ASSIGNMENT-4

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Download all python codes from

https://github.com/ThurpuNaveena/Assignment−4/blob/main/Assignment−4.py

and latex-tikz codes from

https://github.com/ThurpuNaveena/Assignment−4/blob/main/main.tex

1 Question No 2.25

Find the discriminant of the quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ hence find the nature of its roots.

2 SOLUTION

Given $3x^2 - 2x + \frac{1}{3} = 0$ can be expressed as

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -2 & -1 \end{pmatrix} \mathbf{x} + \frac{1}{3} = 0 \tag{2.0.1}$$

Compare given quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ with $ax^2 + bx + c = 0$,we get

$$a = 3, b = -2, c = \frac{1}{3} \tag{2.0.2}$$

$$Discriminant(D) = b^2 - 4ac \tag{2.0.3}$$

$$= (-2)^2 - 4(3)(\frac{1}{3}) \tag{2.0.4}$$

$$= 4 - 4 \tag{2.0.5}$$

$$= 0 \quad (\because D = 0) \tag{2.0.6}$$

Discriminant is zero and the nature of roots of equation $3x^2 - 2x + \frac{1}{3} = 0$. The vertex can be found by $\frac{-b}{2a}$ for the x coordinate

$$\frac{-b}{2a} \tag{2.0.7}$$

$$\mathbf{x} = \frac{1}{3} \tag{2.0.8}$$

$$\mathbf{y} = 3\left(\mathbf{x} - \tfrac{1}{3}\right)^2 \tag{2.0.9}$$

$$\left(\tfrac{1}{3}, 0\right) \tag{2.0.10}$$

vertex $\left(\frac{1}{3}, 0\right)$

The vector form of equation is

$$y = 3x^2 - 2x + \frac{1}{3} \tag{2.0.11}$$

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -2 & -1 \end{pmatrix} \mathbf{x} + \frac{1}{3} = 0 \tag{2.0.12}$$

Thus

$$y = 0 \implies 3x^2 - 2x + \frac{1}{3} = 0 \tag{2.0.13}$$

$$x = \frac{1}{3} \tag{2.0.14}$$

$$x - intercept\left(\tfrac{1}{3}, 0\right) \tag{2.0.15}$$

$$y - intercept\left(0, \tfrac{1}{3}\right) \tag{2.0.16}$$

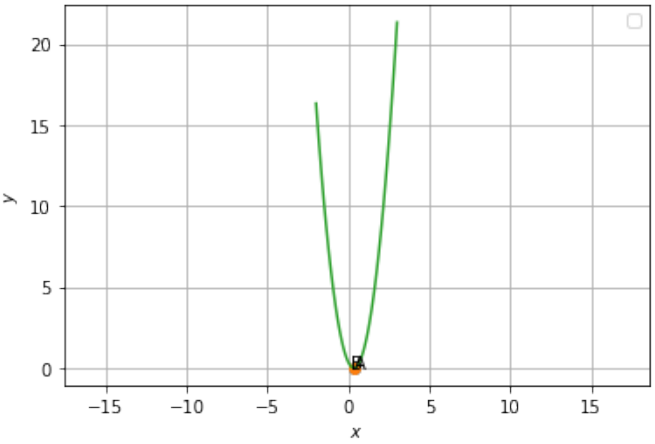

Fig. 2.1: Roots of $3x^2 - 2x + 1/3 = 0$