# Computación y Estructuras Discretas III

Andrés A. Aristizábal P. aaaristizabal@icesi.edu.co Ángela Villota apvillota@icesi.edu.co

Departamento de Computación y Sistemas Inteligentes



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## Agenda del día

- Decision Algorithms and Grammars
  - The decision problem
  - The CYK Algorithm

- The Chomsky's Normal Form
  - Ejercicios

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## The decision problem

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Given two (1) a property  $\mathcal{P}$ , (or a predicate) and (2) a study subject s, (let's say a grammar), the decision problem is to find an algorithm able to answer the **YES/NO** question: does s complies with the property  $\mathcal{P}$ ,?

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### What about grammars?

A decision problem in this context is to find an algorithm able to answer the following: Given a context-free grammar G and a string s, determine whether s can be generated by G.

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- Indeed! there exists an efficient algorithm (polynomial) to solve the question about the belonging of a string to the language generated by a grammar.
- The CYK algorithm (named in honor of the researchers Cocke, Younger, and Kasami) uses dynamic programming to determine whether a given string can be generated by a given CFG. It's a fundamental algorithm in computational linguistics and parsing.

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### The CYK Algorithm

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### Input

- A context-free grammar G in Chomsky Normal Form (CNF), represented as a set of production rules.
- A string s over the alphabet of terminals of G

### Output

- Yes if there exists a derivation of s from the start symbol of G
- No otherwise.

So, when we apply the *CYK* algorithm to a real-world problem, the decision problem is whether a given string can be derived from a given *CFG* according to the rules of the grammar. If the CYK algorithm successfully finds a valid derivation for the string then the answer is **Yes**; otherwise, it's **No**.

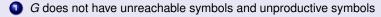


Then, we have to learn how to transform a Grammar in the CNF first!!

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#### **Teorema**

All CFG G has an equivalent in the CNF

Transforming a grammar in the CNF

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