ASSIGNMENT-3

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Download all python codes from

https://github.com/ThurpuNaveena/ASSIGNMENT−3/tree/main/CODES

and latex-tikz codes from

https://github.com/ThurpuNaveena/ASSIGNMENT−3/tree/main

1 QUESTION No-2.36 (a) (Linear forms)

Find the equation of the planes that passes through three points $\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}6\\4\\-5\end{pmatrix},\begin{pmatrix}-4\\-2\\3\end{pmatrix}$

2 Solution

$$\mathbf{R} = \begin{pmatrix}1\\1\\-1\end{pmatrix}, \mathbf{S} = \begin{pmatrix}6\\4\\-5\end{pmatrix} \text{and} \mathbf{T} = \begin{pmatrix}-4\\-2\\3\end{pmatrix} \tag{2.0.1}$$

If the equation of the plane is given by

$$\mathbf{n}^T\mathbf{x} = 1 \tag{2.0.2}$$

$$\begin{pmatrix}1 & 1 & -1\\6 & 4 & -5\\-4 & -2 & 3\end{pmatrix}\mathbf{n} = \begin{pmatrix}1\\1\\1\end{pmatrix} \tag{2.0.3}$$

Row reducing the augmented matrix,

$$\begin{pmatrix}1 & 1 & -1 & 1\\6 & 4 & -5 & 1\\-4 & -2 & 3 & 1\end{pmatrix} \xleftrightarrow{R_2\to R_2-6R_1} \begin{pmatrix}1 & 1 & -1 & 1\\0 & -2 & 1 & -5\\-4 & -2 & 3 & 1\end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_3\to R_3+4R_1} \begin{pmatrix}1 & 1 & -1 & 1\\0 & -2 & 1 & -5\\0 & 2 & -1 & 5\end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2\to \frac{-R_2}{3}} \begin{pmatrix}1 & 1 & -1 & 1\\0 & 1 & \frac{-1}{2} & \frac{5}{2}\\0 & 2 & -1 & 5\end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_1\to R_1-R_2} \begin{pmatrix}1 & 0 & \frac{-1}{2} & \frac{-3}{2}\\0 & 1 & \frac{-1}{2} & \frac{5}{2}\\0 & 2 & -1 & 5\end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_3\to R_3-2R_2} \begin{pmatrix}1 & 0 & \frac{-1}{2} & \frac{-3}{2}\\0 & 1 & \frac{-1}{2} & \frac{5}{2}\\0 & 0 & 0 & 0\end{pmatrix} \tag{2.0.8}$$

$$\begin{pmatrix}1 & 0 & -0.5 & -1.5\\0 & 1 & -0.5 & 2.5\\0 & 0 & 0 & 0\end{pmatrix} \tag{2.0.9}$$

Number of Non zero row =2. Rank of Matrix =2 For the linear equation to have infinite solution. Rank(coefficient matrix) = Rank(Augmented matrix)and both not equal to Rank (Full matrix)

$$\begin{pmatrix}1 & 1 & -1\\6 & 4 & -5\\-4 & -2 & 3\end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow{R_2 \to R_2 - 6R_1} \begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ -4 & -2 & -3 \end{pmatrix} \tag{2.0.11}$$

$$\xleftrightarrow{R_3 \to R_3 + 4R_1} \begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 2 & -1 \end{pmatrix} \tag{2.0.12}$$

$$\xleftrightarrow{R_2 \to \frac{-R_2}{2}} \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & \frac{-1}{2} \\ 0 & 2 & -1 \end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow{R_1 \to R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{-1}{2} \\ 0 & 2 & -1 \end{pmatrix} \tag{2.0.14}$$

$$\xleftrightarrow{R_3 \to R_3 - 2R_2} \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{-1}{2} \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.15}$$

$$\begin{pmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.16}$$

Rank of matrix =3

Rank of coefficient matrix = 0

since the given matrix has infinite solutions,there are infinitely many planes passing through the points which means that they lie on a straight line.

Equation of the line

$$\mathbf{A} = \begin{pmatrix} 1 \\ 6 \\ -4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \tag{2.0.17}$$

$$\mathbf{x} = \mathbf{A} + \lambda (\mathbf{B} - \mathbf{A}) \tag{2.0.18}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 6 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \tag{2.0.19}$$

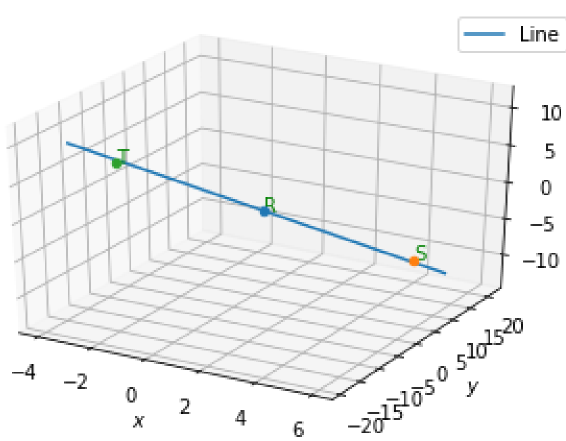

Fig. 2.1: plot of the line