

CHANNEL CODING - FINAL PROJECT

UNIFIED HIGH-SPEED WIRELINE-BASE HOME NETWORKING TRANSCEIVERS

Access networks – In premises networks

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- Standard overview
- Encoder
- Decoder
- Rationale Decoder
- BICM
- Results

An introduction



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- The **G.hn ITU G9960** is a standard prosed by the ITU-T.
- **Designed for:** the transmission of data over premises' wiring.
- The standard **defines:**
 1. the home network architecture and reference models.
 2. the physical layer specification.



Transmission modes



- Each node of the home-network has a different **profile**.
- The standard refers to two different profiles:
 1. **Low-complexity profile (L-CP)**.
 2. **Standard profile (SP)**.
- A node of the network is required to support one profile, at minimum.

Profile name	Domain type	Valid bandplans
L-CP	Power-line baseband	25 MHz
SP	Power-line baseband	50 MHz, 100 MHz
	Telephone-line baseband	50 MHz, 100 MHz
	Coax baseband	50 MHz, 100 MHz
	Coax RF	50 MHz, 100 MHz, 200 MHz

Transmission modes

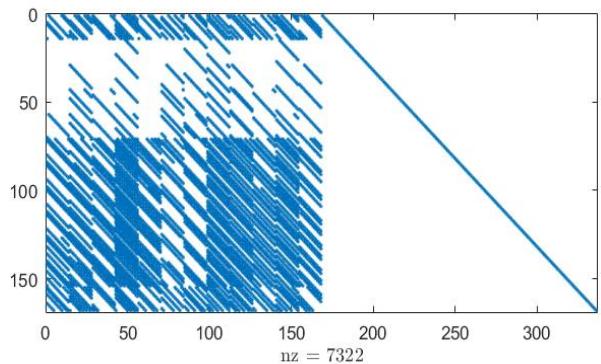
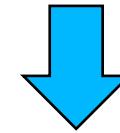
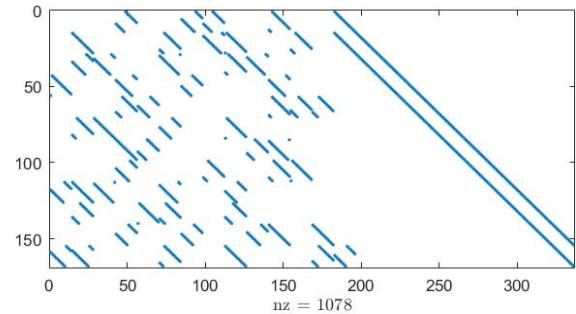
- The parity check matrix H of size $(N-K) \times N$ will have:
 - Rate $\frac{1}{2}$: $N = 1920$ (*LCP,SP*),
 8640 (*SP*).
 - Rate $\frac{2}{3}$: $N = 1440, 6480$ (*SP*)
 - Rate $\frac{5}{6}$: $N = 1152, 5184$ (*SP*)
- Rate $\frac{16}{18}, \frac{20}{21}$ are obtained puncturing the code with rate $\frac{5}{6}$, through different puncturing patterns.

Profile name	FEC rate	FEC block size
L-CP	$\frac{1}{2}$	120 bytes (Payload)
SP	$\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$ $\frac{16}{18}, \frac{20}{21}$	120 and 540 bytes (Payload)

Encoder



- Given H , get its systematic form H_{sys} .

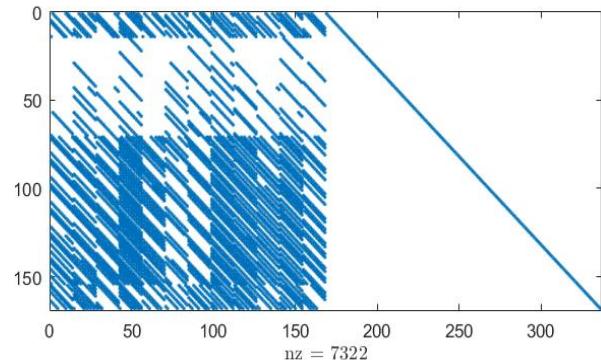
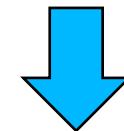
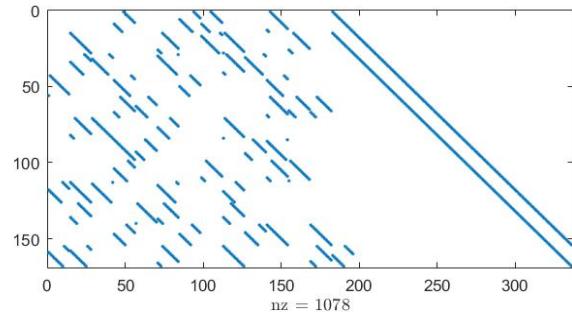


Encoder



1. Given H , get its systematic form H_{sys} .
2. Encoding Procedure for a single word:

- ✓ The K information bits of u are directly copied to the codeword v .
 - ✓ Let A be the submatrix obtained considering $N-K$ rows and the first k columns of H_{sys} ; then, the $N-K$ parity-check bits are computed as $A^* u$.
 - ✓ The final codeword is $v = [u|A u]$
- Preprocessing is done once.



Decoder – a first (naif) approach



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- All the minsum/ sumproduct algorithm have been written in c, and invoked via mex functions.
- There are at least three matrices as big as the H matrix:

Decoder – a first (naif) approach



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Unfeasible in terms of:

- Memory allocation.
- Computational efforts.



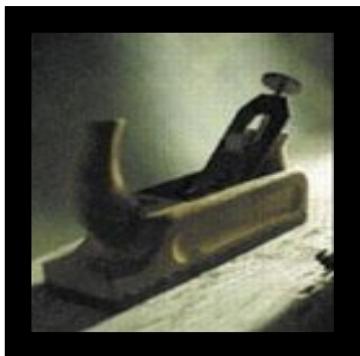
Decoder – a first (naïf) approach

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Decoder – a first (naif) approach



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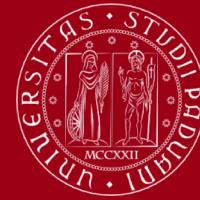


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Decoder – Rationale



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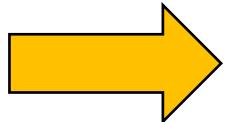
- Preprocess the matrix H in such a way to get all the **useful patterns** from it.

Decoder – Rationale



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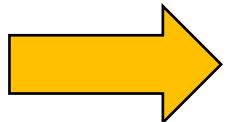
- Preprocess the matrix H in such a way to get all the **useful patterns** from it.



Do only once.

Decoder – Rationale

- Preprocess the matrix H in such a way to get all the **useful patterns** from it.



Do only once.

- From H we'll get:

$$\Psi_{v \rightarrow c} \quad \Psi_{c \rightarrow v}$$

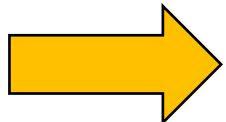
$$\prod_{v \rightarrow c} \quad \prod_{c \rightarrow v}$$

$$\{l_i\}_{i=1}^{N-K} \quad \{j_i\}_{i=1}^N$$

Decoder – Rationale



- Preprocess the matrix H in such a way to get all the patterns from it.



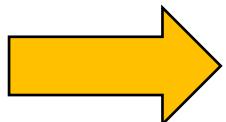
Do only once.

- From H we'll get:

$$\Psi \quad \quad \quad \Psi$$
$$v \rightarrow c \quad c \rightarrow v$$

$$\prod_{v \rightarrow c} \quad \prod_{c \rightarrow v}$$

$$\{l_i\}_{i=1}^{N-K} \quad \{j_i\}_{i=1}^N$$



No need to evaluate directly H in the message passing for the computation of LLRs.



Updates as a linear map



- Let be $\Psi_{c \rightarrow v}$ and $\Psi_{v \rightarrow c}$ the check update matrix and the variable update matrix, respectively:

$$\Psi_{c \rightarrow v} = \begin{bmatrix} I_{l_1}^C & & & 0 \\ & \ddots & \ddots & \\ 0 & & & I_{l_{N-K}}^C \end{bmatrix}$$

$$\Psi_{v \rightarrow c} = \begin{bmatrix} I_{j_1}^C & & & 0 \\ & \ddots & \ddots & \\ 0 & & & I_{j_N}^C \end{bmatrix}$$

- I_s^C denotes the complement of the Identity matrix of size s .
- $\{l_i\}_{i=1}^{N-K}$ and $\{j_i\}_{i=1}^N$ are the number of ones in each raw and column respectively.

LLRs notation



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- Let $\underset{c \rightarrow v}{LLR}, \underset{v \rightarrow c}{LLR} \in \mathbb{F}_2^U$ with U the total numbers of ones (edges) in the H matrix.

$$LLR = \begin{bmatrix} & & \\ & & \\ & \vdots & \\ & & \\ c \rightarrow v & & \end{bmatrix} \begin{array}{l} j_1 \\ \\ \\ \\ j_N \end{array}$$

$$LLR = \begin{bmatrix} & \\ & \\ \vdots & \\ & \end{bmatrix} \quad \begin{array}{l} l_1 \\ \quad \\ \quad \\ \quad \end{array}$$

Updates in minSum and sumProd



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- For both minsum and sumproduct decoder the **variable updates** will have the following expression:

$$LLR = \underset{v \rightarrow c}{\Psi} \cdot \underset{c \rightarrow v}{LLR} + p$$

- In the sum product the **check updates** is:

$$\underset{c \rightarrow v}{LLR} = \left(\underset{c \rightarrow v}{\Psi} \cdot \Phi \left(\underset{v \rightarrow c}{|LLR|} \right) \right) * \text{sgn} \left(\underset{v \rightarrow c}{LLR} \right)$$

- Where $\Phi(x) = \log \left(\frac{e^x + 1}{e^x - 1} \right)$, $\text{sgn}(\cdot)$ is the function that maps each element of the vector in its corresponding sign and p is the vector of prior LLRs.



In order to properly compute the LLRs updates, vectors must be properly ordered...

HOW?

Permutation matrices



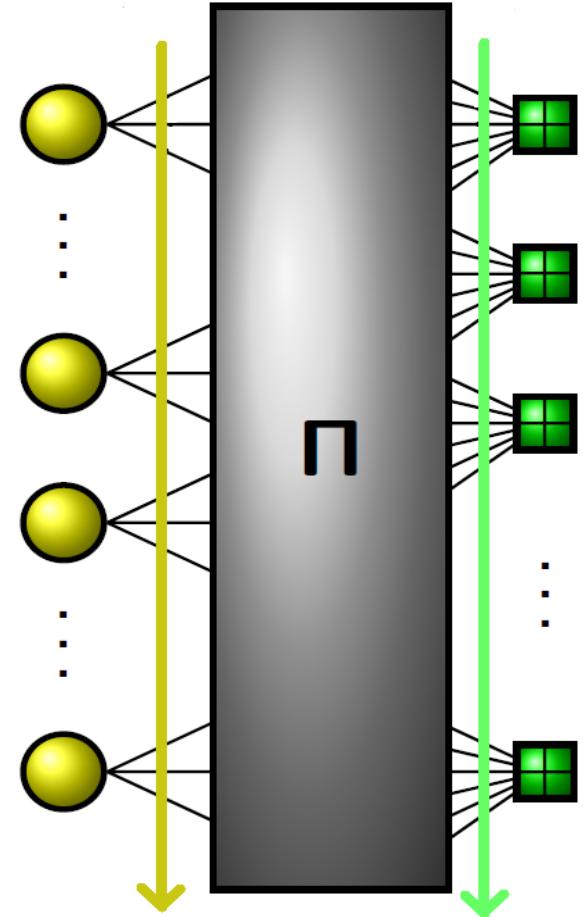
In order to properly compute the LLRs updates, vectors must be properly ordered...

HOW?

We will refer to two different LLR representation:

Check representation (CR)

Variable representation (VR)



Permutation matrices

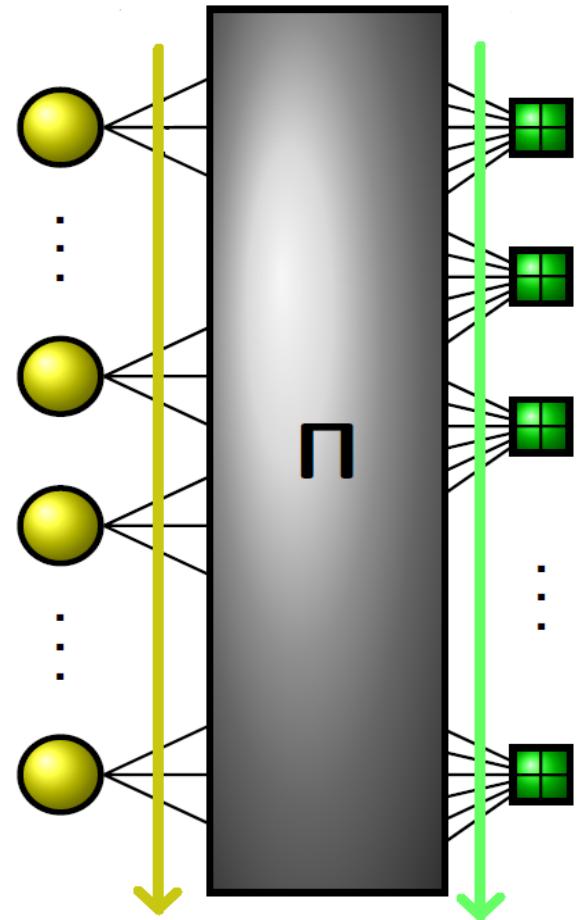


CHECK REPRESENTATION (CR)

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

VARIABLE REPRESENTATION (VR)

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$



Permutation matrices (1/2)

- Let $per : D_U \longrightarrow D_U$ be the permutation map, that associates to each edge in **CR** the corresponding position in the **VR**.
- $D_U = \{1, \dots, U\}$, U the number of ones (edges) in the matrix H .

$$\Pi_{c \rightarrow v} = \begin{bmatrix} e_{per(1)}^\top \\ \vdots \\ e_{per(U)}^\top \end{bmatrix}$$

- Where $e_i \in \mathbb{F}_2^U$ denotes the i -th canonical vector in \mathbb{F}_2^U .

Permutation matrices (2/2)

- If we think of $\prod_{c \rightarrow v}$ as a linear application and we exploit the property of the permutation matrices:

$$\prod_{v \rightarrow c} = \prod_{c \rightarrow v}^{-1} = \prod_{c \rightarrow v}^{\top}$$

- Hence, there is no need to compute directly $\prod_{v \rightarrow c}$ since:

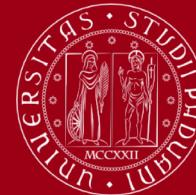
$$\prod_{c \rightarrow v} \longleftrightarrow \prod_{v \rightarrow c}$$

Sum product



```
for it=1:iter
    LLRvc = perMatrVC * LLRvc;           ← Get LLRvc in CR
    curSign = sign(LLRvc);
    curSign(curSign == 0) = 1;
    signLLRcv = GetSign(curSign,nOR);
    tmpP = abs(LLRvc);
    tmpP(tmpP < minf) = 1e-10;
    temp = upMatrCV * PhiMap(tmpP);
    LLRcv = PhiMap(temp).* signLLRcv;      ← Update
    LLRcv = perMatrCV * LLRcv;             ← Get LLRcv in VR
    LLRvc = upMatrVC * LLRcv + LLRprior;   ← Update
    curIndex = 1;
    for j=1:length(nOC)
        if(vPLLRS(j) + sum(LLRcv(curIndex:nOC(j)+curIndex-1)) < 0)
            u_hat(j) = 1;
        else
            u_hat(j) = 0;
        end
        curIndex = curIndex + nOC(j);
    end
    if( mod(Hreal * u_hat,2) == 0)
        break;
    end
end
```

Practical considerations



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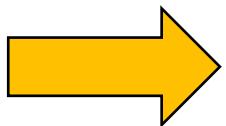
- Product between sparse matrices is optimized in Matlab.
(<https://it.mathworks.com/help/matlab/math/sparse-matrix-operations.html>)
- Only vectors proportional to the number of ones in the check matrix are used.

Mex function for specific tasks have been implemented:

- Arrayfunc command of matlab leads to slow computation.
- Mex function **PhiMap()** improves dramatically the performance.
- Search in the minsum is not a bottle neck since elements to be checked are few (consequence to the fact that H is sparse).

ENCODING

- Grouping sequences of $b = \log_2 M$ bits and then mapping them according to the suitable constellation mappers.
- Both **constellation mappers** (4 and 16 symbols) designed with mex function.



Faster than defining a matlab function and then using the arrayfunc function.

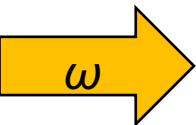
BICM – Implementation perspective



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COMPUTATION OF LLRs FOR DECODER

b0	b1
0	0
0	1
1	0
1	1



Re	Im
-1	-1
1	-1
-1	1
1	1

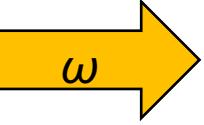
*Given the
rx symbol*



$$LLR = \ln \left(\frac{P(0,0) + P(0,1)}{P(1,0) + P(1,1)} \right)$$

$b_0 \rightarrow v_0$

b0	b1
0	0
1	0
0	1
1	1



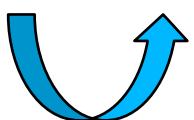
Re	Im
-1	-1
-1	1
1	-1
1	1

*Given the
rx symbol*

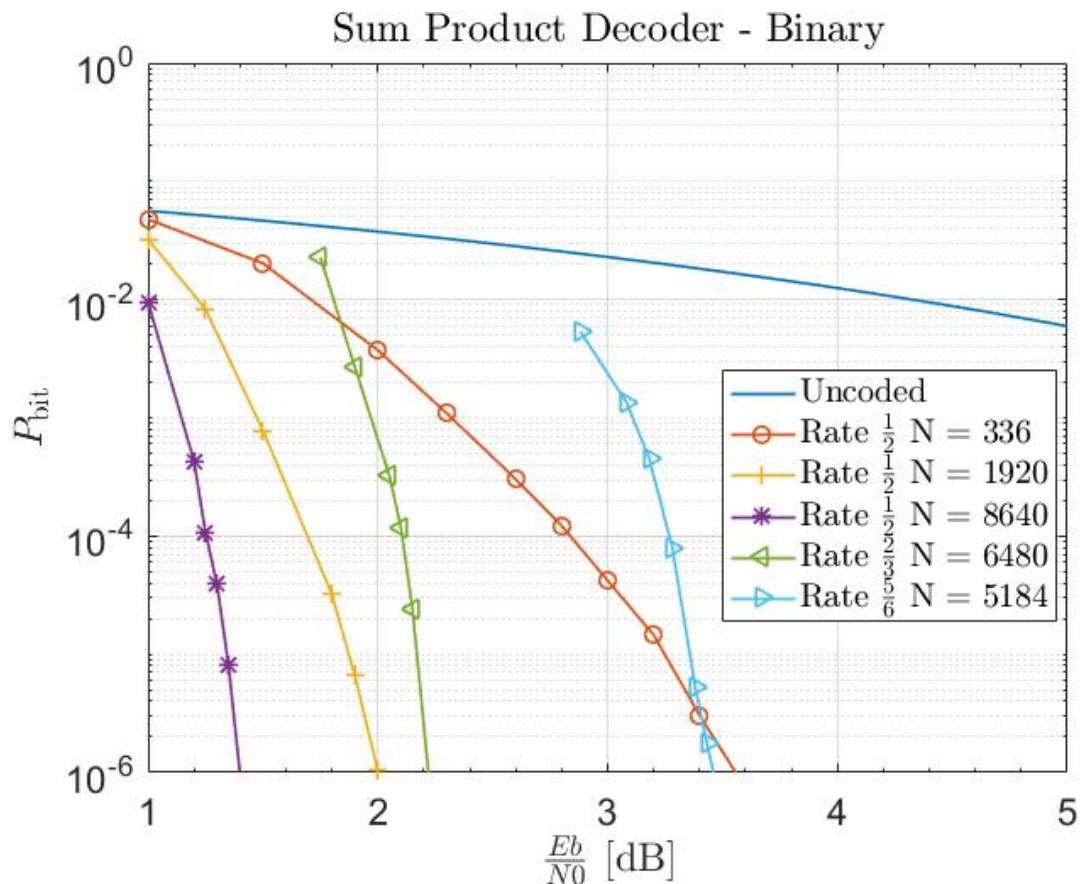


$$LLR = \ln \left(\frac{P(0,0) + P(1,0)}{P(0,1) + P(1,1)} \right)$$

$b_1 \rightarrow v_1$



Results

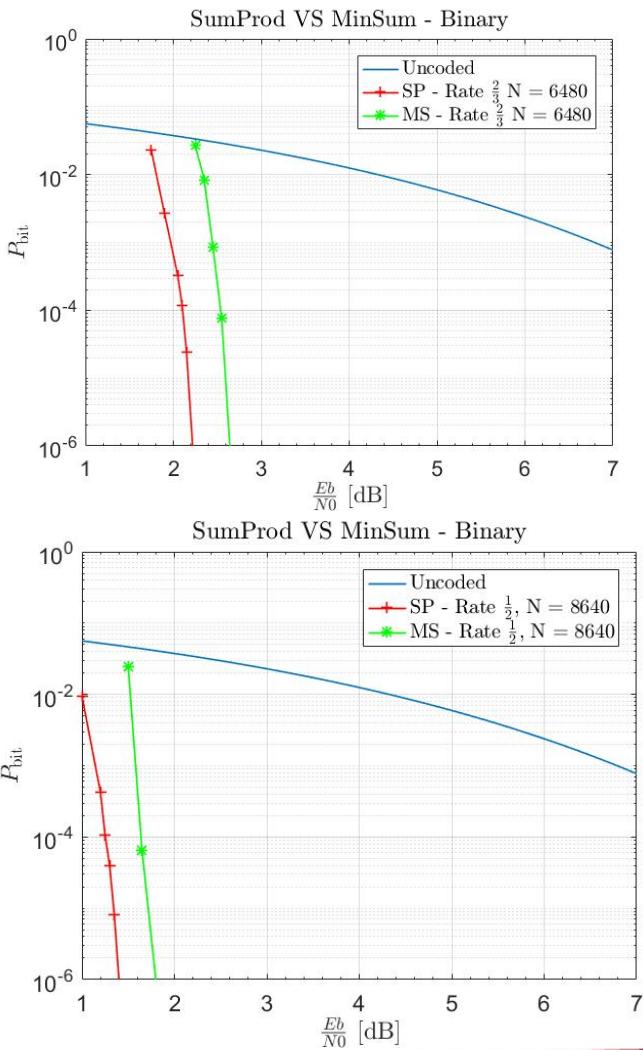
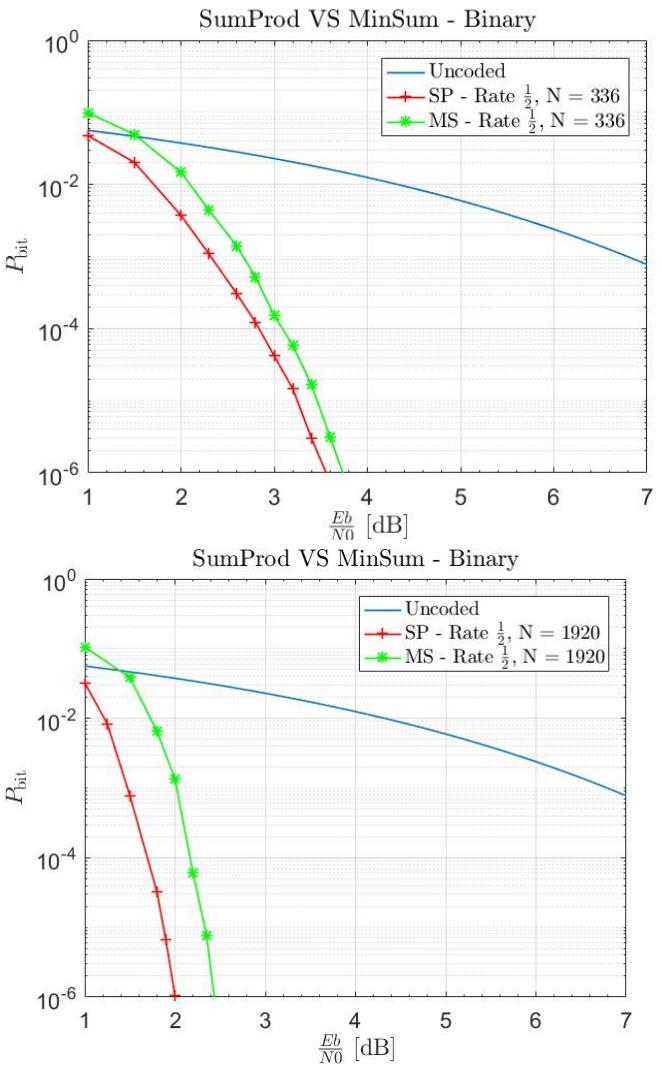


Uncoded case:

$$P_{\text{bit}} = 10^{-6}$$

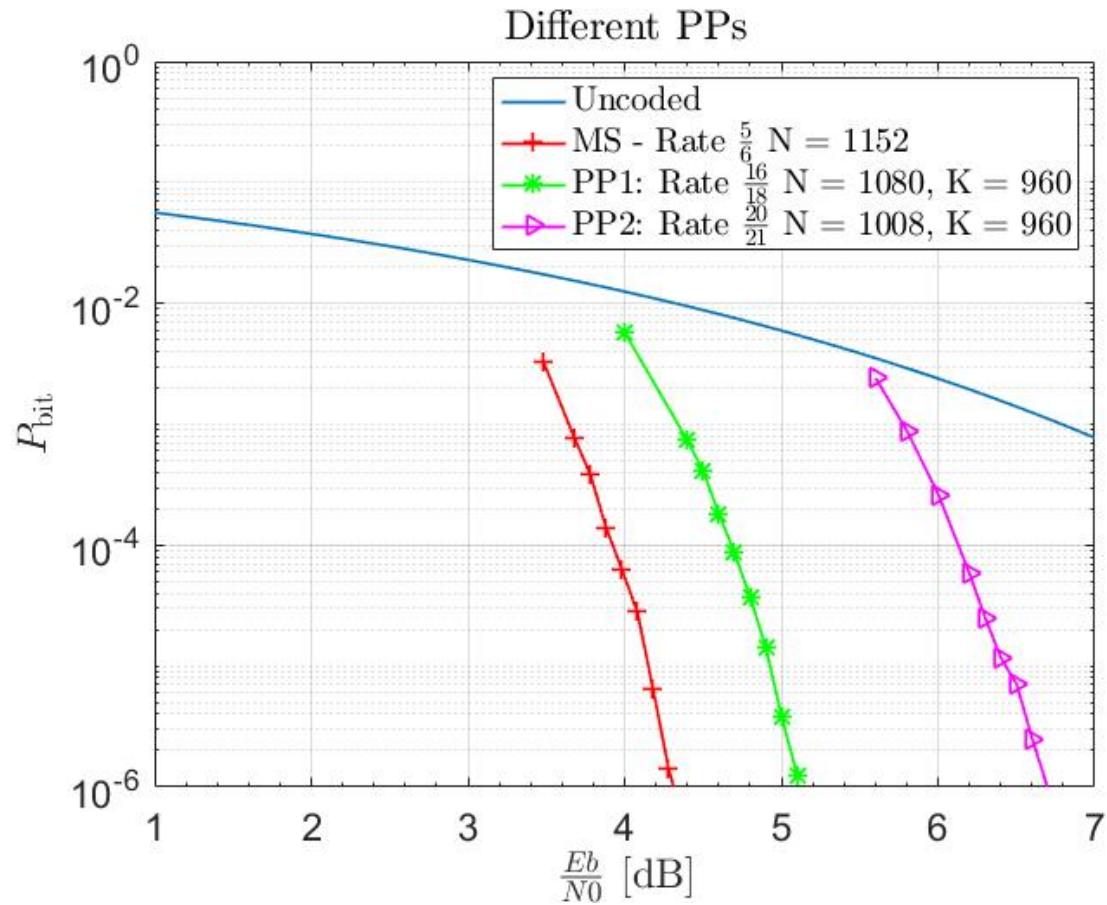
for $\frac{E_b}{N_0} \cong 10,5 \text{ dB}$

Results – MinSum vs SumProd



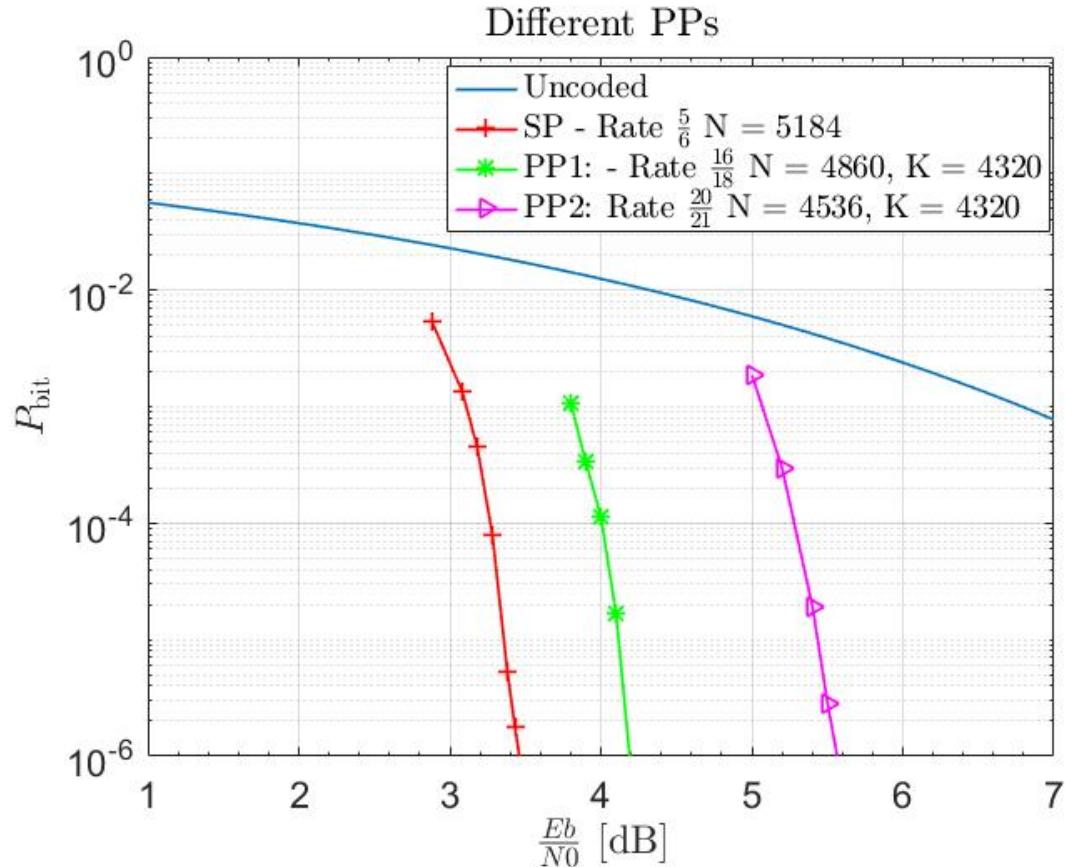
Results – Different Puncturing Patterns

For K = 960:

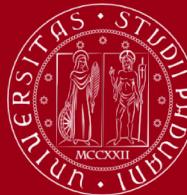


Results - Different Puncturing Patterns

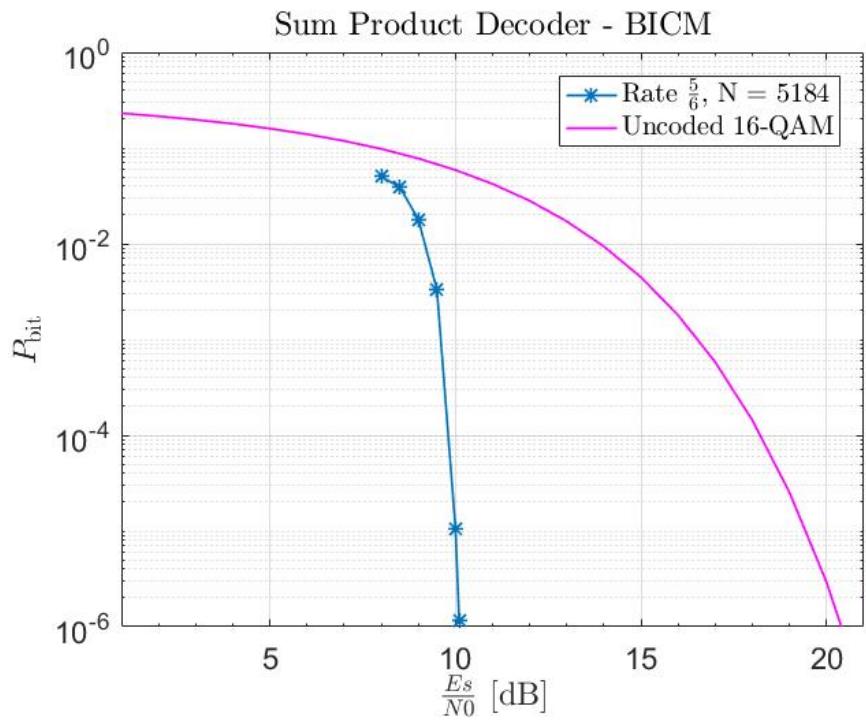
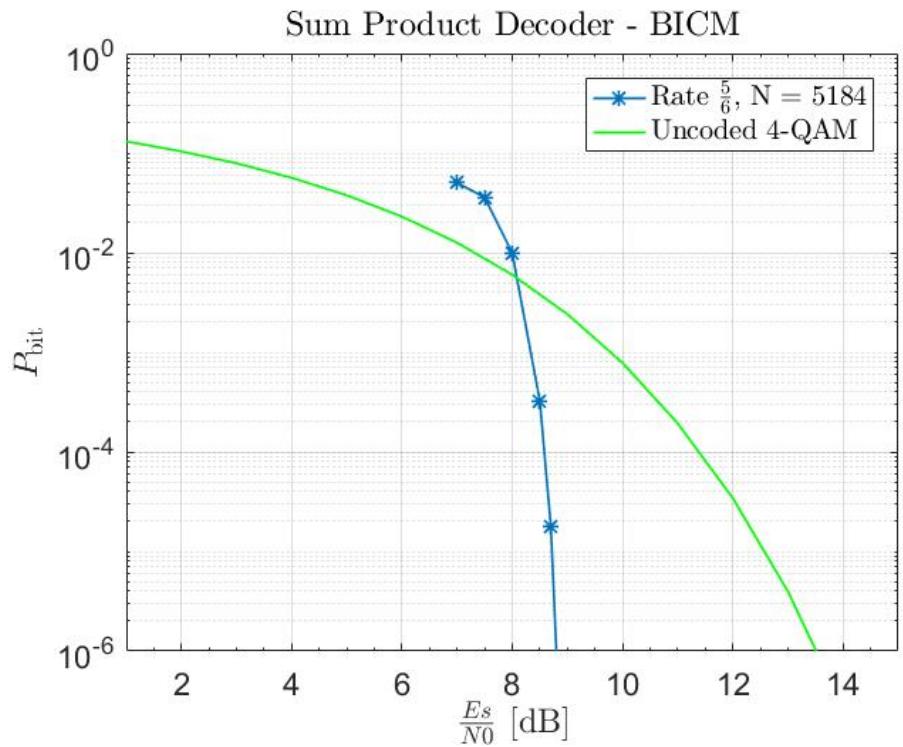
For K = 4320:



Results - BICM



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- Possible extensions/improvements.

Details

- All the images have been created with LaTeX or Matlab.
- The images reported in pag. 8 were taken from two superb books (According to Goodreads): The Pragmatic Programmer and Programming Pearls.
- The image of LDPC decoder have been selected from PDFs of Channel Coding Course (Tomaso Erseghe).