# *Redes de Computadores*

## Delay Models in Computer Networks

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- *What are the common multiplexing strategies?*
- *What is a Poisson process?*
- *What is the Little theorem?*
- *What is a queue?*
- *What is the meaning of service time $1/\mu$ in a queue of packets?*
- *What is the meaning of traffic intensity $\rho$ in a queue model?*
- *What is the probability of a M/M/1 queue being in a given state n ?*
- *What is the mean number of clients in a M/M/1 queue? What is the mean waiting time in a M/M/1 queue? What is the relationship between N and $\rho$ in a M/M/1 queue?*
- *What are the differences between M/M/1 and M/G/1 queues? How to estimate mean number of packets and mean delay in a M/G/1 queue?*
- *How to model a network of transmission lines? How to calculate the mean number of packets and mean delay in this case?*
- *What is a Jackson Network? Why is it important?*

# *Multiplexing Traffic on a Link*

Tubo que permite fazer transfe-rência de C bit/s


- Communication link
  - » Bit pipe with a given capacity C (bit/s)
  - » Link capacity → rate at which bits are transmitted to the link
  - » Link may transport multiplexed traffic streams → Arranjar estratégias para enviar fluxos pelo mesmo canal

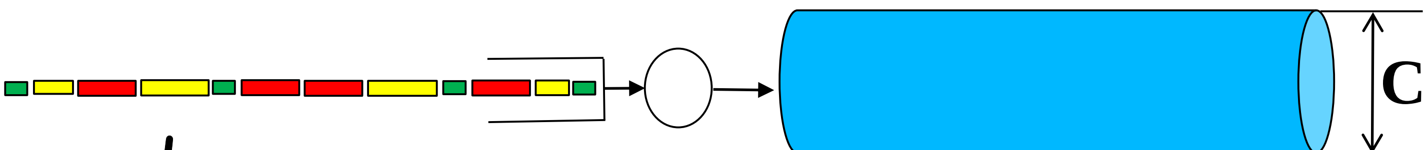
- Multiplexing strategies
  - » Statistical Multiplexing
  - » Frequency Division Multiplexing
  - » Time Division Multiplexing

- Multiplexing strategy affects traffic delay

# *Statistical Multiplexing*

Fila de espera

- Packets of all traffic streams merged in a single queue
- Packets transmitted on a first-come first-served basis
- Time required to transmit a packet of length L ➔ $T_{frame}=L/C$

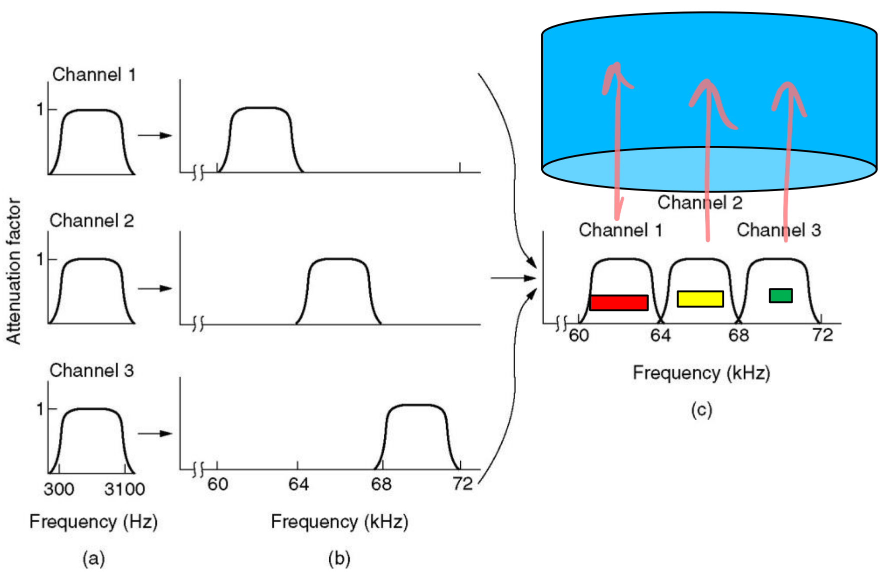



Fila única para todos os pacotes → Todos pelos mesmo cabo

# *FDM – Frequency Division Multiplexing*

- Link capacity C subdivided into ***m*** portions
- Channel bandwidth W subdivided into m channels of W/m Hz
- Capacity of each channel ➔ C/m → Podemos associar um canal a um fluxo de banda
- Time required to transmit a packet of length L ➔ $T_{frame} = Lm/C$

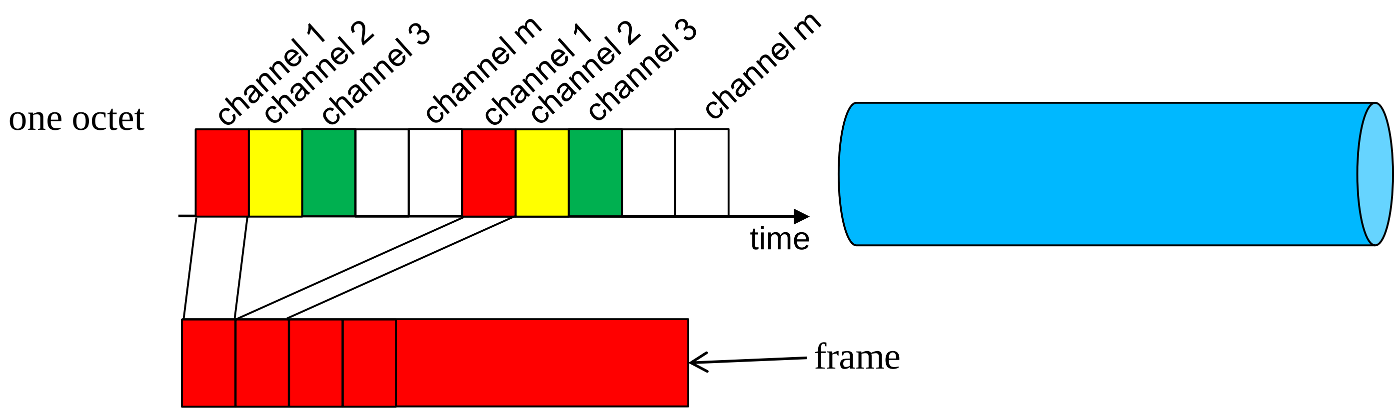
$$\frac{L}{\frac{C}{m}} = \frac{L \times m}{C}$$



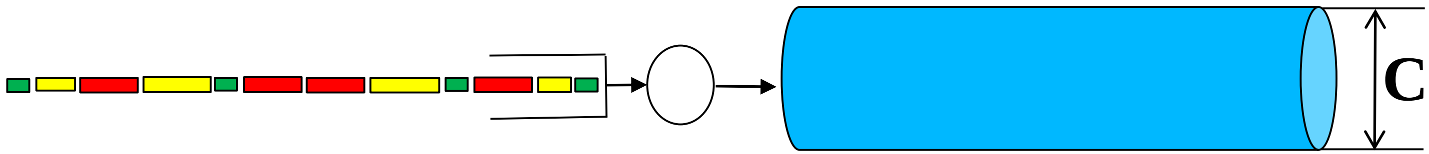
# *TDM – Time Division Multiplexing*

- Time axis divided into ***m*** slots of fixed length
  (usually one octet long)
- Communication ➔ m channels with capacity C/m
- Time required to transmit a packet of length L ➔ $T_{frame}=Lm/C$

Transmitir frames por partes, divididos por canais



# *Delay on Computer Networks*




- Delay
  - » Important performance parameter in computer networks
  - » Characterized using queue models

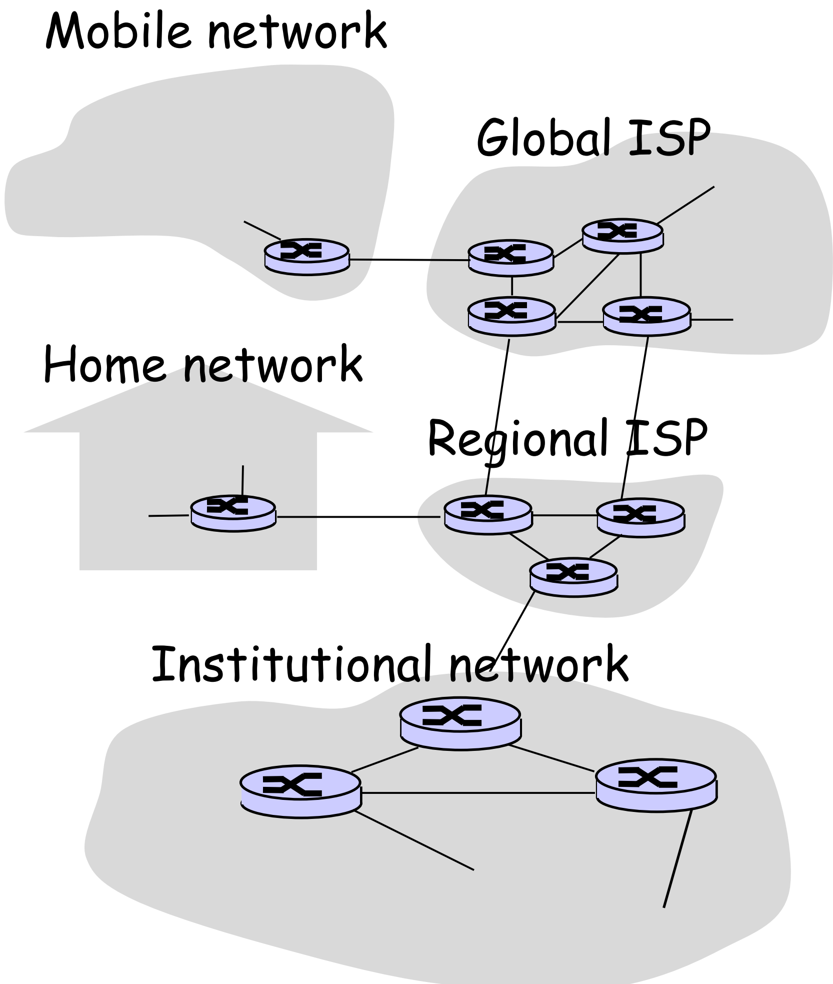
- Queue model
  - » Customers arrive at random times to obtain service
  - » Customer ➔ packet to be transmitted through a link
  - » Serve a packet = transmit a packet
  - » Service time ➔ **packet transmission time =$T_{pac(frame)}$= L/C**

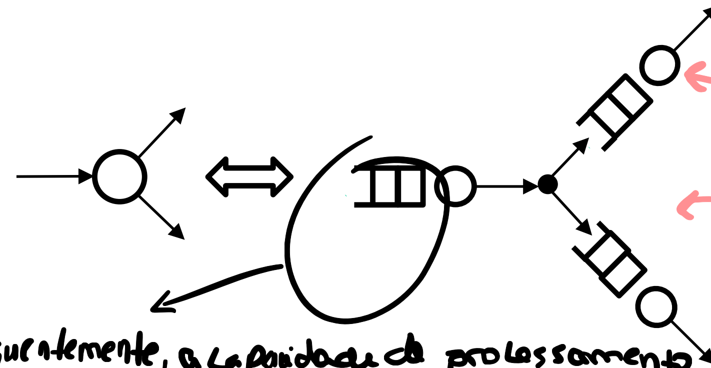
Depende do tamanho do pacote

Depende da capacidade do canal

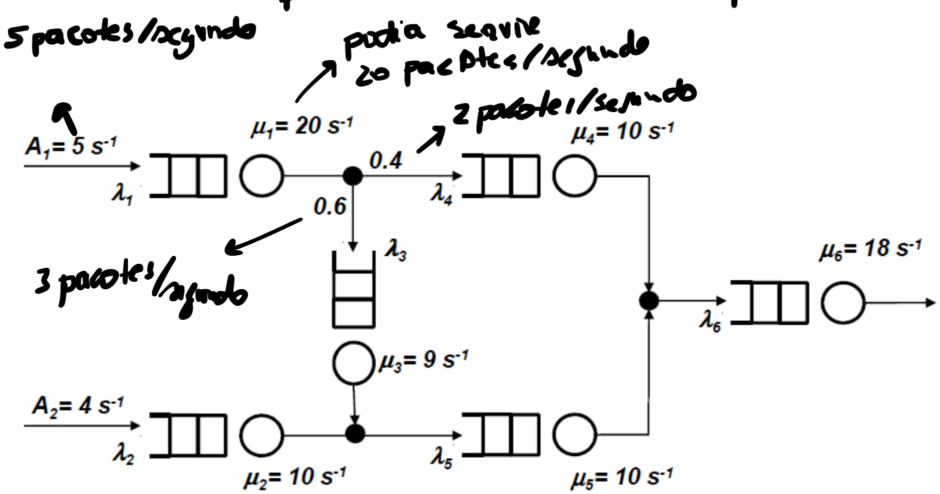
- Queue models enable the quantification of
  - » Average number of customers/packets in the network
  - » Average delay per packet ➔ waiting plus service times

# *Computer Networks Modeled as Queue Networks*



Frequentemente, a capacidade de processamento é tão elevada que só existem filas de espera à saída



# *Poisson Distribution and Poisson Process*

- Poisson distribution with parameter m

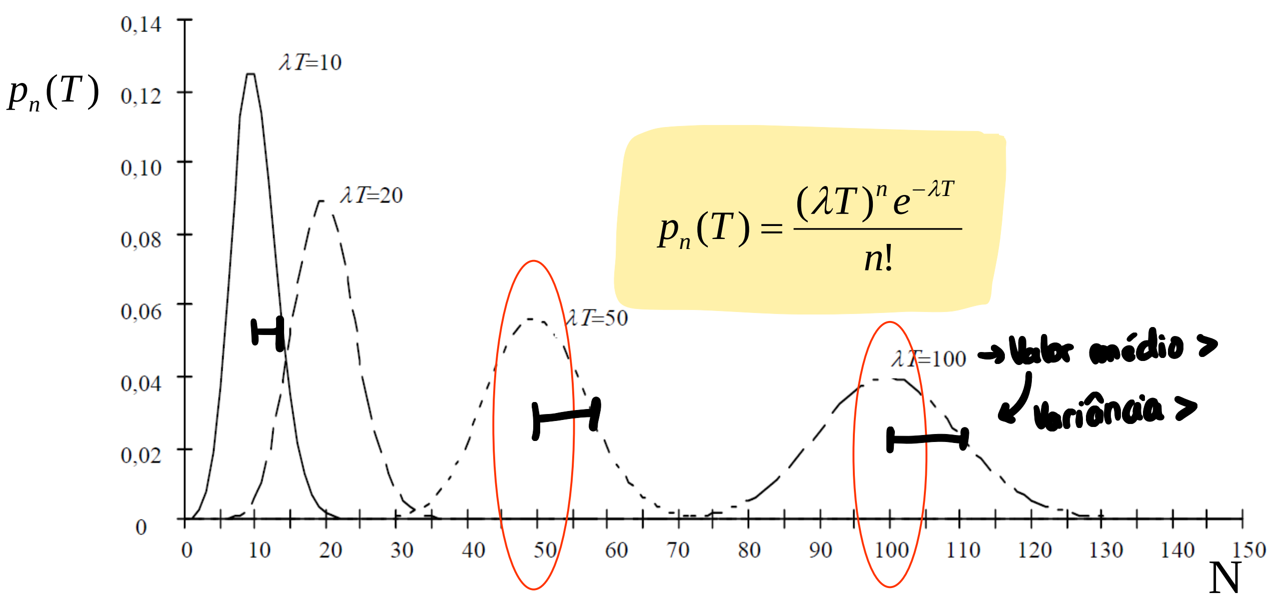
Variância de N

$$P[N=n]=p_n=\frac{m^n e^{-m}}{n!},\quad n=0,1,\ldots \qquad E[N]=Var[N]=m = \lambda T$$

Valor médio de N

↳ Quanto maior o valor médio, maior a variância

- Poisson process
  - » $\lambda T=m$ , (e.g. $\lambda \rightarrow$ arrivals/s )
  - » P[ n arrivals in interval T ] = $p_n(T)=p_n=\dfrac{(\lambda T)^n e^{-\lambda T}}{n!}$ $\qquad E[N]=Var[N]=\lambda T$



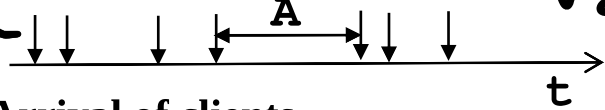

λ → taxa de chegada de pacotes à fila de espera
→ número médio

λT → número de pacotes que chegam num tempo T

# *Inter-Arrival Interval A – Statistical Characterization*

Chegada de um pacote

Intervalo de tempo entre a chegada de clientes consecutivos




**Arrival of clients**

`A – time interval between the arrival of consecutive clients`

Probabilidade de uma variável aleatória = t

Durante t → Ø clientes a chegar

$$p_n(T) = \frac{(\lambda T)^n e^{-\lambda T}}{n!} \rightarrow e^{-\lambda t}$$

n=Ø

$$F_A(t) = P[A \le t] = 1 - P[A > t] = 1 - p_0(t) = 1 - e^{-\lambda t}$$

Derivada

densidade de probabilidade

$$f_A(t) = pdf = \frac{\partial F_A(t)}{\partial t} = \lambda e^{-\lambda t}$$

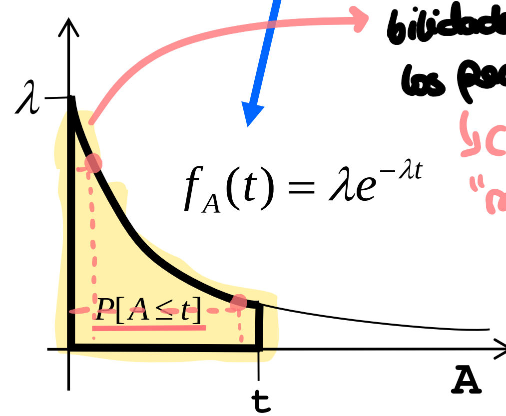
**Exponential distribution**

$$E[A] = 1/\lambda \rightarrow \lambda = 10\,s^{-1} \rightarrow E = \frac{1}{10} = 0{,}1$$

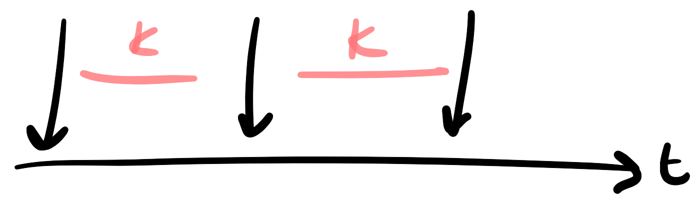
$$Var[A] = 1/\lambda^2 \rightarrow \lambda = 10\,s^{-1} \rightarrow Var = \frac{1}{100} = 0{,}01$$

Muita maior a probabilidade de intervalos pequenos

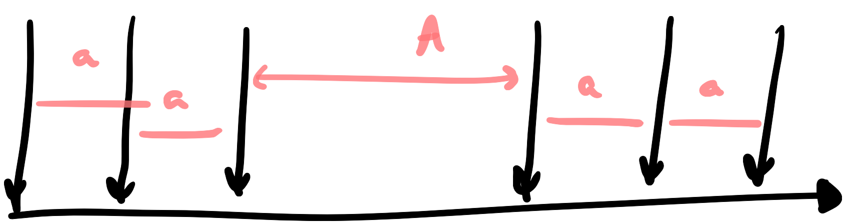
↳ Chegam aos "molhos"



- What is the difference between
  - Deterministic arrivals and
  - Poisson arrivals?




Processo de chegada determinístico
↳ chegada de clientes é constante

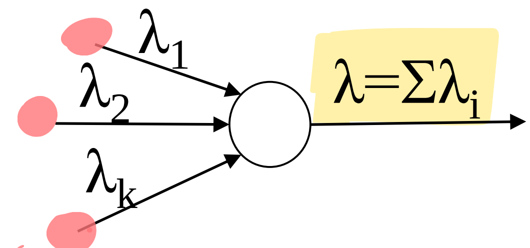



Grande predominância de chegada de pacotes em intervalos de tempo pequenos, alternada com um tempo de espera maior.

De vez em quando, há um tempo de chegada >.

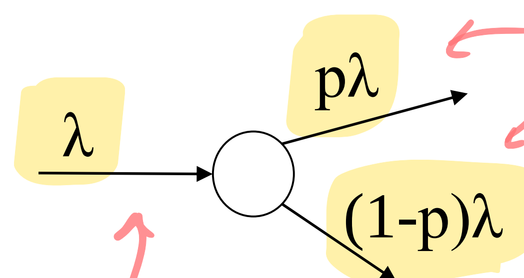
# *Markov Process - Properties*

- Merging Property → Fusão
  - » $A_1, A_2, \dots A_k$ are independent Poisson Processes with rates $\lambda_1, \lambda_2, \dots \lambda_k$
  - » $A = \Sigma A_i$ still is a Poisson process, with rate $\lambda = \Sigma\lambda_i$



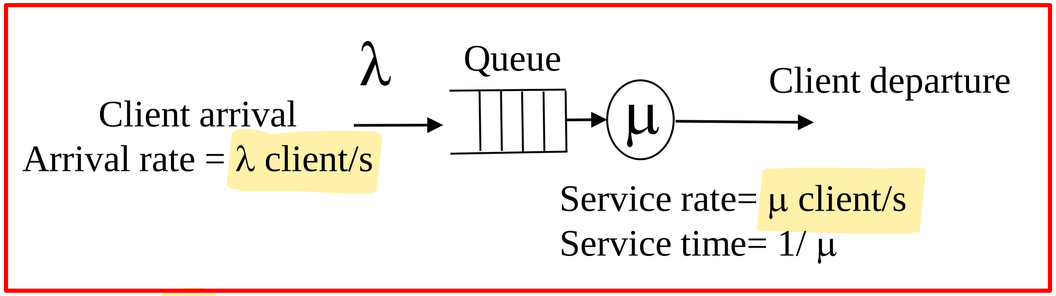

- Splitting property
  - » Packets arrive to a router according to a Poisson Process $(A,\lambda)$
  - » They are routed randomly to two output lines with probabilities **p** and **1-p**
  - » Packets leaving the router still are Poisson Processes, characterized by $(A,p\lambda)$ and $(A,(1-p)\lambda)$

também são




se for Poisson

# *Queue Model* → Representar pacotes que esperam para ser transmitidos




- Queue – model used for
  - » Customers waiting in line
  - » Packets in a network
- Used to determine
  - » Average number of clients in the system → N
  - » Average delay experienced by a client → T
- Queue characterized in terms of
  - » λ - arrival rate of client (average number of clients per time unit)
  - » μ - service rate (average number of clients the server processes per time unit)
  - » ρ=λ/μ – traffic intensity (occupation of the server) → Intensidade de tráfego

$\lambda = 3\,s^{-1}$, $\mu = 10\,s^{-1}$ ; $\rho = \frac{3}{10} = 30\%$ → O servidor está ocupado, em média, 30% do tempo

ρ → Ocupação do servidor

- Kendall notation ➔ **A/S/s/K**
  - » A – arrival statistical process
  - » S – service statistical process
  - » s – number of servers
  - » K – capacity of the system in buffers

# *Little's Theorem* → Teorema de Little → Número de clientes no sistema por segundo

- $N=\lambda T$
  - » N- average number of clients in a system
  - » T – average amount of time a client spends in the system
  - » $\lambda$ – arrival rate of clients to the system

$\lambda = 10 s^{-1}$ → $T = 2s$ → $N = 20$

Tempo que permanece no sistema

- $T=T_w+T_s$
  - » $T_w$ – time a client waits in the queue for being served
  - » $T_s$ – service time
- $N=N_w+N_s$
  - » $N_w$ – number of clients waiting in the queue for being served
  - » $N_s$ – number of clients being served
- $N_w=\lambda T_w$

In Queue

λ → fluxo que atravessa a fila

T = Número de clientes na fila × fluxo que atravessa a fila

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Não depende da taxa de serviço

$$N_w = \lambda T_w \Rightarrow T_w = N_w / \lambda$$

só depende da taxa de chegada de clientes

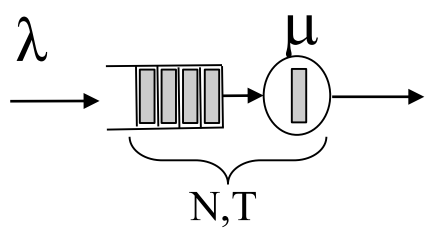
Número de clientes à espera

- The (mean) time a client has to wait before being served ($T_w$) depends on the number of clients waiting ($N_w$) and on the arrival rate of clients ($\lambda$ )

- No dependence on the service rate?! → A taxa de chegada = taxa de saída
- Can you explain it? Não saem + do que aqueles que entram

# *Little's Theorem*

- Can be applied to a single Queue



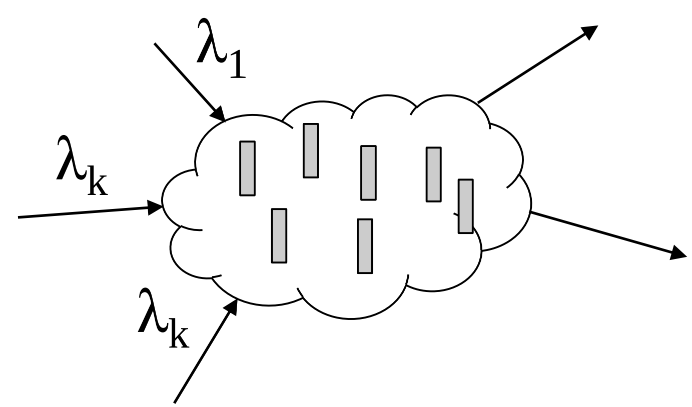

- Can be applied to a complex system

Podemos aplicar little a qualquer sistema

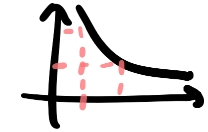
  - » For each stream i → $N_i=\lambda_i T_i$
  - » For the system:

$\lambda=\Sigma\ \lambda_i$ $\quad$ $N=\Sigma\ N_i$

$T=(\Sigma\ N_i)/\ (\Sigma\ \lambda_i)\ \rightarrow\ T=N/\lambda$



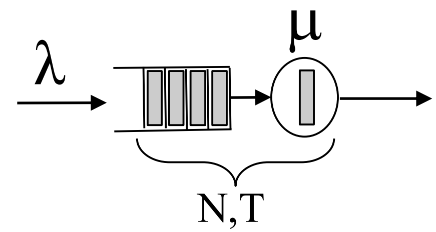
Muitas + ocorrências de tempos pequenos



# *M/M/1 Queue*

Ambos de Poisson

- M/M/1 → Chegada Markov → Serviço Markov ↳ 1 único servidor
  - » Poisson arrival, exponential service time




- Modeled by a Markov Chain
  - » State **k** - k clients in the queue
  - » p(i,j) – probability of transition from state i to state j
  - » When δ → 0

    p(i, i+1)= λδ        p(i, i-1)= μδ

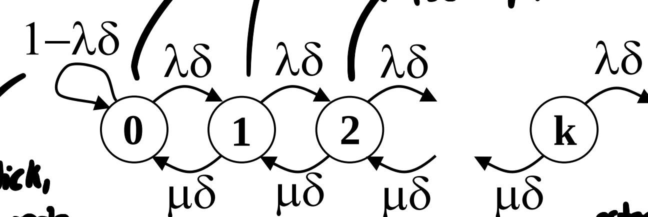
    p(i, i)= 1- λδ–μδ        p(0, 0)= 1- λδ

    p(i, j)=0 for other values i, j

Ø clientes no sistema

1 serviço, Ø em espera

1 serviço, 1 em espera




Sempre que há 1 tick, pode chegar novo cliente, pode sair 1 cliente ou não acontecer nada

→ probabilidade chegar 1

Markov

estado i ↓ estado i+1

$$p(i,i+1) = p_1(\delta) = (\lambda\delta)e^{-\lambda\delta} \approx \lambda\delta$$

$$p(0,0) = p_0(\delta) = e^{-\lambda\delta} \approx 1-\lambda\delta$$

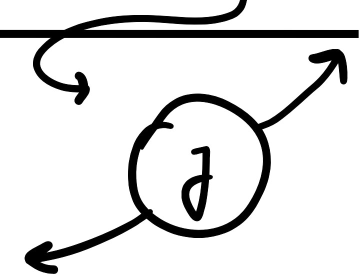
$$p_n(T) = \frac{(\lambda T)^n e^{-\lambda T}}{n!}$$

  - » Birth-death chain
    - – Transitions between adjacent states
    - – λδ and μδ become flow rates between states

→ δ é muito pequeno, é impossível chegarem/saírem 2 clientes num intervalo δ

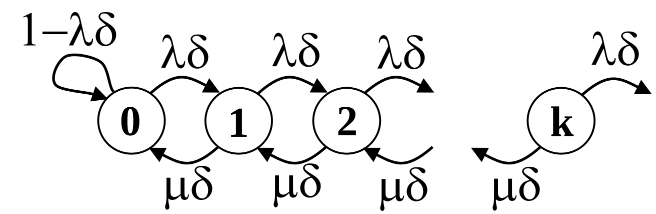
Número de vezes que sai de j
=
Número de vezes que entra em j

# *M/M/1 Queue – Equilibrium Analysis*




- P(j) – probability of the Markov chain be in state j
- Markov Chain - global balance equations

$$P(j)\sum_{\substack{i=0 \\ i\neq j}}^{\infty} p(j,i) = \sum_{\substack{i=0 \\ i\neq j}}^{\infty} P(i)\,p(i,j)$$




- In the case of M/M/1

$$P(0)\lambda\delta = P(1)\mu\delta \quad \Rightarrow P(1) = \rho P(0)$$

$$P(2) = \rho P(1) = \rho^2 P(0)$$

$$P(n) = \rho^n P(0)$$

$$\sum_{i=0}^{\infty} P(i) = 1$$

$$\sum_{i=0}^{\infty} \rho^i P(0) = \frac{P(0)}{1-\rho} = 1$$

$$P(0) = 1-\rho$$

$$\boxed{P(n) = \rho^n (1-\rho)}$$

# *M/M/1 Queue*

- Average Queue size N

Média pesada

$$N = \sum_{n=0}^{\infty} nP(n) = \sum_{n=0}^{\infty} n\rho^n(1-\rho) = \frac{\rho}{1-\rho} \qquad N = \sum_{n=0}^{\infty} nP(n) = \frac{\rho}{1-\rho} = \frac{\lambda/\mu}{1-\lambda/\mu} = \frac{\lambda}{\mu-\lambda}$$

- Average amount of time the client spends in the system, T

  » Little's formula, T=N/λ ➔ $T = \dfrac{1}{\mu-\lambda}$

taxa chegada

taxa serviço

- Average waiting time $T_w$ ➔ $T_w = T - T_s = \dfrac{1}{\mu-\lambda} - \dfrac{1}{\mu} = \dfrac{\rho}{\mu(1-\rho)}$

tempo médio

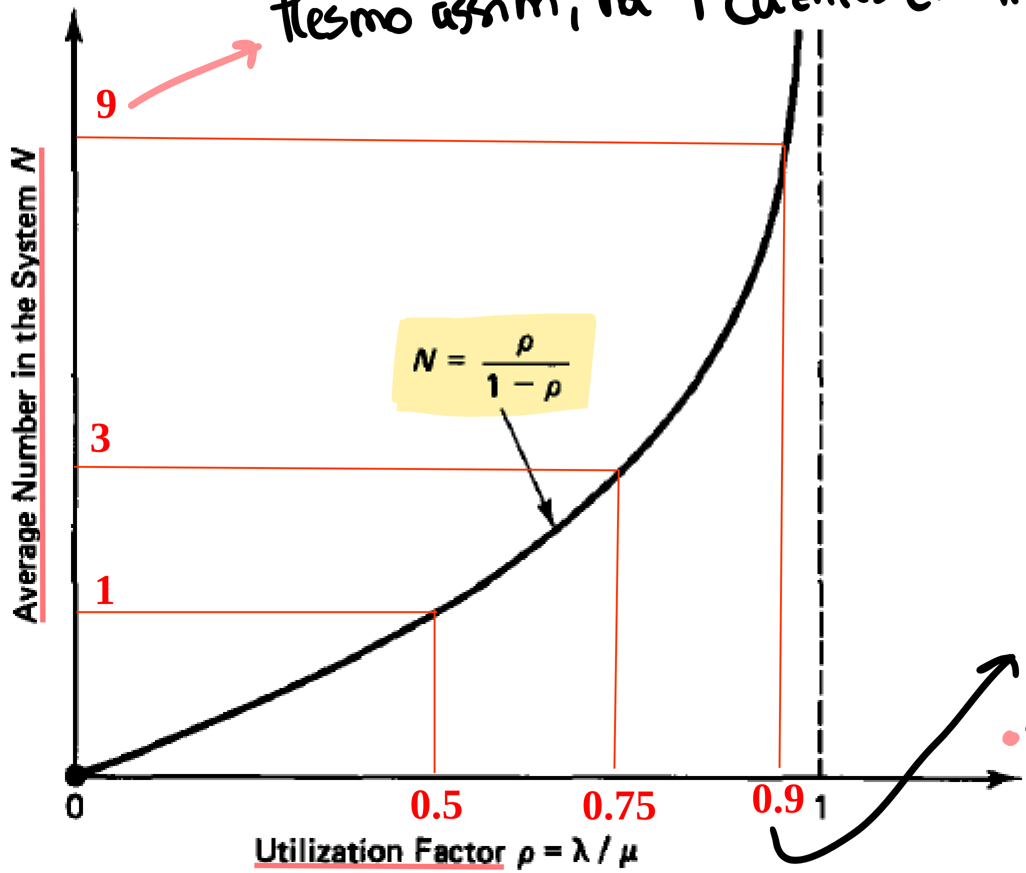
tempo serviço

- Average number of clients waiting in the queue, $N_w$

$$N_w = T_w\lambda = \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu} = N - \rho$$

# *M/M/1 Queue – N=f(ρ)*

Mesmo assim, há 9 clientes em fila



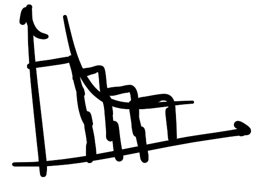

ocupado em 90%.

• 10% do tempo não tem nada para fazer

está livre

**Figure 3.6** The average number in the system versus the utilization factor in the $M/M/1$ system. As $\rho \to 1$, $N \to \infty$.

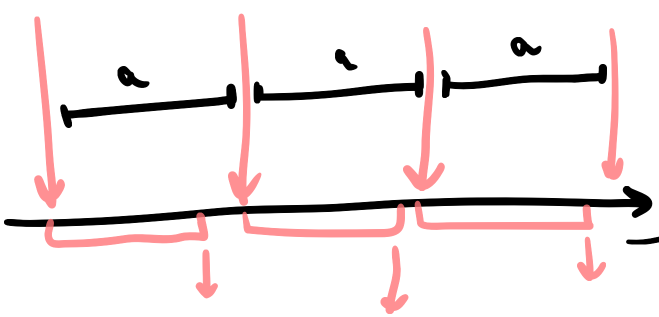
- M/M/1: ρ=0.9 ➔ N=9 → Devido há distribuição da chegada



↳ clientes chegam todos juntos num curto espaço de tempo, logo vai haver fila de espera

- Why have clients to wait if the server is busy only 90% of his time?

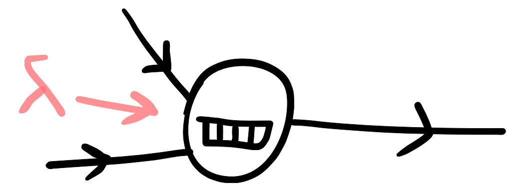
- What would happen for D/D/1, ρ=0.9?




Não ia haver espera, ou esta ia ser = 0.9

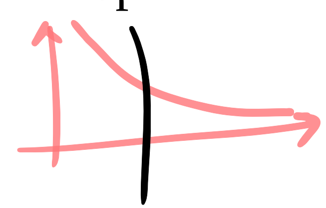
Pois como o tempo é constante, o cliente vai encontrar o servidor vazio e este vai trabalhar ρ e vai estar livre 1-ρ

# *Packet Length, Service Time, Speed*




- » 100 packet/s are required to be transmitted through a link
- » Packets arrive according to a Poisson process
- » Packet lengths are exponentially distributed ➔ $E[L]=10^4$ bit/packet
- » Link has capacity C=10 Mbit/s

Valor médio



- Then
  - » Arrival rate: $\lambda=100$ packet/s
  - » Service rate: $\mu=C/E[L]=10^7/10^4=10^3$ packet/s → taxa serviço: $\frac{\text{capacidade}}{\text{valor médio}}$
  - » $\rho=\lambda/\mu=0.1$, $N=\rho/(1-\rho)=1/9$, $T=N/\lambda=1/900$ s

Número médio de pacotes à espera

- Assume now: $\lambda'=10\lambda$ and $C'=10C$ ➔ $\mu'=10C/E[L]=10\mu$
  - » Then $\rho'=\rho$ and $N'=N$ but $T'=N'/\lambda'=T/10$
    The speed of the system increases! (*)

↳ Como a capacidade aumentou em 10, e o tamanho do pacote manteve, a taxa de envio aumenta em 10

(*) Em termos de permanência de pacotes no sistema, é tudo igual, mas os tempos vão diminuir 10x.

# *M/M/1/B Queue*

- M/M/1 queue has limited capacity (B buffers)
  - » Packets can be lost → se a fila tiver cheia
  - » Probability of packet being lost = P(B) ➔ Queue is full

Probabilidade de ser perdido = Probabilidade de a fila estar cheia

- Analysis similar to M/M/1

Probabilidade da cadeia estar no estado n

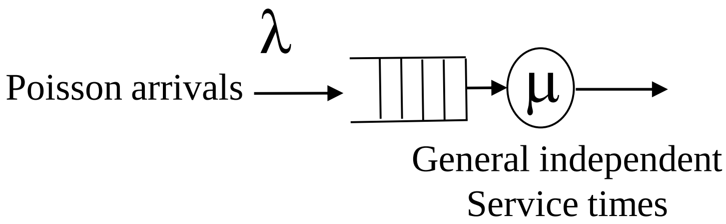
$$\sum_{i=0}^{B} P(i) = 1 \qquad P(n) = \rho^n P(0)$$

$$P(0) = \frac{1-\rho}{1-\rho^{B+1}} \qquad P(B) = \frac{(1-\rho)\rho^B}{1-\rho^{B+1}}$$

- Particular cases

$$\rho = 1, \quad P(B) = \frac{1}{B+1} \qquad \rho >> 1, \quad P(B) \approx \frac{\rho - 1}{\rho} = \frac{\lambda - \mu}{\lambda}$$

# *M/G/1 Queue*




- Poisson arrivals at rate λ

Valor médio do serviço ?

Valor médio do quadrado do serviço ?

- Service time X has arbitrary distribution with given E[X] and E[$X^2$]
  - » Service times Independent and Identically Distributed (IID)
  - » Independent of arrival times
  - » E[service time] =E[X]= 1/μ
  - » Single Server queue

# *M/G/1 Queue – Pollaczek-Khinchin (P-K) Formula*

$$\boxed{T_w = \frac{\lambda E[X^2]}{2(1-\rho)}}$$

- where $\rho = \lambda/\mu = \lambda E[X]$ = line utilization
- From Little's Theorem

\# clientes em espera

» $N_w = \lambda T_w$

tempo de serviço

» $T = T_w + E[X] = T_w + 1/\mu$

tempo de espera

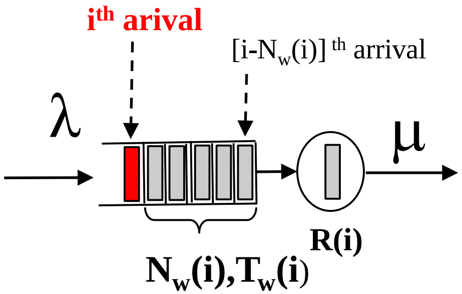
» $\mathbf{N = \lambda T} = \lambda(T_w + 1/\mu) = \mathbf{N_w + \rho}$

\#clientes no serviço

# *M/G/1 Queue – Proof of (P-K) Formula*

$$T_w = \frac{\lambda E[X^2]}{2(1-\rho)}$$

- Let
  - $T_w(i)$ - waiting time in queue of i^th arrival
  - $R(i)$ – residual service time seen by the i^th arrival
  - $N_w(i)$ – number of clients found in queue by the i^th arrival
  - $X(i)$ – service time of the i^th arrival




$$T_w(i) = \sum_{j=i-N_w(i)}^{i-1} X(j) + R(i)$$

$$E[T_w(i)] = T_w = E[N_w(i)] \times E[X(i)] + E[R(i)] = \frac{N_w}{\mu} + E[R(i)]$$

» Using Little's formula

$$T_w = \frac{\lambda T_w}{\mu} + E[R(i)] \qquad T_w = \frac{E[R(i)]}{1-\rho}$$

# M/G/1 Queue – Proof of (P-K) Formula

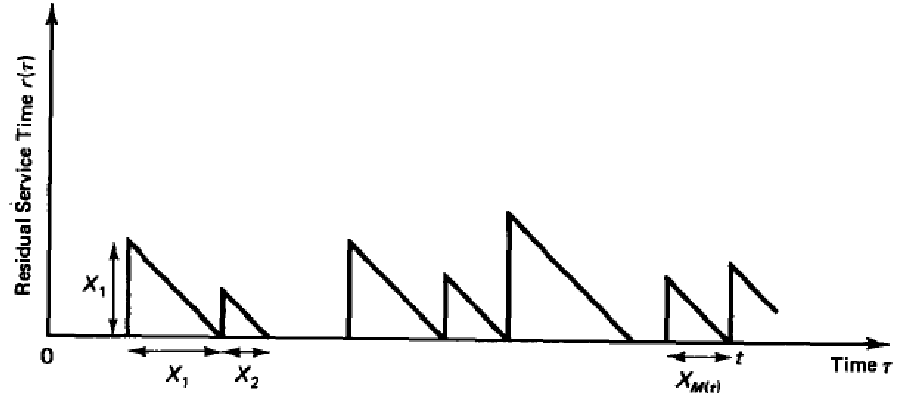



**Figure 3.10** Derivation of the mean residual service time. During period $[0, t]$, the time average of the residual service time $r(\tau)$ is

M(t) – number of clients served by time t

$$E[R(i)] = R_t = \frac{1}{t}\int_0^t r(\tau)\partial\tau = \frac{1}{t}\sum_{i=1}^{M(t)}\frac{X_i^2}{2} = \frac{M(t)}{2t}\sum_{i=1}^{M(t)}\frac{X_i^2}{M(t)}$$

$t \to \infty, \quad \frac{M(t)}{t} = \lambda$ = arrival rate= departure rate

$$E[R(i)] = \frac{\lambda}{2}\sum_{i=1}^{M(t)}\frac{X_i^2}{M(t)} = \frac{\lambda}{2} \times E\left[X^2\right]$$

$$T_w = \frac{E[R(i)]}{1-\rho}$$

$$\boxed{T_w = \frac{\lambda E[X^2]}{2(1-\rho)}}$$

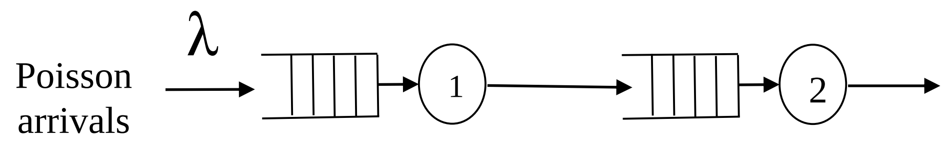
# *M/G/1 Examples*

- Case M/M/1
  - » $E[X] = 1/\mu$ ; $E[X^2] = 2/\mu^2$

$$T_w = \frac{\lambda}{\mu^2(1-\rho)} = \frac{\rho}{\mu(1-\rho)}$$

- Case M/D/1
  - » Deterministic, constant service time $1/\mu$
  - » $E[X] = 1/\mu$ ; $E[X^2] = 1/\mu^2$

$$T_w = \frac{\lambda}{2\mu^2(1-\rho)} = \frac{\rho}{2\mu(1-\rho)}$$

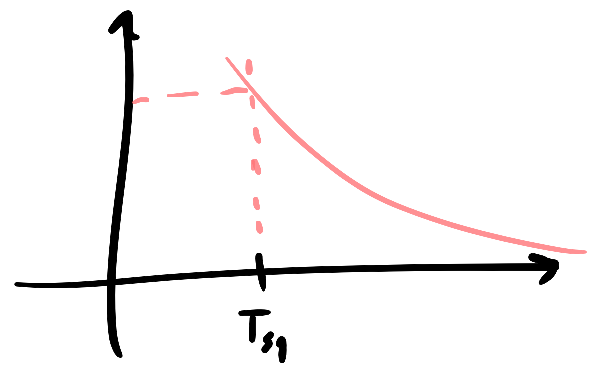



- Assume Queue 1 is M/D/1. → pacotes vão chegando em $\lambda$
  tempo de serviço é determinístico
  ↓
  constante

- Can the arrival of packets to Queue 2 be described as a Poisson process?

Pacote chega → Demora a ser servido

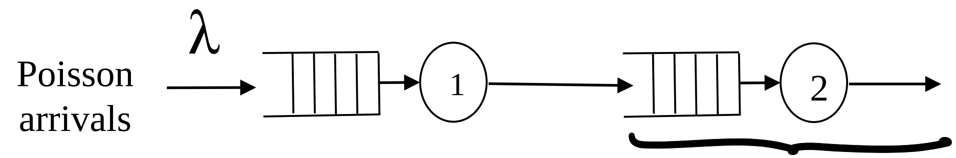

Pacote 3 demora $T_s$ a ser servido
Pacote 2 espera $T_s$ para ser servido
Pacote 1 espera $2 \times T_s$ para ser servido




→ $T_s$ é constante

# *Networks of Transmission Lines - Problems*

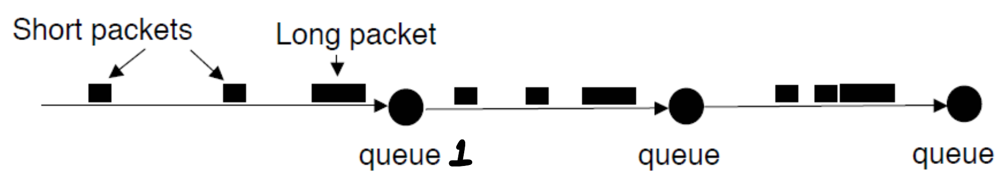
- Case 1
  - Arrival to $Q_1$ → Poisson, $\lambda$
  - Assume contant packet length → $Q_1$= M/D/1 → CONSTANTE
  - Arrival to $Q_2$ is not Poisson; $\lambda_2 < \mu_2$ → $1/\lambda_2 > 1/\mu_2$
  
  → no waiting at $Q_2$




Deixamos de poder aplicar a 2 Markov

- Case 2
  - $Q_1$=M/M/1
  - arrival to $Q_2$ strongly related to packet length
  - long packets require long service at each node
  - shorter packets will catch up long packets → interarrival times change

→ $Q_2$ cannot be modeled as M/M/1




→ Mesmo que queue 1 seja M/M/1, as outras não vão ser → Redes de filas de espera

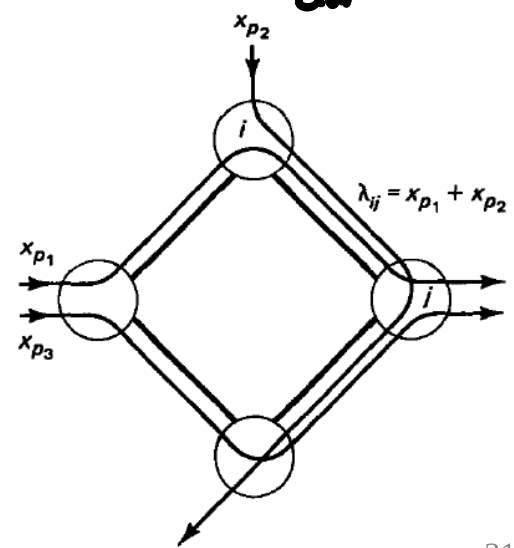
# *Kleinrock Independence Approximation*

- Merging several packet streams on a transmission line restores independence of interarrival times and packet lengths
- M/M/1 can be used to model each communication link
- Approximation good for
  - » systems involving Poisson stream arrivals at the entry points
  - » packet lengths nearly exponentially distributed
  - » densely connected networks
  - » Moderate to heavy traffic loads

Tlis turar um número elevado de fluxo de pacotes na mesma rede – Considerando que todos seguem uma distribuição de Poisson

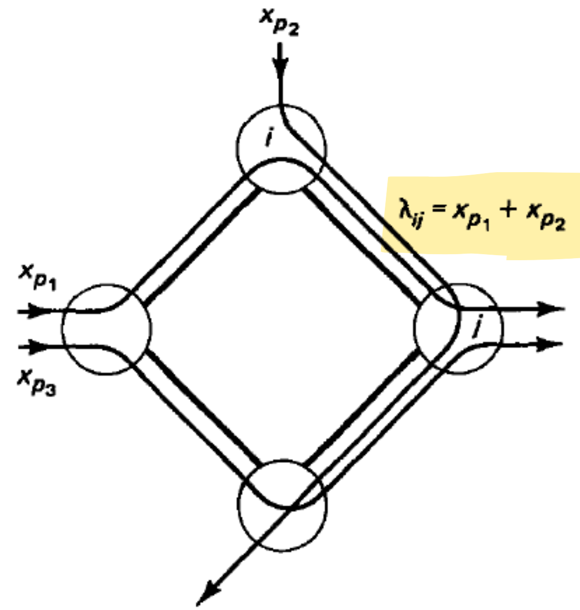
⇓

Podemos assumir que a rede vai seguir uma distribuição exponencial



# *Kleinrock Independence Approximation*

- Let
  taxa de chegada de pacotes em p
  - » $x_p$ = arrival rate of packets along path p
  - » $\lambda_{ij}$ = arrival rate of packets to link (i,j)
  - » $\mu_{ij}$ = service rate on link (i,j)
  ↳ Número de pacotes servidos no link (i,j)




$\lambda_{ij} = x_{p_1} + x_{p_2}$

- Link queues ➔ <u>independent M/M/1 queues</u>

$$\lambda_{ij} = \sum_{\substack{\text{all p traversing} \\ \text{link(ij)}}} x_p \qquad \rho_{ij} = \frac{\lambda_{ij}}{\mu_{ij}} \qquad N_{ij} = \frac{\rho_{ij}}{1-\rho_{ij}}$$

Tudo que atravessa (i,j)

- And
  - » N= Average number of packets in network
  - » T – Average packet delay in network

$$N = \sum_{i,j} N_{ij} \qquad \lambda = \sum_{\text{all paths p}} x_p = \text{total external arrival rate} \qquad T = \frac{N}{\lambda}$$

→ Podemos usar estas informações para calcular o valor médio do desempenho da nossa rede ←

*' → Número de pacotes que vão para fora da rede em análise

Vai mobilizar o nó do grafo:

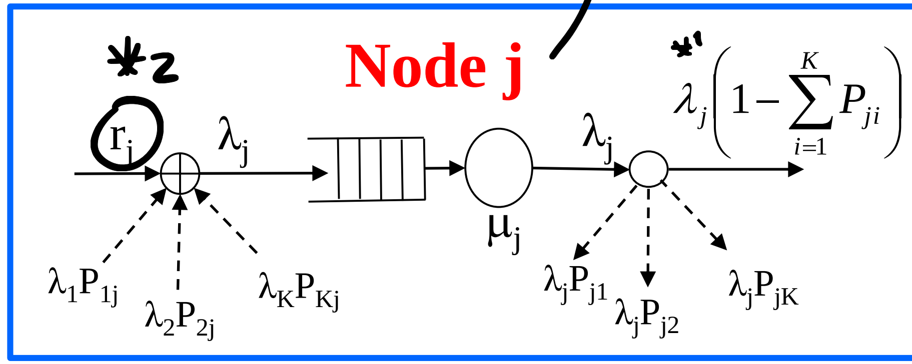
entram e saem os mesmos

# Jackson Networks

- Arrival rate at node j

$$\lambda_j = r_j + \sum_{i=1}^{K} \lambda_i P_{ij} \quad , j = 1,2,\ldots,K$$

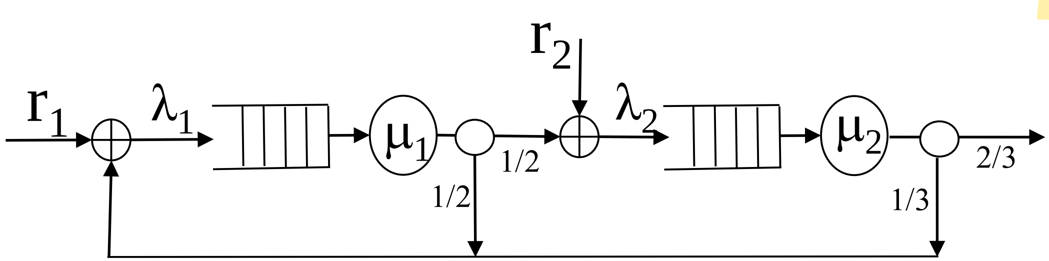
↳ chegada




- Independent routing of packets
  - » When a packet leaves node ***i*** it comes to node ***j*** with probability $\boldsymbol{P_{ij}}$
  - » Packets can loop inside network
  - » Packet leaves the system at node j with probability

*₁

$$P = 1 - \sum_{i=1}^{K} P_{ji}$$




(*₂) Número de pacotes provenientes de outra rede

# *Jackson Networks*

- Let the state of the system be defined by $\vec{n} = (n_1, n_2, \ldots, n_K)$

  $n_j$ – number of clients in $Q_j$

  ↳ número de pacotes que estão na fila k

- Jackson's theorem: $P(\vec{n}) = \prod_{j=1}^{K} P_j(n_j) = \prod_{j=1}^{K} \rho_j^{n_j}(1-\rho_j)$, where $\rho_j = \frac{\lambda_j}{\mu_j}$ (*)

  » State of $Q_j$ ($n_j$) is independent $\left(\prod_{j=1}^{K}\right)$ of state of other queues

  » Similar to independent M/M/1 queues!

  » Similar to Kleinrock's independence

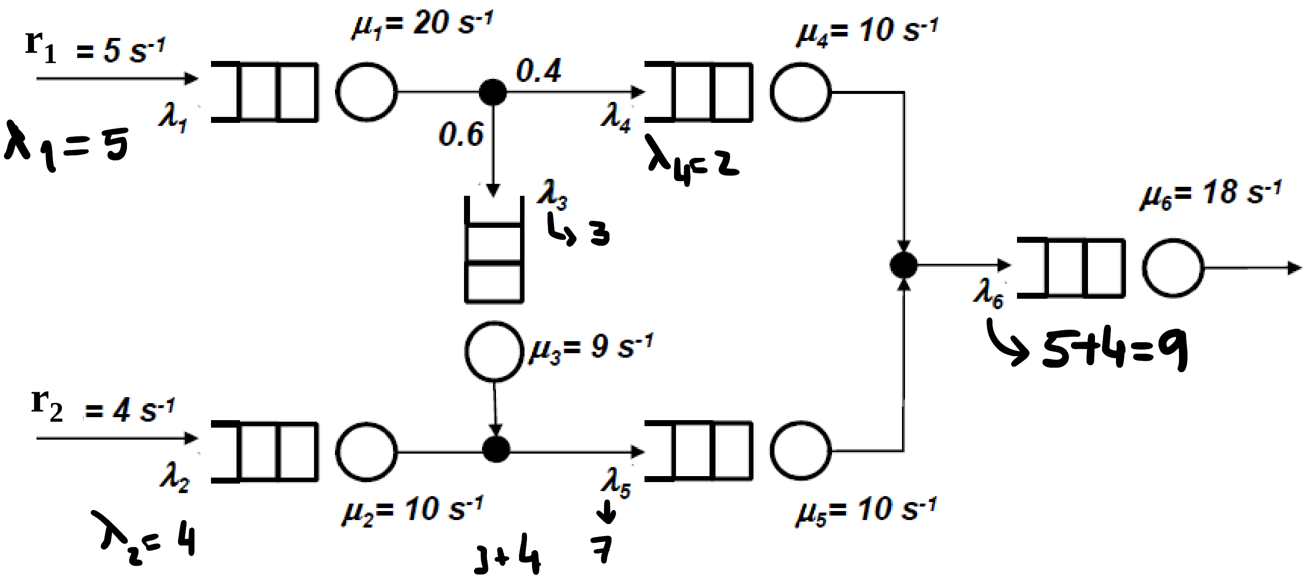
(*) equivalente a termos K filas M/M/1 independentes entre si

- Again, by Little's theorem

$$N_j = \frac{\rho_j}{1-\rho_j} \qquad N = \sum_{j=1}^{K} N_j \qquad \lambda = \sum_{j=1}^{K} r_j \qquad T = \frac{N}{\lambda}$$

# Jackson Network - Example

$\lambda_4 = \lambda_1 \times 0.4 = 2$
$\lambda_3 = \lambda_1 \times 0.6 = \lambda_1 - \lambda_4 = 3$




$$\lambda = \sum_{i=1}^{6} r_i = 9 \text{ s}^{-1}$$

$$N = \sum_{i=1}^{6} N_i = 5.08$$

$$T = \frac{N}{\lambda} = \frac{5.08}{9} = 0.56 \text{ s}$$

| Queue $i$ | $\mathbf{r_i}$ $(s^{-1})$ | $\lambda_i$ $(s^{-1})$ | $\mu_i$ $(s^{-1})$ | $\rho_i = \lambda_i/\mu_i$ | $\mathbf{N_i} = \rho_i/(1-\rho_i)$ |
|---|---|---|---|---|---|
| 1 | 5 | 5 | 20 | 0.25 | 0.33 |
| 2 | 4 | 4 | 10 | 0.40 | 0.67 |
| 3 | - | 3 | 9 | 0.33 | 0.50 |
| 4 | - | 2 | 10 | 0.20 | 0.25 |
| 5 | - | 7 | 10 | 0.70 | 2.33 |
| 6 | - | 9 | 18 | 0.50 | 1 |

soma dos que entram

# *Homework*

1. Review slides

2. Read *Bertsekas&Gallager*
   » Sections 3.1, 3.2, 3.3, 3.5, 3.6, 3.8

3. Answer questions at moodle