Exercises: Distributions

Statistics for Data Science

Master Program in Advanced Analytics

2025/2026

1 Discrete Distributions

1. Consider the following statements. Classify them whether they are true or false.

   (a) If a variable X follows a Binomial Distribution with only one trial, then it is equivalent of following a Bernoulli Distribution.

   (b) A Poisson distribution is appropriate to describe binary outcomes.

   (c) The probability density function describes the probability function of a discrete variable whereas a probability mass function describe the probability mass function of a continuous random variable.

   (d) A variable is discrete if the sample space is countable.

2. Complete the table below:

Distribution	Probability Mass Function	Cumulative Distribution Function
Binomial	**A**	**B**
C	`dpois(x, lambda)`	**D**
	P$(X = x)$	**E**

   Hint: Check help in R Studio for Statistical Distributions.

3. Let *X* be a random variable that follows a Poisson distribution with mean equal to 7. Calculate:

   (a) $P(X \leq 5)$

   (b) $P(X < 5)$

   (c) $P(4 \leq X \leq 16)$

4. Consider 60 tosses of a balanced coin.

   (a) What kind of distribution is appropriate for this setting? Why? **Binomial Distribution**

   (b) Calculate the probability of the following events:

      i. get 20, 25 or 30 times heads

      ii. get less than 30 times heads

      iii. get between 20 and 40 times heads

5. A box contains 6 balls. One of the balls has the number 1; Two balls have the number 2; Two balls have the number 3; one ball has the number 4. Assume that with retrieve two balls out of the box (with replacement). Calculate the probability that neither has the number one.
   Hint: You can define the probability of success as retrieving a ball with number one.

6. Let X be a random variable. In **R** we performed the following instruction:

```
dbinom(4, 10, 0.6)
[1] 0.1115
```

   What is the meaning of this result?

7. Let X be the same random variable of the previous question. Consider the following results:

```
pbinom(4, 10, 0.6)
[1] 0.1662
pbinom(5, 10, 0.6)
[1] 0.3669
```

   Based on the previous output, calculate the probabilities $P(X \leq 5)$, $P(X < 5)$, $P(X > 4)$ e $P(X = 5)$.

2 Continuous Distributions

1. Consider the following statements. Classify them whether they are true or false.

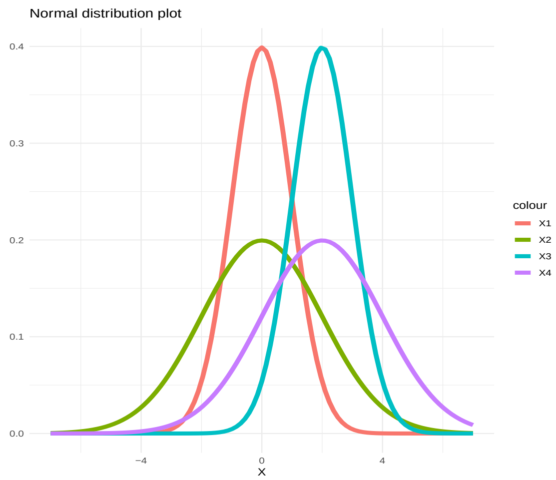
   (a) Let $X_1, X_2, \ldots, X_n$ be n independent random variables having the standard normal distribution, then let $Y = \sum_i^n X_i^2$, Y follows a chi-squared distribution with n degrees of freedom. T

   (b) Let $X_1 \sim \chi^2(v_1)$ and $X_2 \sim \chi^2(v_2)$, then $Y = \frac{X_1/v_1}{X_2/v_2} \sim$ t($V_1 + v_2$). F

   (c) A standard normal distribution has $\mu = 0$ and $\sigma = 1$. T

   (d) The t-distribution is symmetric and bell-shaped, like the normal distribution, but has heavier tails, meaning that it is more prone to producing values that fall closer to its mean. F

2. Consider the following Normal Distribution plots:



   Fill in the gaps with $>$, $<$ or $=$:

   (a) μ_{X_1} <u><</u> μ_{X_4}

   (b) μ_{X_2} <u><</u> μ_{X_3}

   (c) μ_{X_1} <u>=</u> μ_{X_2}

   (d) σ_{X_1} <u><</u> σ_{X_4}

   (e) σ_{X_2} <u>></u> σ_{X_3}

   (f) σ_{X_1} <u><</u> σ_{X_2}

3. Consider $X \sim$ F(4, 300). Find the probabilities below:

   (a) $P(X > 3)$

   (b) $P(X < 3)$

   (c) $P(3 \leq X \leq 6)$

4. Repeat the previous exercise, but now consider $X \sim \chi^2$(4).

5. The following instructions were given in **R**:

```
pnorm(2)
[1] 0.9772
pnorm(2, 1, 1)
[1] 0.8413
pnorm(2, 1, 2)
[1] 0.6915
```

   Indicate the distibutions that are being used and the resulting probabilities.

6. The following instructions were given in **R**:

   ```
   qnorm(0.975)
   [1] 1.96
   qnorm(0.975, 1, 1)
   [1] 2.96
   qnorm(0.975, 1, 2)
   [1] 4.92
   ```

   Indicate the distibutions that are being used and the resulting values.

7. What is the result of `qnorm(pnorm(2))`? Explain.