Modelação de Sistemas Físicos 2020 – 2021

Formulário:

$$v_x(t) = \frac{dx}{dt}$$

$$a_x(t) = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$v_x(t+\delta t) = v_x(t) + \left.\frac{dv_x}{dt}\right|_t \delta t + \frac{1}{2}\left.\frac{d^2v_x}{dt^2}\right|_t \delta t^2 + \frac{1}{3!}\left.\frac{d^3v_x}{dt^3}\right|_t \delta t^3 + \mathcal{O}(\delta t^4)$$

$$\vec{F} = m\,\vec{a}$$

$$W = \int_C \vec{F}\cdot d\vec{r} = \int_{t_0}^{t_1} \vec{F}\cdot\vec{v}\,dt = \frac{1}{2}m\,|\vec{v}_1|^2 - \frac{1}{2}m\,|\vec{v}_0|^2 \qquad \int_C \vec{F}^{(conservativa)}\cdot d\vec{r} = E_{p0} - E_{p1}$$

$$\vec{F}_{res} = -m\,D|\vec{v}|\vec{v} \qquad \vec{F}_{res} = -\frac{C_{res}}{2}A\,\rho_{ar}|\vec{v}|\,\vec{v} \qquad \left|\vec{F}_{rol}\right| = \mu\left|\vec{N}\right|$$

$$\vec{F}_{Magnus} = \frac{1}{2}A\,\rho_{ar}\,r\,\vec{\omega}\times\vec{v} \qquad \vec{F}_{grav} = -G\frac{m\,M}{|\vec{r}|^2}\frac{\vec{r}}{|\vec{r}|} \qquad \vec{F}_{elástica} = -k\,\vec{r}$$

$$\vec{F}_{elet} = -k_e\frac{q\,Q}{|\vec{r}|^2}\frac{\vec{r}}{|\vec{r}|} \qquad \vec{F}_{elet} = q\vec{E}_{elet}$$

$$F_x = -\frac{dE_p}{dx} \qquad E_p = m\,g\,y \qquad E_p = \frac{1}{2}k\,x^2 \qquad E_p = -G\frac{m\,M}{|\vec{r}|}$$

$$P_o = \vec{F}\cdot\vec{v}$$

$$\int_{t_0}^{t_1}\vec{F}(t)\,dt = \vec{p}_1 - \vec{p}_0$$

$$\sum\vec{F}^{ext} = \frac{d\vec{P}}{dt}$$

$$x(t) = A\ \cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{K}{M}} \qquad E = \frac{1}{2}k\,A^2$$

$$E_p(x) = E_p(x_{min}) + \left.\frac{dE_p}{dx}\right|_{x_{min}}\delta x + \frac{1}{2}\left.\frac{d^2E_p}{dx^2}\right|_{x_{min}}\delta x^2 + \frac{1}{3!}\left.\frac{d^3E_p}{dx^3}\right|_{x_{min}}\delta x^3 + \mathcal{O}(\delta x^4)$$

$$x(t) = \frac{a_0}{2} + a_1\cos(\omega t) + b_1\sin(\omega t) + a_2\cos(2\omega t) + b_2\sin(2\omega t) + \cdots$$

$$a_n = \frac{2}{T}\int_t^{t+T} f(t)\cos(n\omega t)\ dt, \quad n = 0,\ 1,\ 2,\ \cdots \qquad b_n = \frac{2}{T}\int_t^{t+T} f(t)\sin(n\omega t)\ dt, \quad n = 1,\ 2,\ \cdots$$

$$x(t) = A\,e^{-\frac{b}{2m}t}\cos(\omega\,t + \phi) \qquad \omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

$$A(\omega_f) = \frac{F_0/m}{\sqrt{\left(\omega_f^2 - \omega_0^2\right)^2 + \left(\frac{b\,\omega_f}{m}\right)^2}}$$

$$y(x,t) = A\,\mathrm{sen}(kx - \omega t + \phi)$$

Grandezas físicas e conversões:

1 polegada = 1 in = 0,39370 m   1 pé = 1 ft = 2,54 cm   1 milha = 1,609344 km

1 rad = 57.29578 graus

1 cv (cavalo − vapor métrico) = 735,4975 W   1 hp (cavalo − vapor inglês) = 745,715 W

$M_{Sol} = \mathrm{M} = 1.989 \times 10^{30}$ kg   1 AU = 1.489×10^{11} m   1 ano = 365,24 dias

$G = 6.67 \times 10^{11}\ \mathrm{m^3/(kg \cdot s^2)} = 4\pi^2\ \mathrm{AU^3/(M \cdot ano^2)}$ $g = 9{,}80\ \mathrm{m/s^2}$

$\rho_{ar} = 1.225\ \mathrm{kg/m^3}$ $v_{som} = 340$ m/s

$c = 299792{,}458$ km/s $= 2{,}99792458 \times 10^8$ m/s

$k_B = 1.380649 \times 10^{-23}$ J/K $= 8.61733 \times 10^{-5}$ eV/K

$\varepsilon_0 = 8{,}854187817 \times 10^{-12}$ F/m $k_e = 1/4\pi\varepsilon_0 = 8{,}98755188 \times 10^9\ \mathrm{N \cdot m^2/C^2}$

$m_e = 9{,}10938356 \times 10^{-31}$ kg

$m_p = 1{,}67262 \times 10^{-27}$ kg $= 1836.151\ m_e$

$m_n = 1{,}67493 \times 10^{-27}$ kg   1 Å $= 10^{-10}$ m

$e = 1{,}602176208 \times 10^{-19}$ C $e/c = 5{,}34428 \times 10^{-28}\ \mathrm{C \cdot m/s}$

Grandezas matemáticas e Transformações Trigonométricas:

e = 2,71828183 $\pi = 3{,}14159265$

$\mathrm{sen}\,(-x) = -\mathrm{sen}\,(x)$ $\mathrm{sen}\,(\pi - x) = \mathrm{sen}\,(x)$ $\mathrm{sen}\left(x \pm \frac{\pi}{2}\right) = \pm\cos(x)$

$\cos(-x) = +\cos(x)$ $\cos\left(x \pm \frac{\pi}{2}\right) = \mp\,\mathrm{sen}\,(x)$

$\mathrm{sen}\,(x \pm y) = \sin x\ \cos y \pm \cos x\ \sin y$ $\cos(x \pm y) = \cos x\ \cos y \mp \mathrm{sen}\,x\ \mathrm{sen}\,y$

$$\sin x\ \cos y = \frac{1}{2}\left[\mathrm{sen}\,(x+y) + \mathrm{sen}\,(x-y)\right]$$

$$\cos x\ \sin y = \frac{1}{2}\left[\mathrm{sen}\,(x+y) - \mathrm{sen}\,(x-y)\right]$$

$$\sin x\ \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

$$\cos x\ \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$\mathrm{sen}^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$ $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

$$\mathrm{sen}\,x \pm \mathrm{sen}\,y = 2\cos\left(\frac{x \mp y}{2}\right)\mathrm{sen}\left(\frac{x \pm y}{2}\right)$$

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ $\cos x - \cos y = 2\,\mathrm{sen}\left(\frac{x+y}{2}\right)\mathrm{sen}\left(\frac{x-y}{2}\right)$