1. Burnside Lemma + Polya enumeration theorem
   Counts the number of inequivalent colorings on n-set under a permutation group.

$$N(C,G) = \frac{1}{|G|} \sum_{f \in G} |C(f)| = \frac{1}{|G|} \sum_{f \in G} k^{\#(f)} = \frac{1}{|G|} \sum_{f \in G} k^{\sum e_i}$$

   G is the equivalent permutation group
   C is all colorings on n-set
   $N(C,G)$ is the count of inequivalent colorings
   $C(f)$ is the stable kernel of permutation f
   k is the number of colors available
   $\#(f)$ is the number of cycles in permutation f
   $e_1 \dots e_n$ is the type of permutation f - it has e_i i-cycles

2. Lucas theorem: $\binom{m}{n} \equiv \prod \binom{m_i}{n_i} \mod p$
   where p is prime, m_i is base-p digits of m

3. Fermat's little theorem: $a^{p-1} \equiv 1 \mod p$ where p is prime
   Euler's theorem: $a^{\phi(n)} \equiv 1 \mod n$ where $gcd(a,n) = 1$

4. Wolstenholme's theorem: $\binom{2p-1}{p-1} \equiv 1 \mod p^3$, $\binom{ap}{bp} \equiv \binom{a}{b} \mod p^3$ where p is prime

5. primality criteria (n is prime iff)

$$\prod_{1 \le k \le n-1} (2^k - 1) \equiv n \mod (2^n - 1)(Ventieghems)$$

$$(n-1)! \equiv -1 \mod n(Wilson)$$

6. Euler's totient function

$$\phi(n) = n \prod_{p|n, p \text{ prime}} (1 - \frac{1}{p})$$

$$\phi(mn) = \phi(m)\phi(n) \text{ if } gcd(m,n) = 1$$

$$\phi(mn) = \phi(m)\phi(n)\frac{d}{\phi(d)} \text{ where } d = gcd(m,n)$$

$$\phi(lcm(m,n))\phi(gcd(m,n)) = \phi(m)\phi(n)$$

$$\sum_{d|n} \phi(d) = n$$

$$\sum_{d|n} \frac{n}{d}\phi(d) = \sum_{k=1..n} gcd(k,n)$$

$$\phi(n)d(n) = \sum_{k=1..n}^{gcd(k,n)=1} gcd(k-1,n)$$

$$\frac{1}{2}n\phi(n) = \sum_{k=1..n}^{gcd(k,n)=1} k$$

$$a \mid b \to \phi(a) \mid \phi(b)$$

$$n \mid \phi(a^n - 1) \text{ for } a, n > 1$$

7. Mobius function

$$\phi(n) = n \sum_{d|n} \frac{\mu(d)}{d}$$

$$\sum_{d|n} \frac{\mu^2(d)}{\phi(d)} = \frac{n}{\phi(n)}$$

8. Nim Lose iff XOR sum is zero