Fuzzy systems

Introduction to fuzzy systems

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Today's goal

Today, we going to introduce...

Fuzzy systems

- A bit of "fuzzy" history
- Basic definitions of fuzzy systems
  - Rules and sets
  - Membership functions
- How to read scientific papers

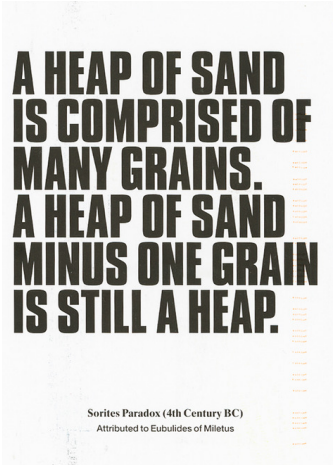
Reading list


W. Pedrycz and F. Gomide, *Fuzzy Systems Engineering –Toward Human-Centric Computing* – 3rd Edition


T.J. Ross, *Fuzzy Logic with Engineering Applications* – 3rd Edition


Foundations of Fuzzy Logic:
`https://www.mathworks.com/help/fuzzy/foundations-of-fuzzy-logic.html`

A "fuzzy" concept



`http://america.pink/sorites-paradox_4100507.html`

The sand is either a heap or is not a heap, without any shades of grey.

Fuzzy logic offers a continuous spectrum of logical states represented in the unit interval of real numbers.

The sand moves smoothly from **"definitely heap"** to **"definitely not heap"**, with shades in the intermediate region.

A "fuzzy" concept

Precision is not truth
Henri E. B. Matisse (1869–1954)

Oxford English Dictionary

Fuzzy: blurred, indistinct; imprecisely defined; confused, vague

Why Fuzzy Systems?

All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life but only to an imagined celestial existence. We must exploit our tolerance for imprecision.
Bertrand Russell (1872 –1970)

Motivations

- The real world is too complicated for precise descriptions to be obtained, therefore approximation (or **fuzziness**) must be introduced to obtain a reasonable model.
- Develop a theory to formulate the human reasoning in a systematic manner and put it into engineering systems.
- Fuzzy systems theory gives the formal tools to reason about **incomplete, imprecise, vague or uncertain information** as a representation language.

Modelling the human reasoning

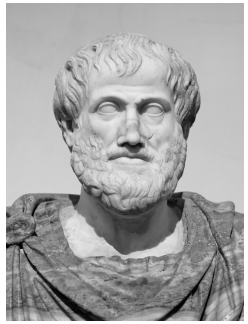
Fuzzy systems transform a human knowledge into a mathematical formula: **knowledge-based or rule-based systems**

- The human brain has the ability to understand linguistic terms, such as *very, partly, etc*, and infer from them.
- In fuzzy sets, an element belongs to a set to a degree, indicating the certainty (or uncertainty) of membership.
- Together with fuzzy logic, fuzzy set theory provides the tools to **develop software products that model human reasoning**.

A historical perspective

The mind versus the logic: Aristotle and Zadeh[1]

- Aristotle (384-322 BC) is a towering philosopher from the ancient Greece.
- He is the father of the field of logic.
- He observed that the validity of any argument can be determined by its structure rather than its content.
- Syllogism: All men are mortal; Socrates is a man; therefore, Socrates is mortal.
- Given the structure of this argument, **as long as the premises are true, then the conclusion is also guaranteed to be true**.



[1] L.I. Perlovsky *The Mind vs. Logic: Aristotle and Zadeh*

A historical perspective

The mind versus the logic: Aristotle and Zadeh

Aristotle's brand of logic dominated this area of thought until the rise of modern propositional logic and predicate logic 2000 years later.

The XX century saw the first developments of alternatives to probability theory and to classical Aristotelian logic as paradigms to address more kinds of uncertainty than just the random kind.

- Jan Lukasiewicz developed a multivalued, discrete logic (circa 1930).
- Arthur Dempster (in the 1960s) developed a theory of evidence including an assessment of ignorance.
- In 1965, **Lotfi Zadeh introduced his seminal idea in a continuous-valued logic: fuzzy set theory**.
- In the 1970s, Glenn Shafer extended Dempster's work to produce a complete theory of evidence dealing with information from more than one source, and Lotfi Zadeh illustrated a possibility theory resulting from special cases of fuzzy sets.

A historical perspective

Pioneers

1972 Japan Fuzzy System Research foundation is founded.

1973 Zadeh introduces a methodology for describing systems using language that incorporates *fuzziness*.

1974 Mamdani of the University of London succeeds with an experimental fuzzy control of a steam engine.

1980 Smith & Co. A/S in Denmark implements fuzzy theory in cement kiln control. It is the first practical implementation of fuzzy theory.

1983 Fuji Electric Co. implements fuzzy theory in the control of chemical injection for water purification plant (Japan's first).

1984 International Fuzzy System Association (IFSA) founded.

1985 1st IFSA Internation Conference.

1987 2nd IFSA international conference and fuzzy logic controlled subway system starts operation in Sendai, Japan.

A historical perspective

Objections

In the early days, fuzzy logic faced some objections:

- "Fuzzy theory is wrong, wrong, and pernicious. What we need is more logical thinking, not less. The danger of fuzzy logic is that it will encourage the sort of imprecise thinking that has brought us so much trouble. Fuzzy logic is the cocaine of science." (Professor W. Kahan, UC Berkeley)
- "Fuzziness is probability in disguise. I can design a controller with probability that could do the same thing that you could do with fuzzy logic." (Prof. M. Tribus (IEEE Institute) on hearing of the fuzzy-logic control of the Sendai subway system 1988).

Most objections to fuzzy logic have faded due to the **success of fuzzy in many different fields of applications**.

Applications

Fuzzy logic has been successfully used in numerous fields such as **control systems engineering, image processing, power engineering, industrial automation, robotics, consumer electronics, optimisation, ...**

- Pattern recognition
  - Image, audio, signal processing, ...
- Qualitative analysis
  - Operation research, management
- Inference
  - Expert systems for planning and prediction, software engineering in medicine, business
- Control (it is probably the most popular)
  - Modeling, identification and control
  - Multivariable control
  - Merging fuzzy control and PLCs
  - Industrial automation

Applications

Everyday life applications

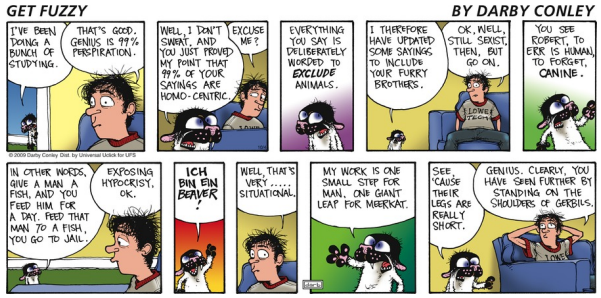


Image credits: http://www.gocomics.com/getfuzzy

Applications

Fuzzy systems at home

Source: https://www.surfexcel.in/

Fuzzy washing mashing were produced for the first time by Matsushita Electric Industrial Company in Japan around 1990

They use a fuzzy system to **automatically set the proper cycle** according to the kind and amount of dirt and the size of the load.

Nowadays the technology is rather common in many brands of washing machine.

Applications

Fuzzy systems at home

Matsushita introduced what it calls a **digital image stabiliser**, based on fuzzy systems.

It stabilises the picture when the hand is shaking.

More...

Fuzzy logic is used in **vacuum cleaner** to measure air dust and adjust the suction power, in **microwave ovens**, to measure temperature, humidity, weight of food to set time and power, in **air conditioning** systems to regulate the temperature, ...

Applications

Fuzzy systems in the streets

- Nissan has patented a **fuzzy automatic transmission** that saves fuel by 12 to 17%, and developed a **fuzzy antilock braking** system.
- Mitsubishi developed a fuzzy omnibus system that controls a car's automatic transmission, suspension, traction, four-wheel steering, four-wheel drive, and air conditioner.
- Hitachi designed Sendai subway in Japan with 16 stations and 13 km of route.

Applications

Advantages

- It uses rules that express imprecision of the real world
- It is easy to understand, test and maintain
- It is easy to prototype
- Parallel evaluation of rules

Disadvantages

- It needs more tests and simulations
- It does not learn easily
- It is difficult to establish the correct rules
- Lack of precise mathematical models

Crisp Vs. Fuzzy logic

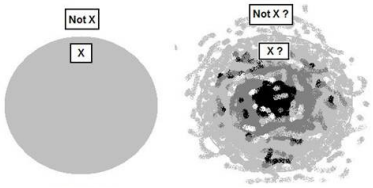
Crisp logic

It is the classical, **two-valued set theory** where an element is either a member of the set or not.

Fuzzy logic

Fuzzy sets are an extension of crisp (two-valued) sets to handle the concept of **partial truth**.

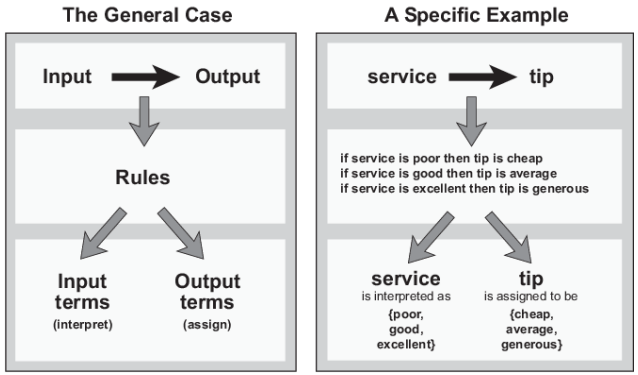
It enables the modelling of the uncertainties of natural language.



Fuzzy rules

- The point of fuzzy logic is to map an input space to an output space, and the primary mechanism for doing this is a list of `IF-THEN` statements called **rules**.
- All rules are evaluated in parallel, and the order of the rules is unimportant.
- The rules themselves are useful because they refer to variables and the adjectives that describe those variables.
- Before you can build a system that interprets rules, you must define all the terms you plan on using and the adjectives that describe them.
- To say that the water is hot, you need to define the range that the water's temperature can be expected to vary as well as what we mean by the word hot.

Fuzzy rules



Source: http://www.mathworks.com/

To interpreter the values in the input vector and, based on some set of rules, assigns values to the output vector, we need to define a method. This is called **fuzzy inference**.

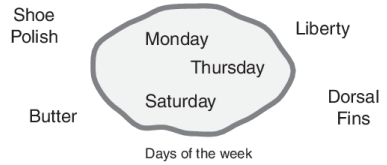
Sets

Crisp logic

A classical set is a container that wholly includes or wholly excludes any given element.

Aristotle's Law of the Excluded Middle

⇝ X must either be in A or not in A.

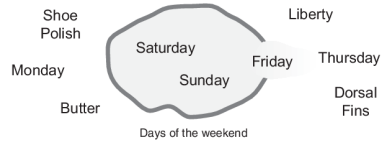


Fuzzy set

A fuzzy set is a set without a crisp, clearly defined boundary.

It can contain elements with only a partial degree of membership.

The truth of any statement becomes a matter of degree.



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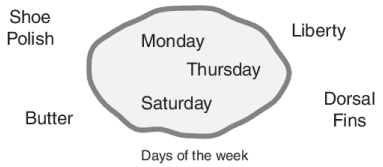
Sets

Crisp logic

The two categories A and not-A, should between them contain the entire universe.

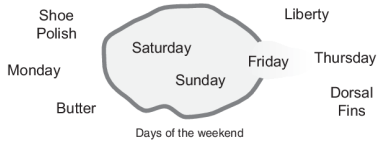
There is no thing that is both a day of the week and not a day of the week.



Fuzzy set

Saturday and Sunday belong to the weekend set, but what about Friday?

It feels like a part of the weekend, but somehow it seems like it should be technically excluded.



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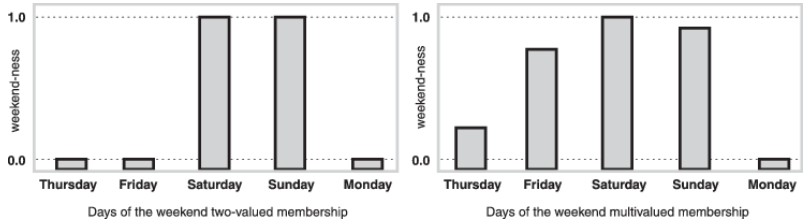
Reasoning[2]

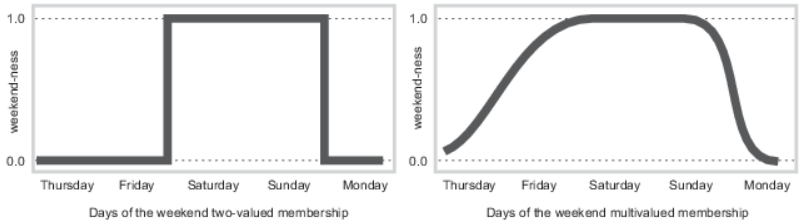


- Reasoning in fuzzy logic is just a matter of generalising the familiar `YES-NO` Boolean logic.
- If you ask the question “Is X a member of set A?” the answer might be yes, no, or any one of a thousand intermediate values in between.

  X might have partial membership in A.
- **Multivalued logic** is in contrast to the more familiar concept of **two-valued logic**.

[2] Source: http://www.mathworks.com/

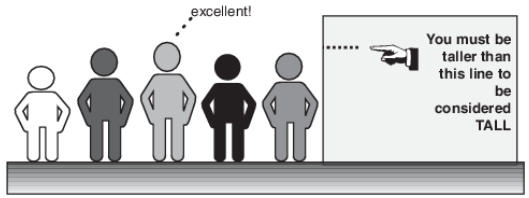
Reasoning



- Making a continuous plot, we are defining the degree to which any given instant belongs in the weekend rather than an entire day.
- Left plot: the weekend-ness truth value jumps discontinuously from 0 to 1.
- Right plot: the curve varies smoothly.
- The curve that defines the weekend-ness of any instant in time is a function that maps the input space (time of the week) to the output space (weekend-ness).

  It is known as a **membership function**.

Membership function

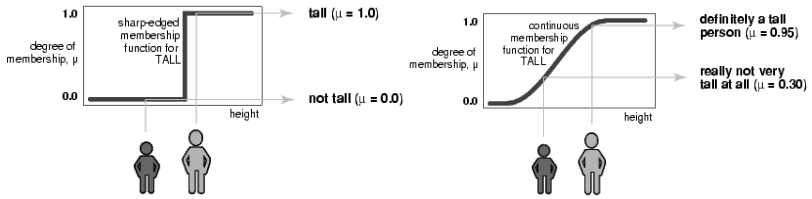


Source: http://www.mathworks.com/

- The set of all potential heights, say from 1 to 2.5 m, is our **universe of discourse**.
- If “tall” is a set defined as heights equal to or greater than 1.8 m, a computer would not recognize an individual of height 1.798 as being a member of the set “tall”.
- It is unreasonable to call one person short and another one tall when they differ in height by the width of a hair.

Membership function

The curve that defines the **degree of membership** between 0 and 1 is called **membership function** and is often indicated with μ.



Source: http://www.mathworks.com/

- A crisp set has a unique membership function.
- A fuzzy set can have an infinite number of μ to represent it. Subjective interpretations are included into fuzzy sets.
- For fuzzy sets the uniqueness is sacrificed, but flexibility is gained because the membership function can be adjusted to maximize the utility for a particular application.

Membership function

The only condition a membership function must really satisfy is that it must vary between 0 and 1.

The function itself can be an arbitrary curve whose shape we can define as a function that suits us from the point of view of simplicity, convenience, speed, and efficiency.

- **Classical set**: $A\{x|x > 6\}$
- **Fuzzy set**: $A\{x, \mu_A|x \in X\}$

  It is an extension of a classical set.

  If X is the universe of discourse and its elements are x, then a fuzzy set A in X is defined as a set of ordered pairs.

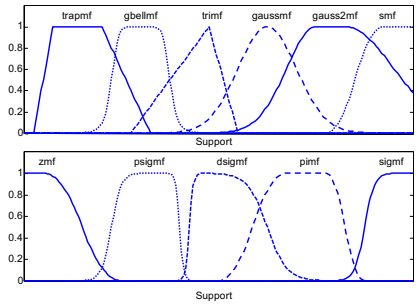
  The membership function μ_A maps each element of X to a membership 0 and 1.

Membership function

The only condition a membership function must really satisfy is that it must vary between 0 and 1.

The function itself can be an arbitrary curve whose shape we can define as a function that suits us from the point of view of simplicity, convenience, speed, and efficiency.

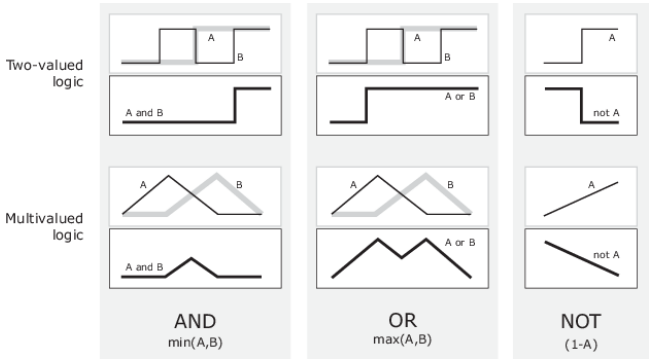
- There is a very wide selection to choose from when selecting μ.
- You can also create your own membership functions.



Logical operations

The most important thing to realize about fuzzy logical reasoning is the fact that it is a superset of standard Boolean logic.

- If we keep the fuzzy values at their extremes of 1 (completely true), and 0 (completely false), standard logical operations will hold.



If-Then rules

Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy logic.

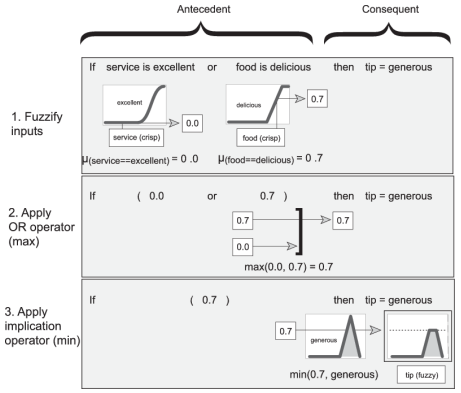
These if-then rule statements are used to formulate the conditional statements that comprise fuzzy logic.

A single fuzzy if-then rule assumes the form

```
If x is A, then y is B
```

- A and B are linguistic values defined by the fuzzy set on the ranges (universes of discourse) X and Y, respectively.
- The `if-part` of the rule "x is A" is called the **antecedent or premise**.
- The `then-part` of the rule "y is B" is called the **consequent or conclusion**.

If-Then rules



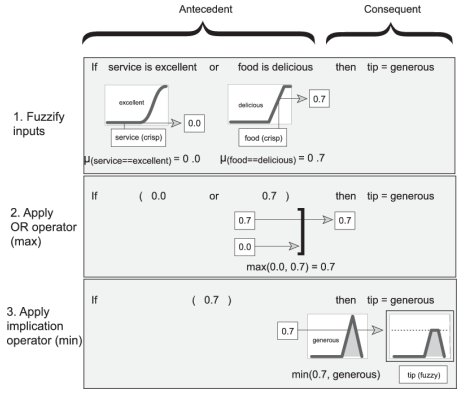

```
If service is good,
then tip is average
```

- `good` is represented as a number between 0 and 1.

  The antecedent is an interpretation that returns a single number between 0 and 1.
- `average` is represented as a fuzzy set.

  The consequent is an assignment that assigns the entire fuzzy set B to the output variable y.

If-Then rules

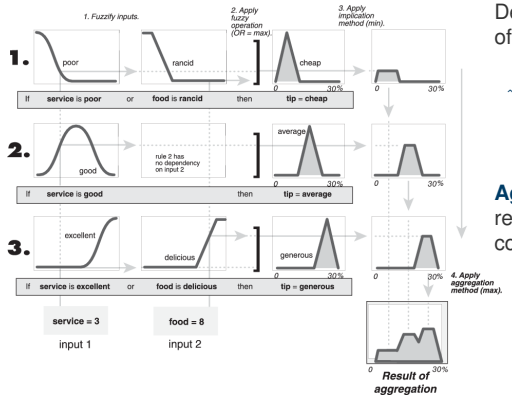


The input to an if-then rule is the current value for the input variable (`service`) and the output is an entire fuzzy set (`average`).

Interpreting an if-then rule involves two steps:

- Evaluation of the antecedent: **Fuzzifying** the inputs and applying any necessary fuzzy operators.
- Application of the result to the consequent: **Implication**

If-Then rules

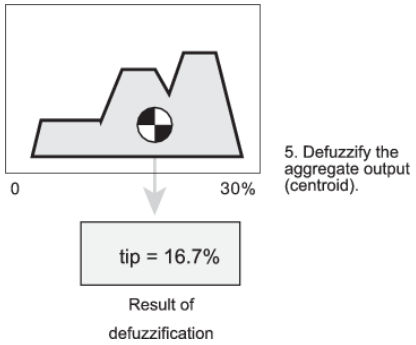


Decisions are based on the testing of all of the rules in a fuzzy inference system

- ⇝ The rules must be combined in some manner in order to make a decision.

Aggregation: the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set.

If-Then rules

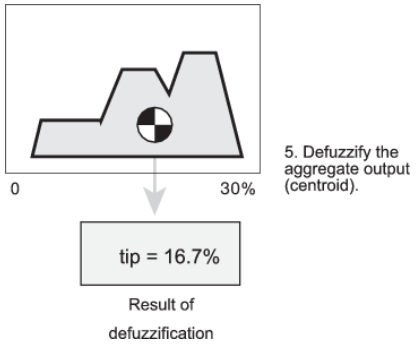


The aggregate of a fuzzy set encompasses a range of output values.

- ⇝ It must be defuzzified in order to resolve a single output value from the set.

The input for the **defuzzification** process is a fuzzy set (the aggregate output fuzzy set) and the output is a single number.

If-Then rules

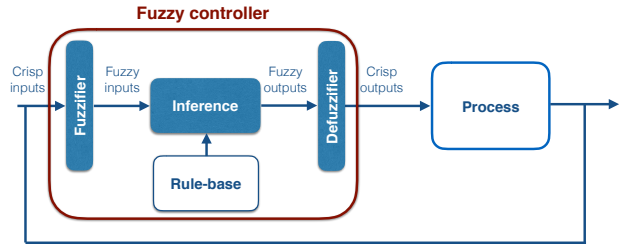


There are different defuzzification methods supported:

- Centroid and bisector
- Middle of maximum.
- Largest of maximum, and smallest of maximum.

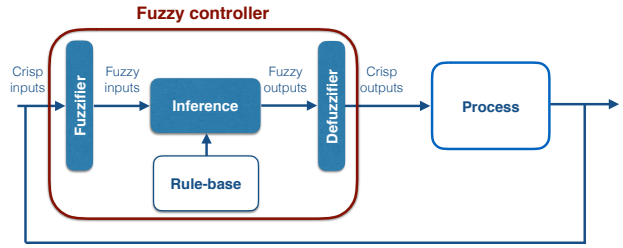
Perhaps the most popular is the **centroid calculation**: It returns the center of area under the curve.

Fuzzy systems



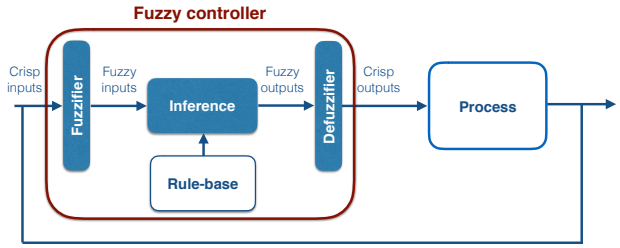
- Embedding the fuzzy system in a closed-loop control system gives a **fuzzy controller**.
- It provides a formal methodology for representing, manipulating, and implementing a human's heuristic knowledge on how to control a system.

Fuzzy systems



1. Define the linguistic variables and terms (⇝ initialization)
2. Construct the membership functions (⇝ initialization)
3. Construct the rule base (⇝ initialization)
4. Convert crisp input data to fuzzy values using the membership functions (⇝ fuzzification)

Fuzzy systems



5. Evaluate the rules in the rule base ($\rightsquigarrow$ inference)
6. Combine the results of each rule ($\rightsquigarrow$ inference)
7. Convert the output data to non-fuzzy values ($\rightsquigarrow$ defuzzification)

Tips for HW4

Suggestions on how to read a scientific paper.

Type of papers

- Original article – information based on original research
- Case reports – usually of a single case
- Technical notes – describe a specific technique or procedure
- Review – detailed analysis of recent research on a specific topic
- Commentary – short article with author's personal opinions
- Editorial – often short review or critique of original articles
- Letter to the Editor – short and on subject of interest to readers

Tips for HW4

Suggestions on how to read a scientific paper.

Structure of papers

- Title
- Abstract
- Introduction
- Methods
- Results
- Discussion/Conclusions
- Acknowledgements
- References

Tips for HW4

Suggestions on how to read a scientific paper.

IMRAD Format

- **I**ntroduction: What was the question?
- **M**ethods: How did the research(s) try to answer it?
- **R**esults: What did the researchers find?
- **A**nd ...
- **D**iscussion: What do the results mean?

Tips for HW4

Suggestions on how to read a scientific paper.

Action to take

1. **Skim**: Read quick the article without taking notes:
   - Read the abstract: it tells the major findings of the article and why they matter.
   - Read first for the “big picture”.
   - Note any terms or techniques you need to define.
   - Write down any questions or parts you do not understand.
   - If you are unfamiliar with any of the key concepts in the article, look them up in the literature.

Tips for HW4

Suggestions on how to read a scientific paper.

Action to take

2. **Re-read**: Read again more carefully especially the “methods” and “results/conclusions” sections:
   - Carefully examine the graphs, tables, and diagrams.
   - Ask yourself questions about the study, such as: What problems does the study address? Why is it important? Is the method good? Are the findings supported by evidence? Are they unique and supported by other work in the field?
   - Make sure you understand the article fully.

   Again... If you are unfamiliar with some concepts, look for them in the literature.

Tips for HW4

Suggestions on how to read a scientific paper.

Action to take

3. **Interpret**: Try to interpret the data first before reading the captions and details.
   - When reading the discussion and results, look for key issues and new findings.
   - Make sure you understand the article fully.
   - Make sure that you have distinguished the main point.

     If not, go over the text again.

Tips for HW4

Suggestions on how to read a scientific paper.

Action to take

4. **Summarize**: Take notes as you read:
   - Taking notes improves recall and comprehension; you may think you’ll remember everything but details will slip away.
   - Describe the article in your own words – to distill the article down to its “scientific essence”.
   - Note the “key points” – purpose of the study/questions asked, assumptions, major findings and conclusions, questions unanswered and any surprises.