Métodos de Apoio à Decisão
Solution of Assignment 1

João Pedro PEDROSO

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Questions 1 and 2

Problem data:

P	set of products	`R C I`
C	set of planets	`Venus Mars Mercury`
O	set of operations for production	`Cleaning Cooking Packing`
N	number of periods to consider in the plan	`12`
$T = \{1, \ldots, N\}$	set of periods to consider in the plan	
v_{cpt}	value in planet c of product p in period t	table 1 of statement
r_{op}	production rate for p in operation o	table 2 of statement
S	maximum quantity that can be sent per planet/period	`1000`

Variables:

z_{pt}	quantity of p to produce in period t, for $p \in P, t \in T$
x_{cpt}	quantity of p to sell in c in period t, for $c \in C, p \in P, t \in T$

Formulation:

$$\text{maximize} \quad \text{profit} = \sum_{c \in C} \sum_{p \in P} \sum_{t \in T} v_{cpt} x_{cpt} \tag{1}$$

$$\text{subject to:} \quad \sum_{p \in P} z_{pt} / r_{op} \leq 1, \qquad \forall o \in O, \forall t \in T, \tag{2}$$

$$z_{pt} = \sum_{c \in C} x_{cpt}, \qquad \forall p \in P, \forall t \in T, \tag{3}$$

$$\sum_{p \in P} x_{cpt} \leq S, \qquad \forall c \in C, \forall t \in T, \tag{4}$$

$$z_{pt} \geq 0, \qquad \forall p \in P, \forall t \in T,$$

$$x_{cpt} \geq 0, \qquad \forall c \in C, \forall p \in P, \forall t \in T.$$

We assume there is one line for each of the operations, used in the sequence `Cleaning, Cooking, Packing` for manufacturing each the products. Constraints (2) ensure that the total capacity of each production line is not exceeded (i.e., usage is at most 100% of capacity). Constraints (3) determine production quantities, based on the sales. Constraints (4) impose a maximum on the monthly delivery for each of the planet/period, due to limited capacity on the shiping vehicles.

Results:

	Production			Sales Mars			Sales Mercury			Sales Venus		
Mês	C	I	R	C	I	R	C	I	R	C	I	R
1	0	1000.0	308.3	0	1000.0	0	0	0	0	0	0	308.3
2	850.0	0	0	850.0	0	0	0	0	0	0	0	0
3	850.0	0	0	850.0	0	0	0	0	0	0	0	0
4	850.0	0	0	850.0	0	0	0	0	0	0	0	0
5	656.1	0	421.9	656.1	0	0	0	0	0	0	0	421.9
6	656.1	0	421.9	656.1	0	0	0	0	0	0	0	421.9
7	0	1200.0	0	0	1000.0	0	0	0	0	0	200.0	0
8	0	1200.0	0	0	200.0	0	0	0	0	0	1000.0	0
9	850.0	0	0	850.0	0	0	0	0	0	0	0	0
10	850.0	0	0	850.0	0	0	0	0	0	0	0	0
11	0	1000.0	308.3	0	1000.0	0	0	0	0	0	0	308.3
12	0	1000.0	308.3	0	1000.0	0	0	0	0	0	0	308.3

The profit with this production plan is 492965.8 solarcoins. As expected, sales quantities are larger for planets with higher prices. For some markets (all products in Mercury, R for Mars, C for Venus) it is not worthy to sell at all.

Question 3

Results: dual variables

The optimal value of the dual variable corresponds to the shadow price associated to each constraint. In particular, for constraints (2) these values are:

Month	Cleaning	Cooking	Packing
1	0	22200.0	0
2	0	29750.0	0
3	0	32300.0	0
4	0	35700.0	0
5	0	34208.8	2631.6
6	0	37519.3	8789.5
7	0	62400.0	0
8	0	78000.0	0
9	0	38250.0	0
10	0	36550.0	0
11	0	18500.0	0
12	0	18500.0	0

Using the relationship $\Delta\text{profit} \approx \mu_i \Delta b_i$, where μ_i is the shadow price associated to constraint i and b_i is the corresponding right-hand side, we observe that `Cooking` is the operation for which consistently, for all periods, a marginal change in the right-hand side Δb_i leads to a higher improvement in the profit. Hence, improvements should be made on the capacity of line `Cooking`.

Question 4

Additional data:

h monthly unit holding cost, in solarcoins

Additional variables:

s_{pt} quantity of p kept in inventory at the end of period t, for $p \in P, t = 0, 1, \dots, N$

New formulation

$$\text{maximize profit} = \sum_{p \in P} \sum_{t \in T} \left[\sum_{c \in C} v_{cpt} x_{cpt} - h s_{pt} \right] \tag{5}$$

subject to (2), (4), and:

$$z_{pt} + s_{p,t-1} = \sum_{c \in C} x_{cpt} + s_{pt}, \qquad \forall p \in P, \forall t \in T, \tag{6}$$

$$s_{p0} = 0, \qquad \forall p \in P, \tag{7}$$

$$z_{pt} \geq 0, \qquad \forall p \in P, \forall t \in T,$$

$$x_{cpt} \geq 0, \qquad \forall c \in C, \forall p \in P, \forall t \in T$$

$$s_{pt} \geq 0, \qquad \forall p \in P, \forall t \in T.$$

Constraint (6) is the bill of materials, determining the inventory at the end of each month; (7) sets inventory to zero at the beginning, for all products. The bill of materials determines that, in each month, the quantity produced plus the quantity kept in stock in the previous month is equal to the quantity delivered plus the quantity kept in stock at the end of that month. We assume that inventory is kept only on Earth.

Results:

	Production			Inventory			Sales Mars			Sales Mercury			Sales Venus		
Month	C	I	R	C	I	R	C	I	R	C	I	R	C	I	R
1	850.0	0	0	850.0	0	0	0	0	0	0	0	0	0	0	0
2	656.1	0	421.9	1506.1	0	421.9	0	0	0	0	0	0	0	0	0
3	656.1	0	421.9	2162.3	0	843.9	0	0	0	0	0	0	0	0	0
4	159.1	714.3	402.6	1321.4	714.3	1246.5	1000.0	0	0	0	0	0	0	0	0
5	0	942.9	396.4	1000.0	1657.1	642.9	321.3	0	0	0	0	0	0	0	1000.0
6	0	942.9	396.4	0	2600.0	0	1000.0	0	0	0	0	39.3	0	0	1000.0
7	0	1200.0	0	0	1800.0	0	0	1000.0	0	0	0	0	0	1000.0	0
8	0	1200.0	0	0	0	0	0	1000.0	0	0	1000	0	0	1000.0	0
9	850.0	0	0	0	0	0	850.0	0	0	0	0	0	0	0	0
10	850.0	0	0	0	0	0	850.0	0	0	0	0	0	0	0	0
11	0	1000.0	308.3	0	0	0	0	1000.0	0	0	0	0	0	0	308.3
12	0	1000.0	308.3	0	0	0	0	1000.0	0	0	0	0	0	0	308.3

The profit associated to this production plan is 560562.0 solarcoins. The increase in profit is due to the additional flexibility introduced by the possibility of keeping end product in stock, which allows producing in advance to supply more profitable markets, in more profitable months.

Correcting your report

The grade for a completely correct work is 2 values (including a mark for the quality of self-evaluation). For self-evaluating your work, start your grade with 0, and add values according to:

- Question 1 (0.3):
  - Assumptions correct and coherent with models: 0.1.
  - Variables and objective correctly defined: 0.1.
  - Constraints correctly defined: 0.1.
- Question 2 (0.3):
  - A solution for your model was obtained with optimization solver: 0.2.
  - Correct solution obtained: 0.1.
- Question 3 (0.2):
  - Description of how to identify the correct production line, making use of dual variables: 0.1.
  - Correct production line (`Cooking`) identified based on the solution obtained with solver: 0.1.
- Question 4 (1.2):
  - Assumptions correct and coherent with your models: 0.2.
  - Variables correctly defined: 0.2.
  - Constraints correctly defined: 0.2.
  - Correct objective: 0.1.
  - Completely correct formulation: 0.1.
  - Correct solution obtained with optimization solver: 0.2.
- Peer reviewing: 0.2 (to be assessed by me).

Additional remarks:

- Prepare your self-evaluation report:
  1. download files for these models: trabalho-1-a.mod trabalho-1-b.mod and data trabalho-1-a.dat (works with both models).
  2. obtain the solution using software GLPK (or other solver) and the models provided, and compare it with your own solution;
  3. comment each grade added;
  4. if your formulation is different of this one and you think it is also correct, provide a careful explanation of your assumptions.
- There is some margin for proposing models different of this one, but attempting to value clearly wrong answers may invalidate the submission.
- Hand your self-evaluation report on the practical class of May 5.