Solution for the exercise - Video 02

x_i	$f(x_i)$
3.0	2.5
4.5	1.0
7.0	2.5
9.0	0.5

No. of Data pts. 4

$\therefore n+1 = 4 \Rightarrow n = 3$

~~No. of intervals = $3n = 3(3) = 9$~~

No. of intervals = $n = 3$

For Quadratic splines } $= 3n$
no. of unknowns

$= 3(3) = 9$

From Condition 1

$$20.25 a_1 + 4.5 b_1 + c_1 = 1.0 \quad — (1)$$

$$20.25 a_2 + 4.5 b_2 + c_2 = 1.0 \quad — (2)$$

$$49 a_2 + 7 b_2 + c_2 = 2.5 \quad — (3)$$

$$49 a_3 + 7 b_3 + c_3 = 2.5 \quad — (4)$$

Total eq^{n}s $= 2n - 2 = 2(3) - 2 = 4$

From Condition 2

$$9 a_1 + 3 b_1 + c_1 = 2.5 \quad — (5)$$

$$81 a_3 + 9 b_3 + c_3 = 0.5 \quad — (6)$$

2 - eq^{n}s.

Total eq^{n}s $= 4 + 2 = 6$

From Condition 3

$$9a_1 + b_1 = 9a_2 + b_2 \quad -(7)$$

$$14a_2 + b_2 = 14a_3 + b_3 \quad -(8)$$

$eq^{n}s = (n-1) = 3-1 = 2$

Total $eq^{n}s = 6+2 = 8$

From Condition 4

$$a_1 = 0 \quad -(9)$$

Total $eq^{n}s = 9$

Represent all 9 $eq^{n}s$ in $eq^{n}s$ 1

$\underline{A}\,\underline{X} = \underline{b}$ form & solve

For $\underline{X}$.

Then

$a_1 = 0$ $\quad b_1 = -1$ $\quad c_1 = 5.5$

$a_2 = 0.64$ $\quad b_2 = -6.76$ $\quad c_2 = 18.46$

$a_3 = -1.6$ $\quad b_3 = 24.6$ $\quad c_3 = -91.3$

$$f(x)=\begin{cases} -x + 5.5 & 3.0 \le x \le 4.5 \\ 0.64x^2 - 6.76x + 18.46 & 4.5 \le x \le 7.0 \\ -1.6x^2 + 24.6x - 91.3 & 7.0 \le x \le 9.0 \end{cases}$$

$f(5) = 0.64(5)^2 - 6.76(5) + 18.46 =$ ✓

$$\int_{4.0}^{7.5} f(x)\,dx = \int_{4.0}^{4.5} (-x+5.5)\,dx + \int_{4.5}^{7.0} (0.64x^2 - 6.76x + 18.46)\,dx$$

$$+ \int_{7.0}^{7.5} (-1.6x^2 + 24.6x - 91.3)\,dx$$

$=$ ✓

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