ASSIGNMENT-5

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Download all python codes from

https://github.com/ThurpuNaveena/Assignment−5/blob/master/codes.py

and latex-tikz codes from

https://github.com/ThurpuNaveena/Assignment−5

1 Question No 2.36

Find the equation of the ellipse,whose length of the Major axis is 20 and foci $=\begin{pmatrix} 0 \\ \pm 5 \end{pmatrix}$

2 Solution

Given that,

$$\text{length of the major axis } = 2a = 20 \tag{2.0.1}$$

$$\therefore \text{Length of semi major axis, } a = 10 \tag{2.0.2}$$

$$\text{Foci} = \mathbf{F} = \begin{pmatrix} 0 \\ \pm 5 \end{pmatrix} \tag{2.0.3}$$

Lemma 2.1. *The standard equation of an ellipse is given by:*

$$\frac{\mathbf{y}^\top D\mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.4}$$

$$\textit{where, } D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{2.0.5}$$

Lemma 2.2. *The coordinates of foci* $\mathbf{F}$ *of ellipse with y-axis as major axis are:*

$$\mathbf{F} = \begin{pmatrix} 0 \\ \pm\left(\sqrt{\frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}}\right) \end{pmatrix} \tag{2.0.6}$$

Also,the length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} \tag{2.0.7}$$

and the length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} \tag{2.0.8}$$

1) From (2.0.7) length of semi-major axis is:

$$\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} = a \tag{2.0.9}$$

$$\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1} = a^2 \tag{2.0.10}$$

$$\implies \lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{a^2} \tag{2.0.11}$$

2) From (2.0.6), the focus of ellipse is given as:

$$\mathbf{F} = \begin{pmatrix} 0 \\ \pm\left(\sqrt{\frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}}\right) \end{pmatrix} \tag{2.0.12}$$

or

$$\|\mathbf{F}\|^2 = \frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} \tag{2.0.13}$$

$$\|\mathbf{F}\|^2 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1} - \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} \tag{2.0.14}$$

3) Putting value of λ_1 from (2.0.11) in above equation, we get:

$$\|\mathbf{F}\|^2 = a^2 - \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} \tag{2.0.15}$$

$$\|\mathbf{F}\|^2 - a^2 = -\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2} \tag{2.0.16}$$

$$\implies \lambda_2 = -\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\|\mathbf{F}\|^2 - a^2} \tag{2.0.17}$$

4) *For finding* λ_1:
   - From (2.0.11) we have:

$$\lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{a^2} \tag{2.0.18}$$

$$\implies \lambda_1 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{100} \ (\because a = 10) \tag{2.0.19}$$

5) *For finding* λ_2*:*
   - Putting value of a from (2.0.2) and $\mathbf{F}$ from (2.0.3) in equation (2.0.17),we get:

$$\lambda_2 = -\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\left(\sqrt{0^2 + 5^2}\right)^2 - 10^2} \tag{2.0.20}$$

$$\lambda_2 = -\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{25 - 100} \quad (2.0.21)$$

$$\lambda_2 = \frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{75} \quad (2.0.22)$$

6) Using lemma (2.1),the standard equation of ellipse is given by :

$$\frac{\mathbf{y}^\top D\mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \quad (2.0.23)$$

$$\implies \frac{\mathbf{y}^\top \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \quad (2.0.24)$$

7) Putting (2.0.19) and (2.0.22) in above equation we get:

$$\implies \mathbf{y}^\top \begin{pmatrix} \frac{1}{100} & 0 \\ 0 & \frac{1}{75} \end{pmatrix} \mathbf{y} = 1 \quad (2.0.25)$$

which is the required equation of ellipse.

8) The Plot of ellipse is:

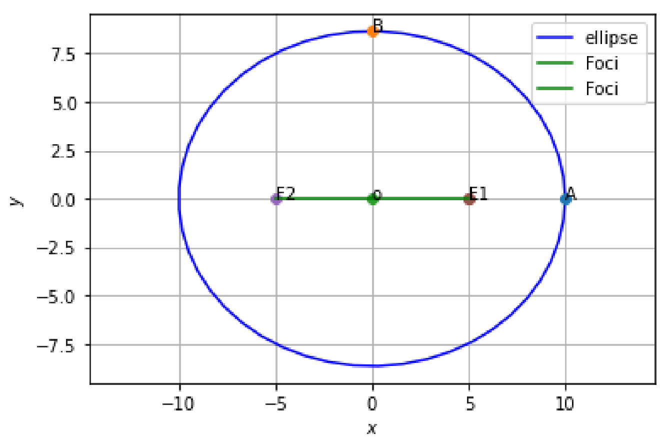

Fig. 2.1: Ellipse $\frac{x^2}{100} + \frac{y^2}{75} = 1$