# CHANNEL CODING - FINAL PROJECT

## UNIFIED HIGH-SPEED WIRELINE-BASE HOME NETWORKING TRANSCEIVERS

Access networks – In premises networks


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DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

UNIVERSITÀ DEGLI STUDI DI PADOVA

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# An introduction

- The **G.hn ITU G9960** is a standard prosed by the ITU-T.
- **Designed for**: the transmission of data over premises' wiring.
- The standard **defines**:

    1. the home network architecture and reference models.
    2. the physical layer specification.

International Telecommunication Union

# Transmission modes

- Each node of the home-network has a different **profile**.
  - The standard refers to two different profile:
    1. **Low-complexity profile** (L-CP).
    2. **Standard profile** (SP).
    - A node of the network is required to support one profile, at minimum.

| Profile name | Domain type | Valid bandplans |
|---|---|---|
| L-CP | Power-line baseband | 25 MHz |
| SP | Power-line baseband | 50 MHz, 100 MHz |
| | Telephone-line baseband | 50 MHz, 100 MHz |
| | Coax baseband | 50 MHz, 100 MHz |
| | Coax RF | 50 MHz, 100 MHz, 200 MHz |

# Transmission modes



- The parity check matrix *H* of size *(N-K) x N* will have:

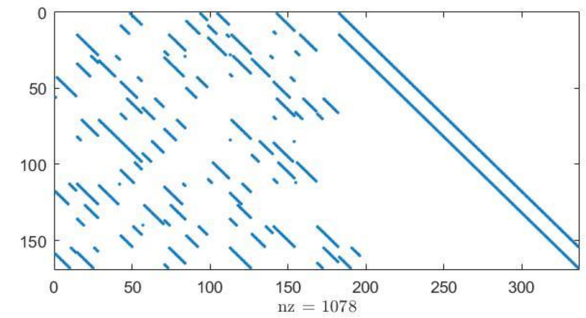
  - Rate $^1/_2$ : *N* = 1920 (*LCP,SP*), 8640 (*SP*).
  - Rate $^2/_3$ : *N* = 1440, 6480 (*SP*)
  - Rate $^5/_6$ : *N* = 1152, 5184 (*SP*)

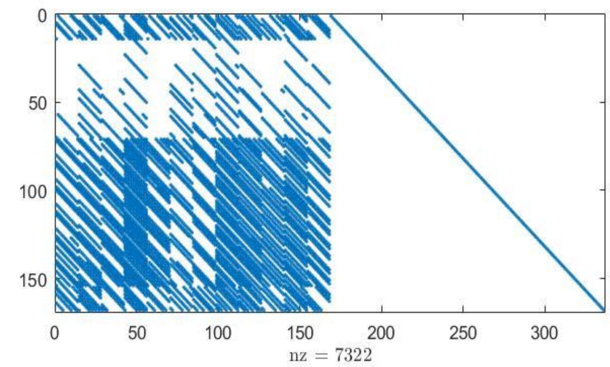
- Rate $^{16}/_{18}$ , $^{20}/_{21}$ are obtained puncturing the code with rate $^5/_6$, through different puncturing patterns.

| Profile name | FEC rate | FEC block size |
|---|---|---|
| L-CP | $^1/_2$ | 120 bytes (Payload) |
| SP | $^1/_2, ^2/_3, ^5/_6$ <br> $^{16}/_{18}$ , $^{20}/_{21}$ | 120 and 540 bytes (Payload) |

# Encoder

1. Given H, get its systematic form Hsys .



# Encoder



1. Given H, get its systematic form $H_{sys}$ .

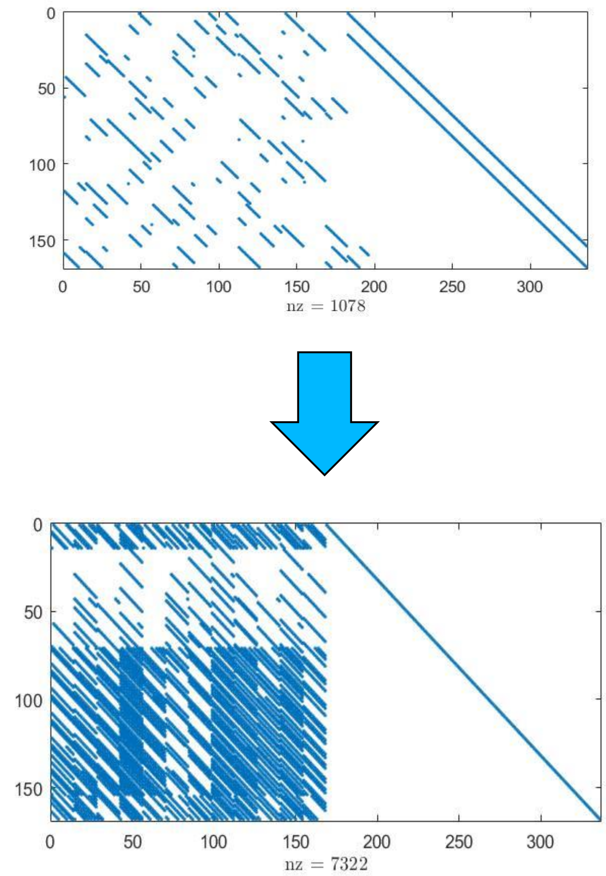
2. Encoding Procedure for a single word:

   ✓ The *K* information bits of *u* are directly copied to the codeword *v*.

   ✓ Let A be the submatrix obtained considering *N-K* raws and the first k colums of $H_{sys}$ ; then, the *N–K* parity-check bits are computed as A * u.

   ✓ The final codeword is *v* = [u|A u]

- Preprocessing is done once.



# Decoder – a first (naif) approach

- All the minsum/ sumproduct algorithm written in c, and invoking via mex functions.

- At least three matrixes as big as the *H* matrix:

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- Computational efforts.

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  ⇒ Do only once.

- From *H* we'll get:

$$\Psi_{v\rightarrow c} \quad \Psi_{c\rightarrow v} \quad \Pi_{v\rightarrow c} \quad \Pi_{c\rightarrow v} \quad \{l_i\}_{i=1}^{N-K} \quad \{j_i\}_{i=1}^{N}$$

# Decoder – Rationale

- Preprocess the matrix *H* in such a way to get all the patterns from it.

→ Do only once.

- From H we'll get:

$$\underset{v\to c}{\Psi} \qquad \underset{c\to v}{\Psi} \qquad \underset{v\to c}{\Pi} \qquad \underset{c\to v}{\Pi} \qquad \{l_i\}_{i=1}^{N-K} \qquad \{j_i\}_{i=1}^{N}$$

→ No need to evaluate directly H in the message passing for the computation of LLRs.

# Updates as a linear map



- Let be $\underset{c\to v}{\Psi}$ and $\underset{v\to c}{\Psi}$ the check update matrix and the variable update matrix, respectively:

$$\underset{c\to v}{\Psi} = \begin{bmatrix} I^C_{l_1} & & 0 \\ & \ddots & \\ 0 & & I^C_{l_{N-K}} \end{bmatrix} \qquad \underset{v\to c}{\Psi} = \begin{bmatrix} I^C_{j_1} & & 0 \\ & \ddots & \\ 0 & & I^C_{j_N} \end{bmatrix}$$

- $I^C_s$ denotes the complement of the Identity matrix of size $s$.

- $\{l_i\}_{i=1}^{N-K}$ and $\{j_i\}_{i=1}^{N}$ are the number of ones in each raw and column respectively.

# LLRs notation

- Let $\underset{c\to v}{LLR}, \underset{v\to c}{LLR} \in \mathbb{F}_2^U$ with $U$ the total numbers of ones (edges) in the H matrix.

$$\underset{c\to v}{LLR} = \begin{bmatrix} \\ \hline \\ \vdots \\ \\ \hline \\ \end{bmatrix} \begin{array}{l} \left.\vphantom{\Big|}\right\} j_1 \\ \\ \\ \\ \left.\vphantom{\Big|}\right\} j_N \end{array} \qquad \underset{v\to c}{LLR} = \begin{bmatrix} \\ \hline \\ \vdots \\ \\ \hline \\ \end{bmatrix} \begin{array}{l} \left.\vphantom{\Big|}\right\} l_1 \\ \\ \\ \\ \left.\vphantom{\Big|}\right\} l_{N-K} \end{array}$$

# Updates in minSum and sumProd



- For both minsum and sumproduct decoder the **variable updates** will have the following expression:

$$\underset{v \to c}{LLR} = \underset{v \to c}{\Psi} \cdot \underset{c \to v}{LLR} + p$$

- In the sum product the **check updates** is:

$$\underset{c \to v}{LLR} = \left( \underset{c \to v}{\Psi} \cdot \Phi\left( \left| \underset{v \to c}{LLR} \right| \right) \right) * \operatorname{sgn}\left( \underset{v \to c}{LLR} \right)$$

- Where $\Phi(x) = \log\left(\frac{e^x+1}{e^x-1}\right)$, $\operatorname{sgn}(\cdot)$ is the function that maps each element of the vector in its corresponding sign and *p* is the vector of prior LLRs.

# Permutation matrixes



In order to compute properly the updates of the LLRs, vectors must be properly ordered…

**HOW?**

# Permutation matrixes



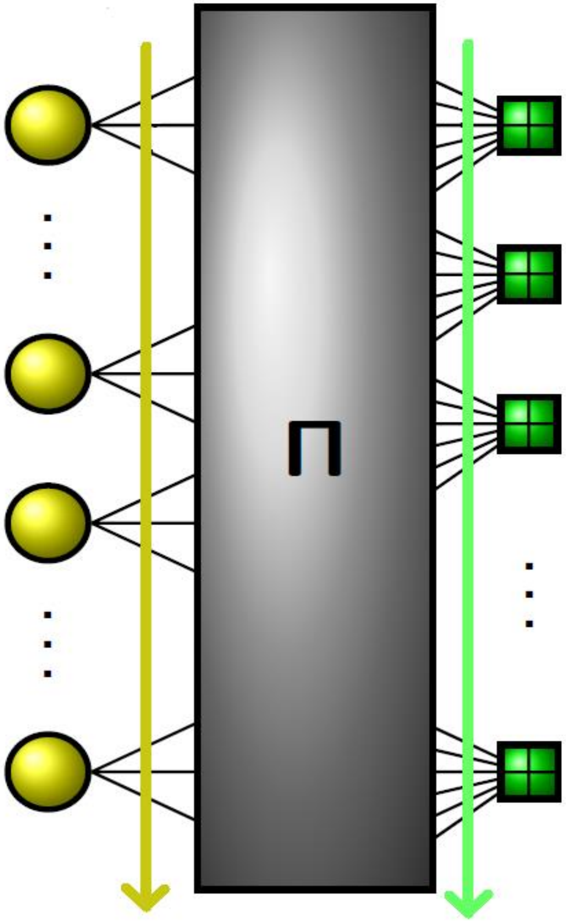
In order to compute properly the updates of the LLRs, vectors must be properly ordered…

**HOW?**

We will refer to two different LLR representation**:**

**Check representation (CR)**

**Variable representation (VR)**



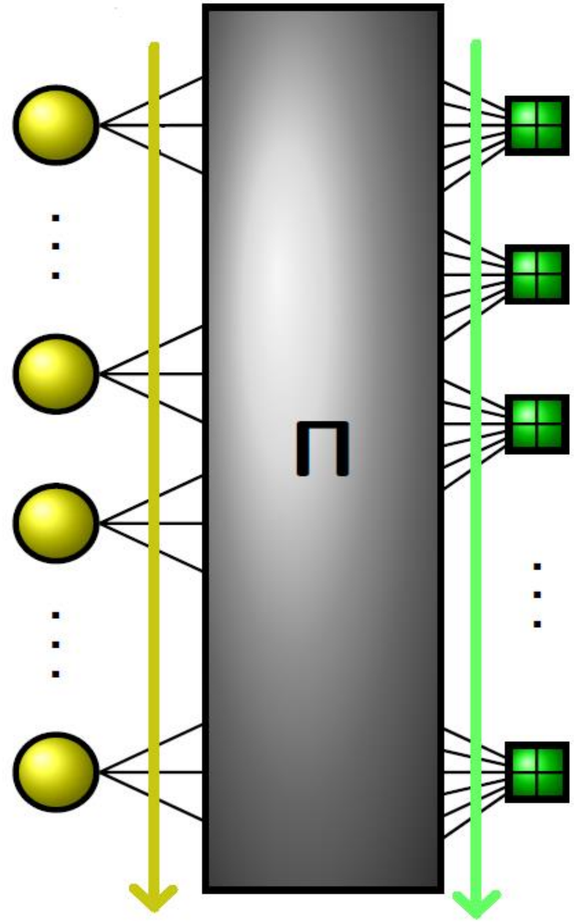
# Permutation matrixes

**CHECK REPRESENTATION (CR)**

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

**VARIABLE REPRESENTATION (VR)**

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$



# Permutation matrixes (1/2)

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- Let be $per : D_U \longrightarrow D_U$ the permutation map, that associate for each edge in **CR** the corrisponding position in the **VR**.

- $D_U = \{1, \ldots, U\}$, $U$ the number of ones (edges) in the matrix H.

$$\prod_{c \to v} = \begin{bmatrix} e_{per(1)}^{\top} \\ \hline \vdots \\ \hline e_{per(U)}^{\top} \end{bmatrix}$$

- Where $e_i \in \mathbb{F}_2^U$ denotes the i-th canonical vector in $\mathbb{F}_2^U$ .

# Permutation matrixes (2/2)



- If we think of $\prod_{c \to v}$ as a linear application and we expoit the property of the permutation matrixes:

$$\prod_{v \to c} = \prod_{c \to v}^{-1} = \prod_{c \to v}^{\top}$$

- Hence, there is no need to compute directly $\prod_{v \to c}$ since:

$$\prod_{c \to v} \longleftrightarrow \prod_{v \to c}$$

# Sum product

```
for it=1:iter
     LLRvc = perMatrVC * LLRvc;
     curSign = sign(LLRvc);
      curSign(curSign == 0) = 1;
      signLLRcv = GetSign(curSign,nOR);
     tmpP = abs(LLRvc);
     tmpP(tmpP < minf) = 1e-10;
     temp = upMatrCV * PhiMap(tmpP);
     LLRcv =  PhiMap(temp).* signLLRcv;
     LLRcv = perMatrCV * LLRcv;
     LLRvc = upMatrVC * LLRcv + LLRprior;
     curIndex = 1;
    for j=1:length(nOC)
         if(vPLLRs(j) + sum(LLRcv(curIndex:nOC(j)+curIndex-1)) < 0)
            u_hat(j) = 1;
         else
            u_hat(j) = 0;
         end
         curIndex = curIndex + nOC(j);
    end
    if( mod(Hreal * u_hat,2) == 0)
        break;
    end
end
```

Get LLRvc in **CR**

Update

Get LLRcv in **VR**

Update

# Practical considerations



- Product between sparse matrixes is optimized in Matlab. (https://it.mathworks.com/help/matlab/math/sparse-matrix-operations.html )
- Only vectors proportional to the number of ones in the check matrix are used.

**Mex function** for specific tasks have been implemented:

- Arrayfunc command of matlab leads to slow computation.
- Mex function **PhiMap()** improves dramatically the performances (Make a number).
- Search in the minsum is not bottle neck since elements to be checked are few (consequence to the fact that $H$ is sparse).

## ENCODING

- Grouping sequences of b $= \log_2 M$ bits and then mapping according to the suitable constellation mappers.

- Both **constellation mappers** (4 and 16 symbos) realizated with mex function.

Faster than define a matlab function and then using arrayfunc.

# BICM – Implementation perpective



## COMPUTATION OF LLRs FOR DECODER

| b0 | b1 |
|---|---|
| 0 | 0 |
| 0 | 1 |
| 1 | 0 |
| 1 | 1 |

$\omega$

| Re | Im |
|---|---|
| -1 | -1 |
| 1 | -1 |
| -1 | 1 |
| 1 | 1 |

*Given the rx symbol*

$$LLR = \ln\left(\frac{P(0,0) + P(0,1)}{P(1,0) + P(1,1)}\right)$$

$b_0 \rightarrow v_0$

| b0 | b1 |
|---|---|
| 0 | 0 |
| 1 | 0 |
| 0 | 1 |
| 1 | 1 |

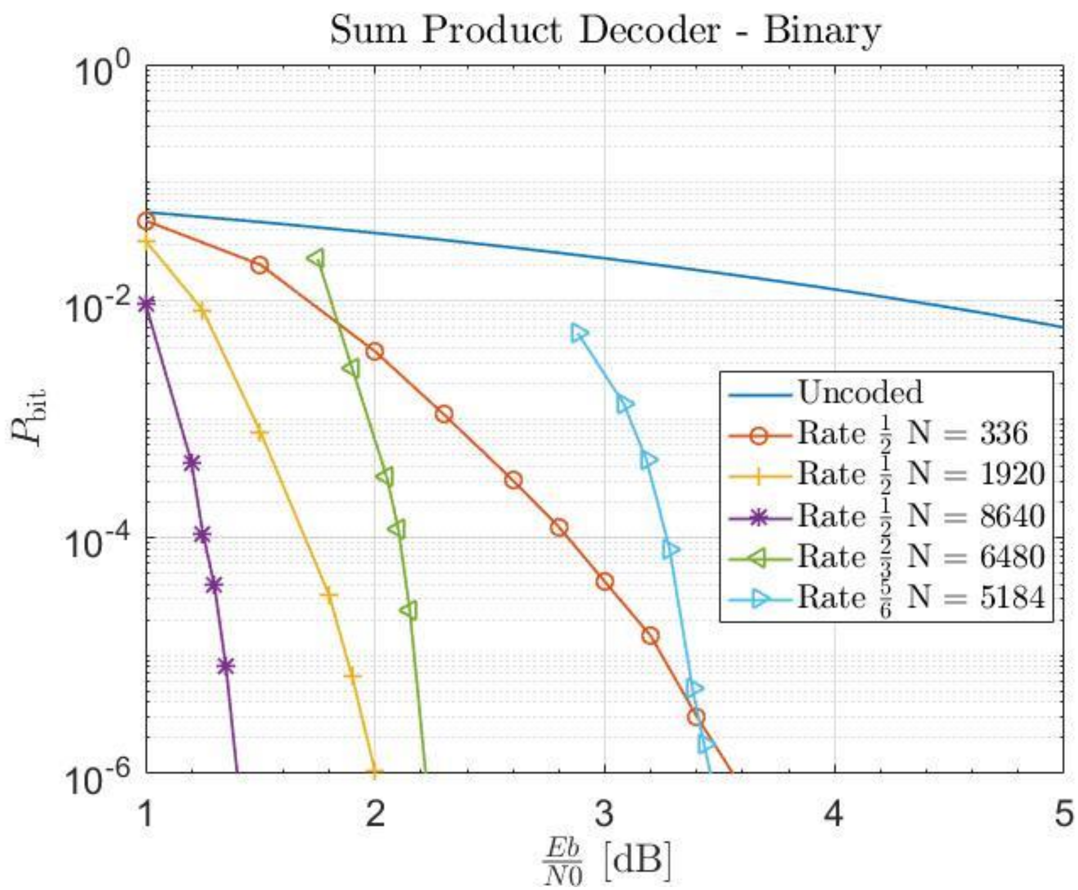
$\omega$

| Re | Im |
|---|---|
| -1 | -1 |
| -1 | 1 |
| 1 | -1 |
| 1 | 1 |

*Given the rx symbol*

$$LLR = \ln\left(\frac{P(0,0) + P(1,0)}{P(0,1) + P(1,1)}\right)$$

$b_1 \rightarrow v_1$




***Uncoded case***:
Pbit = $10^{-6}$
for $\frac{Eb}{N0} \cong 10{,}5$ dB

# Results – MinSum vs SumProd

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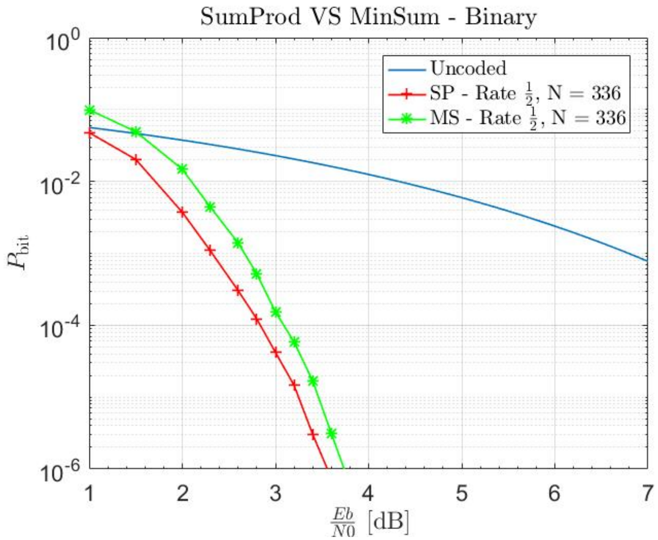
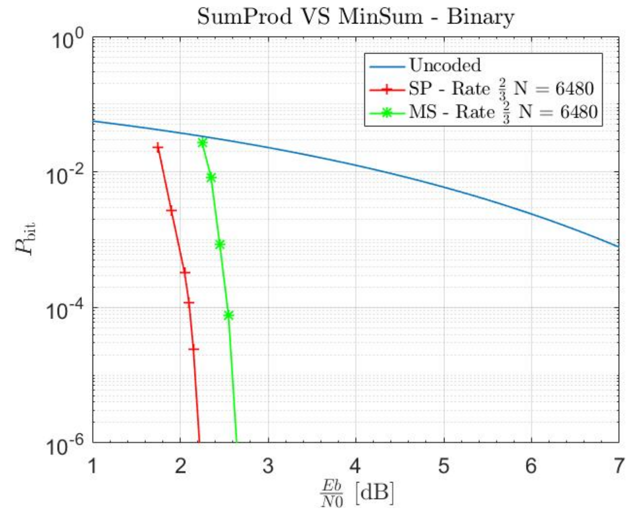
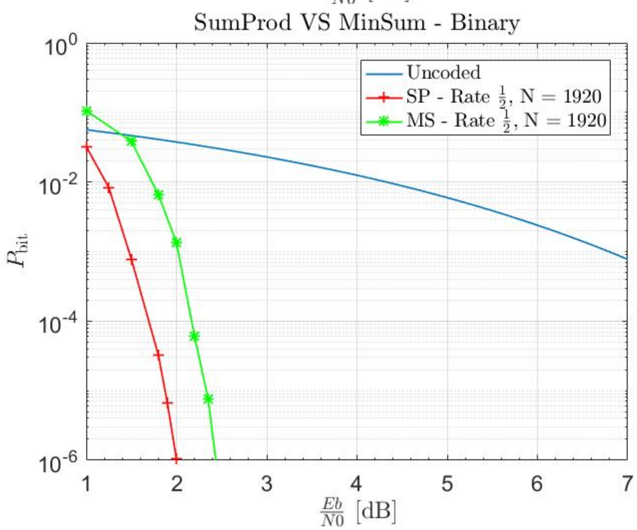
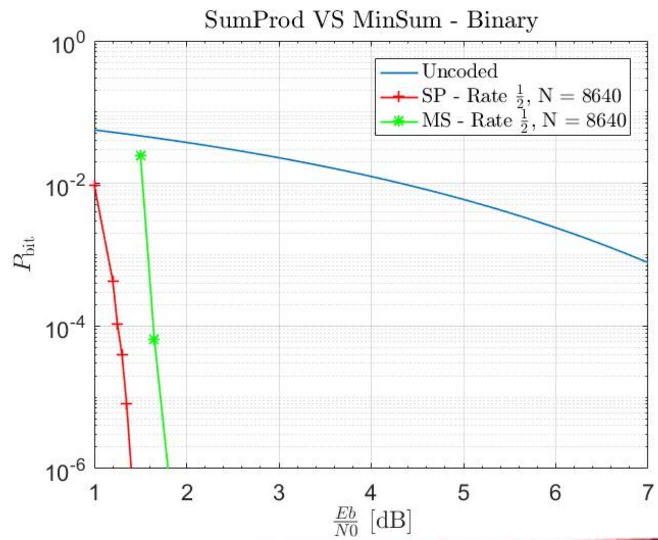


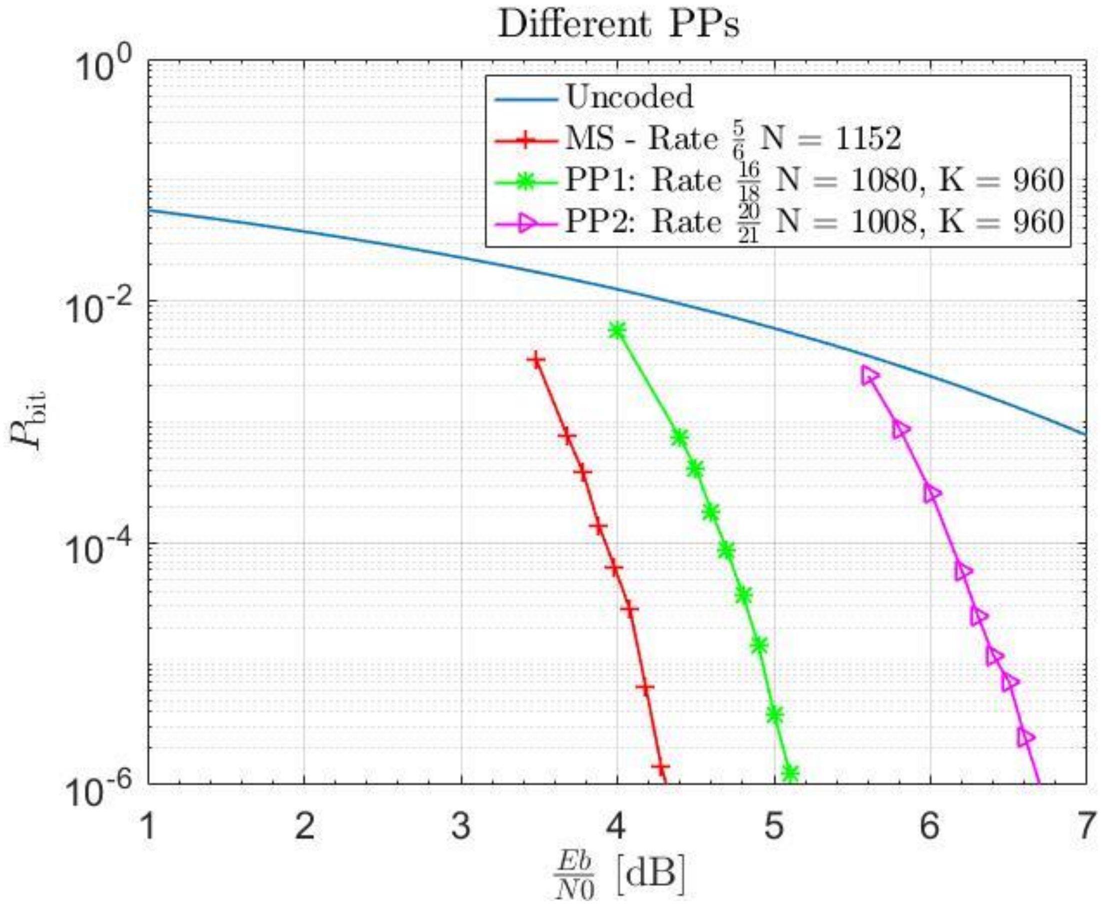



# Results – Different Puncturing Patterns

**For K = 960:**



# Results - Different Puncturing Patterns

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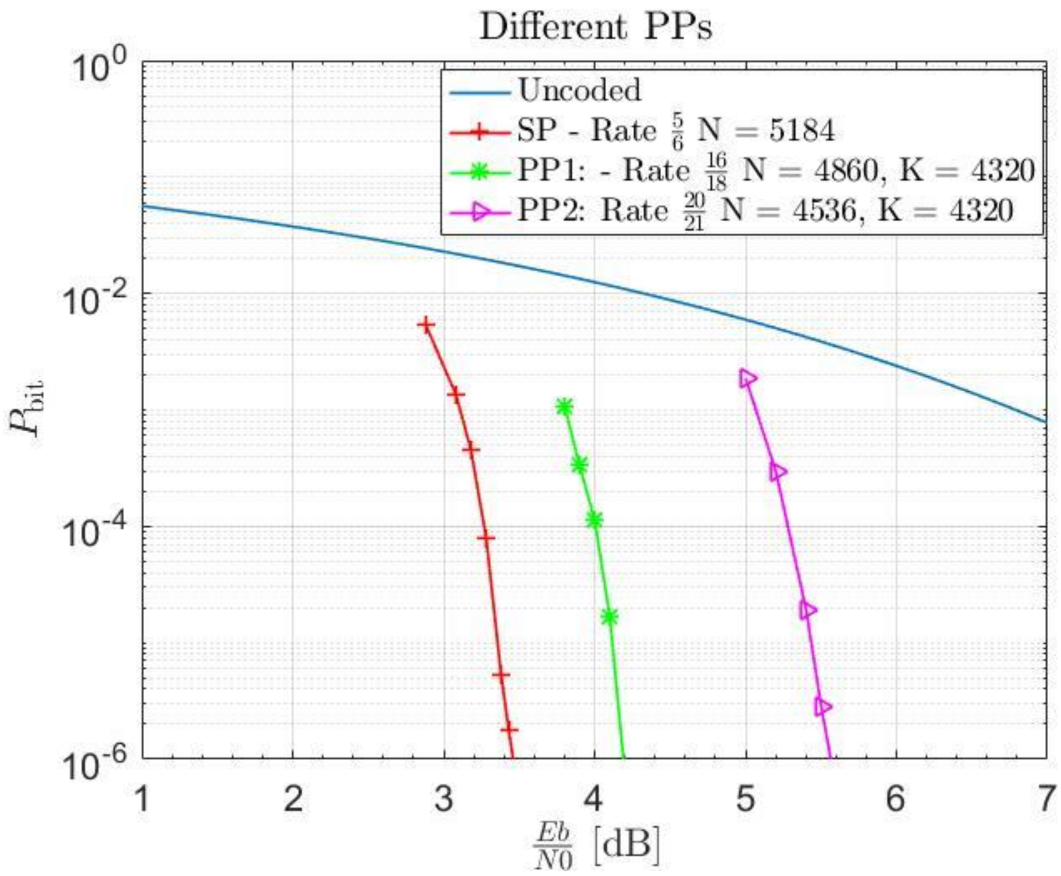
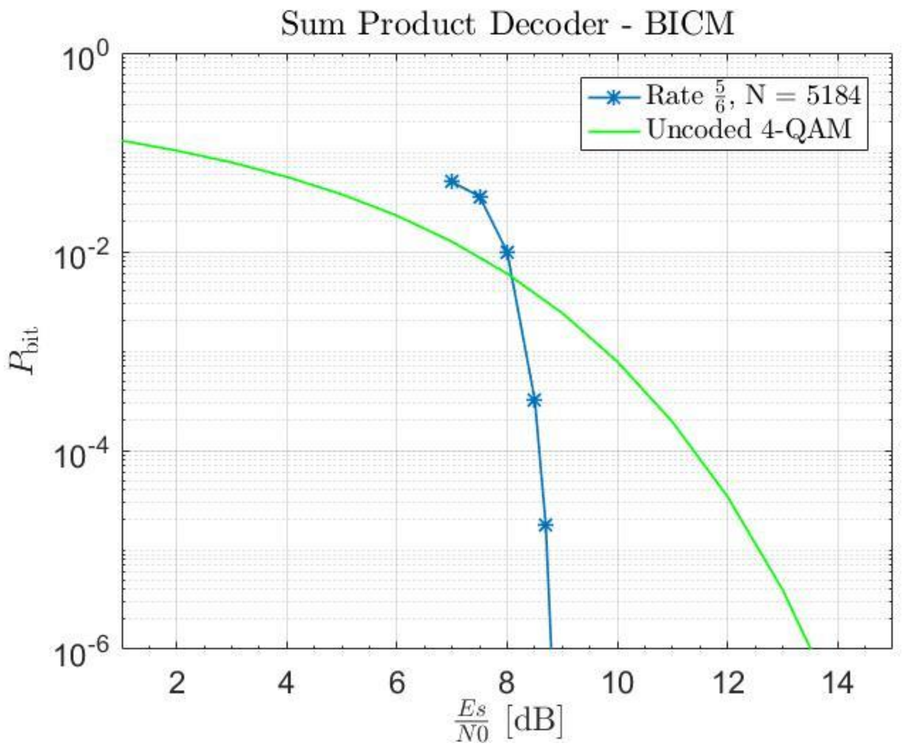
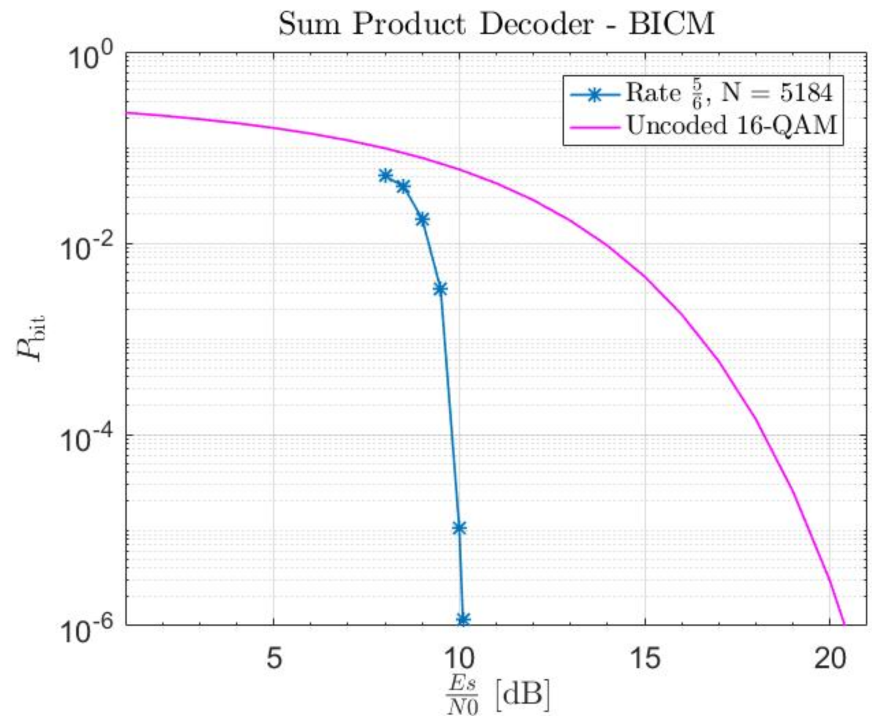
**For K = 4320:**



# Results - BICM





# Conclusions and remarks



- Possible extensions/improvements.

**Details**

- All the images has been created with LaTeX or Matlab.

- The images reported in pag. 8 where taken from two superb books (According to Goodreads): The Pragmatic Programmer and Programming Pearls.

- The images of LDPC decoder have been select from PDFs of Channel Coding Course (Tomaso Erseghe).