In[17]:=
```
f[x_, z_] := b0 + b1 * z + b2 * x
```

In[18]:=
```
∂b0 Sum[(f[xi, zi] - yi)^2, {i, 1, n}]
```

Out[18]=

$$\sum_{i=1}^{n} (2\,b0 + 2\,b2\,x_i - 2\,y_i + 2\,b1\,z_i)$$

In[19]:=
```
∂b1 Sum[(f[xi, zi] - yi)^2, {i, 1, n}]
```

Out[19]=

$$\sum_{i=1}^{n} \left(2\,b0\,z_i + 2\,b2\,x_i\,z_i - 2\,y_i\,z_i + 2\,b1\,z_i^2\right)$$

In[20]:=
```
∂b2 Sum[(f[xi, zi] - yi)^2, {i, 1, n}]
```

Out[20]=

$$\sum_{i=1}^{n} \left(2\,b0\,x_i + 2\,b2\,x_i^2 - 2\,x_i\,y_i + 2\,b1\,x_i\,z_i\right)$$

In[21]:=
```
A = {{n0 + n1, Sz, Sx}, {Sz, Sz2, Sxz}, {Sx, Sxz, Sx2}}
```

Out[21]=

$$\{\{n0 + n1,\ Sz,\ Sx\},\ \{Sz,\ Sz2,\ Sxz\},\ \{Sx,\ Sxz,\ Sx2\}\}$$

In[22]:=
```
A // MatrixForm
```

Out[22]//MatrixForm=

$$\begin{pmatrix} n0 + n1 & Sz & Sx \\ Sz & Sz2 & Sxz \\ Sx & Sxz & Sx2 \end{pmatrix}$$

In[24]:=
```
b = {Sy, Syz, Syx};
```

In[27]:=
```
b // MatrixForm
```

Out[27]//MatrixForm=

$$\begin{pmatrix} Sy \\ Syz \\ Syx \end{pmatrix}$$

In[28]:=
```
x = {b0, b1, b2}
```

Out[28]=

$$\{b0,\ b1,\ b2\}$$

In[29]:=
```
x = Solve[A.x == b, {b0, b1, b2}]
```

Out[29]=

$$\left\{\left\{b0 \to -\frac{-Sxz^2\,Sy + Sx\,Sxz\,Syz + Sxz\,Syx\,Sz - Sx2\,Syz\,Sz + Sx2\,Sy\,Sz2 - Sx\,Syx\,Sz2}{n0\,Sxz^2 + n1\,Sxz^2 - 2\,Sx\,Sxz\,Sz + Sx2\,Sz^2 + Sx^2\,Sz2 - n0\,Sx2\,Sz2 - n1\,Sx2\,Sz2},\ b1 \to -\frac{Sx\,Sxz\,Sy - n0\,Sxz\,Syx - n1\,Sxz\,Syx - Sx^2\,Syz + n0\,Sx2\,Syz + n1\,Sx2\,Syz - Sx2\,Sy\,Sz + Sx\,Syx\,Sz}{n0\,Sxz^2 + n1\,Sxz^2 - 2\,Sx\,Sxz\,Sz + Sx2\,Sz^2 + Sx^2\,Sz2 - n0\,Sx2\,Sz2 - n1\,Sx2\,Sz2},\ b2 \to -\frac{-n0\,Sxz\,Syz - n1\,Sxz\,Syz + Sxz\,Sy\,Sz + Sx\,Syz\,Sz - Syx\,Sz^2 - Sx\,Sy\,Sz2 + n0\,Syx\,Sz2 + n1\,Syx\,Sz2}{n0\,Sxz^2 + n1\,Sxz^2 - 2\,Sx\,Sxz\,Sz + Sx2\,Sz^2 + Sx^2\,Sz2 - n0\,Sx2\,Sz2 - n1\,Sx2\,Sz2}\right\}\right\}$$

In[61]:=
```
A = {{n0 + n1, n1, 0}, {n1, n1, Sx1}, {0, Sx1, Sx2}}
```

Out[61]=
```
{{n0 + n1, n1, 0}, {n1, n1, Sx1}, {0, Sx1, Sx2}}
```

In[31]:=
```
b = {Sy1 + Sy0, Sy1, Syx}
```

Out[31]=
```
{Sy0 + Sy1, Sy1, Syx}
```

In[41]:=
```
x = {b0, b1, b2}
```

Out[41]=
```
{b0, b1, b2}
```

In[68]:=
```
A // MatrixForm
```

Out[68]//MatrixForm=

$$\begin{pmatrix} n0 + n1 & n1 & 0 \\ n1 & n1 & Sx1 \\ 0 & Sx1 & Sx2 \end{pmatrix}$$

In[69]:=
```
b // MatrixForm
```

Out[69]//MatrixForm=

$$\begin{pmatrix} Sy0 + Sy1 \\ Sy1 \\ Syx \end{pmatrix}$$

In[70]:=
```
x // MatrixForm
```

Out[70]//MatrixForm=

$$\begin{pmatrix} b0 \\ b1 \\ b2 \end{pmatrix}$$

In[62]:=
```
beta = Solve[A.x == b, {b0, b1, b2}]
```

Out[62]=

$$\left\{\left\{b0 \to -\frac{-Sx1^2\,Sy0 + n1\,Sx2\,Sy0 - Sx1^2\,Sy1 + n1\,Sx1\,Syx}{n0\,Sx1^2 + n1\,Sx1^2 - n0\,n1\,Sx2},\ b1 \to -\frac{n1\,Sx2\,Sy0 - n0\,Sx2\,Sy1 + n0\,Sx1\,Syx + n1\,Sx1\,Syx}{-n0\,Sx1^2 - n1\,Sx1^2 + n0\,n1\,Sx2},\ b2 \to -\frac{-n1\,Sx1\,Sy0 + n0\,Sx1\,Sy1 - n0\,n1\,Syx}{-n0\,Sx1^2 - n1\,Sx1^2 + n0\,n1\,Sx2}\right\}\right\}$$

In[63]:=
```
beta[[1]][[3]]
```

Out[63]=

$$b2 \to -\frac{-n1\ Sx1\ Sy0 + n0\ Sx1\ Sy1 - n0\ n1\ Syx}{-n0\ Sx1^2 - n1\ Sx1^2 + n0\ n1\ Sx2}$$

In[64]:=
```
beta2 = b2 /. beta[[1]][[3]]
```

Out[64]=

$$-\frac{-n1\ Sx1\ Sy0 + n0\ Sx1\ Sy1 - n0\ n1\ Syx}{-n0\ Sx1^2 - n1\ Sx1^2 + n0\ n1\ Sx2}$$

In[65]:=
```
tauhat = Sy1/n1 - Sy0/n0
```

Out[65]=

$$-\frac{Sy0}{n0} + \frac{Sy1}{n1}$$

In[71]:=
```
tauxhat = Sx1/n1 - -Sx1/n0
```

Out[71]=

$$\frac{Sx1}{n0} + \frac{Sx1}{n1}$$

In[74]:=
```
tauhat - beta2 * tauxhat // FullSimplify
```

Out[74]=

$$\frac{-n1\ Sx2\ Sy0 + n0\ Sx2\ Sy1 - (n0 + n1)\ Sx1\ Syx}{-\left((n0 + n1)\ Sx1^2\right) + n0\ n1\ Sx2}$$

In[77]:=
```
beta1 = b1 /. beta[[1]][[2]]
```

$$-\frac{n1\ Sx2\ Sy0 - n0\ Sx2\ Sy1 + n0\ Sx1\ Syx + n1\ Sx1\ Syx}{-n0\ Sx1^2 - n1\ Sx1^2 + n0\ n1\ Sx2}$$

In[79]:=
```
beta1 // FullSimplify
```

Out[79]=

$$\frac{-n1\ Sx2\ Sy0 + n0\ Sx2\ Sy1 - (n0 + n1)\ Sx1\ Syx}{-\left((n0 + n1)\ Sx1^2\right) + n0\ n1\ Sx2}$$