

ASSIGNMENT-4

T.Naveena

Download all python codes from

<https://github.com/ThurpuNaveena/Assignment-4/blob/main/Assignment-4.py>

and latex-tikz codes from

<https://github.com/ThurpuNaveena/Assignment-4/blob/main/main.tex>

1 QUESTION No 2.25

Find the discriminant of the quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ hence find the nature of its roots.

2 SOLUTION

Given $3x^2 - 2x + \frac{1}{3} = 0$ can be expressed as

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + (-2 \quad -1) \mathbf{x} + \frac{1}{3} = 0 \quad (2.0.1)$$

Compare given quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -2, c = \frac{1}{3} \quad (2.0.2)$$

$$\text{Discriminant}(D) = b^2 - 4ac \quad (2.0.3)$$

$$= (-2)^2 - 4(3)\left(\frac{1}{3}\right) \quad (2.0.4)$$

$$= 4 - 4 \quad (2.0.5)$$

$$= 0 \quad (\because D = 0) \quad (2.0.6)$$

Discriminant is zero and the nature of roots of equation $3x^2 - 2x + \frac{1}{3} = 0$. The vertex can be found by $\frac{-b}{2a}$ for the x coordinate

$$\frac{-b}{2a} \quad (2.0.7)$$

$$x = \frac{1}{3} \quad (2.0.8)$$

$$y = 3\left(x - \frac{1}{3}\right)^2 \quad (2.0.9)$$

$$\left(\frac{1}{3}, 0\right) \quad (2.0.10)$$

vertex $\left(\frac{1}{3}, 0\right)$

The vector form of equation is

$$y = 3x^2 - 2x + \frac{1}{3} \quad (2.0.11)$$

For $\mathbf{x} = \begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix}$ substitute in (2.0.1)

$$\begin{pmatrix} \frac{1}{3} & 0 \end{pmatrix}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix} + (-2 \quad -1) \begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix} + \frac{1}{3} = 0 \quad (2.0.12)$$

Hence $\frac{1}{3}$ is a zero

The graph of the given parabola intersect the x-axis at the one point. Hence it has real and equal roots.

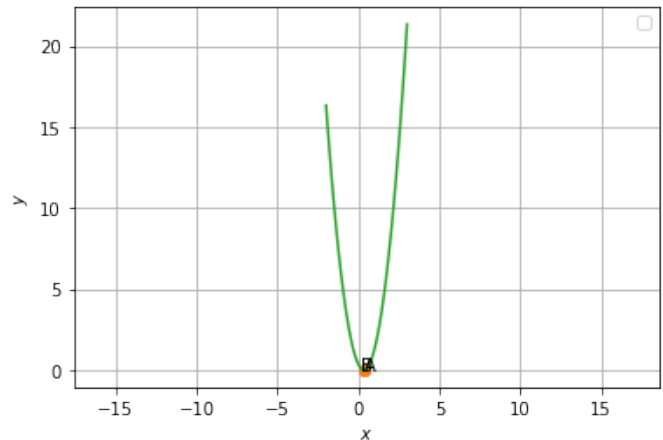


Fig. 2.1: Roots of $3x^2 - 2x + 1/3 = 0$