

**Example:** Solve the following system by using Gauss elimination method.

$$\begin{bmatrix} 3 & -2 & 5 & 0 \\ 4 & 5 & 8 & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 7 & 6 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 4 \\ 5 \\ 7 \end{Bmatrix}$$

**Solution:**

First, let's put the system in this augmented form:

$$\begin{array}{c} \text{1} \quad \text{2} \quad \text{3} \quad \text{4} \quad b \\ \text{1} \left| \begin{array}{cccc|c} 3 & -2 & 5 & 0 & 2 \\ 4 & 5 & 8 & 1 & 4 \\ 1 & 1 & 2 & 1 & 5 \\ 2 & 7 & 6 & 5 & 7 \end{array} \right. \end{array}$$

The red numbers indicate to **rows** and the blue numbers indicates to **columns**

To eliminate the elements under the main diagonal in the **column 1**, fix the row 1 and starting from the **row 2 to 4** and from **column 1** to **b** multiply each element by the factor:  $a(\mathbf{1},\mathbf{1})/a(\mathbf{row},\mathbf{1})$ .

$$a(\mathbf{1},\mathbf{1})/a(\mathbf{2},\mathbf{1}) = 3/4 = 0.75$$

$$a(\mathbf{1},\mathbf{1})/a(\mathbf{3},\mathbf{1}) = 3/1 = 3$$

$$a(\mathbf{1},\mathbf{1})/a(\mathbf{4},\mathbf{1}) = 3/2 = 1.5$$

$$\begin{array}{c} \text{1} \quad \text{2} \quad \text{3} \quad \text{4} \quad b \\ \text{1} \left| \begin{array}{cccc|c} 3 & -2 & 5 & 0 & 2 \\ 3 & \mathbf{3.75} & 6 & 0.75 & 3 \\ 3 & 3 & \mathbf{6} & 3 & 15 \\ 3 & 10.5 & 9 & \mathbf{7.5} & 10.5 \end{array} \right. \end{array}$$

From **row 2 to 4** and from **column 1** to **b** subtract each element from that in the **fixed row 1** and the **same column**.

$$\begin{array}{c} \text{1} \quad \text{2} \quad \text{3} \quad \text{4} \quad b \\ \text{1} \left| \begin{array}{cccc|c} 3 & -2 & 5 & 0 & 2 \\ 0 & \mathbf{-5.75} & -1 & -0.75 & -1 \\ 0 & -5 & \mathbf{-1} & -3 & -13 \\ 0 & -12.5 & -4 & \mathbf{-7.5} & -8.5 \end{array} \right. \end{array}$$

To eliminate the elements under the main diagonal in the **column 2**, fix the row 2 and starting from the **row 3 to 4** and from **column 2** to **b** multiply each element by the factor:  $a(\mathbf{2},\mathbf{2})/a(\mathbf{row},\mathbf{2})$ .

$$a(\mathbf{2},\mathbf{2})/a(\mathbf{3},\mathbf{2}) = -5.75/-5 = 1.15$$

$$a(\mathbf{2},\mathbf{2})/a(\mathbf{4},\mathbf{2}) = -5.75/-12.5 = 0.46$$

$$\begin{array}{c} \text{1} \quad \text{2} \quad \text{3} \quad \text{4} \quad b \\ \text{1} \left| \begin{array}{cccc|c} 3 & -2 & 5 & 0 & 2 \\ 0 & \mathbf{-5.75} & -1 & -0.75 & -1 \\ 0 & \mathbf{-5.75} & \mathbf{-1.15} & -3.45 & -14.95 \\ 0 & \mathbf{-5.75} & -1.84 & \mathbf{-3.45} & -3.91 \end{array} \right. \end{array}$$

From **row 3 to 4** and from **column 2** to **b** subtract each element from that in the **fixed row 2** and the **same column**.

$$\begin{array}{c|cccc|c}
 & 1 & 2 & 3 & 4 & b \\
 1 & 3 & -2 & 5 & 0 & 2 \\
 2 & 0 & -5.75 & -1 & -0.75 & -1 \\
 3 & 0 & 0 & 0.15 & 2.7 & 13.95 \\
 4 & 0 & 0 & 0.84 & 2.7 & 2.91
 \end{array}$$

To eliminate the elements under the main diagonal in the **column 3**, **fix the row 3** and starting from the **row 4 to 4** and from **column 2 to b** multiply each element by the factor:  $a(3,3)/a(4,3)$ .

$$a(3,3)/a(4,3) = 0.15/0.84 = 0.178571$$

$$\begin{array}{c|cccc|c}
 & 1 & 2 & 3 & 4 & b \\
 1 & 3 & -2 & 5 & 0 & 2 \\
 2 & 0 & -5.75 & -1 & -0.75 & -1 \\
 3 & 0 & 0 & 0.15 & 2.7 & 13.95 \\
 4 & 0 & 0 & 0.15 & 0.482143 & 0.519643
 \end{array}$$

From **row 4 to 4** and from **column 3 to b** subtract each element from that in the **fixed row 3** and the **same column**.

$$\begin{array}{c|cccc|c}
 & 1 & 2 & 3 & 4 & b \\
 1 & 3 & -2 & 5 & 0 & 2 \\
 2 & 0 & -5.75 & -1 & -0.75 & -1 \\
 3 & 0 & 0 & 0.15 & 2.7 & 13.95 \\
 4 & 0 & 0 & 0 & 2.217857 & 13.430357
 \end{array}$$

Now, the back substitution stage of the solution:

$$\begin{array}{c|cccc|c}
 & 1 & 2 & 3 & 4 & b \\
 1 & 3 & -2 & 5 & 0 & 2 \\
 2 & 0 & -5.75 & -1 & -0.75 & -1 \\
 3 & 0 & 0 & 0.15 & 2.7 & 13.95 \\
 4 & 0 & 0 & 0 & 2.217857 & 13.430357
 \end{array}$$

Starting from row 4 to 1:

$$x(4) = b(4) / a(4,4) = 13.430357 / 2.217857 = 6.055556$$

$$x(3) = [b(3) - a(3,4) * x(4)] / a(3,3)$$

$$= [13.95 - 2.7 \times 6.055556] / 0.15 = -16$$

$$x(2) = [b(2) - a(2,4) * x(4) - a(2,3) * x(3)] / a(2,2)$$

$$= [-1 - (-0.75) \times 6.055556 - (-1) \times (-16)] / (-5.75) = 2.166667$$

$$x(1) = [b(1) - a(1,4) * x(4) - a(1,3) * x(3) - a(1,2) * x(2)] / a(1,1)$$

$$= [2 - 0 \times 6.055556 - 5 \times (-16) - (-2) \times 2.166667] / 3 = 28.777778$$

So, the solution of the system is:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 28.777778 \\ 2.166667 \\ -16 \\ 6.055556 \end{Bmatrix}$$

To verify the solution, we can multiply [a] by {x} to see if it gives {b}.

$$\begin{bmatrix} 3 & -2 & 5 & 0 \\ 4 & 5 & 8 & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 7 & 6 & 5 \end{bmatrix} \begin{pmatrix} 28.777778 \\ 2.166667 \\ -16 \\ 6.055556 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 5 \\ 7 \end{pmatrix}$$