ENERGY 291 Final Project Report
Optimizing Residential HVAC Thermostat to Minimize Electricity Cost

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Abstract

Heating, ventilation, and air conditioning (HVAC) load represents the majority of the total load demand for the residential sector. Generally, people living inside a residential building would have control over the house temperature and the air conditioner would run to maintain the set temperature. If the users are willing to loosen the house temperature setpoint above and below few degree Fahrenheit, we could model the house as a thermal storage unit and utilize the thermal inertia for load shifting, cost minimization and peak demand minimization. The model will optimize the thermostat's heating and cooling energy profile throughout the day based on forecasted day-ahead outdoor temperatures and TOU (Time of Use) cost of electricity. The model would minimize cost and roll-out the day-ahead optimal thermostat control after receiving the comfort-savings trade-off setting from the customers. As a result, the home owner can save a total of $120 per month by loosening house temperature set point by ±5°F. Also, peak load of HVAC can be lowered by 48% with ±5°F comfort range, from 3200 W to 1655 W, which can relief stress on the grid at peak hours.

1 Introduction

1.1 Problem

One of the most important problems facing utilities and the power sector is the management of peak load demand. The residential sector accounts for nearly 25% of total electricity consumption in the United States. While HVAC load represents more than half the total load demand for the residential sector [1]. As record high temperatures increase with global warming, peak load demand is expected to increase as a result of projected increase use of HVAC during peak summer hours to maintain temperature inside the house.

As a way of motivating consumers to shift load from peak hours, electric utilities introduces time of use (TOU) electricity rates; At times of peak load, on-peak rates are more than double the regular off peak electricity rate. This can cause an economic hardship for home owners at the end of the month when they pay their electric bill.

1.2 Approach

By utilizing the flexibility of controlling thermostat set temperature, HVAC unit has a huge potential of decreasing cost and peak load through demand response. To understand the house thermal model and how the set temperature inside the house is maintained, ambient temperature data and house thermal properties will need to be obtained. The house's thermal properties depends upon the orientation, insulation materials, and the ambient temperature. After gathering all data and assumption, the energy load profile for heating and cooling will be modeled using thermal equations. The house thermal model will then be built to understand the change of indoor temperature with heating and cooling power and change in ambient temperature.

1.3 Measurement of Success

The goal of the project is to model the heating and cooling energy profiles to maintain a specific set temperature. As mentioned before, the two main problems with peak load are the high electricity cost that the home owner endures at the end of the month when they get their electricity bill as a result of on-peak rates, and the peak load stress on the grid that utilities tries to avoid. As such, the optimization model will be most successful if the home owner is able to save more in electricity cost at the end of the month. Also, another measurement of success is the amount of reduction in peak load that the model is able to achieve while maintaining set temperature range.

2 Methods

2.1 House Thermal Model

We model the house thermal system as a LTI (Linear Time Invariant) state model. Temperatures, conductances, thermal masses and heat flows are entirely equivalent to voltages, conductances, capacitors and current flows in the electric circuit analog. That is, the differential equations expressing conservation for energy are the same. We assume that the heater's and cooler's energy flow depends on the current indoor air temperature and a fixed temperature of the coil over which air is blown.
Heater equation

$$\frac{dQ_{heat}}{dt} = (T_{heater} - T_{indoor}) \times Mdot \times c$$

Cooler equation

$$\frac{dQ_{cool}}{dt} = (T_{indoor} - T_{cooler}) \times Mdot \times c$$

Thermal losses/gain of the houses

$$\frac{dQ_{losses}}{dt} = \frac{T_{indoor} - T_{out}}{R_{eq}}$$

House indoor temperature change

$$\frac{dT_{indoor}}{dt} = \frac{1}{M_{air} \times c}\left(\frac{dQ_{heat}}{dt} - \frac{dQ_{cool}}{dt} - \frac{dQ_{losses}}{dt}\right)$$

where, dQ_{heat}/dt is the heat flow from the heater into the room, dQ_{cool}/dt is the heat flow from the cooler out of the room, dQ_{losses}/dt is the heat loss/gain to outside of the house, T_{heater} is the temperature of the heating coil, T_{cooler} is the temperature of the cooling coil, T_{indoor} is the current indoor air temperature, Mdot is the air mass flow rate through the heater, R_{eq} is the equivalent thermal resistance of the house, M_{air} is the mass of air inside the house, c is the heat capacity of air at constant pressure. All the parameter values and units are mentioned at the end.

2.2 Optimization Model

Optimization of heating and cooling energy profile is formulated as a linear optimization problem that can be efficiently solved using the Simplex method. To set up the optimization problem, the decision variables, objective function, and constraints will be explicitly defined below.

2.2.1 Decision Variables

The variables that are being modeled are the indoor Temperature T_{indoor}, energy required for heating Q_{heat}, and energy required for cooling Q_{cool}, shown in units in Table 1.

Table 1: Decision variables

Variable Name	**Domain**	**Unit**
T_{indoor}	R^{25}	^{o}C
Q_{heat}	R^{24}	Joules/hr
Q_{cool}	R^{24}	Joules/hr

2.2.2 Objective Function

The optimization model minimizes the total cost on a real time basis by varying the heating and cooling energy profile of the Air Conditioner.

$$\text{minimize} \sum_{t=1}^{24} cost_{(t)} \times (\eta_{heat} \times Q_{heat(t)} + \eta_{cool} \times Q_{cool(t)}) \times \gamma_{J2KW}$$

where $cost_{(t)}$ is the hourly TOU rate, η_{heat} and η_{cool} is the heating and cooling efficiency of the HVAC system. $Q_{heat(t)}$ and $Q_{cool(t)}$ are the heat tranferred per hour of the HVAC system, and γ_{J2KW} is the Joules/hr to KW conversion factor.

2.2.3 Constraints

The optimization model has nine constraints to satisfy physical and optimization limits. The temperature set-point of 69.8 ^oF or 21 ^oC is set as the base temperature for the modeled customer and this base temperature can be modified if needed. The user can set their tolerable comfort ranges from $\pm$ 0 *to* 5^oF, where 0^oF corresponds to maximum comfort and 5^oF corresponds to maximum savings and are able to tolerate fluctuation between 65 to 75 ^oF with respect to the assumed set-point. We start the day with the indoor temperature at equilibrium with the base temperature and the final temperature the same. This is established in the following constraint

$$T_{indoor(1)} = T_{set}\ ^oC \tag{1}$$

$$T_{indoor(25)} = T_{set}\ ^oC \tag{2}$$

where T_{set} is the base or set-point temperature and T_{indoor} is the decision variable.

The indoor temperature can fluctuate between the tolerance levels and is enforced as

$$T_{set} - \Delta T_{comfort} \leq T_{indoor(t)} \leq T_{set} + \Delta T_{comfort}\ ^oC, \quad t = 1, ..., 25 \tag{3 \& 4}$$

where $\Delta T_{comfort}$ is the comfort range preference of the user.

The thermal inflow and outflow of the house is simplified as a first order difference equation where the change in indoor temperature across the hour is given by

$$M_{air} \times c_{air} \times (T_{indoor(t+1)} - T_{indoor(t)}) = Q_{heat(t)} - Q_{cool(t)} - \frac{T_{indoor(t)} - T_{out(t)}}{R_{eq}}\ Joules/hr\ \forall t \tag{5}$$

where M_{air} (kg) is the mass of air in the house of given dimensions, which is derived as $M_{air} = \rho_{air} \times V_{house}$, where ρ_{air} kg/m^3 is the density of air at sea level and V_{house} is the house volume. c_{air} $J/(kgK)$ is the air specific heat. $Q_{heat(t)}$ *and* $Q_{cool(t)}$ are the energy flow from the HVAC system in Joules/hr. R_{eq} is the house's equivalent thermal resistivity from the insulation materials the calculation is given in as $R_{Wall} \times R_{Window}/(R_{Wall} + R_{Window})$.

The heat energy from the HVAC system can be modeled as a amount of energy the air mass picks up as it moves though a heated or cooled coil. The equation of thermal transfer is given as the following constraint.

$$Q_{heat(t)} \leq Mdot \times c \times (T_{heater(t)} - T_{indoor(t)}) \;\; Joules/hr \; t = 1, ..., 24 \tag{6}$$

$$Q_{cool(t)} \leq Mdot \times c \times (T_{indoor(t)} - T_{cooler(t)}) \;\; Joules/hr \; t = 1, ..., 24 \tag{7}$$

where $Q_{heat(t)}$ and $Q_{cool(t)}$ are the decision variable corresponding to the thermal energy transferred into the house for controlling the temperature. Mdot is the air flow rate of the HVAC system. In a strict thermodynamic system, the heat energy should be exactly equal to the thermal transfer between the coils and air passing through them. However, we relaxed the constraint from " $=$ " to " $\leq$ " to account for the losses while transferring and also to make the model more feasible. The optimized Q_{heat} and Q_{cool} from the solution can be used to control the fan speed (Mdot) to achieve the desired thermal flow into the house.

While the objective is to minimize the total cost, model is also run to minimize the peak load of the HVAC system throughout the day. We need our model's power demand to be less than a specified watt. If the goal is to just minimize the cost, we assume this value to be very large number.

$$Q_{heat(t)} \times \gamma_{J2W} \leq \eta_{heat} \times P_{limit} \; \forall \; t \tag{8}$$

$$Q_{cool(t)} \times \gamma_{J2W} \leq \eta_{cool} \times P_{limit} \; \forall \; t \tag{9}$$

where γ_{J2W} is the conversion factor between Joules/hr to Watt, η_{heat} and η_{cool} are the heating and cooling efficiency, and P_{limit} is the given upper limit of the total power consumed in Watt.

3 Data

The simplified ETP model of the house with outdoor losses can be seen as heat energy passing through a resistor, which is the insulation of the building. Because there is variability in house structure and materials, no houses would have the same thermal resistance. Therefore, simulating a house's thermal model requires the house's physical dimensions and structure, insulation and glazing materials of doors and windows, and their corresponding R values. The model house assumed to be of a basic one story home with 800 square meters, 2 windows, 40 degree roof pitch. The equivalent thermal resistance (Req) value of the materials and values given from the Oregon Residential Reach Code, under Oregon Revised Statute (ORS) 455.500 which is an optional set of standards providing a choice for builders, consumers, contractors, and others [3]. The model house parameters are given in Table 4 in the appendix.

The price per hour data for the entire day was taken from electricity price tariff published by the Portland General Electric website [2]. Time of Use pricing is different for different times of the day, namely On-peak (times when demand is high) of 20.448 ¢ per kWh, Mid-peak of 15.119 ¢ per kWh and Off-peak (times when demand is low) of 4.128 ¢ per kWh. Time of Use pricing periods differ seasonally, because people use power differently in winter than in summer, which changes high-demand times. The various TOU time periods are given in Table 2.

The thermal loss to the house exterior depends on the outdoor temperature. It is difficult to have the exact outside temperature of an individual house. However, the hourly temperature data published by the National Weather Service acted as a close alternative to the ambient outdoor temperature [4]. Analyzing and optimizing the electric heating and cooling profile of HVAC system varies with seasons. Colder winter time temperatures in Oregon result in increased demand for natural gas and electricity to heat homes and buildings. As Oregon summers get warmer, the state is seeing increasing use of air conditioners in the hottest months. As given in the figure 1, we can see that the outdoor temperature droops very low during the winter.

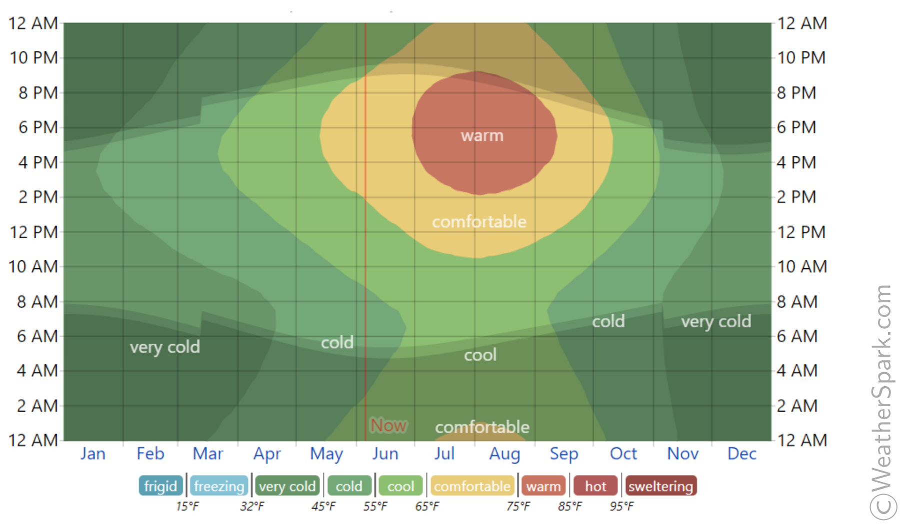

Figure 1: Average Hourly Profile Portland

Table 2: PGE TOU Price

TOU period	**Winter (Nov. 1 – April 30)**	**Summer (May 1 – Oct. 31)**
On-peak	6 to 10 a.m. Mon. – Fri. 5 to 8 p.m. Mon. – Fri.	3 to 8 p.m. Mon. – Fri.
Mid-peak	10 a.m. to 5 p.m. Mon. – Fri. 8 to 10 p.m. Mon. – Fri. 6 a.m.to 10 p.m. Sat.	6 a.m. to 3 p.m. Mon. – Fri. 8 to 10 p.m. Mon. – Fri. 6 a.m. to 10 p.m. Sat.
Off-peak	10 p.m. to 6 a.m. every day 6 a.m. to 10 p.m. Sun. and holidays	10 p.m. to 6 a.m. every day 6 a.m. to 10 p.m. Sun. and holidays

Table 3: Parameter Values and Units

Paramater Name	**Value**	**Domain**	**Unit**
cost	TOU price	R^{24}	cents/KWh
J2KW	2.77778E-07	R	Kwh$\times hr/joules$
T_{set}	20	R	^oC
$T_{comfort}$	2.78	R	^oC
M_{air}	8662.317	R	Kg
Mdot	3600	R	Kg/hr
R_{eq}	6.77E-07	R	$^oC \times hr/joules$
c	1005.4	R	joules/kg-K

4 Results and Discussion

4.1 Cost minimization with and without comfort level

To maintain a constant set indoor temperature inside the house, the HVAC system will have to resist the change of indoor temperature from the losses to the outside through heating or cooling energy. Figure 2 below shows the power for cooling and heating required to maintain the the constant indoor temperature of 21°C.

As an opportunity to save cost, we allow the flexibility of ±5°F allowed range for the indoor temperature to power the HVAC system less and save energy. Figure 3 shows the variability of indoor temperature within the allowed range that results in maximum savings. The heating and cooling power to maintain that is shown in Figure 3b. As a result of the comfort level flexibility, cost of electricity used for heating and cooling went down from $5.23 to $1.23 per day. This can translate to a saved cost for the homeowner of $120 per month for a weather pattern like shown in this specific date.

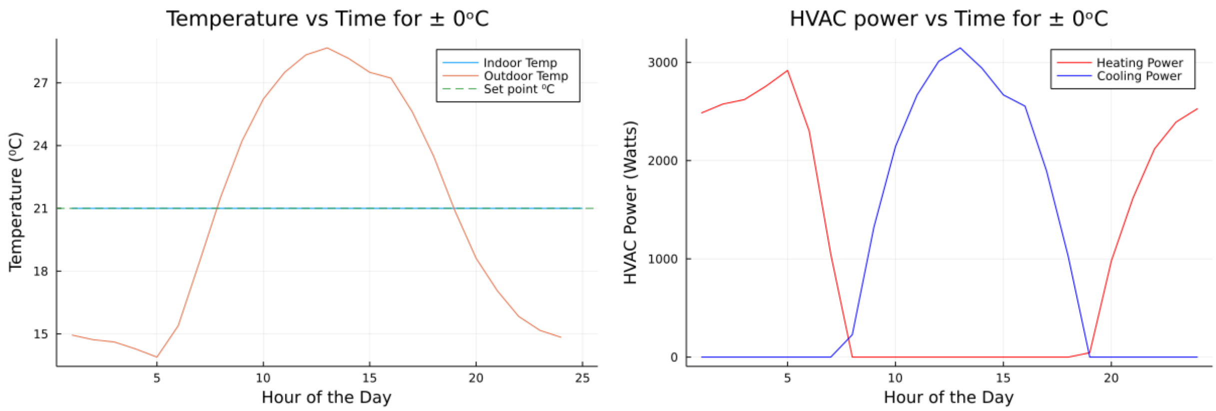

Figure 2: Temperature and power profile without comfort level

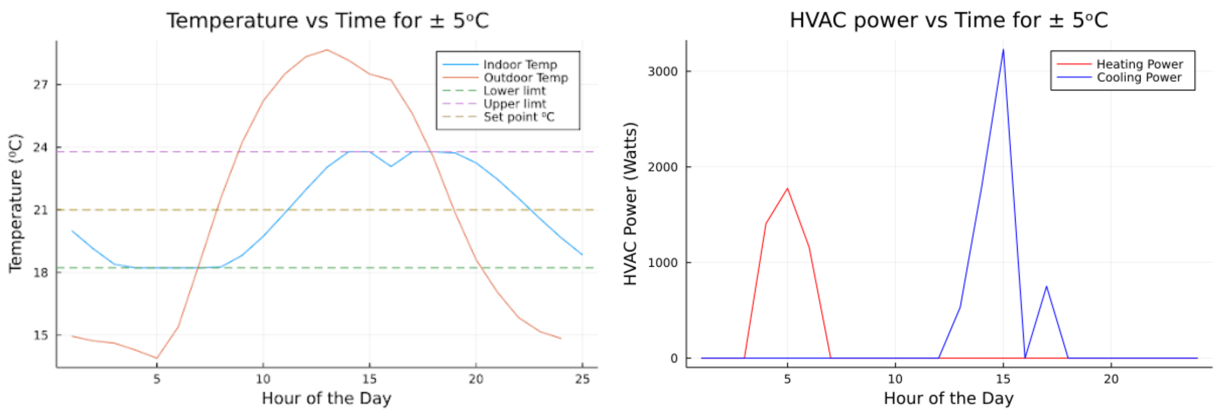

Figure 3: Temperature and power profile with comfort level

4.2 Peak load minimization

The original model optimizes the HVAC power profile that maintains the allowed range of indoor temperature with maximum savings. At some instances, we noticed huge peaks of power demand at certain time of the day. During a hot summer season, peak loads can translate into a sharp increase in aggregated power demand that can stress the power grid and electric utilities.

To find the minimum peak load possible in a given day for the comfort range, we introduce a constraint on maximum HVAC power and keep increasing the maximum demand of Q_{heat} and Q_{cool} until the model is feasible to find the minimum feasible peak load. As a result, we are able to decrease the peak load in the given power profile by 48% from 3200 W to 1655 W as shown in Figure 4. The model is able to do this by operating the HVAC for three hours at low power mode compared to operating at maximum power for one hour. Even though the total cost of electricity slightly goes up (from \$ 1.17 to \$ 1.23), we are able to decrease the peak load in an event where utilities need to curtail demand during high-peak periods. This can be incentivized by utilities with discounted electricity prices or reward customers who participate in demand response programs that reduces their peak load profile.

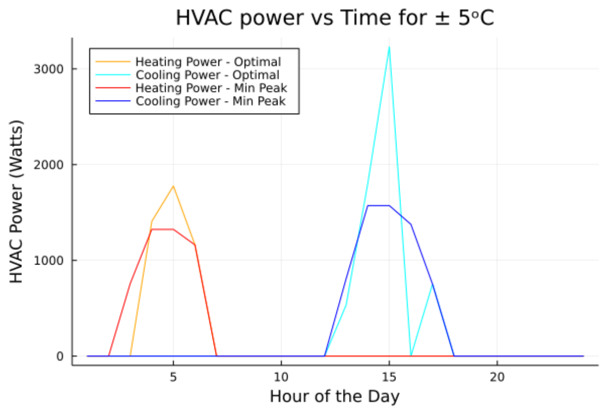

Figure 4: Minimized Peak Load Profile

4.3 Sensitivity Analysis

A sensitivity analysis was done to quantify the changes on the cost of electricity with respect to comfort level range flexibility. As shown in Figure 5, a constant set indoor temperature with zero flexibility translates to a maximum electricity cost of around \$5 per day. The cost decreases as the comfort level range increases as shown in the figure. As the comfort level reaches $\pm 8°F$, the cost approaches zero because the HVAC doesn't operate as the indoor temperature is always within the range.

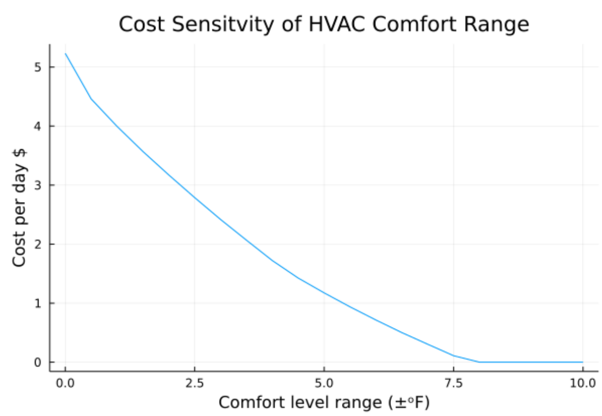

Figure 5: Cost sensitivity with respect to comfort range flexibility

5 Conclusion and Future Work

With HVAC power contributing the most of the total residential demand profile, it is prudent to minimize cost, peak-demand and shift it to higher renewable duration. As summer temperature soars, homeowners can be affected with electricity bill shocks a result of higher HVAC power to cool indoor temperature when the price of electricity is higher. Additionally, from the perspective of the Transmission and Distribution System high-peak periods can stress the power grid and electric utilities with peak demands occurring at the time. The model optimized the heating and cooling load profile of an HVAC system throughout the day and maintained a set comfort range of indoor temperature and minimized the electricity cost. As a result of allowing a flexible comfort range of $\pm 5°$F, the homeowner lowered electricity cost of HVAC power from \$5.23 to \$1.23 per day, which can be translated to \$120 per month. Furthermore, the model also reduced the peak load for the given comfort range, as it swept over increasing peak load till it found the minimum load for which the model became feasible. The model decreased peak load for the given day by 48%, from 3200 W to 1655 W, by operating the HVAC for a longer period at lower power mode.

This study can be used to show the opportunity on lowering electricity bill for homeowners and for the utilities to incentivize demand response programs on HVAC systems to lower peak demand. The model can also be used to optimize the HVAC system over a group of residential buildings that are aggregated to participate in a demand response program. We could in that case, if the residential units are in close proximity and share a single transformer, the optimization problem would have a hard upper limit power of the transformer capacity as a constraint and would be able to optimize over them. As the penetration of solar energy is getting higher, there is a power surplus in the morning and we can utilize the same by giving emphasis on using power during day by reducing the cost in the morning and/or increasing the cost during night. The model can also be integrated with solar PV by making use of network model of energy used and sold and using thermal mass of the house like a storage device.

With rife optimization potential on the behind-the-meter distributed energy resources, we are able to achieve cost saving from the customer's perspective as well as load shedding for the grid. This project is an optimization subset of a large optimization problem spanning multiple devices and various houses in a micro-grid or neighbourhood.

References

[1] Energy use in homes, https://www.eia.gov/energyexplained/use-of-energy/homes.php: :text=Electricity

[2] Time of Use Pricing, https://portlandgeneral.com/about/info/pricing-plans/time-of-use/time-of-use-pricing-home.

[3] https://www.oregon.gov/bcd/codes-stand/Documents/2021-residential-reach-code.pdf

[4] https://www.weather.gov/wrh/timeseries?site=KPDX

[5] “House Thermal Model.” MATLAB and Simulink, https://www.mathworks.com/help/simulink/slref/thermal-model-of-a-house.html.

Appendix

Table 4: Model House Parameters

House Parameter	**Value**
House Length	40 meter
House Width	10 meter
House Height	8 meter
Roof Pitch	40 Degree
Number of Windows	#2
Window Height	1 meter
Window Width	1 meter
Wall Insulation Material	Glass Wool
K Insulation	0.038 Joules/sec/m/C
Wall Thickness	0.4 meter
Window Material	Glass
K Window	0.78 Joules/sec/m/C
Window Thickness	0.01 meter