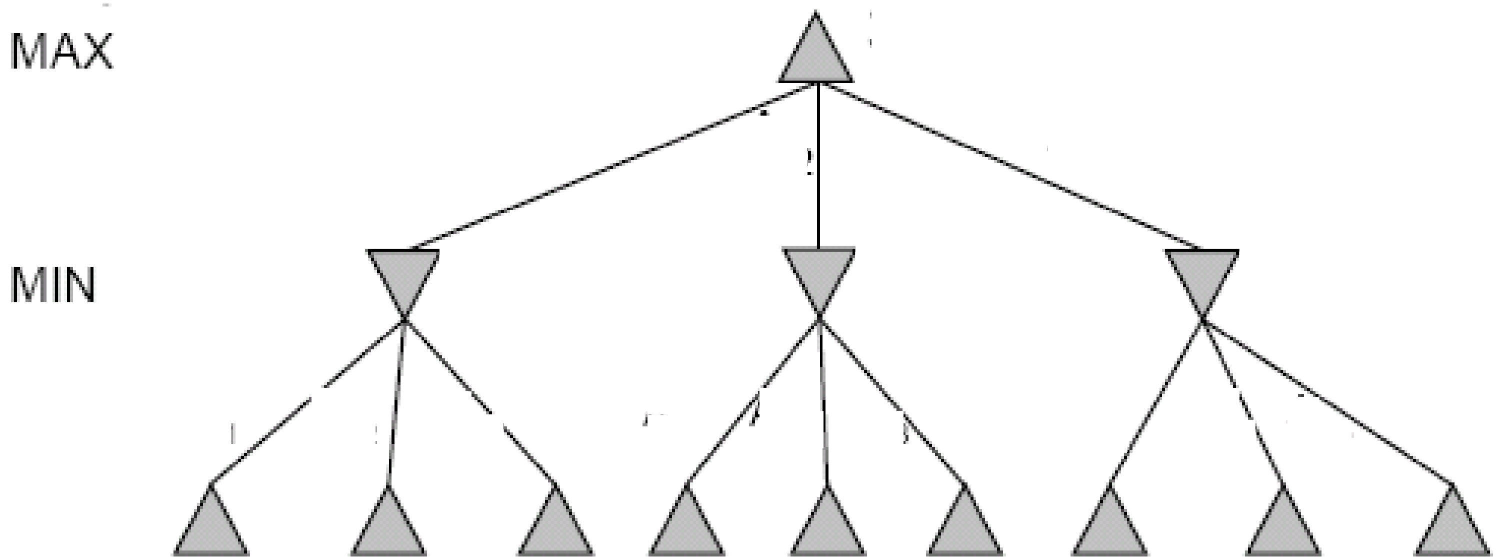
Optimal strategies

- Find the contingent *strategy* for MAX assuming an infallible MIN opponent.
- Assumption: Both players play optimally !!
- Given a game tree, the optimal strategy can be determined by using the minimax value of each node:

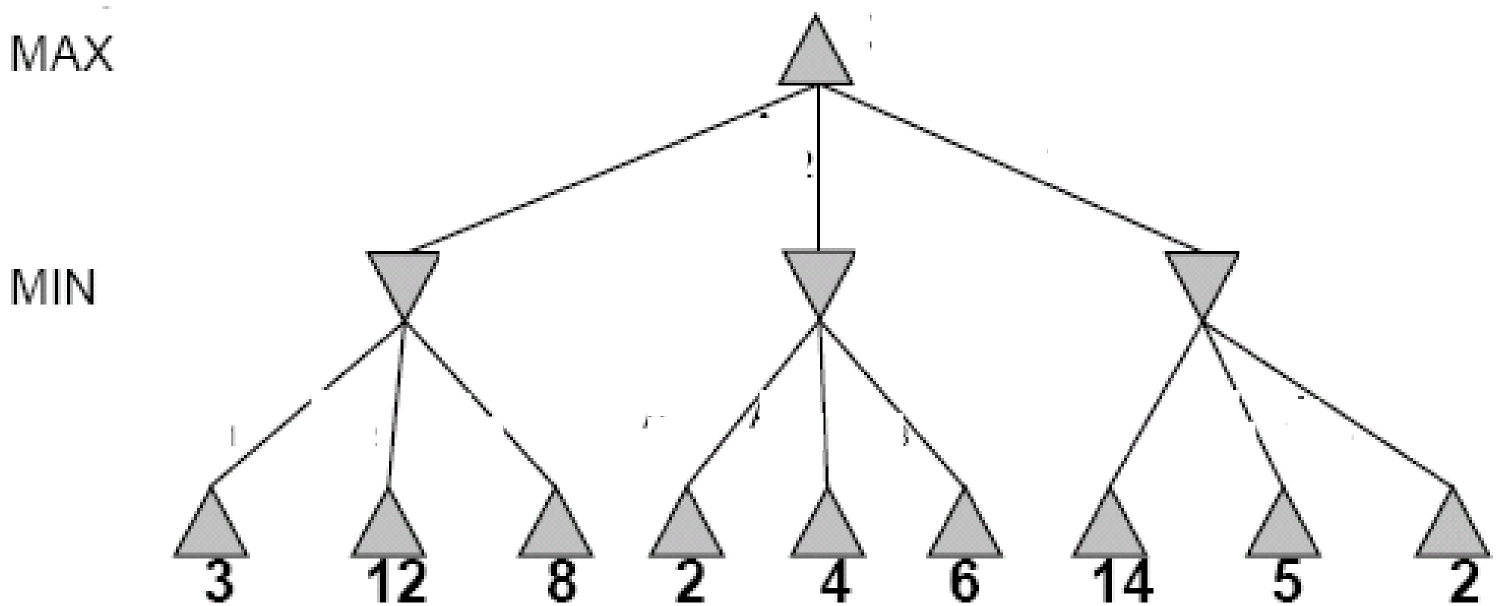
MINIMAX-VALUE(n)=

UTILITY(n)	If n is a terminal
$\max_{s \in successors(n)}$ MINIMAX-VALUE(s)	If n is a max node
$\min_{s \in successors(n)}$ MINIMAX-VALUE(s)	If n is a min node

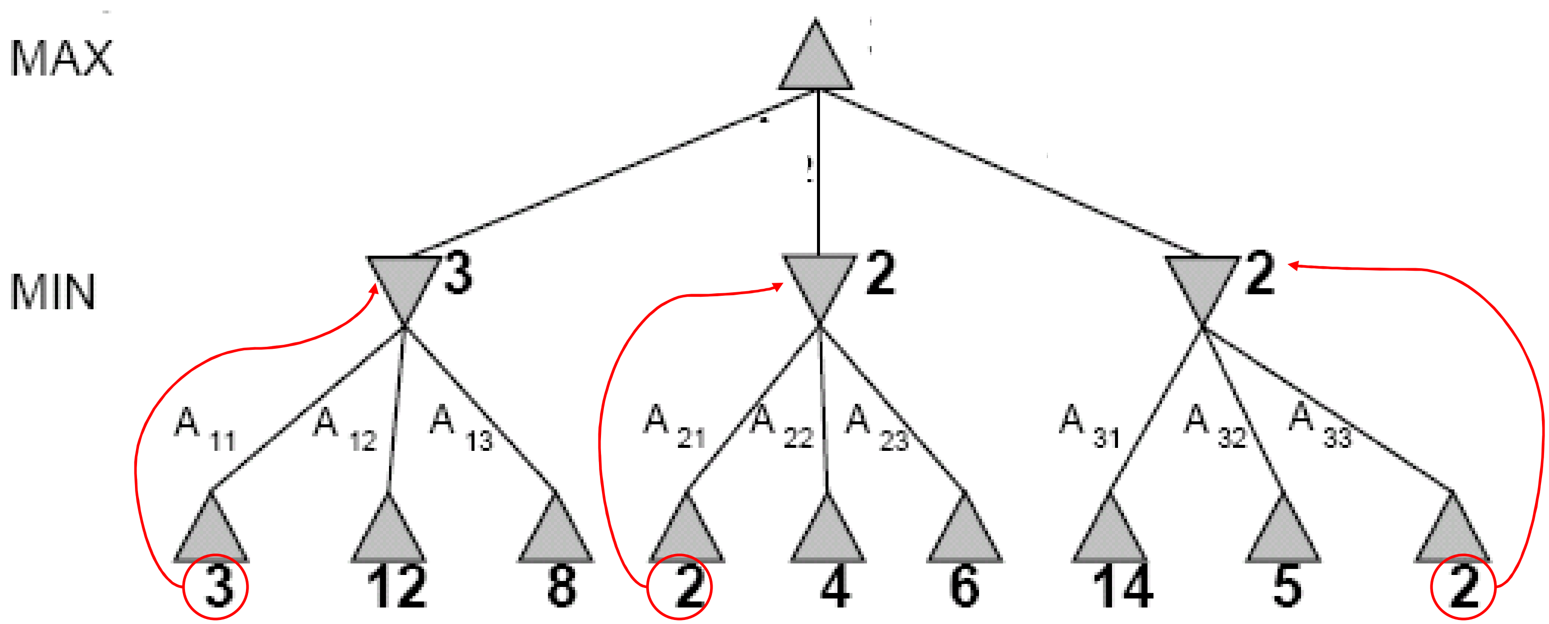
Two-Ply Game Tree



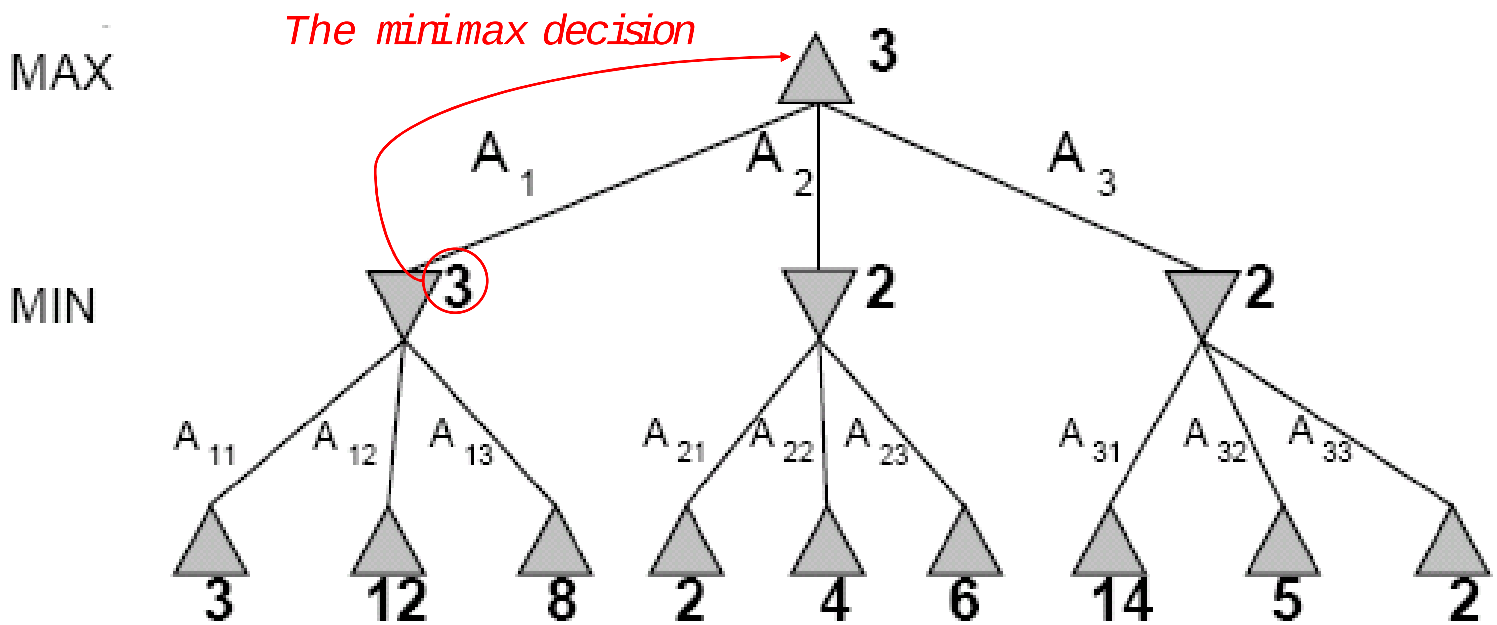
Two-Ply Game Tree



Two-Ply Game Tree



Two-Ply Game Tree

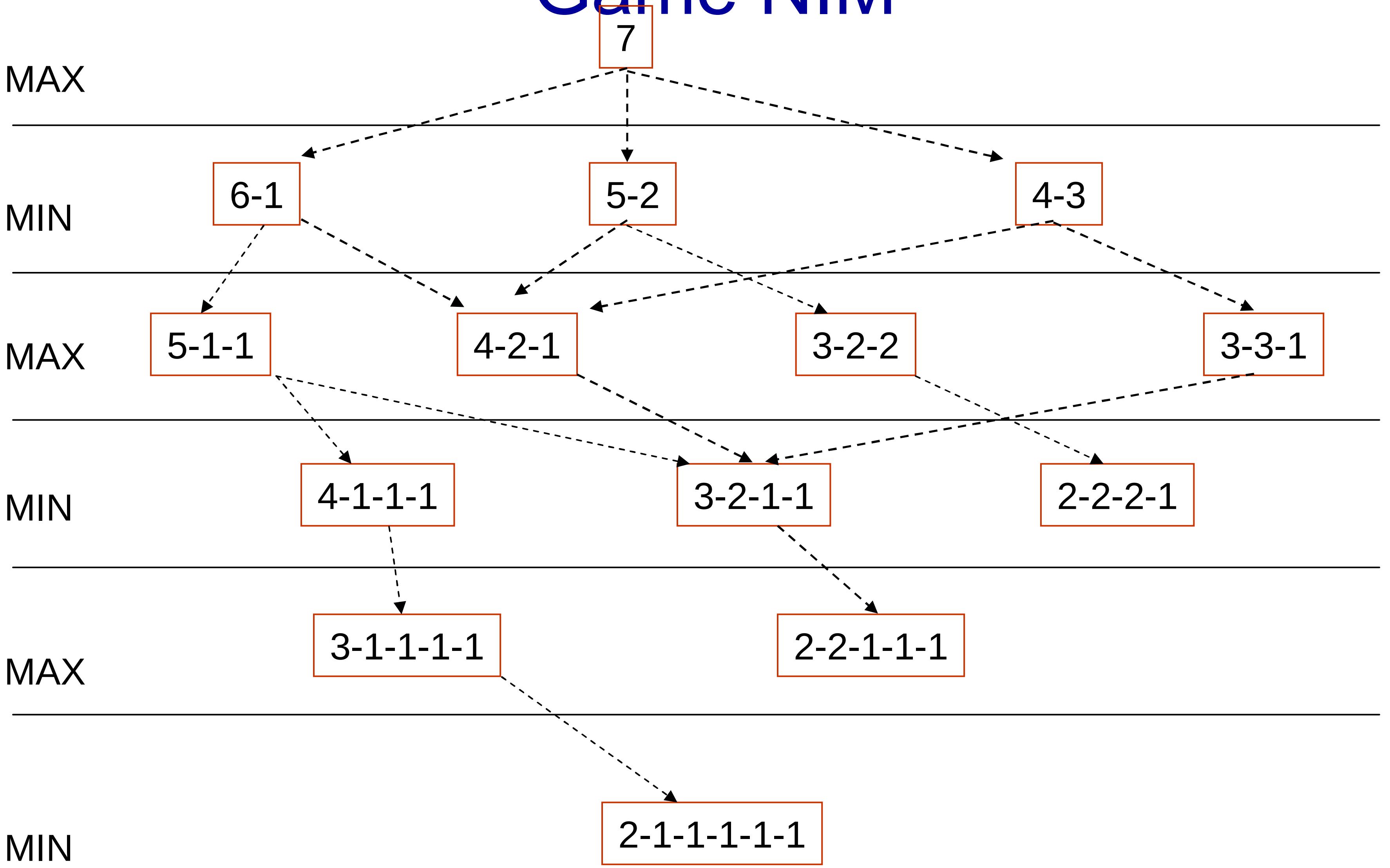


Minimax maximizes the worst-case outcome for max.

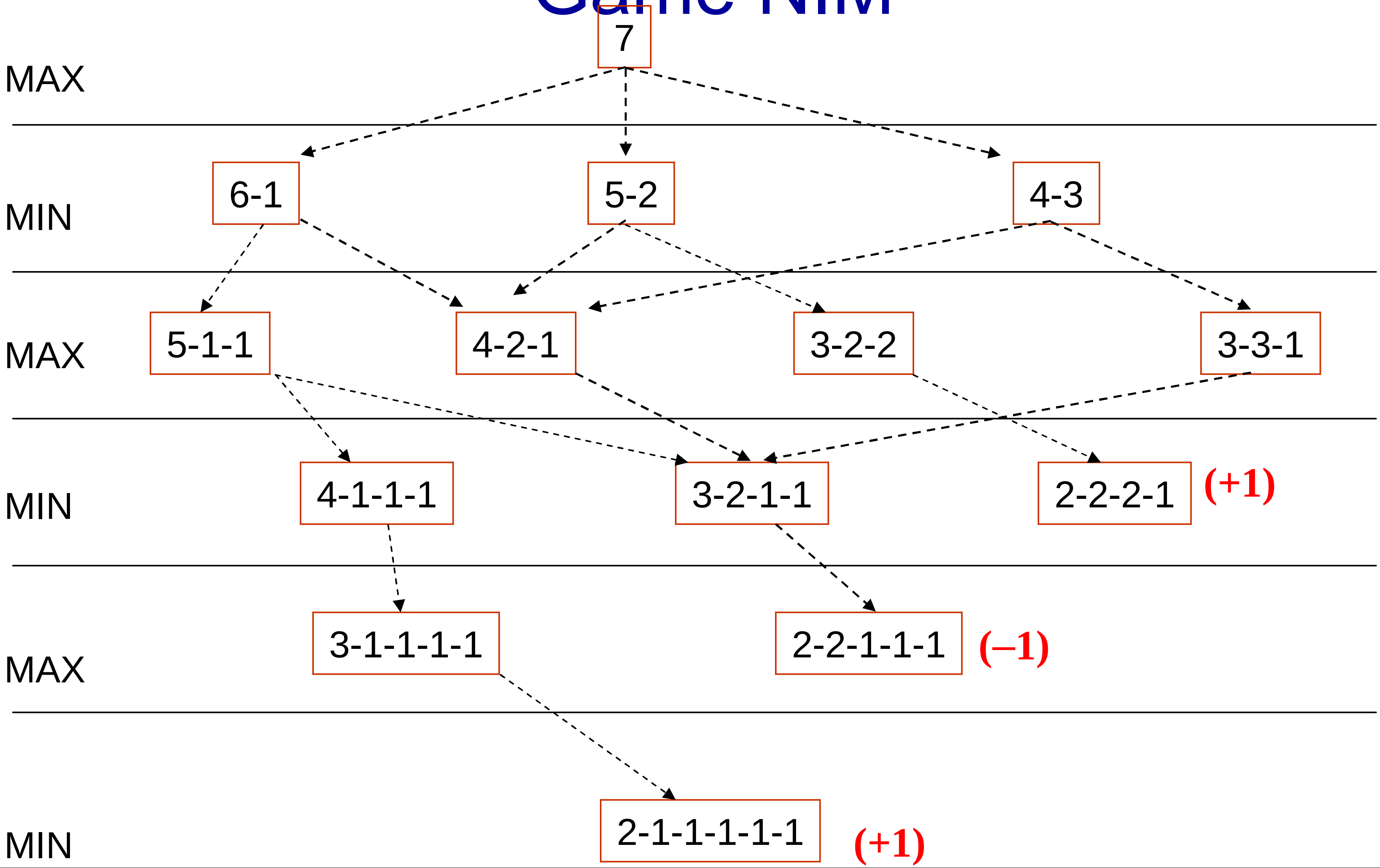
Game NIM

- Given N rods in n (n=1) heaps, two players, take turns
- At each turn, player has to select one heap and separate it into two different heaps having different number of rods
- A player looses the game if he/she can not select a heap for separation.

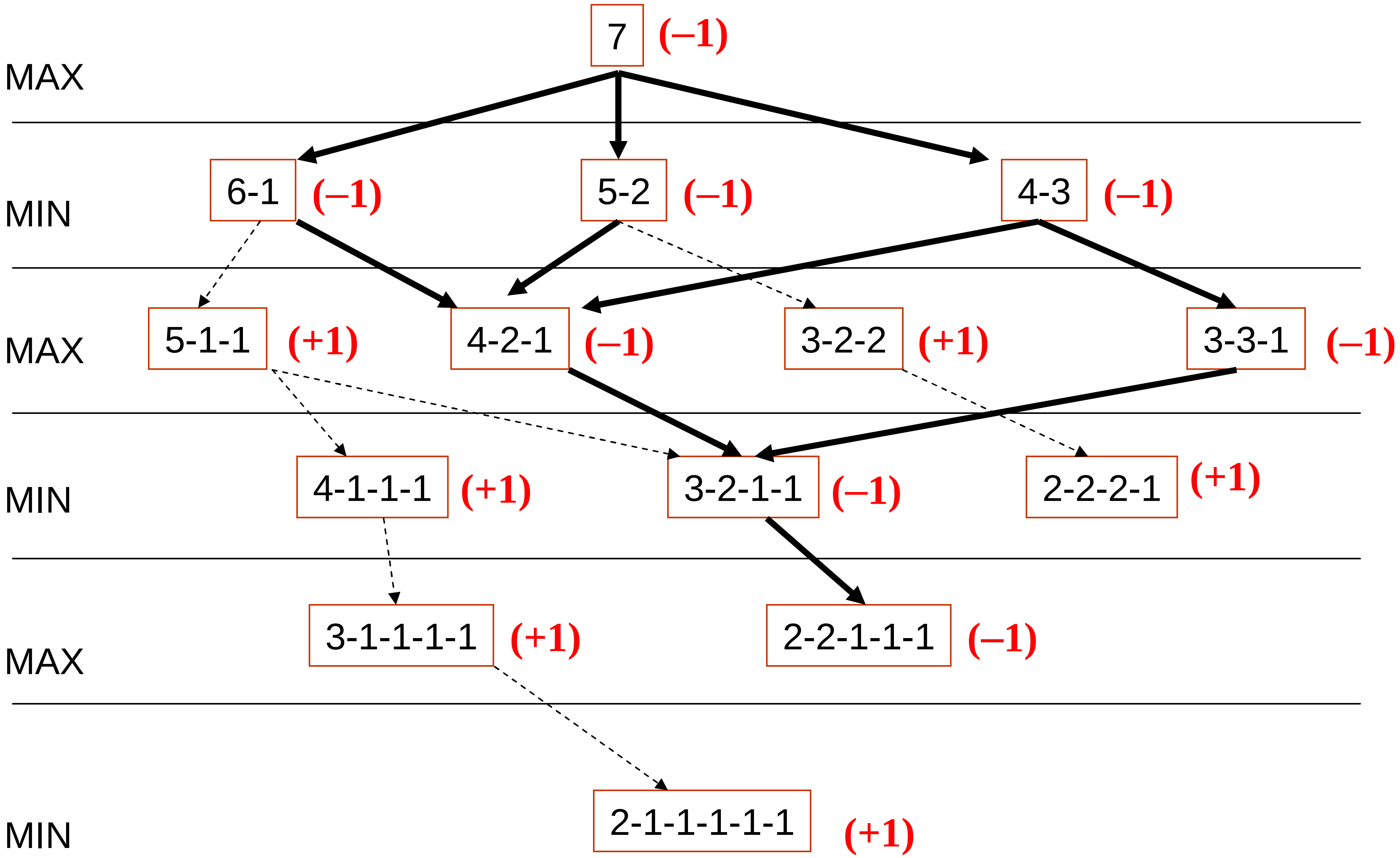
Game NIM


7
MAX
6-1
5-2
4-3
MIN
5-1-1
4-2-1
3-2-2
3-3-1
MAX
4-1-1-1
3-2-1-1
2-2-2-1
MIN
3-1-1-1-1
2-2-1-1-1
MAX
2-1-1-1-1-1
MIN

Game NIM


7
MAX
6-1
5-2
4-3
MIN
5-1-1
4-2-1
3-2-2
3-3-1
MAX
4-1-1-1
3-2-1-1
2-2-2-1
(+1)
MIN
3-1-1-1-1
2-2-1-1-1
(–1)
MAX
2-1-1-1-1-1
(+1)
MIN

Game NIM


7 (–1)
MAX
6-1 (–1)
5-2 (–1)
4-3 (–1)
MIN
5-1-1 (+1)
4-2-1 (–1)
3-2-2 (+1)
3-3-1 (–1)
MAX
4-1-1-1 (+1)
3-2-1-1 (–1)
2-2-2-1 (+1)
MIN
3-1-1-1-1 (+1)
2-2-1-1-1 (–1)
MAX
2-1-1-1-1-1 (+1)
MIN

What if MIN does not play optimally?

- Definition of optimal play for MAX assumes MIN plays optimally: maximizes worst-case outcome for MAX.
- But if MIN does not play optimally, MAX will do even better. [Can be proved.]

Minimax Algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*
  inputs: *state*, current state in game
  $v \leftarrow$ MAX-VALUE(*state*)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for *a*,*s* in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v,MIN-VALUE(s))
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for *a*,*s* in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN(v,MAX-VALUE(s))
  return v

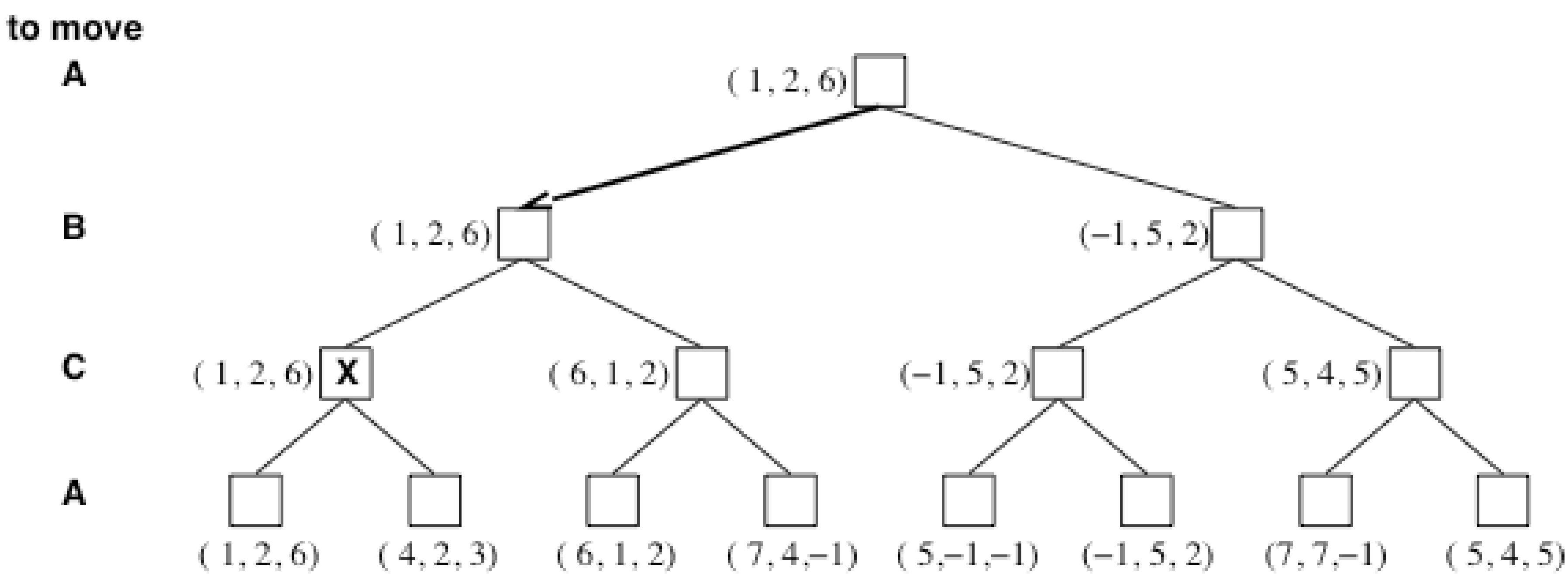
Properties of Minimax

Criterion	Minimax
Complete?	Yes ☺
Time	$O(b^m)$ ☹
Space	$O(bm)$ ☺
Optimal?	Yes ☹

Multiplayer games

- Games allow more than two players
- Solution:
  - Single minimax values become vectors
  - For example, 3 players: A, B, C
  - Values at each node: [V_A, V_B, V_C]
    - ✓ V_A: measure the utility of the state under in viewpoint of A
    - ✓ V_B: measure the utility of the state under in viewpoint of B
    - ✓ V_C: measure the utility of the state under in viewpoint of C
- Multiplayer → Making Alliances

Multiplayer games



Multiplayer games

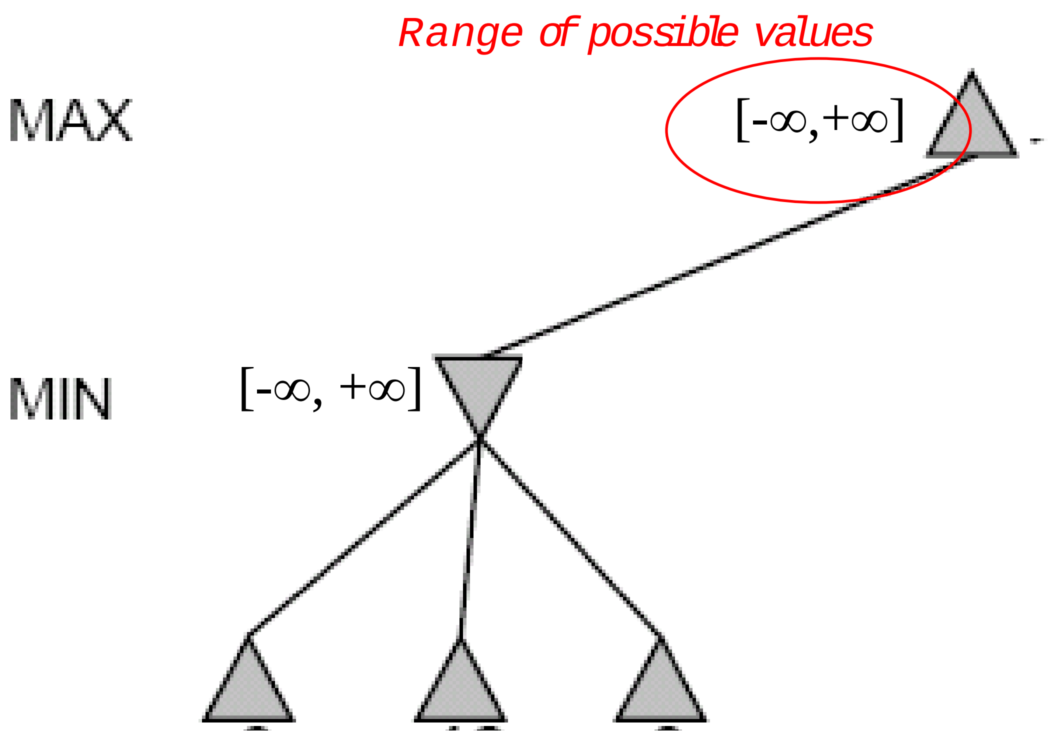
- Multiplayer → Strategy: Making Alliances, look like “Three Kingdoms” in Chinese history
  - A and B are in their weak points
  - C is in its stronger point
  - → It is a reasonable that A and B make a alliance and together attack C
  - ...
  - However, as soon as C weakens the alliance is break down.

Problem of minimax search

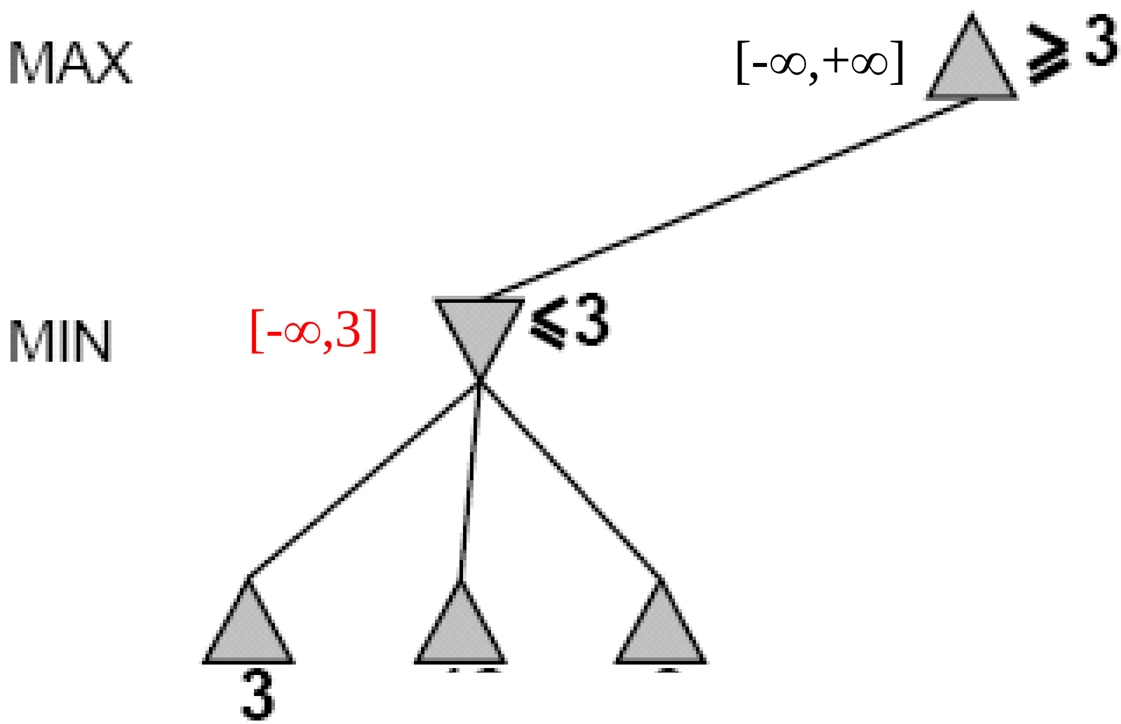
- Number of games states is exponential to the number of moves.
  - Solution: Do not examine every node
  - ==> Alpha-beta pruning
    - ✓ Alpha = value of best choice found so far at any choice point along the MAX path
    - ✓ Beta = value of best choice found so far at any choice point along the MIN path
- Revisit example …

Alpha-Beta Example

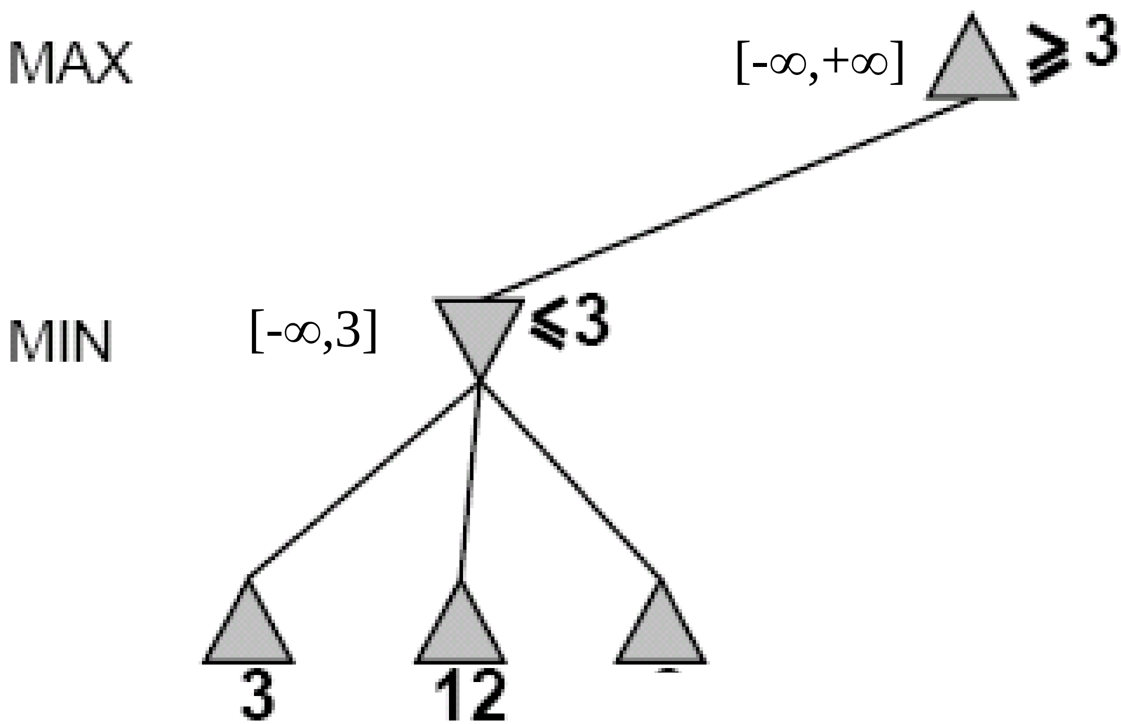
Do DF-search until first leaf



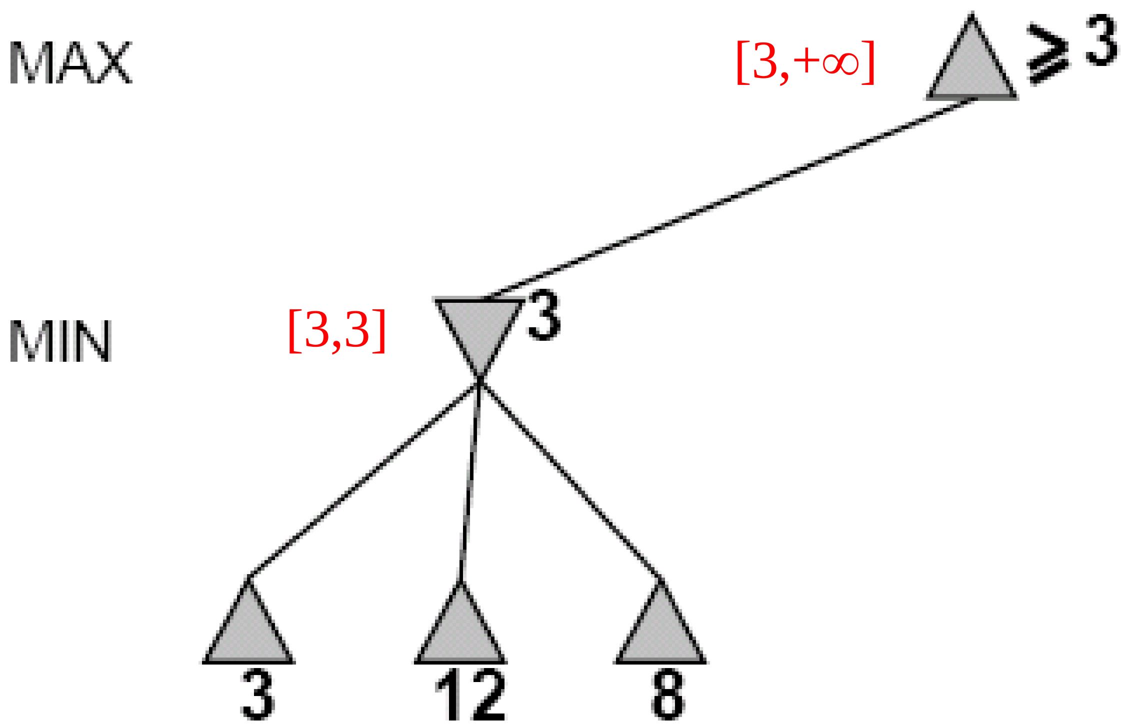
Alpha-Beta Example (continued)



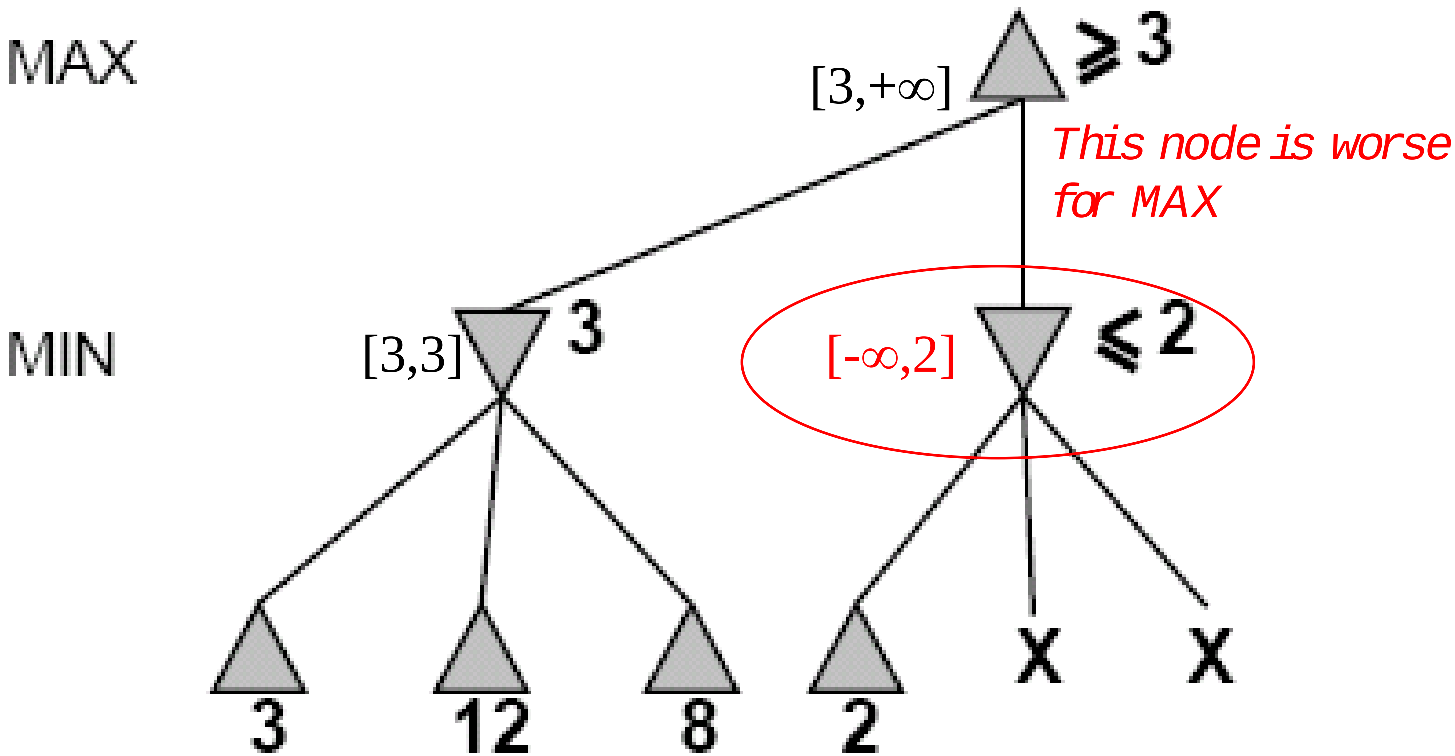
Alpha-Beta Example (continued)



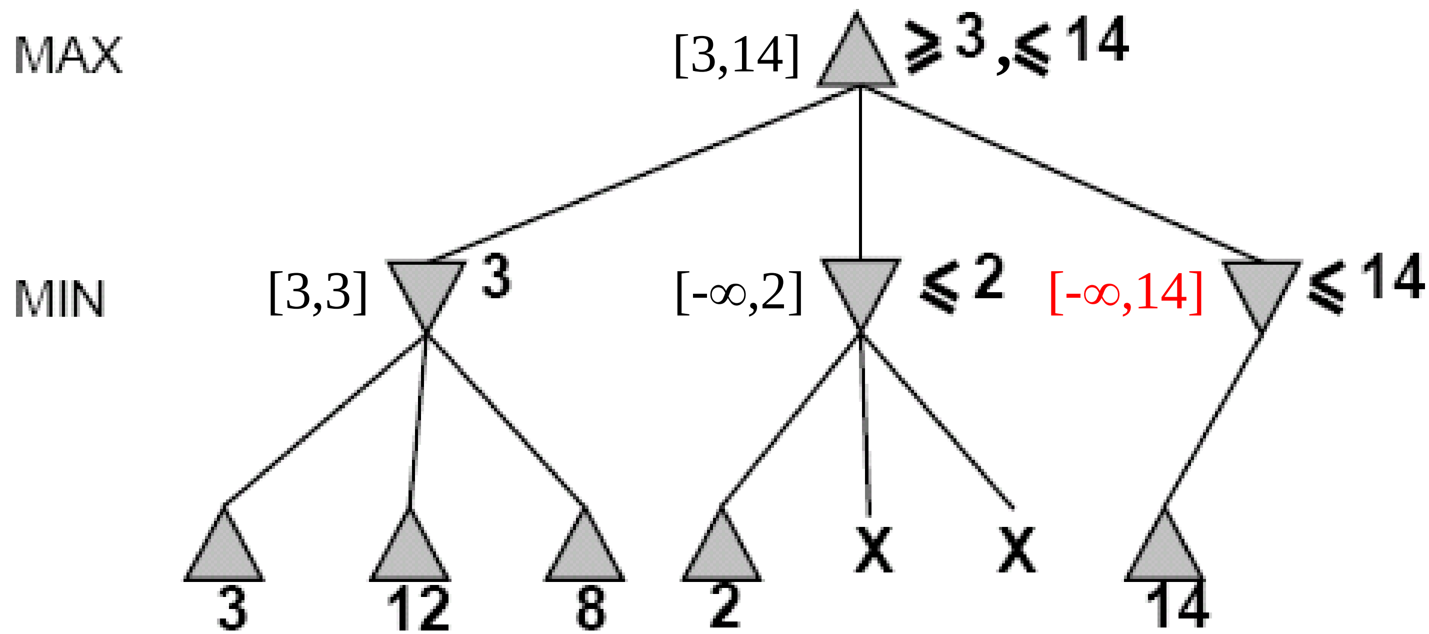
Alpha-Beta Example (continued)



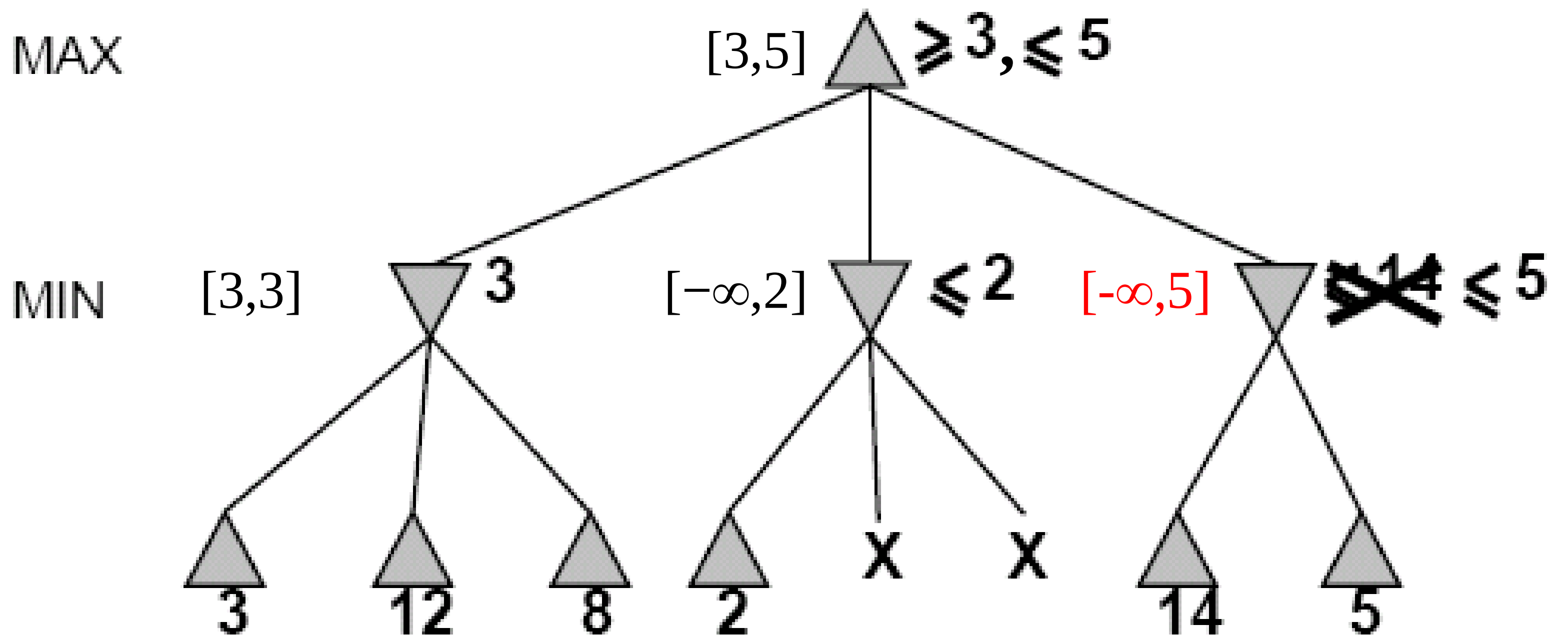
Alpha-Beta Example (continued)



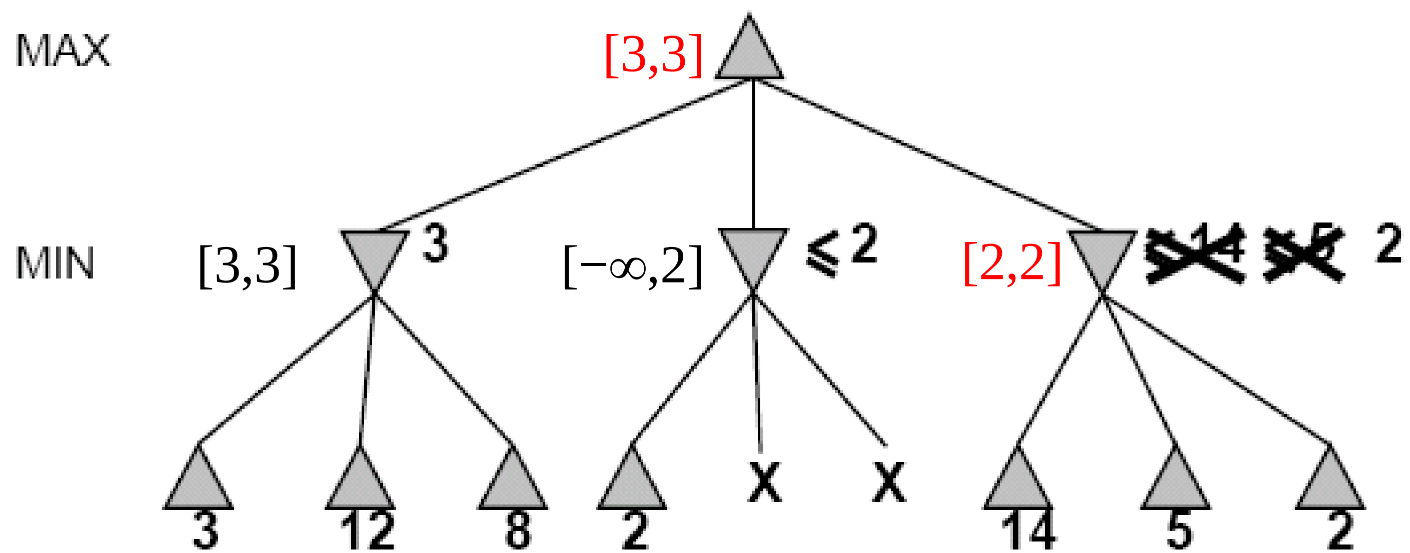
Alpha-Beta Example (continued)



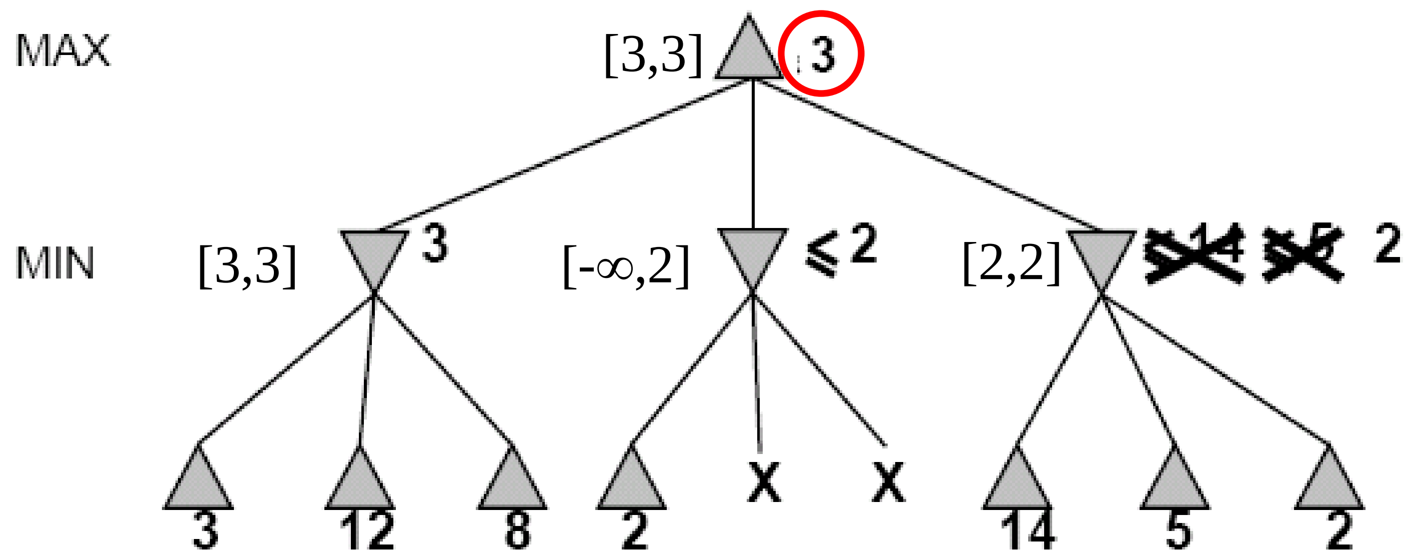
Alpha-Beta Example (continued)



Alpha-Beta Example (continued)



Alpha-Beta Example (continued)



Alpha-Beta Algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  inputs: *state*, current state in game
  $v \leftarrow$ MAX-VALUE(*state*, $-\infty$, $+\infty$)
  return the *action* in SUCCESSORS(*state*) with value v

---

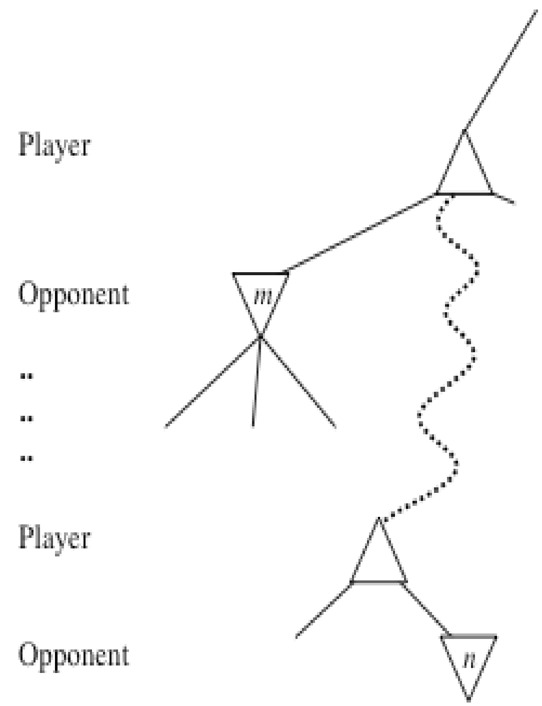
function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for *a*,*s* in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

Alpha-Beta Algorithm

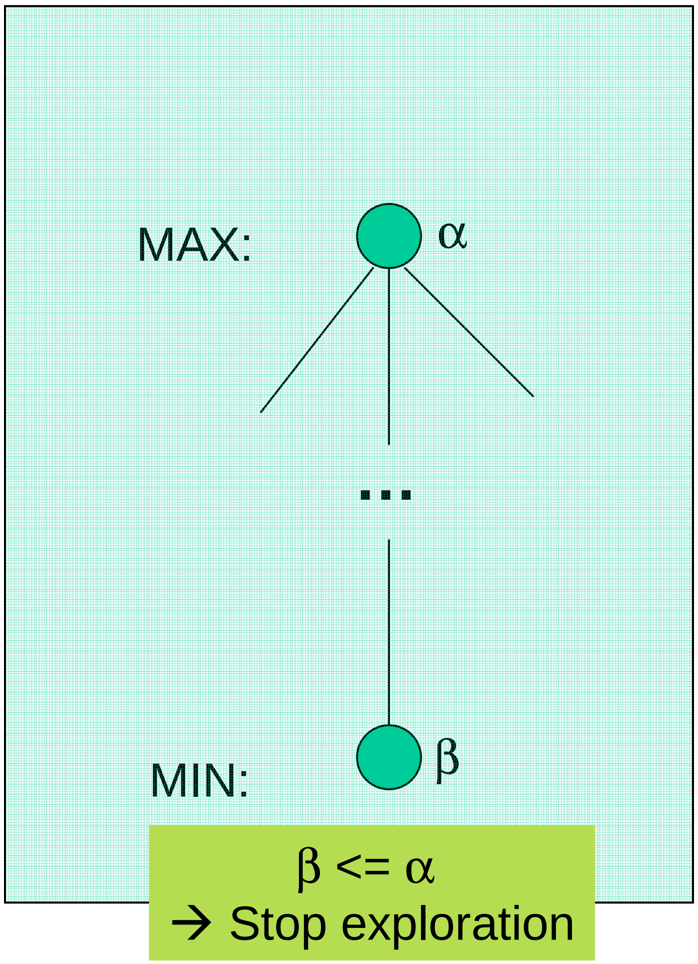
function $\boxed{\text{MIN-VALUE}(\textit{state}, \alpha, \beta)}$ **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for *a,s* in SUCCESSORS(*state*) **do**
    $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s, \alpha, \beta))$
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow \text{MIN}(\beta, v)$
  return v

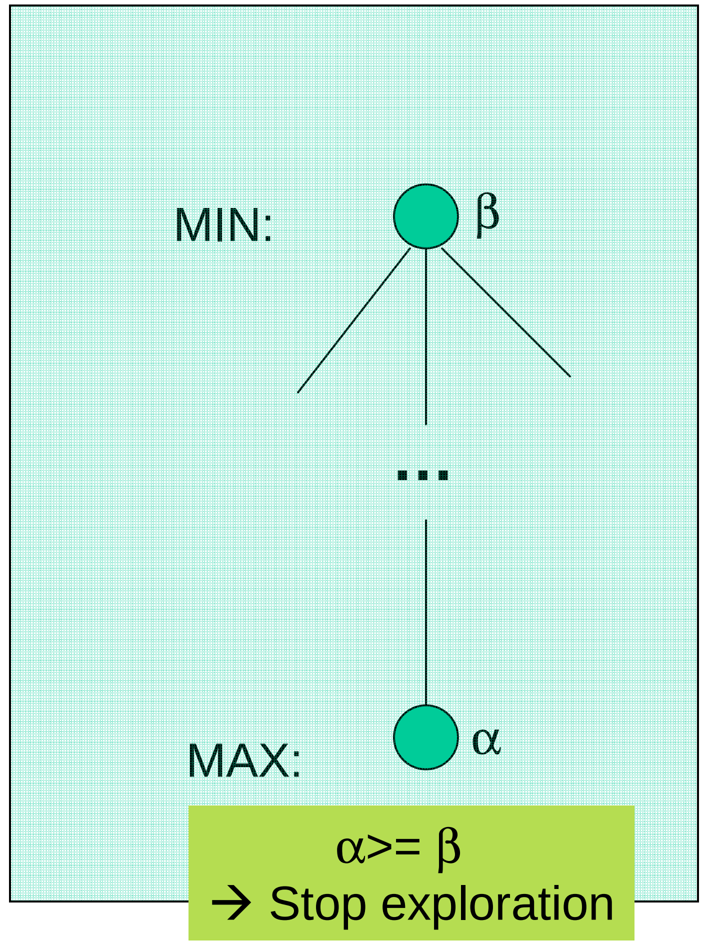
General alpha-beta pruning

- Consider a node *n* somewhere in the tree
- If player has a better choice at
  - Parent node of n
  - Or any choice point further up
- *n* will **never** be reached in actual play.
- Hence when enough is known about *n*, it can be pruned.

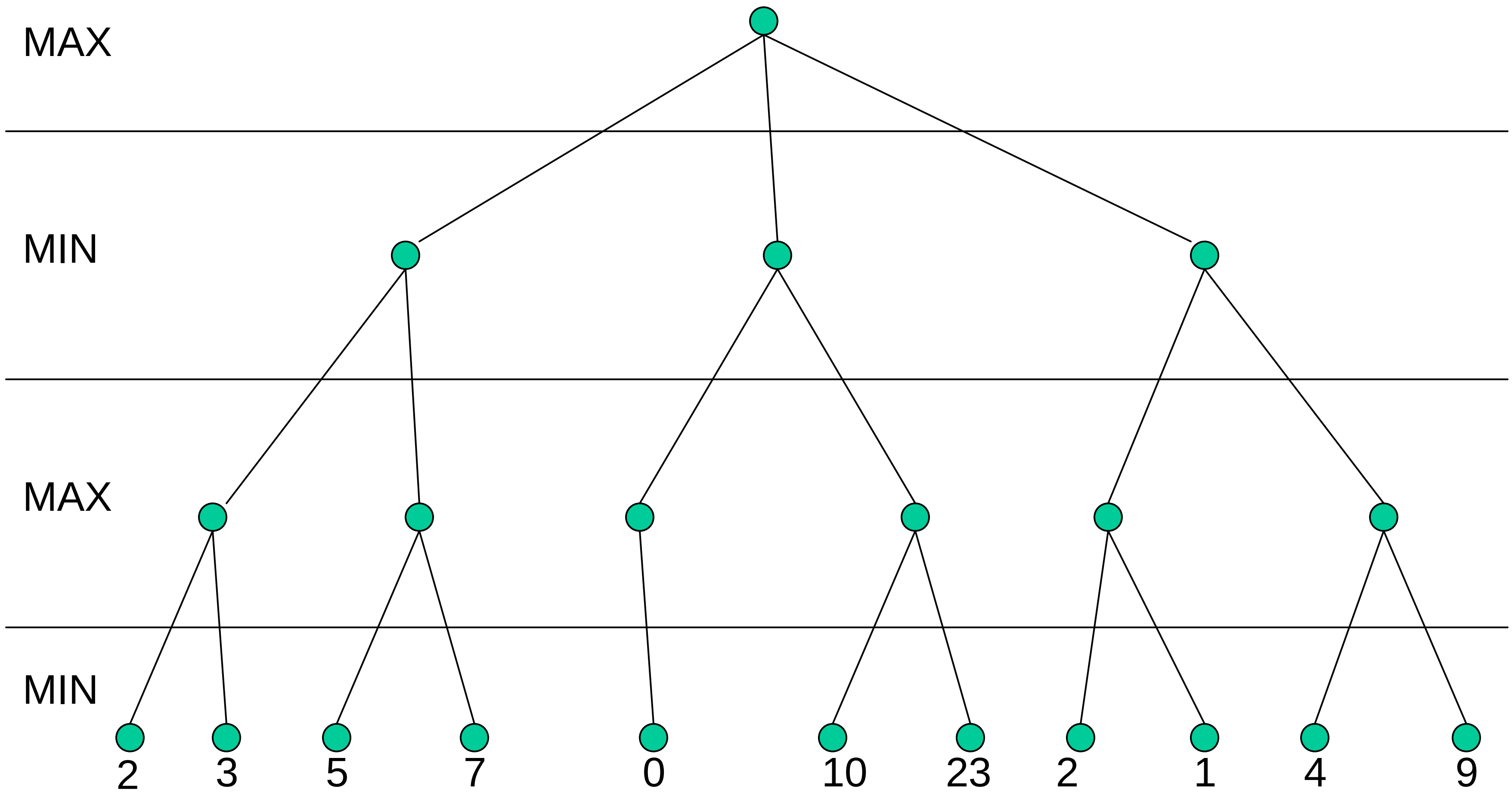


Alpha-Beta Pruning



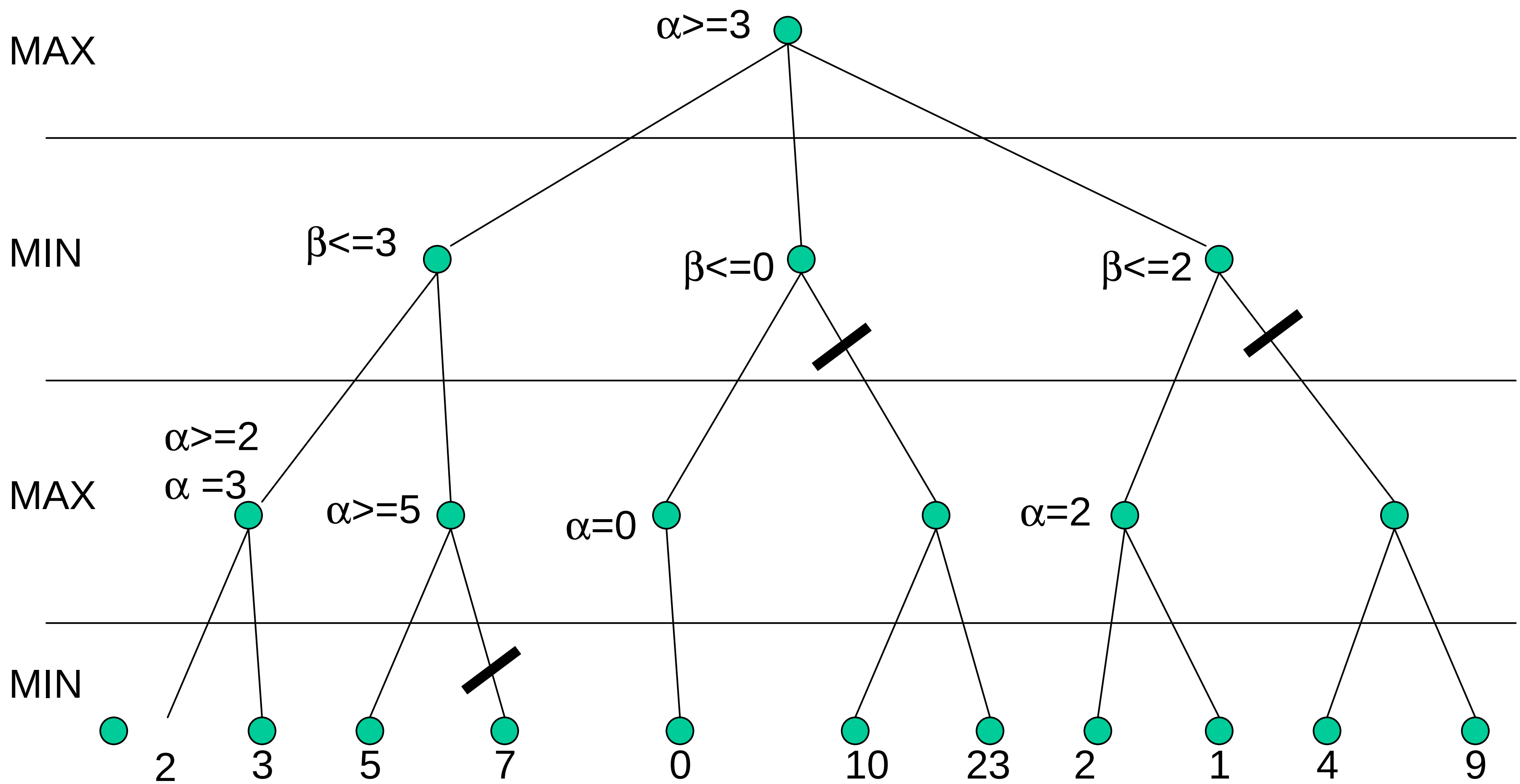


Alpha-Beta Example



Suppose that the search tree has the form as shown in the figure

Alpha-Beta Example



α-β pruning

Final Comments about Alpha-Beta Pruning

- Pruning does not affect final results
- Entire subtrees can be pruned.
- Good move *ordering* improves effectiveness of pruning
- With "perfect ordering," time complexity is $O(b^{m/2})$
  - Branching factor of sqrt(b) !!
  - Alpha-beta pruning can look twice as far as minimax in the same amount of time
- Repeated states are again possible.
  - Store them in memory = transposition table

Games of imperfect information

- Minimax and alpha-beta pruning require too much leaf-node evaluations.
- May be impractical within a reasonable amount of time.
- SHANNON (1950):
  - Cut off search earlier (replace TERMINAL-TEST by CUTOFF-TEST)
  - Apply heuristic evaluation function EVAL (replacing utility function of alpha-beta)

Cutting off search

- Change:
  - **if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

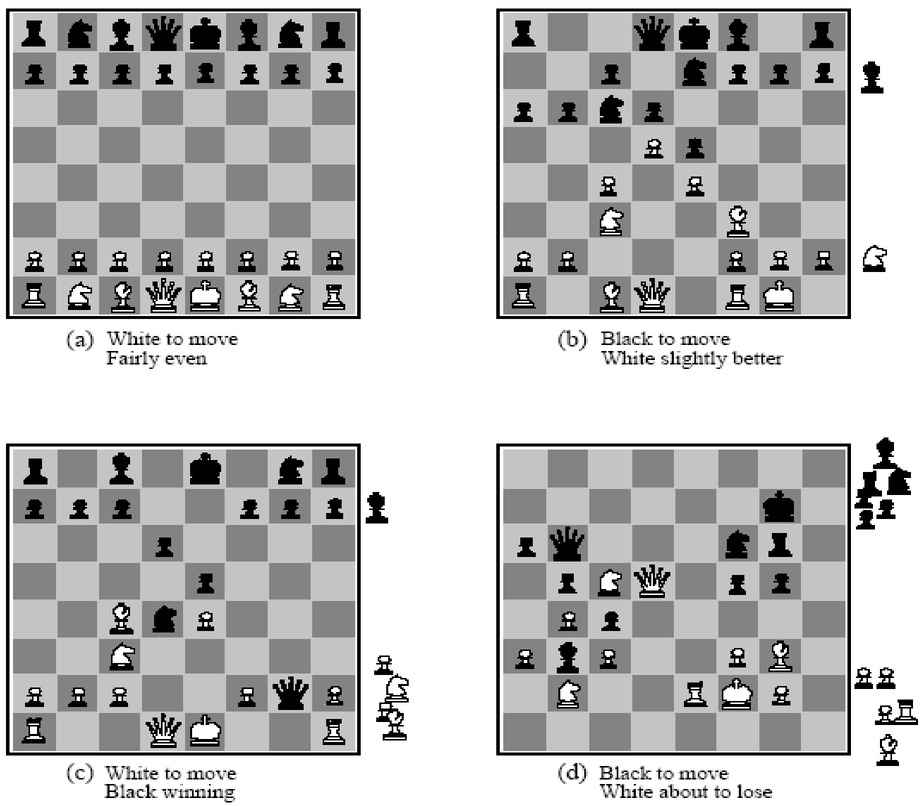
  into
  - **if** CUTOFF-TEST(*state,depth*) **then return** EVAL(*state*)

- Introduces a fixed-depth limit *depth*
  - Is selected so that the amount of time will not exceed what the rules of the game allow.
- When cuttoff occurs, the evaluation is performed.

Heuristic EVAL

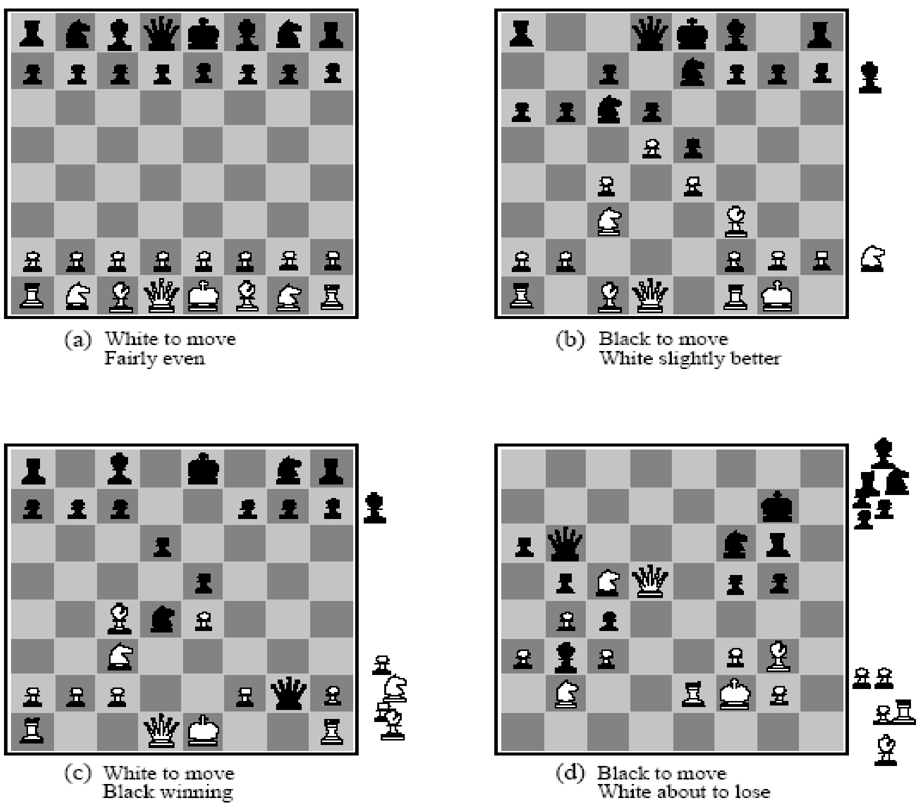
- Idea: produce an estimate of the expected utility of the game from a given position.
- Performance depends on quality of EVAL.
- Requirements:
  - EVAL should order terminal-nodes in the same way as UTILITY.
  - Computation may not take too long.
  - For non-terminal states the EVAL should be strongly correlated with the actual chance of winning.
- Only useful for quiescent (no wild swings in value in near future) states

Heuristic EVAL example



$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

Heuristic EVAL example

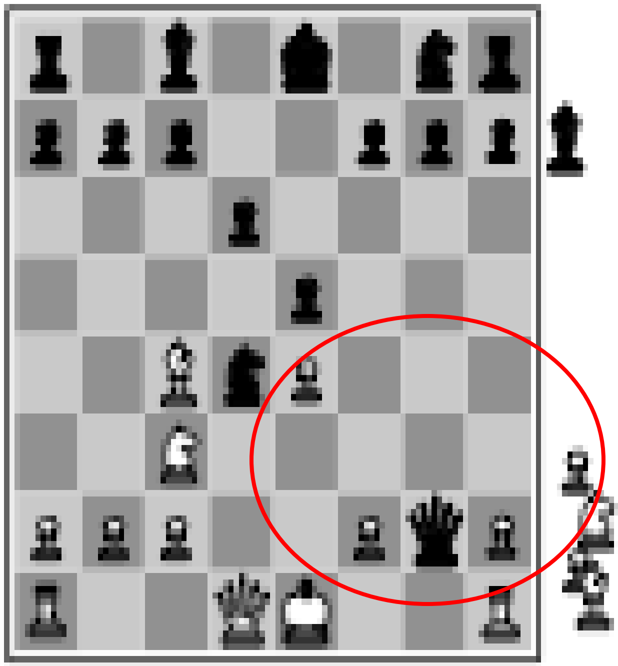


Addition assumes independence

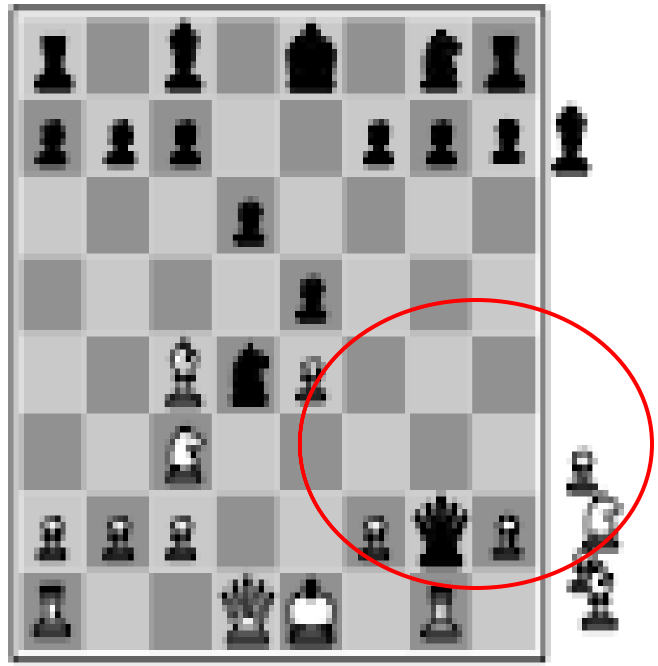
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

Heuristic difficulties

Heuristic counts pieces won



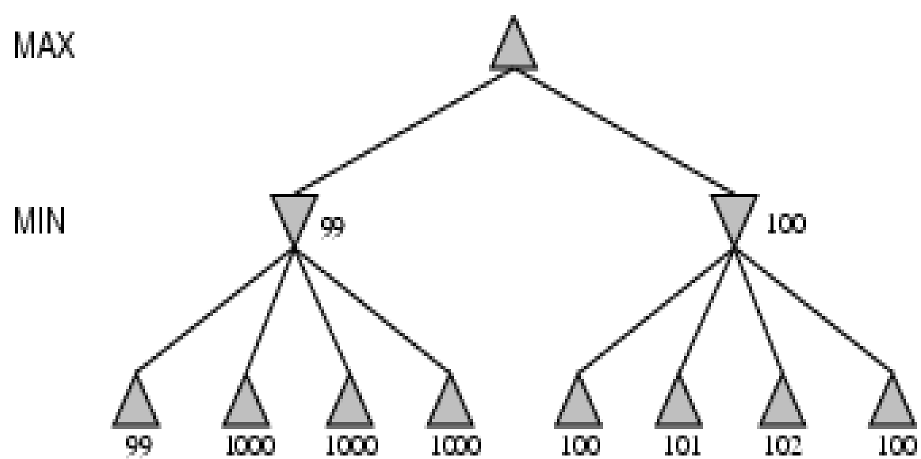
(a) White to move



(b) White to move

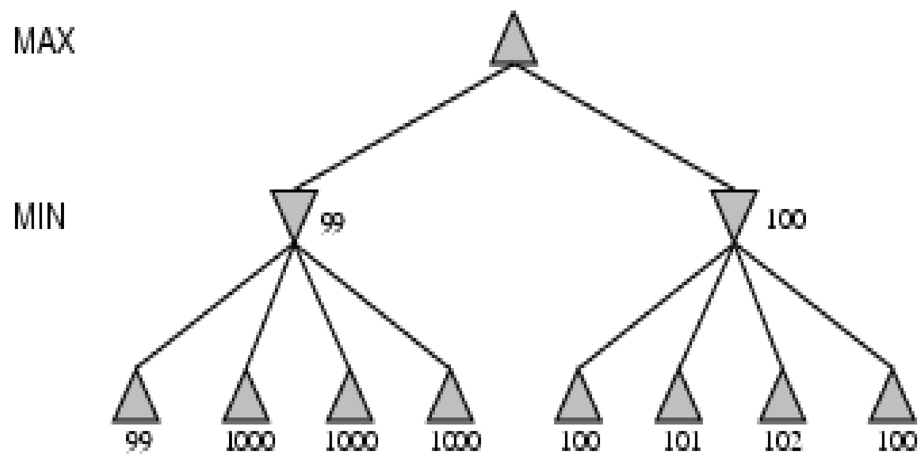
Discussion

- ❖ Examine section on state-of-the-art games yourself
- ❖ Minimax assumes right tree is better than left, yet …
  - Return probability distribution over possible values
  - Yet expensive calculation



Discussion

- Utility of node expansion
  - Only expand those nodes which lead to significanlty better moves
- Both suggestions require meta-reasoning



Summary

- Games are fun (and dangerous)
- They illustrate several important points about AI
  - Perfection is unattainable -> approximation
  - Good idea what to think about
  - Uncertainty constrains the assignment of values to states
- Games are to AI as grand prix racing is to automobile design.