ASSIGNMENT-1

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1 QUESTION NO-2.2

Construct an isosceles triangle whose base is $a = 8$cm and altitude $AD = h = 4$cm.

2 SOLUTION

Given,

$$a = 8, \ h = 4 \qquad (2.0.1)$$

we use the Pythagoras theorem,

$$c^2 = a^2 + b^2 \qquad (2.0.2)$$
$$\implies c^2 = 4^2 + 4^2 \qquad (2.0.3)$$
$$\implies c^2 = 32 \qquad (2.0.4)$$
$$\implies c = 5.6 \qquad (2.0.5)$$

$$\mathbf{b} = \mathbf{c} \qquad (2.0.6)$$

Two sides are equal so $\triangle ABC$ is isosceles triangle
Let the vertices of $\triangle ABC$ and $\mathbf{D}$ be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} p \\ 0 \end{pmatrix} \qquad (2.0.7)$$

Then

$$AB = \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0} \quad (2.0.8)$$
$$BC = \|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \qquad (2.0.9)$$
$$AC = \|\mathbf{A} - \mathbf{C}\|^2 = b^2 \qquad (2.0.10)$$

From (2.0.10),

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \qquad (2.0.11)$$
$$= \mathbf{A}^T\mathbf{A} + \mathbf{C}^T\mathbf{C} - \mathbf{A}^T\mathbf{C} - \mathbf{C}^T\mathbf{A} \qquad (2.0.12)$$
$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \quad \left(\because \mathbf{A}^T\mathbf{C} = \mathbf{C}^T\mathbf{A}\right) \qquad (2.0.13)$$
$$= a^2 + c^2 - 2ap \qquad (2.0.14)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \qquad (2.0.15)$$
$$p = \frac{8^2 + (5.6)^2 - (5.6)^2}{16} \qquad (2.0.16)$$
$$p = 4 \qquad (2.0.17)$$

From (2.0.8),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \qquad (2.0.18)$$
$$\implies q = \pm\sqrt{c^2 - p^2} \qquad (2.0.19)$$
$$\implies q = \pm\sqrt{(5.6)^2 - 4^2} \qquad (2.0.20)$$
$$\implies q = \pm3.9 \qquad (2.0.21)$$

Let the vertices of $\triangle ABC$ and $\mathbf{D}$ be

$$\mathbf{A} = \begin{pmatrix} 4 \\ 3.9 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \qquad (2.0.22)$$
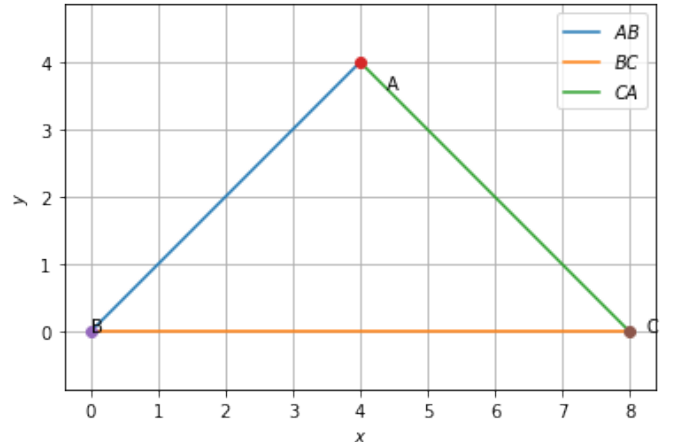

Fig. 2.1: isosceles triangle$\triangle ABC$