CHANNEL CODING - FINAL PROJECT

UNIFIED HIGH-SPEED WIRELINE-BASE HOME NETWORKING TRANSCEIVERS

Access networks – In premises networks

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30th March 2017

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DI PADOVA

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An introduction

- The **G.hn ITU G9960** is a standard proposed by the ITU-T.
- **Designed for**: the transmission of data over premises' wiring.
- The standard **defines**:

1. the home network architecture and reference models.
2. the physical layer specification.

International Telecommunication Union

Transmission modes

- Each node of the home-network has a different **profile**.

- The standard refers to two different profiles:
  1. **Low-complexity profile** (L-CP).
  2. **Standard profile** (SP).

- A node of the network is required to support one profile, at minimum.

Profile name	Domain type	Valid bandplans
L-CP	Power-line baseband	25 MHz
SP	Power-line baseband	50 MHz, 100 MHz
	Telephone-line baseband	50 MHz, 100 MHz
	Coax baseband	50 MHz, 100 MHz
	Coax RF	50 MHz, 100 MHz, 200 MHz

Transmission modes

- The parity check matrix *H* of size *(N-K) x N* will have:

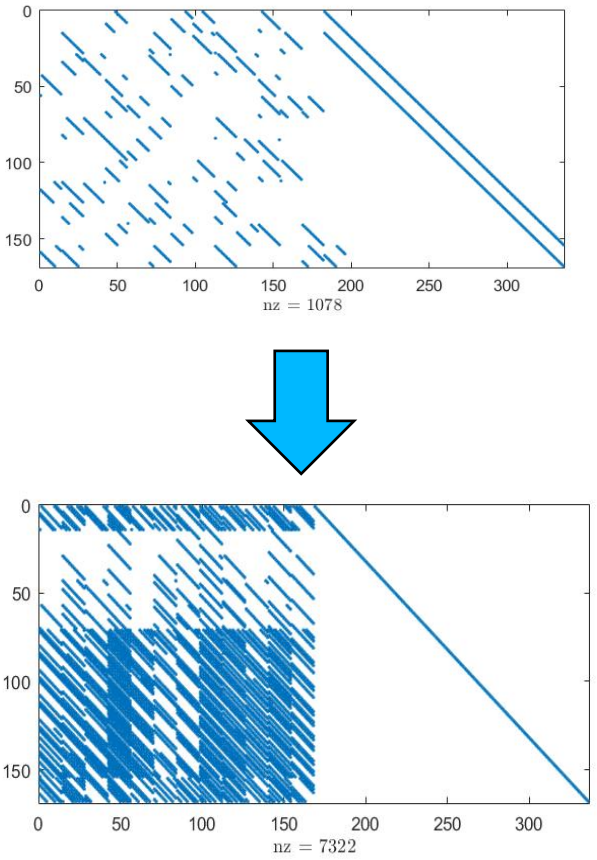
  - Rate $^1/_2$: *N* = 1920 (*LCP,SP*), 8640 (*SP*).
  - Rate $^2/_3$: *N* = 1440, 6480 (*SP*)
  - Rate $^5/_6$: *N* = 1152, 5184 (*SP*)

- Rate $^{16}/_{18}$, $^{20}/_{21}$ are obtained puncturing the code with rate $^5/_6$, through different puncturing patterns.

Profile name	FEC rate	FEC block size
L-CP	$^1/_2$	120 bytes (Payload)
SP	$^1/_2, ^2/_3, ^5/_6$ $^{16}/_{18}, ^{20}/_{21}$	120 and 540 bytes (Payload)

Encoder

1. Given H, get its systematic form H_{sys} .



Encoder

1. Given H, get its systematic form H_{sys} .

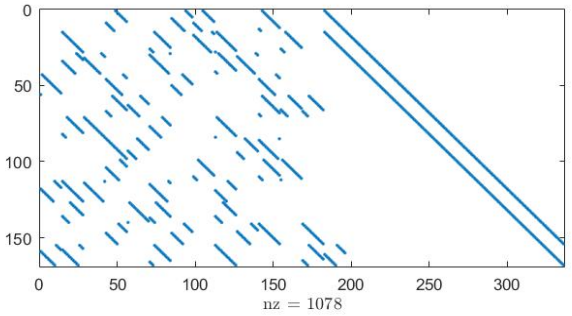
2. Encoding Procedure for a single word:

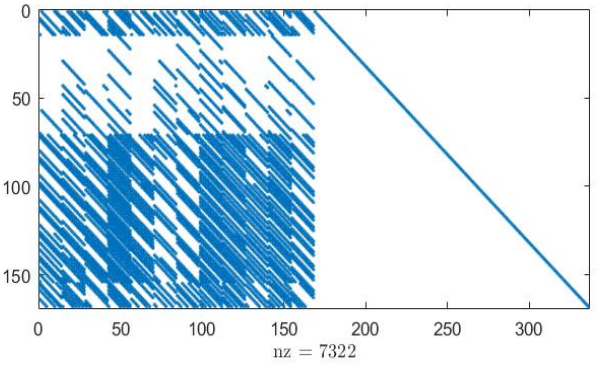
   ✓ The *K* information bits of *u* are directly copied to the codeword *v*.

   ✓ Let A be the submatrix obtained considering *N-K* rows and the first k columns of H_{sys} ; then, the *N–K* parity-check bits are computed as A * u.

   ✓ The final codeword is *v* = [u|A u]

- Preprocessing is done only once.





Decoder – a first (naif) approach

- All the minsum/ sumproduct algorithm have been written in c, and invoked via mex functions.

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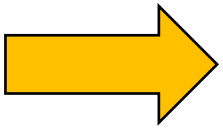

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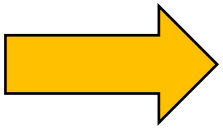

Do it only once.

- From H we get:

$$\underset{v\to c}{\Psi} \quad \underset{c\to v}{\Psi} \quad \underset{v\to c}{\prod} \quad \underset{c\to v}{\prod} \quad \{l_i\}_{i=1}^{N-K} \quad \{j_i\}_{i=1}^{N}$$

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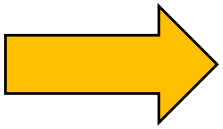

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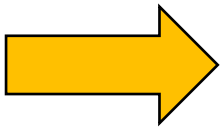

No need to evaluate directly H in the message passing algorithm for the computation of LLRs.

Updates as a linear map

- Let $\underset{c\to v}{\Psi}$ and $\underset{v\to c}{\Psi}$ be the check update matrix and the variable update matrix, respectively:

$$\underset{c\to v}{\Psi} = \begin{bmatrix} I^C_{l_1} & & 0 \\ & \ddots & \\ 0 & & I^C_{l_{N-K}} \end{bmatrix} \qquad \underset{v\to c}{\Psi} = \begin{bmatrix} I^C_{j_1} & & 0 \\ & \ddots & \\ 0 & & I^C_{j_N} \end{bmatrix}$$

- I^C_s denotes the complement of the Identity matrix of size s.
- $\{l_i\}_{i=1}^{N-K}$ and $\{j_i\}_{i=1}^{N}$ are the number of ones in each row and column respectively.

LLRs notation

- Let's define $\underset{c\to v}{LLR}, \underset{v\to c}{LLR} \in \mathbb{F}_2^U$ as the LLRs vectors, with U the total numbers of ones (edges) in the H matrix.

$$\underset{c\to v}{LLR} = \begin{bmatrix} \\ \vdots \\ \\ \\ \vdots \\ \end{bmatrix} \begin{matrix} \} j_1 \\ \\ \\ \\ \} j_N \end{matrix}$$

$$\underset{v\to c}{LLR} = \begin{bmatrix} \\ \vdots \\ \\ \\ \vdots \\ \end{bmatrix} \begin{matrix} \} l_1 \\ \\ \\ \\ \} l_{N-K} \end{matrix}$$

Updates in minSum and sumProd

- For both minsum and sumproduct decoder the **variable updates** will have the following expression:

$$\underset{v\to c}{LLR} = \underset{v\to c}{\Psi} \cdot \underset{c\to v}{LLR} + p$$

- In the sum product the **check updates** is:

$$\underset{c\to v}{LLR} = \left(\underset{c\to v}{\Psi} \cdot \Phi\left(\left|\underset{v\to c}{LLR}\right|\right)\right) * \operatorname{sgn}\left(\underset{v\to c}{LLR}\right)$$

- Where $\Phi(x) = \log\left(\frac{e^x+1}{e^x-1}\right)$, $\operatorname{sgn}(\cdot)$ is the function that maps each element of the vector to its corresponding sign and *p* is the vector of prior LLRs.

Permutation matrices

In order to properly compute the LLRs updates, vectors must be properly ordered…

HOW?

Permutation matrices

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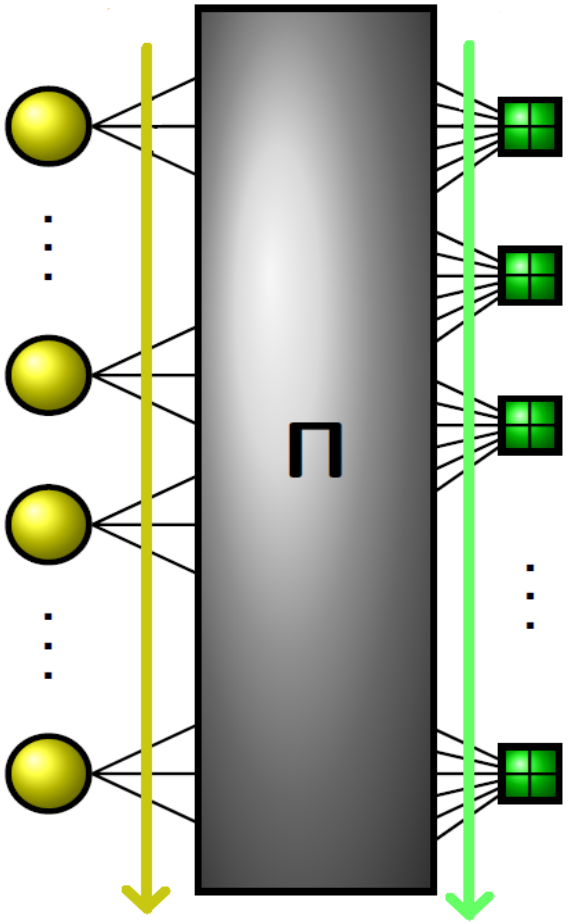
HOW?

We will refer to two different LLR representation**:**

Check representation (CR)

Variable representation (VR)



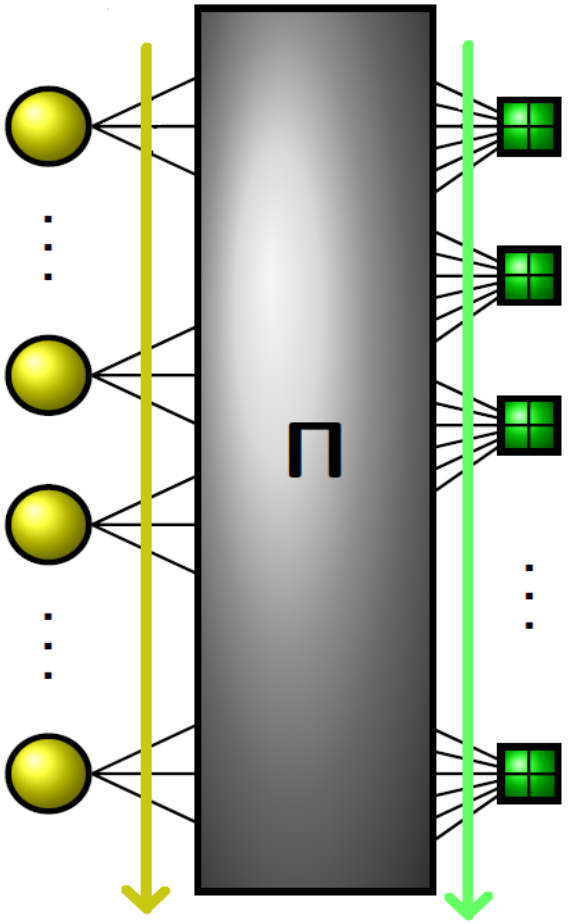
Permutation matrices

CHECK REPRESENTATION (CR)

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

VARIABLE REPRESENTATION (VR)

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$



Permutation matrices (1/2)

- Let $per : D_U \longrightarrow D_U$ be the permutation map that associates to each edge in expressed in the **CR** the corrisponding position in the **VR**.

- $D_U = \{1, \ldots, U\}$, U the number of ones (edges) in the matrix H.

$$\prod_{c \to v} = \begin{bmatrix} e_{per(1)}{}^{\top} \\ \hline \vdots \\ \hline e_{per(U)}{}^{\top} \end{bmatrix}$$

- Where $e_i \in \mathbb{F}_2^U$ denotes the i-th canonical vector in $\mathbb{F}_2^U$.

Permutation matrices (2/2)

- If we think of $\prod_{c \to v}$ as a linear application and we expoit the property of the permutation matrices:

$$\prod_{v \to c} = \prod_{c \to v}^{-1} = \prod_{c \to v}^{\top}$$

- Hence, there is no need to compute directly $\prod_{v \to c}$ since:

$$\prod_{c \to v} \longleftrightarrow \prod_{v \to c}$$

Sum product


```
for it=1:iter
     LLRvc = perMatrVC * LLRvc;
     curSign = sign(LLRvc);
      curSign(curSign == 0) = 1;
      signLLRcv = GetSign(curSign,nOR);
     tmpP = abs(LLRvc);
     tmpP(tmpP < minf) = 1e-10;
     temp = upMatrCV * PhiMap(tmpP);
     LLRcv =  PhiMap(temp).* signLLRcv;
     LLRcv = perMatrCV * LLRcv;
     LLRvc = upMatrVC * LLRcv + LLRprior;
     curIndex = 1;
    for j=1:length(nOC)
         if(vPLLRs(j) + sum(LLRcv(curIndex:nOC(j)+curIndex-1)) < 0)
            u_hat(j) = 1;
         else
            u_hat(j) = 0;
         end
         curIndex = curIndex + nOC(j);
    end
    if( mod(Hreal * u_hat,2) == 0)
        break;
    end
end
```

Get LLRvc in **CR**

Update

Get LLRcv in **VR**

Update

Practical considerations

- Product between sparse matrices is optimized in Matlab. (https://it.mathworks.com/help/matlab/math/sparse-matrix-operations.html)
- Only vectors proportional to the number of ones in the check matrix are used.

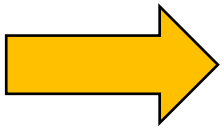
Mex function for specific tasks have been implemented:

- Arrayfunc command of matlab leads to slow computation.
- Mex function **PhiMap()** improves dramatically the performance.
- Search in the minsum is not a bottle neck since elements to be checked are few (consequence to the fact that H is sparse).

ENCODING

- Grouping sequences of b = $\log_2 M$ bits and then mapping them according to the suitable constellation mappers.

- Both **constellation mappers** (4 and 16 symbols) designed with mex function.



Faster than defining a matlab function and then using the arrayfunc function.

COMPUTATION OF THE LLRs FOR DECODER

b0	b1
0	0
0	1
1	0
1	1

ω

Re	Im
-1	-1
1	-1
-1	1
1	1

Given the rx symbol

$$LLR = \ln\left(\frac{P(0,0)+P(0,1)}{P(1,0)+P(1,1)}\right)$$

$b_0 \rightarrow v_0$

b0	b1
0	0
1	0
0	1
1	1

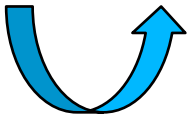
ω

Re	Im
-1	-1
-1	1
1	-1
1	1

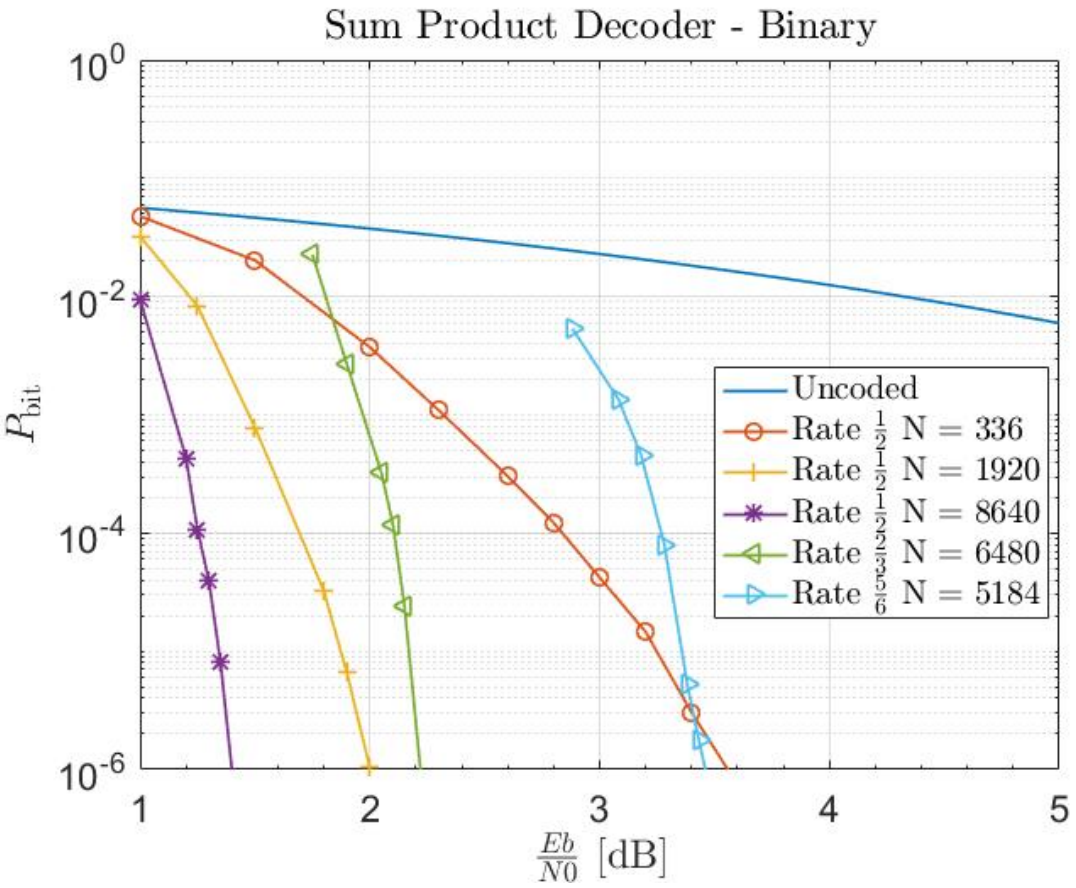
Given the rx symbol

$$LLR = \ln\left(\frac{P(0,0)+P(1,0)}{P(0,1)+P(1,1)}\right)$$

$b_1 \rightarrow v_1$

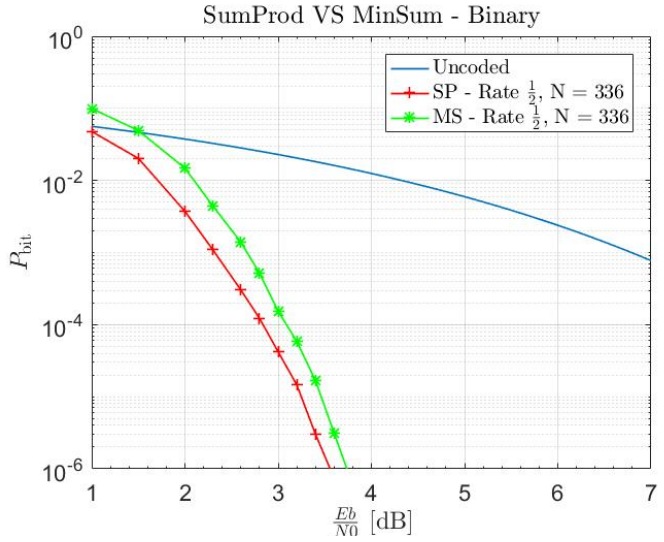


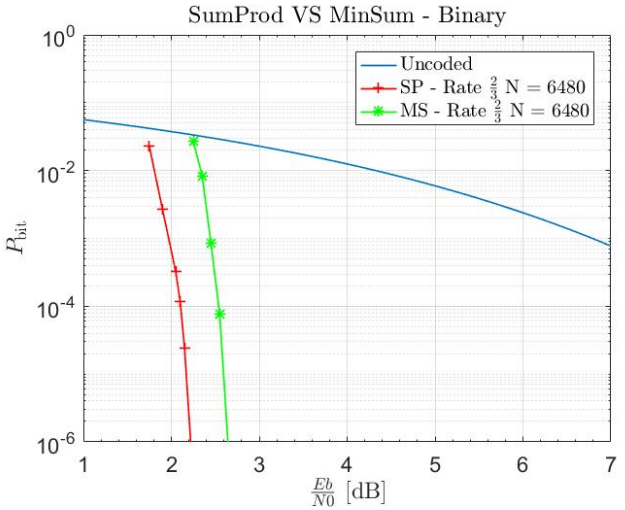
Results

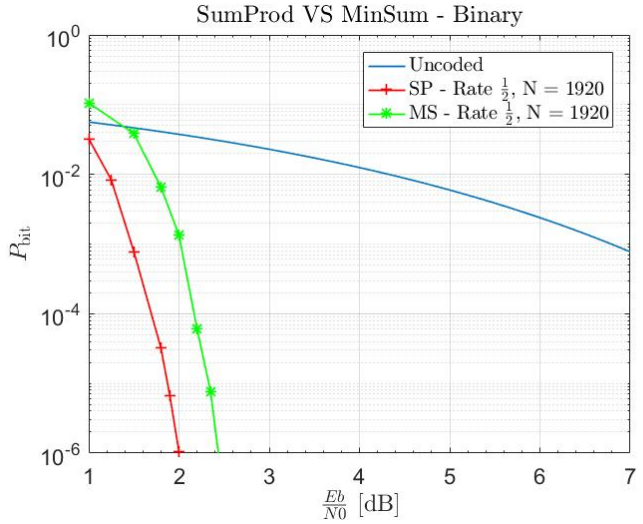


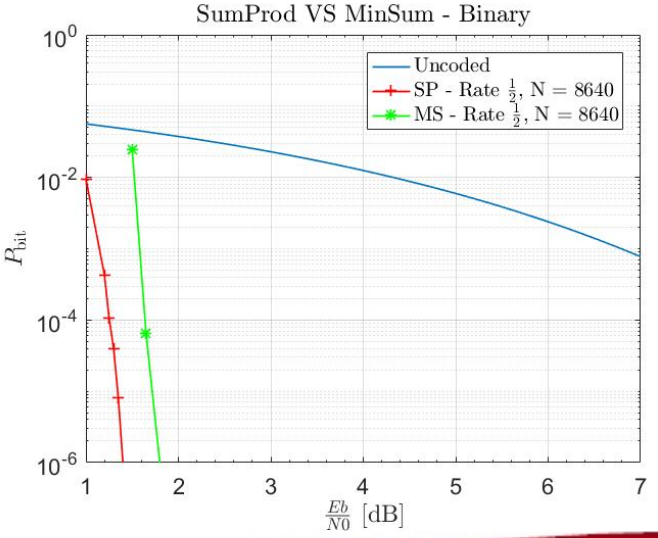
Uncoded case:
Pbit = 10^{-6}
for $\frac{Eb}{N0} \cong$ 10,5 dB

Results – MinSum vs SumProd



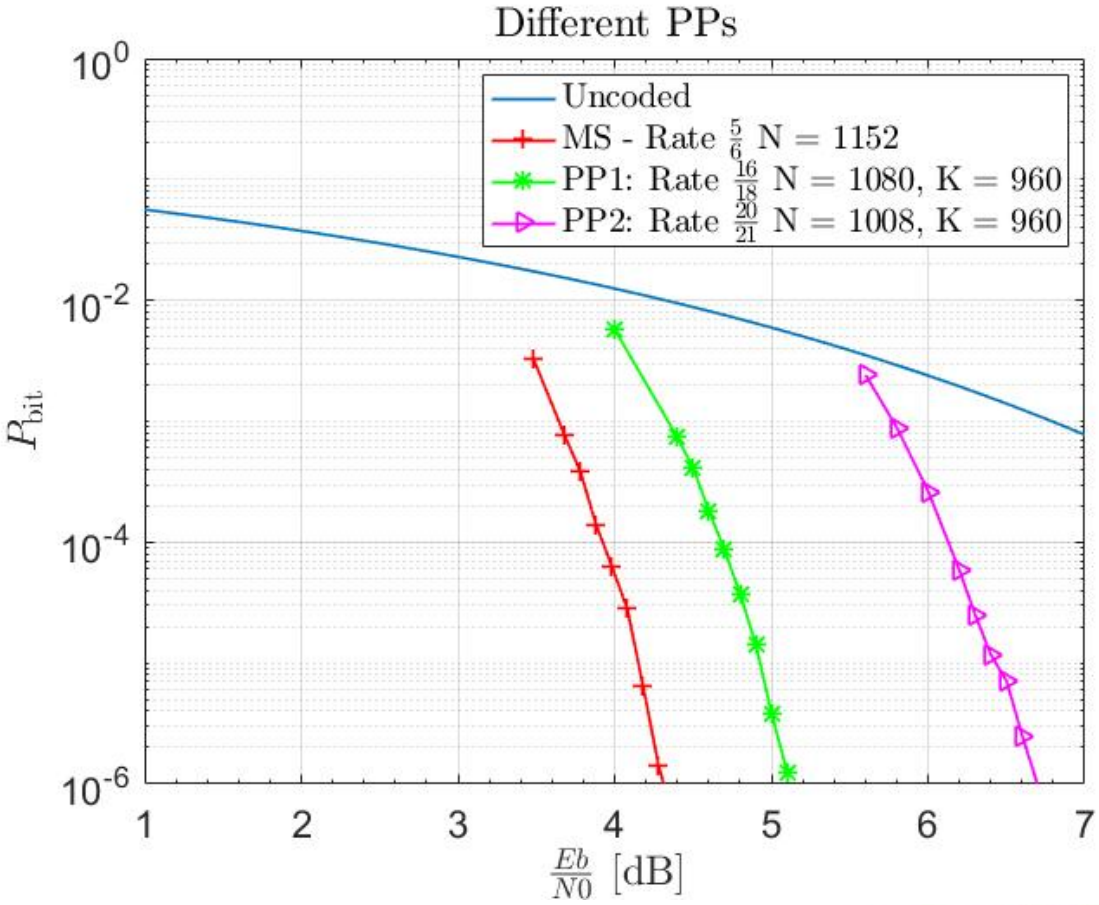






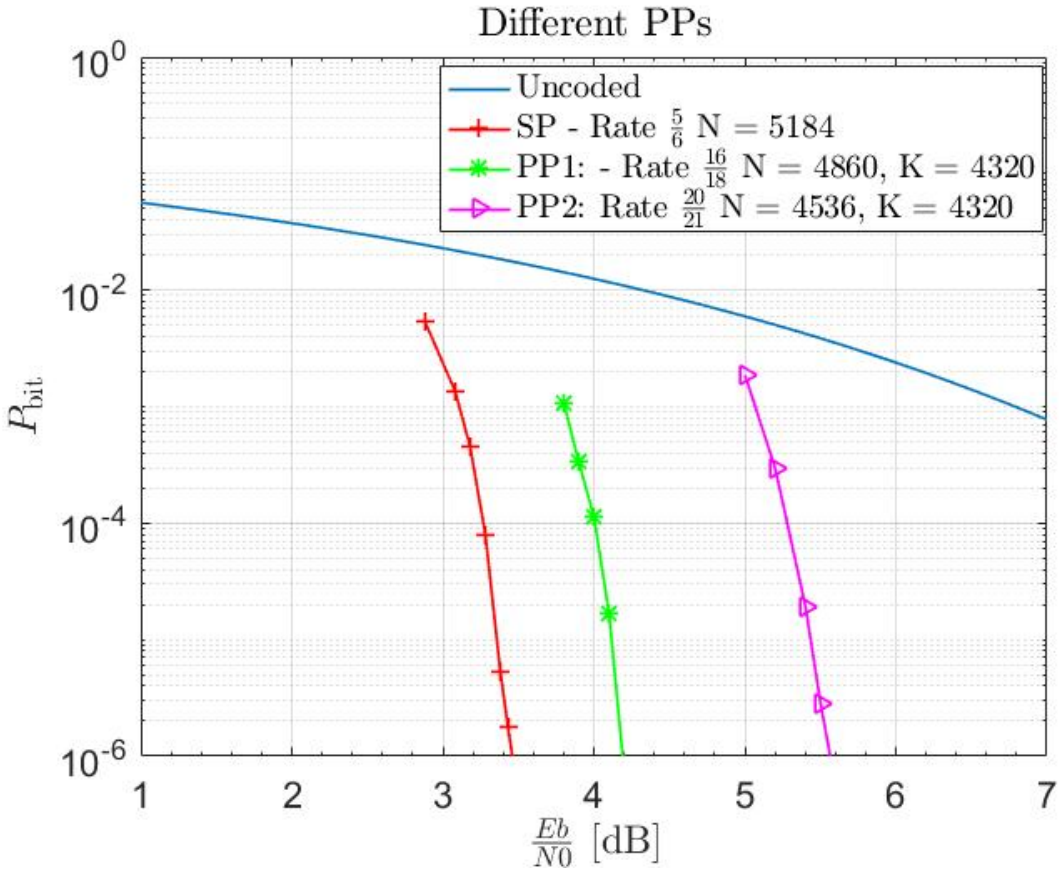
Results – Different Puncturing Patterns

For K = 960:

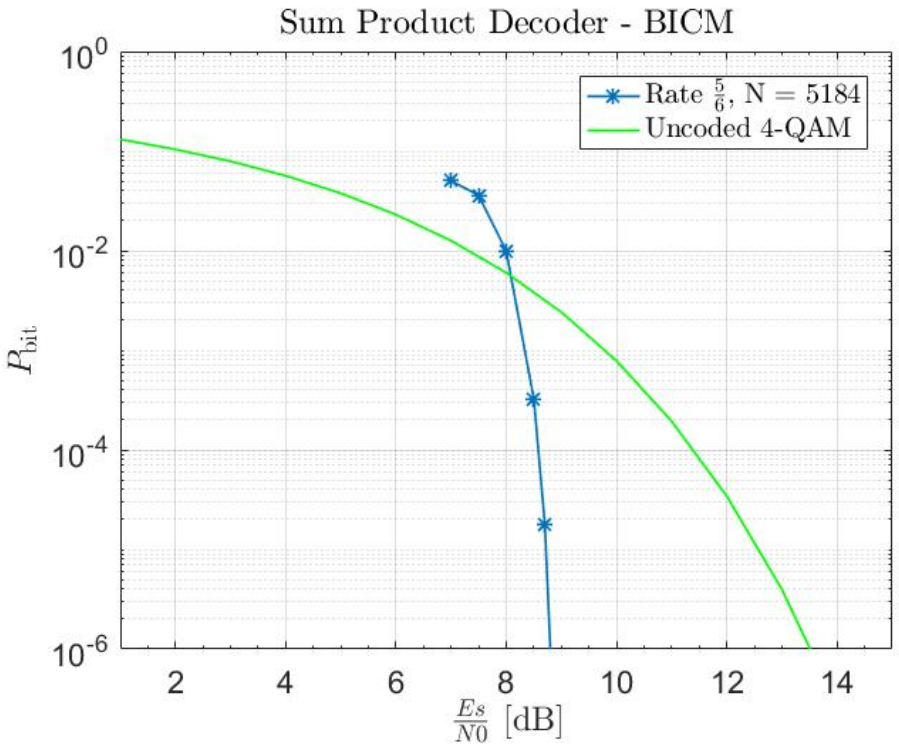


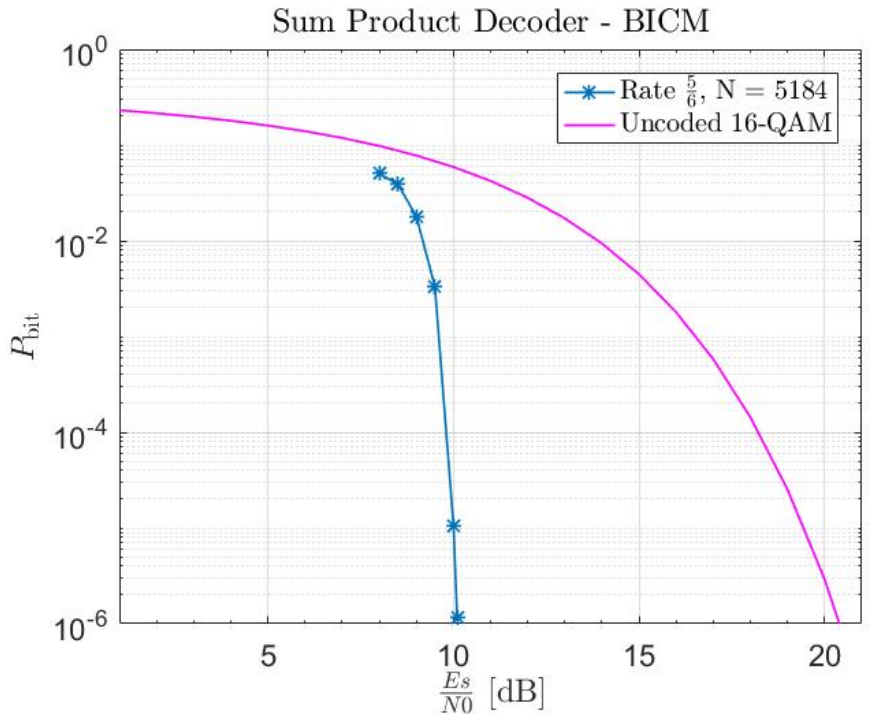
Results - Different Puncturing Patterns

For K = 4320:



Results - BICM





Conclusions and remarks

- Possible extensions/improvements.

Details

- All the images have been created with LaTeX or Matlab.

- The images reported in pag. 8 where taken from two superb books (According to Goodreads): The Pragmatic Programmer and Programming Pearls.

- The image of LDPC decoder have been selected from PDFs of Channel Coding Course (Tomaso Erseghe).