ENERGY 291 Project Report 4
Optimizing Residential Air Conditioning Thermostat to Minimize Electricity Cost

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Background / Motivation

Air conditioning (AC) load represents the majority of the total load demand for the residential sector.Generally, people living inside a residential building would have control over the house temperature and the air conditioner would run to maintain the set temperature. If in case the users are willing to loosen the house temperature setpoint above and below few degree Farenhiet, we could model the house as a thermal storage unit and utlize the thermal inertia for load shifting, cost minimization and peak demand minimization. The house's thermal properties depends upon the orientation, insulation materials, and the ambient temperature. The model will optimize the thermostat's heating and cooling energy profile throughout the day based on forecasted day-ahead outdoor temperatures and TOU (Time of Use) cost of electricity. The model would minimize cost and roll-out the day-ahead optimal thermostat control after receiving the comfort-savings trade-off setting from the customers.

House Thermal model

We model the house thermal system as a LTI (Linear Time Invariant) state model. Temperatures, conductances, thermal masses and heat flows are entirely equivalent to voltages, conductances, capacitors and current flows in the electric circuit analog. That is, the differential equations expressing conservation for energy are the same. We assume that the heater's and cooler's energy flow depends on the current indoor air temperature and a fixed temperature of the coil over which air is blown.

Heater equation

$$\frac{dQ_{heat}}{dt} = (T_{heater} - T_{indoor}) \times Mdot \times c$$

Cooler equation

$$\frac{dQ_{cool}}{dt} = (T_{indoor} - T_{cooler}) \times Mdot \times c$$

Thermal losses/gain of the houses

$$\frac{dQ_{losses}}{dt} = \frac{T_{indoor} - T_{out}}{R_{eq}}$$

House indoor temperature change

$$\frac{dT_{indoor}}{dt} = \frac{1}{M_{air} \times c}\left(\frac{dQ_{heat}}{dt} - \frac{dQ_{cool}}{dt} - \frac{dQ_{losses}}{dt}\right)$$

where, dQ_{heat}/dt is the heat flow from the heater into the room, dQ_{cool}/dt is the heat flow from the cooler out of the room, dQ_{losses}/dt is the heat loss/gain to outside of the house, T_{heater} is the temperature of the heating coil, T_{cooler} is the temperature of the cooling coil, T_{indoor} is the current indoor air temperature, Mdot is the air mass flow rate through the heater, R_{eq} is the equivalent thermal resistance of the house, M_{air} is the mass of air inside the house, c is the heat capacity of air at constant pressure. All the parameter values and units are mentioned at the end.

Optimization Model

We build an optimization model that minimizes the total cost to the customer by varying the heating and cooling energy profile of the Air Conditioner. We converted the first order differential equation to first order difference equation.
Decision variables

$$T_{indoor} \in R^{25}, Q_{heat} \in R^{24}_{+}, Q_{cool} \in R^{24}_{+}$$

$$\text{minimize} \sum_{t=1}^{24} cost_{(t)} \times (Q_{heat(t)} + Q_{cool(t)}) \times J2KW$$

$$\text{subject to } T_{indoor(1)} = T_{set}$$

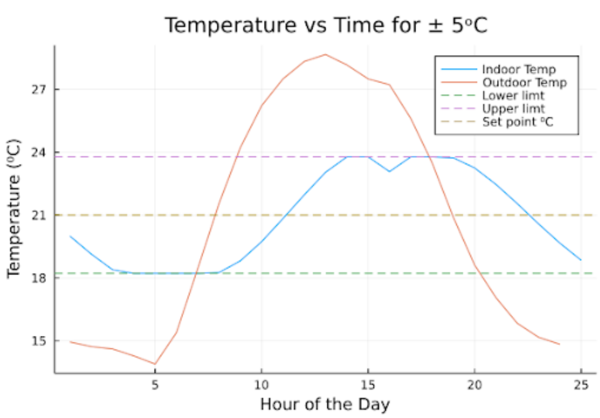
$$T_{set} - \Delta T_{comfort} \leq T_{indoor(t)} \leq T_{set} + \Delta T_{comfort}, \quad t = 1, ..., 25$$

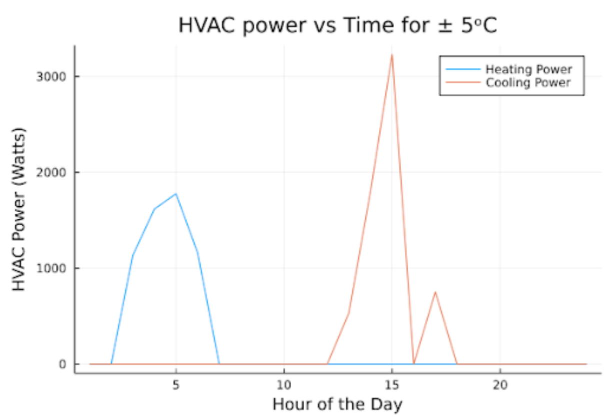
$$M_{air} \times c \times (T_{indoor(t+1)} - T_{indoor(t)}) = Q_{heat(t)} - Q_{cool(t)} - \frac{T_{indoor(t)} - T_{out(t)}}{R_{eq}} \quad t = 1, ..., 24$$

$$Q_{heat(t)} \leq Mdot \times c \times (T_{heater(t)} - T_{indoor(t)}) \quad t = 1, ..., 24$$

$$Q_{cool(t)} \leq Mdot \times c \times (T_{indoor(t)} - T_{cooler(t)}) \quad t = 1, ..., 24$$

Initial Results





Challenges

Thermal model of a house is complex and it depends upon several aspects including orientation of the house, insulation material of walls, floors, ceiling, windows and doors, heat gains from other appliances, and ambient temperature and wind speed outside. For that reason, building the thermal model to calculate the change in indoor temperature due to AC and change of ambient temperature was the most challenging aspect of building the model. We took an iterative approach and kept trying different initial conditions and parameters that resulted in a realistic model.

Parameter definitions and units

Decision Variables

Table 1: Decision variables

Variable Name	**Domain**	**Unit**
T_{indoor}	R^{25}	^oC
Q_{heat}	R^{24}	Joules/hr
Q_{cool}	R^{24}	Joules/hr

Paramater Units and Values

Table 2: Parameter Values and Units

Paramater Name	**Value**	**Domain**	**Unit**
cost	TOU price	R^{24}	cents/KWh
J2KW	2.77778E-07	R	Kwh$\times hr/joules$
T_{set}	20	R	^oC
$T_{comfort}$	2.78	R	^oC
M_{air}	8662.317	R	Kg
Mdot	3600	R	Kg/hr
R_{eq}	6.77E-07	R	$^oC \times hr/joules$
c	1005.4	R	joules/kg-K