Computación y Estructuras Discretas III

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2024-2

Agenda del día

1. **Regular languages and Automata theory**
   - Topic presentation
   - Exercises

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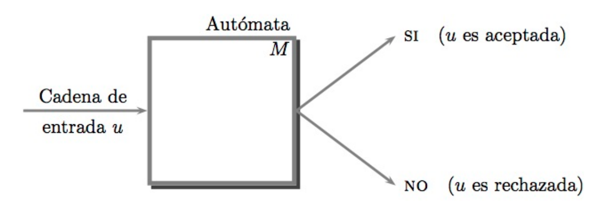
What is a finite automaton?

Finite automata are abstract machines that process input strings, which are either accepted or rejected.



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2. $Q = q_0, q_1, ..., q_n$*, the set of internal states of the automaton.*
3. $q_0 \in Q$*, the initial state.*
4. $F \subseteq Q$*, the set of final or accepting states.* $F \neq \emptyset$.

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4. $F \subseteq Q$*, the set of final or accepting states.* $F \neq \emptyset$.
5. *The transition function of the automaton:*

$$\begin{array}{rcc} \delta : Q \times \Sigma & \longrightarrow & Q \\ (q, s) & \longmapsto & \delta(q, s) \end{array}$$

Example

Let's consider the automaton defined by the following 5 *components:*

$\Sigma = a, b$

$Q = q_0, q_1, q_2$

q_0 *: initial state.*

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Transition function δ :

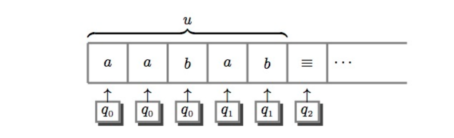
δ	a	b
q_0	q_0	q_1
q_1	q_1	q_2
q_2	q_1	q_1

$$\delta(q_0, a) = q_0 \qquad \delta(q_0, b) = q_1$$
$$\delta(q_1, a) = q_1 \qquad \delta(q_1, b) = q_2$$
$$\delta(q_2, a) = q_1 \qquad \delta(q_2, b) = q_1.$$

Example

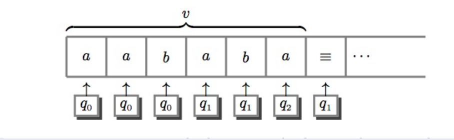
We illustrate the processing of two input strings.

1. $u = aabab$



Since q_2 is an accepting state, the input string u is accepted.

2. $v = aababa$



Since q_1 is not an accepting state, the string v is rejected.

How do we call these previously defined automata?

They are called Deterministic Finite Automata (DFA) because for each state q and for each symbol $a \in \Sigma$, the transition function $\delta(q, a)$ is always defined.

How do we call the language accepted by a finite automaton?

Let M be an automaton, the language accepted by this automaton is denoted as $L(M)$, and it is defined as follows:

$$L(M) := \{u \in \Sigma^* \mid M \text{ terminates processing the input string } u \text{ in a state } q \in F\}.$$

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Regular languages and Automata theory

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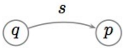
- *The vertices or nodes are the automaton's states*
- *State q is represented by:* 

- *The initial state* q_0 *is represented by:* 

- *The final state q is represented by:* 

- *Transition* $\delta = (q, s) = p$ *is represented as:*



Regular languages and Automata theory

Example

$\Sigma = \{a, b\}$.
$Q = \{q_0, q_1, q_2\}$.
q_0 : *initial state*
$F = \{q_0, q_2\}$, *accepting states*
Transition function δ:

δ	a	b
q_0	q_0	q_1
q_1	q_1	q_2
q_2	q_1	q_1

$\delta(q_0, a) = q_0$ $\quad$ $\delta(q_0, b) = q_1$
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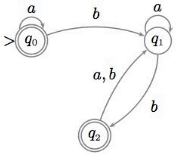

Find a set of strings accepted by this automata

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DFA vs NFA

Deterministic Finite Automaton (DFA)
$\rightarrow$ For every state in the automaton, there is exactly one transition for each possible input symbol in the alphabet.
$\rightarrow$ If the DFA reaches the end of the input string, it accepts the input if it is in a designated accepting state, otherwise, it rejects the input.
s DFA

Non-Deterministic Finite Automaton (NFA)
$\rightarrow$ For a given state and input symbol, there can be multiple possible transitions or no transition at all.
$\rightarrow$ NFA accepts an input string if there exists at least one path through the states that leads to an accepting state, regardless of whether other paths lead to rejection.

Non-Deterministic Finite Automaton

What are the changes on defining a DFA?

Mainly the way we define the transitions as for each state $q \in Q$ and each $a \in \Sigma$, the transition $\delta(q, a)$ can consist of **more than one state or may not be defined**.

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An NFA is defined by $M = (\Sigma, Q, q_0, F, \Delta)$ where:

1. Σ is the tape alphabet.
2. Q is the (finite) set of internal states.
3. $q_0 \in Q$ is the initial state.
4. $\emptyset \neq F \subseteq Q$ is the set of final or accepting states.
5. Let $\wp(Q)$ be the set of subsets of Q,

$$\begin{array}{rcl} \Delta : Q \times \Sigma & \longrightarrow & \wp(Q) \\ (q, s) & \mapsto & \Delta(q, s) = \{q_{i1}, q_{i2}, \ldots, q_{ik}\} \end{array}$$

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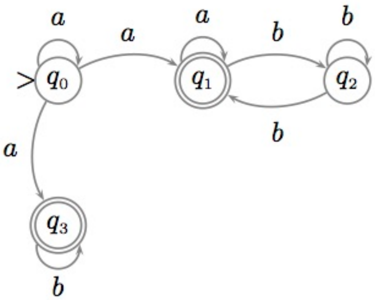
Example

Let M be the following NFA:



Δ	a	b
q_0	$\{q_0, q_1, q_3\}$	$\varnothing$
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q_2	$\varnothing$	$\{q_1, q_2\}$
q_3	$\varnothing$	$\{q_3\}$

How does an FNA process an input string?

An NFA can process an input string $u \in \Sigma^*$ in different ways. Seen in the transition diagram, this means that there can be various paths from the initial state, labeled with the symbols of u.

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What is the notion of acceptance for nondeterministic finite automata?

$$
\begin{aligned}
L(M) &= \text{language accepted or recognized by } M \\
&= \{u \in \Sigma \mid \text{there exists at least one complete computation} \\
&\qquad \text{of } u \text{ that ends in a state } q \in F\}
\end{aligned}
$$

For a string u to be accepted, there must exist some computation in which u is processed completely and ends with M in an accepting state.

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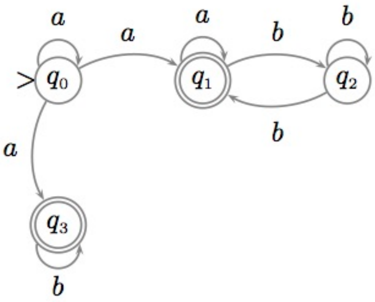

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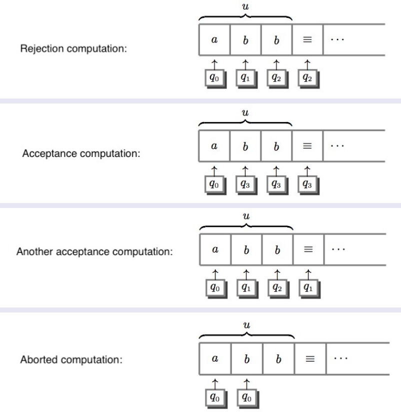
For the string $u = abb$, there are computations that lead to rejection, aborted computations, and computations that end in accepting states.

Regular languages and Automata theory

Example

According to the definition of accepted language, $u \in L(M)$, we have:



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DFA $\rightarrow$ NFA
A DFA $M = (\Sigma, Q, q_0, F, \delta)$ can be considered as an NFA $M' = (\Sigma, Q, q_0, F, \Delta)$ by defining $\Delta(q, a) = \delta(q, a)$ for each $q \in Q$ and each $a \in \Sigma$

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A non-deterministic automaton with λ transitions is a tuple $M = (\Sigma, Q, q_0, F, \Delta)$ where the transition function is defined as $\Delta : Q \times (\Sigma \cup \{\lambda\}) \rightarrow \wp(Q)$.

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Which means, it contains λ transitions.

The λ transition, $\Delta(q, \lambda) = \{p_{i_1}, ..., p_{i_n}\}$, also called null or spontaneous transition, allows transitioning between states without processing any symbol read on the tape.

In a transition diagram, these transitions result in arcs labeled with λ.

When is an input w accepted by an NFA-λ?

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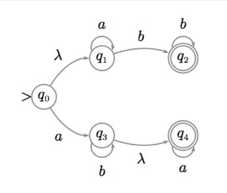
The same as with regular NFAs

A string w is accepted by an NFA-λ if there exists at least one path from the initial state q_0 to an accepting state, whose labels consist exactly of the symbols of w, **BUT** interspersed with zero or more λs.

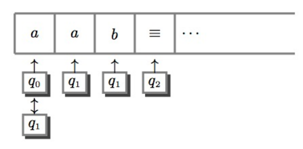
Example

Let M be the following NFA-λ:

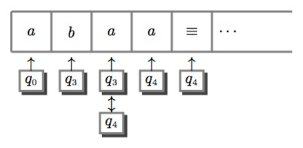


Example

The acceptance computation for $u = aab$ *is:*



The acceptance computation for $v = abaa$ *is:*



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Regular languages and Automata theory

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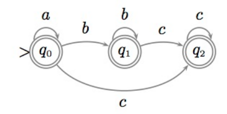
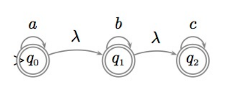
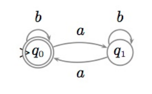


M' *is an NFA-*λ *that accepts* L :

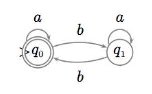


Example

Let M_1 be the following DFA that accepts the language of all strings with an even number of á's over $\Sigma = \{a, b\}$:

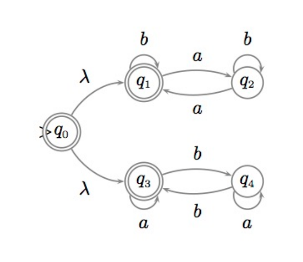


And let M_2 be the following DFA that accepts the language of all strings with an even number of 'b's over $\Sigma = \{a, b\}$:



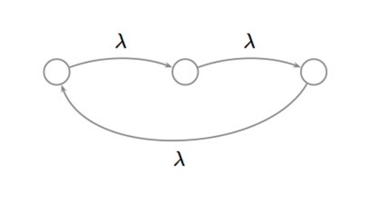
Example

M_3 is the following NFA-λ that accepts the language of all strings with an even number of á's or an even number of 'b's over $\Sigma = \{a, b\}$:



Can infinite loops occur in an NFA-λ?

Unlike DFAs and NFAs, in NFAs-λ, there can be computations that never terminate. This can happen if the automaton enters a state from which there are multiple chained λ transitions that return to the same state.



Are NFAs and NFAs-λ computationally equivalent?

Yes, NFA-λ are computationally equivalent to NFAs.
'Cause, λ transitions can be eliminated by adding transitions that simulate them, without altering the accepted language.
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