1

ASSIGNMENT-3

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Download all python codes from

https://github.com/ThurpuNaveena/ASSIGNMENT -3/tree/main/CODES

and latex-tikz codes from

https://github.com/ThurpuNaveena/ASSIGNMENT -3/tree/main

1 QUESTION No-2.36 (a) (Linear forms)

Find the equation of the planes that passes through three points $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 4 \\ -5 \end{pmatrix}$, $\begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}$

2 Solution

$$\mathbf{R} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 6 \\ 4 \\ -5 \end{pmatrix} \text{ and } \mathbf{T} = \begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}$$
 (2.0.1)

If the equation of the plane is given by

$$\mathbf{n}^T \mathbf{x} = 1 \tag{2.0.2}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
 (2.0.3)

Row reducing the augmented matrix,

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 6 & 4 & -5 & 1 \\ -4 & -2 & 3 & 1 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 6R_1} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 1 & -5 \\ -4 & -2 & 3 & 1 \end{pmatrix}$$
(2.0.4)

$$\stackrel{R_3 \to R_3 + 4R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 1 & -5 \\ 0 & 2 & -1 & 5 \end{pmatrix}$$
(2.0.5)

$$\stackrel{R_2 \to \frac{-R_2}{3}}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & \frac{-1}{2} & \frac{5}{2} \\ 0 & 2 & -1 & 5 \end{pmatrix}$$
(2.0.6)

$$\stackrel{R_1 \to R_1 - R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & \frac{-1}{2} & \frac{-3}{2} \\ 0 & 1 & \frac{-1}{2} & \frac{5}{2} \\ 0 & 2 & -1 & 5 \end{pmatrix}$$
(2.0.7)

$$\stackrel{R_3 \to R_3 - 2R_2}{\longleftrightarrow} \begin{pmatrix}
1 & 0 & \frac{-1}{2} & \frac{-3}{2} \\
0 & 1 & \frac{-1}{2} & \frac{5}{2} \\
0 & 0 & 0 & 0
\end{pmatrix}$$
(2.0.8)

$$\begin{pmatrix} 1 & 0 & -0.5 & -1.5 \\ 0 & 1 & -0.5 & 2.5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(2.0.9)$$

Number of Non zero row =2. Rank of Matrix =2 For the linear equation to have infinite solution. Rank(coefficient matrix) = Rank(Augmented matrix) and both not equal to Rank (Full matrix)

$$\begin{pmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{pmatrix} \tag{2.0.10}$$

$$\stackrel{R_2 \to R_2 - 6R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ -4 & -2 & -3 \end{pmatrix}$$
 (2.0.11)

$$\stackrel{R_3 \to R_3 + 4R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$
(2.0.12)

$$\stackrel{R_2 \to \frac{-R_2}{2}}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & \frac{-1}{2} \\ 0 & 2 & -1 \end{pmatrix}$$
(2.0.13)

$$\stackrel{R_1 \to R_1 - R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{-1}{2} \\ 0 & 2 & -1 \end{pmatrix}$$
(2.0.14)

$$\stackrel{R_3 \to R_3 - 2R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{-1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$
(2.0.15)

$$\begin{pmatrix}
1 & 0 & -0.5 \\
0 & 1 & -0.5 \\
0 & 0 & 0
\end{pmatrix}$$
(2.0.16)

Rank of matrix =3

Rank of coefficient matrix = 0

since the given matrix has infinite solutions, there are infinitely many planes passing through the points which means that they lie on a straight line.

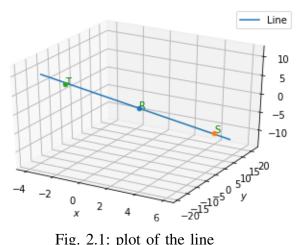


Fig. 2.1: plot of the line