

# ASSIGNMENT-4

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Download all python codes from

<https://github.com/ThurpuNaveena/Assignment-4/blob/main/Assignment-4.py>

and latex-tikz codes from

<https://github.com/ThurpuNaveena/Assignment-4/blob/main/main.tex>

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.0.11)$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.12)$$

$\therefore$  Vertex  $\mathbf{c}$  is given by

$$\begin{pmatrix} -1 & -1 \\ 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} \frac{-1}{3} \\ 1 \\ 0 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \begin{pmatrix} -1 & -1 \\ 3 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} \frac{-1}{3} \\ 1 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \mathbf{c} = \begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix} \quad (2.0.15)$$

Now,

$$\mathbf{p}_1^T \mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ 0 \end{pmatrix} \quad (2.0.16)$$

$$= 0 \quad (2.0.17)$$

and,

$$\mathbf{p}_2^T \mathbf{V} \mathbf{p}_2 = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.18)$$

$$= 3 \quad (2.0.19)$$

$\therefore$

$$(\mathbf{p}_1^T \mathbf{c})(\mathbf{p}_2^T \mathbf{V} \mathbf{p}_2) = 0 \quad (2.0.20)$$

Hence, it has real and equal roots.

## 1 QUESTION No 2.25

Find the discriminant of the quadratic equation  $3x^2 - 2x + \frac{1}{3} = 0$  hence find the nature of its roots.

## 2 SOLUTION

Given  $3x^2 - 2x + \frac{1}{3} = 0$  can be expressed as

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -2 & -1 \end{pmatrix} \mathbf{x} + \frac{1}{3} = 0 \quad (2.0.1)$$

Compare given quadratic equation  $3x^2 - 2x + \frac{1}{3} = 0$  with  $ax^2 + bx + c = 0$ , we get

$$a = 3, b = -2, c = \frac{1}{3} \quad (2.0.2)$$

$$\text{Discriminant}(D) = b^2 - 4ac \quad (2.0.3)$$

$$= (-2)^2 - 4(3)\left(\frac{1}{3}\right) \quad (2.0.4)$$

$$= 4 - 4 \quad (2.0.5)$$

$$= 0 \quad (\because D = 0) \quad (2.0.6)$$

Discriminant is zero and the nature of roots of equation  $3x^2 - 2x + \frac{1}{3} = 0$ .

$$y = 3x^2 - 2x + \frac{1}{3} \quad (2.0.7)$$

$$\Rightarrow 3x^2 - 2x + \frac{1}{3} - y = 0 \quad (2.0.8)$$

$$\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -1 \\ \frac{-1}{2} \end{pmatrix} \mathbf{x} + \frac{1}{3} = 0 \quad (2.0.9)$$

Here,

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -1 \\ \frac{-1}{2} \end{pmatrix}, f = \frac{1}{3} \quad (2.0.10)$$

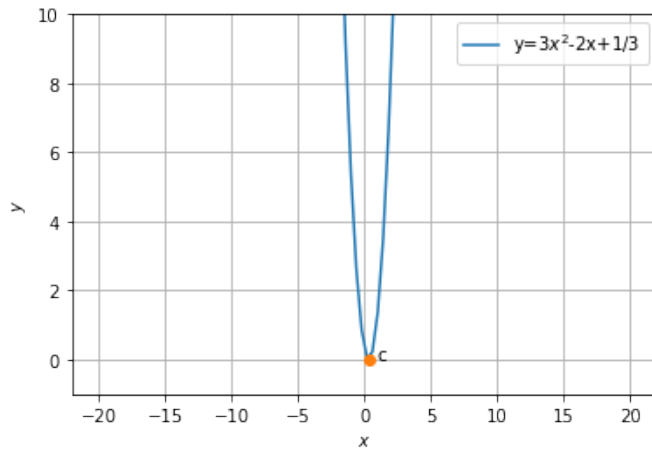


Fig. 2.1: Roots of  $3x^2 - 2x + 1/3 = 0$