ASSIGNMENT-3

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Download all python codes from

https://github.com/ThurpuNaveena/ASSIGNMENT−3/tree/main/CODES

and latex-tikz codes from

https://github.com/ThurpuNaveena/ASSIGNMENT−3/tree/main

1 QUESTION No-2.36 (a) (Linear forms)

Find the equation of the planes that passes through three points $\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}6\\4\\-5\end{pmatrix},\begin{pmatrix}-4\\-2\\3\end{pmatrix}$

2 Solution

$$\mathbf{R}=\begin{pmatrix}1\\1\\-1\end{pmatrix},\mathbf{S}=\begin{pmatrix}6\\4\\-5\end{pmatrix}and\mathbf{T}=\begin{pmatrix}-4\\-2\\3\end{pmatrix} \tag{2.0.1}$$

If the equation of the plane is given by

$$\mathbf{n}^T\mathbf{x}=C, \tag{2.0.2}$$

$$\mathbf{n}^T\mathbf{R}=\mathbf{n}^T\mathbf{S}=\mathbf{n}^T\mathbf{T}=C, \tag{2.0.3}$$

$$(\mathbf{R}-\mathbf{SS}-\mathbf{T})^T\mathbf{n}=0 \tag{2.0.4}$$

after some algebra, using row reduction on the above matrix

$$\begin{pmatrix}-5&-3&4\\10&6&-8\end{pmatrix}\xleftrightarrow{R_1\to 2R_1+R_2}\begin{pmatrix}-10&0&8\\10&6&-8\end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2\to R_2+R_1}\begin{pmatrix}-10&0&8\\0&6&0\end{pmatrix} \tag{2.0.6}$$

Thus

$$\mathbf{n}=\begin{pmatrix}\frac{8}{10}\\0\\1\end{pmatrix}=\begin{pmatrix}\frac{4}{5}\\0\\1\end{pmatrix}=\begin{pmatrix}4\\0\\5\end{pmatrix} \tag{2.0.7}$$

$$C=\mathbf{n}^T\mathbf{T}=-1 \tag{2.0.8}$$

Thus, the equation of the plane is

$$\begin{pmatrix}4&0&5\end{pmatrix}\mathbf{n}=-1 \tag{2.0.9}$$

Alternatively, the normal vector to the plane can be obtained as

$$\mathbf{n}=(\mathbf{R}-\mathbf{S})\times(\mathbf{S}-\mathbf{T}) \tag{2.0.10}$$

The equation of the plane is then obtained from

$$\mathbf{n}^T(\mathbf{x}-\mathbf{T})=[(\mathbf{R}-\mathbf{S})\times(\mathbf{S}-\mathbf{T})]^T(\mathbf{x}-\mathbf{T})\quad=0 \tag{2.0.11}$$

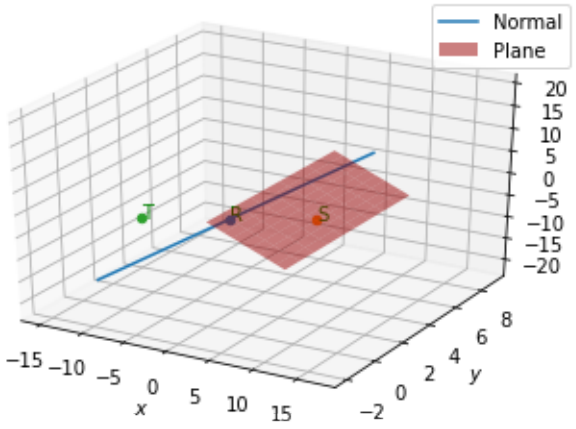

Fig. 2.1: Plot of the plane