ENERGY 191/291 Homework #4, Spring 2022

Part I: Math and algorithms

1. Solve the following linear program using the matrix (revised) simplex method:

$$\text{Max: } f(x) = 3x_1 + 4x_2 + 5x_3 \tag{1}$$

Subject to the following constraints:

$$1x_1 + 2x_2 + 2.5x_3 \leq 14 \tag{2}$$
$$1x_1 + 2x_2 + 1.33x_3 \leq 12 \tag{3}$$
$$4x_1 + 2x_2 + 5x_3 \leq 14 \tag{4}$$
$$2x_1 + 3x_2 + 1x_3 \leq 6 \tag{5}$$
$$x_1, x_2, x_3 \geq 0 \tag{6}$$

(a) Write your initial (unchanging) simplex components **A**, **b**, and **c**. [5 pts]

(b) For each iteration, record the new **B** matrix, as well as $\mathbf{x}_B$ and the objective function value at that iteration. Hint: you should perform inversion and matrix multiplication operations using a basic Simplex method you implement in Julia. Many tutorials can be found online for building the simple Julia code required for matrix inversion and running the necessary tests for each pivot. Submit your Julia code as a file named `final_HW4Q.jl`[15 pts]

(c) State your final optimal solution and objective function value. Which constraints are binding (report using evidence from values of slack and surplus variables)? [5 pts]

(d) Using your final shadow prices as evidence, which constraint should you put in the most effort to loosen? [5 pts]

Part II: Modeling

Optimization of wind power storage with linear battery lifetime

This problem set is based in part on: Donnelly, C. (2010). *Optimizing battery lifetime in wind power storage projects*. Class project, ENERGY 191/291, Stanford University, March 2010.

You are a wind power generator who operates 3 wind farms in south-eastern Wyoming and north-eastern Colorado (see Figure 1.1). These farms are named WIND-A, WIND-B, and WIND-C, and have peak capacities of 60 MW, 15 MW, and 30 MW, respectively. These locations vary in their average wind output over time, as shown in Figure 1.2. For example, site WIND-C is the weakest wind site, with an average speed of 7.2 m/sec and a capacity factor of 28.2%.

Because the wind power often arrives when there is little demand (for example, in the middle of the night) you are exploring a potential wind-power storage site, which would take power from the three wind farms and store it in a single repository for sale when the power is more valuable. Such a storage site would consist of a sodium-sulfur (NaS) battery bank. Your storage site is marked STORE-A in Figure 1.1. Your initial estimate for the amount of energy stored in STORE-A is 200 MWh, but you want to explore other options as well. You will be able to profit from storage if the cost of electricity storage is less than the difference in power price between the middle of the night and the peak daytime price (see Figure 1.3 for a power price time series).

The cost of storing electricity has two components: the loss of electricity during storage (called "round-trip" battery losses), and a capital cost that accounts for the wear and tear on your battery system. The losses of electricity during storage mean that whenever you store 1 MWh of power, less power is available to sell upon discharging the battery.

The capital cost of the battery is governed by the cycle life of your battery, which can be measured as the effective number of "100%" discharge cycles (e.g., MWh discharged from your battery over its life per MWh of battery capacity). Neither of these costs is affected by the length of time for which the power is stored. The values of these parameters are obtained from the literature (Lu, 2009) and tabulated below in Table 1.1. To estimate the cost of capital degradation over the life of the optimized time period, c_{cap}, use the relationship:

$$c_{cap} = C_{cap} \times \frac{D_c}{L_c} \qquad \left[\frac{\$}{\text{MWh}}\right] \tag{1.1}$$

where D_c is the total number of effective discharge cycles, measured as the sum of total discharge over all time periods divided by the battery capacity. For example, if total discharges from a 10 MWh battery are 1700 MWh, D_c would equal 170 effective complete discharges. L_c is the expected number of effective total discharges over the life of the battery and C_{cap} is the capital cost per MWh. Values for these terms are given below in Table 1.1.

Build an optimization model to determine how much power to store and sell on the representative day. Your model should maximize profits, including both effects of round-trip efficiency and the wear and tear associated with charging and discharging the battery. You will want to explore the sensitivity of model results how large of a battery system you install.

Data

Power outputs from your three farms for a three representative days (September 1-3rd, 2004), as well as the value of power as a function of time, as plotted in Figures 1.2 and 1.3 are given in an at-

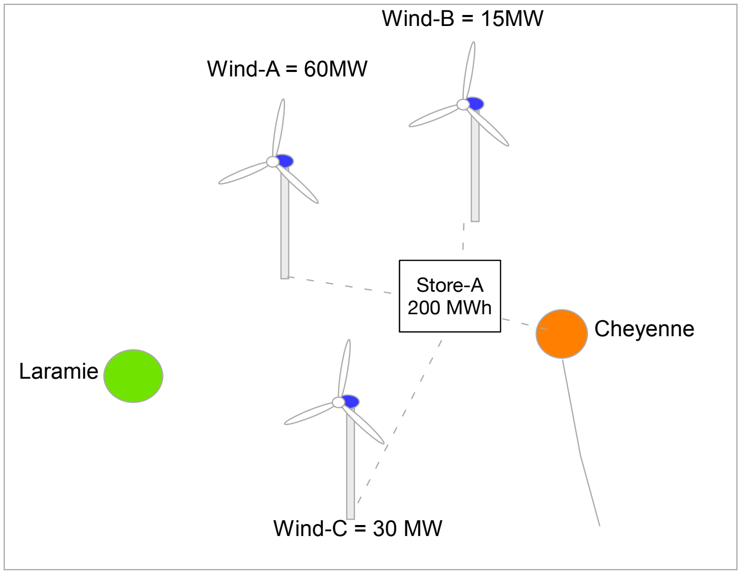

Figure 1.1: Schematic diagram of locations of wind farms and proposed storage site. This is a cartoon schematic representing approximate location of data sources as accessed from National Renewable Energy datasets.

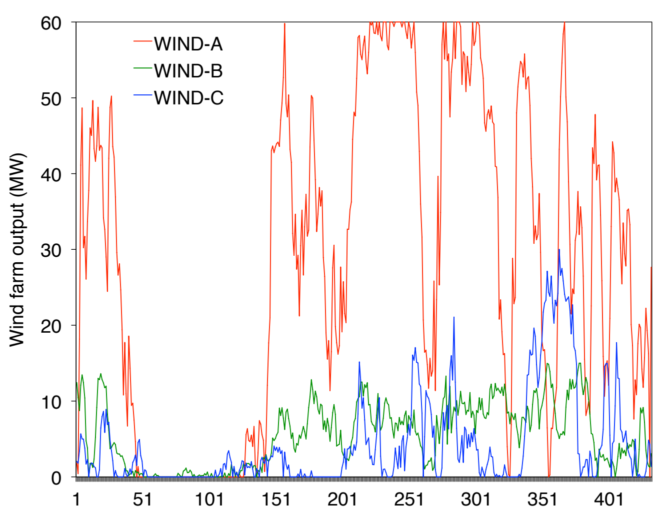

Figure 1.2: Wind power output from three farms. Source: National Renewable Energy Laboratory, Western Wind Resources Dataset.

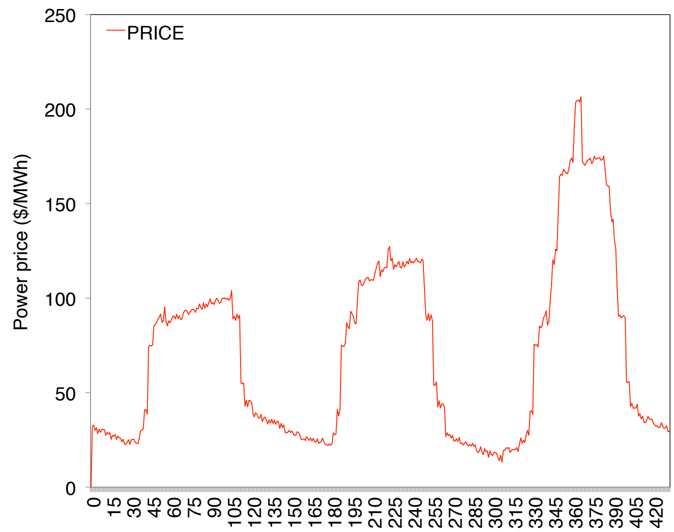

Figure 1.3: Power price as a function of time. Note the much higher power price at peak times of day. Source: Author assumption.

tached spreadsheet. These data are provided for 10 minute increments. The parameters associated with your battery (capital costs, decay parameter, etc.) are given in Table 1.1.

Table 1.1: Parameters for battery storage system. Source: Data from (Lu, 2009) (Makarov 2008) (EPRI 2003).

Parameter	Symbol	Value	Unit
Capital costs	C_{cap}	250,000	[$/MWh capacity]
Lifetime constant	L_c	3000	[MWh lifetime stored into battery per MWh of capacity]
Round-trip losses	W	0.0645	[MWh/MWh stored]

Questions

1. Write the optimization problem in condensed mathematical form. Clearly note decision variables, constraints, and objective function. Define indices clearly. Also, be sure to list the units for all terms. [20 pts]

2. Program your model in Julia/JuMP to determine the amount of power stored in the battery at all times. Generate a plot of the amount stored as a function of time. Report your values for the total profit (revenues - capital costs) for storing power over time. For this question, name your file `final_HW4Q2.jl` [25 pts]

   Hints: assume that the battery loss occurs upon discharging the battery, not upon charging the battery. Also, you can compute battery capital costs by calculating the fraction of available battery cycles that are used up during your modeled time period. Again assume that capital cost occurs upon discharge.

3. Your result should show strange behavior: the model finds an optimal solution that involves draining the entire battery in one time step. This would be detrimental to your batteries and

overload your local power transmission system which takes power from the battery to the grid. Institute a limit such that the battery can only be drained by 10 MWh in any 10-minute time step. How does this affect your optimal solution? For this question, name your file `final_HW4Q3.jl` [10 pts]

4. Explore the sensitivity of your results to the battery bank size. Keep the same model from above (including discharge rate constraint) but vary the size of batteries by increments of 100 MWh from 0 to 1000 MWh. Generate a plot of total energy storage and profits over the 3 days as a function of battery size. At what size does the increased size of the battery not result in more energy stored? [10 pts]

5. The price path and wind outputs only include a few days of data. Explain in a few sentences how using a more complete dataset might affect your results. Do you think your results would be more or less sensitive to the capital cost if you used a whole year's worth of data? [5 pts]