Computación y Estructuras Discretas III

Andrés A. Aristizábal P. aaaristizabal@icesi.edu.co
Angela Villota apvillota@icesi.edu.co

Departamento de Computación y Sistemas Inteligentes
UNIVERSIDAD ICESI

2024-2

1. **Grammars and Languages**
   - Previous Exercises
   - CYK Algorithm
   - Exercises

Agenda del día

1. **Grammars and Languages**
   - Previous Exercises
   - CYK Algorithm
   - Exercises

What is the algorithm to find generating variables in a CFG?

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INITIALIZE:
  $\mathbf{GEN} := \{A \in V \mid \exists \text{ rule } A \to w,\ w \in \Sigma^*\}$
REPEAT:
  $\mathbf{GEN} := \mathbf{GEN} \cup \{A \in V \mid \exists \text{ rule } A \to w,\ w \in (\Sigma \cup \mathbf{GEN})^*\}$
UNTIL:
  There are no more variables added to **GEN**

Example

Find the generating variables from this grammar:

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$$G : \begin{cases} S \to ACD \mid bBd \mid ab \\ A \to aB \mid aA \mid C \\ B \to aDS \mid aB \\ C \to aCS \mid CB \mid CC \\ D \to bD \mid ba \\ E \to AB \mid aDb \end{cases}$$

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Solution

$$\mathbf{GEN_1} = \{S, D\}.$$

Grammars and Languages

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Grammars and Languages

What is the algorithm to find reachable variables in a CFG?

Grammars and Languages

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INITIALIZE:
REACH $:= \{S\}$
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REACH $:=$ **REACH** $\cup \{A \in V \mid \exists$ producción $B \to uAv,\ B \in$ **REACH** y $u, v \in (V \cup \Sigma)^* \}$
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There are no more variables added to **REACH**

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$$G : \begin{cases} S \rightarrow aS \mid AaB \mid ACS \\ A \rightarrow aS \mid AaB \mid AC \\ B \rightarrow bB \mid DB \mid BB \\ C \rightarrow aDa \mid ABD \mid ab \\ D \rightarrow aD \mid DD \mid ab \\ E \rightarrow FF \mid aa \\ F \rightarrow aE \mid EF \end{cases}$$

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$$\textbf{REACH}_1 = \{S\}.$$

Grammars and Languages

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Grammars and Languages

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Grammars and Languages

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Remove the useless variables from G.

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Solution

1. *We execute the algorithm to find generating variables.*

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1. *We execute the algorithm to find generating variables.*
2. *We eliminate non generating variables.*

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1. *We execute the algorithm to find generating variables.*
2. *We eliminate non generating variables.*
3. *We execute the algorithm to find reachable variables.*

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Remove the useless variables from G.

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4. *We eliminate non reachable variables.*

Grammars and Languages

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Grammars and Languages

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2. *The only non generating variable from G is A. Therefore, G is equivalent to the following grammar G_1 :*

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Grammars and Languages

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1. *We execute the second algorithm.*

Solution

$$G_1 : \begin{cases} S \longrightarrow SBS \mid BC \mid Bb \\ B \longrightarrow aBCa \mid b \\ C \longrightarrow aC \mid abb \\ D \longrightarrow ab \\ E \longrightarrow aS \\ F \longrightarrow aDb \mid aF \end{cases}$$

1. *We execute the second algorithm.*

$$\textbf{REACH}_1 = \{S\}.$$

Grammars and Languages

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$$\begin{aligned} \textbf{REACH}_\mathbf{1} &= \{S\}. \\ \textbf{REACH}_\mathbf{2} &= \{S\} \cup \{B, C\} = \{S, B, C\}. \end{aligned}$$

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2. *Variables D, E, F are non reachable. Therefore, G is equivalent to the following grammar G_2, that does not have useless variables.*

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$$G_1 : \begin{cases} S \to SBS \mid BC \mid Bb \\ B \to aBCa \mid b \\ C \to aC \mid abb \\ D \to ab \\ E \to aS \\ F \to aDb \mid aF \end{cases}$$

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Grammars and Languages

Solution

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Grammars and Languages

What is a λ rule?

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Definition

A rule of the form $A \rightarrow \lambda$ is called a λ rule.

Grammars and Languages

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A rule of the form $A \to \lambda$ is called a λ rule.

What is a nullable variable?

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Definition

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A variable A is nullable if A $\stackrel{*}{\Longrightarrow} \lambda$.

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What is the algorithm to find nullable variables in a CFG?

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What is a nullable variable?

Definition

A variable A is nullable if $A \stackrel{*}{\Longrightarrow} \lambda$.

What is the algorithm to find nullable variables in a CFG?

INITIALIZE:

$\quad$ **NULL** $:= \{A \in B \mid A \to \lambda$ is a rule$\}$

REPEAT:

$\quad$ **NULL** $:=$ **NULL** $\cup \{A \in V \mid \exists$ rule $A \to w,\ w \in ($**NULL**$)^*\}$

UNTIL:

$\quad$ There are no more variables added to **NULL**

Grammars and Languages

Can you build a CFG without λ productions?

Theorem

Given a CFG G, can we construct an equivalent CFG G' to G without λ productions, except (possibly) $S \rightarrow \lambda$?

Grammars and Languages

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Example

Remove the λ rules from the following grammar G.

Grammars and Languages

Can you build a CFG without λ productions?

Theorem

Given a CFG G, can we construct an equivalent CFG G' to G without λ productions, except (possibly) $S \to \lambda$?

Example

Remove the λ rules from the following grammar G.

$$G : \begin{cases} S \to AB \mid ACA \mid ab \\ A \to aAa \mid B \mid CD \\ B \to bB \mid bA \\ C \to cC \mid \lambda \\ D \to aDc \mid CC \mid ABb \end{cases}$$

Grammars and Languages

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1. *We execute the algorithm to find the nullable variables.*
2. *We remove λ productions except for $S \to \lambda$, adding new productions that simulate the effect of the eliminated λ productions. That is, for each production $A \to u$ in G, we add productions of the form $A \to v$ obtained by removing from the string u one, two, or more nullable variables present, in all possible ways.*

Grammars and Languages

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Grammars and Languages

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Grammars and Languages

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A production of the form $A \rightarrow B$ where A and B are variables is called a unit production.

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Given a CFG G, it is possible to construct an equivalent to G CFG G$'$ without unit productions.

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Solution

1. *We execute the algorithm to find the unit variables.*
2. *We remove the unit productions, adding for each variable A of G the (non-unit) productions of the variables contained in the unit set* ***UNIT***(A).

Grammars and Languages

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Grammars and Languages

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Grammars and Languages

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Grammars and Languages

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Grammars and Languages

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2. *By removing the unit productions we obtain an equivalent grammar G' :*

$$G' : \begin{cases} S \to AS \mid AA \mid BA \mid \lambda \\ A \to aA \mid a \\ \\ \\ \end{cases}$$

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$$G : \begin{cases} S \to AS \mid AA \mid BA \mid \lambda \\ A \to aA \mid a \\ B \to bB \mid bC \mid C \\ C \to aA \mid bA \mid B \mid ab \end{cases}$$

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Grammars and Languages

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5. The resulting productions (different from $S \to \lambda$) should be of the form $A \to a$ or of the form $A \to w$, where $a \in \Sigma$, $w \in V^*$ and $|w| \geq 2$. These last ones can be simulated with productions of the form $A \to BC$ or $A \to a$. First, introduce, for each $a \in \Sigma$, a new variable T_a with the only production $T_a \to a$. Then, introduce new variables, with binary productions, to simulate the desired productions.

How can a production be simulated using only simple and binary productions?

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Simulating the production $A \rightarrow abBaC$ with binary and simple productions.

Grammars and Languages

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We introduce the variables T_a and T_b, and the productions $T_a \rightarrow a$ and $T_b \rightarrow b$. Then $A \rightarrow abBaC$ is simulated as:

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Grammars and Languages

How can a production be simulated using only simple and binary productions?

Example

Simulating the production $A \longrightarrow abBaC$ with binary and simple productions.

Solution

We introduce the variables T_a and T_b, and the productions $T_a \longrightarrow a$ and $T_b \longrightarrow b$. Then $A \longrightarrow abBaC$ is simulated as:

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Grammars and Languages

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As in this case there are no useless variables, we do not remove any variables.

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Grammars and Languages

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Grammars and Languages

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$\mathbf{UNIT_1}(B) = \{B\}$.

Solution

$$G_1 : \begin{cases} S \rightarrow AB \mid aBC \mid SBS \mid B \mid aB \\ A \rightarrow aA \mid C \mid a \\ B \rightarrow bbB \mid b \\ C \rightarrow cC \mid c \end{cases}$$

3. *We compute the unit sets for each of the variables in G_1 :*

$$\textbf{UNIT}_\mathbf{1}(S) = \{S\}.\ \textbf{UNIT}_\mathbf{2}(S) = \{S\} \cup \{B\} = \{S, B\}.\ \textbf{UNIT}_\mathbf{3}(S) = \{S, B\} \cup \{\ \} = \{S, B\}.$$
$$\textbf{UNIT}_\mathbf{1}(A) = \{A\}.\ \textbf{UNIT}_\mathbf{2}(A) = \{A\} \cup \{C\} = \{A, C\}.\ \textbf{UNIT}_\mathbf{3}(A) = \{A, C\} \cup \{\ \} = \{A, C\}.$$
$$\textbf{UNIT}_\mathbf{1}(B) = \{B\}.\ \textbf{UNIT}_\mathbf{2}(B) = \{B\} \cup \{\ \} = \{B\}.$$

Solution

$$G_1 : \begin{cases} S \to AB \mid aBC \mid SBS \mid B \mid aB \\ A \to aA \mid C \mid a \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

3. *We compute the unit sets for each of the variables in G_1 :*

$\textbf{UNIT}_\textbf{1}(S) = \{S\}$. $\textbf{UNIT}_\textbf{2}(S) = \{S\} \cup \{B\} = \{S, B\}$. $\textbf{UNIT}_\textbf{3}(S) = \{S, B\} \cup \{\ \} = \{S, B\}$.
$\textbf{UNIT}_\textbf{1}(A) = \{A\}$. $\textbf{UNIT}_\textbf{2}(A) = \{A\} \cup \{C\} = \{A, C\}$. $\textbf{UNIT}_\textbf{3}(A) = \{A, C\} \cup \{\ \} = \{A, C\}$.
$\textbf{UNIT}_\textbf{1}(B) = \{B\}$. $\textbf{UNIT}_\textbf{2}(B) = \{B\} \cup \{\ \} = \{B\}$.
$\textbf{UNIT}_\textbf{1}(C) = \{C\}$.

Solution

$$G_1 : \begin{cases} S \to AB \mid aBC \mid SBS \mid B \mid aB \\ A \to aA \mid C \mid a \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

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$\mathbf{UNIT_1}(A) = \{A\}$. $\mathbf{UNIT_2}(A) = \{A\} \cup \{C\} = \{A, C\}$. $\mathbf{UNIT_3}(A) = \{A, C\} \cup \{\ \} = \{A, C\}$.
$\mathbf{UNIT_1}(B) = \{B\}$. $\mathbf{UNIT_2}(B) = \{B\} \cup \{\ \} = \{B\}$.
$\mathbf{UNIT_1}(C) = \{C\}$. $\mathbf{UNIT_2}(C) = \{C\} \cup \{\ \} = \{C\}$.

Solution

$$G_1 : \begin{cases} S \rightarrow AB \mid aBC \mid SBS \mid B \mid aB \\ A \rightarrow aA \mid C \mid a \\ B \rightarrow bbB \mid b \\ C \rightarrow cC \mid c \end{cases}$$

3. *We compute the unit sets for each of the variables in G_1:*

$$\mathbf{UNIT_1}(S) = \{S\}.\ \mathbf{UNIT_2}(S) = \{S\} \cup \{B\} = \{S, B\}.\ \mathbf{UNIT_3}(S) = \{S, B\} \cup \{\ \} = \{S, B\}.$$
$$\mathbf{UNIT_1}(A) = \{A\}.\ \mathbf{UNIT_2}(A) = \{A\} \cup \{C\} = \{A, C\}.\ \mathbf{UNIT_3}(A) = \{A, C\} \cup \{\ \} = \{A, C\}.$$
$$\mathbf{UNIT_1}(B) = \{B\}.\ \mathbf{UNIT_2}(B) = \{B\} \cup \{\ \} = \{B\}.$$
$$\mathbf{UNIT_1}(C) = \{C\}.\ \mathbf{UNIT_2}(C) = \{C\} \cup \{\ \} = \{C\}.$$

We remove the unit productions and add, for each variable A in G, the (non-unit) productions of the variables contained in the unit set $UNIT(A)$, obtaining the equivalent grammar G_2:

Solution

$$G_1 : \begin{cases} S \to AB \mid aBC \mid SBS \mid B \mid aB \\ A \to aA \mid C \mid a \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

3. *We compute the unit sets for each of the variables in G_1:*

$\mathbf{UNIT_1}(S) = \{S\}$. $\mathbf{UNIT_2}(S) = \{S\} \cup \{B\} = \{S, B\}$. $\mathbf{UNIT_3}(S) = \{S, B\} \cup \{\,\} = \{S, B\}$.
$\mathbf{UNIT_1}(A) = \{A\}$. $\mathbf{UNIT_2}(A) = \{A\} \cup \{C\} = \{A, C\}$. $\mathbf{UNIT_3}(A) = \{A, C\} \cup \{\,\} = \{A, C\}$.
$\mathbf{UNIT_1}(B) = \{B\}$. $\mathbf{UNIT_2}(B) = \{B\} \cup \{\,\} = \{B\}$.
$\mathbf{UNIT_1}(C) = \{C\}$. $\mathbf{UNIT_2}(C) = \{C\} \cup \{\,\} = \{C\}$.

We remove the unit productions and add, for each variable A in G, the (non-unit) productions of the variables contained in the unit set $UNIT(A)$, obtaining the equivalent grammar G_2:

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ \\ \\ \end{cases}$$

Solution

$$G_1 : \begin{cases} S \rightarrow AB \mid aBC \mid SBS \mid B \mid aB \\ A \rightarrow aA \mid C \mid a \\ B \rightarrow bbB \mid b \\ C \rightarrow cC \mid c \end{cases}$$

3. *We compute the unit sets for each of the variables in G_1:*

$\mathbf{UNIT_1}(S) = \{S\}$. $\mathbf{UNIT_2}(S) = \{S\} \cup \{B\} = \{S, B\}$. $\mathbf{UNIT_3}(S) = \{S, B\} \cup \{\,\} = \{S, B\}$.
$\mathbf{UNIT_1}(A) = \{A\}$. $\mathbf{UNIT_2}(A) = \{A\} \cup \{C\} = \{A, C\}$. $\mathbf{UNIT_3}(A) = \{A, C\} \cup \{\,\} = \{A, C\}$.
$\mathbf{UNIT_1}(B) = \{B\}$. $\mathbf{UNIT_2}(B) = \{B\} \cup \{\,\} = \{B\}$.
$\mathbf{UNIT_1}(C) = \{C\}$. $\mathbf{UNIT_2}(C) = \{C\} \cup \{\,\} = \{C\}$.

We remove the unit productions and add, for each variable A in G, the (non-unit) productions of the variables contained in the unit set $UNIT(A)$, obtaining the equivalent grammar G_2:

$$G_2 : \begin{cases} S \rightarrow AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \rightarrow aA \mid a \mid cC \mid c \\ \\ \end{cases}$$

Solution

$$G_1 : \begin{cases} S \to AB \mid aBC \mid SBS \mid B \mid aB \\ A \to aA \mid C \mid a \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

3. *We compute the unit sets for each of the variables in G_1:*

$\mathbf{UNIT_1}(S) = \{S\}$. $\mathbf{UNIT_2}(S) = \{S\} \cup \{B\} = \{S, B\}$. $\mathbf{UNIT_3}(S) = \{S, B\} \cup \{\,\} = \{S, B\}$.
$\mathbf{UNIT_1}(A) = \{A\}$. $\mathbf{UNIT_2}(A) = \{A\} \cup \{C\} = \{A, C\}$. $\mathbf{UNIT_3}(A) = \{A, C\} \cup \{\,\} = \{A, C\}$.
$\mathbf{UNIT_1}(B) = \{B\}$. $\mathbf{UNIT_2}(B) = \{B\} \cup \{\,\} = \{B\}$.
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We remove the unit productions and add, for each variable A in G, the (non-unit) productions of the variables contained in the unit set $UNIT(A)$, obtaining the equivalent grammar G_2:

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \to aA \mid a \mid cC \mid c \\ B \to bbB \mid b \end{cases}$$

Solution

$$G_1 : \begin{cases} S \rightarrow AB \mid aBC \mid SBS \mid B \mid aB \\ A \rightarrow aA \mid C \mid a \\ B \rightarrow bbB \mid b \\ C \rightarrow cC \mid c \end{cases}$$

3. *We compute the unit sets for each of the variables in G_1:*

$$\begin{aligned} &\mathbf{UNIT_1}(S) = \{S\}.\ \mathbf{UNIT_2}(S) = \{S\} \cup \{B\} = \{S, B\}.\ \mathbf{UNIT_3}(S) = \{S, B\} \cup \{\ \} = \{S, B\}. \\ &\mathbf{UNIT_1}(A) = \{A\}.\ \mathbf{UNIT_2}(A) = \{A\} \cup \{C\} = \{A, C\}.\ \mathbf{UNIT_3}(A) = \{A, C\} \cup \{\ \} = \{A, C\}. \\ &\mathbf{UNIT_1}(B) = \{B\}.\ \mathbf{UNIT_2}(B) = \{B\} \cup \{\ \} = \{B\}. \\ &\mathbf{UNIT_1}(C) = \{C\}.\ \mathbf{UNIT_2}(C) = \{C\} \cup \{\ \} = \{C\}. \end{aligned}$$

We remove the unit productions and add, for each variable A in G, the (non-unit) productions of the variables contained in the unit set $UNIT(A)$, obtaining the equivalent grammar G_2:

$$G_2 : \begin{cases} S \rightarrow AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \rightarrow aA \mid a \mid cC \mid c \\ B \rightarrow bbB \mid b \\ C \rightarrow cC \mid c \end{cases}$$

Solution

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \to aA \mid a \mid cC \mid c \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

Solution

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \to aA \mid a \mid cC \mid c \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

4. *We introduce new variables for each* $a \in \Sigma : T_a, T_b, T_c$ *and the new transitions:* $T_a \to a, T_b \to b, T_c \to c$. *This is done so that all productions are of the form* $A \to a$ *or of the form* $A \to w$, *where* $a \in \Sigma$, $w \in V^*$ *and* $|w| \geq 2$. *Thus, we arrive at the grammar* G_3 :

Solution

$$G_2 : \begin{cases} S \to AB \,|\, aBC \,|\, SBS \,|\, aB \,|\, bbB \,|\, b \\ A \to aA \,|\, a \,|\, cC \,|\, c \\ B \to bbB \,|\, b \\ C \to cC \,|\, c \end{cases}$$

4. *We introduce new variables for each $a \in \Sigma$: T_a, T_b, T_c and the new transitions: $T_a \to a$, $T_b \to b$, $T_c \to c$. This is done so that all productions are of the form $A \to a$ or of the form $A \to w$, where $a \in \Sigma$, $w \in V^*$ and $|w| \geq 2$. Thus, we arrive at the grammar G_3 :*

$$G_3 : \begin{cases} S \to AB \,|\, T_aBC \,|\, SBS \,|\, T_aB \,|\, T_bT_bB \,|\, b \end{cases}$$

Solution

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \to aA \mid a \mid cC \mid c \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

4. *We introduce new variables for each $a \in \Sigma$: T_a, T_b, T_c and the new transitions: $T_a \to a$, $T_b \to b$, $T_c \to c$. This is done so that all productions are of the form $A \to a$ or of the form $A \to w$, where $a \in \Sigma$, $w \in V^*$ and $|w| \geq 2$. Thus, we arrive at the grammar G_3 :*

$$G_3 : \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \end{cases}$$

Solution

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \to aA \mid a \mid cC \mid c \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

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$$G_3 : \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_bB \mid b \\ \\ \\ \\ \end{cases}$$

Solution

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \to aA \mid a \mid cC \mid c \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

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Solution

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \to aA \mid a \mid cC \mid c \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

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$$G_3 : \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_bB \mid b \\ C \to T_cC \mid c \\ T_a \to a \\ \\ \end{cases}$$

Solution

$$G_2 : \begin{cases} S \to AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \to aA \mid a \mid cC \mid c \\ B \to bbB \mid b \\ C \to cC \mid c \end{cases}$$

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$$G_3 : \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_bB \mid b \\ C \to T_cC \mid c \\ T_a \to a \\ T_b \to b \end{cases}$$

Solution

$$G_2 : \begin{cases} S \rightarrow AB \mid aBC \mid SBS \mid aB \mid bbB \mid b \\ A \rightarrow aA \mid a \mid cC \mid c \\ B \rightarrow bbB \mid b \\ C \rightarrow cC \mid c \end{cases}$$

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$$G_3 : \begin{cases} S \rightarrow AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_bB \mid b \\ C \rightarrow T_cC \mid c \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

Solution

$$G_3 : \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_bB \mid b \\ C \to T_cC \mid c \\ T_a \to a \\ T_b \to b \\ T_c \to c \end{cases}$$

Solution

$$G_3 : \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_bB \mid b \\ C \to T_cC \mid c \\ T_a \to a \\ T_b \to b \\ T_c \to c \end{cases}$$

5. *Finally, we introduce new variables, with binary productions, to simulate the productions of the form $A \to w$, where $w \in V^*$ and $|w| \geq 2$, and we obtain the equivalent grammar G_4 :*

Solution

$$G_3 : \begin{cases} S \rightarrow AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_bB \mid b \\ C \rightarrow T_cC \mid c \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

5. *Finally, we introduce new variables, with binary productions, to simulate the productions of the form $A \rightarrow w$, where $w \in V^*$ and $|w| \geq 2$, and we obtain the equivalent grammar G_4 :*

$$G_4 : \begin{cases} S \rightarrow AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \end{cases}$$

Solution

$$G_3 : \begin{cases} S \rightarrow AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_bB \mid b \\ C \rightarrow T_cC \mid c \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

5. *Finally, we introduce new variables, with binary productions, to simulate the productions of the form* $A \rightarrow w$, *where* $w \in V^*$ *and* $|w| \geq 2$, *and we obtain the equivalent grammar* G_4 :

$$G_4 : \begin{cases} S \rightarrow AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ \\ \\ \\ \\ \\ \end{cases}$$

Solution

$$G_3 : \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_bB \mid b \\ C \to T_cC \mid c \\ T_a \to a \\ T_b \to b \\ T_c \to c \end{cases}$$

5. *Finally, we introduce new variables, with binary productions, to simulate the productions of the form $A \to w$, where $w \in V^*$ and $|w| \geq 2$, and we obtain the equivalent grammar G_4 :*

$$G_4 : \begin{cases} S \to AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_3 \mid b \\ \\ \\ \\ \\ \end{cases}$$

Solution

$$G_3 : \begin{cases} S \rightarrow AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_bB \mid b \\ C \rightarrow T_cC \mid c \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

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$$G_4 : \begin{cases} S \rightarrow AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_3 \mid b \\ C \rightarrow T_cC \mid c \end{cases}$$

Solution

$$G_3: \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_bB \mid b \\ C \to T_cC \mid c \\ T_a \to a \\ T_b \to b \\ T_c \to c \end{cases}$$

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$$G_4: \begin{cases} S \to AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_3 \mid b \\ C \to T_cC \mid c \\ T_1 \to BC \end{cases}$$

Solution

$$G_3 : \begin{cases} S \to AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_bB \mid b \\ C \to T_cC \mid c \\ T_a \to a \\ T_b \to b \\ T_c \to c \end{cases}$$

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$$G_4 : \begin{cases} S \to AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \\ A \to T_aA \mid a \mid T_cC \mid c \\ B \to T_bT_3 \mid b \\ C \to T_cC \mid c \\ T_1 \to BC \\ T_2 \to BS \end{cases}$$

Solution

$$G_3 : \begin{cases} S \rightarrow AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_bB \mid b \\ C \rightarrow T_cC \mid c \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

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$$G_4 : \begin{cases} S \rightarrow AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_3 \mid b \\ C \rightarrow T_cC \mid c \\ T_1 \rightarrow BC \\ T_2 \rightarrow BS \\ T_3 \rightarrow T_bB \end{cases}$$

Solution

$$G_3 : \begin{cases} S \rightarrow AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_bB \mid b \\ C \rightarrow T_cC \mid c \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

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Solution

$$G_3 : \begin{cases} S \rightarrow AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_bB \mid b \\ C \rightarrow T_cC \mid c \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

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$$G_4 : \begin{cases} S \rightarrow AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_3 \mid b \\ C \rightarrow T_cC \mid c \\ T_1 \rightarrow BC \\ T_2 \rightarrow BS \\ T_3 \rightarrow T_bB \\ T_a \rightarrow a \\ T_b \rightarrow b \end{cases}$$

Solution

$$G_3 : \begin{cases} S \rightarrow AB \mid T_aBC \mid SBS \mid T_aB \mid T_bT_bB \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_bB \mid b \\ C \rightarrow T_cC \mid c \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

5. *Finally, we introduce new variables, with binary productions, to simulate the productions of the form* $A \rightarrow w$, *where* $w \in V^*$ *and* $|w| \geq 2$, *and we obtain the equivalent grammar* G_4 :

$$G_4 : \begin{cases} S \rightarrow AB \mid T_aT_1 \mid ST_2 \mid T_aB \mid T_bT_3 \mid b \\ A \rightarrow T_aA \mid a \mid T_cC \mid c \\ B \rightarrow T_bT_3 \mid b \\ C \rightarrow T_cC \mid c \\ T_1 \rightarrow BC \\ T_2 \rightarrow BS \\ T_3 \rightarrow T_bB \\ T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{cases}$$

Agenda del día

1. **Grammars and Languages**
   - Previous Exercises
   - CYK Algorithm
   - Exercises

What is the CYK algorithm?

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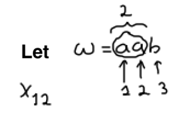

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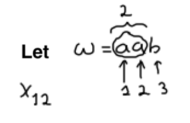

- That is, X_{ij} = set of variables A such that $A \stackrel{+}{\Longrightarrow} a_i a_{i+1} \cdots a_{i+j-1}$.

Grammars and Languages

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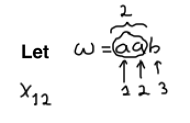

- That is, X_{ij} = set of variables A such that $A \overset{+}{\Longrightarrow} a_i a_{i+1} \cdots a_{i+j-1}$.
- By determining the sets X_{ij}, the possible ways to derive substrings of w that allow building a derivation of the complete string w are obtained.

Grammars and Languages

- The table is filled column by column, from top to bottom; the first column ($j = 1$) corresponds to substrings of length 1, the second column ($j = 2$) corresponds to substrings of length 2, and so on. The last column ($j = n$) corresponds to the only substring of length n in w, which is the string w itself.

Grammars and Languages

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- Variables that derive terminals at different positions in the string are found (first column).

Grammars and Languages

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Grammars and Languages

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- Variables that derive terminals at different positions in the string are found (first column).
- Since j represents length, when $j = 1$, we find the variables that generate strings of length 1, and those are terminals.
- It follows that $w \in L(G)$ if and only if $S \in X_{1n}$.

Example

$$G: \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases} \qquad w = a\,a\,b$$

Grammars and Languages

Example

$$G: \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases} \qquad w = a\,a\,b$$

$$X_{11} = \{A\}$$

$$w = \underset{\substack{\uparrow \\ 1}}{\overbrace{a}^{1}}\ \underset{\substack{\uparrow \\ 2}}{a}\ \underset{\substack{\uparrow \\ 3}}{b}$$

Grammars and Languages

Example

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$$w = \underset{\substack{\uparrow \\ 1}}{\overbrace{a}^{1}}\ \underset{\substack{\uparrow \\ 2}}{a}\ \underset{\substack{\uparrow \\ 3}}{b}$$

$$X_{21} = \{A\}$$

$$w = \underset{\substack{\uparrow \\ 1}}{a}\ \underset{\substack{\uparrow \\ 2}}{\overbrace{a}^{1}}\ \underset{\substack{\uparrow \\ 3}}{b}$$

Grammars and Languages

Example

$$G: \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases} \qquad w = a\,a\,b$$

$$X_{11} = \{A\}$$

$$w = \begin{array}{ccc} \overbrace{a}^{1} & a & b \\ \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{array}$$

$$X_{21} = \{A\}$$

$$w = \begin{array}{ccc} a & \overbrace{a}^{1} & b \\ \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{array}$$

$$X_{31} = \{B\}$$

$$w = \begin{array}{ccc} a & a & \overbrace{b}^{1} \\ \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{array}$$

Grammars and Languages

Example

$$G: \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases} \qquad w = a\ a\ b$$

$X_{11} = \{A\}$

$$\begin{array}{cccc} w = & \overbrace{a}^{1} & a & b \\ & \uparrow & \uparrow & \uparrow \\ & 1 & 2 & 3 \end{array}$$

$X_{12} = \{A\}$

$$\begin{array}{ccc} w = & \overbrace{a\ a}^{2} & b \\ & \uparrow\ \ \uparrow & \uparrow \\ & 1\ \ \ 2 & 3 \end{array}$$

AA

$X_{21} = \{A\}$

$$\begin{array}{cccc} w = & a & \overbrace{a}^{1} & b \\ & \uparrow & \uparrow & \uparrow \\ & 1 & 2 & 3 \end{array}$$

$X_{31} = \{B\}$

$$\begin{array}{cccc} w = & a & a & \overbrace{b}^{1} \\ & \uparrow & \uparrow & \uparrow \\ & 1 & 2 & 3 \end{array}$$

Example

$$G: \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases} \qquad w = a\,a\,b$$

$X_{11} = \{A\}$

$$w = \overbrace{a}^{1}\,a\,b \qquad \begin{matrix} \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{matrix}$$

$X_{12} = \{A\}$

$$w = \overbrace{a\,a}^{2}\,b \qquad \begin{matrix} \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{matrix}$$

AA

$X_{21} = \{A\}$

$$w = a\,\overbrace{a}^{1}\,b \qquad \begin{matrix} \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{matrix}$$

$X_{22} = \{S\}$

$$w = a\,\overbrace{a\,b}^{2} \qquad \begin{matrix} \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{matrix}$$

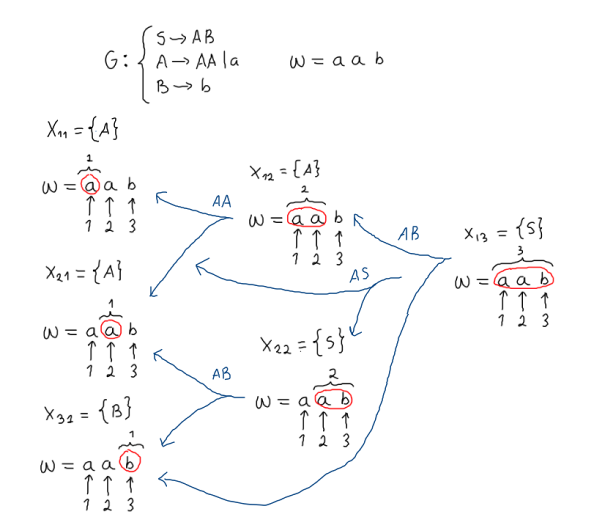
AB

$X_{31} = \{B\}$

$$w = a\,a\,\overbrace{b}^{1} \qquad \begin{matrix} \uparrow & \uparrow & \uparrow \\ 1 & 2 & 3 \end{matrix}$$

Grammars and Languages

Example



What is the pseudocode for the CYK algorithm?

What is the pseudocode for the CYK algorithm?

Algoritmo CYK

INPUT:
Gramámar G in CNF and a string of n terminals $w = a_1 a_2 \cdots a_n$.

INITIALIZE:
$j = 1$. For each i, $1 \leq i \leq n$,
$X_{ij} = X_{i1}$:= set of variables A such that $A \to a_i$

REPEAT:
$j := j + 1$. For each i, $1 \leq i \leq n - j + 1$,
$X_{ij} :=$ set of variables A such that $A \to BC$ es
a production of G, with $B \in X_{ik}$ y $C \in X_{i+k, j-k}$,
taking into account all k such that $1 \leq k \leq j - 1$.

UNTIL: $j = n$.

OUTPUT: $w \in L(G)$ if and only if $S \in X_{1n}$.

Example

Let G be the grammar

$$G : \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases}$$

Example

Let G be the grammar

$$G : \begin{cases} S \to AB \\ A \to AA \,|\, a \\ B \to b \end{cases}$$

and the string $w = aab$*. Use the CYK algorithm to determine if* $w \in L(G)$ *or not.*

Solution

		$J=1$	$J=2$	$J=3$
a	$i=1$			
a	$i=2$			
b	$i=3$			

$$G: \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases}$$

Solution

$$\begin{array}{llll} & & J=1 & J=2 \quad J=3 \\ a & i=1 & \{A\} & \\ a & i=2 & \{A\} & \\ b & i=3 & \{B\} & \end{array}$$

$$X_{11} = \{A\}$$
$$X_{21} = \{A\}$$
$$X_{31} = \{B\}$$

$$G: \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases}$$

Grammars and Languages

Solution

		$j=1$	$j=2$	$j=3$
a	$i=1$	$\{A\}$	$\{A\}$	
a	$i=2$	$\{A\}$	$\{S\}$	
b	$i=3$	$\{B\}$		

$$G : \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases}$$

$X_{11} = \{A\}$

$X_{21} = \{A\}$

$X_{31} = \{B\}$

$$X_{ij} \longrightarrow X_{ik}\, X_{i+k\ j-k}$$

$j=2 \quad 1 \leq k \leq 1$

$i=1$

$X_{12} \to X_{11} X_{21}$

$X_{12} \to \{A\}\{A\} = \{AA\}$

$X_{12} = \{A\}$

$i=2$

$X_{22} \to X_{21} X_{31}$

$X_{22} \to \{A\}\{B\} = \{AB\}$

$X_{22} = \{S\}$

Grammars and Languages

Solution

$$G : \begin{cases} S \to AB \\ A \to AA \mid a \\ B \to b \end{cases}$$

		J = 1	J = 2	J = 3
a	i = 1	{A}	{A}	{S}
a	i = 2	{A}	{S}	
b	i = 3	{B}		

$X_{11} = \{A\}$

$X_{21} = \{A\}$

$X_{31} = \{B\}$

$$X_{ij} \longrightarrow X_{ik}\, X_{i+k\; j-k}$$

$j = 2 \quad 1 \le k \le 1$

$i = 1$

$X_{12} \to X_{11} X_{21}$

$X_{12} \to \{A\}\{A\} = \{AA\}$

$X_{12} = \{A\}$

$i = 2$

$X_{22} \to X_{21} X_{31}$

$X_{22} \to \{A\}\{B\} = \{AB\}$

$X_{22} = \{S\}$

$J = 3 \quad 1 \le k \le 2$

$i = 1$

$X_{13} \to X_{11} X_{22} \cup X_{12} X_{31}$

$X_{13} \to \{A\}\{S\} \cup \{A\}\{B\} = \{AS, AB\}$

$X_{13} = \{S\}$

$$S \in X_{13}$$

Example

Let G be the grammar

$$G : \begin{cases} S \rightarrow BA \mid AC \\ A \rightarrow CC \mid b \\ B \rightarrow AB \mid a \\ C \rightarrow BA \mid a \end{cases}$$

Example

Let G be the grammar

$$G : \begin{cases} S \to BA \mid AC \\ A \to CC \mid b \\ B \to AB \mid a \\ C \to BA \mid a \end{cases}$$

and the string $w = bbab$. Use the CYK algorithm to determine if $w \in L(G)$ or not.

Solution

$$G: \begin{cases} S \to BA \mid AC \\ A \to CC \mid b \\ B \to AB \mid a \\ C \to BA \mid a \end{cases}$$

		$j=1$	$j=2$	$j=3$	$j=4$
b	$i=1$				
b	$i=2$				
a	$i=3$				
b	$i=4$				

Grammars and Languages

Solution

		$j=1$	$j=2$	$j=3$	$j=4$
b	$i=1$	$\{A\}$			
b	$i=2$	$\{A\}$			
a	$i=3$	$\{B,C\}$			
b	$i=4$	$\{A\}$			

$$X_{11} = \{A\}$$
$$X_{21} = \{A\}$$
$$X_{31} = \{B,C\}$$
$$X_{41} = \{A\}$$

$$G : \begin{cases} S \rightarrow BA \mid AC \\ A \rightarrow CC \mid b \\ B \rightarrow AB \mid a \\ C \rightarrow BA \mid a \end{cases}$$

Solution

		$J=1$	$J=2$	$J=3$	$J=4$
b	$i=1$	$\{A\}$	$\{\}$		
b	$i=2$	$\{A\}$	$\{B,S\}$		
a	$i=3$	$\{B,C\}$	$\{S,C\}$		
b	$i=4$	$\{A\}$			

$$G : \begin{cases} S \to BA \mid AC \\ A \to CC \mid b \\ B \to AB \mid a \\ C \to BA \mid a \end{cases}$$

$X_{11} = \{A\}$
$X_{21} = \{A\}$
$X_{31} = \{B,C\}$
$X_{41} = \{A\}$

$$X_{ij} \longrightarrow X_{ik}\, X_{i+k\; j-k}$$

$j=2 \quad 1 \leq k \leq 1$

$i=1$

$X_{12} \to X_{11} X_{21}$
$X_{12} \to \{A\}\{A\} = \{AA\}$
$X_{12} = \{\}$

$i=2$

$X_{22} \to X_{21} X_{31}$
$X_{22} \to \{A\}\{B,C\} = \{AB, AC\}$
$X_{22} = \{B,S\}$

$i=3$

$X_{32} \to X_{31} X_{41}$
$X_{32} \to \{B,C\}\{A\} = \{BA, CA\}$
$X_{32} = \{S,C\}$

Grammars and Languages

Solution

$$G: \begin{cases} S \to BA \mid AC \\ A \to CC \mid b \\ B \to AB \mid a \\ C \to BA \mid a \end{cases}$$

		$J=1$	$J=2$	$J=3$	$J=4$
b	$i=1$	$\{A\}$	$\{\}$	$\{B\}$	
b	$i=2$	$\{A\}$	$\{B,S\}$	$\{S,C\}$	
a	$i=3$	$\{B,C\}$	$\{S,C\}$		
b	$i=4$	$\{A\}$			

$X_{11} = \{A\}$
$X_{21} = \{A\}$
$X_{31} = \{B,C\}$
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$$X_{ij} \longrightarrow X_{ik}\, X_{i+k\; j-k}$$

$j=2 \quad 1 \le k \le 1$

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$X_{22} \to X_{21} X_{31}$
$X_{22} \to \{A\}\{B,C\} = \{AB, AC\}$
$X_{22} = \{B,S\}$

$i=3$

$X_{32} \to X_{31} X_{41}$
$X_{32} \to \{B,C\}\{A\} = \{BA, CA\}$
$X_{32} = \{S,C\}$

$J=3 \quad 1 \le k \le 2$

$i=1$

$X_{13} \to X_{11} X_{22} \cup X_{12} X_{31}$
$X_{13} \to \{A\}\{B,S\} \cup \{\}\{B,C\} = \{AB, AS\}$
$X_{13} = \{B\}$

$i=2$

$X_{23} \to X_{21} X_{32} \cup X_{22} X_{41}$
$X_{23} \to \{A\}\{S,C\} \cup \{B,S\}\{A\} = \{AS, AC, BA, SA\}$
$X_{23} = \{S,C\}$

Solution

$$G: \begin{cases} S \to BA \mid AC \\ A \to CC \mid b \\ B \to AB \mid a \\ C \to BA \mid a \end{cases}$$

		J=1	J=2	J=3	J=4
b	i=1	{A}	{ }	{B}	{S,C}
b	i=2	{A}	{B,S}	{S,C}	
a	i=3	{B,C}	{S,C}		
b	i=4	{A}			

$X_{11} = \{A\}$
$X_{21} = \{A\}$
$X_{31} = \{B,C\}$
$X_{41} = \{A\}$

$$X_{ij} \to X_{ik} X_{i+k\, j-k}$$

$j=2 \quad 1 \leq k \leq 1$

$i=1$

$X_{12} \to X_{11} X_{21}$
$X_{12} \to \{A\}\{A\} = \{AA\}$
$X_{12} = \{\ \}$

$i=2$

$X_{22} \to X_{21} X_{31}$
$X_{22} \to \{A\}\{B,C\} = \{AB, AC\}$
$X_{22} = \{B,S\}$

$i=3$

$X_{32} \to X_{31} X_{41}$
$X_{32} \to \{B,C\}\{A\} = \{BA, CA\}$
$X_{32} = \{S,C\}$

$J=3 \quad 1 \leq k \leq 2$

$i=1$

$X_{13} \to X_{11} X_{22} \cup X_{12} X_{31}$
$X_{13} \to \{A\}\{B,S\} \cup \{\ \}\{B,C\} = \{AB, AS\}$
$X_{13} = \{B\}$

$i=2$

$X_{23} \to X_{21} X_{32} \cup X_{22} X_{41}$
$X_{23} \to \{A\}\{S,C\} \cup \{B,S\}\{A\} = \{AS, AC, BA, SA\}$
$X_{23} = \{S,C\}$

$J=4 \quad 1 \leq k \leq 3$

$i=1$

$X_{14} \to X_{11} X_{23} \cup X_{12} X_{32} \cup X_{13} X_{41}$
$X_{14} \to \{A\}\{S,C\} \cup \{\ \}\{S,C\} \cup \{B\}\{A\} = \{AS, AC, BA\}$
$X_{14} = \{S,C\}$

$S \in X_{14}$

Agenda del día

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