ASSIGNMENT-4

T.Naveena

Download all python codes from

https://github.com/ThurpuNaveena/Assignment−4/blob/main/Assignment−4.py

and latex-tikz codes from

https://github.com/ThurpuNaveena/Assignment−4/blob/main/main.tex

1 Question No 2.25

Find the discriminant of the quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ hence find the nature of its roots.

2 SOLUTION

Given $3x^2 - 2x + \frac{1}{3} = 0$ can be expressed as

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -2 & -1 \end{pmatrix} \mathbf{x} + \frac{1}{3} = 0 \tag{2.0.1}$$

Compare given quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ with $ax^2 + bx + c = 0$,we get

$$a = 3, b = -2, c = \frac{1}{3} \tag{2.0.2}$$

$$Discriminant(D) = b^2 - 4ac \tag{2.0.3}$$

$$= (-2)^2 - 4(3)(\frac{1}{3}) \tag{2.0.4}$$

$$= 4 - 4 \tag{2.0.5}$$

$$= 0 \quad (\because D = 0) \tag{2.0.6}$$

Discriminant is zero and the nature of roots of equation $3x^2 - 2x + \frac{1}{3} = 0$.

$$y = 3x^2 - 2x + \frac{1}{3} \tag{2.0.7}$$

$$\implies 3x^2 - 2x + \frac{1}{3} - y = 0 \tag{2.0.8}$$

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -1 \\ \frac{-1}{2} \end{pmatrix} \mathbf{x} + \frac{1}{3} = 0 \tag{2.0.9}$$

Here,

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -1 \\ \frac{-1}{2} \end{pmatrix}, f = \frac{1}{3} \tag{2.0.10}$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.11}$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p_1}^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.12}$$

$\therefore$Vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} -1 & -1 \\ 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} \frac{-1}{3} \\ 1 \\ 0 \end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix} -1 & -1 \\ 3 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} \frac{-1}{3} \\ 1 \end{pmatrix} \tag{2.0.14}$$

$$\implies \mathbf{c} = \begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix} \tag{2.0.15}$$

Now,

$$\mathbf{p_1}^T \mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix} \tag{2.0.16}$$

$$= 0 \tag{2.0.17}$$

and,

$$\mathbf{p_2}^T \mathbf{V} \mathbf{p_2} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.18}$$

$$= 3 \tag{2.0.19}$$

$\because$

$$(\mathbf{p_1}^T \mathbf{c})(\mathbf{p_2}^T \mathbf{V} \mathbf{p_2}) = 0 \tag{2.0.20}$$

Hence,it has real and equal roots.

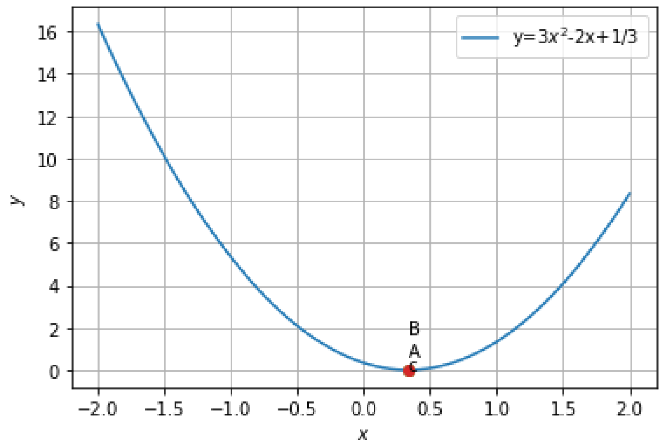

Fig. 2.1: Roots of $3x^2 - 2x + 1/3 = 0$